Demand-Based Expected Returns

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Abstract

We develop a framework to extract heterogeneous investors' subjective beliefs by combining option prices and portfolio holdings. We show how to recover investor-specific expectations of returns and risk, consensus beliefs, and belief dispersion. Applying it to S&P 500 options' buy–sell order data, we find that subjective expected returns and Sharpe ratios vary by investor type and depend on portfolio composition. Beliefs inferred from prices alone display strong counter-cyclicality, whereas those incorporating holdings can reverse sign, exhibit muted cyclicality, and align with professional survey expectations under market-timing strategies. Our results highlight the value of holdings data in belief recovery.

Keywords: subjective expected returns, subjective risk, options, holdings, recovery

JEL Classification: G12, G40

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1 Introduction

Canonical estimates of market expected returns derived from option prices tend to rise during crisis periods and exhibit substantial variation over time. In contrast, survey-based estimates are typically less volatile and may display pro-cyclical, counter-cyclical, or a-cyclical patterns depending on the investor's type and sophistication level (see, e.g., Greenwood and Shleifer [2014], Nagel and Xu [2023], and Dahlquist and Ibert [2024]). While price-based measures generally overlook investors' actual holdings, a growing body of research highlights the strong link between portfolio positions and investor beliefs.[1] Motivated by this literature, our paper emphasizes the importance of incorporating investors' holdings to better understand the heterogeneous nature of subjective expected returns and perceived risks. Options holdings, in particular, offer unique insights, as it is well established that investors have a "fundamental" need to trade options (see, e.g., Ross [1976]; Hakansson [1979]).

In this paper, we present a simple theoretical framework for extracting investors' heterogeneous subjective beliefs from option prices and holdings data under the no-arbitrage assumption. Our approach allows for real-time inference of individual investors' expected market returns and perceived risks at a granular level and we show how to aggregate these beliefs to obtain consensus expected returns and quantify belief dispersion across market participants.

Empirically, we document significant heterogeneity in the resulting expected return estimates across different types of investors. More importantly, we find that expected returns derived from holdings' data can diverge in several intriguing ways from those inferred from prices alone. Using transaction-level data on buy and sell orders for S&P 500 index options, we show that the subjective expected returns of retail and institutional investors are significantly smaller and less cyclical compared to those inferred from price data alone. In contrast, expected returns of market makers more closely follow price-based measures. Intuitively, institutional investors like hedge funds often buy insurance in the option market via long-put positions, which lowers their exposure to market downturns. Conversely, market makers are mostly net suppliers of deep-out-of-the-money puts to public investors, see, e.g., Gârleanu, Pedersen, and Poteshman [2008] and Chen, Joslin, and Ni [2019]. Increased market exposure via

[1] For example, Giglio et al. [2021] find a robust relationship between expected returns and portfolio holdings using survey data. Beutel and Weber [2023] provide causal evidence on this connection through experimental studies. Additionally, Egan, MacKay, and Yang [2024] show that belief heterogeneity accounts for much of the variation in household portfolio choices.

the short put position leads to higher expected returns for market makers compared to customers' expected return. In line with the survey literature, we conclude that the dynamic features of subjective measures of expected returns inferred jointly from option prices and holdings can vary greatly across investors.

Our approach is based on the assumption of an arbitrage-free market, where prices can be expressed as the expected value of future payoffs discounted by investor's stochastic discount factor (SDF) M_i. The expectation is taken under a probability measure $\mathbb{P}_i$, which represents the investor's subjective belief, while the SDF reflects the investor's risk preferences. Existing methods typically extract agents' beliefs from a cross-section of option prices under certain assumptions about the SDF, overlooking valuable information available at granular levels, such as portfolio holdings, trading flows, or open interest. In contrast, our "demand-based" belief recovery extracts $\mathbb{P}_i$ by leveraging investor-level data on option holdings, in addition to option prices. Specifically, we consider investors with heterogeneous beliefs, who can hold different portfolios invested in the aggregate market index and a family of options written on the index.

Our main theoretical result asserts that subjective expectations under belief $\mathbb{P}_i$, such as subjective expected market returns, can be directly inferred from investors' portfolio holdings and a cross-section of option prices, which fully determine the risk-neutral probability measure $\mathbb{Q}$ (Breeden and Litzenberger [1978]). Since holdings are observable at the investor level, we can derive measures of both subjective expected returns and risks in real time for each investor type. Moreover, we demonstrate how to recover a consensus belief and quantify belief dispersion across investors. For instance, we show that when all agents are unconstrained and hold their own growth-optimal portfolios, aggregate market clearing implies a consensus expected market return that coincides with the risk-neutral variance.

Our model ties each investor's stochastic discount factor to both index and option returns, so inferred expected returns and risks vary with option-portfolio composition and weight. By matching option prices with daily Open–Close buy/sell records from the Chicago Board of Option Exchange (CBOE), we recover beliefs that align with observed holdings.

Our empirical analysis produces the following main findings. First, variations in option portfolio holdings influence investors' ex-ante exposure to market risk, resulting in significant differences in the expected market return characteristics across investors and over time. Our findings reveal that the expected returns inferred for market makers are more correlated with price-based measures of expected returns than with those for customers. Customers' expected returns tend to be smaller and may even turn negative in some cases. This difference can be partly attributed to the fact that in our sample, customers are predominantly long puts, providing protection against market downturns.

Second, our framework allows us to recover subjective measures of investors' perceived risk and their risk-return tradeoffs. Compared to expected returns, customers' inferred risks show a stronger correlation with price-based measures, though they are consistently slightly higher. In contrast, market makers' inferred risks more closely align with price-based measures. As a result, customers tend to exhibit significantly lower and more volatile inferred Sharpe ratios compared to market makers.

Finally, we extend our empirical framework along two dimensions. First, we explore the relationship between our demand-based expected return time-series and survey-based measures of expected market returns. We find that demand-based expected returns can be reconciled with survey expectations under economically plausible market timing strategies and with only moderate aggregate option exposure relative to investors' market allocations. Second, we extend our theory and study the effect of time-varying risk aversion. We find that in order to align demand-based expected returns to the survey data, risk aversion coefficients do not have to deviate too much from our benchmark case, where we set risk aversion equal to one (hence, log utility).

Related Literature. This paper is related to several important strands of the literature. Starting from the seminal work of Ross [2015], a growing literature has studied how to recover investors' beliefs using information provided by the cross-section of option prices; see, e.g., Borovička, Hansen, and Scheinkman [2016], Schneider and Trojani [2019] and Jensen, Lando, and Pedersen [2019], among others, for recent refinements of the Ross [2015] recovery theorem. Similarly, Pazarbasi, Schneider, and Vilkov [2024] derive non-parametric bounds on belief dispersion from option prices. Our approach is distinct, as we focus on incorporating demand- and price-based option information in the empirical recovery of investor-specific beliefs.

Another strand of the literature investigates properties of investor beliefs that are consistent with (i) a set of pricing constraints, and (ii) specific belief properties inferred from survey data while assuming an initial physical probability, see, e.g., Chen, Hansen, and Hansen [2020], Ghosh and Roussellet [2023], and Korsaye [2024], among others. These approaches do not incorporate demand-based information and are not designed for granular, real-time applications since in practice, recovery involves the joint estimation of transition probabilities under the physical belief, parametric stochastic discount factor models of investor preferences, and minimal divergence adjustments to match the target belief properties imposed by survey data.

Our paper is also related to the literature that examines the properties of option demand among heterogeneous investors. Chen, Joslin, and Ni [2019] document how variations in the net demand for deep OTM put options between intermediaries and public investors may be partly driven by intermediaries' constraints. Farago, Khapko, and Ornthanalai [2021] propose a heterogeneous agent model to explain index put trading volumes. Almeida and Freire [2022] link pricing kernel puzzles to various option demand effects. We complement this literature by offering a simple framework that enables us to estimate the beliefs of both intermediaries and end users using price and holdings data.

A closely related body of literature uses option prices to recover proxies of expected returns for a single investor without including demand-based information. For example, Martin [2017] and Gao and Martin [2021] derive lower bounds on expected market returns and expected log market returns. These bounds correspond to the subjective expected return and expected log return, respectively, of an investor who maximizes long-run growth, is fully invested in the market, and does not trade options. Both bounds can be directly computed in real time from option prices as two distinct measures of risk-neutral variance. Extensions of these bounds, as discussed in Schneider and Trojani [2019] and Chabi-Yo and Loudis [2020], among others, are expressed as specific functions of multiple option-implied moments. Tetlock [2023] examines an investor, who invests in the market and a set of synthetic higher-moment payoffs created with option-replicating portfolios. He shows that the expected market return and variance, according to this investor's beliefs, can be expressed as linear combinations of option-implied moments, with weights reflecting the investments in each replicating portfolio. Since these weights are not directly observable, he estimates them using a statistical approach that maximizes the predictive power of the investor's subjective variance for actual realized variance. Unlike this literature, we construct subjective measures of investor expected return and risk that fully leverage real-time information from option prices and holdings. We demonstrate that the dynamics and statistical properties of the inferred expected returns depend on quantity data in important ways. Specifically, we show that the properties of these inferred returns can differ significantly from those derived using only option price information or price data combined with synthetic option portfolio weights estimated from time-series market return data.

Our paper also contributes to the empirical literature examining the beliefs of heterogeneous investors through survey data. Dahlquist and Ibert [2024] document substantial heterogeneity in asset managers' expectations, while Giglio et al. [2021] investigate the link between retail investors' beliefs and their portfolio choices. Meeuwis et al. [2022] further show that political orientation shapes households' beliefs and their allocations to risky assets. Our approach differs by inferring investor beliefs from a combination of option prices and holdings data, available in real time and at a higher frequency than traditional survey measures.

More broadly, our paper is motivated by the demand-based asset pricing literature, initiated by the seminal work of Koijen and Yogo [2019]. Similar to our approach, asset demand systems impose economic constraints that match holdings data with price data, while also incorporating market-clearing equilibrium conditions. While that literature primarily focuses on how heterogeneous investor demands influence asset prices, our focus is on recovering subjective expected returns from observable option price and demand patterns.

Outline. The remainder of the paper is organized as follows. The key intuition is that holdings data provide valuable insight into investors' beliefs. We begin by illustrating this intuition in Section 2. Section 3 presents our theoretical framework, showing how subjective expected returns and risks can be inferred from data on prices and holdings. Section 4 reports and discusses our main empirical findings. All proofs and additional mathematical details are provided in the Appendix, with further results available in an Online Appendix.

2 Illustrative Example

The core idea of our paper is that portfolio holdings encode information about investors' beliefs and perceptions of risk. To build intuition, we start with a simple example showing how option positions relate to the shape of an investor's stochastic discount factor (SDF) as a function of market returns—and, by extension, to her views about the market. In particular, we examine a set of investors with differing beliefs, each of whom holds her growth-optimal portfolio—that is, the portfolio that maximizes expected long-run wealth (see, e.g., Long [1990]).

As is well-known, the return of the growth-optimal portfolio is the reciprocal of investor's stochastic discount factor: $M^\star = 1/R^\star$, where $R^\star$ is the return of the optimal portfolio according to the investor's subjective view. Specifically, although investors may share identical preferences, their optimal portfolios can differ due to differences in their beliefs. To establish a benchmark, one might consider the setting in Martin [2017], where an investor's belief leads her to hold a growth-optimal portfolio fully invested in the market. In this case, the SDF of the investor takes the simpler form $M^0 := 1/R$, where R is the return on the market index. This choice will serve as our benchmark throughout the remainder of the paper. A-priori, however, there is no reason to exclude other traded assets (such as options) from the optimal portfolio. For example, it is well-known that options are non-redundant securities held for hedging purposes.[2] Therefore, the corresponding optimal portfolio will generally include non-zero positions in index options, leading to $R^\star \neq R$, and in turn $M^\star \neq M^0$.

Figure 1 visualizes the ratio $M^0/M^\star$ as a function of the market return R, for an investor invested in calls (left panel) and puts (right panel) of different moneyness. For illustration purposes, we assume that the investor's portfolio consists of a fixed proportion of 90% invested in the index and 10% in an equally weighted portfolio of calls or puts, with options having the same maturity but different strikes.[3] Notice that even with a relatively moderate investment in options and small variations of market returns, the ratio $M^0/M^\star$ can substantially depart from 1 for at-the-money (ATM) and especially out-of-the-money (OTM) calls and puts. For example, for a market excess return of +20% (-20%), $M^0/M^\star$ increases to 6 (5) for OTM calls (puts).

Next, we illustrate in Figure 2 the potential implications of investors' option demand for the time-series properties of their subjective expected returns. To this end, we plot the time-series of the expected market return of different investors with SDF $M^\star$ from Figure 1, alongside the expected return of the benchmark investor with SDF M^0. Panel A plots the expected returns of investors who hold calls in addition to the index. Not too surprisingly, given the relatively time-invariant nature of the index and option investment in our illustrative example, different expected return series exhibit a substantial degree of comovement, with expected returns that tend

[2]Theoretically, economies with heterogeneous beliefs typically lead to non-trivial net demands for options. For example, Buraschi, Trojani, and Vedolin [2014] show that agents with more pessimistic views about future economic growth demand out-of-the-money (OTM) puts from more optimistic agents. In markets with trading frictions, Johnson, Liang, and Liu [2016] show that the high demand for index options is primarily due to the transfer of unspanned crash risk.

[3]In later sections, we make use of transaction-level data from the CBOE to precisely track actual option portfolio holdings in real-time.

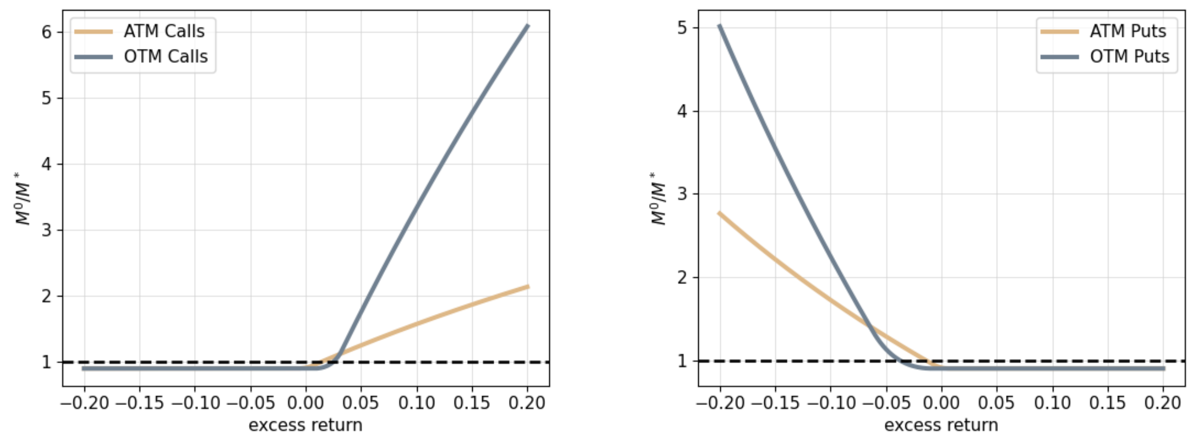


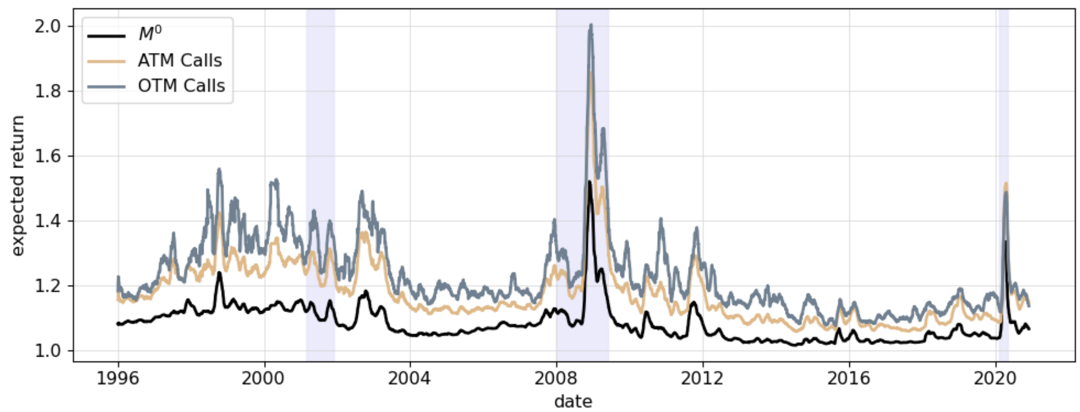
Figure 1. Ratio between Benchmark SDF M^0 and $M^\star$
Notes: This figure plots $M^0/M^\star$ as a function of the excess return on the market. $M^\star = 1/R^\star$, where $R^\star$ is the return of a portfolio investing 90% of the wealth in the underlying and 10% in an equally-weighted portfolio of calls (left plot) and puts (right plot) with different moneyness. ATM options have $|\Delta| \in (0.4, 0.6]$. OTM options have $|\Delta| \in [0.1, 0.4]$.

to increase (decrease) in bad (good) times and are quite volatile. Consistent with our previous evidence, the expected returns of investors who trade call options are also considerably higher.

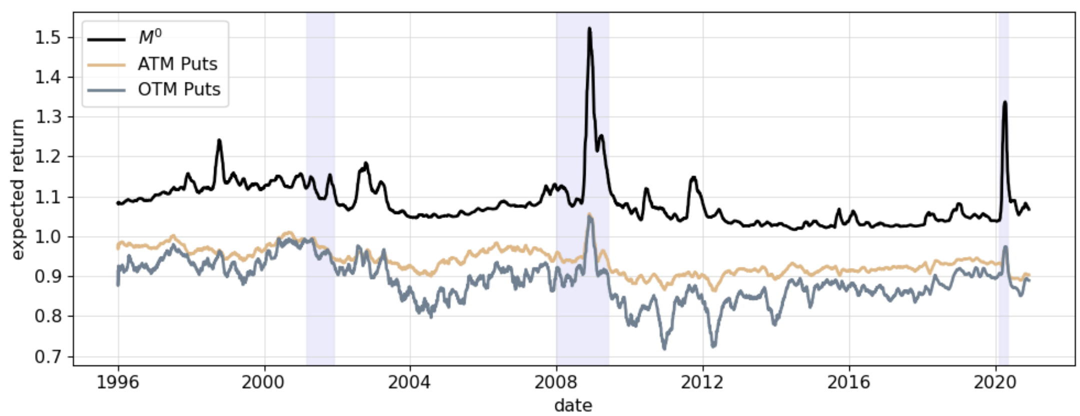
We can contrast the above patterns with the inferred expected returns of investors who are long puts. As seen in Panel B, decreasing exposure to downside market risk through long put positions lowers the corresponding expected return, turning negative for most of the sample. As discussed earlier, this feature reflects the fact that put investors, ceteris paribus, have more left-skewed market beliefs, making their subjective risk-return tradeoff for holding just the index suboptimal when put options are available. Equivalently, their long put positions directly reflect their pessimistic views about the market, and are therefore associated with lower expected returns.

To summarize, even moderate investments in put options can lead to partly procyclical expected return patterns. Overall, this suggests that both the magnitude and cyclicality of investors' expected returns can depend in complex ways on option type, option moneyness, and the relative importance of option versus market index allocations in an investor's optimal portfolio.[4] In the following, we develop a theory to formalize our intuition.

[4]We study more complex option trading strategies such as collars and straddles in the appendix. In general, the conclusion remains the same: the statistical properties of expected returns depend on the composition of investors' portfolios.



A. Calls



B. Puts

Figure 2. Time-Series of Expected Market Return

Notes: This figure plots the expected market return recovered from different stochastic discount factors. In each panel, we plot the expected return recovered by $M^0 = 1/R$, as well as by $M^\star = 1/R^\star$, where $R^\star$ is the return of a portfolio investing 90% in the index and 10% in an equally-weighted portfolio of either ATM or OTM options for calls (Panel A) and puts (Panel B). Frequency is daily, horizon is monthly. Time series are 30-days moving averages. Values are annualized. Grey areas indicate NBER recession periods.

3 Theoretical Framework

We now introduce a simple theoretical framework to explain how (i) subjective expected returns can be empirically recovered from options prices and holdings data, (ii) to aggregate heterogeneous beliefs into a consensus belief, and (iii) to measure the associated belief heterogeneity.

Consider an investor, labeled i, with logarithmic preferences, who has access to three types of assets: a risk-free asset, a risky asset with forward return R, and a cross-section of options written on the risky asset. Let $\mathbb{E}^i[\cdot]$ denote this investor's subjective conditional expectation over possible states of return R, under her subjective probability belief $\mathbb{P}_i$. The investor's subjective belief may or may not align with the true underlying data-generating process. In this paper, we assume the risky asset is the S&P 500 index and the options are European calls and puts on this index. Finally, we denote by $\boldsymbol{R}^e = \boldsymbol{R} - \mathbf{1}$ the vector of excess forward returns, which includes the excess returns on both the index and the options, and by $\mathbb{Q}$ the (forward) pricing measure such that $\mathbb{E}^{\mathbb{Q}}[\boldsymbol{R}^e] = \mathbf{0}$.

3.1 Subjective Expected Returns

Our goal is to recover investor's belief $\mathbb{P}_i$, under the simple framework introduced above, to infer moments of market returns, such as the expected return $\mathbb{E}^i[R]$ or perceived market risk $\mathbb{V}ar^i(R)$. Since investors have logarithmic utility, an agent-specific SDF that prices all assets from the perspective of agent i is given by:

$$M_i = \left(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e\right)^{-1}, \tag{1}$$

where $\boldsymbol{\theta}_i$ is the vector of (optimal) portfolio weights for investment in the market index and the options by investor i; see, e.g., Long [1990]. Note that SDF M_i is the reciprocal of the return of the growth-optimal portfolio, since it maximizes expected long-run growth of the investor i's wealth, and such that:

$$\mathbb{E}^i[M_i \boldsymbol{R}^e] = \mathbf{0}.$$

We can then define a change of measure, $\frac{d\mathbb{P}_i}{d\mathbb{Q}} = M_i^{-1}$ and write the subjective expected market return for investor i as:

$$\mathbb{E}^i[R] = \mathbb{E}^{\mathbb{Q}}[M_i^{-1}R] = \mathbb{E}^{\mathbb{Q}}\left[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)\, R\right] = 1 + \mathbb{C}ov^{\mathbb{Q}}\left(\boldsymbol{\theta}_i'\boldsymbol{R}, R\right) \ . \tag{2}$$

Equation (2) is the central identity studied in this paper. It links investor-specific beliefs about market returns to the risk-neutral expectation of a particular payoff, which depends solely on the market return, option returns, and investor i's holdings of these assets. Furthermore, while equation (2) is derived under the assumption of a log investor, we discuss below to what extent it can serve as a useful lower bound for expected returns when this assumption does not hold.

Equation (2) nests the main result in Martin [2017], who derives a lower bound for the expected return of an investor with risk aversion of at least one. In his setting, the subjective expected return coincides with the risk-neutral variance (SVIX). The key distinction from Martin [2017] is that we do not assume that the investor's optimal portfolio is fully allocated to the market, treating options as redundant assets. In our paper, we argue that options are non-redundant securities often held for fundamental trading purposes, such as crash insurance.

While identity (2) holds exactly for the expected market return of a log utility investor, one may wonder how it is affected by a violation of this assumption. Following Martin [2017], we can show that it provides in general a lower bound for expected market returns if the following negative covariance condition (NCC) holds:

$$\mathbb{C}ov^i(M_i(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e), R) \leq 0\,, \tag{3}$$

where the covariance is formed under agent's belief $\mathbb{P}_i$. Clearly, this NCC always holds with equality for a log utility investor, as in this case $M_i = (1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{-1}$. Conversely, for an investor with power utility and risk aversion γ, who is fully invested in the market and not holding options, this NCC holds if and only if $\gamma \geq 1$.[5]

The validity of the NCC when allowing for options can depend on various economic factors, such as the characteristics of the market price of nonlinear risks or the relative size of the optimal option investment compared to the index investment. Since this quantity is not directly observable in the data, our empirical study bounds it to ensure

[5] Martin [2017] demonstrates that the NCC holds in various prominent macro-finance asset pricing models and Back, Crotty, and Kazempour [2022] show that the bounds are valid also conditionally (but can show slackness) in the data.

it remains economically "small." This approach implies that option trading ideally leads to only moderate perturbations of the NCC relative to the benchmark case studied in Martin [2017].

Given NCC (3), the following lower bound on investor's expected market return follows:

$$\mathbb{E}^i[R] \geq \frac{\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)R]}{\mathbb{E}^{\mathbb{Q}}[1+\boldsymbol{\theta}_i'\boldsymbol{R}^e]} = 1 + \mathbb{C}ov^{\mathbb{Q}}\left(\boldsymbol{\theta}_i'\boldsymbol{R}, R\right) \ , \tag{4}$$

since $\mathbb{E}^{\mathbb{Q}}[1+\boldsymbol{\theta}_i'\boldsymbol{R}^e] = 1$. The right-hand side of inequality (4) represents the expected return in equation (2), which is derived under the log utility assumption. Therefore, the expected return in the log utility setting provides a lower bound for the expected return of an agent who holds the same optimal portfolio $\boldsymbol{\theta}_i$ but potentially has non-log utility preferences that satisfy the NCC condition (3).

3.2 Subjective Risk

Our framework can easily be extended to study not only expected returns, but also subjective higher moments of market returns such as investors' perceived market risk, allowing us to explore the properties of the subjective risk-return trade-off.[6] Using the same logic applied in the derivation of equation (2), we obtain the following expression for the subjective second moment of market returns:

$$\mathbb{E}^i[R^2] = \mathbb{E}^{\mathbb{Q}}\left[R^2\right] + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, R^2) \ . \tag{5}$$

In equation (5), the second moment of the market return is the sum of two terms: the second risk-neutral moment of market returns and the risk-neutral covariance between market returns and the investor's optimal portfolio return. By definition, the first term represents the price of a realized second-moment payoff, while the second term captures the risk premium of realized second-moment payoffs. This premium is given by a risk-neutral coskewness coefficient, which measures the covariance between the investor's optimal portfolio return and the squared market return. In the special case where the investor is optimally invested only in the market, the investor's subjective risk coincides with risk-neutral market coskewness.

By combining formulas (2) and (5) for the first two subjective moments of market

[6]While the relationship between conventional measures of realized risk and returns is typically ambiguous and weak, recent literature has consistently reported a stronger positive association when using survey-based measures. For example, Couts, Goncalves, and Loudis [2023] find this in the context of the aggregate stock and bond markets, while Jensen [2024] observes it for individual stocks.

returns, we can decompose the investor's subjective market variance as follows.

Proposition 1. *Investor's subjective variance can be decomposed as:*

$$\mathbb{V}ar^{i}(R) = \mathbb{V}ar^{\mathbb{Q}}(R) + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, V(R)) - (\mathbb{E}^{i}[R-1])^2, \tag{6}$$

where $V(R) := (R-1)^2$ is the SVIX realized variance payoff.

Proposition 1 provides insight into the joint impact of an investor's portfolio holdings on subjective expected return and risk. For example, an investor who is fully invested only in the market index has a subjective market variance given by:

$$\mathbb{V}ar^{i}(R) = \mathbb{V}ar^{\mathbb{Q}}(R) + \mathbb{C}ov^{\mathbb{Q}}(R, V(R)) - (\mathbb{V}ar^{\mathbb{Q}}(R))^2 \,. \tag{7}$$

Specifically, the investor's subjective variance is lower than the implied variance when the leverage effect, captured by the risk-neutral covariance term $\mathbb{C}ov^{\mathbb{Q}}(R, V(R))$, is not larger than the squared implied variance. Since the leverage effect is typically negative in the data, the subjective variance for these investors tends to be smaller than the implied variance.

More generally, given equation (6), the overall effect on subjective variance depends on the combined impact of the investor's optimal portfolio on the squared subjective equity premium, as well as the co-leverage effect between the investor's portfolio return and the market's realized variance, as reflected by the covariance term $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, V(R))$.

3.3 Consensus Belief and Belief Disagreement

Thus far, we have explored how to recover the expected returns of heterogeneous individual investors. A natural next step is to define a consensus belief within such a cross-section of heterogeneous investors. To this end, let's assume that we observe the portfolios of all investors when both the index (in positive net supply) and the option market (in zero net supply) clear.[7] For brevity, we denote by $R_i = 1 + \boldsymbol{\theta}_i'\boldsymbol{R}^e$ the (forward) return on the i-th investor's wealth over the given horizon, where the wealth of investor i at time $t = 0, T$ is indicated by W_{it}. Given the market index value I_t at time t, the index and option market clearing condition yield that $I_t = W_t := \sum_i W_{it}$ and and $R = \frac{I_T}{I_0} = \frac{W_T}{W_0} = \sum_i \frac{W_{i0}}{W_0} R_i =: \sum_i w_i R_i$ with w_i being the $i-$th investor's share of the

[7]We study a second case, when only a subset of portfolios is observable in the Internet Appendix.

aggregate wealth. The consensus market expected return among all investors is then defined by:

$$\bar{\mathbb{E}}[R] := \sum_i w_i \mathbb{E}^i[R] \,. \tag{8}$$

The next proposition characterizes the consensus belief in an economy where markets clear and all investors optimally allocate their wealth between the index and option markets.

Proposition 2. *Let investor j be a log investor optimally invested in the index and in the option market. Investor j's expected market return is then given by:*

$$\mathbb{E}^j[R] = 1 + \mathbb{C}ov^{\mathbb{Q}}(R_j, R) \,. \tag{9}$$

If both index and option markets clear, then

$$\mathbb{E}^j[R] = 1 + w_j \mathbb{V}ar^{\mathbb{Q}}(R_j) + \sum_{i \neq j} w_i \mathbb{C}ov^{\mathbb{Q}}(R_j, R_i) \,. \tag{10}$$

Finally, if index and option markets clear and all investors are log investors optimally invested in both markets, then the consensus belief is given by:

$$\bar{\mathbb{E}}[R] = 1 + \mathbb{V}ar^{\mathbb{Q}}(R) \,. \tag{11}$$

The first two identities in Proposition 2 demonstrate that the expected return perceived by each investor is a wealth-weighted average of all risk-neutral covariances between the return on her optimally allocated wealth and the returns of all other investors in the economy. An obvious special case of Proposition 2 occurs when all investors have identical beliefs. In this case, the market equity premium as perceived by all agents is given by the SVIX, see equation (11). Intuitively, the consensus expected return aligns with the risk-neutral implied variance because, as the option market clears, the impact of option holdings disappears in aggregate, leading investors to collectively hold the market.

In addition to the consensus belief, we can also calculate the associated degree of (scaled) belief disagreement:[8]

$$\mathbb{D}(R) = \frac{1}{\bar{\mathbb{E}}[R]} \sum_i w_i \left| \mathbb{E}^i[R] - \bar{\mathbb{E}}[R] \right| . \tag{12}$$

4 Empirical Analysis

This section outlines the data used and the empirical implementation of our main theoretical results. We begin with an arbitrage-free cross-section of options and a forward probability measure $\mathbb{Q}$, which reproduces as an expected payoff the price of any payoff replicable through a delta-hedged option portfolio; see, e.g., Acciaio et al. [2016].[9]

4.1 Data

To empirically implement our theory, we utilize the CBOE Open-Close dataset, which provides daily buy and sell volumes of SPX options since 1996, separated by position type (opening/closing) and origin: (i) customer, (ii) broker-dealer, (iii) firm, and (iv) market maker. Following common practice, we aggregate the daily volumes for the last three categories into cumulative positions and label them as “market makers.”[10] Customers include both retail and institutional investors.[11] Our dataset spans January 1996 to December 2020. The "broker-dealer" label is available only from 2011 and represents less than 3% of the trades.

The volume data does not include pricing information. To address this, we obtain end-of-day bid-ask prices from the OptionMetrics database and use the best closing bid and ask prices to compute mid-point prices. We then merge the CBOE Open-Close dataset with the price data and apply standard filters from Bakshi, Cao, and Chen [1997]. Specifically, we exclude option contracts that: (i) have a price below 3/8, (ii) have an implied volatility smaller than 0.1% or greater than 1, (iii) exhibit a bid

[8] We scale $\mathbb{D}(R)$ by the consensus expected market return to isolate belief heterogeneity shocks from the mechanical effects of shocks to consensus market return expectations.

[9] Specifically, this also implies $\mathbb{E}^{\mathbb{Q}}[\boldsymbol{R}^e] = \mathbf{0}$.

[10] According to the CBOE Regulatory Circular, firms are defined as "OCC clearing member firm proprietary accounts". Therefore, we aggregate firms and broker-dealers with market makers, as they predominantly trade against public customers, despite not being formally designated as intermediaries.

[11] It is not possible to determine the exact fraction of trades originating from retail investors, as all orders sent to the CBOE are routed through intermediaries, meaning even the CBOE cannot identify end-user investors. However, estimates suggest that retail investor trading volume was less than 10% before the COVID-19 pandemic, see, e.g., Han [2024].

Table 1. Summary statistics monthly options data.

	Calls				Puts			
	K/S_t		Holdings		K/S_t		Holdings	
	All	OTM	All	OTM	All	OTM	All	OTM
mean	1.02	1.04	-20,607	-9,133	0.89	0.95	65,696	21,890
std	0.09	0.03	57,496	21,140	0.14	0.03	94,310	41,301
min	0.19	1.00	-675,460	-153,762	0.11	0.71	-534,099	-203,685
median	1.02	1.03	-14,879	-5,688	0.92	0.96	46,813	12,862
max	1.82	1.57	494,353	132,687	2.67	1.00	1,282,355	358,137

Notes: This table reports summary statistics for the options data with 30 days maturity. The relative moneyness K/S_t is computed over the single option contracts that are traded on every date. «Holdings» are defined as the total customers' opening/closing buy orders minus sell orders cumulated for every option from the issuance to every trading date, and it is aggregated over all the traded options on a single day. For each variable, the first column refers to the full dataset, while the second to OTM options (with $0.1 \leq |\Delta| \leq 0.4$) only. Data runs from January 1996 to December 2020.

price greater than the ask price, (iv) have a relative bid-ask spread larger than 1/2, or (v) are traded in fewer than 5 units. We also filter out instances where the sum of transactions across investors is not zero, which occurs in less than 2% of cases. Additionally, no-arbitrage filters are applied.

We use monthly horizons, as the most liquid options in the full sample expire approximately thirty days from the observation date.[12] We compute all quantities at a daily frequency, then aggregate them into monthly moving averages to account for the high turnover in option holdings.[13] On each day, we separately interpolate the implied volatility, option delta, and investor holdings for calls and puts, using a grid of strike prices for the required maturity.[14] The grid points consist of the different strikes actively traded at that time.

Table 1 presents summary statistics of our option data, specifically for customers' net demand for calls and puts. Holdings are defined as the total buy orders minus the total sell orders, aggregated daily and cumulated over time from the option's issuance (for further details, see Section 4.2). On average, customers are net sellers of call options and net buyers of put options. In our empirical analysis, we focus on out-of-the-money (OTM) options with $|\Delta| \in [0.1, 0.4]$, as these account for the largest

[12]Additional details on option volumes and trading can be found in Appendix B.
[13]This aggregation reflects, among other things, the diversity within the "Customer" label, which encompasses various individual agents.
[14]When extrapolation is necessary, we use the nearest value outside the convex hull in terms of strikes and maturities.

share of trading volume (57%, see Appendix B). Notice that we exclude very deep-OTM options (with $|\Delta| < 0.1$) because they are more likely to be affected by interpolation errors.

4.2 Implementation with Option Holdings

We implement equation (2) using the Carr and Madan [2001] formula. Specifically, note that

$$f_i(R) := (1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e) R \,,$$

depends only on R and $\boldsymbol{\theta}_i$, because $\boldsymbol{R}^e$ only depends on R. Denoting by k an option's relative moneyness and by $x(k)$ the option's normalized payoff relative to the index forward price, we then compute equation (2) as follows from our option price and holding data:

$$\begin{aligned}
\mathbb{E}^i[R] &= \mathbb{E}^{\mathbb{Q}}[f_i(R)] \\
&\approx \mathbb{E}^{\mathbb{Q}}\left[f_i(1) + f_i'(1)(R-1) + \sum_k f_i''(k)\, x(k)\, \Delta k \right] \\
&= f_i(1) + \sum_k f_i''(k) \mathbb{E}^{\mathbb{Q}}\left[\, x(k) \,\right] \Delta k
\end{aligned}$$

The computation of this expected return proxy relies on the vector $\boldsymbol{\theta}_i$ of investor's i portfolio weights, which defines the investor's total holdings of the index and each available option as a fraction of the investor's wealth. In contrast, our database reports the total daily opening and closing positions on every option contract for a corresponding fixed maturity. Daily opening and closing positions represent shocks to total option demand and their sum reproduces the daily changes (flows) in the holdings of each option contract. In order to recover total daily portfolio holdings of each option contract for a corresponding maturity, we aggregate over time its opening and closing positions from issuance up to the current date. To this end, we exploit the fact that the CBOE Open-Close Database explicitly assigns a unique identification number to every option contract, which can be used to track it day-by-day from issuance to expiration.

Figure 3 displays the monthly time series of total holdings by customers in OTM put and call options with one month to expiration.[15] Since options are in zero net supply,

[15]We present a more granular breakdown across moneyness in the appendix.

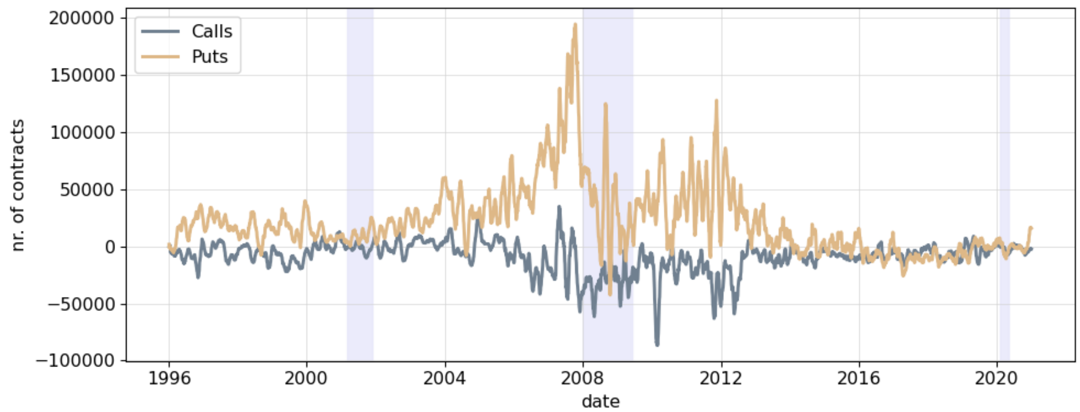


Figure 3. Customers' Holdings of monthly OTM options

Notes: This figure plots the 30-day moving average of customers' portfolio holdings of OTM calls and puts from 1996 to 2020. Options holdings are the sum of opening and closing positions on the same contract that customers enter from issuance. Grey areas indicate NBER recession periods.

market makers' holdings simply mirror those of the customers, and we do not report them separately.[16] We observe that customers are typically long OTM puts and short OTM calls, with the number of put positions generally exceeding the number of call positions (on average $+2.2 \times 10^4$ contracts/day for puts and -0.9×10^4 contracts/day for calls). The dynamics of option holdings show heteroskedasticity and significant persistence, which are closely linked to periods of market distress. For example, in the lead-up to the 2008 financial crisis, we observe a spike in customers' aggregate holdings of OTM puts in September 2007, followed by a temporary drop, and then another increase in August 2008 before declining again. The total number of call and put contracts held is positively correlated, with an unconditional correlation of approximately 24%. These patterns suggest that the aggregate option portfolio held by customers may function as insurance against a long position in the market index, with long OTM puts providing protection and short OTM calls used to finance the portfolio's insurance.

Notice that the patterns uncovered in our holdings data contrast with daily option *flows* which typically display large variations with both positive and negative signs. For example, Chen, Joslin, and Ni [2019] document that during the Great Financial Crisis of 2008, market makers shifted from being daily net sellers of deep OTM options to daily net buyers, based on daily option opening positions aggregated across all maturities. In contrast to their study which studies flows, our paper focuses on options

[16] Additional information on option holdings, including details along the moneyness and time to maturity dimensions, as well as a comparison with opening positions, is provided in Appendix B.

with a 30-day time to maturity in order to construct coherent SDFs and expected return proxies with price and holdings data of sufficiently liquid options.[17]

4.3 Subjective Expected Returns

Next, we examine the properties of investors expected returns, as inferred from equation (2). To do this, recall that the portfolio vector $\boldsymbol{\theta}_i$ represents the investor's portfolio weights in both the market index and each available option. Although we have data on option holdings, we lack information regarding index holdings. Consequently, we investigate the properties of the investor's subjective expected returns under different assumptions about the unobserved index holdings. To this end, we parameterize investor's portfolio holdings as a fraction of her total wealth as follows:

$$\boldsymbol{\theta}_i := \begin{pmatrix} \theta_{iI} \\ (1-\theta_{iI})\boldsymbol{\omega}_i \end{pmatrix} , \tag{13}$$

where $\boldsymbol{\omega}_i$ is a vector of option portfolio holdings expressed in percentage of the total investor's wealth that is not allocated to the index investment. Denoting by $\boldsymbol{O}^e$ ($\boldsymbol{O}$) the subvector of option excess returns (gross returns), equation (2) reads:

$$\begin{aligned} \mathbb{E}^i[R] &= 1 + \theta_{iI}\mathbb{E}^{\mathbb{Q}}[(R-1)R] + (1-\theta_{iI})\mathbb{E}[\boldsymbol{\omega}_i'\boldsymbol{O}^e R] \\ &= 1 + \theta_{iI}\mathbb{V}ar^{\mathbb{Q}}(R) + (1-\theta_{iI})\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, R) . \end{aligned} \tag{14}$$

If all wealth is invested in the index ($\theta_{iI} = 1$), the investor's market equity premium is equal to the SVIX. More generally, if an investor's wealth is not fully allocated to the market index, the equity premium is given by a linear combination of the SVIX and the risk-neutral covariance between market returns and the return of the optimally invested option portfolio.

The simple decomposition in equation (14) provides an intuitive understanding of how the investor's expected return can deviate from the benchmark when $\theta_{iI} = 1$. First, deleveraging relative to a full market investment (e.g. $\theta_{iI} \in (0, 1)$) reduces the direct market exposure in the optimal portfolio, thus lowering the contribution of the SVIX to the equity premium. Second, the nature of the optimal option investment can further reduce the expected return or offset this reduction, depending on the sign

[17] We also include a broader range of moneyness, excluding only the very deep-OTM options (with $|\Delta| < 0.1$). Appendix B reports more details on the properties of the time series of daily option opening positions built with our data following the methodology in Chen, Joslin, and Ni [2019].

of the covariance term $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, R)$: A negative (positive) covariance contributes negatively (positively) to the premium. This is intuitive, as a negative (positive) covariance suggests that the option investment is hedging (leveraging) the existing market investment, reflecting a more pessimistic (optimistic) view about future market returns. Specifically, if the investor's option portfolio consists only of long put (call) positions, the covariance is negative (positive), and the option's contribution to the equity premium is negative (positive), consistent with the illustrative examples of Section 2.

We gather from Figure 4 Panel A, that the covariance term is almost always negative, with a few exceptions (mostly between 2017 and 2019).[18] This suggests that customers' option portfolios act as a natural hedge against a positive market index exposure. Consequently, the term $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, R)$ also represents in equation (14) the lowest possible equity premium across customers' index investments $\theta_{cI} \in [0, 1]$. This term also exhibits significant time-varying behavior and negative skewness, marked by occasional large negative spikes, especially during periods of market distress and at disruptive market events, such as, e.g., the Bank of America rescue. Two key drivers explain this variability: (i) changes in the option positions within customers' portfolio weights $\boldsymbol{\omega}_c$ and (ii) fluctuations in the valuations of OTM options, which are encapsulated in the evolving risk-neutral distribution $\mathbb{Q}$. When compared to the contribution of variance term $\mathbb{V}ar^{\mathbb{Q}}(R)$, these fluctuations introduce greater volatility and reduce the persistence of the associated expected return component in equation (14). For example, the covariance term exhibits a standard deviation of 0.66, compared to 0.06 for $\mathbb{V}ar^{\mathbb{Q}}(R)$, and no first-order autocorrelation. As a consequence, these properties naturally affect not only the level, but also the cyclicality properties of investors' overall expected returns inferred from equation (14).

4.3.1 Customers' Expected Returns

Recall that the condition for equation (14) to be compatible with an arbitrage-free market is strict positivity of investor's optimal portfolio return:

$$1 + \boldsymbol{\theta}_c'\boldsymbol{R}^e = 1 + \theta_{cI}(R - 1) + (1 - \theta_{cI})\boldsymbol{\omega}_c'\boldsymbol{O}^e > 0 \, ,$$

$\mathbb{Q}-$almost surely, using investor's SDF $M_c = 1/(1 + \boldsymbol{\theta}_c'\boldsymbol{R}^e)$, and the fact that in an arbitrage-free market probabilities $\mathbb{P}_c$ and $\mathbb{Q}$ are equivalent. We can use this fact to

[18] To save space, we relegate detailed summary statistics to Appendix **??**.

discipline the choice of the unobservable market index investment θ_{cI}, by imposing the no-arbitrage condition $\theta_{cI} \in \Theta(\mathbb{Q}, \boldsymbol{\omega}_c)$, where:

$$\Theta(\mathbb{Q}, \boldsymbol{\omega}_c) := \{\theta_I : \mathbb{Q}(1 + \theta_I(R-1) + (1-\theta_I)\boldsymbol{\omega}_c'\boldsymbol{O}^e) > 0) = 1\} . \tag{15}$$

Specifically, given the smallest nonnegative fraction of customers' index investment compatible with an arbitrage-free market:

$$\bar{\theta}_{cI} := \inf\{\theta_{cI} : \theta_{cI} \geq 0 \text{ and } \theta_{cI} \in \Theta(\mathbb{Q}, \boldsymbol{\omega}_c)\} ,$$

we parameterize admissible customers' index investments $\theta_{cI} \in [\bar{\theta}_{cI}, 1]$ as:[19]

$$\theta_{cI}(\alpha) = \bar{\theta}_{cI} + \alpha\,(1 - \bar{\theta}_{cI})\,; \quad \alpha \in [0, 1] . \tag{16}$$

We examine how different values of the parameter α, that is, different levels of index investment, affect expected returns and shape the resulting time-varying effective index investments, denoted by $\theta_{cI}(\alpha)$. The time-series of customers' inferred expected market returns is displayed in Figure 4 Panel B, while Table 2 (Panel A) provides summary statistics.

Given the typically negative covariance term $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, R)$ for customers, Figure 4 documents that greater investments in options, or lower α, are usually associated with a lower expected return. The most significant expected return corrections relative to the benchmark case $\theta_{cI} = 1$ occur during periods when either the belief distortion $\mathbb{C}ov(\boldsymbol{\omega}_c'\boldsymbol{O}, R)$ is large (such as during market distress) or when the minimal market investment $\bar{\theta}_{cI}$ is low (e.g. between 2003 and 2006). While the benchmark average expected return is around 8% per year, we observe that it is already reduced by half for an investor holding on average only about 3% of her wealth in options ($\alpha = 0.9$). Moreover, it should be noted that the conditional expected return of investors with a relatively moderate average investment in options of about 1% can even become negative ($\alpha = 0.95$).

Finally, as the proportion of option investments increases, the volatility of customers' expected returns rises, leading to less persistent estimates of expected returns. For example, while the first autocorrelation coefficient for the benchmark expected return series is 0.82, it drops to 0.58 when $\alpha = 0.8$. This suggests that the associated

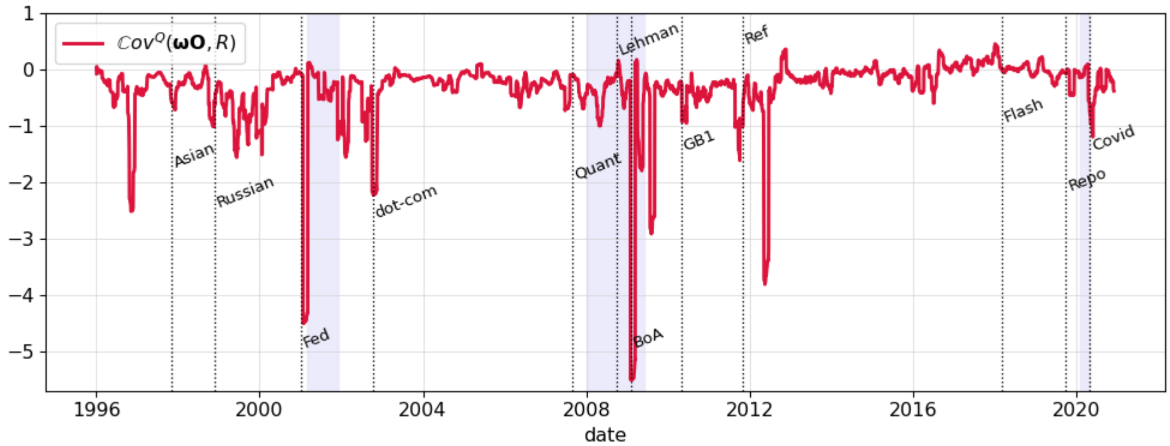
[19] Since investor's SDF M_i may not necessarily be monotonic in R, there may also exist a maximum index investment threshold compatible with an arbitrage-free market. However, in our data this threshold is always greater than 1. In contrast, minimum threshold $\bar{\theta}_{iI}$ is always strictly positive.

Table 2. Summary statistics of expected returns $\mathbb{E}^i[R]$.

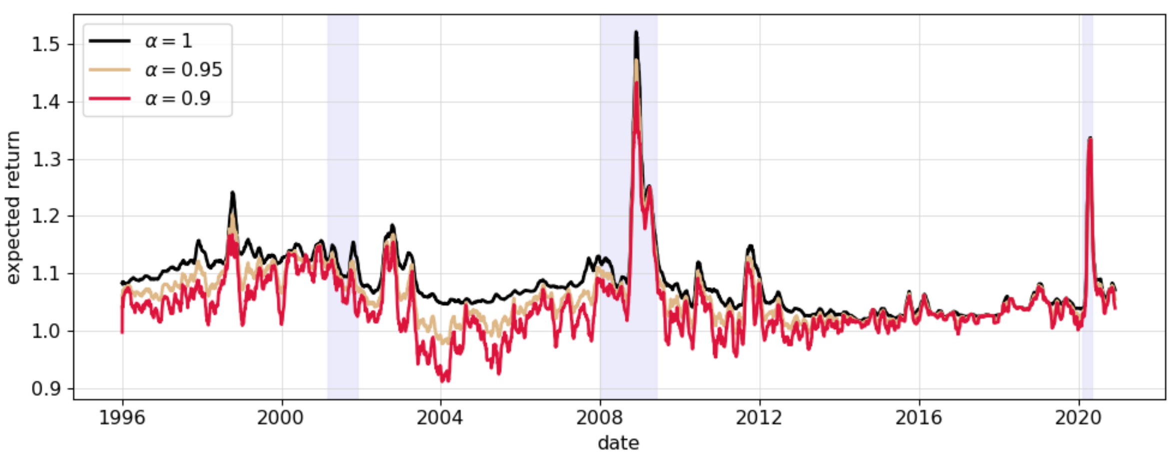
α	mean	std	min	q25	q50	q75	max	corr (%)	AR(1)	θ_{cI} (avg.)
1	1.082	0.058	1.017	1.042	1.069	1.114	1.521	100	0.82	1
Panel A: Customers										
95%	1.061	0.056	0.977	1.024	1.049	1.080	1.472	96	0.77	0.99
90%	1.041	0.058	0.911	1.011	1.032	1.062	1.433	85	0.69	0.97
80%	1.004	0.071	0.792	0.970	1.006	1.038	1.366	59	0.58	0.94
50%	0.913	0.124	0.520	0.838	0.933	1.001	1.337	20	0.51	0.86
0	0.810	0.194	0.258	0.679	0.838	0.960	1.345	5	0.49	0.72
Panel B: Market Makers										
95%	1.077	0.062	0.986	1.032	1.068	1.112	1.527	98	0.86	0.99
90%	1.071	0.067	0.949	1.023	1.066	1.113	1.539	94	0.87	0.98
80%	1.062	0.081	0.880	1.008	1.063	1.116	1.568	85	0.85	0.96
50%	1.044	0.128	0.639	0.966	1.060	1.125	1.661	69	0.76	0.90
0	1.039	0.208	0.397	0.919	1.065	1.148	1.867	62	0.62	0.81

Notes: This table reports summary statistics for the 30-day moving average of annualized expected returns recovered by different portfolios built on customers' (Panel A) and market makers' (Panel B) options positions. «corr» is the correlation with the SVIX. «θ_{cI} (avg.)» is the average investment in the index for each time-series. Portfolios differ across the amount of wealth invested in the index, expressed as a function of α. The case $\alpha = 1$ is the benchmark recovered by M^0 ($\theta_{cI}(1) = 1$). The case $\alpha = 0$ corresponds to the minimum investment in the index compatible with the no-arbitrage condition ($\theta_{cI}(0) = \bar{\theta}_{cI}$). Data is daily and values are annualized. Data runs from January 1996 to December 2020.

measure of $\mathbb{E}^c[R]$ becomes more volatile and less persistent, reflecting shocks from both option prices and option demands. Importantly, this second effect is absent in benchmark expected return proxies that rely solely on the SVIX. Furthermore, while the correlation with this benchmark expected return is still positive, it is already only 0.58 for average option investments of about 6% ($\alpha = 0.8$), indicating a expected return cyclicality that may differ considerably in the presence of option investments.



A. Belief Distortion



B. Subjective Expected Returns

Figure 4. Customers' Belief Distortion and Expected Return

Notes: Panel 4A plots the time-series of the monthly covariance term $\mathbb{C}\text{ov}^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, R)$, where $\boldsymbol{\omega}_c$ are customers' option holdings. Major events marked: Asian financial crisis (Oct. '97), Russian financial crisis (Nov. '98), first Fed rate cut (Jan. '01), Nasdaq low (Oct. '02), quant-fund crisis (Aug. '07), Lehman bankruptcy (Sep. '08), BoA rescue (Jan. '09), Greek bailout installment (Apr. '10), Greek referendum call (Oct. '11), Flash crisis (Feb. '18), repo spike (Sept. '19), COVID-19 peak (Apr. '20). Panel 4B plots the time-series of the expected market return implied by customers' options holdings for different levels of index investment $\theta_{cI}(\alpha)$. $\alpha = 1$ corresponds to the monthly expected return recovered by the benchmark $M^0 = 1/R$ $(\theta_{cI}(\alpha) = 1)$. Frequency is daily, horizon is monthly, values are annualized. Data are 30-day moving averages. Gray bars indicate NBER recessions.

4.3.2 Market Makers' Expected Returns

To recover market makers' expected return, $\mathbb{E}^m[R]$, we follow a similar approach to that used in the previous section, with the added refinement of incorporating empirical evidence that intermediaries typically manage their risk-return tradeoff using delta-hedged derivative strategies; see, for example, Baltussen, Jerstegge, and Whelan [2024] for S&P 500 options, and Amayaa et al. [2024] and Dim, Eraker, and Vilkov [2025] for 0DTE options. Consequently, we model market makers' option portfolios as delta-hedged and parameterize their return on optimally invested wealth as follows:

$$\begin{aligned} 1+\boldsymbol{\theta}_m'\boldsymbol{R}^e &= 1+\theta_{mI}(R-1)+(1-\theta_{mI})\boldsymbol{\omega}_m'(\boldsymbol{O}^e-\boldsymbol{\Delta}(R-1)) \\ &= 1+(\theta_{mI}-(1-\theta_{mI})\boldsymbol{\omega}_m'\boldsymbol{\Delta})(R-1)+(1-\theta_{mI})\boldsymbol{\omega}_m'\boldsymbol{O}^e \ , \end{aligned}$$

where θ_{mI} now represents the market maker's fraction of wealth invested in the index, excluding the portion allocated for delta-hedging purposes, and $\boldsymbol{\Delta}$ is the vector of option deltas corresponding to the option return vector $\boldsymbol{O}$. Similar to the previous section, we discipline the choice of the market makers' index investment with the no-arbitrage condition $\theta_{mI}\in\Theta(\mathbb{Q},\boldsymbol{\omega}_m,\boldsymbol{\Delta})$, where:

$$\Theta(\mathbb{Q},\boldsymbol{\omega}_m,\boldsymbol{\Delta}) := \{\theta_I : \mathbb{Q}(1+(\theta_I-(1-\theta_I)\boldsymbol{\omega}_m'\boldsymbol{\Delta}))(R-1)+(1-\theta_I)\boldsymbol{\omega}_m'\boldsymbol{O}^e)>0)=1\} \ .$$

Given the smallest nonnegative fraction of market makers' index investment compatible with an arbitrage-free market:

$$\bar{\theta}_{mI} := \inf\{\theta_{mI} : \theta_{mI}\geq 0 \text{ and } \theta_{mI}\in\Theta(\mathbb{Q},\boldsymbol{\omega}_m,\boldsymbol{\Delta})\} \ ,$$

we finally parameterize admissible market makers' index investments $\theta_{mI}\in[\bar{\theta}_{mI},1]$ as:

$$\theta_{mI}(\alpha)=\bar{\theta}_{mI}+\alpha\,(1-\bar{\theta}_{mI}) \ ; \quad \alpha\in[0,1] \ . \tag{17}$$

Similar to the previous section, we study the implications of various index investment configurations $\theta_{mI}(\alpha)$. The time series of market makers' inferred expected market returns is displayed in Figure 5, while Table 2 (Panel B) provides summary descriptive statistics for these variables.

Since during most of our sample period market makers take opposite positions in options relative to customers ($\boldsymbol{\omega}_m=-\boldsymbol{\omega}_c$), they often maintain short positions in OTM puts and long positions in OTM calls. This evidence is especially pronounced until around 2013 and results in negative delta hedged positions that contribute negatively to market makers' expected returns. Conversely, the positive covariance term $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_m'\boldsymbol{O}, R)$ for market makers contributes positively to expected market returns during these periods, producing a positive net contribution of the delta-hedged option portfolio to market makers' expected returns.[20] Despite this evidence, we find that the total impact on market makers' expected returns relative to the benchmark case $\theta_{mI} = 1$ remains relatively modest until around 2013, as illustrated in Figure 5.[21]

More pronounced negative deviations from the benchmark expected return emerge between 2014 and 2020. During this period, the expected return contribution from the delta-hedged option portfolio can become negative. Relative to the benchmark, this leads to lower overall expected returns for market makers, which exhibit a declining trend until around 2018, followed by an upward trend until approximately 2020. Interestingly, while the overall impact on the average expected return relative to the benchmark is moderate across the full sample, we find that after 2014, market makers' expected return can turn negative for average option holdings as low as 1% of the invested wealth ($\alpha = 0.95$). Furthermore, Table 2 (Panel B) highlights that significant differences in market makers' equity premium persistence (as measured by the first-order autocorrelation coefficient) and its correlation with the benchmark expected return emerge when the average index allocation is below 90% over the entire sample period ($\alpha \leq 0.5$).

[20]Recall that $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_m'\boldsymbol{O}, R) = -\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, R)$ since $\boldsymbol{\omega}_m = -\boldsymbol{\omega}_c$.

[21]This is primarily due to the fact that the minimal index investment $\overline{\theta}_{mI}$ necessary to maintain an optimal portfolio in line with no-arbitrage conditions during these periods is typically quite large. Consequently, the admissible parametric index investment $\theta_{mI}(\alpha)$ can only deviate modestly from the benchmark value $\theta_{mI} = 1$. Relative to the benchmark, this feature results in small negative expected return contributions, which are largely offset by the positive contributions due to the delta-hedged option portfolio.

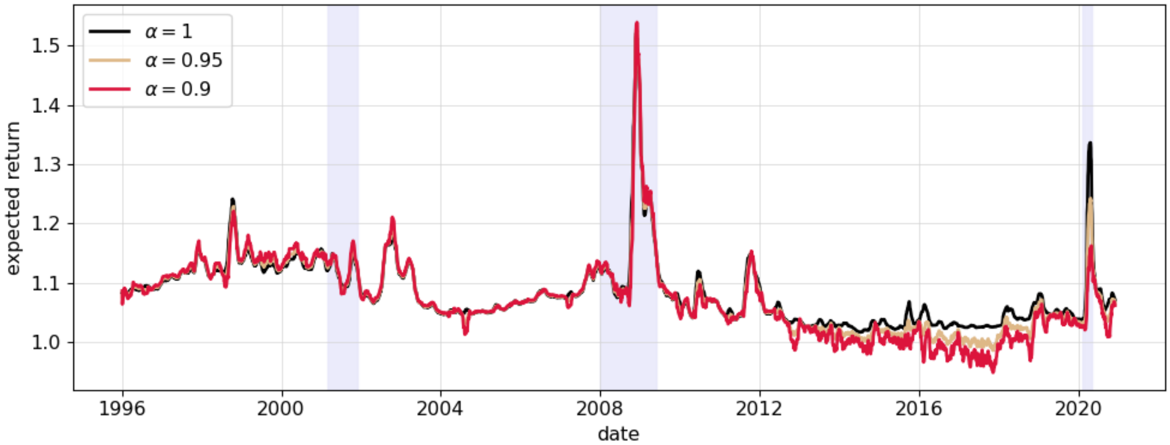


Figure 5. Market Makers subjective expected returns

Notes: This figure the time-series of the expected market return implied by market makers' options holdings for different levels of index investment $\theta_{mI}(\alpha)$. $\alpha = 1$ corresponds to the monthly expected return recovered by the benchmark $M^0 = 1/R$ ($\theta_{mI}(1) = 1$). Frequency is daily, horizon is monthly, values are annualized. Data are 30-days moving averages. Gray bars indicate NBER recessions.

4.4 Subjective Risk-Return Tradeoff

Using Proposition 1, the investor's subjective market variance is given by:

$$\mathbb{V}ar^i(R) = \mathbb{V}ar^{\mathbb{Q}}(R) + \theta_{iI}\mathbb{C}ov^{\mathbb{Q}}(R, V(R)) + (1-\theta_{iI})\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, V(R)) - (\mathbb{E}^i[R-1])^2 \,. \tag{18}$$

It follows that the composition of an investor's portfolio influences subjective variance through two primary channels. First, it affects the squared subjective market equity premium. Second, it shapes the risk-neutral co-leverage characteristics of the optimal portfolio return with respect to the realized market variance, as captured by the term:

$$\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, V(R)) = \theta_{iI}\mathbb{C}ov^{\mathbb{Q}}(R, V(R)) + (1-\theta_{iI})\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, V(R)) \,. \tag{19}$$

As shown in Table 3 and Figures 6A–6C, the contribution of the first channel to subjective variance is an order of magnitude smaller than that of the second. Consequently, for practical purposes, the co-leverage term in equation (19) usually drives the deviation of subjective variance from implied variance. This term is determined by two components: the market leverage coefficient $\mathbb{C}ov^{\mathbb{Q}}(R, V(R))$ and the co-leverage coefficient $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, V(R))$, which captures the comovement between the returns of the option portfolio and realized market variance.

Empirically, the first leverage component is negative, as expected, and an order of magnitude smaller than the implied variance, as shown in Figure 6B. Conversely, the sign and magnitude of the second co-leverage component depend on the structure of the option portfolio. For instance, a portfolio that replicates a long (short) position in realized market variance $V(R)$ will exhibit a positive (negative) covariance. Consequently, the overall impact on subjective variance remains ambiguous ex-ante, as it is determined by the relative strength and direction of both the market leverage component and the option portfolio's co-leverage component.

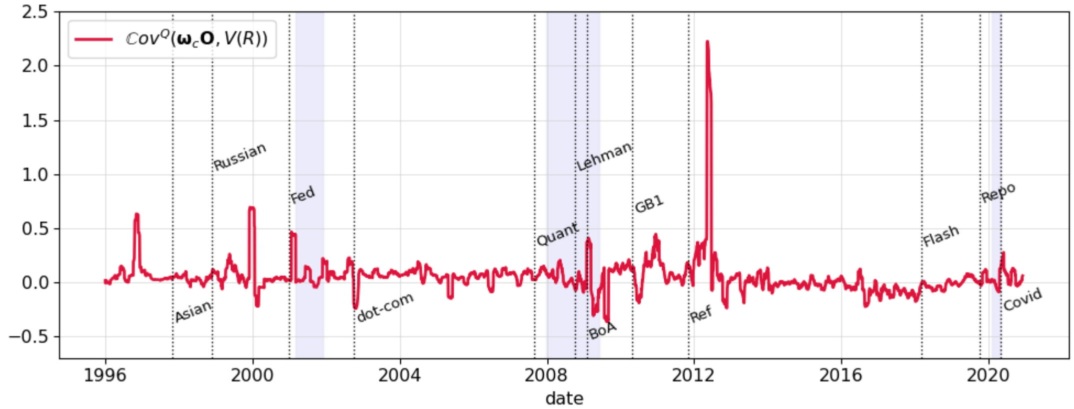
As shown in Figure 6A, the customers' co-leverage component $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, V(R))$ is largely positive, with rare exceptions, and typically an order of magnitude greater than the market implied variance, especially prior to 2013. As a result – even for quite small allocations to options – customers' subjective variance in Figure 7A is generally higher than the benchmark variance perceived by an investor fully invested in the market alone ($\alpha = 1$). This feature is more pronounced in the sample period before 2013, because there customers' index allocations more easily deviate from the benchmark. Symmetrically, the co-leverage component $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_m'\boldsymbol{O}, V(R))$, which does not incorporate delta hedging effects, is usually negative and large before 2013, a period in which market makers' index allocations only moderately deviate from the benchmark. Instead, it can become positive after 2013, when makers' index allocations more substantially deviate from the benchmark. In such instances, the contribution of the delta hedging portfolio to the subjective variance, embedded in co-leverage component $\mathbb{C}ov^{\mathbb{Q}}(-\boldsymbol{\omega}_m'\boldsymbol{\Delta}R, V(R))$ is also small. Therefore, as shown in Figure 8A, the market makers' subjective variance is also generally larger than the benchmark variance of an investor fully invested in the market, especially after 2013.

Finally, the joint effects of a generally lower expected market return and higher market volatility typically result in uniformly lower and substantially more volatile Sharpe ratios, as shown in Figures 7B and 8B, compared to those of an investor fully invested in the market. This pattern is particularly pronounced for customers before 2013 and for market makers after 2013.

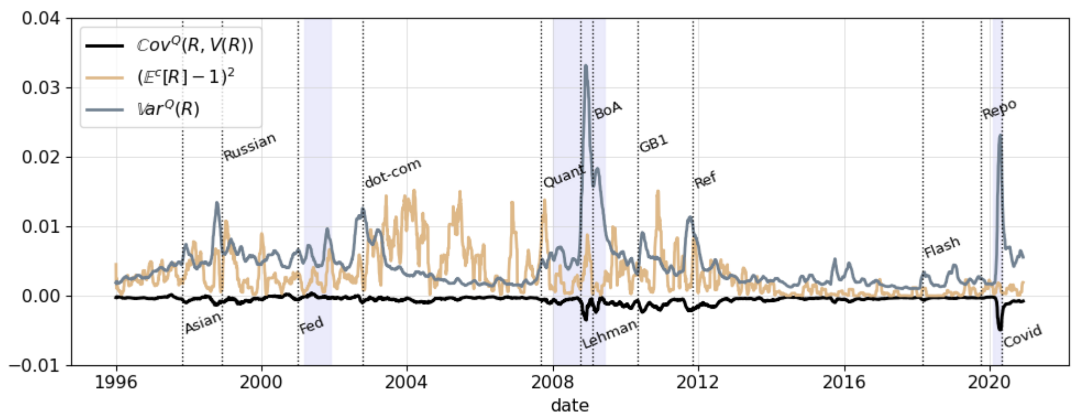
Table 3. Components subjective variance.

	mean	std	min	q25	q50	q75	max	corr (%)
$\mathbb{V}ar^{\mathbb{Q}}(R)$	.0044	.0038	.0010	.0021	.0035	.0055	.0332	100
$\mathbb{C}ov^{\mathbb{Q}}(R, V(R))$	-.0006	.0005	-.0049	-.0007	-.0004	-.0003	.0004	-63
$\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, V(R))$	.0520	.1828	-.3634	-.0148	.0358	.0883	2.225	4
$\mathbb{C}ov^{\mathbb{Q}}(-\boldsymbol{\omega}_m'\boldsymbol{\Delta}R, V(R))$	.0007	.0017	-.0014	.0000	.0002	.0006	.0158	34
$(\mathbb{E}^c[R]-1)^2$	.0029	.0031	.0000	.0006	.0019	.0041	.0152	12
$(\mathbb{E}^m[R]-1)^2$	.0013	.0018	.0000	.0001	.0006	.0019	.0192	27

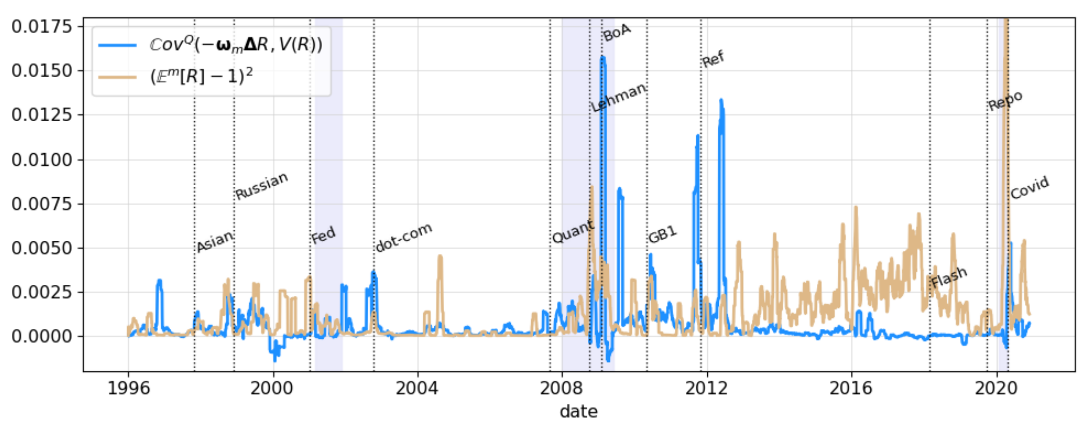
Notes: This table reports summary statistics for the main components of the subjective variance as in eq. (18). For both Customers and Market Makers, $\mathbb{E}^i[R]$ is the perceived expected return when $\alpha = 0$ ($\theta_{cI} = \bar{\theta}_{cI}$ and $\theta_{mI} = \bar{\theta}_{mI}$). «corr» is the correlation with the SVIX. Data are the 30-days moving averages, values are monthly. Data runs from January 1996 to December 2020.



A. Customers' Option Leverage coefficient



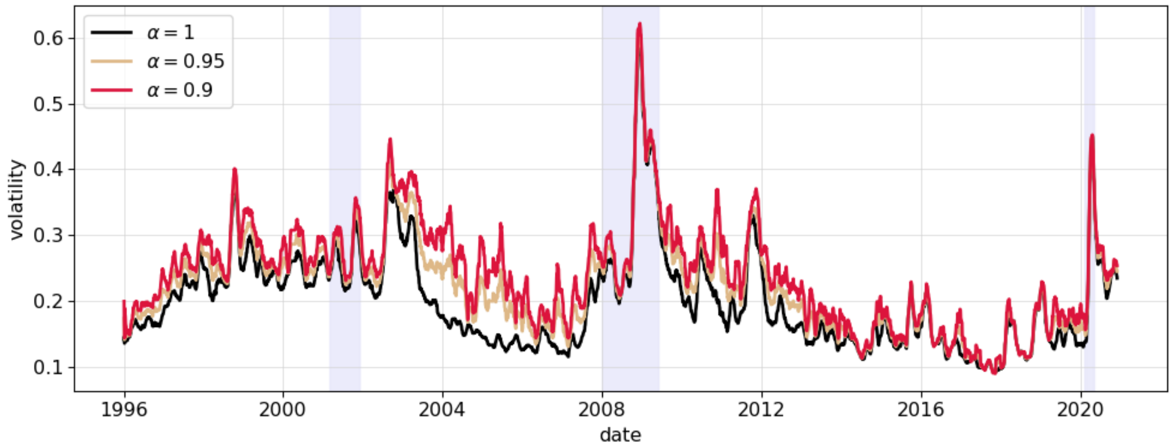
B. Market Leverage Coefficient, Risk neutral Variance and squared Customers' Equity Premium



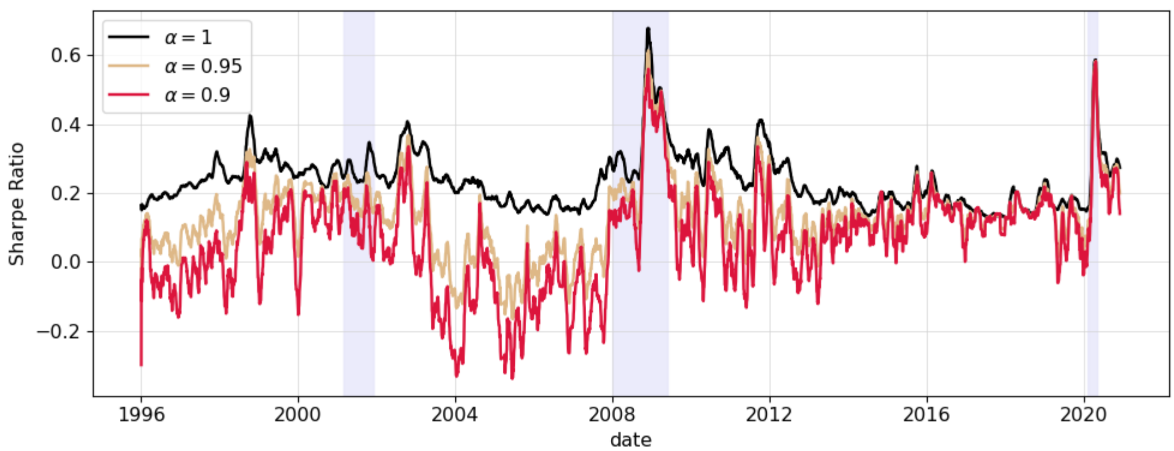
C. Market Makers' Leverage Coefficient from Δ-hedging and squared Market Makers' Equity Premium

Figure 6. Components of the subjective variance $\mathbb{V}ar^i(R)$

Notes: Panel 6A plots the time series of the co-leverage $\mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\omega}_c'\boldsymbol{O}, V(R))$ implied by the Customers' option holdings. Panel 6B plots the co-leverage $\mathbb{C}ov^{\mathbb{Q}}(R, V(R))$ implied by a zero investment in options, the Customers' risk premium $(\mathbb{E}^c[R]-1)^2$ for $\alpha = 0$, and the risk neutral variance $\mathbb{V}ar^{\mathbb{Q}}(R)$. Panel 6C plots the co-leverage $\mathbb{C}ov^{\mathbb{Q}}(-\boldsymbol{\omega}_m'\boldsymbol{\Delta}R, V(R))$ implied by Market Makers' delta-hedging, and the Market Makers' risk premium $(\mathbb{E}^m[R]-1)^2$ for $\alpha = 0$. Data are 30-days moving averages. Values are monthly. Gray bars indicate NBER recessions. Some major events are highlighted in the graph: the Asian financial crisis (Oct. '97); the Russian financial crisis (Nov. '98); the first Fed rate cut (Jan. '01); Nasdaq lowest value («dot-com», Oct. '02); quant-strategy hedge fund crisis (Aug. '07); the Lehman bankruptcy (Sep. '08); the Bank of America rescue («BoA», Jan. '09); Greek bailout installment («GB1», Apr. '10); call for Referendum about financial aid in Greece («Ref», Oct. '11); a Flash crisis (Feb. '18); sharp repo rate increase (Sept. '19); Covid-19 pandemic (Apr. '20).



A. Customers' Subjective Volatility



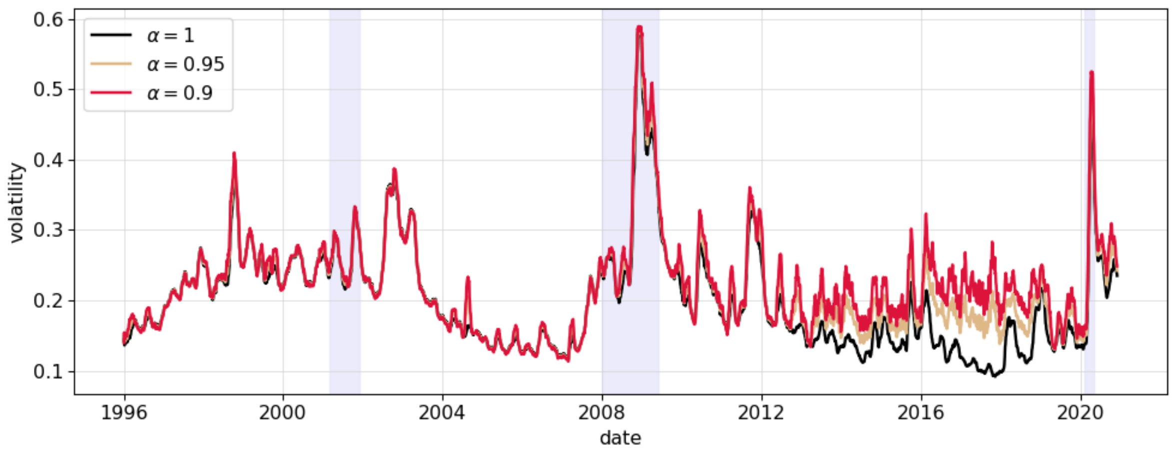
B. Customers' Subjective Sharpe Ratio

Figure 7. Subjective Volatility and Sharpe Ratio of Customers
Notes: This figure plots the time-series of the subjective volatility (Panel 7A) and the subjective Sharpe Ratio (Panel 7B) as recovered through the SDF supported by the observed option positions of Customers, for different levels of the underlying investment indexed by α. $\alpha = 1$ corresponds to $\theta_{cI} = 1$. Frequency is daily, horizon is monthly, values are annualized 30-days moving averages. Gray bars indicate NBER recessions.

Table 4. Summary statistics of subjective volatility and Sharpe ratio.

	α	mean	std	min	q25	q50	q75	max	corr (%)
				Panel A: Customers					
Volatility	1	0.199	0.074	0.092	0.142	0.178	0.238	0.583	90
	95%	0.221	0.075	0.091	0.166	0.216	0.258	0.602	87
	90%	0.238	0.080	0.090	0.178	0.237	0.284	0.621	81
	80%	0.265	0.091	0.089	0.196	0.265	0.322	0.654	70
	50%	0.322	0.124	0.084	0.229	0.319	0.401	0.727	52
	0	0.381	0.165	0.074	0.263	0.369	0.478	0.902	39
Sharpe Ratio	1	0.237	0.081	0.124	0.176	0.225	0.279	0.678	84
	95%	0.136	0.109	-0.165	0.073	0.135	0.190	0.613	61
	90%	0.067	0.141	-0.338	-0.023	0.082	0.151	0.581	43
	80%	-0.040	0.199	-0.792	-0.183	-0.008	0.109	0.580	23
	50%	-0.271	0.335	-1.196	-0.546	-0.226	0.014	0.580	0
	0	-0.575	0.540	-1.985	-1.003	-0.518	-0.106	0.591	-13
				Panel B: Market Makers					
Volatility	1	0.199	0.074	0.092	0.142	0.178	0.238	0.583	90
	95%	0.213	0.070	0.115	0.165	0.198	0.244	0.586	86
	90%	0.223	0.071	0.113	0.176	0.215	0.253	0.589	78
	80%	0.240	0.078	0.111	0.189	0.234	0.276	0.605	64
	50%	0.273	0.106	0.100	0.197	0.258	0.334	0.727	35
	0	0.306	0.148	0.081	0.192	0.271	0.402	0.843	15
Sharpe Ratio	1	0.237	0.081	0.124	0.176	0.225	0.279	0.678	84
	95%	0.203	0.111	-0.106	0.136	0.213	0.276	0.689	79
	90%	0.188	0.135	-0.208	0.104	0.213	0.282	0.700	72
	80%	0.171	0.175	-0.380	0.061	0.206	0.297	0.726	63
	50%	0.159	0.273	-0.734	-0.014	0.251	0.358	0.811	50
	0	0.188	0.422	-1.484	-0.075	0.346	0.492	0.994	40

Notes: This table reports summary statistics for the 30-day moving average of annualized volatility and Sharpe ratio recovered by different portfolios built on customers' (Panel A) and market markers' (Panel B) options positions. «corr» is the correlation with the SVIX. Portfolios differ with respect to the fraction of wealth invested in the index, expressed as a function of α. The case $\alpha = 1$ is the benchmark case recovered by M^0 ($\theta_{cI}(1) = 1$). The case $\alpha = 0$ corresponds to the minimum investment in the index compatible with the no-arbitrage condition ($\theta_{cI}(0) = \bar{\theta}_{cI}$). Data is daily, values are annualized. Data runs from January 1996 to December 2020.



A. Market Makers' Subjective Volatility

B. Market Makers' Subjective Sharpe Ratio

Figure 8. Subjective Volatility and Sharpe Ratio of Market Makers
Notes: This figure plots the time-series of the subjective volatility (Panel 8A) and the subjective Sharpe Ratio (Panel 8B) as recovered through the SDF supported by the observed option positions of Market Makers, for different levels of the underlying investment indexed by α. $\alpha = 1$ corresponds to $\theta_{mI} = 1$. Frequency is daily, horizon is monthly, values are annualized 30-days moving averages. Gray bars indicate NBER recessions.

4.5 Consensus Beliefs and Disagreement

We now turn to constructing a consensus belief from the time series of expected returns for customers and market makers. As defined in equation (8), the consensus belief is a wealth-weighted average of the agents' individual expected returns:

$$\bar{\mathbb{E}}[R] = \sum_{i=c,m} w_i \mathbb{E}^i[R].$$

As is well recognized in heterogeneous-agent models, speculative behavior arising from divergent beliefs can significantly alter the endogenous wealth distribution. In extreme cases, one group may be driven out of the market altogether, and an equilibrium may not exist; see, e.g., Yan and Xiong [2010]. In our earlier analysis, we did not explicitly prevent such divergence. To ensure a stable wealth distribution across both agents, we impose a constraint that equalizes total wealth between customers and market makers.[22]

In addition to the consensus belief, we also calculate a measure of belief disagreement as in equation (12). The resulting time series for $\bar{\mathbb{E}}[R]$ and $\mathbb{D}(R)$ are presented in Figure 9, with summary statistics provided in Table 5, for the benchmark case where $\theta_{cI} = \theta_{mI} = \bar{\theta}_I$, a choice that ensures that the total wealth is equally shared between customers and market makers. The consensus belief shown in Figure 9A is typically

[22]More specifically, we first define the market index allocation that satisfies the no-arbitrage condition for both customers and market makers as:

$$\bar{\theta}_I := \max(\bar{\theta}_{cI}, \bar{\theta}_{mI}). \tag{20}$$

Second, we parameterize a common admissible index allocation for both customers and market makers as:

$$\theta_I(\alpha) = \bar{\theta}_I + \alpha(1 - \bar{\theta}_I), \quad \alpha \in [0, 1]. \tag{21}$$

Finally, we constrain differences between the index allocations of customers and market makers, denoted by $\theta_{iI}(\alpha)$ $(i = c, m)$, by imposing the following constraint:

$$\left| \frac{\theta_I(\alpha) - \theta_{iI}(\alpha)}{1 - \theta_I(\alpha)} \right| \leq \epsilon_i, \tag{22}$$

where $\epsilon_i > 0$ is a small constant. This constraint ensures that the fractions of wealth not allocated to index investment, given by $1 - \theta_I(\alpha)$ and $1 - \theta_{iI}(\alpha)$, remain sufficiently similar:

$$1 - \epsilon_i \leq \frac{1 - \theta_{iI}(\alpha)}{1 - \theta_I(\alpha)} \leq 1 + \epsilon_i, \tag{23}$$

guaranteeing that the actual wealth of both investors under their optimal allocation can be made economically comparable.

Table 5. Summary statistics of consensus belief and belief disagreement for $\theta_{cI} = \theta_{mI} = \bar{\theta}_I$

	mean	std	min	q25	q50	q75	max	corr (%)
$\bar{\mathbb{E}}[R]$	1.075	0.052	1.015	1.039	1.065	1.105	1.488	99
$\mathbb{D}(R)$	0.024	0.021	0	0.008	0.018	0.035	0.134	72

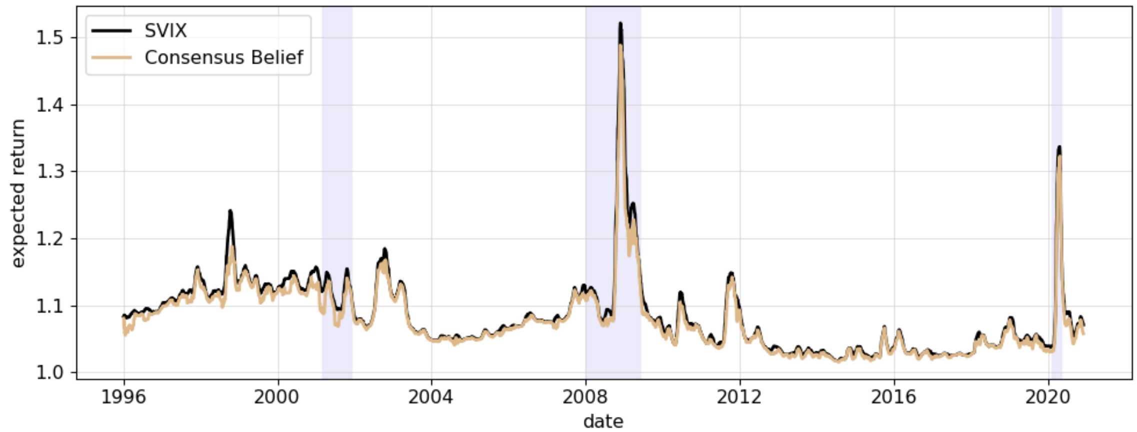
Notes: This table reports summary statistics for the consensus belief $\bar{\mathbb{E}}[R]$ and the belief disagreement $\mathbb{D}(R)$ as defined in the main text, computed when the wealth fraction invested in the index is equal to $\bar{\theta}_I = \max(\bar{\theta}_{cI}, \bar{\theta}_{mI})$ for both customers and market makers. Data are annualized daily values. «corr» is the correlation with the SVIX. Data runs from January 1996 to December 2020.

slightly smaller than the SVIX, because, customers and market makers have in aggregate a net zero allocation to options and they are also slightly under-invested in the index:

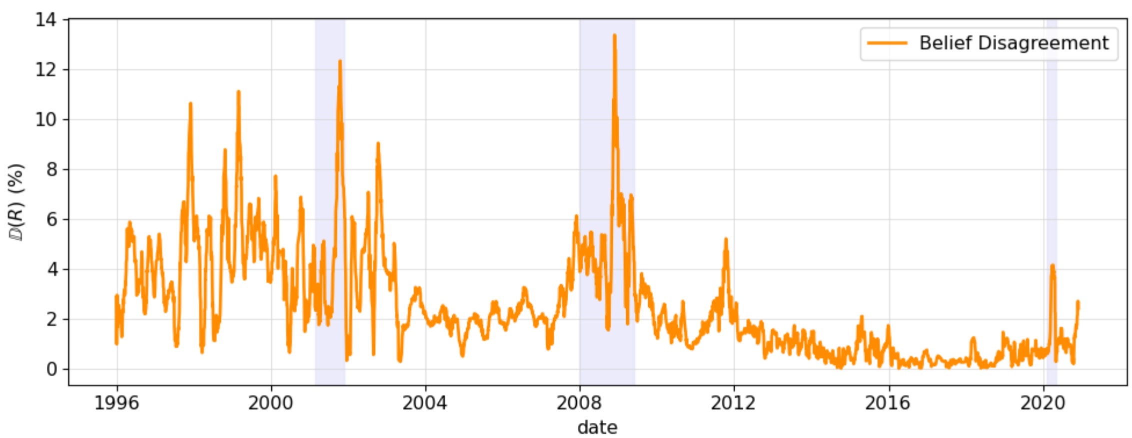
$$\bar{\mathbb{E}}(R) = 1 + \bar{\theta}_I \mathbb{V}ar(R), \tag{24}$$

under parameterization $\theta_{cI} = \theta_{mI} = \bar{\theta}_I \leq 1$. Specifically, since the underlying no-arbitrage condition for both customers and market makers requires that $\bar{\theta}$ remains within a narrow range around 1, this economy is designed to produce a consensus belief that mirrors the cyclical properties of the SVIX, as confirmed also by the summary statistics in Table 5.

The belief disagreement index, shown in Figure 9B, exhibits in part countercyclical dynamics, with the two largest spikes occurring during the first two recessions in our sample. Interestingly, under the given parameterization, the disagreement between customers and market makers is not particularly large during the COVID-19 pandemic. However, it is notably larger earlier in the sample, especially before the burst of the Nasdaq bubble. Table 5 further highlights that $\mathbb{D}(R)$ is partially positively correlated with the consensus belief, and thus with SVIX, but may also experience large spikes at different time points than these variables.



A. Consensus Belief among investors



B. Belief Disagreement among investors

Figure 9. Consensus belief and disagreement between Customers and Market Makers, $\theta_{cI} = \theta_{mI} = \bar{\theta}_I$
Notes: This figure plots the consensus belief (Panel 9A) and the belief disagreement (Panel 9B) as defined in the main text. Frequency is daily, horizon is monthly. Values are annualized 30-day moving averages. Gray bars indicate NBER recessions.

5 Extensions

In this section, we study two possible extensions of our framework. We first show how to use survey data to infer optimal holdings and then study the effect of time-varying risk aversion.

5.1 Survey Data

Up to this point, our empirical approach has relied on assumptions about investors' optimal index allocations. We now extend the analysis by incorporating survey data to infer hypothetical optimal index holdings for log-utility investors that are consistent with both (i) the observed dynamics of option prices and positions, and (ii) the evolution of survey-based expectations. This allows us to evaluate whether demand-based expected returns—when aligned with survey expectations—can be rationalized by economically plausible optimal portfolios that also satisfy key constraints, such as the no-arbitrage conditions outlined in equation (15).

Figure 10 presents expected return time series from three survey sources we consider: the Livingston Survey (Federal Reserve Bank of Philadelphia), the Graham and Harvey CFO Survey [Ben-David, Graham, and Harvey 2013], and the individual investor series compiled by Nagel and Xu [2023].[23] The Livingston Survey is conducted semi-annually, in June and December, and polls a broad group of economists from financial and non-financial institutions, academia, labor organizations, government agencies, and insurance companies. The Graham and Harvey survey collects quarterly data from corporate financial officers regarding their expectations for S&P 500 returns. The series by Nagel and Xu [2023] is also available at a quarterly frequency. It extends the UBS/Gallup investor survey both backward and forward in time by integrating data from other sources, including the Conference Board Survey and the Michigan Survey of Consumers.[24]

[23]We do not know the identities of market makers and customers but since they most likely represent a mix between highly sophisticated as well as less sophisticated investors we rely on the three surveys that best represent these groups.

[24]These data were obtained from Zhengyang Xu's website.

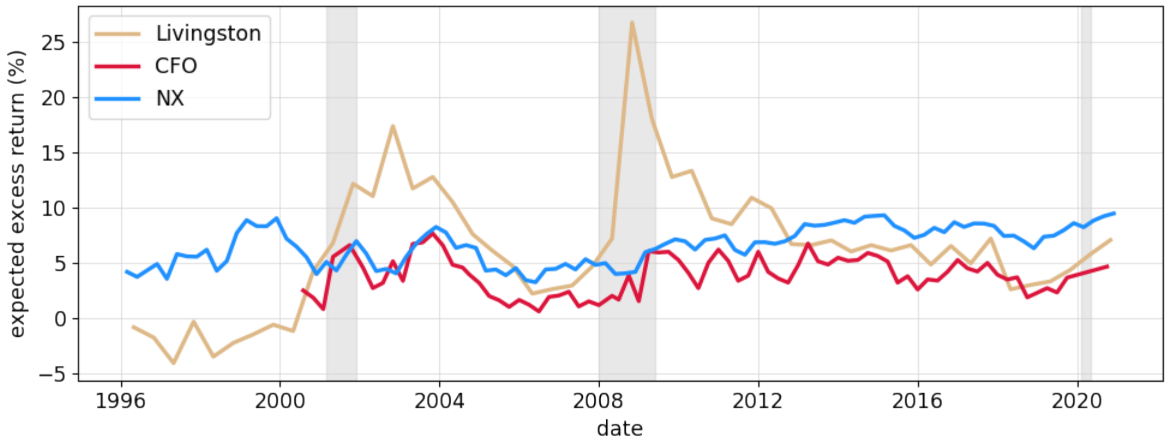


Figure 10. Subjective market equity premia from Survey data

Notes: This figure plots the time-series of survey market equity premia (in percentage) as in the survey data from Livingston, Ben-David, Graham, and Harvey [2013] and Nagel and Xu [2023]. Frequency is quarterly (semi-annual for Livingston), horizon is yearly, values are annualized. Data runs from January 1996 to December 2020. Gray bars indicate NBER recessions.

5.1.1 Holdings Implied by Survey Expected Returns

The surveys we consider are conducted at a lower frequency than our monthly option price and holdings data, and they provide estimates of the one-year-ahead expected return on the S&P 500 index. Since one-year-maturity options suffer from insufficient liquidity to reliably construct demand-based expected returns at that horizon, we adopt a simpler approach. Specifically, we compute the average monthly demand-based expected returns over the sampling window of each survey – i.e., over a semi-annual window for the Livingston survey and a quarterly window for both the Nagel and Xu [2023] series and the Graham and Harvey survey. These average expected returns depend on a single unknown parameter—the index allocation θ_{iI}—which can be calibrated to align the demand-based expected return with the corresponding survey-based expected return.

More specifically, given a survey-implied expected return $\mathbb{E}^s[R]$, we exploit equation (14) to solve for the index allocation θ_{iI} such that:

$$\mathbb{E}^s[R] = 1 + \theta_{iI}\overline{\mathbb{V}ar}^{\mathbb{Q}}(R) + (1 - \theta_{iI})\overline{\mathbb{C}ov}^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, R)\,, \tag{25}$$

where $\overline{\mathbb{V}ar}^{\mathbb{Q}}(R)$ and $\overline{\mathbb{C}ov}^{\mathbb{Q}}(\boldsymbol{\omega}_i'\boldsymbol{O}, R)$ denote the average risk-neutral variance of market returns and the average risk-neutral covariance between the investor's option position and the market return, respectively, computed over the survey's sampling window.

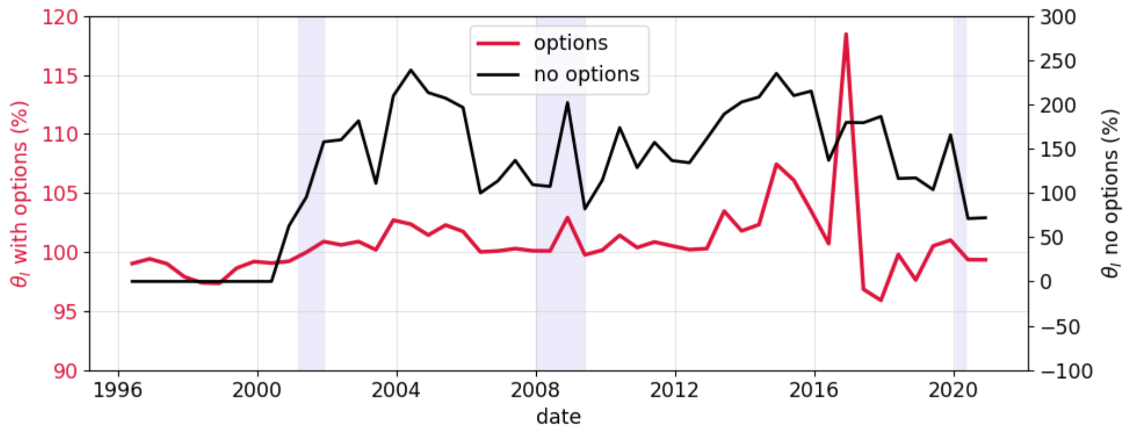
We solve equation (25) under two different scenarios: (i) one that imposes zero net investment in options, i.e., a pure allocation between the index and the risk-free asset ($\boldsymbol{\omega}_i = \mathbf{0}$), and (ii) another that incorporates observed option holdings into the construction of demand-based expected returns. In both cases, we report the corresponding no-arbitrage bounds on index allocations, as defined by the set in equation (15).[25]

Figure 11 presents the time series of index allocations for both cases. Two key findings emerge. First, in the absence of option investments, aligning demand-based expected returns with survey-based expectations requires highly volatile index allocations that often imply extreme leverage—reaching levels of approximately 250% across all surveys. Second, these allocations frequently hit the theoretical upper bound on index investment required to ensure consistency with no-arbitrage conditions—a phenomenon observed in 44%, 12%, and 29% of the cases for the Livingston, CFO, and NX surveys, respectively. When this constraint binds, a perfect alignment with survey-based expected returns becomes infeasible.

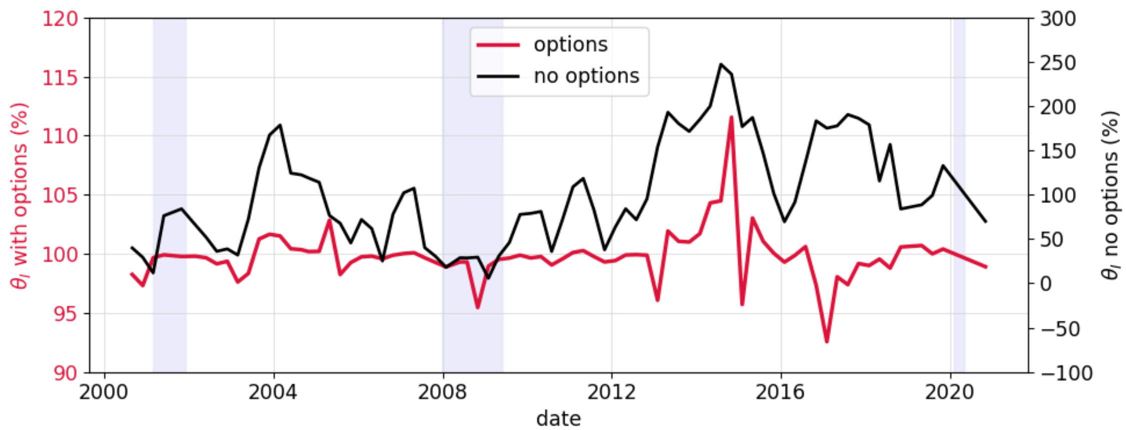
Second, once option holdings are incorporated, aligning demand-based expected returns with survey-based expectations yields index allocations that are smoother over time and remain close to a 100% investment in the index. The resulting demand-based expected returns are also more consistent with no-arbitrage conditions: the no-arbitrage bounds are violated in only 4%, 5%, and 8% of cases for the Livingston, CFO, and NX surveys, respectively.

Overall, the evidence indicates that incorporating option holdings enhances the consistency between survey-based and model-implied expected returns, doing so in a manner that is more readily justified by economically plausible index allocation dynamics and no-arbitrage conditions.

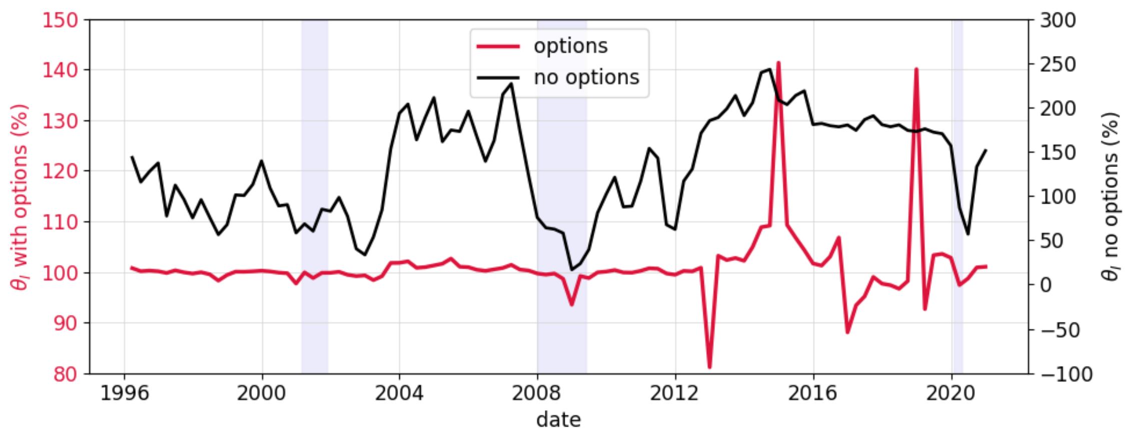
[25]Apart from these no-arbitrage bounds, we do not impose additional constraints on the admissible index allocations required to align demand-based and survey-expected returns, meaning that leveraged index positions with $\theta_{iI} > 1$ are permitted in principle. In such cases, the investor's option holdings relative to total wealth are represented by a weight vector $(1 - \theta_{iI})\boldsymbol{\omega}_i$ with $1 - \theta_{iI} < 0$.



A. Index investment for Livingston survey



B. Index investment for CFO survey



C. Index investment for NX survey

Figure 11. Survey-data implied index investment

Notes: This figure plots the index investment θ_{iI} that is necessary to match the expected market return from the Livingston survey (Panel 11A), the CFO survey (Panel 11B) and NX survey data (Panel 11C). The index investment is computed for the investor with option holdings $\boldsymbol{\omega}_c$ (left scale, in red) and for the investor who takes no position in options (right scale, in black). Whenever the optimal θ_{iI} violates the no-arbitrage bounds, we replace the value with the bound itself. Frequency is quarterly (semi-annual for Livingston). Gray bars indicate NBER recessions.

5.1.2 Subjective Risk-Return Tradeoff and Survey Data

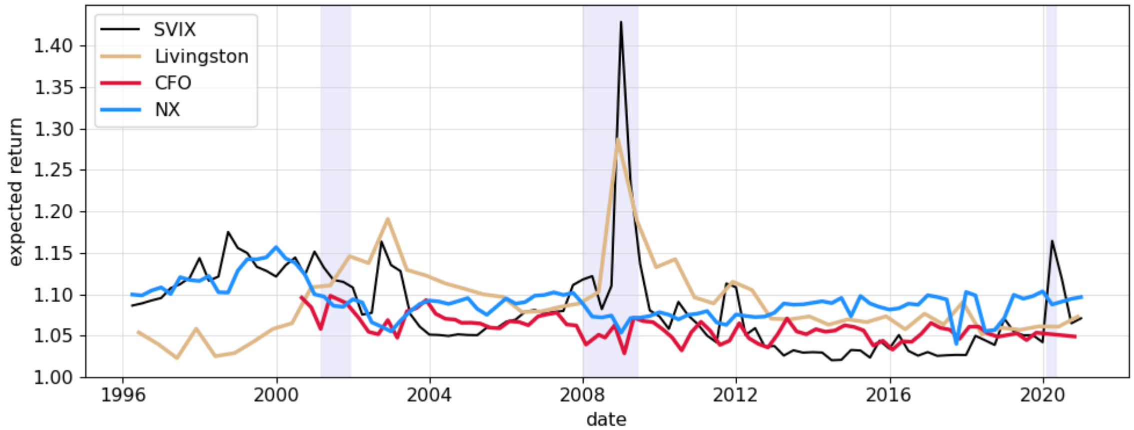
Figure 12 plots the time series of subjective expected returns, market return variances, and risk-return tradeoffs, measured by Sharpe ratios.[26] For comparison, we also report these quantities for a log investor fully invested in the index $(\theta_{iI} = 1)$.

Overall, we find that the demand-based subjective variances implied by survey expected returns exhibit remarkably similar levels and cyclicality across the different surveys, quite closely matching the subjective variance of a log-utility investor fully invested in the index $(\theta_{iI} = 1)$. As a result, the documented differences in survey expected returns translate into a corresponding heterogeneity in both the levels and dynamics of the resulting subjective Sharpe ratios.

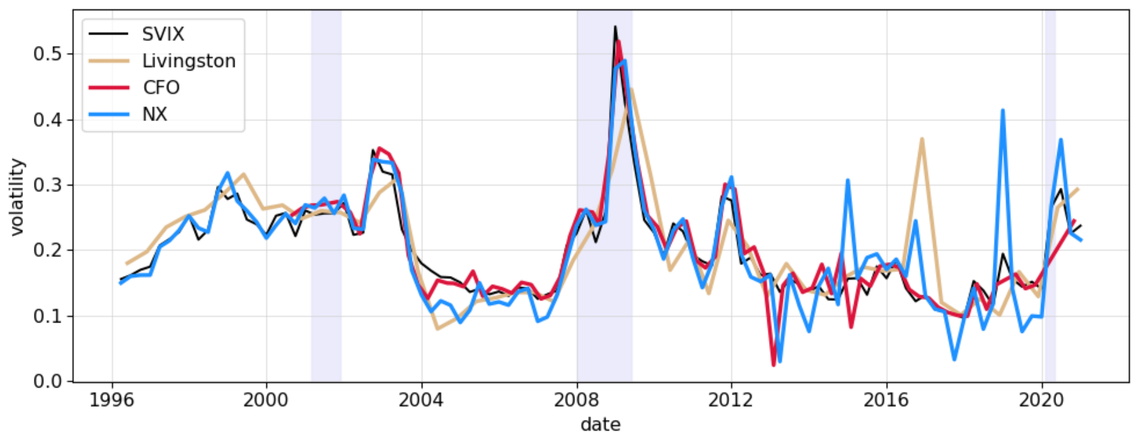
Among the subjective demand-based Sharpe ratios of investors holding options, the CFO survey typically yields the most pessimistic values, while the Livingston and NX surveys tend to produce the most optimistic ratios—during the periods 2001–2013 and 2013–2020, respectively. Notably, the cyclicality of these Sharpe ratios appears largely unrelated to recessions, with the exception of the Livingston survey, whose Sharpe ratios show some alignment with recessionary periods. In contrast, the subjective Sharpe ratios of a log-utility investor fully invested in the index are generally lower—often substantially so—and exhibit much less volatility over time. These benchmark Sharpe ratios display more pronounced countercyclicality, consistent with the countercyclical behavior of the corresponding expected returns.

In summary, our findings suggest that the dynamic behavior of expected returns and Sharpe ratios—when inferred under the assumption that investors are fully invested in the market—may be fragile. Even moderate option exposure can significantly influence both the level and the cyclicality of these measures, potentially aligning them more closely with patterns observed in survey data and consistent with economically plausible market-timing behavior.

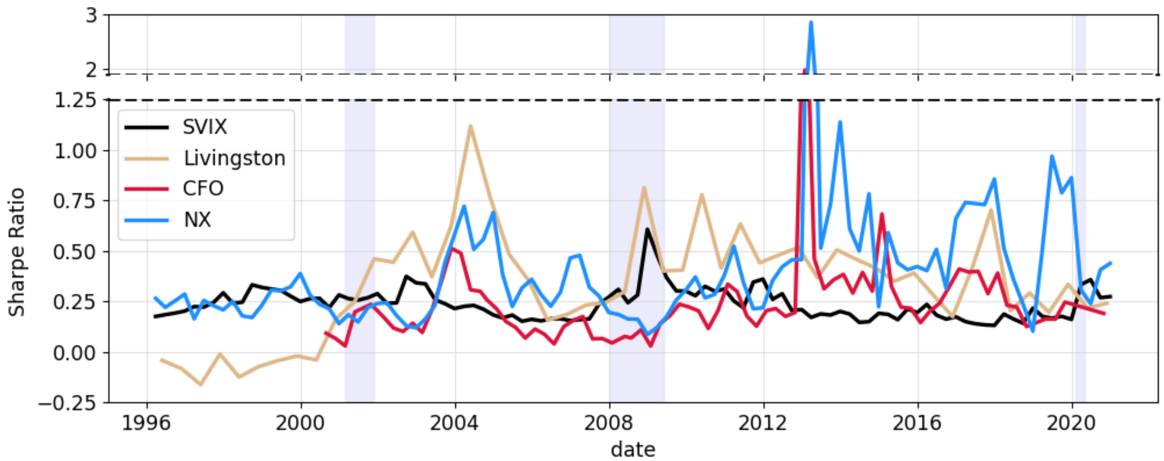
[26] We report the corresponding SDFs in the Appendix.



A. Demand-based subjective expected returns aligned from survey expected returns



B. Demand-based subjective volatility aligned from survey expected returns



C. Demand-based subjective Sharpe ratio aligned from survey expected returns

Figure 12. Demand-based moments $\mathbb{E}^i[R]$, $\mathbb{V}ol^i(R)$, and Sharpe ratio $SR^i(R)$ aligned from survey data

Notes: This figure plots the demand-based market expected return (Panel 12A), market volatility (Panel 12B) and market Sharpe ratio (Panel 12C) as reconstructed from survey data, considering investors with the same option holdings as the customers in our data and associated index investment from Figure 11 satisfying no-arbitrage constraints. The benchmark case reproduced under a log utility investor fully invested in the market index (SVIX) is also displayed for comparison. Frequency is quarterly (semi-annual for Livingston), values are annualized. Gray bars indicate NBER recessions.

5.2 Time-Varying Risk Aversion

Our basic framework can be naturally extended to accommodate SDF specifications that do not assume a log-utility investor. For example, consider a power utility SDF of the form:

$$M_i \propto (1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{-\gamma} , \tag{26}$$

where γ denotes the relative risk aversion parameter. The corresponding subjective expectation of a generic payoff $f(R)$ is given by:

$$\mathbb{E}^i[f(R)] = \frac{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma} f(R)]}{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma}]} , \tag{27}$$

where the change of measure from the risk-neutral probability to the investor's subjective belief now reads:

$$\frac{d\mathbb{P}^i}{d\mathbb{Q}} = \frac{(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma}}{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma}]} . \tag{28}$$

The subjective expected market return takes the following form:

$$\mathbb{E}^i[R] = 1 + \frac{\mathbb{C}ov^{\mathbb{Q}}((1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma}, R)}{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i' \boldsymbol{R}^e)^{\gamma}]} . \tag{29}$$

This extended framework allows us to more broadly investigate the economic implications of key modeling assumptions needed to recover theory-based subjective expected returns that are consistent with survey data. To study the effect of including holdings data, we consider two different settings: (i) Price-based expected returns for investors with time-varying risk aversion who are fully invested in the market, (ii) demand-based expected returns for investors with constant risk aversion who dynamically allocate across both the index and option markets. Using our earlier parameterizations of portfolio weights, setting (i) yields the following expected return:

$$\mathbb{E}^i[R] = 1 + \frac{\mathbb{C}ov^{\mathbb{Q}}(R^{\gamma}, R)}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}]} , \tag{30}$$

which depends solely on the risk aversion parameter γ. In this setting, γ is the only free parameter available to align survey-based expectations with model-implied expected returns, which are constrained to be strictly positive and monotonically increasing in the risk aversion parameter. Figure 13 presents the time series of time-varying risk aversion parameters needed to reconcile model-implied expected returns with survey data under the assumptions of setting (i). By construction, the negative expected returns reported in the Livingston survey prior to 2000 cannot be matched within this framework, even under very low values of risk aversion. Furthermore, while for all surveys the median implied risk aversion parameter is not too different from $\gamma = 1$, consistent with logarithmic utility preferences, the recovered values of γ are often substantially higher and exhibit pronounced time variation during the second half of the sample. This pattern is especially evident in the NX and partly the Livingston surveys, which report relatively high and volatile expected returns over the same period. Finally, the implied risk aversion parameters tend to be lower during recessions—especially in the CFO and NX surveys—indicating a counter-cyclical pattern in risk tolerance that may appear economically counterintuitive.

Based on the earlier parameterizations of portfolio weights, setting (ii) yields the following expected return:

$$\mathbb{E}^i[R] = 1 + \frac{\mathbb{C}ov^{\mathbb{Q}}\left((1+\theta_{iI}(R-1)+(1-\theta_{iI})\boldsymbol{\omega}'\boldsymbol{O}^e)^{\gamma}, R\right)}{\mathbb{E}^{\mathbb{Q}}\left[(1+\theta_{iI}(R-1)+(1-\theta_{iI})\boldsymbol{\omega}'\boldsymbol{O}^e)^{\gamma}\right]}\,, \tag{31}$$

which in principle permits an alignment with survey-based expected returns through a time-varying index exposure θ_{iI}, even under the assumption of a constant risk aversion parameter γ. This formulation allows us to analyze the behavior of the time-varying index exposures θ_{iI} across different values of the constant risk aversion parameter γ.

Unlike the benchmark case of $\gamma = 1$, where expected returns exhibit a relatively straightforward dependence on portfolio weights, the model-implied expected returns for $\gamma \neq 1$ are highly nonlinear functions of the index allocation θ_{iI}, for which there is no a priori guarantee of monotonicity with respect to either θ_{iI} or γ.[27] This nonlinearity poses additional challenges for aligning demand-based expected returns with survey data under general risk preferences. In particular, when the risk aversion parameter satisfies $\gamma \neq 1$, even substantial deviations from the benchmark index allocation $\theta_{iI} = 1$—which corresponds to zero investment in options—can preclude an exact reconciliation between demand-based and survey-implied expected returns.

Figure 14 summarizes the characteristics of index allocations and demand-based expected returns that result from an alignment with survey-based expected returns under general risk preferences. First, we find that even for general levels of risk aversion

[27] Lemma 1 and its proof in Appendix A characterize these nonlinearities more precisely.

($\gamma \neq 1$), violations of the no-arbitrage bounds on index allocations are less frequent than those reported in Section 5.2 using time-varying risk aversions and fixed index allocations $\theta_{iI} = 1$.[28]

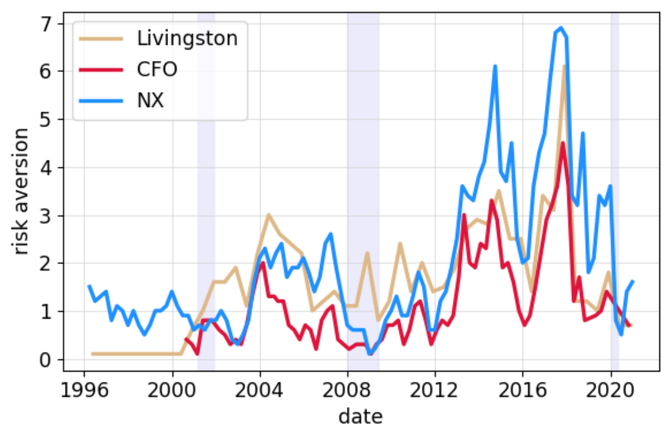
Second, across the considered broad range of admissible index allocations $\theta_{iI} \in [0.5, 1.5]$ and levels of risk aversion in Figure 14, the allocations that minimize the discrepancy between survey- and demand-based expected returns tend to cluster around the benchmark value $\theta_{iI} = 1$, exhibiting both a median close to this benchmark and low dispersion. The exception is the case $\gamma = 1.5$, which shows a lower median allocation and notably higher variability.

Third, with the exception of the benchmark log-utility case – and, to a lesser extent, the case of $\gamma = 1.25$ – all other settings generally exhibit a substantial misalignment between survey- and demand-based expected returns. This misalignment is reflected in consistently sizable relative errors with respect to the survey-implied equity premia and is economically undesirable, as these represent the smallest achievable discrepancies over a wide range of admissible index allocations, $\theta_{iI} \in [0.5, 1.5]$.[29]

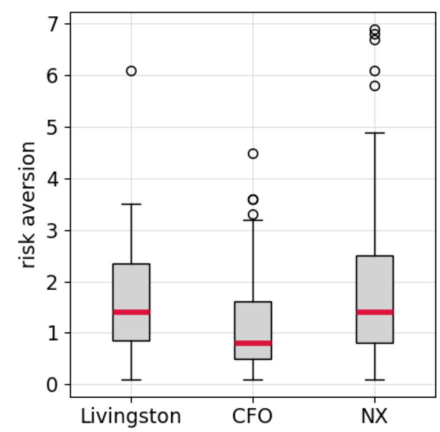
Overall, the evidence in this section reinforces the conclusion that demand-based expected returns generated under the benchmark log-utility specification can be reconciled with survey-based beliefs through an economically plausible market timing behavior and relatively modest option exposures.

[28] On average, no-arbitrage violations occurred in less than 6% of cases across surveys and considered values of risk aversion, except for $\gamma = 1.25$, where violations arose in 24% (Livingston), 17% (CFO), and 22% (NX) of the cases, respectively.

[29] Unreported results for higher levels of risk aversion show even larger discrepancies.



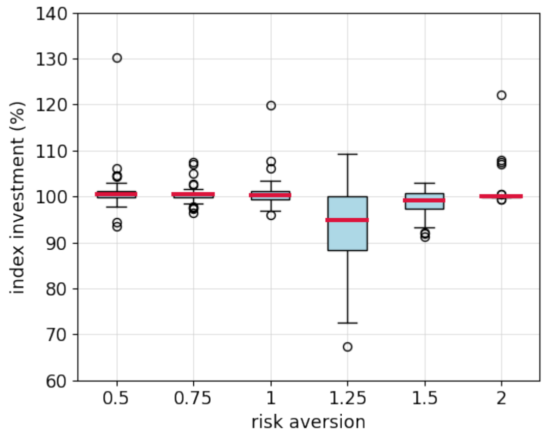
A. Time Series



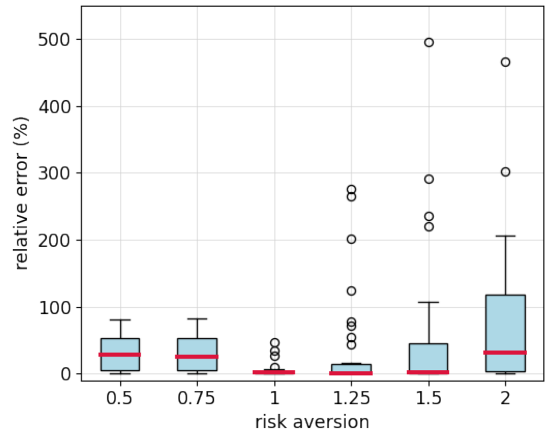
B. Statistics

Figure 13. Time-varying Risk Aversion based on Survey Data

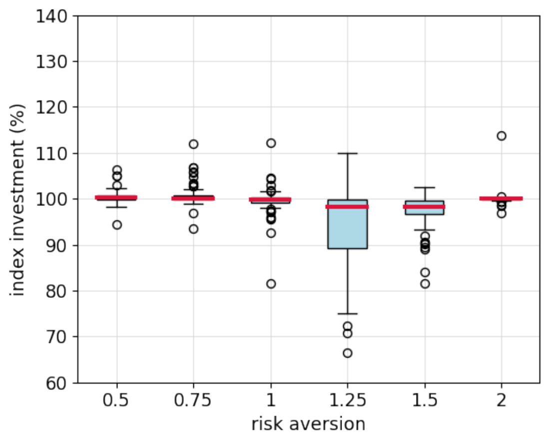
Notes: This figure plots the risk aversion γ that is necessary to match the expected market return from the survey data, assuming a full investment in the index (no options nor risk-free investment). We consider a discrete range of admissible values for $\gamma \in [0, 8]$. Expected returns are matched with a (numerical) relative error of less than 2%. Gray bars indicate NBER recessions.



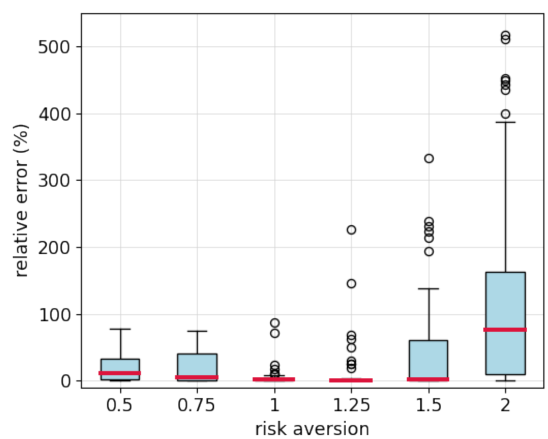
A. θ_{iI} Livingston Survey



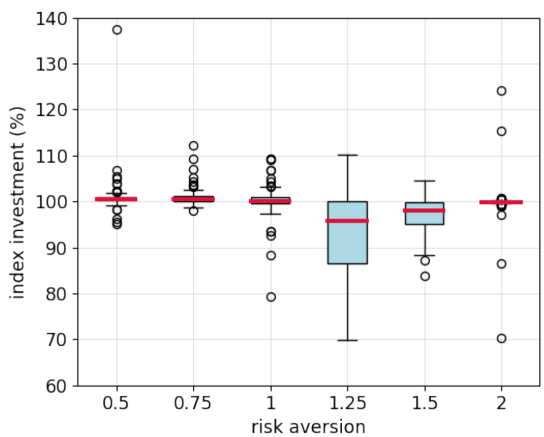
B. Relative error Livingston Survey



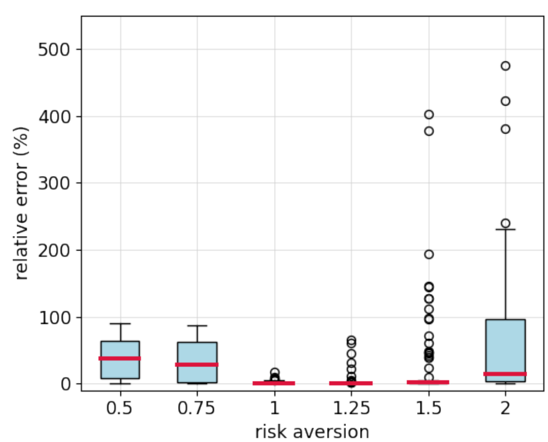
C. θ_{iI} CFO Survey



D. Relative error CFO Survey



E. θ_{iI} NX Survey



F. Relative error NX Survey

Figure 14. Relative error when matching survey data risk premia for different risk aversions

Notes: The left panels of the figure display the distribution of index allocations θ_{iI} required – together with customers' option positions $(\boldsymbol{\omega}_i)$ – to match the survey-implied risk premia across different levels of risk aversion γ. The right panels show the corresponding distributions of the relative errors in these risk premia, which arise due to imperfect matching. The optimal index allocation θ_{iI} is selected from a discrete grid of 1,000 points over the interval $[0.5, 1.5]$. Observations that violate the no-arbitrage condition are excluded from the analysis.

6 Conclusion

A well-established literature has documented the pronounced counter-cyclicality and high variability of expected market return proxies implied by option prices. More recently, a growing body of research has emphasized systematic differences in survey-based return expectations, particularly in dependence of investor sophistication.

This paper develops a simple framework that leverages demand-side data to recover investor beliefs. By jointly analyzing option holdings and prevailing market prices, we provide a unified approach to inferring individual, demand-based expected returns and subjective risk assessments. Among sophisticated investors acting as liquidity providers or demanders, we find that both expected returns and subjective Sharpe ratios can vary substantially—differing in magnitude, dynamics, and cyclicality—even with moderate option exposures relative to overall market allocations. Moreover, these demand-based expected returns can be further reconciled with survey-based beliefs under economically plausible market timing strategies.

In general, we conclude that expected return proxies derived exclusively from market prices may overlook substantial heterogeneity in investor beliefs and impose overly restrictive assumptions on the dynamics of those beliefs. While our current framework abstracts from trading frictions, such frictions likely play a central role in shaping how portfolio holdings respond to investor expectations. Prior research (e.g., Giglio et al. [2021]) shows that this responsiveness tends to increase as trading costs decline. In future work, we plan to extend our demand-based framework to incorporate trading frictions, enabling a more comprehensive analysis of how expectations are formed and reflected in investor portfolios.

A Proofs

Proof of Proposition 1. From equation (2) and equation (5), we obtain:

$$\begin{aligned}
\mathbb{V}ar^i(R) &= \mathbb{E}^{\mathbb{Q}}\left[R^2\right] + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, R^2) - (1 + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, R))^2 \\
&= \mathbb{V}ar^Q(R) + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, R^2 - 2R) - (\mathbb{E}^i[R-1])^2 \\
&= \mathbb{V}ar^Q(R) + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, R^2 - 1 - 2(R-1)) - (\mathbb{E}^i[R-1])^2 \\
&= \mathbb{V}ar^Q(R) + \mathbb{C}ov^{\mathbb{Q}}(\boldsymbol{\theta}_i'\boldsymbol{R}, V(R)) - (\mathbb{E}^i[R-1])^2 \, .
\end{aligned}$$

This concludes the proof. ■

Proof of Proposition 2. We first obtain, using investor j optimality conditions for investment in the index and in the option market:

$$\mathbb{E}^j[R] = \mathbb{E}^{\mathbb{Q}}[R_j R] = 1 + \mathbb{C}ov(R_j, R) \, .$$

Therefore,

$$\bar{\mathbb{E}}[R] - 1 = \sum_j w_j \mathbb{E}^j[R-1] = \sum_j w_j \mathbb{C}ov^{\mathbb{Q}}(R_j, R) = \mathbb{C}ov^{\mathbb{Q}}\left(\sum_j w_j R_j, R\right) \, .$$

Since under the given market clearing condition $R = \sum_i w_i R_i$, we finally obtain:

$$\mathbb{E}^j[R] = 1 + \mathbb{C}ov(R_j, R) = 1 + w_j \mathbb{V}ar^{\mathbb{Q}}(R_j) + \sum_{i \neq j} w_i \mathbb{C}ov^{\mathbb{Q}}(R_j, R_i) \, ,$$

and

$$\bar{\mathbb{E}}[R] - 1 \quad = \quad \mathbb{V}ar^{\mathbb{Q}}(R) \, .$$

This concludes the proof. ■

Lemma 1. *The subjective expected return*

$$\mathbb{E}^i[R] = \frac{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i'\boldsymbol{R}^e)^\gamma R]}{\mathbb{E}^{\mathbb{Q}}[(1 + \boldsymbol{\theta}_i'\boldsymbol{R}^e)^\gamma]} \, , \tag{32}$$

can be approximated to the first order in $1-\theta_{iI}$ *as:*

$$\mathbb{E}^i[R] \approx \frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}]}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}]} + \gamma(1-\theta_{iI})\left(\frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}X]}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}]} - \frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}]\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X]}{(\mathbb{E}^{\mathbb{Q}}[R^{\gamma}])^2}\right) , \tag{33}$$

where $X := \boldsymbol{\omega}_i'\boldsymbol{O}^e - (R-1)$*. Up to the second order in* $1-\theta_{iI}$*, it can be approximated as:*

$$\mathbb{E}^i[R] \approx \frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}] + \gamma(1-\theta_{iI})\mathbb{E}^{\mathbb{Q}}[R^{\gamma}X] + \frac{\gamma(\gamma-1)(1-\theta_{iI})^2}{2}\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X^2]}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}] + \gamma(1-\theta_{iI})\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X] + \frac{\gamma(\gamma-1)(1-\theta_{iI})^2}{2}\mathbb{E}^{\mathbb{Q}}[R^{\gamma-2}X^2]} . \tag{34}$$

Proof. We expand around $\theta_{iI} = 1$ the subjective expected return:

$$\mathbb{E}^i[R] = \frac{\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma}R]}{\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma}]} . \tag{35}$$

To this end, we write:

$$(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma} = (R + (1-\theta_{iI})(\boldsymbol{\omega}_i'\boldsymbol{O}^e - (R-1)))^{\gamma} =: (R + (1-\theta_{iI})X)^{\gamma} ,$$

and make use of the binomial expansion:

$$(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma} = R^{\gamma}\left(1 + \gamma(1-\theta_{iI})\frac{X}{R} + \frac{\gamma(\gamma-1)(1-\theta_{iI})^2}{2}\left(\frac{X}{R}\right)^2 + \dots\right) .$$

The denominator in equation (35) can then be written as:

$$\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma}] = \mathbb{E}^{\mathbb{Q}}[R^{\gamma}] + \gamma(1-\theta_{iI})\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X] + \frac{\gamma(\gamma-1)(1-\theta_{iI})^2}{2}\mathbb{E}^{\mathbb{Q}}[R^{\gamma-2}X^2] + \dots$$

Analogously, the numerator can be written as:

$$\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma}R] = \mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}] + \gamma(1-\theta_{iI})\mathbb{E}^{\mathbb{Q}}[R^{\gamma}X] + \frac{\gamma(\gamma-1)(1-\theta_{iI})^2}{2}\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X^2] + \dots$$

To the first-order in $1-\theta_{iI}$, we thus obtain:

$$\frac{1}{\mathbb{E}^{\mathbb{Q}}[(1+\boldsymbol{\theta}_i'\boldsymbol{R}^e)^{\gamma}]} \approx \frac{1}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}]} - \gamma(1-\theta_{iI})\frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X]}{(\mathbb{E}^{\mathbb{Q}}[R^{\gamma}])^2} .$$

Therefore, to the first-order in $1-\theta_{iI}$ it also follows:

$$\mathbb{E}^i[R] \approx \frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}]}{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}]} + \gamma(1-\theta_{iI})\left(\frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma}X]}{\mathbb{E}[R^{\gamma}]} - \frac{\mathbb{E}^{\mathbb{Q}}[R^{\gamma+1}]\mathbb{E}^{\mathbb{Q}}[R^{\gamma-1}X]}{(\mathbb{E}[R^{\gamma}])^2}\right) .$$

This concludes the proof. ■

B Data Appendix

This section provides additional information on the properties of option holdings in our data, and a comparison with daily demand flows computed as daily opening positions.

B.1 Options Trading Volume

Figure B.1 plots the times series of the benchmark expected return $\mathrm{E}^0[R]$ in the data. Figure B.2 plots the time series of the total trading volume for 30-day put and call OTM options, respectively. Figure B.3 depicts the empirical unconditional distribution of options' trading volume across moneyness, as measured by the option's Delta. As is evident from the figure, most trading is concentrated in out-of-the-money options. Figure B.4 depicts the empirical unconditional distribution of options' trading volume across times to maturity. It shows that over 50% of all trading happens in options with expiry less than 45 days.

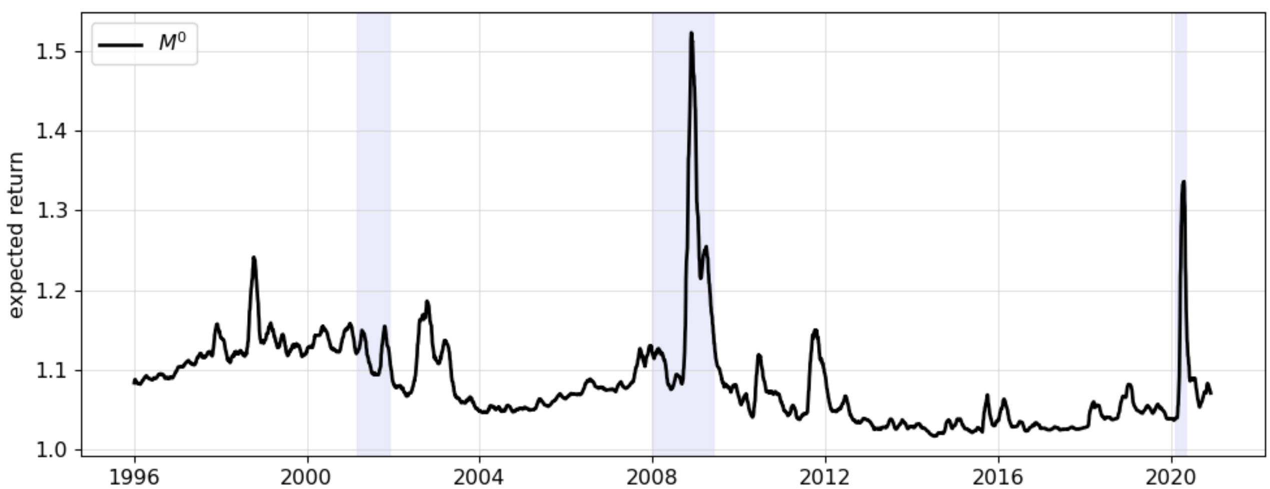


Figure B.1. Expected return $\mathrm{E}^0[R]$

Notes: This figure plots the time-series of expected return $\mathrm{E}^0[R]$, i.e., the expected return recovered with SDF $M^0 = 1/R$. The graph shows the 30-days moving average of daily recovered expected returns for the horizon of one month. Gray bars indicate NBER recessions.

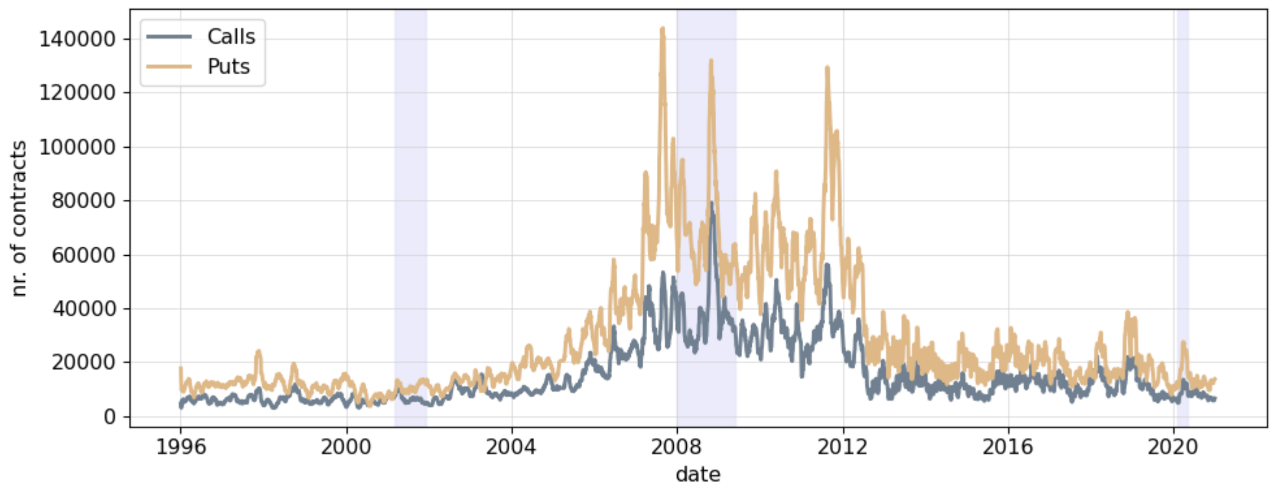


Figure B.2. Trading Volume in 30-days OTM options across time

Notes: This figure plots the time-series of the trading volume in OTM calls and puts, expiring in 30 days, from 1996 to 2020. Data are thirty-days moving average of daily observations. Gray bars indicate NBER recessions.

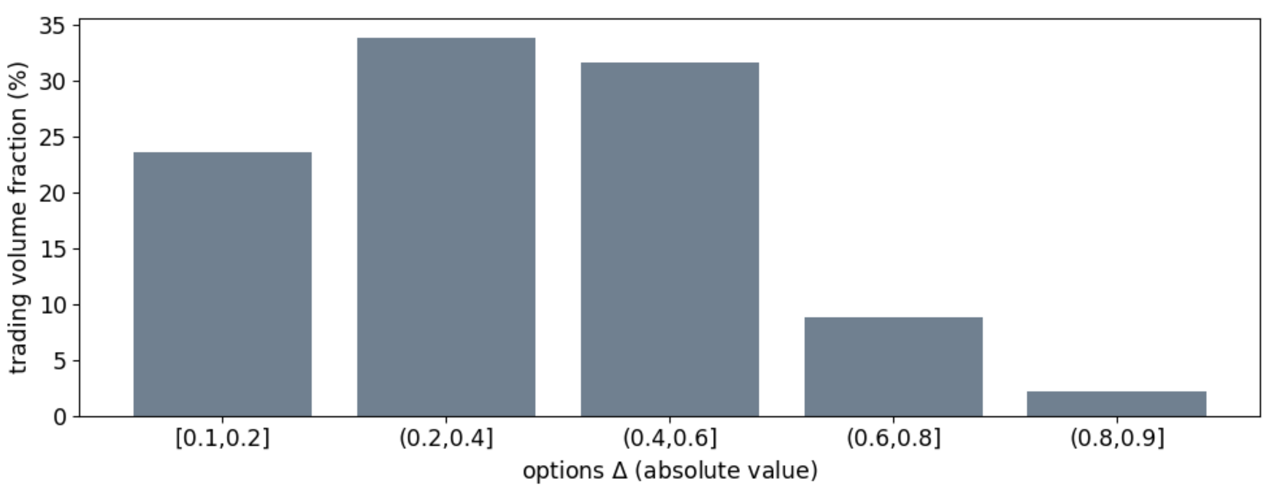


Figure B.3. Trading Volume in 30-days OTM options across moneyness

Notes: This figure plots the distribution of the trading volume in calls and puts expiring in 30 days, across different levels of moneyness identified by their $|\Delta|$. Each value is reported as fraction of the total trading volume. Data runs from 1996 to 2020.

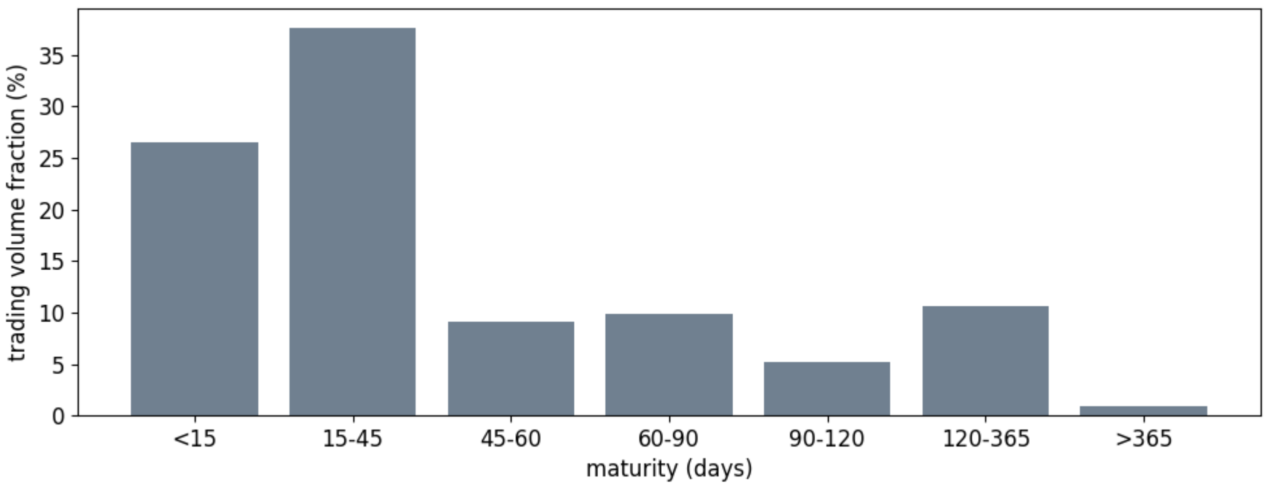
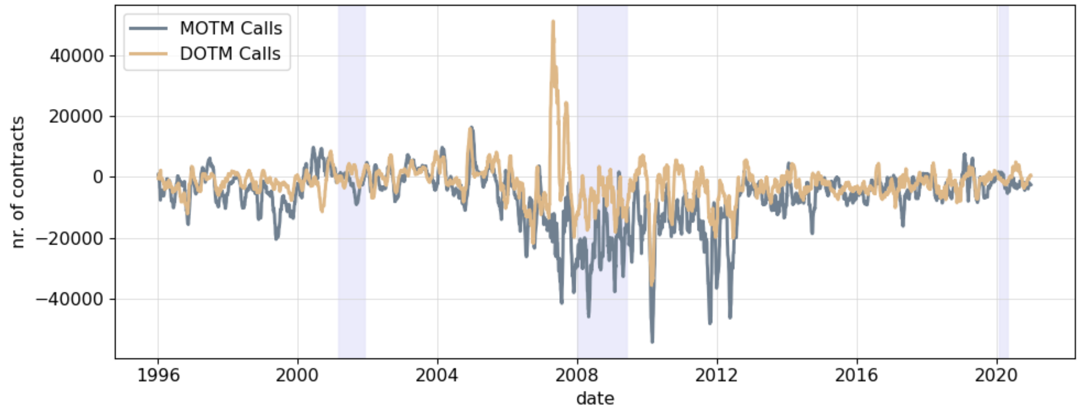


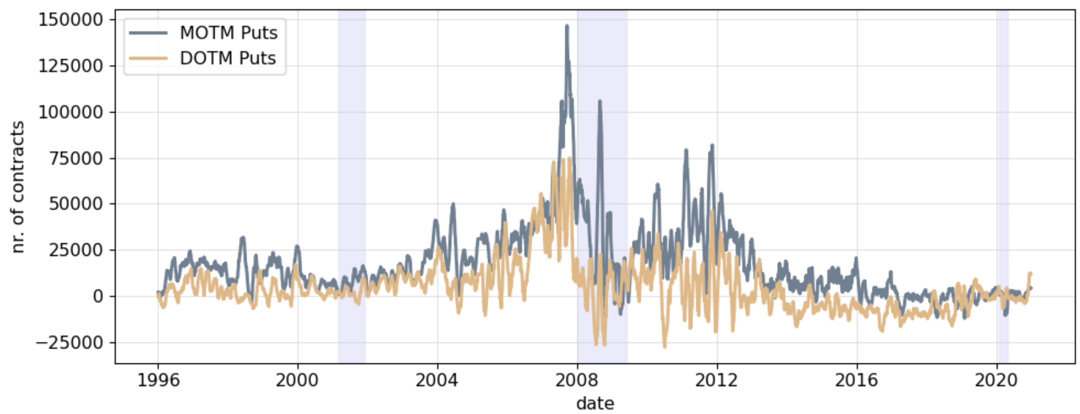
Figure B.4. Trading Volume in OTM options across maturity
Notes: This figure plots the distribution of the trading volume in OTM calls and OTM puts, across different times to maturity. Each value is reported as fraction of the total trading volume. Data runs from 1996 to 2020.

B.2 Option Holdings By Moneyness

Figures B.5A–B.5B provide a more granular look by stratifying the data along the moneyness dimension. Specifically, they distinguish between mild-OTM options with $|\Delta| \in (0.2, 0.4]$ and deep-OTM options with $|\Delta| \in [0.1, 0.2]$. Customers generally hold long positions in both mild-OTM and deep-OTM puts, with the number of mild-OTM puts typically exceeding the number of deep-OTM puts. Similarly, customers were often short on both mild-OTM and deep-OTM calls, with mild-OTM calls outnumbering deep-OTM calls. However, an interesting pattern emerges at the end of 2007, when customers held larger long positions in deep-OTM calls than short positions in mild-OTM calls.



A. OTM Calls



B. OTM Puts

Figure B.5. Customers' Holdings of monthly OTM options

Notes: This figure plots the 30-day moving average of customers' portfolio holdings of OTM calls and puts from 1996 to 2020. Options holdings are the sum of opening and closing positions on the same contract that customers enter from issuance. Mild-OTM options («MOTM») have $|\Delta| \in (0.2, 0.4]$. Deep-OTM options («DOTM») have $|\Delta| \in [0.1, 0.2]$. Grey areas indicate NBER recession periods.

B.3 Options Demand Flows

We can compare the time-series of our option holding proxies, shown in Figure 3, with the time series of daily opening option positions, i.e., demand flows. Figure B.6 illustrates this time series by aggregating demand flows from all deep-out-of-the-money options with maturities ranging between 7 to 500 days, as outlined in Chen, Joslin, and Ni [2019]. Unlike the holding proxies in Figure 3, demand flows more often switch signs. While customers generally display positive demand flows – indicating an increase in holdings of OTM options – there are instances where negative demand flows emerge, particularly during crisis events such as the Lehman Brothers default or the Euro and Repo crises. Chen, Joslin, and Ni [2019] attribute this behavior to market makers' constraints becoming more binding during such periods, compelling them to act as net buyers of market insurance. These patterns are largely absent in our aggregated option holdings, which encompass all moneyness and are restricted to a 30-day time to maturity.

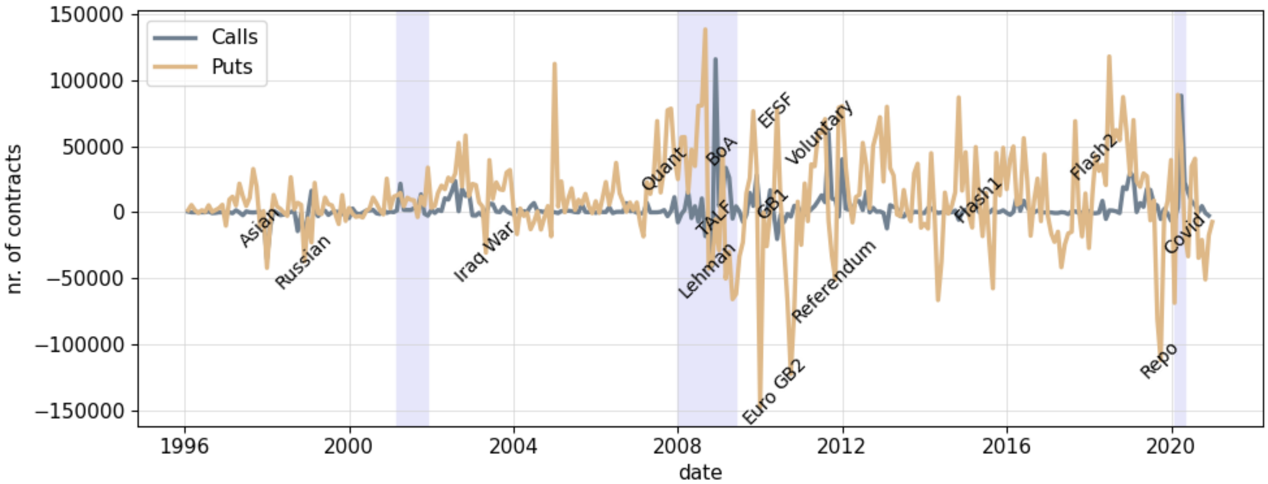


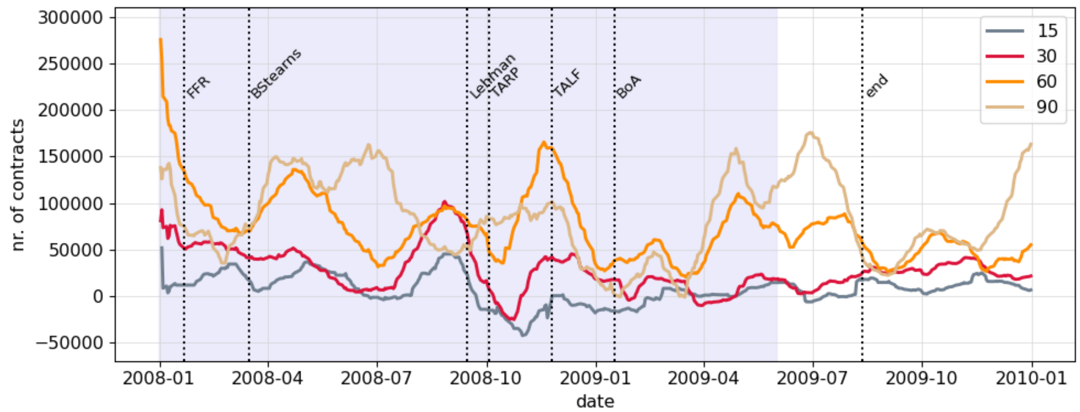
Figure B.6. Customers' demand flows for deep-OTM options at any maturity
Notes: This figure plots the time-series of our proxy for customers' demand flows on deep-OTM calls and puts (with $K/S_t \leq 0.85$ for puts and $K/S_t \geq 1.15$ for calls), at any maturity between 7 and 500 days. Option demand flows are the sum of opening positions on the same contract recorded every day. Daily data are summed over monthly basis. Gray bars indicate NBER recessions. Some major events are highlighted in the graph: the Asian financial crisis (Oct. '97); the Russian financial crisis (Nov. '98); Iraq war (Apr. '03); the quant-strategy hedge fund crisis (Aug. '07); the Lehman bankruptcy (Sep. '08); creation of TALF (Nov. '08); the Bank of America rescue («BoA», Jan. '09); the Euro crisis induced by Greek debt crisis (Dec. '09); Greek bailout installments («GB1», Apr. '10; «EFSF», May '10; «GB2», Sept. '10); agreement to Voluntary Greece bondholder role (Jun. '11); call for Referendum about financial aid in Greece (Oct. '11); two Flash crisis (Apr. '15 and Feb. '18); sharp repo rate increase (Sept. '19); Covid-19 pandemic (Apr. '20).

B.4 Option Holdings During the Financial Crisis

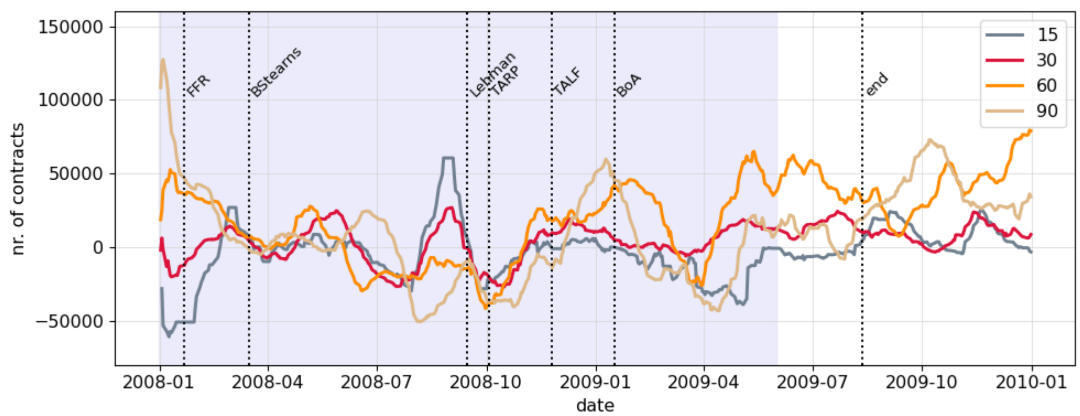
Here we analyze customers' option holdings across various maturities during the Great Financial Crisis. We focus on options with expirations of 15, 30, 60, and 90 days, and examine data separately for mild- and deep-OTM puts.

Figure B.7 presents the time series of these holdings. Our findings show that, generally, customers maintain long positions in mild-OTM options at all maturities, except for a brief period following the Lehman default. During this time, they sold shorter-term puts and increased their holdings in three-month options. Prior to and immediately after the crisis, the different series exhibit a notrivial positive co-movement, with seasonality effects being especially pronounced for longer-term options.

The holdings of deep-OTM puts show a distinct pattern. On certain dates, such as right before the Lehman bankruptcy, customers were long on short-term puts and short on long-term ones, suggesting they anticipated imminent market turbulence. During the height of the crisis, they took negative positions in options of all maturities through the end of 2008. Afterward, they resumed holding long-term puts, maintaining this position until about April 2009.



A. Mild-OTM Puts



B. Deep-OTM Puts

Figure B.7. Customers' Holdings of monthly OTM puts during the Great Financial Crisis
Notes: This figure plots the time-series of our proxy for customers' portfolio holdings of OTM puts during the period around the Financial Crisis in 2008. Options expire in 15, 30, 60 or 90 days. Options holdings are the sum of opening and closing positions on the same contract that customers enter from issuance. Mild-OTM options have $|\Delta| \in (0.2, 0.4]$. Deep-OTM options have $|\Delta| \in [0.1, 0.2]$. The plots display the thirty-days moving average of the holdings. Grey areas indicate NBER recession periods. Some important events are highlighted: the Fed. Market Open Committee lower the fed fund rate (Jan. 22, '08); Bear Stearns acquired by JP Morgan (Mar. 16, '08); Lehman default (Sept. 15, '08); creation of TARP (Oct. 3, '08); creation of TALF (Nov. 25, '08); rescue of Bank of America (Jan. 16, '09); Fed suggests the worst of recession is over (Aug. 12, '09).

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