A Macro-Finance Model of Mortgage Structure: Financial Stability & Risk Sharing*

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Abstract

Mortgage structure matters not only for monetary policy transmission, but also for financial stability. In an adjustable-rate mortgage (ARM) regime, interest rate rises cause higher default rates due to increases in mortgage payments. In a fixed-rate mortgage (FRM) regime, households are protected, but banks are potentially more exposed to rate rises. To evaluate these competing mechanisms under different mortgage regimes, we build a quantitative model with flexible mortgage contract structures, borrowers, and an intermediary sector. Our approach captures borrowers' endogenous default decisions and intermediaries' equilibrium pricing effects on mortgage rates and risk premia, reflecting the interaction between interest rate and credit risks, and intermediary net worth. We find that financial stability risks are "U-shaped" in mortgage structure: while ARM payments are more sensitive to interest rates, defaults happen in states when intermediary net worth is high, resulting in lower risk premia in constrained states of the world compared to the benchmark FRM economy. As a result, an intermediate mortgage fixation length minimizes the volatility of intermediary net worth and improves the sharing of aggregate risks. Our findings have implications for mortgage design, macroprudential, and monetary policy.

JEL: E52, G21, G28, R31, E44.

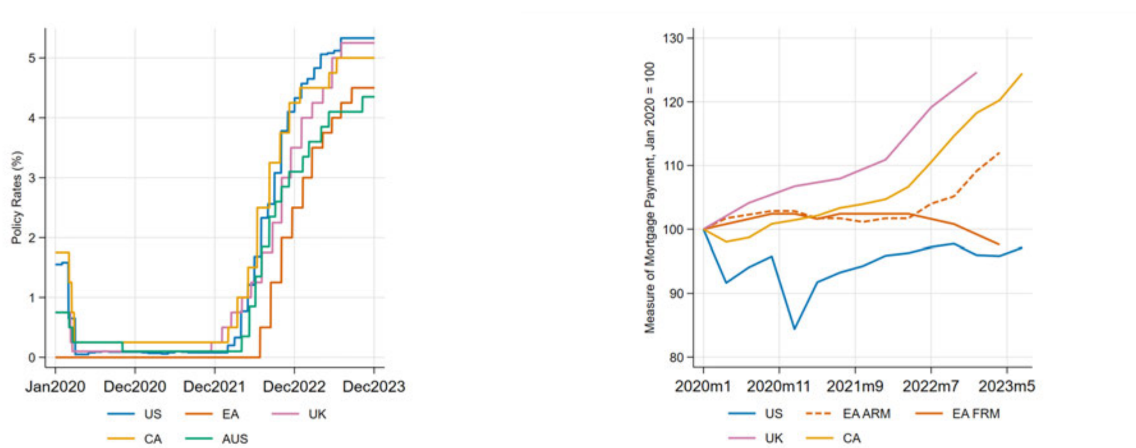
Keywords: mortgages, financial stability, interest rate risk, credit risk, fixed-rate, adjustable-rate mortgages, risk sharing, intermediary asset pricing, household finance

1 Introduction

Mortgage structure matters for macroeconomic outcomes. It directly affects the transmission of monetary policy, since adjustable-rate mortgages (ARMs) reset more immediately compared to fixed-rate mortgages (FRMs) (e.g. Calza et al., 2013; Di Maggio et al., 2017; Fuster and Willen, 2017). In this paper, we show that mortgage structure also matters for financial stability risks. Differences in mortgage structure were brought into sharp relief by the global monetary tightening cycle between 2022 to 2023, which led to a divergence in mortgage payments despite similar policy rate increases across countries, as shown in Figure 1. Panel (a) illustrates that policy rates increased by comparable amounts of 300 to 400 basis points between 2022 to 2023 across countries, while Panel (b) shows that measures of average mortgage payments in FRM economies, including the US and some Euro Area countries, remained stable, but payments in ARM economies such as the UK and Canada saw increases of up to 25% by the beginning of 2023.

Figure 1: Comparison of Policy Rates and Mortgage Payments, 2020–2023

(a) Policy Rates (b) Measure of Mortgage Payments



Notes: Panel (a) shows main monetary policy rates for the US, Euro Area (EA), United Kingdom (UK), Canada (CA), and Australia (AUS). Panel (b) shows measures of average mortgage payments. Euro Area fixed-rate mortgage markets (EA FRM) aggregates France and Germany, adjustable-rate mortgage markets (EA ARM) aggregates Finland, Italy and Portugal. Data sources: US: mortgage debt service ratio (DSR) from FRED; EA: total DSR from BIS; UK: average expected monthly mortgage payment from the FCA; CA: average monthly scheduled outstanding mortgage payments from CMHC.

The differential mortgage payment sensitivity to rate changes highlights differential financial stability risks and risk-sharing properties across mortgage structures. Interest rate rises in an

ARM economy lead to increases in mortgage payments for households, raising defaults and bank credit losses. In contrast, in an FRM regime, households are protected from rate rises, but banks are potentially more exposed to interest rate risk.

How would one evaluate the effect of mortgage structure on financial stability and risk sharing between households and financial intermediaries? A naïve starting point could be to consider a mortgage structure that aims to offset the cash flow sensitivity of bank liabilities, in particular deposits, yielding a "zero duration" financial system where interest rate risks are perfectly hedged. However, this naïve approach ignores several channels that likely arise in equilibrium. First, narrowly focusing on interest rate risk neglects that changes in interest rates also affect credit risk, since households make endogenous decisions to default, and these decisions differ across macroeconomic environments and mortgage structures (Campbell and Cocco, 2015). For instance, rising rates and ARM payments can trigger default for liquidity-constrained households, while this effect is absent for FRMs. Second, financial intermediaries' willingness to hold mortgages and their mortgage pricing, in particular of risk premia, depends on intermediary net worth, such that overall financial stability depends on interest rate and credit risk, and the correlation of these risks with intermediary net worth.

To tackle these conceptual challenges and to evaluate financial stability and risk sharing under different mortgage regimes in equilibrium, we build a quantitative macro-finance model with flexible mortgage contract structures, borrowers, and a financial sector. We then calibrate the model to the US FRM economy as a benchmark, and compare it to counterfactual economies with alternative mortgage structures. In the model, there are two types of households, borrowers who borrow to finance their housing purchases, and savers, who own intermediaries ("banks"). Households face idiosyncratic income shocks. Borrowers and banks trade in two financial markets: deposits and mortgages. We model realistic and flexible mortgage payment structures. In an FRM regime, mortgages have fixed payments. In an ARM regime, mortgages are issued with fixed payments in an initial teaser stage, and subsequently convert to floating payments (a fixed spread over the contemporaneous risk-free rate) with some probability, to reflect varying fixed-rate lengths in typical adjustable-rate mortgages.

Since we are interested in the greater default sensitivity of borrowers to interest rates in the ARM economy, it is important to incorporate a realistic notion of liquidity-driven default

(Gerardi et al., 2018; Ganong and Noel, 2022), where households default because they cannot afford to make increased mortgage payments. To do so, we follow Diamond et al. (2022) and split household decision-making into two stages with a cash-in-advance-type constraint. In the consumption stage, households may use only their liquid assets – income and deposit holdings – to consume and make housing and mortgage payments, with the option to default. Default can provide liquidity, but lowers subsequent wealth. In the ensuing trading stage, households make portfolio decisions to allocate their wealth between deposits, housing, and Lucas trees, and they can adjust their mortgage balance by taking out a new mortgage. Banks lend in the mortgage market subject to a leverage constraint, financing their loan portfolios with savers' equity and deposits, which are risk-free one-period bonds held by households and also elastically demanded by outside investors. The deposit rate does not necessarily move one-for-one with the policy rate, to which ARMs are indexed. Our reduced form model of imperfect pass-through is consistent with banks' market power in deposit markets (Drechsler et al., 2017) and time-varying liquidity premia due to the opportunity cost of holding money (Nagel, 2016; Krishnamurthy and Li, 2022).

To solve the model, we follow Diamond and Landvoigt (2021) and Diamond et al. (2022) to show that, despite idiosyncratic and undiversifiable risks, borrowers make identical choices per unit of wealth. This removes the borrower wealth distribution as an infinite-dimensional state variable, making the model tractable.

We evaluate the US fixed-rate market regime under a counterfactual with fully adjustable-rate mortgages as well as varying intermediate fixation lengths. We also consider how our results change with greater pass-through of policy rates to the cost of bank funding, and when we vary intermediaries' effective risk aversion. Our benchmark FRM economy and ARM counterfactual produce empirically valid and differential responses to a rise in rates: In the FRM economy, mortgage payments remain stable, with a small reduction in defaults since holding on to the current mortgage becomes more valuable. In contrast, in the ARM economy, mortgage payments rise, causing a spike in default rates and reduction in house values.

In the FRM economy, when the policy rate goes up, banks' interest income remains unchanged. The cost of deposit funding – the banks' interest expenses – increases, albeit less than the policy rate. Taken together, this leads to a drop in banks' net interest margin. Banking becomes less profitable, despite slight offsetting decreases in credit losses. However, fixed-rate mortgages have a long duration. In response to higher rates, the market value of bank assets falls (Jiang et al., 2024). With both lower cash flows and lower asset values, the net worth of the banking sector declines. More constrained banks demand higher compensation to take on mortgage risk, a key implication of intermediary-based asset pricing models (Elenev et al., 2016; Diamond and Landvoigt, 2021).

In contrast, in the ARM economy, the net interest margin of banks increases as mortgages are indexed to the policy rate rather than the deposit rate. Therefore, banks become more profitable even as credit losses rise. Moreover, because the increase in cash flows outpaces the increase in the relevant discount rate (the deposit rate), adjustable-rate mortgages effectively have negative duration: their value increases with higher rates. With higher cash flows and higher asset values, the net worth of the banking sector increases.

Next, we turn to evaluating financial stability outcomes by solving counterfactual economies ranging from a pure ARM counterfactual with annual rate resets to economies with increasingly long fixation lengths to the benchmark fully fixed-rate economy. We find that financial stability risks, measured using a range of metrics including our preferred measure – volatility of intermediaries' return on equity (ROE) – are "U-shaped" in mortgage structure: ROE volatility is highest in a pure ARM economy where intermediary net worth is very sensitive to interest changes, leading to large negative duration. Volatility is less high in an FRM economy as sticky deposits provide a hedge to the large positive duration of fully fixed-rate mortgages.

Because mortgage are risky, the sensitivity of bank assets to interest rates depends not just on the policy rate but also on expected losses and risk premia, which vary over time with intermediary net worth. In the FRM economy, risk premia are higher than in the ARM economy when intermediaries are constrained, which typically occurs in a high-rate environment. In contrast, in the ARM economy intermediaries are constrained when both interest rates and default rates are low. As interest rates rise, defaults in the ARM economy typically rise as well, coinciding with states when intermediary net worth is high, which provides a "hedge" from a net-worth perspective. As a result, intermediary ROE volatility is minimized by balancing both forces in ARM and FRM economies, around an intermediate fixation length of 3 years.

Intuitively, an intermediate fixation length reduces the cyclicality of default and net worth with respect to interest rates. While ARM economies exhibit strong pro-cyclical default and FRM economies exhibit somewhat counter-cyclical default, intermediate fixation lengths result in lower cyclicality overall, reducing volatility.

We evaluate other factors that drive the relationship between mortgage structure and financial stability in counterfactual economies. Removing the role of risk premia in a counterfactual with risk-neutral intermediaries indeed improves financial stability of the FRM economy relative to the ARM one. On the other hand, increasing deposit sensitivity exacerbates financial stability risks as it increases duration mismatch on the liability side with fixed-rate mortgages on the asset side, consistent with (Drechsler et al., 2017). These forces affect which contract structure minimizes intermediary ROE volatility, but do not overturn the "U-shape" result.

Lastly, we assess the way mortgage structure determines how risks are shared between households. To quantify the degree of risk sharing, we compare the variance of individual consumption growth relative to aggregate borrower consumption growth as a measure of intra-borrower risk-sharing of idiosyncratic risks, and the variance of aggregate consumption growth of borrowers relative to savers as a measure of risk-sharing of aggregate risks between borrowers and savers. Mortgage structure most strongly affects the sharing of interest rate risk between borrowers and savers with aggregate risk-sharing optimized at a fixation length of 3 years, which also minimizes the volatility of intermediary ROE. In this economy with low effective mortgage duration and default rates that respond little to interest rates, rate shocks have the weakest redistributive effect.[1] However, low exposure to aggregate risk leads borrowers to endogenously choose higher exposure to idiosyncratic risk, highlighting a somewhat subtle downside in equilibrium.

Our work has implications for monetary policy and macroprudential regulation of financial stability risks. The paper provides a framework for how changes in interest rates differentially affect financial stability depending on mortgage structure. It thus helps formalize mechanisms that affect linkages between monetary policy and financial stability. We propose a flexible modeling framework to study the effect of mortgage structure on financial stability, which takes into account endogenous household default decisions, interaction effects between interest rate and credit risk, and the capitalization of the banking system. Our findings highlight how intermediate fixation lengths, common in many countries, can balance sources of volatility in

[1] See e.g. Auclert (2019).

both pure ARM and FRM structures.

Related Literature Our paper makes several contributions to the existing literature. First, we assess macroeconomic implications of different mortgage contract designs, similar to Garriga et al. (2017); Greenwald et al. (2019); Campbell et al. (2021); Guren et al. (2021), but focusing on the novel channel of interest rate and credit risk sharing between households and banks. Conceptually, we thus integrate features of existing quantitative macro-models with financial intermediaries such as Elenev et al. (2016); Diamond et al. (2022), as well as recent work by Sanchez Sanchez (2023), who studies mortgage choice in a counterfacual economy without government guarantees, into a framework with flexible mortgage structures and liquidity-driven default, matching empirical evidence (Gerardi et al., 2018; Ganong and Noel, 2022). Our mechanism is closely related to Campbell and Cocco (2015) who show that fixed- and adjustable-rate mortgages default in different macroeconomic states of the world, and we integrate this intuition into a macroeconomic framework with a banking sector.

We contribute to existing work on mortgage choice (Campbell and Cocco, 2003; Koijen et al., 2009; Badarinza et al., 2018; Liu, 2022) as well as optimal mortgage contract design (Piskorski and Tchistyi, 2010; Campbell, 2012; Eberly and Krishnamurthy, 2014; Mian and Sufi, 2015; Piskorski and Seru, 2018). Our work is further related to papers that emphasize the role of the mortgage market (Scharfstein and Sunderam, 2016; Di Maggio et al., 2017; Fuster and Willen, 2017; Greenwald, 2018; Chen et al., 2020; Di Maggio et al., 2020; Berger et al., 2021; Eichenbaum et al., 2022) and financial intermediaries (Wang, 2018; Di Tella and Kurlat, 2021; Wang et al., 2022; Diamond et al., 2024) on monetary policy transmission.

The paper further offers a novel lens to interpret linkages between monetary policy and financial stability (Adrian and Shin, 2008; Hanson et al., 2011; Stein, 2012; Borio, 2014; Jiménez et al., 2014; Garriga and Hedlund, 2018; Smets, 2018; Caballero and Simsek, 2019; Martinez-Miera and Repullo, 2019; Ajello et al., 2022; Boyarchenko et al., 2022; Gomes and Sarkisyan, 2023), highlighting that mortgage structure can mediate how changes in interest rates affect financial stability.

Lastly, we contribute to a growing body of work on the financial stability implications (Jiang et al., 2024; Drechsler et al., 2023; Haas, 2023; Varraso, 2023; Begenau et al., 2024; DeMarzo et al., 2024) and transmission mechanism (Fonseca and Liu, 2023; Greenwald et al., 2023) of recent rate rises.

2 Motivating Facts on Mortgage Structure

This section illustrates variation in mortgage structure across a range of different countries which motivates the counterfactual mortgage structures that we study using our model.

2.1 Mortgage Structure Across Countries

There is substantial variation in mortgage market systems and contract structures across countries (Campbell, 2012; Badarinza et al., 2016).[2] Figure 2 shows the average fixed-rate length across countries from different data sources.

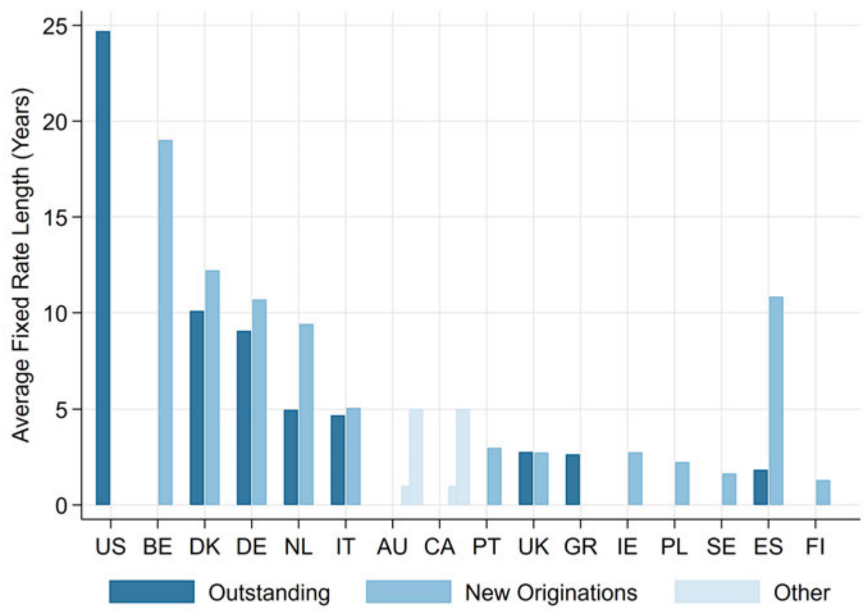
A striking fact noted by Campbell (2012) is that the US appears as an outlier in international comparison, with an average fixed-rate length of almost 25 years, driven by the reliance on 30-year FRMs and 15-year FRMs.[3] The US is followed by a group of countries including Denmark, Germany, Belgium, and the Netherlands, which offer mortgages with fixation lengths of up to 30 years, but the average mortgage outstanding has a length typically closer to 10 years. For Belgium, data is available only for new mortgage originations, which have been close to 20 years. The vast majority of all remaining mortgage markets have fixed-rate lengths between 2 to 5 years, including countries such as Australia, Canada, the UK, Ireland, Portugal, Greece, and Spain. Other Scandinavian countries such as Finland, Sweden, and Norway (the latter with no data on average fixed-rate lengths) are typically thought of as originating many pure adjustable-rate mortgages, with rates resetting at least every year.

Even within the common currency Euro Area, countries vary from longer-term fixed-rate

[2] In this paper, we will not take a stance on the drivers of the underlying structure and take prevalent contract structures as given. Reasons that have been put forward to explain cross-country heterogeneity in mortgage structure include historical path dependence, the availability of long-term mortgage funding, historical inflation experiences (Badarinza et al., 2018), as well as variation in underwriting standards and the role of credit risk (Liu, 2022).

[3] The only country with a comparable average fixed-rate length is typically thought of as France. While data for average fixation lengths is not available for France, the typical mortgage is a 30-year fixed-rate mortgage according to the European Mortgage Federation.

Figure 2: Average Mortgage Fixed-Rate Lengths Across Countries



Notes: "Outstanding" reflect data from Badarinza et al. (2016) ("BCR") as of 2013, while "New Originations" reflect data from the European Mortgage Federation for new mortgage originations, as of 2023Q1, from the EMF Quarterly Review of European Mortgage Markets 2023 Q2. Figure adapted from Liu (2022).

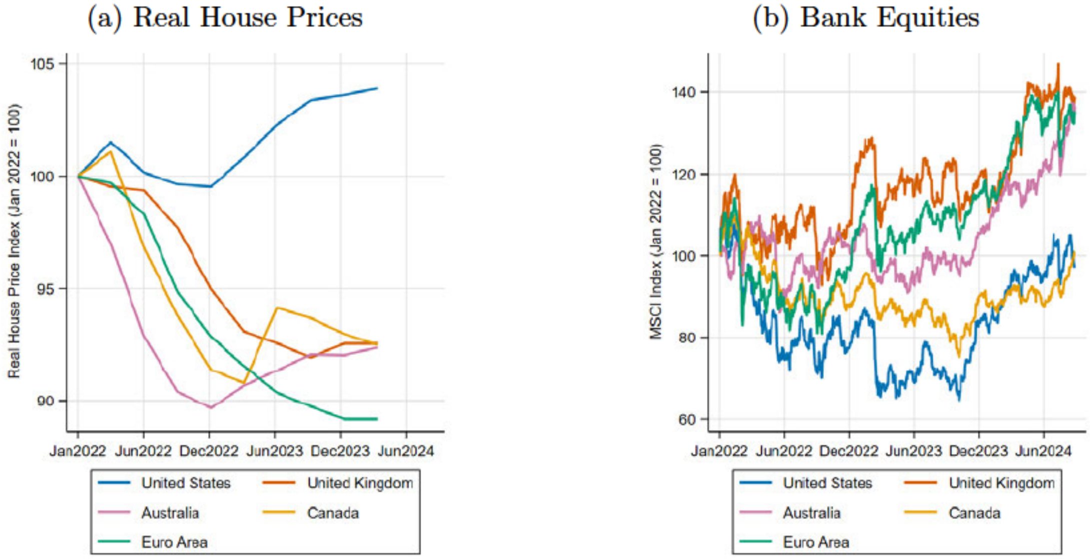
mortgage systems (such as Germany and France) to largely variable-rate mortgage systems such as Finland, Greece, Ireland and Portugal, which is reflected in the divergence in mortgage payments in 2022 in Figure 1.[4]

As a result, mortgages typically exist on a spectrum from fully adjustable-rate mortgages common in countries such as Finland, Sweden and Norway which reset every year (or depending on contract terms, even semi-annually), to short fixation periods of two to five years common in many countries including the UK, Canada, Australia and most Eurozone countries, to the 30-year fixed-rate mortgage common in the US. We think of mortgages with short fixation periods as sitting between pure ARM and FRM structures from an interest rate risk perspective, as these will allow households to fix their mortgage rate for some, but typically not all, of the term over which the mortgage is repaid.[5]

[4] Spain has seen much longer fixation lengths in newly originated mortgages compared to past mortgage originations, likely a result of government interventions in 2022 that allow conversion of adjustable to fixed-rate mortgages, aimed at protecting vulnerable borrowers from interest rate rises, see e.g. Financial Times, November 2022.

[5] Thus fixation length is a distinct feature and different from the choice of the loan repayment window, which

Figure 3: House Prices and Bank Equities



Notes: This figure shows the real house price (Panel (a)) and bank equity indices (Panel (b)) across countries indexed to January 2022. Data: OECD, MSCI bank indices from Bloomberg.

2.1.1 Cross-Country Variation in Other Economic Outcomes, 2022–2023

Figure 1 Panel (b) highlights how mortgage mortgage payments diverged between 2022 and 2023, depending on the underlying mortgage structure. Figure 3 shows other economic outcomes, real house prices (Panel (a)) and bank equity prices (Panel (b)) that also suggest some divergence. The US experienced much stronger house price growth than the UK, Australia, Canada, and Euro Area, whose real house price indices declined by 7 to 11 per cent between 2022 and 2023, while the US index rose by about 4 per cent. Somewhat similarly, UK, Australia, and Euro Area bank equities outperformed bank equity indices in the US (and also Canada) by almost 40 per cent, despite a strong US equity market performance in non-bank equities over that same time period. While these cross-country facts are merely suggestive, we can later compare these to predicted differences in economic outcomes when we vary mortgage structure in the model.

However, mortgage structure is certainly not the only economic fundamental that differs across these countries. To assess how differential mortgage structures lead to differences in economic outcomes, financial stability, and risk-sharing properties more formally, in the next section we develop and calibrate a quantitative model of an FRM economy, and evaluate coun-

is typically 30 years on average for most countries (see Liu (2022) for a more detailed discussion).

terfactual economies with a pure ARM structure as well as intermediate fixation lengths.

3 Model

In this section, we develop a rich quantitative dynamic model of the mortgage market. The key qualitative insights also emerge from a stylized two-period model, which we relegate to Appendix C.

Time is infinite and discrete $t = 0, 1, \ldots$. The economy is populated by continuums of two types of households with preferences over housing and non-durables – borrowers labeled B indexed by $i \in [0, \ell]$ and savers labeled S indexed by $i \in (\ell, 1]$.

Households' utility function is given by

$$\sum_{t=0}^{\infty} \beta^t u^B(c_t^i, h_{t-1}^i)$$
$$u^B(c_t^i, h_{t-1}^i) = \frac{\left((c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta}\right)^{1-\gamma} - 1}{1-\gamma}$$

where β is the discount factor, θ governs the share of housing in the utility function, and γ is the coefficient of relative risk aversion.

The aggregate supply of houses is exogenous and fixed at $\bar{H}$, with a fraction α_H owned by borrowers while the remaining fraction $1 - \alpha_H$ belongs to savers. Only borrowers trade houses. Each unit of housing requires a maintenance payment of δ_h every period to prevent its full depreciation.

Non-durable goods are produced by a continuum $k \in [0, 1]$ of Lucas trees, whose aggregate yield each period is given by Y_t. Borrowers own a total of α trees, while savers owns the remaining $1 - \alpha$. Each type of agent can trade trees within their type, but not across types. The yield of borrower-owned trees is subject to an idiosyncratic shock ε_t^i, which is i.i.d. across borrowers and time. Saver-owned trees are not subject to idiosyncratic shocks. Therefore, each household's income is given by

$$y_t^i = s_{t-1}^i (Y_t + \varepsilon_t^i) \qquad \forall i \in [0, \ell]$$
$$y_t^i = s_{t-1}^i Y_t \qquad \forall i \in (\ell, 1]$$

where s_{t-1}^i is the share of trees owned by each agent type at the start of period t, so that $\int_0^\ell s_{t-1}^i di = \alpha$ and $\int_\ell^1 s_{t-1}^i di = 1 - \alpha$.

In addition to trading houses, borrowers trade in two financial markets – deposits and mortgages. Deposits are one-period risk-free bonds, while mortgages are long-term, defaultable, and may have fixed or adjustable payments.

Their counterparties in these markets are banks labeled I (short for "intermediaries"). Banks are firms who issue equity to saver households.

3.1 Borrowers

Following Diamond et al. (2022), we split each period into two subperiods – *consumption* and *trading*. In the consumption subperiod, shocks are realized, and borrowers make mortgage payments or default. In the trading subperiod, all households make portfolio choices.

Borrowers enter the period with a house h_{t-1}^i, a mortgage with outstanding balance m_{t-1}^i, and deposits d_{t-1}^i. They receive income y_t^i after the realization of aggregate and idiosyncratic income shocks.

Mortgage Regimes We consider two mortgage regimes. In the fixed-rate mortgage regime (FRM), the outstanding balance of the mortgage implies a fixed mortgage payment $x_t^i = \iota_f + \delta \bar{q}^m$ per unit of mortgage m_{t-1}^i, where ι_f denotes the interest component and the principal component is normalized to a fraction δ of the steady-state mortgage price $\bar{q}^m$. In the adjustable-rate mortgage regime (ARM), the mortgage payment is determined by whether or not the adjustable rate mortgage is in its *teaser* stage τ.

In the teaser stage, ARM payments are fixed at $(\iota_\tau + \delta \bar{q}^m) m_{t-1}^i$ with ι_τ the initial fixed "teaser" rate of the mortgage. After the teaser stage, the mortgage payment is determined by the policy (risk-free) rate r_t^f plus the spread ι_a on adjustable-rate mortgages.

$$x_t^i = \begin{cases} \iota_\tau + \delta \bar{q}^m, & \mathbb{1}_\tau = 1 \\ r_t^f + \iota_a + \delta \bar{q}^m & \mathbb{1}_\tau = 0 \end{cases}$$

An adjustable-rate mortgage is always issued in the teaser stage and it becomes a regular ARM with probability π_τ at the end of the second (trading) subperiod. Therefore, the expected duration of the teaser stage, or "fixation period," is $\frac{1}{\pi_\tau}$.

After payments are made, the mortgage balance decreases by δ, such that the remaining balance is $(1-\delta)m_{t-1}^i$.

Consumption Stage In the consumption stage, households use income y_t^i and their deposits holdings d_{t-1}^i to make mortgage payments $x_t^i m_{t-1}^i$ and housing maintenance payments $\delta_H h_{t-1}^i$.

Households can choose to default. If they default by failing to make the mortgage payment, they lose their house and their mortgage balance is written off. They also lose a fraction λ of their endowment of Lucas trees and face a continuous idiosyncratic shock to their post-default value function. In other words, default carries both a pecuniary and a non-pecuniary cost.

A household that repays the mortgage faces a consumption-stage budget constraint given by:

$$c_t^{i,nd} + x_t^i m_{t-1}^i + \delta_H h_{t-1}^i + a_t^i = y_t^i + d_{t-1}^i$$

where $a_t^i \geq 0$ is the household's holdings of intra-period deposits that a household can bring into the trading stage in lieu of consuming. It enters the trading stage with wealth:

$$w_t^{i,nd} = a_t^i - (1-\delta)m_{t-1} q_t^m + p_t^h h_{t-1}^i + p_t^s s_{t-1}^i$$

where q_t^m is the price of the mortgage, p_t^h is the price of housing, and p_t^s is the price of the Lucas trees. The nonnegativity constraint $a_t^i \geq 0$ operates similarly to cash-in-advance or working capital constraints, requiring borrowers to have enough liquidity to finance their consumption before being able to rebalance their portfolios by selling assets or borrowing.

A household that defaults faces a budget constraint given by:

$$c_t^{i,d} = y_t^i + d_{t-1}^i$$

Having expunged their mortgage, lost their house, and given up a fraction of future income, it enters the trading stage with wealth:

$$w_t^{i,d} = (1-\lambda) p_t^s s_{t-1}^i$$

The default decision depends on the utility of consumption plus the continuation value as represented by the trading stage value function $V_t^i(w_t^i, \mathcal{Z}_t)$, where $\mathcal{Z}_t$ denotes state variables exogenous to an individual borrower.

Denote the value of default by $V^{i,d}$ and the value of repayment by $V^{i,nd}$. The value of making the mortgage payment is given by:

$$V_t^{i,nd}(d_{t-1}^i, m_{t-1}^i, \mathbb{1}_\tau, h_{t-1}^i, \epsilon_t^i, \mathcal{Z}_t) = \max_{a_t^i \geq 0} u^B(c_t^{i,nd}, h_{t-1}^i) + V(w_t^{i,nd}, \mathcal{Z}_t)$$

while the value of default is given by:

$$V_t^{i,d}(d_{t-1}^i, m_{t-1}^i, \mathbb{1}_\tau, h_{t-1}^i, \epsilon_t^i, \mathcal{Z}_t) = u^B(c_t^{i,d}, h_{t-1}^i) + V(w_t^{i,d}, \mathcal{Z}_t)$$

subject to the budget constraints and wealth evolution equations above. Households default iff:

$$\eta_t^i V_t^d(\cdot) > V_t^{nd}(\cdot)$$

where η_t^i is the household's idiosyncratic default shock.

Trading Stage In the trading stage households make portfolio decisions. They allocate their wealth w_t^i between deposits d_t^i, housing h_t^i, and Lucas trees s_t^i. They can also revise their mortgage balance from $(1-\delta)m_{t-1}^i$ to m_t^i at current price q_t^m.

Borrowers are subject to a cost of deviating from a target loan-to-value ratio, given by $\Phi\left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - L\bar{T}V\right)$. This cost, rebated $\mathcal{R}_t^i$ to the household in proportion to wealth to neutralize income effects, captures the notion of a mortgage rate schedule in reduced form and rules out equilibria in which borrowers take on LTV ratios >> 1 at very high rates in the expectation that they will likely default.

The trading stage budget constraint is given by:

$$w_t^i + \mathcal{R}_t^i = \frac{d_t^i}{1 + r_t^d} + q_t^m m_t^i + p_t^h h_t^i + p_t^s s_t^i + \Phi\left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - L\bar{T}V\right)$$

and the value function is:

$$V(w_t^i, \mathcal{Z}_t) = \max_{d_t^i, h_t^i, s_t^i, m_t^i} \beta \mathrm{E}_t \left[\max\left\{\max_{a_t^i \geq 0} u^B(c_{t+1}^{i,nd}, h_t^i) + V(w_{t+1}^{i,nd}, \mathcal{Z}_t), \eta_t^i \left(u^B(c_{t+1}^{i,d}, h_t^i) + V(w_{t+1}^{i,d}, \mathcal{Z}_t)\right)\right\}\right]$$

where the innermost maximization indicates the optimal consumption-savings choice in next period's consumption stage, the middle maximization indicates the default decision, and the outermost maximization indicates portfolio choices in the current period.

3.2 Banks

Banks are owned by savers so maximize the stream of dividends discounted at the saver's stochastic discount factor.

They lend in the mortgage market, financing their loan portfolios with equity and deposits, which are risk-free one-period bonds held by borrowers and outside investors. Outside investors have perfectly elastic demand for deposits at a price of $\frac{1}{1+r_t^d}$. The deposit rate r_t^d may differ from the policy rate r_t^f to which adjustable mortgages are indexed. Recent work has shown that changes to policy rates do not pass through one-for-one to deposits, complicating banks' exposure to interest rate risks.[6] We model the relationship in reduced form as

$$r_t^d = (\bar{r}^f - \alpha_d) + \beta_d(r_t^f - \bar{r}^f)$$

with $\alpha \geq 0$ and $\beta_d \in (0, 1]$. The parameter α_d captures the average spread between policy and deposit rates, while β_d capture the degree of deposit rate sensitivity to policy rate deviations

[6] E.g., Nagel (2016), Drechsler et al. (2017), and Krishnamurthy and Li (2022)

from its mean. When $\alpha_d = 0$ and $\beta_d = 1$, the two rates are always equal.

Banks portfolios are perfectly diversified and hence identical across banks, so we can write the bank's problem without i subscripts. They enter a period with a stock of outstanding mortgages m^I_{t-1}, of which a fraction F^η_t default. On mortgages that do not default, banks receive a payment x_t per unit of mortgage m^I_{t-1} and have an ex-payment value $(1-\delta)q^m_t$.

Mortgage defaults lead lenders to seize the house, on which they must make a maintenance payment before selling it in foreclosure at a price $p_t(1-\zeta)$ per unit, where ζ represents a foreclosure cost. The total foreclosure proceeds are

$$\int_0^\ell \mathbb{1}^i_{\text{default}} h^i_{t-1} p_t((1-\zeta)-\delta_H)di$$

The payoff per unit of mortgage is therefore:

$$\mathcal{X}_t = (1-F^\eta_t)(x_t + (1-\delta)q^m_t) + \int_0^\ell \mathbb{1}^i_{\text{default}} \frac{h^i_{t-1}}{m^I_{t-1}} p_t((1-\zeta)-\delta_H)di$$

Running the intermediation technology is costly. Banks must pay a fraction ν of the value of their mortgage portfolio as intermediation costs. Their net worth is then given by:

$$w^I_t = (1-\nu)\mathcal{X}_t m^I_{t-1} + d^I_{t-1}$$

where negative values of d^I_t represent borrowing by the lender.

Banks use their equity deposits to finance dividends and mortgage purchases, maximizing

$$\max_{m^I_t, d^I_t} \mathrm{E}_t\left[\sum_{s=t}^{\infty} \mathcal{M}^S_{t,s}\mathrm{Div}_t\right]$$

subject to a budget constraint:

$$w^I_t = \frac{d^I_t}{1+r^d_t} + q^m_t m^I_t + \mathrm{Div}_t$$

and a capital requirement:

$$-d_t \leq \xi(\kappa\bar{q}^m + (1-\kappa)q_t^m)m_t^I$$

where ξ represents the maximum leverage ratio and κ represents the fraction of the mortgage portfolio that is carried at book value on the lender's balance sheet. A value of $\kappa = 1$ indicates that mark-to-market losses on the mortgage portfolio do not tighten leverage constraints, while $\kappa = 0$ indicates a fully mark-to-market regime.

3.3 Savers

Saver households have the same preferences as borrowers, but receive income from their shares of Lucas trees free from idiosyncratic risk. As owners of bank equity, they also receive net dividends from the banks. Finally, they are rebated lump-sum the costs associated with mortgage default – both the pecuniary cost of default faced by borrowers and the foreclosure cost faced by banks – as well as the cost of intermediation. Their budget constraint is simply:

$$c_t^s = \text{Div}_t + \frac{\alpha}{\ell}Y_t + \text{Rebates}_t.$$

3.4 Equilibrium

Given the exogenous processes for aggregate income Y_t and risk-free rate r_t^f and given the idiosyncratic income shocks ε_t^i and ARM reset shocks $\mathbb{1}_\tau$, and the idiosyncratic default shocks η_t^i, an equilibrium is a set of borrower household allocations $\{c_t^i, h_t^i, s_t^i, m_t^i, d_t^i, a_t^i\}_{t=0}^\infty$, borrower default decisions $\{\mathbb{1}_d\}_{t=0}^\infty$ bank allocations $\{\text{Div}_t, m_t^I, d_t^I\}_{t=0}^\infty$, saver allocations $\{c_t^S\}_0^\infty$, and prices $\{p_t^h, p_t^s, q_t^m\}_{t=0}^\infty$ such that each agent maximizes their value function subject to their constraints, and the following market-clearing conditions hold:

1. The mortgage market clears:

$$(1-\ell)m_t^I = M_t^B \equiv \int_0^\ell m_t^i di$$

2. The borrower housing market clears:

$$\alpha_H \bar{H} = H_t^B \equiv \int_0^\ell h_t^i di$$

3. The market for borrower Lucas trees shares clears:

$$\alpha = \int_0^\ell s_t^i di$$

Note that the elastic demand for deposits by outside investors at rate r_t^f implies that the deposit market within the model does not need to clear.

Appendix A contains the derivation of the equilibrium conditions and the solution to the model.

4 Calibration

We calibrate the model at an annual frequency in two steps. Table 1 displays parameters whose values we choose outside the model based on external sources. Table 2 displays "internally" calibrated parameters, whose values are chosen so that the model with fixed-rate mortgages ($\pi_\tau = 0$) matches moments estimated in the data. We discuss each set of parameters in turn.

Stochastic Environment Aggregate dynamics of the model are governed by shocks to aggregate income Y_t and the interest rate r_t^f. In our baseline calibration, we abstract away from income shocks, setting $Y_t = 1$. The risk-free rate process is parameterized by an AR(1) process with mean μ_r, standard deviation σ_r, and persistence ρ_r, calibrated to match the dynamics of the 1-year Treasury constant maturity rate from 1987 to 2024. We estimate a mean rate of 0.031, an unconditional standard deviation of 0.010, and a persistence of 0.656. The standard deviation and persistence parameters imply the standard deviation of interest rate shocks.

Table 1: Externally Calibrated Parameters

Parameter		**Value**
Panel A: Stochastic Processes		
Mean of risk-free rate process	μ_r	0.031
Std. dev. of risk-free rate process	σ_r	0.010
Persistence of risk-free rate process	ρ_r	0.656
Probability of low idiosyncratic income shock (ϵ_L)	π_L	0.058
Idiosyncratic income drop in low state	ϵ_L	-0.456
Idiosyncratic income increase in high state	ϵ_H	Set such that $\mathrm{E}[\epsilon] = 0$
Panel B: Deposit Rates		
Deposit spread w.r.t. base interest rate	α_d	0.018
Deposit sensitivity w.r.t. base interest rate	β_d	0.340
Panel C: Borrowers and Savers		
Borrower population share	ℓ	0.400
Borrower income share	α	0.600
Borrower housing share	α_h	0.500
Risk aversion	γ	1.5
Panel D: Housing, Mortgages and Banks		
Housing maintenance payment	δ_h	0.020
Mortgage rate reset probability	π_τ	0.000
Deviation from target LTV cost	ϕ	0.050
Max. leverage ratio	ξ	0.920
Share at book value	κ	0.000

We normalize the idiosyncratic income shocks to have a mean of 0, which means that they are governed by two parameters. The probability of a low income realization π_L is set to 0.058, which is the average post-war unemployment rate. The magnitude of the low income shock ϵ_L is set based on the Ganong and Noel (2019) estimates of the income loss from unemployment. They find that income loss occurs gradually over the first year as unemployment insurance expires. Since our model is annual, we average the income loss in months after UI kicks in as reported in Figure 2, Panel A of that paper, producing a value of -0.456. The high income shock ϵ_H is set to ensure that the expected value of the idiosyncratic income shock is zero.

Deposit Rates Bank deposit rates are lower than risk-free rates, such as T-Bill and Fed Funds, on average and adjust less than one for one with those rates. We estimate deposit rates using quarterly Call Reports data from 1987 to 2024 as the ratio of interest expense to previous quarter's balance on all non-time deposits. The main role deposits play in our model is liquidity – they are the only asset that can be liquidated to finance consumption in the consumption stage. Time deposits incur penalties for liquidation before maturity, motivating their exclusion.

We set α_d to the average spread between the Fed Funds rate and the deposit rate of 0.018.

It often takes multiple quarters for deposit rates to adjust after a change in the Fed Funds rate. Our specification of r_t^d as a linear function of r_t^f does not allow for such intertia, and contemporaneous responses of deposit rates may understate the sensitivity of the deposit rate to the risk-free rate. We estimate a VAR(1) of Fed Funds and deposit rates and set $\beta_d = 0.340$, the peak of the deposit rate impulse response to a one-unit shock to the Fed Funds rate.

Population, Income, and Housing Shares Using 2023 SCF data, we set $\ell = 0.400$ to the approximate share of homeowners that have a mortgage LTV of at least 30%. Given this definition of borrowers, $\alpha = 0.600$ and and $\alpha_h = 0.500$ are set to the approximate shares of income and housing, respectively, held by borrowers in the SCF data.

Banks Banks are subject to a capital requirement that limits their leverage. We set the maximum leverage ratio ξ to 0.920, which is the maximum Tier 2 capital ratio for banks under Basel III. This calibration effectively assumes a mortgage risk weight of 100%, which is the standard risk weight for residential mortgages. In the baseline calibration, we set the book value share κ to 0.000, meaning that mortgages are held at market value.

Borrower Preferences, Housing, and Defaults Housing maintenance payments as a fraction of housing are set to 0.020 based on the post-war average residential housing depreciation rate. Our model does not include housing investment, so the maintenance payment can be thought of as investment needed to offset depreciation and maintain housing stock at its steady-state value.

We set household risk aversion γ to 1.5, a standard value in the literature.

The remaining set of borrower preference and default-related parameters are calibrated internally. Panel A of Table 2 displays four parameters that must be calibrated jointly. We set patience β to 0.969, which yields a mortgage/income ratio of 148.83% given the values of other parameters, matching its value in the 2023 SCF. The value of housing to income is determined in equilibrium by the present value of user costs parameterized by the utility weight on housing θ, discounted at the rate implied by β and the probability of losing the house in foreclosure (i.e,

Table 2: Internally Calibrated Parameters

Parameter		Value	Target	Value (FRM Bench)
Panel A: Borrowers				
Borrower patience	β	0.969	Mortgage/income	148.83
Housing utility weight for borrowers	θ	0.183	Housing/income	260.59
Std. dev of idiosyncratic default shock	σ_η	0.045	Default rate	2.23
Income loss upon default	λ	0.148	Deposits/income	23.91
Panel B: Intermediaries				
Foreclosure cost	ζ	0.520	LGD	16.97
Banker intermediation cost	ν	0.036	Mortgage rates	0.059
Principal payment share	δ	0.036	Mortgage duration	6.9

default rate). We set θ to 0.183 such that, at the target default rate and given the calibrated value of β, the value of housing/income matches 260.59% in the SCF.

Housing- and mortgage-to-income ratios imply a LTV ratio of approximately 60%. The mapping of this ratio into default rates depends on two parameters – the standard deviation of the idiosyncratic default shock σ_η and the share of future income lost in default λ. The pecuniary cost of default motivates agents to hold deposits so that they can decrease their default probability in the event of a low income realization. We set σ_η to 0.045 and λ to 0.148 to match the average 2003-2023 flow into 90-day delinquency in the New York Fed's Quarterly Report on Household Debt and Credit (QRHDC) of 2.23%, and the deposits-to-income ratio of 23.91% in the SCF.

Mortgages In our model, there are no idiosyncratic shocks to home values, so in the cross-section defaulting households have the same LTV ratios as non-defaulting households. Given the LTV ratio implied by the calibration of housing and mortgage-to-income ratios, we set foreclosure cost ζ to 0.520, which implies a loss given default (LGD) of 16.97%. This is consistent with the average LGD in the data, computed as average charge-off rate on mortgages held by depository institutions, from the St. Louis Fed FRED database, divided by the average default rate from the NY Fed QRHDC.

The mortgage interest payment in the FRM economy ι_f is set so that the steady-state mortgage price $\bar{q}^m$ is equal to 1, and thus ι_f can be interpreted as the steady-state mortgage yield, or par rate. The historical average rate is 0.059. In the model, the mortgage yield, defined as the discount rate, which discounts expected future cash flows to par, depends on (1) the intermediary's cost of funding, a leverage-weighted average of the equity cost of capital implied by β and the deposit cost of capital $\bar{r} - \alpha_d$, (2) expected losses, a function of the default rate and LGD, and (3) the cost of intermediation parameterized by ν. Given a calibration that matches target default rates and LGD, we set ν to 0.036 so that $\bar{q}^m = 1$ at $\iota_f = 0.059$.

In counterfactual exercises with adjustable rate mortgages, we set $\iota_a = \iota_f - \bar{r}$, making payments the same on average. In the baseline calibration, we set interest payments in the teaser/fixed stage of an ARM ι_τ to ι_f so that the end of the fixation period does not cause a jump in payments.[7]

Borrowers in our model do not endogenize the effect of their demand on *their*, rather than the equilibrium, mortgage rate.[8] As a result, at low equilibrium rates, they may face an incentive to take on a large mortgage that implies a high default probability and hence a low expected cost of borrowing. One way to address this issue is to set a maximum LTV constraint, that would be slack in steady state but bind in some states of the dynamic model. To simplify model solution, we follow a different approach and impose a per-housing-dollar quadratic cost of deviating from the steady-state book LTV ratio $\frac{\phi}{2}\left(\frac{q_t^m}{p_t h_t^i} - L\bar{T}V\right)^2$. We set ϕ to a small positive value, 0.050. It has negligibly small effects on equilibrium dynamics but improves our ability to solve the model by ruling out equilibria with counterfactually high LTV ratios.

The last mortgage contract feature is the fraction of the principal paid in each period, δ. This parameter determines the duration of the mortgage, which we set to match the duration of a 30-year fixed rate mortgage in the data. Our model generates an endogenous reduction in duration relative to its contractual value that occurs because of default, but we do not capture the reduction due to moving-induced prepayments. To calculate the correct target duration in the data, we compute an amortization schedule for a 30-year fixed rate mortgage with a rate of ι_f and an annual prepayment probability of 6%, close to the unconditional annual moving probability of mortgage borrowers reported by Fonseca and Liu (2023). This procedure yields δ equal to 0.086, which implies a duration of 6.9 years. We describe the procedure in more detail in Appendix B.

[7] In the data, teaser rates are often set lower such that a jump does occur, but we abstract from this feature in the baseline to develop intuition about the effects of stochastic, rather than predictable, rate changes.

[8] Models with an endogenous debt schedule and long-term debt must tackle dilution incentives and the optimal contract can be difficult to solve. In our framework, such a model would be intractable.

4.1 Model Solution

The model is solved numerically using the global Transition Function Iteration method of Elenev et al. (2021). Our main experiments compare the performance of the economy across a range of mortgage fixation lengths parameterized by π_τ. When this parameter is equal to 0, the economy is in a fully fixed-rate mortgage (FRM) regime. At the other extreme when π_τ is equal to 1, the economy is in an adjustable-rate mortgage (ARM) regime where mortgage payments reset every year. For each economy considered below, we simulate 16 paths of 5,000 periods each after discarding the first 1,000 and report unconditional moments of the long simulation. We also consider impulse responses to interest rate shocks at the stochastic steady state of each model.

5 Results

We first show impulse responses to interest rate shocks at the stochastic steady state of the FRM and ARM economy, and discuss main mechanisms. We then report unconditional moments of a long simulation, grouped by financial stability outcomes, borrowers and consumption, and risk-sharing outcomes across a range of counterfactual mortgage structures.

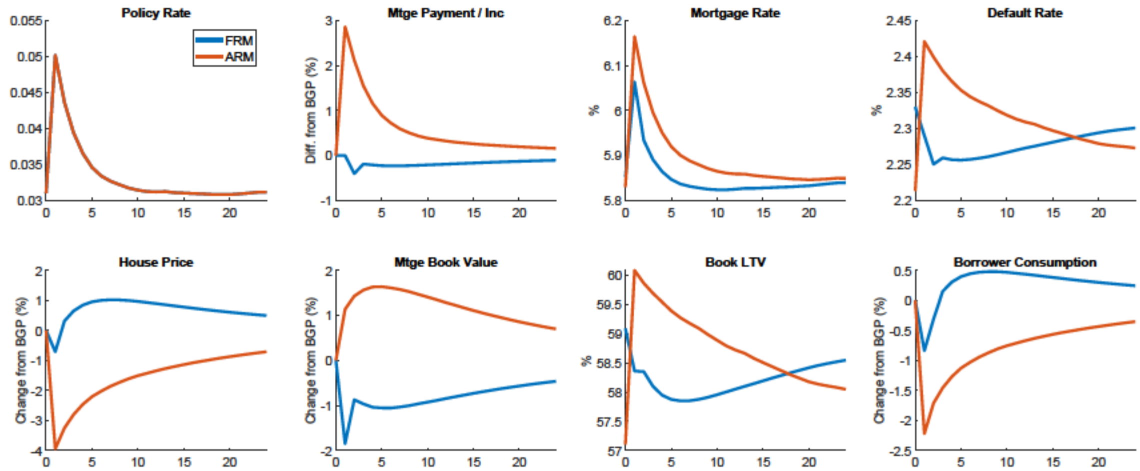
5.1 Rate Shock Impulse Responses and Mechanisms

First, to understand the role of mortgage structure in the transmission of interest rates, we analyze impulse responses of the FRM and ARM economies to a positive shock to the policy rate r^f_t.[9] Figures 4 displays the results for borrower variables, while 5 displays the results for banks.

How does the economy respond to an increase in the policy rate from 3.1% to 5%? The effects vary with the underlying mortgage structure. To contrast the differential effects, we focus on a pure FRM economy where interests rates do not reset with a pure ARM economy

[9] To compute impulse responses, we initialize the economy at the stochastic steady state of a long simulation at $t = 0$ and compute its $t = 1$ transition given a particular realization of exogenous variables. Subsequently, we let the economy evolve stochastically, simulating 5,000 paths of 25 years each. The average path constitutes the plotted impulse response.

Figure 4: Impulse Responses to a Positive Interest Rate Shock: Borrowers



Notes: Impulse Response Functions for a positive shock to the interest rate r_t^f. "FRM" (blue) denotes an economy in which mortgage payments remain fixed at $\iota_f + \delta\bar{q}^m$. "ARM" (red) denotes an economy with a rate fixation length of 1 year ($\pi_\tau = 1.0$) in which mortgage payments mortgages increase with rates $r_t^f + \delta\bar{q}^m$.

where interest rates every year below, but we subsequently expand the comparison and study differing intermediate fixation lengths.

Borrowers When rates are fixed ("FRM"), mortgage payments remain unchanged on impact and borrower liquidity is unaffected. Mortgage rates go up, but existing borrowers are shielded from the increase. In addition, holding on to their current mortgage becomes more valuable because the alternative is more expensive. As a result, borrowers are less likely to default for strategic reasons. At the same time, new borrowers face higher mortgage rates and are less likely to take out a loan, decreasing the aggregate mortgage balance and driving down demand for housing. The net effect of lower defaults and less available credit is a slight decrease in house prices,[10] a larger decrease in mortgage credit and hence future payments, and a persistent decrease in default rates due to lower LTVs.

In contrast, when mortgage payments reset every year ("ARM"), borrowers face higher payments immediately. The liquidity burden of higher payments causes a spike in default rates with higher defaults persisting as long as rates and hence payments remain higher. The reduc-

[10] We do not model explicit mortgage lock-in effects (Fonseca and Liu, 2023) and their impact on house prices in an FRM economy, see e.g. Fonseca et al. (2024).

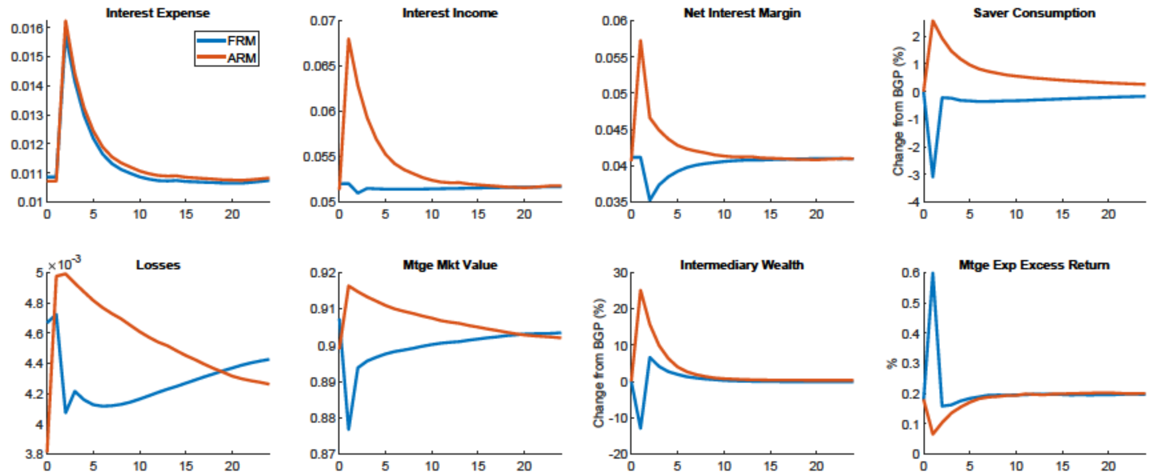
tion in household liquidity has two consequences for credit demand. On one hand, persistently higher default rates lower house prices restricting the available supply of mortgage collateral. But on the other hand, the need to spend a larger share of their liquid assets on mortgage payments disproportionately reduces borrower consumption relative to wealth. The desire to smooth consumption raises demand for credit. On net, the demand effect wins out, resulting in higher credit demand which leads to larger mortgage balances and a bigger increase in mortgage rates relative to the FRM economy.

Banks The different dynamics of default and credit demand have consequences for the financial sector. Figure 5 plots key banking variables in output units, to aid comparison. When rates go up, the cost of deposit funding – the banks' interest expenses – also increase, though less than one for one. When mortgage rates are fixed, interest income remains unchanged, leading to a drop in banks' net interest margin. Banking becomes less profitable, despite the slight offsetting decrease in credit losses discussed above (due to borrowers defaulting less because their low-rate mortgage becomes more valuable). Moreover, fixed-rate mortgages have a long duration. In response to higher rates, the market value of long-dated bank assets falls. With both lower cash flows due to smaller net interest margins, and lower asset values due to higher discount rates, the net worth of the banking sector declines, as does the consumption of their equity-holders – the savers.

More constrained banks demand higher compensation to take on mortgage risk, a result common to intermediary-based asset pricing models. The spike in risk premia, i.e. expected excess returns on mortgages, amplifies mortgage duration, further contributing to market value losses of banks as it increases discount rates.

In contrast, in the ARM economy, higher rates lead to higher mortgage payments. Since mortgages are indexed to the policy rather than the deposit rate, the net interest margin of banks increases as mortgage rates received rise by more than deposit rates paid. Banks become more profitable even though credit losses rise due to a rise in defaults. Intuitively, banks' credit losses in the ARM economy are more "hedged" across states since they precisely arise in states of the world where cash flows from mortgage payments are high. Moreover, because the increase in cash flows outpaces the increase in the relevant discount rate (deposit rate), adjustable rate

Figure 5: Impulse Responses to a Positive Interest Rate Shock: Banks



Notes: Impulse Response Functions for a positive shock to the interest rate r_t^f. "FRM" (blue) denotes an economy in which mortgage payments remain fixed at $\iota_f + \delta\bar{q}^m$. "ARM" (red) denotes an economy with a rate fixation length of 1 year ($\pi_\tau = 1.0$) in which mortgage payments mortgages increase with rates $r_t^f + \delta\bar{q}^m$.

mortgages effectively have negative duration. Their value increases with higher rates. With higher cash flows and higher asset values, the net worth of the banking sector increases, yielding higher dividends for their saver equity-holders.

The increase in intermediary net worth lowers mortgage risk premia. But risk premia are non-linear in intermediary net worth. An improved capital position of already healthy banks in the ARM economy does not reduce risk premia much, but a deterioration in the capital position with impaired balance sheets in the FRM economy leads to a sharp spike in risk premia.

5.2 Financial Stability

We can now compare financial stability outcomes in the benchmark FRM economy with several counterfactual economies where we vary mortgage fixation length. To do so, we solve the model and simulate outcomes for economies with values of $\pi_\tau \in [0, 1]$ where π_τ reflects the annual probability of the rate resetting, and $1/\pi_\tau$ the (expected) fixation length. For instance, a full ARM economy has a rate that resets every year with $\pi_\tau = 1.0$, a 10-year fixed-rate mortgage economy has $\pi_\tau = 0.1$, while the full FRM economy has $\pi_\tau = 0$.

We consider three summary measures of financial stability that we compute across different mortgage structures, varying from fully adjustable (fixation length of 1 year) to fully fixed (infinite fixation length), shown in Figure 6. Panel (a) shows the volatility of banks' mortgage returns. Since banks only hold mortgages in our setting, this is equivalent to the banks' volatility of return on assets (ROA). As such, it only measures income volatility, ignoring the volatility of expenses (or, similarly, volatility on the asset side without considering the liability side), and also ignores leverage effects. Next, Panel (b) shows the duration of intermediary net worth, measured as the negative of the regression coefficient of log wealth on interest rates. Net worth duration reflects by how much intermediary net worth (in per cent) declines in response to a 1 percentage point increase in rates. The pure ARM economy with a fixation length of one year has large negative duration, meaning that net worth increases substantially when interest rates go up (and vice versa). The pure FRM economy with an infinite fixation length has moderate positive duration, meaning net worth declines when rates go up.

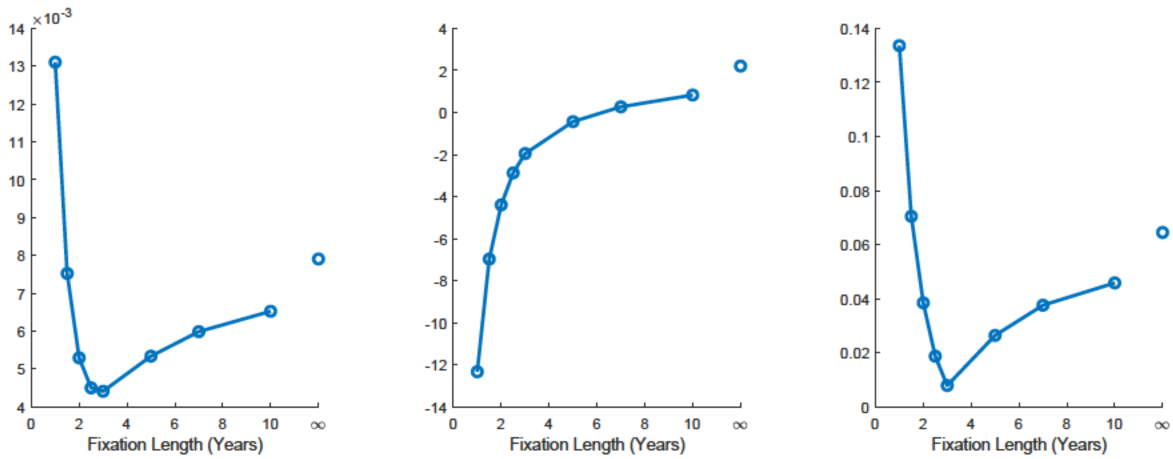
Net worth duration would be a comprehensive measure of risk if net worth was only driven by contemporaneous rates, as captured by the regression. However, the R^2 of the duration regression is only 0.164, suggesting that there are dynamic and persistent effects of rate changes on net worth that are not captured by this measure. As a result, our preferred measure of financial stability is the volatility of banks' return on equity (ROE), shown in Panel (c). This measure captures the combined equilibrium effects of asset and liability-side volatility as well as leverage on the volatility of intermediary net worth.[11]

Both mortgage return volatility and the volatility of banks' ROE in Panel (a) and (c) of Figure 6 reveal a "U-shape" pattern, that is, volatility measures are higher on both extremes of mortgage structure, fully adjustable or fully fixed, than at an intermediate fixation length. In an FRM economy (infinite fixation length), volatility comes from market value fluctuations due to the long duration of mortgages. Shorter fixation lengths decrease duration, but they also increase the volatility of cash flows. In a full ARM structure, mortgage payments and thus cash flows are very sensitive to interest rate changes, more so than deposit expenses. This leads intermediary equity to have strongly negative duration (Panel (b)). Overall, risks to financial stability are minimized at an intermediate fixation length of approximately 3 years. We discuss potential drivers behind this result in the following section.

[11] This is consistent with the intuition in Meiselman et al. (2023), who show that banks' ROE is a strong predictor for systematic tail risks.

Figure 6: Measures of Financial Stability Across Mortgage Structures

(a) Mortgage Return Vol. (b) Net Worth Duration (c) ROE Volatility



Notes: The x-axis reflects an annual rate reset probability of $\pi_\tau \in \{1, 0.7, 0.5, 0.3, 0.25, 0.2, 0.15, 0.1, 0\}$, which corresponds to fixed-rate lengths of approximately 1, 1.43, 2, 3.33, 4, 5, 6.67, 10 and ∞, respectively.

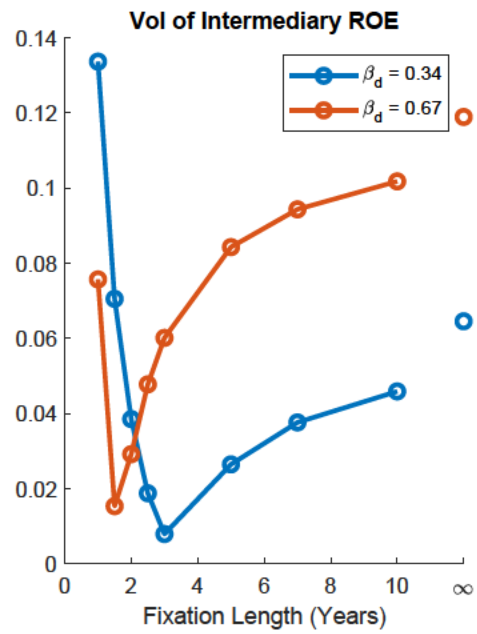
5.2.1 What Drives the U-Shape Result?

What explains the U-shaped pattern of financial stability risks across mortgage structure? We explore two mechanisms: first, the role of deposit rate sensitivity with respect to rate changes, and second, endogenous default and credit losses and their interaction with intermediary net worth, which affect risk premia.

Deposit Sensitivity Figure 7 reveals that the lower volatility advantage of the full FRM benchmark stems primarily from the calibrated degree of deposit stickiness with $\beta_d = 0.34$. $\beta_d \leq 1$ governs the pass-through of interest rate changes to deposit rates, and thus affects how much less sensitive interest expense (as measured by deposit rates) is to rate shocks than interest income, where we assume that the pass-through from interest rates to floating mortgage payments is equal to one. In a counterfactual where deposits are more sensitive to policy rates ($\beta_d = 0.67$), ROE volatility under an FRM structure can be higher than under an ARM structure. Intuitively, a greater deposit sensitivity aligns the duration of liabilities under an ARM structure more with the duration of assets, while the reverse is true in an FRM regime, consistent with findings by Drechsler et al. (2017).

To summarize, the fixation length at which volatility is smallest depends, in part, on the aver-

Figure 7: ROE Volatility by Deposit Sensitivity

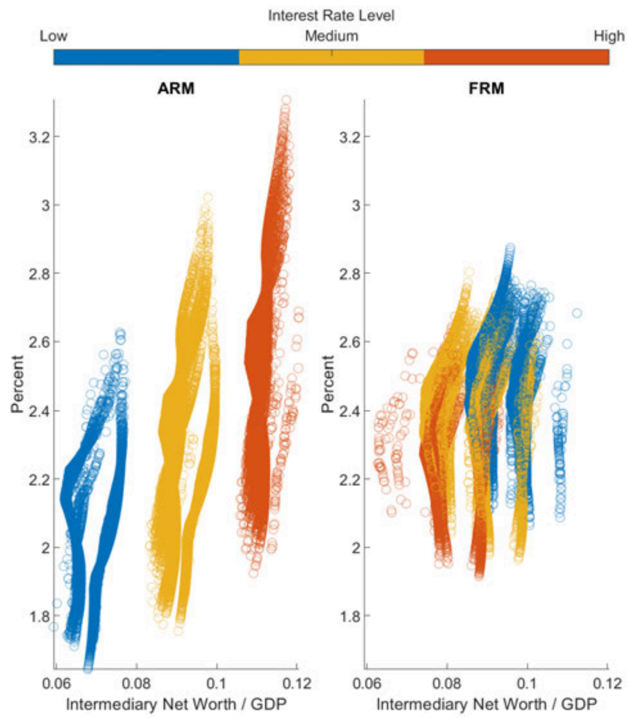


age sensitivity of contractual mortgage payments minus deposit expenses. However, our model reveals additional equilibrium forces that complicate volatility-minimizing fixation length.

Mortgage Default and Risk Premia First, mortgage structure affects the level and volatility of default rates, as well as the sign of their covariance with interest rates. Second, mortgage structure changes the dynamics of mortgage risk premia, which contribute to the volatility of mortgage returns and ROE as "discount rate" news.

To assess these effects, we return to a comparison of the full-ARM vs. full-FRM economy. Figure 8 illustrates substantial differences between FRM and ARM economies when comparing default rates across interest rate levels (color) and intermediary net worth to GDP (x-coordinate): Consistent with Campbell and Cocco (2015), default occurs in different macroeconomic states across mortgage structures, and is more rate sensitive in the ARM economy. With ARMs, the level of defaults is highest at high levels of interest rates (in orange). However, these are also states of the world in which intermediary net worth is relatively high given higher cash flows from higher mortgage payments. Thus banks' default losses are typically more hedged in an ARM economy, as they coincide with high levels of intermediary net worth. Overall, intermediary net worth is positively correlated with the level of interest rates.

Figure 8: Default Rates By Level of Interest Rate and Intermediary Net Worth

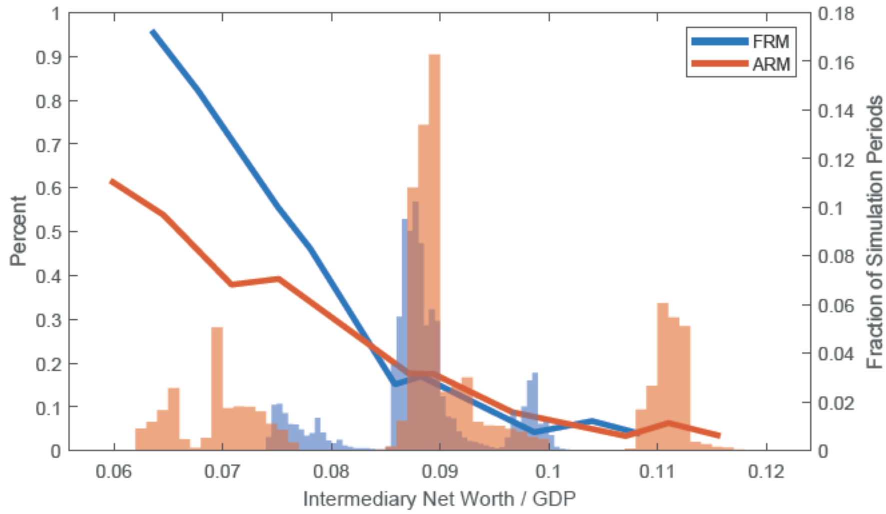


Notes: This figure shows simulation scatter plots of default rates by the level of intermediary net worth (x-axis) and interest rates (color: blue is low, yellow is medium, orange is high). The left plot shows the ARM economy, while the right plot shows the FRM economy.

In contrast, intermediary net worth is somewhat negatively correlated with interest rates in the FRM economy. Rate hikes push intermediary net worth lower. There is a small hedging effect in the FRM economy as well, since rate hikes push default rates lower as shown in the impulse responses, but the lower sensitivity of defaults to rates in the FRM economy means the hedging force is smaller in the FRM world. Figure 9 shows that the weaker hedging effect makes risk premia (left y-axis) more sensitive to intermediary net worth (x-axis) when it is low. In the FRM economy (blue), risk premia are very high when rates are high and intermediary net worth is low, while in the ARM economy (red) risk premia are only moderately high when rates are low.

However, Figure 9 also shows that ARMs make intermediary net worth more volatile on average, with a higher probability of being in a low intermediary net worth state compared to the FRM economy (shown as frequency distribution of simulation periods on the right-hand y-axis), meaning that risk premia in the ARM economy are not necessarily lower on average.

Figure 9: Mortgage Risk Premia Decrease with Intermediary Net Worth



Notes: This figure shows simulated scatter plots of mortgage excess returns by the level of intermediary net worth, for ARM and FRM economies.

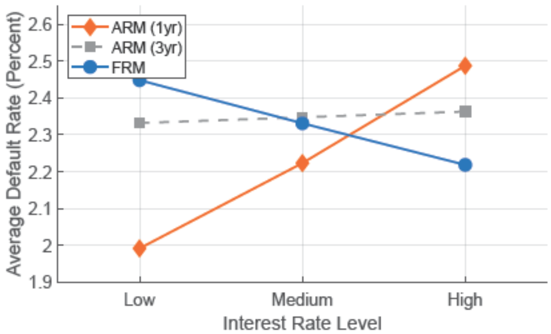
In sum, fixation length affects the cyclicality of mortgage risk contributing both to the credit risk and risk premium components of intermediary ROE. These equilibrium forces highlight that, to minimize intermediary risk, it is insufficient to align the timing of *expected* interest income and expenses.

5.3 Intermediate Fixation Length

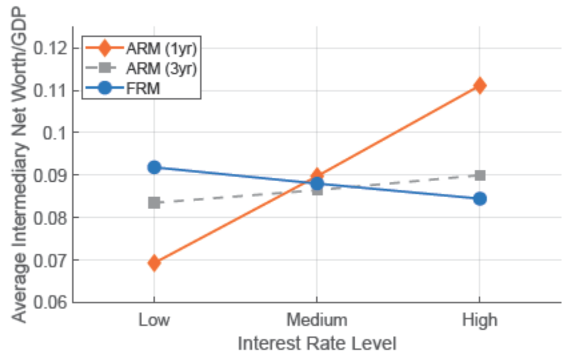
Figure 6 shows that intermediary net worth volatility is minimized around a fixation length of 3 years, and the mechanisms above suggests that such an intermediate fixation length balances somewhat opposing forces in both ARM and FRM economies. To directly compare outcomes in an economy with an intermediate fixation length of 3 years, we turn to Figure 10, which shows the average default rate (Panel a) and average level of intermediary net worth to GDP (Panel b) by different interest rate levels. While default is steeply increasing in the level of interest rates in the full-ARM economy (with a fixation length of one year), and somewhat decreasing in the level of interest rates in the full-FRM economy, default is relatively stable across the intermediate fixation length economy ("ARM 3yr"). Similarly, intermediary net worth is also steeply increasing in rates in the ARM economy and weakly decreasing in rates in the FRM

Figure 10: Default and Net Worth by Level of Interest Rate

(a) Default Rate



(b) Intermediary Net Worth to GDP



Notes: This figure shows simulation-based average rates of default and levels of intermediary net worth to GDP across different levels of interest rates, for the full-ARM (1-year fixation length), intermediate ARM (3-year fixation length), and full-FRM (infinite fixation length) economies.

economy, and broadly stable in the intermediate fixation length economy. The results suggest that an intermediate fixation length broadly balances the different mechanisms in both extremes of mortgage structure, fully adjustable or fully fixed, such that defaults are less sensitive to the level of interest rates, and net worth also becomes less sensitive to the level of interest rates, making the intermediary sector more stable across states of the world with different interest rate levels. In other words, mortgage structure affects the cyclicality of default and net worth with respect to interest rates, as we explore further below.

5.4 Borrowers and Consumption

Our results on financial stability illustrate the risks to which savers, as bank equity holders, are exposed. For a full understanding of risk sharing, we compare outcomes for both borrowers and banks in Table 3, where the top panel reports bank-related moments, including those shown graphically in Figure 6, and the bottom panel reports those associated with borrower decisions. The results for our baseline low deposit sensitivity calibration are presented in the first three columns. The first and third columns report results for the full-ARM and full-FRM economies, respectively, corresponding with the left-most and right-most points in the graphs above. The middle column reports results for an economy in which the fixation length is 3 years, the point at which intermediary net worth volatility is minimized.

Mortgage structure determines not only the degree but also the nature of borrowers' exposure to interest rate risk, which ultimately determines default and portfolio choices. In ARM economies, mortgage payment-to-income (PTI) ratios increase in rates, exposing borrowers to liquidity risks, as seen in the "PTI (OLS coef.)" which reports the coefficient of a regression of PTI on interest rates. A 1 percentage point increase in rates corresponds to a 1.66 percentage point increase in PTI in the ARM (1yr) economy, but only a 0.48 percentage point increase in the ARM (3yr) economy, and a -0.14 percentage point change in the FRM economy as households delever. But rate shocks also have wealth effects, which determine borrowers' strategic default behavior. Higher interest rates always lower house prices on impact, but the extent to which they affect the value of the mortgage – and, hence, LTV ratios, depends on the fixation length. In the FRM economy, high rates lead to low mortgage values. This creates mildly countercyclical LTV ratios (reflected in a negative "LTV (OLS coef.)", analogously defined to the PTI regression coefficient), and, together with stable payments, yields countercyclical default rates (negative "Default Rate (OLS coef.)", which is consistent with the impulse responses showing a decrease in default rates when rates go up. As fixation length decreases, mortgage duration drops and eventually flips sign. In the full-ARM economy, rate hikes lead not only to higher house prices but higher mortgage values, which implies strongly procyclical LTV ratios. Together with procyclical payments, this leads to procyclical default rates, which are more volatile than in the FRM economy. Conversely, default rates can be mildly countercyclical in the FRM economy. At intermediate fixation lengths, default rates are close to acyclical with respect to interest rates, and are least volatile.

Higher exposure to interest rate risk in ARM economies lowers both the supply and the demand for credit. Together with more expensive mortgages due to higher risk premia ("Excess ROA"), volatile default rates cause households to reduce their demand for credit and expand precautionary saving. Relative to the FRM economy, in the full-ARM economy, average mortgage debt falls both relative to income (DTI) and relative to house prices (LTV), while deposits to income increase. As a result, less indebted borrowers default less often on average. The opposite is true for the safer ARM (3yr) economy.

Differences in risk exposures and indebtedness have implications for consumption. Fewer mortgages mean a smaller banking sector, with reduced dividends lowering saver consumption

Table 3: Measures of Financial Stability

Deposit Sensitivity:	Low ($\beta_d = 0.34$)			High ($\beta_d = 0.67$)		
Mortgage Structure:	ARM (1yr)	ARM (3yr)	FRM	ARM (1yr)	ARM (3yr)	FRM
Excess ROE (mean)	2.06	1.72	1.83	1.77	1.83	2.12
ROE (st. dev.)	13.36	0.79	6.45	7.57	6.01	11.89
Excess ROA (mean)	0.21	0.18	0.19	0.18	0.20	0.22
ROA (st. dev.)	1.31	0.44	0.79	1.06	1.08	1.50
Fraction of constraint binding	39.02	92.72	72.53	63.62	56.75	46.45
Duration of bank net worth	-12.33	-1.95	2.21	-9.57	0.08	4.96
PTI (OLS coef.)	1.66	0.48	-0.14	1.55	0.34	-0.26
LTV (OLS coef.)	2.44	0.23	-0.99	1.59	-0.72	-1.82
Default Rate (mean)	2.23	2.35	2.33	2.29	2.31	2.26
Default Rate (std. dev.)	0.26	0.03	0.14	0.15	0.14	0.27
Default Rate (OLS coef.)	0.14	0.02	-0.05	0.10	-0.03	-0.09
DTI (mean)	148.83	151.27	150.91	150.01	150.65	149.63
LTV (mean)	57.58	59.43	59.15	58.56	58.88	58.05
Deposits / Income (mean)	23.91	23.37	23.41	23.66	23.57	23.76

Notes: Unconditional moments from a long simulation of the model. Except for the duration of bank net worth, all quantities are reported in percent. Rows marked "OLS coef." report the coefficient of a regression of the variable on the policy rate r_t^f.

(Panel A) in the ARM economy. While the banking sector is more volatile, its smaller size makes its returns a relatively smaller part of saver consumption, leading to decreased unconditional consumption volatility across time. However, conditional on a particular state of the economy, the volatility of consumption growth – which determines the price of risk in asset pricing models – goes up, consistent with the higher risk premia in the ARM economy discussed above.

The effect on borrowers is the opposite in the ARM economy. With less debt, their interest burden is smaller, and they suffer the pecuniary consequences of default less often. This results in higher average consumption. Having to make larger payments in high rate regimes, borrowers in the ARM economy have higher unconditional consumption volatility, but their endogenous delevering results in the conditional volatility – driven mainly by idiosyncratic shocks – to go down.

Table 4: Consumption Measures

Deposit Sensitivity:	Low ($\beta_d = 0.34$)			High ($\beta_d = 0.67$)		
Mortgage Structure:	ARM (1yr)	ARM (3yr)	FRM	ARM (1yr)	ARM (3yr)	FRM
Panel A: Savers						
Cons. (mean)	49.76	50.09	50.05	49.93	50.00	49.88
Cons. gr. (st. dev.)	1.69	0.54	1.76	0.63	2.11	2.95
Cond. vol of cons. gr.	1.49	0.27	1.33	0.43	1.52	2.32
Panel B: Borrowers						
Cons. (mean)	47.32	47.00	47.05	47.17	47.10	47.24
Cons. gr. (st. dev.)	16.92	17.42	17.42	17.10	17.29	17.07
Cond. vol of cons. gr.	11.01	11.24	11.19	11.19	11.18	11.04

Notes: Unconditional moments from a long simulation of the model.

5.5 Risk Sharing

Lastly, we assess how mortgage structure determines how risks are shared between households. To quantify the degree of risk sharing, it is instructive to consider a hypothetical complete markets benchmark. A social planner subject to rate shocks but not to any of the economy's frictions would insure households fully against idiosyncratic shocks and award each household a constant fraction of overall consumption. In other words, the difference $\Delta \log c_t^i - \Delta \log c_t^j$ between consumption growth rates of any two households i and j would be zero in all periods.[12]

We can then measure the quality of risk sharing by the unconditional variance of differences in consumption growth rates between households. Recall that borrower households are subject to undiversifiable idiosyncratic risk, while saver households are not. We can define two scale-free measures of risk-sharing:

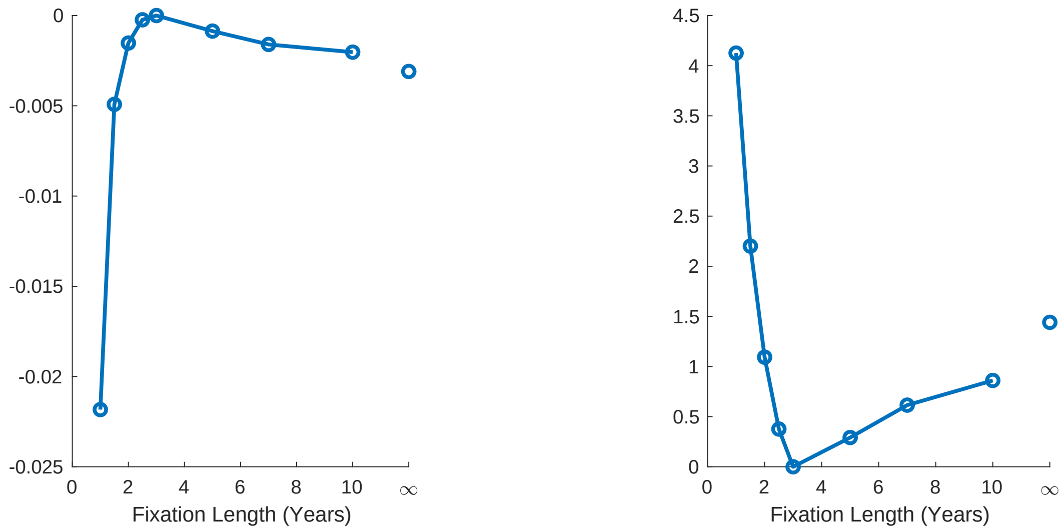
1. Higher values of $\mathcal{R}_{iB} = \text{Var}_0[\Delta \log c_t^i - \Delta \log C_t^B]$, where C_t^B is aggregate consumption of borrowers, indicate worse *intra-borrower* risk-sharing;

2. Higher values of $\mathcal{R}_{BS} = \text{Var}_0[\Delta \log C_t^B - \Delta \log C_t^S]$, where C_t^S is aggregate consumption of borrowers, indicate worse risk-sharing *between borrowers and savers*;

[12]See Appendix A.5 for derivations. Moreover, the planner would optimize the overall economy's exposure to rate shocks. The planner would choose a net deposit position of the economy with respect to the rest of the world to satisfy the consumption-savings Euler equation of the representative agent, whose consumption would be equal to the aggregate consumption of the economy. We also derive these results in Appendix Appendix A.5, but since these effects turn out to be quantitatively negligible, we do not report these separately.

Figure 11: Measures of Risk Sharing across Mortgage Structures

(a) Intra-Borrower ($\mathcal{R}_{iB}$) (b) Between Borrowers and Savers ($\mathcal{R}_{BS}$)



Notes: $\mathcal{R}_{iB}$ measures the variance of individual consumption growth relative to aggregate consumption growth, and $\mathcal{R}_{BS}$ measures the variance of aggregate consumption growth of borrowers relative to savers. In each panel, $\mathcal{R}$ is reported in deviations from the level in the ROE volatility-minimizing economy.

Figure 11 reports the results in standard deviations from the level in the ROE volatility-minimizing economy.[13]

Considering Panel (b), intermediate mortgage fixation lengths lead to the best attainable risk-sharing arrangements between borrowers and savers as $\mathcal{R}_{BS}$ is minimized at a fixation length of 3 years. With low effective mortgage duration and default rates that respond little to interest rates, rate shocks have the least redistributive effects. They affect the consumption of borrowers and savers similarly, leading to low $\mathcal{R}_{BS}$.

However, low exposure to aggregate risk leads borrowers to endogenously choose higher exposure to idiosyncratic risk (Panel (a)). At intermediate fixation lengths, they choose the largest mortgages, and hence the largest mortgage payments, should they choose to make them rather than defaulting. When payments constitute a larger fraction of liquid income, the effect of idiosyncratic income shocks on consumption is amplified. Moreover, higher mortgage balances lead to a higher probability of default. Since consumption levels in and out of default are dif-

[13]At a fixation length of 3 years, $\mathcal{R}_{iB}$ is 0.17, and $\mathcal{R}_{BS}$ is 0.005. Since these measures are scale-free, the level of undiversifiable idiosyncratic risk faced by borrowers is considerably larger than aggregate risk shared between borrowers and savers, consistent with many macroeconomic models.

ferent, a higher probability of default leads to higher consumption volatility. This is reflected in the higher $\mathcal{R}_{iB}$ at intermediate fixation lengths.

Overall, mortgage structure most strongly affects the sharing of interest rate risk between borrowers and savers, with the best attainable outcome occurring at an intermediate fixation length of 3 years. The findings on idiosyncratic risk sharing between borrowers highlight a somewhat subtle downside: a more efficient (aggregate) risk-sharing arrangement leads borrowers to take on more idiosyncratic risk, which the mortgage structures under consideration cannot diversify away.

5.6 Counterfactuals

Lastly, we evaluate factors that affect the financial stability and risk-sharing properties of different mortgage structures by considering additional counterfactual economies.

5.6.1 Deposit Sensitivity

A major source of financial stability risk in the ARM economy is the large difference between high sensitivity of mortgage payments to policy rates and the low sensitivity of deposit rates, calibrated to match the empirical evidence. How different do the trade-offs between ARMs and FRMs look when the pass-through of interest rate changes to deposit rates is higher? We consider a counterfactual in which we double the calibrated benchmark sensitivity of $\beta_d = 0.34$ to $\beta_d = 0.67$ in the fourth through sixth columns of Tables 3 (Financial Stability) and 4 (Consumption).

With more volatile deposit rates at which banks fund themselves, the FRM economy becomes substantially riskier (third vs. sixth columns). Bank equity duration more than doubles, the volatility of both asset and equity returns increases considerably, and banks demand a larger compensation for the risk of holding mortgages. As before, a more volatile economy and more expensive mortgages lead to lower borrower indebtedness, lower default rates, and higher consumption.

The effect of switching from FRMs to ARMs in the high deposit sensitivity counterfactual is opposite to that in the baseline experiment. When policy rates substantially pass through to deposit rates, a mortgage structure in which payments are indexed to the policy rate improves financial stability, reducing the volatility of bank balance sheets and the risk premia associated with them and stimulating mortgage credit. Intuitively, the asset and liability side of bank balance sheets are better aligned with ARMs when deposit rates fluctuate more strongly with interest rates.

5.6.2 Intermediary Risk Aversion and Risk Premia

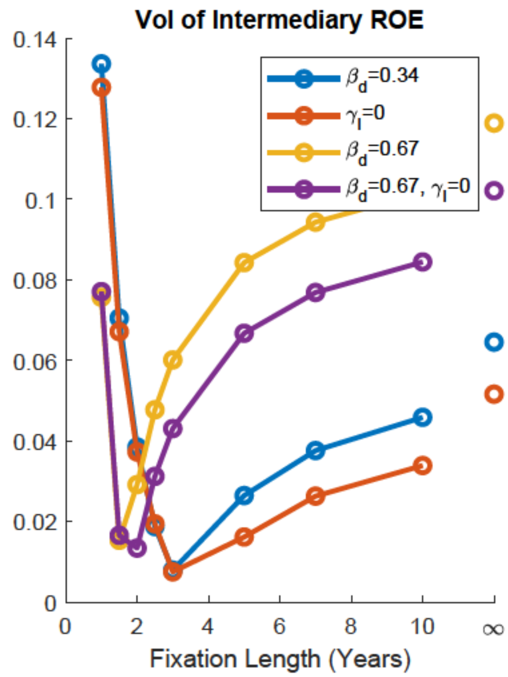
We can further consider an economy with risk-neutral intermediaries and interaction effects with deposit sensitivity. In Figure 12, we compute the volatility of intermediary ROE across different fixation lengths, but setting the coefficient of risk aversion for savers $\gamma_s = 0$ (in yellow), and also allowing for higher deposit sensitivity with $\beta_d = 0.67$ and risk-neutral savers (in purple).[14] Because savers own intermediaries, this comparison illustrates the effect of abstracting away from intermediary frictions. Risk-neutral pricing makes fixed-rate mortgages more attractive and reduces ROE volatility at longer fixation lengths. Consistent with the intuition developed above, without risk premia, reductions in intermediary net worth due to increases in interest rates play less of a role in the FRM economy. When we allow for both higher deposit sensitivity and risk-neutral savers, economies with 10-year fixed-rate mortgages have similar ROE volatility compared to full ARM economies, but ROE volatility is still higher compared to an economy with volatility-minimizing 2-year fixed-rate mortgages.

5.6.3 Other Factors

To summarize, from a mortgage market design perspective, the sensitivity of deposit rates to interest rates and intermediary risk aversion play a substantial role when evaluating which mortgage structure is preferable from a financial stability and risk-sharing perspective. Other factors that we aim to consider are how sensitive default is to an increase in mortgage payments, which could for instance reflect how stringent the underlying recourse regime is, and the sources of aggregate output shocks.

[14] We keep borrower risk aversion at the baseline level of both agents $\gamma = 1.5$.

Figure 12: ROE Volatility - Higher Deposit Sensitivity, Risk-Neutral Intermediaries



6 Conclusion

This paper highlights the role of mortgage structure for financial stability and risk sharing between households and financial intermediaries. To evaluate these effects in equilibrium, we build a quantitative model with flexible mortgage contract structures, borrowers, and an intermediary sector. Borrowers endogenously default for liquidity and net worth-related reasons, and default is more sensitive to interest rates in the adjustable-rate mortgage regime. In addition, intermediary distance to capital constraints affects equilibrium mortgage pricing. As a result, our model captures complex interaction effects between interest rate and credit risk, and intermediary net worth.

Our results highlight differential financial stability implications with respect to interest rate changes across mortgage structures. In an ARM economy, interest rate rises lead to higher mortgage payments for households, increasing default rates and reducing house prices. However, banks in an ARM economy benefit from higher net interest margins and increasing asset values, which protects cash flows despite rising credit losses. In contrast, in an FRM economy, households are shielded from payment increases, reducing defaults, but banks face higher interest expenses and falling asset values, leading to a decline in net worth and profitability.

On net, we find that financial stability risks are "U-shaped" in mortgage structure: while ARM payments are more sensitive to interest rates, defaults happen in states when intermediary net worth is high, resulting in lower risk premia in constrained states compared to the benchmark FRM economy. As a result, an intermediate mortgage fixation length minimizes the volatility of intermediary net worth and optimizes risk-sharing of aggregate risk.

Overall, our findings have implications for monetary policy and macroprudential regulation. Our model provides a framework for understanding how changes in policy rates affect financial stability differentially across mortgage structures. Our paper informs optimal mortgage design that aims to improve financial stability and risk-sharing between households and financial intermediaries.

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A Model Derivations

A.1 Borrowers

The complete borrower's problem is given by:

$$V(w_t^i, \mathcal{Z}_t) = \max_{d_t^i, h_t^i, s_t^i, m_t^i} \beta \mathrm{E}_t \left[\max \left\{ \max_{a_t^i \geq 0} u^B(c_{t+1}^{i,nd}, h_t^i) + V(w_{t+1}^{i,nd}, \mathcal{Z}_t), \eta_t^i \left(u^B(c_{t+1}^{i,d}, h_t^i) + V(w_{t+1}^{i,d}, \mathcal{Z}_t) \right) \right\} \right] \tag{1}$$

where

$$u^B(c_t^i, h_{t-1}^i) = \frac{\left[(c_t^i)^{1-\theta} (h_{t-1}^i)^\theta \right]^{1-\gamma}}{1-\gamma}$$

such that

$$w_t^i + \mathcal{R}_t^i = \frac{d_t^i}{1+r_t^d} + q_t^m m_t^i + p_t^h h_t^i + p_t^s s_t^i + \Phi\left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - L\bar{T}V \right) \tag{2}$$

$$c_t^{i,nd} + x_t^i m_{t-1}^i + \delta_H h_{t-1}^i + a_t^i = s_{t-1}(Y_t + \epsilon_t^i) + d_{t-1}^i \tag{3}$$

$$c_t^{i,d} = s_{t-1}(Y_t + \epsilon_t^i) + d_{t-1}^i \tag{4}$$

$$w_t^{i,nd} = a_t^i - (1-\delta) m_{t-1}^i q_t^m + p_t^h h_{t-1}^i + p_t^s s_{t-1}^i \tag{5}$$

$$w_t^{i,d} = (1-\lambda) p_t^s s_{t-1}^i \tag{6}$$

$$a_t^i \geq 0 \tag{7}$$

where $\mathcal{R}_t^i$ is a rebate of the LTV adjustment cost Φ proportional to wealth w_t^i. With this parametrization, the adjustment cost does not have income effects.

Notice that $u(c,h)$ is homogeneous of degree $1-\gamma$ in c and h and that all constraints are linear in wealth w_t^i in the sense that if a given allocation is feasible for a wealth of 1, then w_t^i times that allocation is feasible for a wealth of w_t^i. By Proposition 1 of Diamond and Landvoigt (2021), these two properties imply that the borrower's value function can be decomposed into $\frac{(w_t^i)^{1-\gamma}}{1-\gamma}$ and a term $v(\mathcal{Z})$ that only depends on state variables exogenous to the borrower.

For a given choice g_t^i, define $\hat{g}_t^i = \frac{g_t^i}{w_t^i}$. Then, the value function can be rewritten as:

$$v(\mathcal{Z}_t)\frac{(w_t^i)^{1-\gamma}}{1-\gamma} = \max_{\hat{d}_t^i,\hat{h}_t^i,\hat{s}_t^i,\hat{m}_t^i} \beta \mathrm{E}_t \Bigg[\max \Bigg\{ \max_{\hat{a}_t^i \geq 0} (w_t^i)^{1-\gamma} u^B(\hat{c}_{t+1}^{i,nd}, \hat{h}_t^i) + v(\mathcal{Z}_{t+1}) \frac{(w_t^i \hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma}, \\ \eta_t^i \left(u^B(\hat{c}_{t+1}^{i,d}, \hat{h}_t^i) + v(\mathcal{Z}_{t+1}) \frac{(w_t^i \hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma} \right) \Bigg\} \Bigg]$$

Divide both sides by $(w_t^i)^{1-\gamma}$ and drop i subscripts on hatted trading stage choice variables following the proposition cited above, getting the following recursion:

$$v(\mathcal{Z}_t) = (1-\gamma) \max_{\hat{d}_t,\hat{h}_t,\hat{s}_t,\hat{m}_t} \beta \mathrm{E}_t \Bigg[\max \Bigg\{ \max_{\hat{a}_t \geq 0} u^B(\hat{c}_{t+1}^{i,nd}, \hat{h}_t) + v(\mathcal{Z}_{t+1}) \frac{(\hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma}, \\ \eta_t^i \left(u^B(\hat{c}_{t+1}^{i,d}, \hat{h}_t) + v(\mathcal{Z}_{t+1}) \frac{(\hat{w}_{t+1}^{i,d})^{1-\gamma}}{1-\gamma} \right) \Bigg\} \Bigg]$$

such that

$$1 = \frac{\hat{d}_t}{1+r_t^d} + q_t^m \hat{m}_t + p_t^h \hat{h}_t + p_t^s \hat{s}_t + \Phi\left(\frac{q_t^m \hat{m}_t^i}{p_t^h \hat{h}_t^i} - L\bar{T}V \right) - \hat{\mathcal{R}}_t \quad (8)$$

$$\hat{c}_t^{i,nd} + x_t^i \hat{m}_{t-1} + \delta_H \hat{h}_{t-1} + a_t^i = \hat{s}_{t-1}(Y_t + \epsilon_t^i) + \hat{d}_{t-1} \quad (9)$$

$$\hat{c}_t^{i,d} = \hat{s}_{t-1}(Y_t + \epsilon_t^i) + \hat{d}_{t-1} \quad (10)$$

$$\hat{w}_t^{i,nd} = \hat{a}_t^i - (1-\delta)\hat{m}_{t-1} q_t^m + p_t^h \hat{h}_{t-1} + p_t^s \hat{s}_{t-1} \quad (11)$$

$$\hat{w}_t^{i,d} = (1-\lambda) p_t^s \hat{s}_{t-1} \quad (12)$$

$$\hat{a}_t^i \geq 0 \quad (13)$$

$$(14)$$

The remaining dependence on i is in consumption stage shock realizations and choices, which enter the value function through the continuation values inside the expectations operator. Therefore, if we can write the consumption stage problem as a function of state variables exogenous to the borrower and i.i.d. idiosyncratic shocks, we will have confirmed the validity of our aggregation.

No Default Branch Consumption Decision If the borrower chooses not to default, they choose $\hat{c}_t^{i,nd}$ and $\hat{a}_t^i$ to maximize $u^B(\hat{c}_{t+1}^{nd}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd})^{1-\gamma}}{1-\gamma}$ subject to the budget constraint (9), wealth evolution (11), and the non-negative intraperiod savings constraint (13). The first order condition for $\hat{a}_t^i$ is:

$$u_c^B(\hat{c}_{t+1}^{i,nd}, \hat{h}_t) = v(\mathcal{Z}_{t+1})\left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} + \kappa_{t+1}^{i,nd}$$

where $\kappa_{t+1}^{i,nd}$ is the Lagrange multiplier on the nonnegativity constraint (13). We will use the functions $\hat{c}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i)$ and $\hat{w}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i)$ to explicitly denote the dependence of the consumption decision on the idiosyncratic realizations borrower's income and the mortgage regime.

Default Decision Given the consumption decision above, a household decides to default iff

$$\underbrace{u^B(\hat{c}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i), \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i))^{1-\gamma}}{1-\gamma}}_{v^{nd}(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon_t^i, \mathbb{1}_\tau^i)} < \eta_t^i \underbrace{\left[u^B(\hat{y}_t^i + \hat{d}_{t-1}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{d}(y_t^i))^{1-\gamma}}{1-\gamma}\right]}_{v^d(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon_t^i)}$$

This expression implies that there exist a default threshold $\eta^*(\epsilon_t^i, \mathbb{1}_\tau^i)$ at which the household is indifferent between defaulting and not defaulting. Which side of the threshold leads to a default vs. no-default decision depends on the sign of the value function, which depends on whether or not $\gamma > 1$. For the rest of these derivations, assume that $\gamma > 1$, the more common case, in which case value functions are negative, and so the default region is given by $[0, \eta^*(y_t^i, \mathbb{1}_\tau^i)]$.

Using the Law of Iterated Expectations, we can separate the conditional expectation E_t in the definition of the value function into an expectation over the realization of aggregate shocks $\mathrm{E}_t^{\mathcal{Z}}[\cdot]$, the expectation over the realizations of i.i.d. idiosyncratic shocks to income ϵ_t^i and reset probability $\mathbb{1}_\tau^i$ denoted by $\mathrm{E}_i[\cdot]$, and the expectation over i.i.d. default utility shocks η^i denoted by $\mathrm{E}_\eta[\cdot]$. Let F_η denote the c.d.f. of the η^i distribution. Then the expectation in the value function can be written as:

$$\mathrm{E}_t^{\mathcal{Z}}\left[\mathrm{E}_i\left[F_\eta(\eta^*(\epsilon,\tau))\mathrm{E}_\eta\left[\eta_t^i\left(u^B(\hat{s}_{t-1}(Y_t+\epsilon_t^i)+\hat{d}_{t-1},\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma}\right)|\eta_t^i > \eta^*(\epsilon_t^i, \mathbb{1}_\tau^i)\right]\right.\right.$$
$$\left.\left. + \left(1 - F_\eta(\eta^*(\epsilon_t^i, \mathbb{1}_\tau^i))\right)\left(u^B(\hat{c}_{t+1}^{nd}(\epsilon_t^i, \mathbb{1}_\tau^i), \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(\epsilon_t^i, \mathbb{1}_\tau^i))^{1-\gamma}}{1-\gamma}\right)\right]\right]$$

Since idiosyncratic shocks are i.i.d., they affect the household problem only through the laws of motion for wealth, admitting aggregation.

If ϵ_t^i idiosyncratic shocks were continuous, the nested expectations above imply integration over a non-rectangular region of (ϵ_t^i, η_t^i), which can be challenging to calculate numerically. Instead, we model shocks to ϵ_t^i as discrete. Shocks to the ARM stage $\mathbb{1}_\tau^i$ are already Bernoulli. In this case, the expectation $\mathrm{E}_i[\cdot]$ above can be written as:

$$\sum_{\tau\in\{0,1\}} \sum_{\epsilon\in\mathcal{E}} \mathcal{P}_\epsilon(\epsilon_t^i = \epsilon)\mathcal{P}_\tau(\tau_t^i = \tau)\times$$
$$\mathrm{E}_i \Bigg[F_\eta(\eta^*(\epsilon,\tau))\mathrm{E}_\eta\left[\eta_t^i | \eta_t^i > \eta^*(\epsilon,\tau)\right] \left(u^B(\hat{s}_{t-1}(Y_t+\epsilon) + \hat{d}_{t-1}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma} \right)$$
$$+ (1 - F_\eta(\eta^*(\epsilon,\tau))) \left(u^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(\epsilon,\tau))^{1-\gamma}}{1-\gamma} \right) \Bigg]$$

Note that conditional on default, the borrower's value function does not depend on the specific realization of the utility penalty, meaning that $u^B(\hat{s}_{t-1}(Y_t+\epsilon)+\hat{d}_{t-1}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma}$ can be brought outside the $\mathrm{E}_\eta[\cdot]$ expectation.

Distribution of η Shocks Let $\log \eta_t^i \sim \mathcal{N}\left(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2\right)$. This implies that the average penalty for default is purely pecuniary and governed by λ, while the dispersion of η shocks given by σ_η governs the sensitivity of default rates to economic conditions.

The log-normal distribution admits a simple expression for the partial expectation of the default penalty:

$$F_\eta^-(\epsilon,\tau) \equiv F_\eta\left(\eta^*(\epsilon,\tau)\right)\mathrm{E}_\eta\left[\eta_t^i | \eta_t^i \leq \eta^*(\epsilon,\tau)\right] = \int_0^{\eta^*(\epsilon,\tau)} \frac{\eta}{\sigma_\eta\sqrt{2\pi}} \exp\left(-\frac{(\log\eta^*(\epsilon,\tau) + \sigma_\eta^2/2)^2}{2\sigma_\eta^2} \right) d\eta$$
$$= \Phi\left(\frac{\log\eta^*(\epsilon,\tau) - \sigma_\eta^2/2}{\sigma_\eta} \right)$$

As well as for the survival probability:

$$\tilde{F}_\eta(\epsilon,\tau) \equiv 1 - F_\eta\left(\eta^*(\epsilon,\tau)\right) = 1 - \Phi\left(\frac{\log \eta^*(\epsilon,\tau) + \sigma_\eta^2/2}{\sigma_\eta}\right) = \Phi\left(\frac{-\log \eta^*(\epsilon,\tau) - \sigma_\eta^2/2}{\sigma_\eta}\right)$$

Therefore, for a given ϵ and τ, the continuation value of the borrower's problem can be written as:

$$F_\eta^-(\epsilon,\tau) v_t^d(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon) + \tilde{F}_\eta(\epsilon,\tau) v_t^{nd}(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon, \tau)$$

where

$$\eta^*(\epsilon,\tau) = \frac{v_t^d(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon)}{v_t^{nd}(d_t^i, h_t^i, s_t^i, m_t^i, \epsilon, \tau)}$$

ARM Reset Probability For a given individual mortgage, the probability of an ARM reset is π_τ *conditional* on it still being in the teaser stage. Since we are not tracking the distribution of mortgages, we can only calculate the unconditional probability of being in the floating (rather than teaser) stage: $\mathcal{P}_\tau(\tau_t^i = 0) = \pi_\tau \left[1 - \mathcal{S}_t\right] + \mathcal{S}_t$, where $\mathcal{S}_t = \mathcal{P}(\tau(m_t^i) = 0)$ is the share of currently outstanding mortgages that have already reset.

We can define this share recursively. Suppose that at the start of the current period, before reset shocks have been realized, the share was $\mathcal{S}_{t-1}$. As a result of reset shocks, there are now $\pi_\tau \left[1 - \mathcal{S}_{t-1}\right] \hat{m}_{t-1}$ new floating rate mortgages. As a result of balance decay, the balances of these mortgages are $1-\delta$ of what they used to be. Newly issued mortgages are all in the teaser stage so do not enter the numerator. Therefore, the share of already reset mortgages is:

$$\mathcal{S}_t = \frac{\left(\mathcal{S}_{t-1} + \pi_\tau \left[1 - \mathcal{S}_{t-1}\right]\right)(1-\delta)\hat{m}_{t-1}}{\hat{m}_t}$$

This aggregation also implies that the teaser vs. floating stage status of a mortgage is randomly reshuffled between households during the trading stage, so that there is no persistence to their mortgage status. This is necessary for aggregation.

A.1.1 First Order Conditions

Preliminaries For a generic choice variable g, write the continuation value of the borrower's problem as:

$$\mathrm{E}_t\left[\underbrace{\left(\int_0^{\eta^*(g)}\eta dF_\eta(\eta)\right)}_{F_\eta^-(g)} v_{t+1}^d(g) + \underbrace{\left[1 - F_\eta(\eta^*(g))\right]}_{\tilde{F}_\eta(g)} v_{t+1}^{nd}(g)\right]$$

Differentiating with respect to g yields and collecting terms:

$$\mathrm{E}_t\left[\frac{\partial v_{t+1}^d(g)}{\partial g}F_\eta^-(g) + \frac{\partial v_{t+1}^{nd}(g)}{\partial g}\tilde{F}_\eta(g) + f_\eta(\eta^*(g))\frac{\partial \eta^*(g)}{\partial g}\left(-\eta^*(g)v_{t+1}^d(g) + v_{t+1}^{nd}(g)\right)\right]$$

Plugging in the default condition $v_{t+1}^{nd}(g) = \eta^*(g)v_{t+1}^d(g)$ leads the last term to become zero:

$$\mathrm{E}_t\left[\frac{\partial v_{t+1}^d(g)}{\partial g}F_\eta^-(g) + \frac{\partial v_{t+1}^{nd}(g)}{\partial g}\tilde{F}_\eta(g)\right]$$

Which is the expression we will use to calculate the first order conditions below.

Define the LTV adjustment cost Φ to be $\Phi(x) = \frac{\phi}{2}x^2$.

Denote by μ_t the Lagrange multiplier on the time t budget constraint (8).

Deposits Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of (inter-period) deposits $\hat{d}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V_{t+1}^{d}}{\partial \hat{d}_t} = u_c^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,d}\right)^{-\gamma} \frac{\partial \hat{w}_{t+1}^{i,d}}{\partial \hat{d}_t^i} = u_c^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t)$$
$$\frac{\partial V_{t+1}^{nd}}{\partial \hat{d}_t} = u_c^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} \frac{\partial \hat{w}_{t+1}^{i,nd}}{\partial \hat{d}_t^i} = u_c^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t)$$

The FOC for (inter-period) deposits $\hat{d}_t^i$ is then given by:

$$\frac{\mu_t}{1+r_t^d} = \beta \mathrm{E}_t \left[F_\eta^-(\epsilon,\tau) u_c^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t) + \tilde{F}_\eta(\epsilon,\tau) u_c^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t) \right]$$

Lucas Tree Shares Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of Lucas tree shares $\hat{s}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V_{t+1}^{d}}{\partial \hat{s}_t} = u_c^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t)(Y_t+\epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,d}\right)^{-\gamma} (1-\lambda) p_{t+1}^s$$
$$\frac{\partial V_{t+1}^{nd}}{\partial \hat{s}_t} = u_c^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t)(Y_t+\epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} p_{t+1}^s$$

The FOC for shares $\hat{s}_t^i$ is then given by:

$$\mu_t p_t^s = \beta \mathrm{E}_t \Bigg[F_\eta^-(\epsilon,\tau) \left(u_c^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t)(Y_t+\epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,d}\right)^{-\gamma} (1-\lambda) p_{t+1}^s \right)$$
$$+ \tilde{F}_\eta(\epsilon,\tau) \left(u_c^B(\hat{c}_{t+1}^{nd}(\epsilon,\tau), \hat{h}_t)(Y_t+\epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} p_{t+1}^s \right) \Bigg]$$

Houses Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of houses $\hat{h}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V^d_{t+1}}{\partial \hat{h}_t} = u^B_h(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t)$$
$$\frac{\partial V^{nd}_{t+1}}{\partial \hat{h}_t} = u^B_h(\hat{c}^{nd}_{t+1}(\epsilon), \hat{h}_t) - u^B_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t)\delta_H + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} p^h_{t+1}$$

The FOC for houses $\hat{h}^i_t$ is then given by:

$$\mu_t p^h_t = \Phi_h \frac{q^m_t \hat{m}^i_t}{(\hat{h}^i_t)^2} + \beta \mathrm{E}_t \Big[F^-_\eta(\epsilon, \tau) u^B_h(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t)$$
$$+ \tilde{F}_\eta(\epsilon, \tau) \left(u^B_h(\hat{c}^{nd}_{t+1}(\epsilon), \hat{h}_t) - u^B_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t)\delta_H + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} p^h_{t+1} \right) \Big]$$

Mortgages Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of houses $\hat{m}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V^d_{t+1}}{\partial \hat{m}_t} = 0$$
$$\frac{\partial V^{nd}_{t+1}}{\partial \hat{m}_t} = u^B_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t) x^i_t + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} (1-\delta) q^m_{t+1}$$

The FOC for shares $\hat{s}^i_t$ is then given by:

$$\mu_t q^m_t \left(1 - \Phi_m \frac{q^m_t}{p^m_t \hat{h}^i_t}\right) = \beta \mathrm{E}_t \left[\tilde{F}_\eta(\epsilon, \tau) \left(u^B_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t) x^i_t + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} (1-\delta) q^m_{t+1} \right) \right]$$

A.1.2 Market-Clearing Conditions and Aggregation

To calculate intermediary wealth and market clearing, we must integrate over the distribution of borrower shocks. First, note that identical choices by borrowers in per-wealth units mean that for any quantity g^i_t that is a function of borrower choices, we can express it is a product of the common per-wealth choice $\hat{g}_t$ and aggregate borrower wealth w^B_t:

$$\int_0^\ell g_t^i di = \hat{g}_t \int_0^\ell w_t^i di = \hat{g}_t w_t^B$$

Aggregate share of defaulting mortgages F_t^η is given by:

$$F_t^\eta = \int_0^\ell \mathbb{1}_d^i di = \sum_{\tau \in \{0,1\}} \sum_{\epsilon \in \mathcal{E}} \mathcal{P}_\epsilon(\epsilon_t^i = \epsilon) \mathcal{P}_\tau(\tau_t^i = \tau) F_\eta(\eta^*(\epsilon, \tau))$$

Aggregate per-unit mortgage payment x_t is given by:

$$x_t = \mathrm{E}_i[x_t^i | \eta^i \leq \eta^{*,i}(\epsilon_t^i, \mathbb{1}_\tau^i)]$$

For other quantities,

- Mortgages: $\int_\ell^1 m_t^I di = \int_0^\ell m_t^i di$ implies $M_t^I = \hat{m}_t W_t^B$
- Borrower Tree Shares: $\alpha = \hat{s}_t W_t^B$
- Houses: $\bar{H} = \hat{h}_t W_t^B$

Finally, the law of motion for aggregate borrower wealth is:

$$\begin{aligned} W_{t+1}^B &= \int_0^\ell w_{t+1}^i di \\ &= W_t^B \mathrm{E}_i \left[\tilde{F}_\eta((\epsilon_t^i, \mathbb{1}_\tau^i)) \hat{w}_{t+1}^{i,d}(\epsilon_t^i) + F_\eta((\epsilon_t^i, \mathbb{1}_\tau^i)) \hat{w}_{t+1}^{i,nd}(\epsilon_t^i, \mathbb{1}_\tau^i) \right] \end{aligned}$$

A.2 Banks

A.2.1 Problem

Banks are not subject to idiosyncratic shocks and are ex-ante identical. As a result, we can solve the problem for the representative *aggregate* bank. Denote aggregate quantities with capital letters. The bank's complete problem is given by:

$$V^I(W_t^I, \mathcal{Z}_t) = \max_{\text{Div}_t^I, D_t^I, M_t^I} \text{Div}_t^I + \text{E}_t\left[\mathcal{M}_{t+1}^S V^I(W_{t+1}^I, \mathcal{Z}_{t+1})\right] \tag{15}$$

subject to

$$W_t^I = \frac{D_t^I}{1+r_t^d} + q_t^m M_t^I + \text{Div}_t^I \tag{16}$$

$$W_{t+1}^I = (1-\nu)\mathcal{X}_{t+1} M_t^I + D_t^I \tag{17}$$

$$D_t \leq \xi\left(\kappa \bar{q}^m + (1-\kappa) q_t^m\right) M_t^I \tag{18}$$

where $\mathcal{X}_t$ is the aggregate mortgage payment per unit of mortgage debt given borrowers' choices:

$$\mathcal{X}_t = \tilde{F}_t^\eta(x_t + (1-\delta)q_t^m) + E_i\left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i)\frac{h_{t-1}^i}{M_{t-1}^I} p_t((1-\zeta) - \delta_h)\right]$$

Since default decisions do not depend on wealth levels and since housing choices $h_t^i = \hat{h}_t w_t^i$ are proportional to borrower wealth for all borrowers,

$$E_i\left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i) h_{t-1}^i\right] = E_i\left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i)\right] E_i\left[h_{t-1}^i\right] = F_t^\eta H_{t-1}^B = F_t^\eta \alpha_h$$

. As a result, the mortgage payoff can be written:

$$\mathcal{X}_t = \tilde{F}_t^\eta(x_t + (1-\delta)q_t^m) + F_t^\eta \frac{\alpha_h}{M_t^I} p_t((1-\zeta) - \delta_h)$$

A.2.2 First Order Conditions

Mortgages The FOC for mortgages M_t^I is given by:

$$q_t^m = \mu_t^L \xi\left(\kappa \bar{q}^m + (1-\kappa) q_t^m\right) + E_t\left[\mathcal{M}_{t+1}^S \mathcal{X}_{t+1}\right]$$

where μ_t^L is the Lagrange multiplier on the leverage constraint (18).

Deposits The FOC for deposits D_t^I is given by:

$$\frac{1}{1+r_t^d} = \mu_t^L + \mathrm{E}_t\left[\mathcal{M}_{t+1}^S\right]$$

Note that absent occasionally binding borrowing constraints $V_t^I = W_t^I$. But in their presence, this doesn't hold.

A.3 Savers

Likewise, we write and solve the representative saver's problem using aggregate quantities. For symmetry, we define saver wealth inclusive of their Lucas Tree shares and housing, even though neither is tradeable by them.

$$V^S(W_t^S, \mathcal{Z}_t) = \max_{C_t^S, E_t} u(C_t^S, H_t^S) + \beta E_t[V^S(W_{t+1}^S, \mathcal{Z}_{t+1})]$$

subject to

$$W_t^S = p_t^s S_t^S + p_t^h H_t^S + E_t p_t^e + C_t^S \tag{19}$$

$$W_{t+1}^S = S_t^S(p_{t+1}^s + Y_t) + H_t^S(p_{t+1}^h - \delta_h) + E_t(p_{t+1}^e + \mathrm{Div}_{t+1}^I) + \mathcal{R}_{t+1}^S \tag{20}$$

where $\mathcal{R}_{t+1}$ are (1) borrower costs of default, parametrized by λ, (2) banks' foreclosure costs, parametrized by ζ, and (3) banks' intermediation costs, parametrized by ν, rebated lump-sum:

$$\mathcal{R}_t^S = F_t^\eta\left(\lambda p_t^s \alpha + \zeta p_t^h \alpha_h\right) + \nu \mathcal{X}_t M_t^I$$

The first order condition for bank equity E_t is

$$p_t^e = \mathrm{E}_t\left[\beta\left(\frac{C_{t+1}^S}{C_t^S}\right)^{-\gamma}(\mathrm{Div}_{t+1} + p_{t+1}^e)\right]$$

which implies the saver's stochastic discount factor $\mathcal{M}^S_{t+1} = \beta\left(\frac{C^S_{t+1}}{C^S_t}\right)^{-\gamma}$.

Normalize the supply of bank shares E_t to 1. Then, iterating on both the bank's value function and the saver's FOC for bank equity, we get that $V^I_t = \text{Div}_t + p^e_t$.

A.4 Resource Constraint

In this section, we verify that aggregate consumption and housing investment are financed by the aggregate output of Lucas trees and by changes in the net deposit position of the economy.

Define aggregate borrower consumption in terms of conditional expectations of individual consumption:

$$\begin{aligned} C^B_t &= W^B_{t-1}\mathrm{E}_i\left[F_\eta(\eta^{*,i})\hat{c}^{i,nd}_t + \tilde{F}_\eta(\eta^{*,i})\hat{c}^{i,d}_t\right] \\ &= W^B_t\left(F^\eta_t\mathrm{E}_i\left[\hat{c}^{i,d}_t|\eta^i \leq \eta^{*,i}\right] + \tilde{F}^\eta_t\mathrm{E}_i\left[\hat{c}^{i,nd}_t|\eta^i > \eta^{*,i}\right]\right) \end{aligned}$$

From the consumption stage budget constraints:

$$\begin{aligned} \mathrm{E}_i\left[\hat{c}^{i,d}_t|\eta^i \leq \eta^{*,i}\right] &= \hat{s}_{t-1}\mathrm{E}_i\left[Y_t + \epsilon^i_t|\eta^i \leq \eta^{*,i}\right] + \hat{d}_{t-1} \\ \mathrm{E}_i\left[\hat{c}^{i,nd}_t|\eta^i > \eta^{*,i}\right] &= \hat{s}_{t-1}\mathrm{E}_i\left[Y_t + \epsilon^i_t|\eta^i > \eta^{*,i}\right] + \hat{d}_{t-1} - \hat{m}_{t-1}x_t - \delta_h\hat{h}_{t-1} - \mathrm{E}_i\left[\hat{a}^i_t|\eta^i > \eta^{*,i}\right] \end{aligned}$$

From the no-default branch wealth evolution equation, we get that intra-period savings $\hat{a}^i_t = \hat{w}^{i,nd}_t - p^h_t\hat{h}_{t-1} - p^s_t\hat{s}_{t-1} + (1-\delta)q^m_t\hat{m}_{t-1}$. Furthermore, observe that

$$\tilde{F}^\eta_t\mathrm{E}_i\left[Y_t + \epsilon^i_t|\eta^i > \eta^{*,i}\right] + F^\eta_t\mathrm{E}_i\left[Y_t + \epsilon^i_t|\eta^i \leq \eta^{*,i}\right] = Y_t + \mathrm{E}_i[\epsilon^i_t] = Y_t$$

Define aggregate borrower deposits $D^B_t = W^B_t\hat{d}_t$. Use market-clearing in Lucas trees and housing to write $W^B_t\hat{s}_t = \alpha$ and $W^B_t\hat{h}_t = \alpha_h$. Use market-clearing in mortgages to write $W^B_t\hat{m}_t = M^I_t$. Assembling,

$$\begin{aligned} C^B &= \alpha Y_t + D^B_{t-1} \\ &\quad + \tilde{F}^\eta_t\left[\alpha_h(p_h - \delta_h) + \alpha p^s_t - M^I_{t-1}\left(x_t + (1-\delta)q^m_t\right) - W^B_{t-1}\mathrm{E}_\tau\left[\hat{w}^{i,nd}_t|\eta > \eta^{*,i}\right]\right] \end{aligned}$$

Recall that $W_t^B = W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^i\right]$. We can break up the expectation as follows:

$$\mathrm{E}_i \left[\hat{w}_t^i\right] = \tilde{F}_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i}\right] + F_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i}\right]$$

Solving for the aggregate wealth of non-defaulters $\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | > \leq \eta^{*,i}\right]$,

$$\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i}\right] = W_t^B - W_{t-1}^B F_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i}\right]$$

Use the default-branch wealth evolution equation and market clearing in Lucas trees to substitute

$$W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i}\right] = (1-\lambda) p_t^s \alpha$$

Multiply the trading stage budget constraint by W_t^B and plug in market-clearing conditions to get

$$W_t^B = \frac{D_t^B}{1+r_t^d} - q_t^m M_t^I + p_t^h \alpha_h + p_t^s \alpha$$

Combining,

$$\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i}\right] = \frac{D_t^B}{1+r_t^d} + q_t^m M_t^I + p_t^h \alpha_h + p_t^s \alpha - F_t^\eta (1-\lambda) p_t^s \alpha$$

Plugging back into the expression for C^B,

$$\begin{aligned}
C^B &= \alpha Y_t + D_{t-1}^B - \frac{D_t^B}{1+r_t^d} + q_t^m M_t^I - p_t^h \alpha_h - p_t^s \alpha + F_t^\eta (1-\lambda) p_t^s \alpha \\
&\quad + \tilde{F}_t^\eta \left[\alpha_h (p_h - \delta_h) + \alpha p_t^s - M_{t-1}^I \left(x_t + (1-\delta) q_t^m\right)\right] \\
&= \alpha Y_t + D_{t-1}^B - \frac{D_t^B}{1+r_t^d} + q_t^m M_t^I - F_t^\eta (p_h \alpha_h + \lambda p_t^s \alpha) \\
&\quad - \tilde{F}_t^\eta \left[\delta_h \alpha_h + M_{t-1}^I \left(x_t + (1-\delta) q_t^m\right)\right]
\end{aligned}$$

This expression admits an economic interpretation. Borrowers earn income from their Lucas trees αY_t and deposits D_{t-1}^B. Those repaying their mortgages – a fraction $\tilde{F}_t^\eta$ – expend resources on housing maintenance $\delta_h\alpha_h$ and mortgage payments $M^I_{t-1}\left(\mathrm{E}_\tau\left[x^i_t|\eta > \eta^{*,i}\right] + (1-\delta)q^m_t\right)$. Those who default – a fraction $\tilde{F}^\eta_t$ – lose the value of their houses $p_h\alpha_h$ and a fraction λ of the value of their Lucas trees $p^s_t\alpha$. In the trading stage, they take out new mortgages $q^m_t M^I_t$ and make new deposits $\frac{D^B_t}{1+r^d_t}$.

Next, consider saver consumption. From the budget constraint and wealth evolution equation of savers,

$$C^S_t = S^S_{t-1}(p^s_t + Y_t) + H^S_{t-1}(p^h_t - \delta_h) + E_{t-1}(p^e_t + \mathrm{Div}^I_t) + \mathcal{R}^S_t - p^s_t S^S_t - p^h_t H^S_t - E_t p^e_t$$

Plug in market clearing conditions $E_t = 1$, $S^S_t = 1-\alpha$, $H^S_t = 1-\alpha_h$, to get

$$C^S_t = (1-\alpha)Y_t - (1-\alpha_h)\delta_h + \mathrm{Div}^I_t + \mathcal{R}^S_t$$

From the budget constraint for banks,

$$\mathrm{Div}^I_t = (1-\nu)\mathcal{X}_t M^I_{t-1} + D^I_{t-1} - \frac{D^I_t}{1+r^d_t} - q^m_t M^I_t$$

Plugging for Div^I_t and $\mathcal{R}_t$ and collecting terms,

$$C^S_t = (1-\alpha)Y_t - (1-\alpha_h)\delta_h + \mathcal{X}_t M^I_{t-1} + D^I_{t-1} - \frac{D^I_t}{1+r^f_t} - q^m_t M^I_t + F^\eta_t\left(\lambda p^s_t\alpha + \zeta p^h_t\alpha_h\right)$$

Next, subtitute the definition of $\mathcal{X}_t$:

$$\begin{aligned} C^S_t = (1-\alpha)Y_t + D^I_{t-1} - \frac{D^I_t}{1+r^d_t} - q^m_t M^I_t + F^\eta_t\lambda p^s_t\alpha \\ - (1-\alpha_h)\delta_h + \tilde{F}^\eta_t(x_t + (1-\delta)q^m_t)M^I_{t-1} + F^\eta_t p_t\alpha_h(1-\delta_h) \end{aligned}$$

Define aggregate deposits as $D_t = D^B_t + D^I_t$. Then, adding C^B_t and C^S_t and collecting terms, we get the resource constraint:

$$\underbrace{C^B_t + C^S_t}_{\text{Aggregate Consumption}} + \underbrace{\delta}_{\text{Housing Investment}} = \underbrace{Y_t}_{\text{Output}} + \underbrace{D_{t-1} - \frac{D_t}{1+r^d_t}}_{\Delta\text{Net Foreign Assets}}$$

A.5 Risk Sharing Measures

A.5.1 Complete Markets Benchmark

An unconstrained social planner chooses allocations for each agent that are proportional to the weight that the planner puts on the utility of that agent.

Define the social welfare problem:

$$\max_{\{\{(c_t^i, h_{t-1}^i)\}_{i=0}^1\}_{t=0}^\infty} E_0 \left[\int_{i=0}^1 \lambda_i \sum_{t=1}^\infty \beta^t \frac{\left((c_t^i)^{1-\theta}(h_t^i)^\theta\right)^{1-\gamma} - 1}{1-\gamma} di \right]$$

such that the resource constraints for each good, in each period and each state of the world are satisfied:

$$\int_0^1 c_t^i = Y_t \quad \forall t, s^t$$
$$\int_0^1 h_{t-1}^i = \bar{H} \quad \forall t, s^t$$

where every variable x_t is implicitly a function of the random variable s^t, denoting the history of the economy up to time t.

Assign μ_t and ν_t as Lagrange multipliers to each of the constraints at time t, history s^t, respectively. Use $\pi(s^t)$ to denote the density of the unconditional history distribution at a given s^t. The first order condition for consumption for agent i at time t are:

$$\lambda_i \beta^t \pi(s^t) \left((c_t^i)^{1-\theta}(h_{t-1}^i)^\theta\right)^{-\gamma} (1-\theta)(c_t^i)^{-\theta}(h_{t-1}^i)^\theta = \mu_t$$

The first order condition for housing for agent i at time $t-1$ are:

$$\lambda_i \beta^t \pi(s^t) \left((c_t^i)^{1-\theta}(h_{t-1}^i)^\theta\right)^{-\gamma} \theta(c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta-1} = \nu_{t-1}$$

Dividing them by each other, we get the optimal MRS between consumption and housing for a given state of the world, which is the same for all households:

$$\frac{c_t^i}{h_{t-1}^i} = \frac{1-\theta}{\theta}\frac{\mu_t}{\nu_{t-1}}$$

Substitute for housing in the consumption FOC:

$$h_{t-1}^i = \frac{\theta}{1-\theta}\frac{\mu_t}{\nu_{t-1}}c_t^i$$

$$\lambda_i\beta^t\pi^(s^t)\left((c_t^i)\left[\frac{\theta}{1-\theta}\frac{\mu_t}{\nu_{t-1}}\right]^\theta\right)^{-\gamma}(1-\theta)^{1-\theta}\theta^\theta = \mu_t^{1-\theta}\nu_{t-1}^\theta$$

Dividing the consumption FOCs for agents i and j at time t by each other, we get:

$$\frac{\lambda_i}{\lambda_j}\left(\frac{c_t^i}{c_t^j}\right)^{-\gamma} = 1$$

which means that the ratio of consumptions is constant over time and states of the world at:

$$\frac{c_t^i}{c_t^j} = \left(\frac{\lambda_i}{\lambda_j}\right)^{-1/\gamma}$$

Rewrite as:

$$c_t^i = \left(\frac{\lambda_i}{\lambda_j}\right)^{-1/\gamma} c_t^j$$

Integrate both sides with respect to i to get aggregate time t consumption:

$$C_t \equiv \int_0^1 c_t^i di = \lambda_j^{\frac{1}{\gamma}} c_t^j \int_0^1 \lambda_i^{-\frac{1}{\gamma}} di$$

Which implies that a given household's consumption c_t^j is a constant fraction of aggregate consumption C_t:

$$c_t^j = \frac{\lambda_j^{-\frac{1}{\gamma}}}{\int_0^1 \lambda_i^{-\frac{1}{\gamma}} di} C_t$$

The same argument applies to housing, i.e. it can be shown that the planner's optimal allocation of housing to agent j

$$h_{t-1}^{j} = \frac{\lambda_j^{-\frac{1}{\gamma}}}{\int_0^1 \lambda_i^{-\frac{1}{\gamma}} di} \bar{H}$$

is constant over time.

Since complete markets implement the planner allocation, this means that in a frictionless economy the volatility of the ratio of consumptions is zero. This likewise implies that each agent's consumption grows at the same rate. Formally, take the log:

$$\log c_t^i - \log c_t^j = -\frac{1}{\lambda} \left(\log \lambda_i - \log \lambda_j\right)$$

Let $\Delta \log c_t^i$ is defined as $\log c_t^i - \log c_{t-1}^i$. Then the log of the ratio of consumption growth rates is:

$$\begin{aligned}
\Delta \log c_t^i - \Delta \log c_t^j &\equiv (\log c_t^i - \log c_{t-1}^i) - (\log c_t^j - \log c_{t-1}^j) \\
&= (\log c_t^i - \log c_t^j) - (\log c_{t-1}^i - \log c_{t-1}^j)
\end{aligned}$$

Then in complete markets, it must be true that

$$\mathcal{R}_{ij} = \text{Var}_0 \left[\Delta \log c_t^i - \Delta \log c_t^j\right] = 0$$

We refer to $\mathcal{R}_{ij}$ as a measure of "internal" risk sharing. In an incomplete markets economy, $\mathcal{R}_{ij} \geq 0$ and $\mathcal{R}_{ij}$ serves as a measure of risk sharing between households, with lower values denoting better risk sharing.

A.5.2 Complete Markets Open Economy

The open economy version of the complete markets model is similar to the closed economy version, except that the planner can now trade a risk-free bond with the rest of the world. The planner's problem is:

$$\max_{\{\{(c_t^i,h_{t-1}^i)\}_{i=0}^1,b_t\}_{t=0}^\infty} \mathrm{E}_0\left[\int_{i=0}^1 \lambda_i \sum_{t=1}^\infty \beta^t \frac{\left((c_t^i)^{1-\theta}(h_{t-1}^i)^\theta\right)^{1-\gamma}-1}{1-\gamma} di\right]$$

such that

$$\int_0^1 c_t^i + \frac{b_t}{1+r_t^d} = Y_t + b_{t-1} \quad \forall t, s^t$$
$$\int_0^1 h_{t-1}^i = \bar{H} \quad \forall t, s^t$$

The derivations above still hold. But now there is an additional choice variable of the planner. Bonds $b_t(s^t)$ show up in the resource constraint for t, s^t and in the resource constraints for all t, s^{t+1} that are reachable from s^t. Denote this set of possible states as $s_{t+1}|s^t$ and the . Then the additional first order condition for the bond is:

$$\frac{\mu_t}{1+r_t^d}\pi(s^t) = \int_{s_{t+1}|s^t} \pi(s^{t+1})\mu_{t+1}$$

Rearranging,

$$1 = (1+r_t^d)\int_{s_{t+1}|s^t} \pi(s_{t+1}|s^t)\frac{\mu_{t+1}}{\mu_t} = (1+r_t^d)\mathrm{E}_t\left[\frac{\mu_{t+1}}{\mu_t}\right]$$

where $\pi(s_{t+1}|s^t)$ denotes the conditional density of s_{t+1} given s^t, and where the second equality stems from the definition of a conditional expectation with $E_t[\cdot]$ denoting $\mathrm{E}[\cdot|s^t]$.

Plug in the FOC for consumption for the multipliers:

$$1 = (1+r_t^d)E_t\left[\left(\frac{c_{t+1}^i}{c_t^i}\right)^{-\gamma(1-\theta)-\theta}\left(\frac{h_t^i}{h_{t-1}^i}\right)^{\theta(1-\gamma)}\right]$$

Recall that for any agent, the optimal housing allocation is constant and the growth rate of consumption is equal to the aggregate consumption growth rate. Then the above equation simplifies to:

$$1 = (1 + r_t^d) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right]$$

The problem admits aggregation, i.e. the planner's optimal choice of bonds is independent of the resource allocation problem.

Take logs

$$0 = \log(1 + r_t^d) + \log E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right]$$

and define

$$\mathcal{R}_{agg} = \text{Var}_0 \left[\log(1 + r_t^d) + \log E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right] \right] \geq 0$$

as the "external" risk sharing measure. In complete markets, $\mathcal{R}_{agg} = 0$, while in incomplete markets larger values of $\mathcal{R}_{agg}$ indicate worse risk sharing between households in the economy and the rest of the world.

A.5.3 Internal Risk Sharing in our Model

In our model, there are two kinds of households: borrowers with consumption denoted by c_t^i and savers, with consumption denoted by c_t^S and identical across all savers. Let $C_t^B = \int_0^\ell c_t^i$ denote aggregate borrower consumption and $C_t^S = (1 - \ell) c_t^s$ denote aggregate saver consumption.

Borrowers are unconditionally identical, meaning internal risk sharing is summarized fully by two risk-sharing measures $\mathcal{R}_{iB}$ and $\mathcal{R}_{BS}$, where $\mathcal{R}_{iB}$ is the variance of the ratio of consumption growth rates between borrower i and the aggregate borrower, and $\mathcal{R}_{BS}$ is the variance of the ratio of aggregate consumption growth rates between borrowers and savers.

Recall, we can write borrower i's consumption at time t, c_t^i, as the product of borrower consumption per unit of wealth $\hat{c}_t^i$ and borrower wealth at time $t-1$, w_{t-1}^i. Consumption per unit of wealth only depends on the identity of the borrower i through the realizations of iid shocks to $\mathcal{S}_t^i = (\epsilon_t^i, \tau_t^i, \eta_t^i)$.

Write the log growth rate of borrower i's consumption as:

$$\Delta \log c_t^i = \log \hat{c}_t(\mathcal{S}_t^i) - \log \hat{c}_{t-1}(\mathcal{S}_{t-1}^i) + \log \hat{w}_{t-1}(\mathcal{S}_{t-1}^i)$$

where $\hat{w}_{t-1}(\mathcal{S}_{t-1}^i)$ represents the growth rate in wealth $\Delta \log w_{t-1}^i$, which also depends on the identity of the borrower i only through the realiations of iid shocks.

The definition of $\mathcal{R}_{iB}$ is $\mathrm{Var}_0[\Delta \log c_t^i - \Delta \log C_t^B]$. Using the law of total variance,

$$\mathcal{R}_{iB} = \mathrm{Var}_0\left[\mathrm{E}_t\left[\Delta \log c_t^i - \Delta \log C_t^B\right]\right] + \mathrm{E}_0\left[\mathrm{Var}_t\left[\Delta \log c_t^i - \Delta \log C_t^B\right]\right]$$

where the conditional moments Var_t and E_t are taken cross-sectionally with respect to realizations of idiosyncratic shocks. Simplifying,

$$\mathcal{R}_{iB} = \mathrm{Var}_0\left[\mathrm{E}_t\left[\Delta \log c_t^i\right] - \Delta \log C_t^B\right] + \mathrm{E}_0\left[\mathrm{Var}_t\left[\Delta \log c_t^i\right]\right]$$

Finally, $\mathcal{R}_{BS}$ is defined as $\mathrm{Var}_0[\Delta \log C_t^B - \Delta \log C_t^S]$.

B Calibration Details

B.1 Mortgage Payments and Duration

Recall, a fixed rate mortgage issued at time 0 pays $\iota_f + \delta \bar{q}^m$ in the first period, $(1-\delta)(\iota_f + \delta \bar{q}^m)$ in the second period, and so on.

Define a mortgage yield to maturity ytm as the discount rate which discounts mortgage cash flows to the mortgage price.

It is easy to see that

$$q^m(ytm) = \sum_{t=1}^{\infty}(1-\delta)^{t-1}\frac{\iota_f + \delta \bar{q}^m}{(1+ytm)^t} = \frac{\iota_f + \delta \bar{q}^m}{ytm + \delta}$$

and that therefore the mortgage is priced to par when $ytm = \iota_f$. We calibrate the model by setting ι_f to the steady state equilibrium ytm, thus ensuring that $\bar{q}^m = 1$.

Define duration of the fixed rate mortgage as the negative semi-elasticity of the mortgage price with respect to the yield to maturity. We have:

$$-\left.\frac{\partial q^m/q^m}{\partial ytm}\right|_{\iota_f} = \frac{1}{\iota_f + \delta}$$

An adjustable rate mortgage makes fixed rate payments $\iota_\tau + \delta\bar{q}^m$, $(1-\delta)(\iota_\tau + \delta\bar{q}^m)$, etc. until (stochastic) reset time τ and pays adjustable payments $(1-\delta)^{\tau-1}\left(r^f_\tau + \iota_a + \delta\bar{q}^m\right)$, $(1-\delta)^\tau\left(r^f_{\tau+1} + \iota_a + \delta\bar{q}^m\right)$, etc. after.

In the baseline calibration, we set $\iota_\tau = \iota_f$ and $\iota_a = \iota_f - \bar{r}$. Then the per-remaining-balance portion of the ARM payment can be written as $\iota_f + (r^f_t - \bar{r})$.

Let $\mathbb{1}^{\text{fixed}}_{1,\ldots,t}$ be a random indicator variable equal to 1 if the mortgage is still in the fixed/teaser stage at time t. So the expected ARM cash flow at time t is then given by:

$$\mathrm{E}_0\left[(1-\delta)^{t-1}\left[\mathbb{1}^{\text{fixed}}_{1,\ldots,t}(\iota_f + \delta\bar{q}^m) + \left(1 - \mathbb{1}^{\text{fixed}}_{1,\ldots,t}\right)\left(\iota_f + r^f_t - \bar{r} + \delta\bar{q}^m\right)\right]\right]$$

Collecting terms,

$$(1-\delta)^{t-1}\left((\iota_f + \delta\bar{q}^m) + \mathrm{E}_0\left[\left(1 - \mathbb{1}^{\text{fixed}}_{1,\ldots,t}\right)\left(r^f_t - \bar{r}\right)\right]\right)$$

Let $\mathbb{1}^{\text{adj}}_s$ be an indicator equal to 1 if a mortgage that resets from fixed to floating at time s. Then

$$\mathbb{1}^{\text{fixed}}_{1,\ldots,t} = \prod_{s=1}^{t}\left(1 - \mathbb{1}^{\text{adj}}_s\right)$$

The realization of morgage resets $\mathbb{1}^{\text{adj}}_s$ are independent of each other, and also independent from the realizaton of future indexation rates r^f_t. Every period, the probability that a mortgage still in the teaser stage resets to the adjustable stage, i.e. $\mathrm{E}_0[\mathbb{1}^{\text{adj}}_s]$ is π_τ. So the expected ARM cash flow at time t can be written as:

$$(1-\delta)^{t-1}\left((\iota_f + \delta\bar{q}^m) + \left(1 - (1-\pi_\tau)^t\right)\left(\mathrm{E}_0[r^f_t] - \bar{r}\right)\right)$$

Define yield to maturity ytm for an adjustable rate mortgage as the (risk-neutral) discount rate that discounts expected ARM cash flows to the mortgage price. We have:

$$\begin{aligned} q^m(ytm) &= \sum_{t=1}^{\infty}(1-\delta)^{t-1}\frac{\iota_f+\delta\bar{q}^m}{(1+ytm)^t}+\sum_{t=1}^{\infty}(1-\delta)^{t-1}\left(1-(1-\pi_\tau)^t\right)\frac{\mathrm{E}_0[r_t^f]-\bar{r}}{(1+ytm)^t} \\ &= \frac{\iota_f+\delta\bar{q}^m}{ytm+\delta}+\left(\mathrm{E}_0[r_t^f]-\bar{r}\right)\left(\frac{1}{ytm+\delta}-\frac{1-\pi_\tau}{1+ytm-(1-\delta)(1-\pi_\tau)}\right) \end{aligned}$$

The first term is the price of the FRM. The second term is equal to zero because $\mathrm{E}_0[r_t^f]=\bar{r}$ in equilibrium. In steady state, risk neutral and physical measures coincide because there are no risk premia, so $ytm=\iota_f$ prices the mortgage to par ($\bar{q}^m=1$) just as before.

To define the duration of the adjustable mortgage, write the price of the mortgage as a function of ytm and future rate r:

$$q^m(ytm,r)=\frac{\iota_f+\delta}{ytm+\delta}+(r-\bar{r})\left(\frac{1}{ytm+\delta}-\frac{1-\pi_\tau}{ytm-(1-\delta)(1-\pi_\tau)}\right)$$

Then, consider the change in mortgage price due to a *parallel* shift in all interest rates, i.e. when $\frac{\partial r}{\partial ytm}=1$. Formally, duration is given by $-\frac{\partial q^m(ytm,r(ytm))/q^m(ytm,r(ytm))}{\partial ytm}$ evaluated at $ytm=\iota_f$ and r such that $q^m(\iota_f,r)=0$, which means $r=\bar{r}$. Taking derivatives,

$$\begin{aligned} \frac{\partial q^m(ytm,r(ytm))}{\partial ytm} = &-\frac{1}{ytm+\delta}+(r-\bar{r})\frac{\partial}{\partial ytm}\left(\frac{1}{ytm+\delta}-\frac{1-\pi_\tau}{1+ytm-(1-\delta)(1-\pi_\tau)}\right) \\ &+\frac{\partial r}{\partial ytm}\left(\frac{1}{ytm+\delta}-\frac{1-\pi_\tau}{1+ytm-(1-\delta)(1-\pi_\tau)}\right) \end{aligned}$$

Imposing $r=\bar{r}$ leads the second term to drop out. Imposing $ytm=\iota_f$ and $\frac{\partial r}{\partial ytm}=1$, we get:

$$-\frac{\partial q^m(ytm,r(ytm))}{\partial ytm}\frac{1}{q^m(ytm,r(ytm))}\bigg|_{ytm=\iota_f,r=\bar{r}}=\frac{1-\pi_\tau}{1+\iota_f-(1-\delta)(1-\pi_\tau)}$$

which is the steady-state contractual duration of the ARM with a reset probability of π_τ.

Note that at $\pi_\tau=0$, the expression simplifies to $1/(\iota_f+\delta)$, which is the FRM duration. At $\pi_\tau=1$, i.e. a reset occuring with probability at the time of the first cash flow (one year after issuance), the duration is 0.

C Two-Period Model

In this section, we illustrate the main differences in the allocation of risk between fixed-rate mortgage (FRM) and adjustable-rate mortgage (ARM) regimes using a two-period model with time indexed by $t = 0, 1$.

C.1 Borrowers

A continuum of borrowers indexed by i is endowed with equal initial wealth w_0 and have preferences over $t = 1$ consumption, residual $t = 1$ wealth, and housing.

Borrowers allocate their initial wealth to deposits d_i, mortgages m_i, houses h_i, and Lucas trees s_i to maximize $E[u(c_{1,i}, w_{1,i}, h_i)]$, the expectation of their $t = 1$ utility kernel given by:

$$U(c, w, h) = (1 - \beta) \log c + \beta \log w + \beta\theta \log h \tag{21}$$

subject to the $t = 0$ budget constraint:

$$w_0 = p_0^h h_i + p_0^s s_i + q d_i - q_0^m m_i$$

Because ex-ante borrowers are identical in terms of their wealth and distributions of $t = 1$ shocks, they will make identical portfolio decisions, and so we will drop i subscripts on h, s, d, and m.

$t = 1$ consists of two subperiods. In the first subperiod ("morning"), borrowers are exposed to an *idiosyncratic* income shock $\epsilon_i \in \{\epsilon_L, \epsilon_H\}$ to the yield of their Lucas tree y_i, such that $y_i = 1 + \epsilon_i$.

Borrowers use their liquid assets – income and deposits – to make mortgage payments and consume. Any excess liquid assets can be carried over into the second subperiod ("afternoon") and constitute remaining borrower wealth along with the value of their housing, trees, and mortgages.

Crucially, borrowers cannot trade these illiquid assets until the afternoon, meaning they cannot obtain additional liquidity in the morning to finance their mortgage payments and consumption. Their only way to increase liquid assets at $t = 1$ is to default on the mortgage payment. To characterize the borrowers' default decision, consider the two branches of their decision tree.

If borrowers do not default, they solve a simple consumption-savings problem:

$$\max_{a_i \geq 0} (1-\beta)\log(\ell_i - a_i) + \beta \log(\omega_i + a_i)$$

where

$$\ell_i = y_i s + d - xm$$

is their stock of liquid assets after making the mortgage payment xm_i, and

$$\omega = p_1^s s + p_1^h h - q_1^m (1-\delta) m$$

is their illiquid wealth, consisting of Lucas trees, houses, and remaining fraction $1-\delta$ of their mortgage balance, all at $t = 1$ prices.

The borrowers per-unit mortgage payment x is given by $\iota + \delta$, where ι represents the interest payment and δ represents the principal payment. In an FRM regime, ι is fixed, while in an ARM regime, $\iota = r + \iota_a$ is a fixed spread over the prevailing short rate r. Therefore, shocks to r constitute the second, *aggregate*, source of risk in the economy.

The optimal unconstrained choice of intraperiod savings equates the marginal utility of consumption $(1-\beta)/c_i$ with the marginal utility of wealth β/w_i, yielding the following expression for intraperiod savings:

$$a_i^* = \max\{0, \beta\ell_i - (1-\beta)\omega\}$$

is increasing in liquid assets and decreasing in illiquid wealth, such that when liquid assets are low – either because of a bad income realization or because ARM mortgage payments increase due to a rate hike – the borrower is constrained. She would like to borrow from her illiquid wealth to finance additional consumption at the expense of future wealth, but she cannot do so directly. The only way to accomplish this is to default, gaining liquid assets xm at the expense of losing housing wealth p_1^h and a fraction of non-housing wealth $\lambda p_1^s s$, as well as extinguishing the remaining principal $(1-\delta)q_i^m m$. A positive value of λ represents pecuniary costs of default in addition to foreclosure, e.g. partial recourse, costs of being locked out of the financial market for some amount of time, etc.

Additionally, defaulting comes with a non-pecuniary stochastic default penalty $\eta_i \sim F_\eta$, such that a household defaults iff

$$u(c_i^{nd}, w_i^{nd}, h) < u(c_i^d w_i^d, h) + \eta_i$$

where the no-default consumption and wealth are given by

$$\begin{aligned} c_i^{nd} &= y_i s + d - xm - a_i^* \\ w_i^{nd} &= a_i^* + p_1^s s + p_1^h h - q_1^m (1-\delta) m \end{aligned}$$

and the default consumption and wealth are given by

$$\begin{aligned} c_i^d &= y_i s + d \\ w_i^d &= (1-\lambda) p_1^s s \end{aligned}$$

Borrowers optimally default if the realization of η_i is above a threshold value η_i^*, which depends on both idiosyncratic and aggregate shocks and is given by

$$\eta_i^* = (1-\beta)\log\frac{c_i^{nd}}{c_i^d} + \beta\log\frac{w_i^{nd}}{w_i^d}$$

implying a survival probability $F_i = F(\eta_i^*)$.

For a borrower with a large stock of liquid assets s and d, the ratio of no-default to default consumption is close to 1, and so her default decision will be largely strategic, i.e. based on the change in wealth due to default. When λ is low, such a borrower will "send in the keys" to a property underwater.

But when the stock of liquid assets is smaller, the driver of default will be the liquidity borrowers can unlock in high marginal utility states by foregoing the mortgage payment xm, even if this default leads to lower future wealth. Empirical evidence, e.g., Ganong and Noel (2022), suggests that this is the primary reason borrowers default. In our model, a rise in ARM mortgage payments due to interest rate hikes together with interaction effects with drivers of strategic default can lead to significant amplification.

We are now ready to characterize the borrowers' $t = 0$ problem. Denoting the Lagrange multiplier on (shadow value of relaxing) the budget constraint by μ, we can write the Euler equation for deposits as:

$$q\mu = \mathrm{E}\left[(1-F_i)\frac{1-\beta}{c_i^{nd}} + F_i\frac{1-\beta}{c_i^d}\right]$$

The marginal cost of deposits is given by $q\mu$, while the marginal benefit is given by the expected marginal utility of consumption, with the expectation taken over aggregate interest rate shocks, idiosyncratic income shocks, and idiosyncratic default penalty shocks, which enter the problem exclusively through the default probabilities F_i that they imply.

The Euler equation for houses equates the marginal cost of housing $p_0^h\mu$ against the marginal benefit, which consists of the user cost $\beta\theta/h$ and the marginal contribution of housing to wealth, which borrowers receive only if they do not default:

$$p_0^h \mu = \mathrm{E}\left[\frac{\beta\theta}{h} + \beta(1-F_i)\frac{p_1^h}{w_i^{nd}}\right]$$

By obtaining a mortgage, borrowers relax their budget constraint by q_0^m (worth $q_0^m \mu$ to them) at $t = 0$. The marginal cost at $t = 1$ consists of the two terms. First, mortgage payments enter the marginal utility of consumption. Second, the remaining mortgage balance enters the marginal utility of wealth. Both terms apply only if the borrower does not default:

$$q_0^m \mu = \mathrm{E}\left[(1-F_i)\left(\frac{1-\beta}{c_i^{nd}}x + \frac{\beta}{w_i^{nd}}(1-\delta)q_i^m\right)\right]$$

Finally, Lucas trees have a utility cost of $p_0^s \mu$ and yield marginal benefits of all four types – consumption and wealth in both no-default and default branches:

$$p_0^s \mu = \mathrm{E}\left[(1-F_i)\left(\frac{1-\beta}{c_i^{nd}}y_i + \frac{\beta}{w_i^{nd}}p_1^s\right) + F_i\left(\frac{1-\beta}{c_i^d}y_i + \frac{\beta}{w_i^d}(1-\lambda)p_1^s\right)\right]$$

To close the borrower side of the model, we assume that houses and Lucas trees are in fixed unit supply $H = \int h di = S = \int s di = 1$.

C.2 Lenders

Lenders are perfectly competitive and risk-neutral financial intermediaries (an assumption we will relax in the fully dynamic model). They raise funds in the form of deposits D^I at price q from borrower households as well as in wholesale markets. They use these funds to make mortgage loans M^I at price q^m to households.

Each performing mortgage yields a $t = 1$ cash flow of x to the lender, as well as having a remaining ex-payment value of $(1-\delta)q_1^m$. When borrowers default, lenders foreclose on the house, yielding a per-house value $p_1^h(1-\zeta)$ net of foreclosure costs $\zeta \geq 0$. The remaining mortgage balance is extinguished.

Let $F_I = \int F_i di$ denote the aggregate default rate (taking an expectation over idiosyncratic income realizations of borrowers). Then the total $t = 1$ of the lenders mortgage portfolio is:

$$\mathcal{X} M^I = (1-\nu)\left[(1-F_I)(x + (1-\delta)q_1^m)M^I + F_I p_1^h H(1-\zeta)\right]$$

where $\nu \geq 0$ denotes the lenders' operating costs as a fraction of the mortgage portfolio. The lender's $t = 0$ problem is to maximize their profit:

$$\max_{D^I, M^I} \mathrm{E}[(1-\nu)\mathcal{X} M^I - D^I]$$

subject to the budget constraint $qD^I = q_0^m M^I$.

Competition between lenders yields a zero-profit condition:

$$q(1-\nu)\mathrm{E}[\mathcal{X}] = q_0^m$$

C.3 Equilibrium

To close the model, we assume that outside investors supply short-term funding elastically at exogenous price q. We also assume exogenous $t = 1$ asset prices p_1^h, p_1^s and q_1^m. This allows us to directly vary the sensitivity of these prices to interest rate shocks r and thus to decompose the effects of these rate shocks onto default rates and lender profits into cash flow effects on mortgage payments x and valuation effects through asset prices.

Given these prices and initial endowment w_0, the competitive equilibrium is defined as a set of time 0 portfolio choices s, h, d, m, D^I, M^I, time 0 prices q_0^m, p_0^h, p_0^s, time 1 consumption and intraperiod savings decisions $\{c_i^{nd}, a_i, c_i^d\}$, and time 1 default decisions $\mathbb{1}_i$ for each realization of idiosyncratic and aggregate shocks such that households solve their optimization problems as characterized by the optimality conditions above, lenders satisfy the zero-profit condition and budget constraint, and markets clear: $h = 1$, $s = 1$, $m = M^I$.

To discipline our characterization, we proceed in two steps. First, we solve a "steady state" of the model, in which we assume constant interest rates, i.e. $\bar{r} = \frac{1}{q} - 1$, and constant prices $q_0^m = q_1^m$, $p_0^h = p_1^h$, $p_0^s = p_1^s$. Second, we solve four different models with interest rate shocks.

1. FRM (Constant Prices): Mortgage payment x is fixed at $\bar{r} + \iota + \delta$, with ι normalized such that time 0 mortgage price $q_0^m = 1$. Since FRMs are a long duration asset, time 1 mortgage prices are inversely proportional to interest rates: $q_1^m = \frac{\bar{r}+\iota+\delta}{r+\iota+\delta}$, with δ governing the duration and hence sensitivitity of mortgage prices to rate shocks. Unlike mortgage prices, house prices p_1^h and Lucas tree price p_1^s remain fixed at p_0^h and p_0^s, respectively, i.e., in this economy we assume no pass-through of rate shocks to real asset prices. With payments and real asset prices remaining fixed, the only effect of interest rate shocks in this economy is on the market value of mortgages.

2. FRM: The mortgage market is the same as in (1). However, we now allow house and Lucas tree prices to vary with interest rates. Both assets can be thought of as perpetuities, and hence a risk-neutral expectation of their cash flows can be written as $p_0^j \bar{r}$ for $j \in \{h, s\}$. After a change in interest rates, the new present value of these cash flows is $p_0^j \frac{\bar{r}}{r^j}$, where $r^j = (1 - \phi_j)\bar{r} + \phi_j r$ is the discount rate appropriate for asset j. Here, the parameter ϕ_j governs the degree of interest rate pass-through to asset j. $\phi_j = 0$ corresponds to the economy described in (1), while $\phi_j > 0$ implies that asset prices fall when interest rates rise. Because asset prices may affect default rates and lender profits, we renormalize the mortgage rate spread ι to ensure that q_0^m remains at 1.

3. ARM (Constant Prices): Mortgage payment x is now a spread over the short rate, $x = r + \iota + \delta$. As in (1), we assume no pass-through of rate shocks to real asset prices. Moreover, because mortgage coupons adjust with rates, the mortgage duration is now 0, so $q_1^m = q_0^m$ for all realizations of r. The only effect of interest rate shocks in this economy is on the mortgage payment. Like in (2), we renormalize ι to ensure $q_0^m = 1$.

4. ARM: The mortgage market is the same as in (3) and real asset prices respond as in (2). As in the other economies, ι is renormalized to ensure $q_0^m = 1$.

C.4 Numerical Example Parametrization

The effect of interest rates on mortgage markets and financial stability depends on the mortgage regime and the degree to which rate shocks are passed through to the prices of real assets. To illustrate this, we consider a numerical example. For most parameters, we choose values consistent with the calibration of the dynamic model in the subsequent section. For other parameters for which the two-period model provides clearer guidance, we provide a rough calibration as follows:

Interest Rates The time 0 interest rate is $\bar{r} = 0.01$, implying a bond price q of approximately 0.99. Time 1 interest rates are normally distributed with a mean of $\bar{r}$ and a standard deviation of 0.01.

Households Household discount factor β is 0.985. Setting it to a value below q implies that absent liquidity constraints, household would not hold deposits. Borrowers value housing services at 10% of their non-housing consumption. A negative income shock leads to a 3/4 drop in income and occurs with probability of 5%. This unlikely but sharp decline in income represents the liquidity consequences of losing a job or, e.g., incurring a large medical expense. In a two-period model with CRRA preferences, initial wealth w_0 is a key determinant of portfolio choices. We set w_0 to produce a mortgage loan-to-value ratio of 80% in the FRM (Fixed Prices) economy, keeping it constant across our experiments so that results can be comparable.

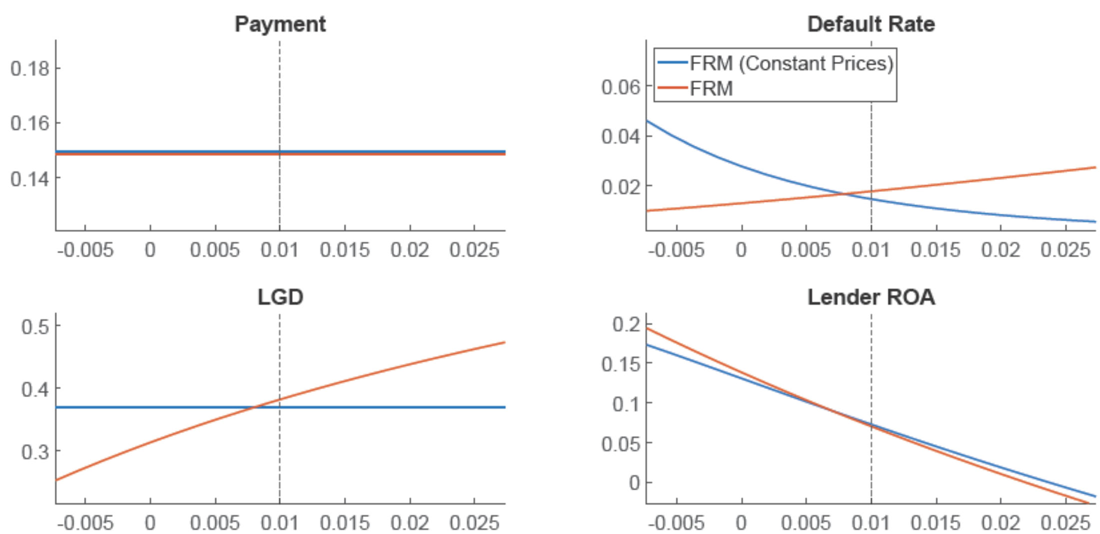
Default The pecuniary penalty of default λ is 0.1, meaning that households' future income declines by 10% as a result of default. Utility costs of default are normally distributed with mean 0 and standard deviation of 0.2125. Together, these parameters imply default rates of 1.6%-2.2% in line with recent empirical estimates.

Lenders Lenders' operating costs ν are set to 0.06, and foreclosure costs ζ are 0.5. These parameters directly effect mortgage rates and losses given default. The mortgage duration δ is set to 0.07. At mortgage rates implied by ν, this value of δ yields a FRM mortgage duration of approximately 7 years, consistent with the effective duration of mortgages in the US.

C.5 Two-Period Model Results

Figures 13 and 14 highlight the key intuition from the model, by showing how selected outcome variables in $t = 1$ respond to interest rate shocks. Figure 13 plots the responses for the two FRM economies (1) with constant asset prices and (2) with asset prices responding to rate shocks, while Figure 14 shows the results for the two ARM economies (3) and (4).

Figure 13: One-Period Outcomes in Response to Interest Rate Shock: FRM



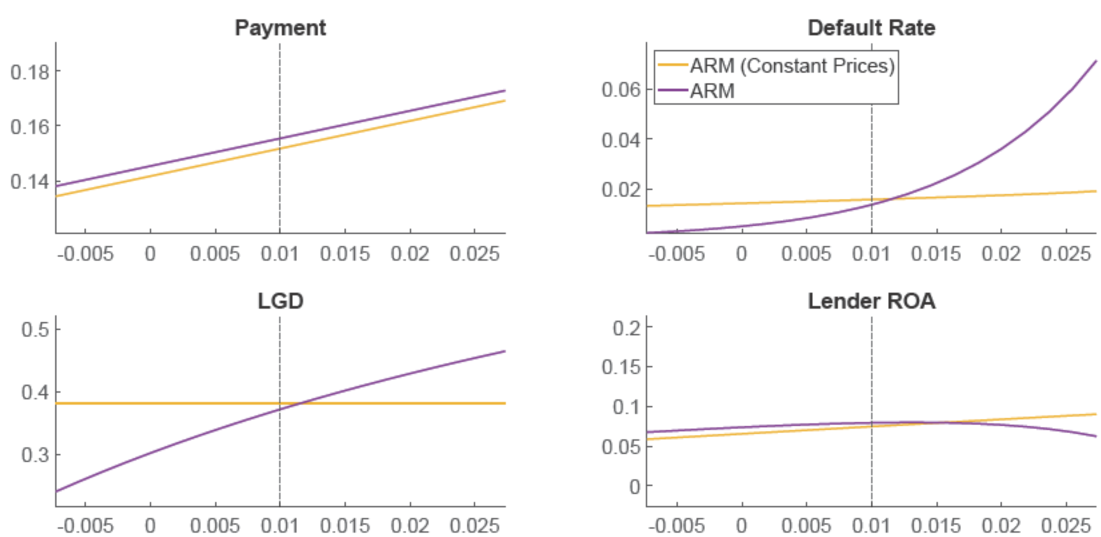
Notes: This figure shows the $t = 1$ responses to a range of interest rate realizations, for mortgage payments ("Payment"), losses given default ("LGD"), mortgage default rates ("Default Rate"), and lender return on assets ("Lender ROA"). The vertical dashed line represents the mean interest rate $\bar{r}$.

First, consider the FRM economies in Figure 13. Intuitively, payments do not respond to rate shocks as a key feature of fixed-rate mortgage contracts, and so borrowers' liquidity is unaffected. Default rates respond to interest rate shocks only to the extent the borrower's strategic incentives are altered. A drop in the mortgage price due to higher rates makes default less attractive — prevailing mortgage rates are higher, making the borrower's existing mortgage with a lower fixed-rate more valuable, which the borrower would have to give up in case of default. In the FRM economy with constant house and Lucas tree prices, this leads to lower default rates as interest rates rise. Loss given default (LGD) remains unchanged, leading to lower credit losses for lenders. Fixed-rate mortgages are a long-duration asset, and so the value of surviving mortgages decreases with interest rates. The increase in discount rates offsets the increase in cash flows coming from slightly lower defaults, leading to a lower return on the lender's mortgage portfolio when rates increase. The reverse is true based on the same mechanisms in the case of a rate cut.

In the FRM economy where asset prices are flexible, house and Lucas tree prices fall when rates rise. Thus there are two forces affecting default incentives. A rise in morgage rates still makes it valuable to hold on to a below-market-rate mortgage. But now, higher discount rates mean that the price of assets lost in default — the house, and a fraction of future income — are lower, making default more attractive. The two forces largely offset each other. In our numerical example, on net, the latter force prevails, leading to a small increase in default rates due to rate hikes.

What does this mean for lenders? Consider losses given default. With fixed prices, LGD is unchanged by rate shocks. When prices are flexible, rate hikes lower the value of collateral and thus increase losses given default. Rather than hedging interest rate risk in long-duration FRMs, credit risk now amplifies it, leading to a steeper relationship between interest rate risk and lenders ROA.

Figure 14: One-Period Outcomes in Response to Interest Rate Shock: ARM



Notes: This figure shows the $t = 1$ responses to a range of interest rate realizations, for mortgage payments ("Payment"), losses given default ("LGD"), mortgage default rates ("Default Rate"), and lender return on assets ("Lender ROA"). The vertical dashed line represents the mean interest rate $\bar{r}$.

The economies with ARMs look considerably different in Figure 14. Even if asset prices stay fixed at t=1, the rate hike-driven increase in mortgage payments causes an increase in defaults as a larger fraction of households cannot afford the higher payments. In contrast, from the lender's perspective, the value of now short-duration surviving mortgages remains constant, as does LGD, since prices are fixed. Their total return responds to rates entirely through cash flows. Higher payments collected on performing loans outweigh the increase in defaults, and returns increase in rates.

Comparing FRMs with ARMs holding prices fixed conveys the traditional FRM vs. ARM intuition. Under the FRM regime, rate hikes benefit households who are protected from rate rises, at the expense of lenders, while this is reversed in the ARM economy.

However, this simple intuition becomes more nuanced when we allow prices to adjust in the economy. In the ARM economy where prices respond to shocks, in response to a rate hike, not only do borrower payments go up, but the cost of default also goes down in present value terms. Moreover, there is no offsetting mortgage rate value channel, i.e. unlike with fixed-rate mortgages, existing borrowers do not benefit from holding a low-rate mortgage in a high-rate environment. As a result, default rates increase substantially, as do losses-given default.

While the lender still benefits from higher payments collected on performing loans, the increase in losses due to higher default rates and LGD leads to a lower net return if the rate hike is large enough. However, the source of these losses is markedly different: while FRM lenders experience losses stemming from interest rate risk, ARM lenders experience losses primarily stemming from credit risk induced by rate rises. The magnitude of these losses depend both on the size of the interest rate shock but also on the level of household debt. An alternative parametrization in which the $t = 0$ LTVs are closer to 100% than to 80% can generate loss-driven low returns on mortgages of the same magnitude as the rate-driven low returns in the FRM economy. In work going forward, we will quantitatively evaluate under what conditions one force dominates the other from a financial stability perspective.

The discussion above also abstracts away from lenders' funding costs. The change in ROA is an upper bound on their unlevered return on equity (ROE) because it represents the case where funding costs are unchanged (e.g., because lenders have a high degree of market power in deposits). The increase in interest income collected by ARM lenders gives them a negative exposure to interest rate risk, i.e. negative duration. Were their funding costs to increase one for one with rates, the asset and liability effects would offset each other, and their portfolio would be immunized, leaving borrowers to bear all the interest rate risk. However, the last set of results makes it clear that even as ARMs immunize lenders from interest rate risk, they increase their exposure to credit risk.

The quantitative model we develop in the paper allows us to explore these trade-offs in a more realistic setting, where lenders face funding costs and households face a broader set of risks. We will also be able to evaluate the implications of these trade-offs for optimal monetary policy.