Interest Rate Risk and Cross-Sectional Effects of Micro-Prudential Regulation

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Abstract

This paper investigates the financial stability consequences of banks' interest rate risk exposure and uninsured deposit funding share. We develop a model incorporating insured and uninsured deposits, interest rate-sensitive securities, and credit-risky loans to understand how banks respond to interest rate risk and the potential for deposit runs. The model delivers the concentration of uninsured deposits in larger banks and examines how banks' portfolio- and funding choices impact financial stability. When banks anticipate volatile bond returns, they seek exposure to this interest rate risk. We study the effects of recent Federal Reserve rate hikes on banks and analyze micro-prudential policy tools to enhance the banking sector's resilience. Higher liquidity requirements that target uninsured deposits are effective at curbing run risk of large banks, but cause misallocation in the lending market. Size-dependent capital requirements are equally effective at mitigating run risk, with minimal unintended consequences.

JEL: G12, G15, F31.

Keywords: Uninsured Deposits, Interest Rate Risk, Financial Stability, Micro-prudential Regulation, Bank Concentration, Bank Portfolio Choices

1 Introduction

In 2022, the Federal Reserve (Fed) raised the policy rate to levels not seen since 2008 to combat unprecedented levels of inflation, leading to a rare realization of an interest rate shock. When rates go up, banks' fixed income portfolio-like positions, such as loans and securities, decline in market value.[1] Coupled with a run on uninsured deposits, these declines led to the failure of major banks like Silicon Valley Bank (SVB) and First Republic.[2]

This paper proposes a framework for analyzing banks' portfolio- and funding choices across the bank size distribution. It then evaluates how banking regulations can enhance financial stability. By focusing on micro-prudential regulations and examining the cross-section of banks, this study identifies and quantifies the characteristics of banks that choose higher exposure to interest rate risk or susceptibility to run risk, and how various regulatory policies contribute to their safety.

Our model setup is guided by three stylized facts about differences in banks' portfolio choices and funding characteristics. First, the banking sector is highly concentrated, with 50% of aggregate assets owned by banks in the top 0.3th percentile in terms of asset size. Second, larger banks choose a higher share of uninsured deposit funding. Together, these two facts imply that uninsured deposit funding is also very concentrated. 90% of aggregate uninsured deposits are issued by the top 10% of banks. Third, smaller banks have a higher share of cash and securities holdings compared to large banks.

To parsimoniously analyze the economic drivers of differences in bond shares and uninsured funding across banks, we build a two-period model with a cross-section of banks and a representative household. The model matches these three stylized facts. Banks choose investments into credit risky loans and interest rate sensitive bonds. We assume that banks are ex-ante heterogeneous in their lending productivity, which scales a decreasing returns to scale production technology. In the second period, bank lending is further exposed to an aggregate and idiosyncratic productivity (credit) shock. Banks fund assets by issuing insured and uninsured deposits as well as equity. A capital requirement and a liquidity requirement constrain bank

[1] E.g., Hirtle (1997); Landier, Sraer, and Thesmar (2013); Begenau, Piazzesi, and Schneider (2020); English, Van den Heuvel, and Zakrajšek (2018); Paul (2023)

[2] See Jiang, Matvos, Piskorski, and Seru (2024) for recent work on the drivers of bank failures during this time.

leverage and portfolio choices. Households derive utility from consumption in periods 0 and 1 and from liquidity services from banks. Banks differ in their liquidity service productivity levels as well. We model liquidity services as a constant elasticity of substitution (CES) aggregator over quantities of insured and uninsured deposits scaled by banks' individual deposit productivity. Hence, households view deposits at different banks as differentiated products. Lastly, banks have the option to default when they cannot repay depositors in the second period. Bank default causes bankruptcy costs that lower aggregate consumption. While some deposits are covered by deposit insurance, funded with lump-sum taxes on households, uninsured depositors are not necessarily be repaid in full. As a result, they may choose to run. We model endogenous run decisions as in Dávila and Goldstein (2023).

We calibrate the model to regulatory data for U.S. commercial banks, focusing on the post-Great Financial Crisis (GFC) period after 2009. Our calibration uses data from bank regulatory filings (FFIEC 031/041) on asset and uninsured deposit concentration, bond share, and realized deposit rates. We jointly calibrate the three sets of parameters that govern loan productivity, deposit productivity, and liquidity preferences. The production technology parameters are crucial in capturing the concentration within the banking sector. Our model differentiates between asset concentration and loan concentration, which is influenced by the curvature of the loan production function and the parameter that governs ex-ante loan productivity differences across banks. Calibration of these parameters targets the standard deviation of the loan and asset distribution, and the aggregate loan return. We assume that a bank's deposit productivity perfectly correlates with its lending productivity with varying slope. To calibrate the parameter governing banks' deposit productivity, the model targets the bond share of the smallest 20% and the top 1% of banks by asset size. The final set of parameters governs liquidity preferences. Our calibration of these parameters targets the average transaction and time deposit rates, the average uninsured deposit share, and the Gini coefficient of uninsured deposits.[3]

Our paper presents the following findings. First, our model rationalizes the cross-sectional facts about banks' security shares, uninsured deposit funding share, assets, and uninsured deposit concentration as follows. A bank's scale is optimally determined by its two production technologies, each featuring decreasing returns in lending and liquidity services. How banks

[3] Due to data limitations, transaction and time deposit rates are used as proxies for insured and uninsured deposit rates.

fund themselves and how much banks invest in bonds depends on the relative strength of their lending and deposit business. Banks can fund their assets with insured and uninsured deposits. Our calibration implies that banks' profit margins are higher in the insured deposit market. When lending is relatively unprofitable given the bank's lending productivity, it turns to bonds to back its insured deposit business, the main source of value for small banks.[4] In contrast, banks with a highly productive lending technology must also tap into the uninsured deposit market to fund their scale. This way, a high-loan-productive bank avoids giving away too much profit margin in the insured market. In our model, small banks hold bonds to support a relatively more profitable deposit business. Large banks hold bonds for precautionary reasons to manage run risk, which uninsured deposits expose them to. As in the data, the largest banks – with more uninsured deposit funding and hence higher run risk exposure– hold optimally more bonds than mid-size banks.

Second, we use our framework to study the effects of an unanticipated interest rate shock on the likelihood of bank failures. The model allows us to distinguish between solvency defaults and run defaults. Solvency defaults are more common in less productive small banks. Run defaults can occur only at banks that fund themselves with uninsured deposits, affecting the largest banks. Yet, we show that in the baseline calibration, run defaults do not occur in equilibrium because large banks hold enough bonds to hedge against run risk. But when banks are hit by an unanticipated rate hike, small banks' solvency default rates rise to over 10%, and large banks are suddenly exposed to a 1% run default probability. These findings illustrate that the combination of uninsured deposit funding and unexpectedly low bond return realizations can significantly increase the financial fragility of large banks.

We then solve a version of the model in which banks anticipate highly volatile bond returns to allow them to hedge against the effects of raising interest rates. Interestingly, we find that, rather than choose a more conservative portfolio as a precaution, banks across the board increase risk-taking in response to higher expected bond return volatility. While a riskier portfolio of assets increases a bank's uninsured deposit rate, it also delivers higher asset returns. The increased asset risk is not perfectly reflected in a bank's funding cost due the mispricing of insured deposits. For small banks without uninsured deposits, the reach-for-yield motive

[4] Hanson, Shleifer, Stein, and Vishny (2015) argue that the traditional banks' business model is well described by this deposit-driven view.

dominates, leading to them choosing a *higher* bond share. For large banks whose loans are very productive and whose uninsured deposit rates reflect some asset risk, an increase in bond risk makes bonds too unattractive both as an investment and as a means to hedge liquidity (run) risk. Given the ex-ante much riskier portfolio choice across the board, an anticipated rate hike leads to more default than an unanticipated one. Without the precautionary buffer of bonds, the rate hike also leads to more run defaults at large banks, while small banks – having chosen a higher bond share – are more likely to default for solvency reasons. Our third result suggests that the rational anticipation of interest rate risk can increase banks' portfolio risk.

Finally, we use the model as a laboratory to study the micro-prudential effects of several policies. We find that unconditional increases in capital requirements reduce solvency default risk of small banks, and run-based default risk of large banks. However, capital requirements also restrict the supply of liquidity. For any significant reduction in default risk, the second effect dominates and the unconditional policy does not improve aggregate welfare. The model suggests other policies that are effective at curbing run risk of large banks. First, heightened liquidity requirements that explicitly condition on uninsured deposits greatly reduce run risk at the largest banks that have most of the uninsured deposits. Yet, this policy also causes misallocation in the loan market by distorting the portfolio choices of large and small banks. Second, a capital requirement that increases in bank size reduces leverage of large banks and thus mitigates their run-induced default risk, while having minimal effects on other banks' choices and the allocation of loans and bonds across banks. Thus, we find that size-dependent capital regulation is a powerful tool for limiting run exposure of systemic banks.

Related Literature Our work is at the intersection of banking, asset pricing, and macro-finance. Since the fall of Silicon Valley Bank, there has been renewed interest in the financial stability consequences of banks' funding structure and interest rate risk exposure (e.g., Jiang et al., 2024; DeMarzo, Krishnamurthy, and Nagel, 2024; Drechsler et al., 2023; Haddad, Hartman-Glaser, and Muir, 2023; Chang, Cheng, and Hong, 2023). Our focus is on building a quantitative framework to jointly analyze banks' funding and portfolio choices and their financial stability consequences. Building on the insights from prior literature, we choose as key features of our framework banks' interest rate risk exposure (e.g., Landier, Sraer, and Thesmar, 2013; Begenau, Piazzesi, and Schneider, 2020; English, Van den Heuvel, and Zakrajšek, 2018; Paul, 2023), large concentration of the banking sector (e.g., Corbae and D'Erasmo, 2013; Corbae and D'Erasmo, 2020), financial fragility through run risk (Robatto, 2019; Dávila and Goldstein, 2023), and uninsured deposit funding share (e.g., Jiang et al., 2020, 2024). We contribute to the literature by synthesizing these essential features of the modern banking system into a tractable model with heterogeneous banks that can serve as a building block for larger-scale macro-prudential analysis along the lines of Elenev, Landvoigt, and Van Nieuwerburgh (2021). Relative to other recent heterogeneous banking models (e.g., Corbae and D'Erasmo, 2013; Egan, Hortaçsu, and Matvos, 2017; Corbae and D'Erasmo, 2021; Jamilov, 2021; Begenau and Landvoigt, 2022; Begenau et al., 2024; Coimbra and Rey, 2024; d'Avernas et al., 2024) our model jointly rationalizes the cross-sectional differences between banks' funding choice–insured vs. uninsured deposits–and portfolio choice–interest rate sensitive bonds vs. credit risky loans, and focuses on micro-prudential policy analysis.

The next section presents key stylized facts that guide our modeling choices. Section 3 presents our two period model of the bank size distribution. We discuss the model's calibration in Section 4. Section 5 analyzes the model mechanism and discusses policy experiments.

2 Stylized Facts

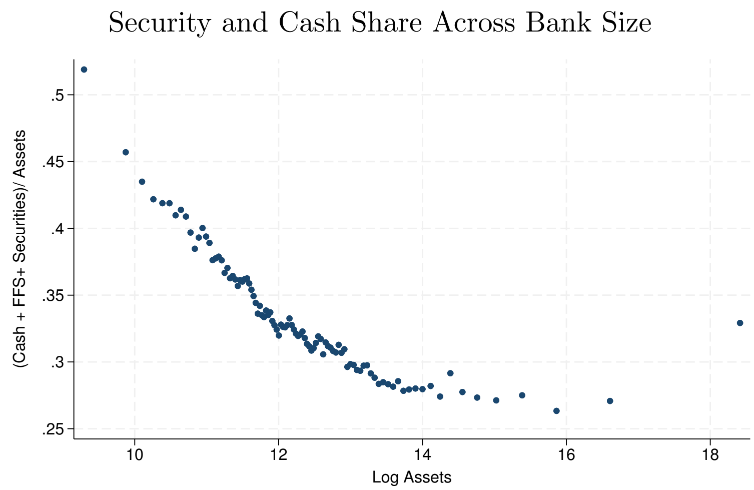
This section presents facts about the cross-section of US commercial bank securities and uninsured deposit funding that are central to our modeling framework. We use bank call report data from 2010Q1 to 2022Q4 to capture the post-GFC era changes in banks' regulatory environment.[5]

Securities Share We study differences in banks' portfolio choices over the size distribution. Smaller banks hold more cash and securities on their balance sheet compared to larger banks. Figure 1 presents a binned scatter plot of the securities and cash shares over bank size as measured by logged assets. To construct this plot, we compute each bank's asset share in securities, cash, and federal funds sold and repo assets for each quarter between 2010Q1 and

[5]The data are from bank level call forms FFIEC 031 and FFIEC 041.

2022Q4. For each log asset percentile, we calculate the average cash and securities share. The security share is generally declining in size, but increases slightly for the largest banks.

Figure 1: **Stylized Facts of the Cross-Section of Banks**



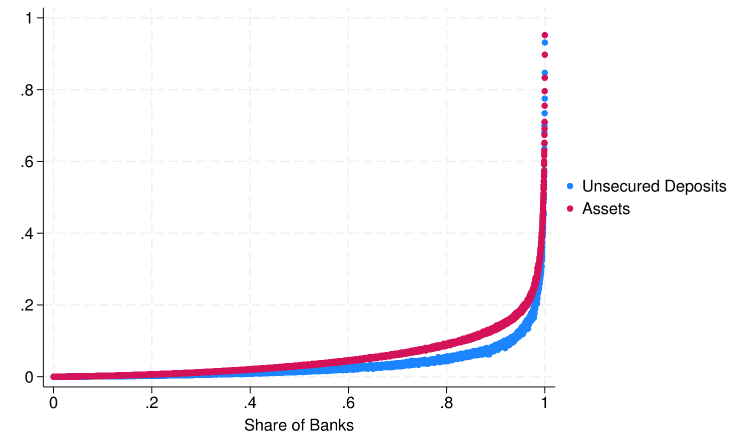
Notes: This figure presents average securities share (Panel A) and the average maturity of securities as a binscatter plot, with 100 bins. The securities share is the ratio of the sum of cash, federal funds sold, repo assets, and securities over assets. The data are from bank call reports, filing forms FFIEC 031/041 using a bank-quarter panel from 2010Q1 to 2022Q4.

Asset and Uninsured Deposits Concentration The banking sector is highly concentrated (Panel A of Figure 2. Most assets and uninsured deposits are held by a small fraction of banks. In addition, Panel B of Figure 2 shows that uninsured deposits represent a larger fraction of large banks' domestic deposit funding. The average uninsured deposit share of deposits increases from about 10% for the smallest banks to about 40% for the largest banks. Since the banking sector is very concentrated, uninsured deposits dollars are disproportionately concentrated in the largest banks (Panel A).

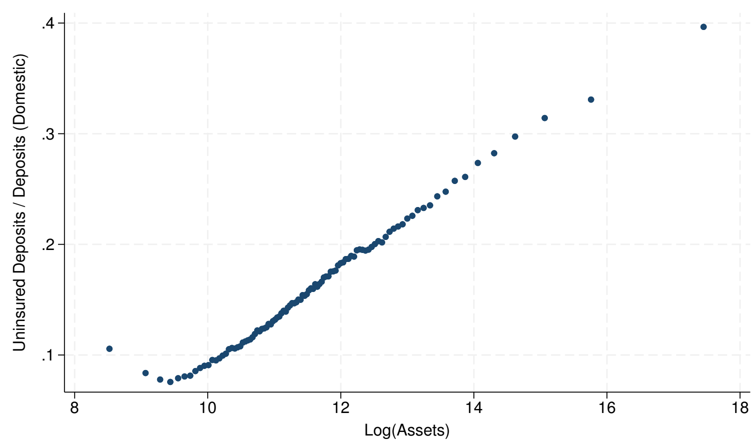
Deposit Rates Figure 3 presents a scatter plot of transaction deposit rates in Panel A and time deposit rates in Panel B. The largest banks pay higher deposit rates compared to the smallest banks, but the largest banks pay less than large mid-size banks. We choose transaction and time deposit rates as data on realized interest expenses on insured and uninsured deposits are unfortunately not available for transaction accounts and are very limited for time deposits.

Figure 2:

Panel A: Concentration of Uninsured Deposits and Assets



Panel B: The Uninsured Deposit Share in the Cross-Section

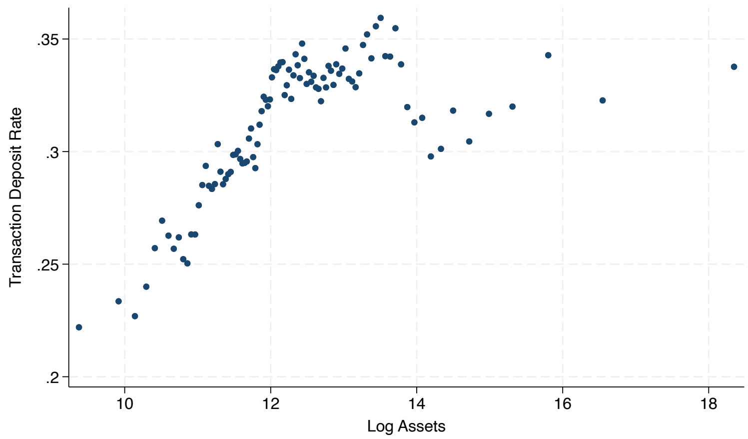


Notes: Panel A shows the concentration of unsecured deposits and assets. We compute the cumulative aggregate share of each held by a given share of banks and plot the curves. Panel B presents the uninsured share over the bank size distribution as a binscatter plot, with 100 bins. The uninsured share is the ratio of uninsured domestic deposits over domestic deposits. Uninsured deposits are deposit accounts with more than $250 thousands. The data are from bank call filing forms FFIEC 031/041 using a bank-quarter panel from 2010Q1 to 2022Q4.

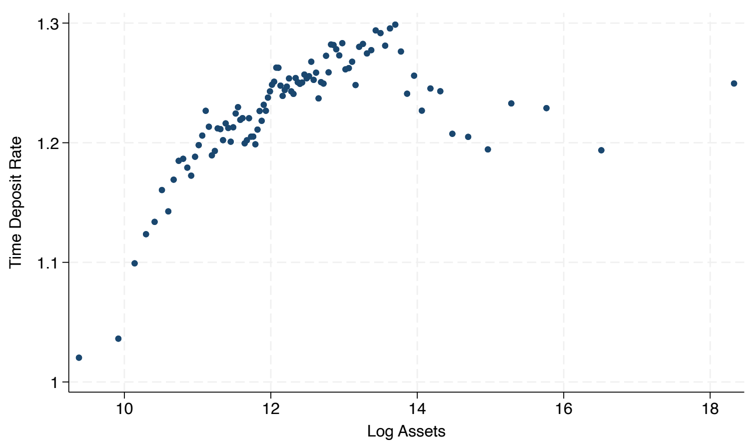
Mark-to-Market Losses and Bank Equity It is rare for interest rate risk to be visibly realized in bank portfolios since they are mainly unmarked. The interest rate hike period after the pandemic has allowed a rare glimpse into the interest rate sensitivity of bank portfolios. For securities, bank call reports require banks to fill in the fair value of their unmarked securities (hold-to-maturity securities). When we revalue assets by just subtracting the book value of securities adding back their fair value, we can recalculate an implied market value of equity.

Figure 3:

Panel A: Transaction Deposit Rates



Panel B: Time Deposit Rates



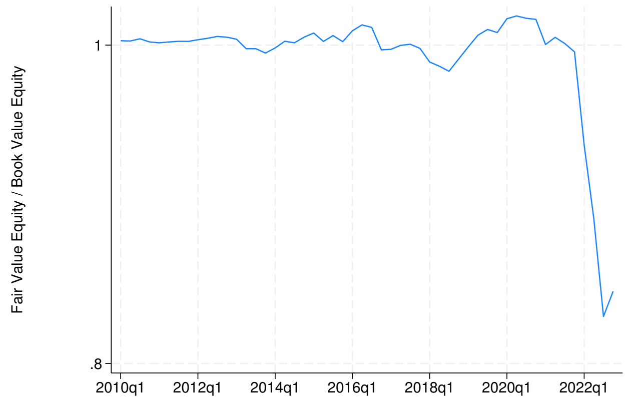
Notes: This figure shows binned scatter plots of deposit rates in percentage points over bank size as measured by log assets. The data are from bank call reports, filing forms FFIEC 031/041 using a bank-quarter panel from 2010Q1 to 2022Q4. Transaction deposit rates (Panel A) are computed as the sum of domestic deposit interest rate expense less domestic time deposit expense divided by transaction deposits at the beginning of the period. Transaction deposits are domestic deposits less domestic time deposits. Time deposit rates (Panel B) are computed as the interest expense on time deposits divided by the beginning of period time deposits.

Panel A of Figure 4 shows the aggregate market value of equity normalized by the aggregate book equity since 2010. The rate hike period led to large unrealized losses in banks' securities portfolio, leading to a drop in equity due to security losses alone of nearly 20%.

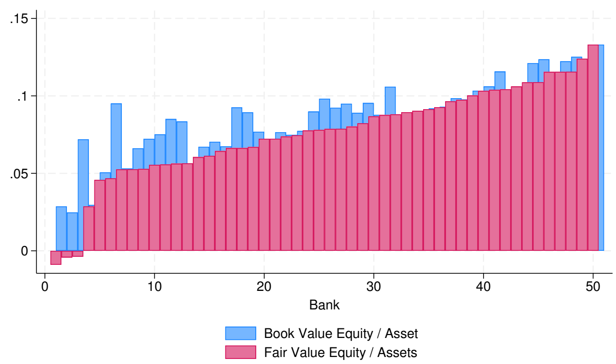
Panel B shows the mark-to-market losses of the 50 largest banks in 2022 Q3, calculated similarly to for Panel A but expressed as a share of assets. For comparison, we also plot the

Figure 4:

Panel A: Security Revaluation at Market Prices: Time Series



Panel B: Mark-to-Market Losses of the 50 Largest Banks in 2022 Q3



Notes: The data are from bank call reports, filing forms FFIEC 031/041 using a bank-quarter panel from 2010Q1 to 2022Q4.

regulatory book value of equity as a fraction of assets. Most of the large banks sustained losses on their securities portfolios, leading to large unrealized losses in equity.

In the next section, we build a model that is consistent with the concentration of assets and uninsured deposits and the cross-sectional differences in the security share and deposit rates.

3 Model of the Cross-section of Banks

3.1 Environment and Timing

There are two types of agents, households and banks. Households are identical, and we solve the problem of a representative household. Banks are heterogeneous in their productivity of lending and providing deposits to households. Banks are indexed by i on the continuum $[0, 1]$. The model has two dates, 0 and 1. At date 0, households invest in bank liabilities, and banks decide on lending and capital structure. At date 1, there are two subperiods. First, aggregate and idiosyncratic bank productivity shocks are realized, and uninsured depositors decide whether to "run" on the bank and demand their balances to be paid. In case of a run, banks have to liquidate assets to pay out these early withdrawals.

Thereafter, banks can decide to default, in which case they are liquidated by the deposit insurance agency. Insured deposit payoffs are safe and backed by taxation. Uninsured deposits are risky and only receive a stochastic recovery value depending on bank assets.

3.2 Households

There is a unit mass of identical households. Households maximize utility over consumption C_t in both periods $t \in \{0, 1\}$, and over liquidity services at time 0 that are provided by bank deposits. They maximize the utility function

$$U(C_0, C_1, \{D_i^I\}, \{D_i^U\}) = \log(C_0) + \psi \log\left(H\left(\{D_i^I\}, \{D_i^U\}\right)\right) + \beta \log\left(\mathrm{E}\left[C_1^{1-\varphi}\right]^{1/(1-\varphi)}\right) \quad (1)$$

with the liquidity preference function

$$H\left(\{D_i^I\}, \{D_i^U\}\right) = \left[\alpha\left(\int_0^1 (A_i^D D_i^I)^{\rho_I} di\right)^{\eta/\rho_I} + (1-\alpha)\left(\int_0^1 (A_i^D D_i^U)^{\rho_U} di\right)^{\eta/\rho_U}\right]^{1/\eta}, \quad (2)$$

where $\{D_i^I\}_{i\in[0,1]}$ are insured, and $\{D_i^U\}_{i\in[0,1]}$ are uninsured deposits. Household enjoy liquidity services provided by both types of deposits, with α being the relative weight on insured deposits and η governing the elasticity of substitution between types. Banks have different productivity in liquidity services provision given by A_i^D. Further, banks have local monopolies in deposit markets, implying that their deposits are not perfect substitutes within the insured and uninsured categories. The degree of substitutability in each type is governed by ρ_j, for $j \in \{I, U\}$, respectively.

At date 0, households are endowed with initial wealth W_0, and they choose how much of this wealth to consume and how much to invest in equity $\{S_i\}$ and deposits $\{D_i^I\}, \{D_i^U\}$ of all banks. The household budget constraint at date 0 is

$$C_0 = W_0 - T + \int_i \Pi_i di - \int_i p_i S_i di - \int_i q_i^I D_i^I di - \int_i q_i^U D_i^U di, \tag{3}$$

where p_i is the price of equity and q_i^j for $j \in \{I, U\}$ is the price of insured and uninsured deposits of bank i. Households pay lump-sum taxes T that are needed to pay for bailouts of insured deposits. They further receive time-0 profits of all banks Π_i. Consumption at time 1 consists of the payoff of all securities bought at time 0:

$$C_1 = \int_i D_i^I di + \int_i \mathbb{I}_{\text{nd}_i} S_i \text{Div}_i di + \int_i D_i^U \mathcal{P}_i^U di. \tag{4}$$

Insured deposits pay off with certainty, while only non-defaulting banks, indicated by the binary variable $\mathbb{I}_{\text{nd}_i}$, pay dividends to equity holders. The payoff of uninsured deposits depends on both aggregate and idiosyncratic risk through banks' default decision and the potential realization of runs. These factors are encapsulated in $\mathcal{P}_i^U$ defined in equation (18) below. Time-1 consumption is exposed to aggregate risk and hence a random variable at time 0.

3.3 Banks

There is a continuum of banks of mass one, indexed by i. Banks are ex-ante heterogeneous in their cost of producing deposits and in their lending productivity, denoted by the pair of parameters $(A_i^D, A_i^K) \sim G$. Banks can invest in two types of assets, bonds and loans (= capital). The capital price at time 0 is normalized to 1 and the aggregate return to capital R_K is risky: $R_K \in \{R_{K,1}, \dots, R_{K,n_K}\}$ with probability vector π_K. Bonds are also exposed to aggregate risk, with bond return realizations $R_B \in \{R_{B,1}, \dots, R_{B,n_B}\}$ and probability vector

π_B, analogous to capital. The aggregate payoffs of both assets are independent. Banks further receive multiplicative i.i.d. shocks ϵ_i to their loan production at time 1. A bank that extends K_i loans at time zero therefore receives total loan payoff

$$R_K A_i^K \epsilon_i K_i^{1-\kappa}, \tag{5}$$

where κ governs the degree of decreasing returns in lending.

Banks can decide to default. If a bank defaults, its pays out a dividend of zero and a fraction ξ of its output is lost in the bankruptcy proceedings. The remaining output $(1-\xi)A_i^K \epsilon_i K_i^{1-\kappa}$ is allocated proportionally to the recovery of insured and uninsured deposits. Uninsured deposits pay out this recovery value to households, while insured deposits pay out 1. Payouts of insured deposits for defaulting banks are funded by the government, which raises lump-sum taxes on households at date 0 to cover the shortfall between recovery and full payout in expectation.

Banks' uninsured deposits are subject to runs. In particular, we assume that fraction $1-\phi$ of uninsured deposits is "runnable," while remaining fraction ϕ is not.[6] Banks can pay out early withdrawals by liquidating bonds or loans to outside investors. Bonds can always be liquidated at par, i.e. at value R_B. However, capital can only be liquidated with fire-sale discount $\delta < 1$.

To summarize, the timing of events is

0. Time 0: Banks choose portfolio $\{K_i, B_i, D_i^U, D_i^I\}$. Households choose portfolio of bank securities.

1a. Time 1: Aggregate and idiosyncratic productivity shocks are realized and observed. Uninsured depositors decide whether to run. Banks subject to runs choose quantity of capital and bonds to sell in order to cover deposit outflow D_i^U.

1b. Bank default decision. All assets pay out.

[6]The non-runnable fraction ϕ represents other bank liabilities that are junior to deposits. This share of uninsured deposits is only senior to bank equity and will receive the residual recovery value of bank assets in bankruptcy. We will also consider too-big-too-fail guarantees for these deposits.

3.4 Bank Problem

We will solve the bank problem backwards, starting at the run stage 1a. in the time line above.

Problem with run. We consider the problem of a bank that experiences a run. The bank needs to decide on the quantities of loans $\hat{K}_i$ and bonds $\hat{B}_i$ to liquidate in order to pay out running depositors. We summarize the portfolio of the bank by $\mathcal{A}_i = (B_i, K_i, D_i^I, D_i^U)$. The aggregate state $\mathcal{R} = (R_K, R_B)$ consists of the realized loan and bond returns. The bank solves the optimization problem

$$V(\mathcal{A}_i, \epsilon_i, \mathcal{R}) = \max_{0 \leq \hat{K}_i \leq K_i, 0 \leq \hat{B}_i \leq B_i} \max\{0, A_i^K \epsilon_i R_K (K_i - \hat{K}_i)^{1-\kappa} + R_B(B_i - \hat{B}_i) - D_i^I - \phi D_i^U\} \quad (6)$$

subject to

$$\delta R_K \hat{K}_i + R_B \hat{B}_i \geq (1-\phi) D_i^U. \quad (7)$$

The bank maximizes its post-run dividend to shareholders in (6). However, if this dividend is negative, banks take advantage of limited liability and go into bankruptcy. The constraint in (7) states that the bank must pay out balances of running depositors $(1-\phi)D_i^U$ by selling capital or bonds. We assume that banks always choose portfolios such that they can fully pay out running depositors, i.e. the constraint (7) can always be met.

Assumption 1. *All banks choose portfolios $\mathcal{A}_i$ that satisfy constraint* (7)*.*

This assumption is not restrictive. If $\kappa > 0$, banks have decreasing returns in loan payoffs with an Inada condition in the production function. For a hypothetical bank that has sold off its complete loan portfolio in a fire sale such that $K_i - \hat{K}_i = 0$, the inframarginal unit of loans has an infinite marginal payoff. This is not compatible with optimality for fairly general conditions, implying that banks will have enough assets to avoid selling all capital. To simplify the solution to the problem given by (6) – (7), we further make the following assumption.

Assumption 2. *Conditional on a run, banks first liquidate their bond holdings to pay out running depositors. Only if bond holdings are insufficient to cover deposit withdrawals, banks also liquidate capital holdings.*

This assumption implies that banks will choose $\hat{B}_i = \min\left\{\frac{(1-\phi)D_i^U}{R_B}, B_i\right\}$ and only resort to selling capital at fire sale discount δ if $(1-\phi)D_i^U > R_B B_i$. While we outright assume this liquidation pecking order for simplicity, we should note that it is also optimal for banks for a wide range of parameters.[7]

A key property of our model is that banks choose their exposure to run risk by issuing uninsured deposits. A bank that issues only insured deposits does not experience any runs. Furthermore, it is important to note that even though banks have sufficient assets to pay out running uninsured depositors, the remaining assets after these payouts may be insufficient to pay out the bank's remaining debts. These consist of non-running uninsured and insured deposits, $\phi D_i^U + D_i^I$. We define optimal policy functions $\hat{B}_i^* = \hat{B}_i(\mathcal{A}_i, \epsilon_i, \mathcal{R})$ and $\hat{K}_i^* = \hat{K}_i(\mathcal{A}_i, \epsilon_i, \mathcal{R})$ for fire sale quantities characterized in Appendix B.2. Inserting these optimal choices into the bank dividend in (6) implies the existence of the default threshold

$$\overline{\epsilon}_i = \frac{D_i^I + \phi D_i^U - (B_i - \hat{B}_i^*)}{A_i^K R_K (K_i - \hat{K}_i^*)^{1-\kappa}}, \tag{8}$$

such that banks with $\epsilon_i > \overline{\epsilon}_i$ do not default even conditional on experiencing a run.

Problem without run. Now we consider a bank that does not experience a run. This bank's dividend payment to households is

$$\max\{0, A_i^K \epsilon_i R_K K_i^{1-\kappa} + B_i - D_i^U - D_i^I\},$$

which implies a threshold

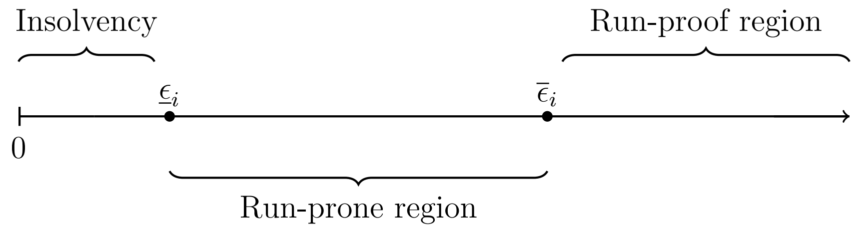
$$\underline{\epsilon}_i = \frac{D_i^U + D_i^I - B_i}{A_i^K R_K K_i^{1-\kappa}}. \tag{9}$$

Banks with idiosyncratic shocks $\epsilon_i < \underline{\epsilon}_i$ will default even if they do not experience a run.

Run coordination. Consistent with the thresholds for idiosyncratic productivity in (8) and (9), we assume that uninsured depositors do not run on banks with $\epsilon_i > \overline{\epsilon}_i$, since these banks are always solvent. Further, we assume that the regulator shuts down banks with $\epsilon_i < \underline{\epsilon}_i$ before

[7] For a given δ, deviations from this pecking order can arise for banks with large capital holdings K_i, but low loan productivity realizations ϵ_i. For such banks, the effective marginal product of loans after the run, $A_i^K \epsilon_i (1-\kappa) K_i^{-\kappa}$, can be lower than the fire sale price δ.

Figure 5: Endogenous run-prone region as function of idiosyncratic risk



depositors can run. This leaves banks in the interval $\overline{\epsilon}_i \geq \epsilon_i \geq \underline{\epsilon}_i$ vulnerable to runs. These banks will default if they experience a run, but not otherwise. We assume that depositors choose to run on such banks conditional on realization of a bank-specific Bernoulli variable ς_i that takes on value 1 with probability π (a sunspot).[8]

Time-0 Problem. Define the household SDF M, derived in Appendix B.1. Then at time 0, bank i solves

$$\max_{K_i, B_i, D_i^U, D_i^I} \Pi_i + \mathrm{E}\left[M\left(\mathbb{I}_{\epsilon_i \geq \overline{\epsilon}_i} + \mathbb{I}_{\overline{\epsilon}_i \geq \epsilon_i \geq \underline{\epsilon}_i}\mathbb{I}_{\varsigma_i = 0}\right)\left(A_i^K \epsilon_i R_K K_i^{1-\kappa} + B_i - D_i^U - D_i^I\right)\right] \tag{10}$$

subject to the budget constraint

$$p_i + q_i^I(D_i^I)D_i^I + q_i^U(\mathcal{A}_i)D_i^U = K_i + pB_i + \Pi_i, \tag{11}$$

the leverage constraint

$$D_i^I + D_i^U \leq \theta^K K_i + \theta^B B_i, \tag{12}$$

and the liquidity constraint

$$\theta^D(D_i^I + D_i^U) \leq B_i. \tag{13}$$

Banks choose their portfolio of assets to maximize the sum of time-0 profits and the present value of time-1 dividends to their shareholders, the households. The bank maximizes dividends for states of the world in which the bank does not default, which are selected by the indicator functions based on the thresholds in (8) and (9). The objective (10) highlights that banks suffering a run, always default. The buget constraint in (11) states that banks raise funds

[8]The run game follows the structure laid out in Dávila and Goldstein (2023).

through sales of equity p_i and both types of deposits. Deposit prices are bank-specific, since banks have market power and idiosyncratic default risk. Banks internalize the effects of their portfolio choice on these bond prices, a key mechanism the shapes their optimal choices. Banks spend the funds raised on purchases of loans and bonds, and on profit payouts. The model contains two features giving rise to non-zero economic profits, which are decreasing returns to scale in lending and market power in deposit markets. The leverage constraint in (12) captures real-world bank equity capital requirements, allowing for different risk weights on loans and bonds. Finally, constraint (13) reflects liquidity regulation such as the liquidity coverage ratio (LCR) that requires banks to hold a fraction of their assets in liquid assets (bonds).

3.4.1 Recovery Value

When a bank defaults, creditors have a claim on the remaining value of bank assets. A fraction ξ of assets is lost in the bankruptcy process. The remainder is allocated proportionally to insured and uninsured deposits. The recovery value for insured deposits is used by the insurance fund to partially cover its expenses for paying out insured depositors. The recovery value of uninsured deposits is paid out to uninsured deposit holders. We separately derive recovery values conditional on whether a bank experienced a run.

No Run. In this case, assets after bankruptcy are $(1-\xi)(\epsilon_i R_K A_i^K K_i^{1-\kappa} + B_i)$. Fractions $D_i^I/(D_i^I + D_i^U)$ and $D_i^U/(D_i^I + D_i^U)$, respectively, are allocated to the recovery value of insured and uninsured bonds. The recovery value per dollar of deposits is thus for both insured and uninsured deposits

$$r_i^{nr} = \frac{(1-\xi)(\epsilon_i R_K A_i^K K_i^{1-\kappa} + B_i)}{D_i^I + D_i^U}. \tag{14}$$

Run. If there is a run, assets after bankruptcy are

$$(1-\xi)(\epsilon_i A_i^K R_K (K_i - \hat{K}_i)^{1-\kappa} + B_i - \hat{B}_i).$$

Since fraction $1-\phi$ of uninsured deposit holders have been paid out already after a run, the recovery value applies to insured deposits and the non-running uninsured deposits:

$$r_i^r = \frac{(1-\xi)\left(\epsilon_i A_i^K R_K (K_i - \hat{K}_i)^{1-\kappa} + B_i - \hat{B}_i\right)}{D_i^I + \phi D_i^U}. \tag{15}$$

3.5 Equilibrium

Expected insurance payouts for insured deposits are for each bank

$$T_i = D_i^I \mathrm{E}\left[F(\underline{\epsilon}_i)(1-\mathrm{E}(r_i^{nr})) + \pi\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right)(1-\mathrm{E}(r_i^r))\right].$$

Total taxes are $T = \int_i T_i di$. The government raises the amount of revenue needed to pay for bailouts in expectation.[9]

Bonds are supplied elastically by the government at price p. Loans are supplied by borrowers elastically at a price of 1. Insured deposit, uninsured deposit, and equity markets clear for each bank: supply of these securities by banks must equal demand by households at prices q_i^I, q_i^U, and p_i, respectively.

3.6 Equilibrium Conditions

We provide a full derivation of all equilibrium conditions in the model appendix. Below we summarize the model's implications for the choices of insured and uninsured deposits, and the sources of bank profits.

[9]Depending on the aggregate state, actual bailout expenses may deviate from this expected expenditure.

3.6.1 Household and Bank Deposit Choices

Households. Households purchase insured and uninsured deposits of all banks. In Appendix B.1, we derive household first-order conditions for deposits of bank i as

$$q_i^I = \psi \mathcal{H}^I(D_i^I)C_0 + \mathrm{E}[M], \tag{16}$$

$$q_i^U = \psi \mathcal{H}^U(D_i^U)C_0 + \mathrm{E}\left[M\mathcal{P}^U(\mathcal{A}_i)\right], \tag{17}$$

where the payoff to uninsured deposits is

$$\begin{aligned} \mathcal{P}^U(\mathcal{A}_i) =& 1 - F(\bar{\epsilon}_i) + F(\underline{\epsilon}_i)\mathrm{E}[r_i^{nr}|\epsilon_i < \underline{\epsilon}_i] \\ &+ (1-\pi)\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right) + \pi\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right)\left(1 - \phi + \phi\mathrm{E}[r_i^r|\underline{\epsilon}_i \le \epsilon_i \le \bar{\epsilon}_i]\right), \end{aligned} \tag{18}$$

with F being the c.d.f. of ϵ_i. Insured deposit prices in (16) have a certain payoff of 1 at time 1. In addition, they deliver a liquidity benefit $\mathcal{H}^I(D_i^I)$ given in equation (25) in Appendix B.1 to households that depends on their quantity – deposits have diminishing marginal benefits. Uninsured deposits in (17) deliver an analogous liquidity benefit $\mathcal{H}^U(D_i^U)$ contained in equation (24) in the appendix. Furthermore, uninsured deposits have a stochastic payoff at date 1 given in equation (18) that depends on aggregate risk and banks' idiosyncratic shock ϵ_i. The first line of (18) accounts for banks that are either run-proof with a shock realization $\epsilon_i > \bar{\epsilon}_i$, or fundamentally insolvent even absent a run $\epsilon_i < \underline{\epsilon}_i$. In the former case, uninsured deposits pay out in full, and in the latter case they pay the no-run recovery value in (14). The second line of (18) accounts for banks in the run-prone region with $\epsilon_i \in [\underline{\epsilon}_i, \bar{\epsilon}_i]$: these banks do not experience a run with probability $1-\pi$, in which case uninsured deposits pay out in full. A run occurs with probability π, in which case the fraction $1-\phi$ of running uninsured depositors is paid out in full, but fraction ϕ of inactive depositors only receives the post-run recovery value from (15). Equation (18) clarifies that the uninsured payoff depends on the complete portfolio $\mathcal{A}_i = (K_i, B_i, D_i^I, D_i^U)$ of the bank, since these choices impact default thresholds and recovery values.

Banks. When banks choose their portfolios, they internalize the effects of their choices on the prices of insured and uninsured deposits. Taking bank first-order conditions for (K_i, B_i, D_i^I, D_i^U)

therefore involves differentiating (16) and (17). For example, the the FOC of bank i for insured deposits is

$$q_i^I = \mu_i + \theta^D \lambda_i - \frac{\partial q^I(D_i^I)}{\partial D_i^I} D_i^I - \frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^I} D_i^U + \mathrm{E}\left[M(1 - F(\underline{\epsilon}_i) - \pi(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i))\right]. \quad (19)$$

In this expression, μ_i is the Lagrange multiplier on the bank's leverage constraint and λ_i is the multiplier on the liquidity constraint. The rightmost term reflects that bank equity owners only care about payoffs in states without bank default. The two partial derivatives in the middle arise as banks take into account the effect of their insured deposit issuance on the household valuation of insured and uninsured deposits – these terms differentiate the household demand functions in (16) and (17). Specifically, each bank internalizes that issuing more insured deposits will decrease households' marginal liquidity benefit and thus raise the interest rate it has to pay on insured deposits ($\frac{\partial q^I(D_i^I)}{\partial D_i^I} < 0$). Further, each bank internalizes that issuing more insured deposits will raise its leverage and default risk, which in turn means that it must pay higher interest on its *uninsured* deposits ($\frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^I} < 0$). All other bank first-order conditions and the derivatives of household demand functions are in Appendix B.2.

4 Calibration

We calibrate the model to bank-level data from regulatory filings (FFIEC 031/041) from 2010q1 to 2022q4. We jointly calibrate the key parameters in Panel A of Table 1 as they shape how bank productivity and size distribution interact with households' liquidity demand. We can structure this description around how and which parameters affect (1) banks' production technology, (2) banks' deposit productivity, (3) and households' liquidity preference.

The production technology parameters are helpful for capturing the concentration of the banking sector. Our model allows for a distinction between asset concentration and loan concentration. Both the loan production function curvature κ and the parameter Z_{shape}, which governs the loan productivity distribution across banks, influence how concentrated the banking sector is. We target both the standard deviation of log loans (1.521) and log assets (1.427). The parameter μ_{RK} determines the average aggregate return on capital R^K, which scales the average aggregate loan return. Note that given the concentration in the banking sector and decreasing returns to scale, the average loan return is distinct from the average aggregate return on capital R^K. We compute a value-weighted loan return in the data as the ratio of the sum of aggregate interest income on loans and leases and loan sale income minus provisions for loan losses and the loan operating expense share over the beginning of period aggregate loans.

The second set of parameters governs heterogeneity in banks' deposit productivity. Our baseline model assumes that a bank's deposit productivity is perfectly correlated with its lending productivity, with a higher sensitivity for below-median and a lower sensitivity for above-median loan productivity. A higher deposit productivity sensitivity for below-median productive banks in terms of lending means that these banks would want to hold a higher bond share to back their deposit franchise. To calibrate A_{-}^{D} and A_{+}^{D}, we target the bond share of the bottom 20% smallest banks by asset size and the bond share of the top 1% of banks. We define the bond share as the ratio of the sum of cash, federal funds sold, repo assets, and securities over assets. Bank size is determined by assets.

The final set of jointly calibrated parameters governs liquidity preferences. The parameter ψ is the weight of liquidity in the household utility function. As such, it scales the liquidity premium that drives a wedge between the risk-free rate and deposit rates. The CES elasticity of substitution parameter ρ, which governs the degree to which insured deposits are differentiated products, also matters for the level of deposit rates (ceteris paribus). When insured deposits are relatively more differentiated relative to uninsured deposits, households accept a lower interest rate on insured deposits relative to uninsured deposits. The ideal empirical targets for ψ and ρ would, therefore, be the insured and uninsured deposit rates. Unfortunately, rates by insurance status are only available for time deposits and only before 2009 or after 2016 when bank call reports reported the interest expense on time deposits broken out by deposit insurance limit.[10] Given the data limitation, we choose to target the transaction deposit rates and time deposit rates to roughly capture insured and uninsured deposit rates. We compute the transaction deposit rate as interest expenses on domestic deposits less domestic time deposit

[10]Between 2009 and 2016, interest expenses were reported for time deposits with balances above and below \$100,000, but the deposit insurance limit had increased to \$250,000 at the end of 2008. In addition, large demand or savings deposits accounts may also exceed the deposit insurance limit. RateWatch reports banks offer rates by deposit products, but offer rates on select products are not informative about what banks actually pay overall on their deposit accounts.

Table 1: Key Parameters

Panel A: Name	Jointly Calibrated Parameters Description	Value	Data	Model
	Production technology			
κ	Production fct curvature	0.123	Std(Log Loans) = 1.521	
μ_{RK}	Mean capital return	1.085	Agg Loan Return = 2.742%	
Z_{shape}	Shape of loan prd dist std = shape × scale	0.13	Std(Assets) =1.427	
	Deposit technology			
A^D_-	Slope of Deposit Productivity −	1.1	Bottom 20% bond share= 0.410	
A^D_+	Slope of Deposit Productivity +	0.275	Top 1% bond share = 0.328	
	Liquidity preference			
ψ	Weight on liquidity	0.1	Time deposit rate = 1.228%	
α	Weight on insured deposits	0.8	Uninsured deposit share = 0.419	
ρ	EoS b/w insured deposits	0.855	Transaction deposit rate =0.312%	
ρ_U	EoS b/w uninsured deposits	0.96	Gini uninsured = 0.963	

Panel B: Name	Externally Calibrated Parameters Description	Value	Data Source
β	Discount factor	0.99	1% risk-free return
γ	Risk aversion	2	Standard value in literature
r	Bond return	0.0123	Avg. bond return
Z_{mean}	Mean loan productivity	1	Normalization
Z_{loc}	Minimum loan productivity	0.6	ELS implied 0.48
Z_{scale}	Max - min loan productivity	1.55	ELS implied 1.54
μ_{RB}	Mean bond return	1	Normalization
σ_{RB}	Deviation of bond return	0.035	Vol of 5-year UST
σ_{RK}	Deviation of capital return	0.045	Vol of corporate bond portfolio
$\sigma_{\epsilon \ln}$	Volatility of idio. capital shocks	0.12	Vol of idio. bank equity return
ϕ	Uninsured deposit haircut in runs	0.1	Long term debt share = 0.074
δ	1 - Firesale costs of capital	0.675	Moody's
ξ	Share lost in recovery	0.25	Moody's

Notes: This table presents the internally (jointly) calibrated model parameters (Panel A) and the parameters set directly to external moments in the data (Panel B). Uninsured share is the share of uninsured deposits in total domestic deposits. ELS stands for Egan, Lewellen, and Sunderam (2022). Details on Panel A parameters are in the text. The sample period is from 2010 to 2022 unless otherwise stated. Data sources are bank call report filings. Details on Panel B parameters are in Appendix Section A1.

interest expense over beginning of period transaction deposit accounts, which are domestic non-time deposit accounts. The time deposit rate is the interest expense on domestic time deposits divided by the beginning of period domestic time deposit accounts. Both rates are annualized. The parameter α is the weight on insured deposits in the liquidity aggregator, and therefore determines the insured deposit share, and likewise our target the uninsured deposit share. We compute the uninsured share as the share of uninsured domestic deposits over total domestic deposits. Uninsured deposits deposit account values with balances over \$250,000 minus the number of uninsured accounts times \$250,000. We choose the Gini coefficient of uninsured deposits as a target for the elasticity of substitution parameter for uninsured deposits ρ_U. When households perceive uninsured deposits as relatively undifferentiated, only the most productive banks will find it profitable to issue uninsured deposits. As a result, uninsured deposits will be only issued by the largest banks since large banks are the most productive banks in our model, leading to the Gini coefficient as a natural target for ρ_U.

We discuss the externally calibrated parameters listed in Panel B in Appendix Section A.

5 Results

In this section, we first describe the model mechanism, then discuss how an unanticipated interest rate shock and a partial bailout guarantee affect the banking system, and finally, we discuss policy implications.

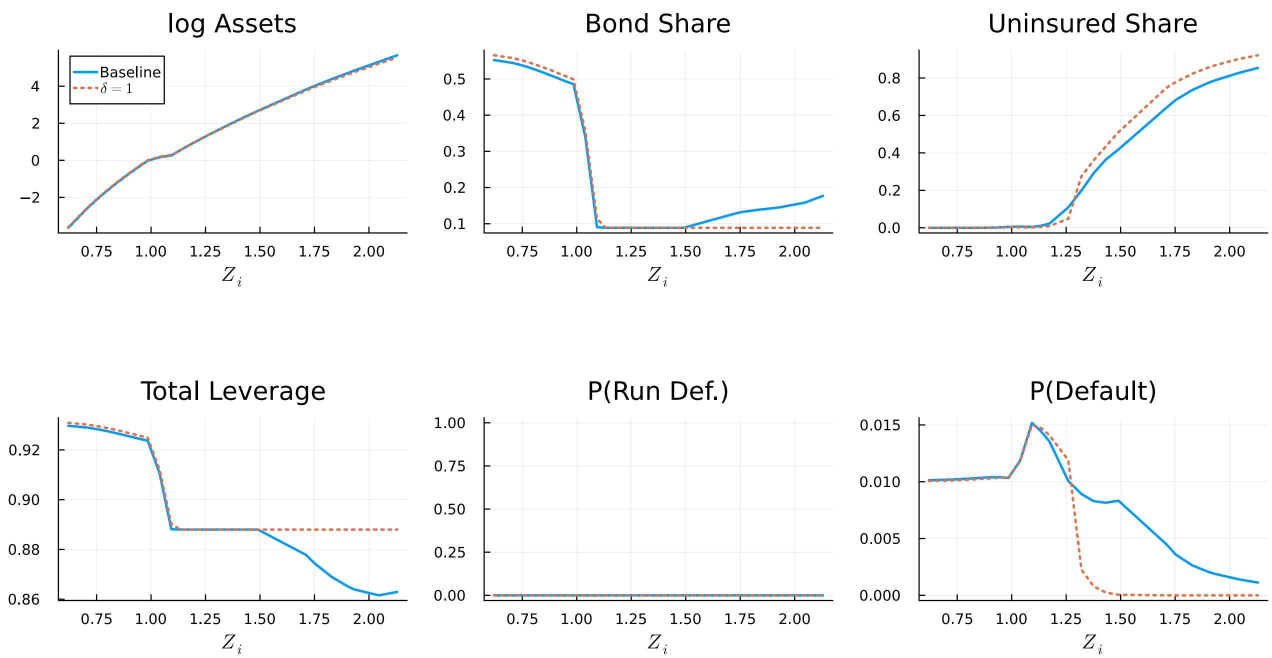
5.1 Key Model Mechanisms

We now describe the key mechanisms that allow the model to match the empirically observed high concentration of uninsured deposits among the largest banks. High shares of uninsured deposits expose the largest banks to the possibility of runs.

5.1.1 The Role of Runs

We first analyze how run risk affects bank choices and the industry equilibrium, both ex-ante and in terms of realized risks. To do so, we consider a counterfactual version of the model where capital can be sold without a firesale discount during runs ($\delta = 1$), allowing banks to fully repay depositors in case of a run. In this special case, runs do not induce banks to realize losses. Thus, with $\delta = 1$, depositors have no reason to run in the first place, and runs will not occur in equilibrium irrespective of banks' portfolio composition.[11]

Figure 6: Equilibrium in Baseline and Without Runs



Notes: Top row: "log Assets" is the logarithm of the sum of capital and bonds, "Bond Share" is the fraction of bonds in the asset portfolio, and "Uninsured Share" is the fraction of uninsured deposits in all deposits. Bottom row: "Total Leverage" is the sum of insured and uninsured deposits divided by assets, "P(Run Def.)" is the unconditional probability of banks experiencing a run and defaulting as a result, and "P(Default)" is the unconditional probability of default across all aggregate payoff states.

Figure 6 shows six outcome variables in the cross-section of banks, indexed by asset productivity Z_i on the x-axis. The blue line shows the calibrated model "Baseline," while the red dotted line shows a counterfactual model with no fire sale discount, $\delta = 1$. Banks choose similar scales in terms of total assets, although the largest, most productive banks are slightly smaller in the baseline model with run risk. The presence of run risk has a noticeable effect on banks' bond share: absent this risk, small banks choose to hold more bonds, while large banks only hold bonds mandated by liquidity regulation. This is contrary to the baseline model, where

[11] Alternatively, we could set the probability of the sunspot for runs π to zero. Results look almost identical to the ones for $\delta = 1$. Varying δ is computationally more convenient.

the largest banks voluntarily hold more bonds than required. The share of uninsured bonds is most strongly affected by the presence of run risk. While the least productive banks do not issue any uninsured deposits in either model, the uninsured share is more steeply increasing in the $\delta = 1$ equilibrium, and it is higher for the largest banks. Without run risk, all banks are at a binding leverage constraint. In the baseline model, the largest banks leave a buffer to the constraint – this is despite the fact that their greater bond share would allow them higher leverage compared to the $\delta = 1$ model. These effects on bond holdings, uninsured deposits, and leverage reflect precautionary portfolio decisions of banks to insure against the risk of runs.

The mid panel of the bottom row shows that runs do not occur in equilibrium of either economy. With $\delta = 1$, the absence of runs is an immediate consequence of the fundamental lack of a run motive. However, $\delta = 0.665$ in the baseline model implies substantial fire sale losses in case of a run. The fact that no runs occur even in this model is exactly due to banks' precautionary behavior: they issue fewer uninsured deposits (to reduce run exposure), hold more bonds (to hedge fire sale losses in capital), and reduce leverage (to limit default risk).

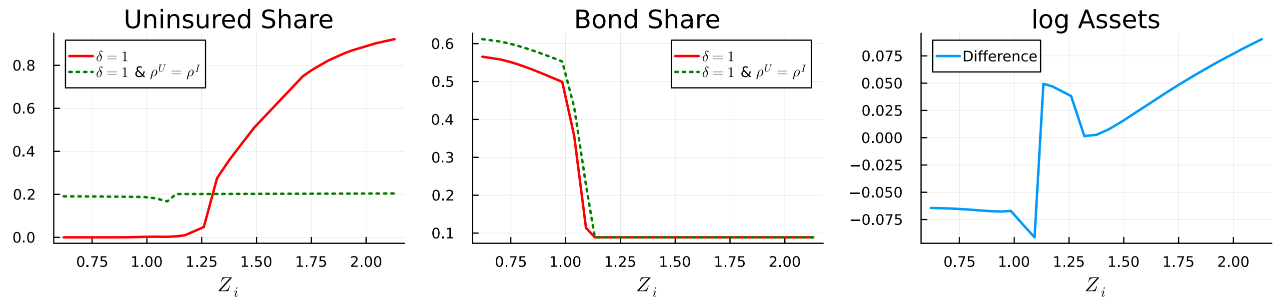
Since run-induced defaults do not occur in either model, all realized defaults shown in the bottom right panel are purely solvency driven. Interestingly, small differences in scale and portfolio composition of large banks cause fewer solvency defaults in the high delta economy. This is a general equilibrium effect: with $\delta = 1$, relatively less productive banks have more uninsured deposits, making it less profitable for the largest banks to scale up their assets. As a result, the largest banks in the no-firesale loss ($\delta = 1$) economy are roughly 3% smaller than in the baseline. Their smaller scale, combined with diminishing returns in capital, is sufficient to lower solvency defaults to zero for the largest banks.

5.1.2 Differential Market Power in Insured versus Uninsured

The model generates a realistic concentration of uninsured deposits among the largest banks. As in the data, the uninsured share is strongly increasing in bank size. To isolate the role of product differentiation in generating this cross-sectional pattern, we consider a simple counterfactual economy in which the degree of production differentiation is identical in both deposit types, i.e. $\rho^U = \rho^I = 0.855$.

Figure 7 shows the effects of this parameter change. As we can see in the top left panel, the

Figure 7: Equilibrium With Symmetric Deposit Market Power



Notes: 'Uninsured Share" is the fraction of uninsured deposits in all deposits. "Bond Share" is the fraction of bonds in the asset portfolio. "log Assets" is the logarithm of the sum of capital and bonds.

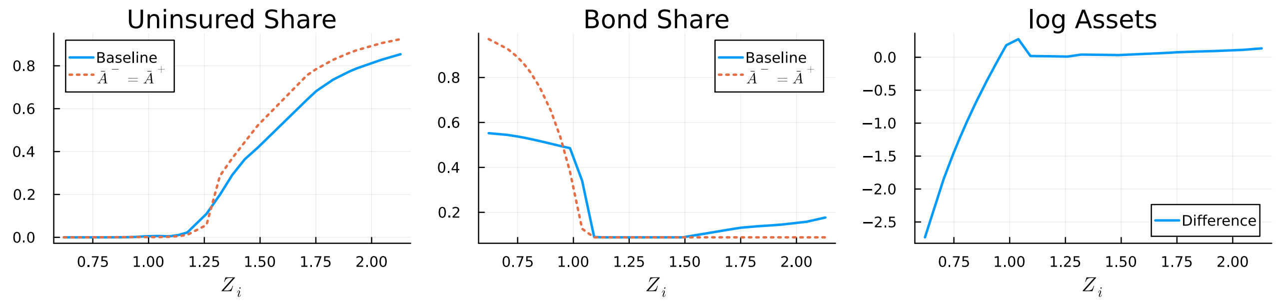
uninsured share is flat at 20% across the size distribution when $\rho^U = \rho^I$. The different nature of competition in the uninsured market, in turn, affects bond shares, leverage, profits, and default rates. The top right panel shows log assets in the $\delta = 1$ baseline minus log assets of the model with equal ρ^j. This difference shows that with symmetric market power in both deposit types, the bank size distribution is more equal: low-productivity banks are relatively larger, and high-productivity banks are smaller compared to the baseline with more differentiation in insured deposits. This comparison highlights that the cross-sectional pattern in uninsured deposits generated by the model is not due to run risk or bailout guarantees, since neither of these channels are present in the $\delta = 1$ economy. Rather by comparing the one-parameter change in ρ^U relative to the high-delta economy, we can clearly see that the differential degree of market power in markets for insured and uninsured deposits is at the core of the model's ability to create the right cross-sectional allocation of uninsured deposits.

5.1.3 Deposit Productivity Heterogeneity

The baseline model features heterogeneity in deposit productivity that is perfectly correlated with loan productivity. However, the loading of deposits on loan productivity is asymmetric above and below the median. Deposit productivity of below-median banks is $\bar{A}^- = 1.1$ times their loan productivity, while for above-median banks the loading is $\bar{A}^+ = 0.25$.

Figure 8 compares the baseline calibration to a model in which the deposit productivity loading is symmetric below and above the mean at $\bar{A}^- = \bar{A}^+ = 0.25$. The direct consequence

Figure 8: Equilibrium With Symmetric Deposit Productivity



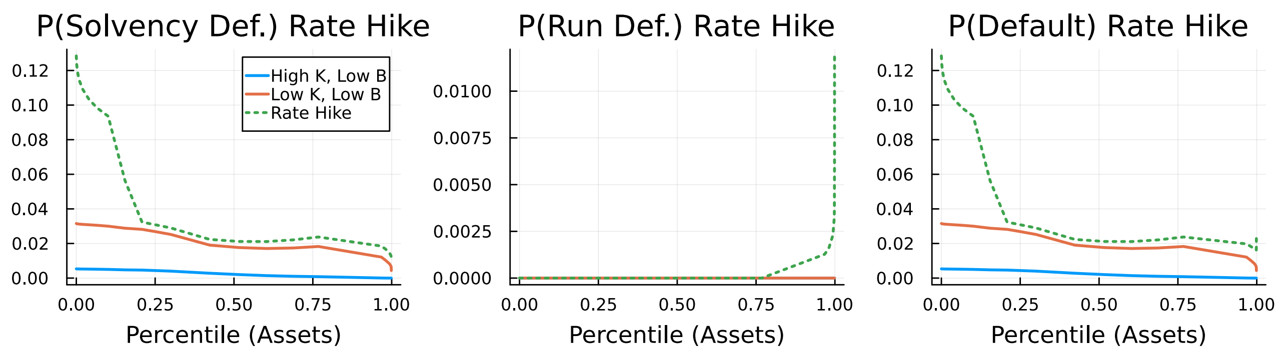
Notes: 'Uninsured Share" is the fraction of uninsured deposits in all deposits. "Bond Share" is the fraction of bonds in the asset portfolio. "log Assets" is the logarithm of the sum of capital and bonds.

of this parameter change is that the deposit productivity of banks below the median is now declining much less rapidly in loan productivity than in the baseline. The rightmost plot in Figure 8 shows that low-productivity banks are much smaller in the baseline economy as a result. In the counterfactual economy with $\bar{A}^- = 0.25$, banks with low loan productivity have relatively much higher deposit productivity. It is optimal for them to issue larger quantities of insured deposits to exploit their market power. Since these banks only have access to low productivity loans, they instead rely on bonds to scale up their balance sheet, resulting in a much higher bond share as can be seen in the middle panel of Figure 8. The main effect of lower $\bar{A}^-$ on large banks is in the uninsured deposit share depicted in the left panel. Even though the above-median banks are not directly affected by the parameter change, general equilibrium forces cause them to increase their uninsured deposit share. In the symmetric $\bar{A}^j$ model, low-productivity banks are much larger, and they supply substantially more insured deposits in total. Since insured and uninsured deposits are imperfect substitutes in aggregate, greater insured supply from small banks creates demand for greater uninsured supply from large banks.

5.2 Effects of An Unanticipated Rate Hike

We next analyze the role of interest rate risk on the fragility of the banking sector. Agents in the model anticipate substantial interest rate return risk: the return on bonds has a standard deviation of 3.5%. What if bond returns drop lower than agents anticipate, for example, following a surprising interest rate hike by the central bank? To answer this question, we study how default risk across the bank size distribution is affected by an unanticipated low return realization of 10% below the mean.[12]

Figure 9: Default Probabilities After Rate Hike



Notes: "P(Solvency Def.)" is the default rate due to insolvency ($\epsilon_i < \underline{\epsilon}_i$). "P(Run Default)" is the default rate caused by runs. "P(Default)" is the sum of insolvency and run-induced defaults.

Figure 9 displays bank default rates across the size distribution for different realizations of aggregate returns on loans and bonds. The left panel shows defaults purely based on solvency risk, i.e. when $\epsilon_i < \underline{\epsilon}_i$. When the loan return is high, and the bond payoff is low, as shown by the solid blue line, solvency defaults are small and slightly declining in size. When both returns are low, as depicted by the red line, solvency default rates are around 3% for the smallest banks and decline to around 0.5% for the largest banks. When interest rates unexpectedly increase such that the return on bonds is -10% (dashed green line), solvency defaults for the smallest banks jump significantly to over 10%; solvency defaults for banks above the 25th percentile of the asset distribution, however, are hardly changed relative to the low anticipated value of bond returns. This is because smaller banks hold a substantially larger fraction of their assets in interest-rate-sensitive bonds.

Run defaults in the middle panel are zero even for low realizations of both returns (red line). The low unexpected return of -10%, however, causes run risk among the largest banks, where most of the uninsured deposits have been issued. Consequently, the run default rate jumps to 1% for these banks. Total defaults in the right panel are the sum of both components. After a

[12]The numerical implementation discretizes the bond return with two equal-probability realizations. The lowest return realization in the expectation set is thus 3.5% below the mean.

rate hike, defaults are higher at all points in the size distribution. However, the model predicts the largest increase among small banks that are most exposed to bond returns due to their high bond portfolio share. The model also generates positive default rates among the largest banks, which are exposed to run risk due to their high uninsured share. These results demonstrate that the combination of uninsured deposits and low bond returns can amplify the default risk of large banks.

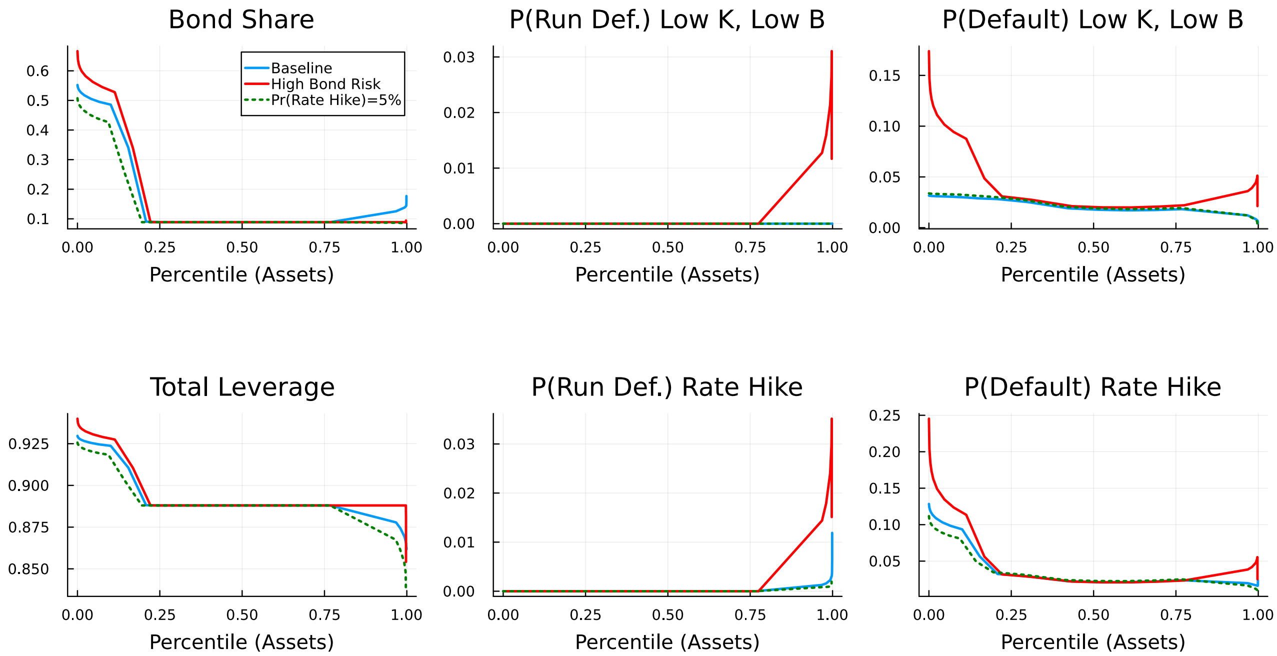
5.3 Were Banks "Surprised" By Rate Hikes?

Do these results hold up if banks anticipate greater bond return risk, or more technically, if lower (and higher) bond return realizations are in the expectation set of banks, rather than arriving as "MIT shocks?"

Symmetric Bond Return Risk. We explore this question in Figure 10, which compares the baseline economy to one in which bond returns are much more volatile at 8.5% standard deviation (compared to 3.5% in the baseline), in the red line labeled "High Bond Risk." The middle and right panels of the top row show default rates conditional on an anticipated low loan/low bond return shock. In the baseline model, this shock does not cause run defaults. In the economy with high bond risk, however, even this anticipated shock causes a 3% spike in run defaults for large banks. This rise in run-induced defaults is, of course, partially due to a much lower realization of bond returns, but it is also caused by different ex-ante portfolio choices of banks. In particular, large banks in the high bond risk economy do not have voluntary bond holdings (top left) and do not keep a precautionary buffer to their maximum allowed leverage (bottom left). Furthermore, small banks in the high-risk economy choose to *increase* their bond share. This risk-seeking behavior results in substantially higher solvency default rates for small banks.

The middle and right panels in the bottom row show how these different portfolio choices play out when the unanticipated rate hike occurs. Run and solvency default rates in the high bond risk economy look similar to those in the row above, which displays the anticipated shock. They are much higher compared to the baseline economy, where agents expect bond return volatility to be just 3.5%.

Figure 10: Default Probabilities After Rate Hike With Higher Bond Risk



Notes: "Bond Share" is the fraction of bonds in the asset portfolio. "Total Leverage" is the sum of insured and uninsured deposits divided by assets."P(Run Default)" is the default rate caused by runs. "P(Default)" is the sum of insolvency and run-induced defaults.

These results highlight that rational anticipation of higher symmetric bond return risk by banks does not necessarily lead to safer portfolios that can absorb greater interest rate fluctuations without incurring solvency and run risk. Banks in the model have several competing motives governing their portfolio choice. On the one hand, banks internalize that risky portfolio choices affect the interest rates they have to pay on uninsured deposits – a mechanism that on its own creates a precautionary motive. On the other hand, banks enjoy government insurance on a large fraction of their deposit base, which, combined with limited liability, creates a motive for seeking risky returns. The fact that banks choose to tolerate higher default risk in the high-bond risk economy indicates that, as the volatility of asset returns increases, the risk-seeking channel partially offsets the precautionary motive.

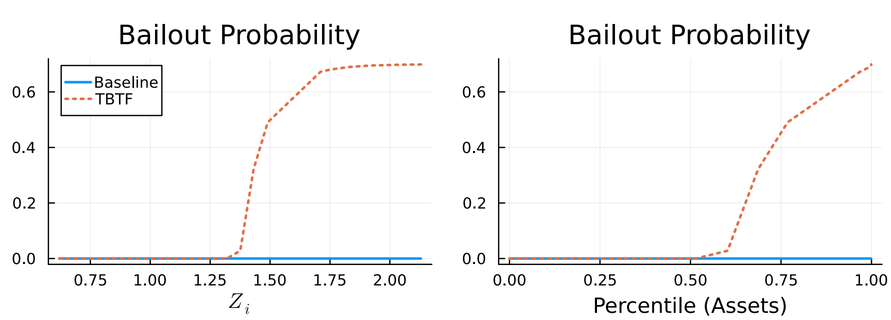
Downside Bond Return Risk. What if banks instead anticipate the actual rate hike with a 5% probability? The green dashed line labeled "Pr(Rate Hike)=5%" shows this scenario. Relative to the high bond risk economy, banks now anticipate asymmetric downside risk in bond returns that is equal in size to the realized rate hike. This change in expectations shifts the mean bond return, causing both small and large banks to downscale their bond shares. Especially large banks reduce leverage substantially in this economy, which works to reduce run and overall default risk. Since bonds are no longer useful as insurance for runs, banks instead use leverage to manage their risk.

Taken together, both scenarios imply that bank exposure to interest rate risk is consistent with rational portfolio choice that anticipates high bond return volatility. Only if banks have expectations of asymmetric downside (but not upside) bond return risk, our model implies that they would optimally choose safer portfolios.

5.4 Partial Guarantees for Uninsured Deposits

Implicit bailout guarantees for large banks that are deemed "too big too fail" (TBTF), are commonly considered as explanation for uninsured deposit concentration at these large banks (O'Hara and Shaw, 1990). The intuition is straightforward: if the "uninsured" deposits of large banks also enjoy significant government guarantees, then depositors should be more willing to hold them. As we established in Section 5.1.2, the baseline version of our model does not rely on this explanation. However, we can easily incorporate probabilistic bailout guarantees of uninsured deposits that are increasing in bank size.

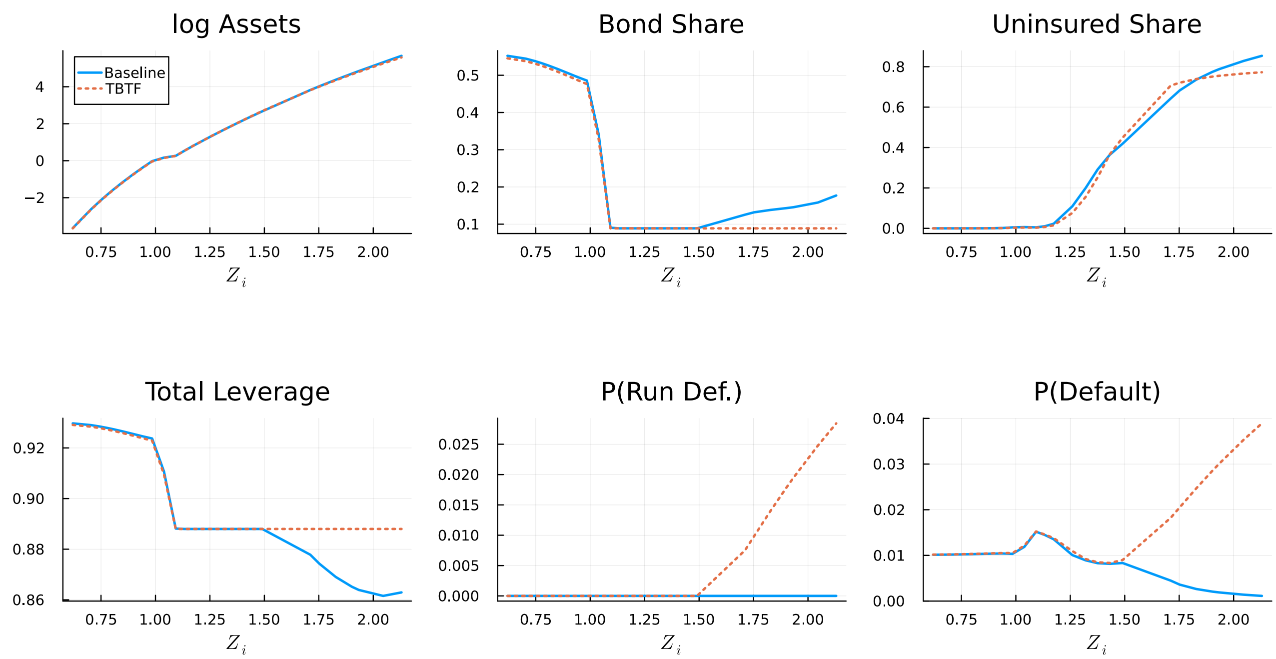
Figure 11: Size-dependent Bailout Probability



Notes: Left panel: size-dependent bailout probability for uninsured deposits by bank productivity Z_i. Right panel: size-dependent bailout probability for uninsured deposits by bank asset distribution percentile.

In Figure 11, we show the size-dependent bailout probability that we feed into the model. The probability is zero for banks below the asset median and then increases in balance sheet size until topping out at 70% for the largest banks. In Figure 12, we see how this bailout probability affects bank choices and outcomes. Most strikingly, too-big-too-fail bailouts raise the run risk of large banks significantly, as the bottom mid-panel shows. This is despite large banks choosing fewer uninsured deposits – the increase in risk happens as a result of a lower bond share and higher leverage. While bailout guarantees do not explain the allocation of uninsured deposits, they are a powerful source of risk-taking for large banks.

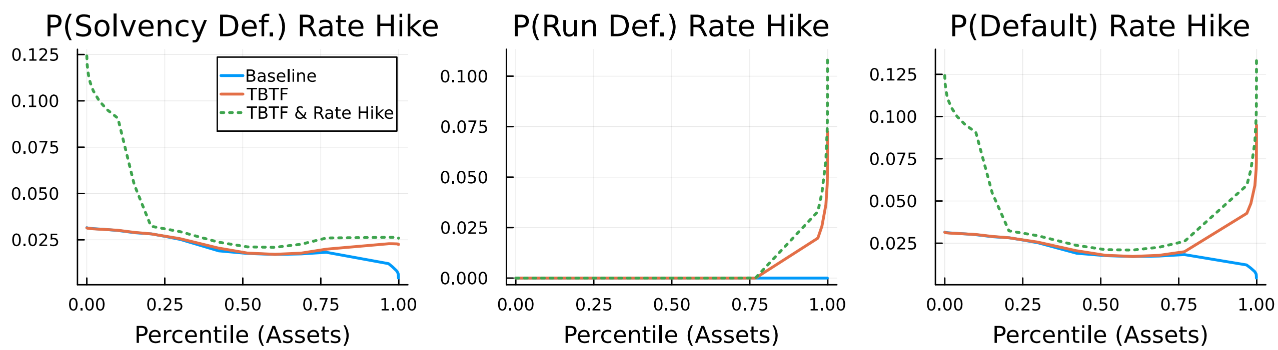
Figure 12: Equilibrium in Baseline and With Bailouts



Notes: Top row: "log Assets" is the logarithm of the sum of capital and bonds, "Bond Share" is the fraction of bonds in the asset portfolio, and "Uninsured Share" is the fraction of uninsured deposits in all deposits. Bottom row: "Total Leverage" is the sum of insured and uninsured deposits divided by assets, "P(Run Def.)" is the unconditional probability of banks experiencing a run and defaulting as a result, and "P(Default)" is the unconditional probability of default across all aggregate payoff states.

We further decompose the increase in default risk in Figure 13, which also displays the effect of an unanticipated rate hike combined with too-big-to-fail guarantees. The red and blue solid lines in the graph show the default probability in the baseline and TBTF versions of the model, respectively, for the worst possible aggregate payoff state. As we know from previous results, even low aggregate return realization for loans and bonds does not cause run-based defaults in the baseline model. However, in the model with bailout guarantees, the run default rate in the bad (expected) state is close to 7.5%. The dashed green line shows default probabilities to the unanticipated rate hike in the model with bailouts. Run-default rates for the largest banks go above 10% in this model; from Figure 9, we recall that this default rate only rises to 1% in the baseline model. Thus, TBTF bailout guarantees amplify the run risk of big banks by factor 10 in case of a large surprise rate increase.

Figure 13: Bank Default Risk With Bailouts



Notes: "P(Solvency Def.)" is the default rate due to insolvency ($\epsilon_i < \underline{\epsilon}_i$). "P(Run Default)" is the default rate caused by runs. "P(Default)" is the sum of insolvency and run-induced defaults.

5.5 Policy Implications

We use the model as a laboratory for evaluating various policy proposals. In the following subsections, we will consider standard size-independent regulation and policies that explicitly condition on bank size. For all policy counterfactuals, we use the model with TBTF guarantees as baseline.

Unconditional capital requirements. Equity capital requirements for risky assets are widely regarded as a powerful tool for mitigating risk-taking by banks and have been studied extensively in the literature. We begin our policy analysis by studying variations in the equity capital requirement for loans, which is captured by the maximum leverage parameter θ in the model. Table 2 shows different aggregate outcomes for different values of θ around the baseline value of 0.88 (corresponding to a 12% risk-weighted capital charge on loans).

In line with other studies, capital requirements govern a trade-off between consumption and

Table 2: Varying the capital requirement on loans

Outcome	θ						
	86%	86.5%	87%	87.7%	**88%**	88.5%	89%
1. Loans	-1.052	-0.818	-0.565	-0.293	1.613	0.316	0.655
2. C0	0.013	0.012	0.009	0.005	2.072	-0.007	-0.017
3. E(C1)	0.097	0.075	0.052	0.027	2.201	-0.029	-0.061
4. E(DWL)	-52.177	-42.007	-29.992	-16.020	0.004	18.149	38.502
5. SD(MPK)	-12.525	-9.684	-6.694	-3.492	0.003	3.874	8.229
6. Liquidity	-1.520	-1.133	-0.750	-0.372	1.020	0.365	0.723
7. Welfare	-0.026	-0.016	-0.008	-0.003	1.518	-0.001	-0.005
8. C1 (Rate Hike)	0.108	0.083	0.058	0.032	2.122	-0.052	-0.109
9. Welfare (Rate Hike)	-0.020	-0.012	-0.005	0.000	1.483	-0.016	-0.037

Notes: This table shows changes in aggregate model outcomes as the maximum allowed leverage on loans θ is varied around its baseline value of 88%. The *levels* of listed outcomes for the baseline model are shown in the $\theta = 88\%$ column of the table. The columns to the left and right show *percentage changes* of the same moments relative to baseline. Moments: 1. Loans – aggregate lending $\mathrm{E}_i[K]$, 2. C_0 – time-0 consumption, 3. $\mathrm{E}[C_1]$ – time-1 expected consumption, 4. E[DWL] – time-1 aggregate expected DWL, including default bankruptcy costs and firesale losses during runs, 5. SD[MPK] – expected standard deviation in marginal product of capital across banks at time 1, 6. Liquidity – liquidity utility H, 7. Welfare – total time-0 expected utility, 8. C1 (Rate Hike) – Time-1 consumption conditional on unanticipated rate hike. 9. Welfare (Rate Hike) – Time-0 welfare conditional on unanticipated rate hike at time 1.

liquidity provision. Row 1 shows that tightening the capital requirement by 2% (to $\theta = 86\%$) restricts lending by 1%, while relaxing the requirement by 1% (to $\theta = 89\%$) increases lending by 0.65%. At the same time, a tighter requirement reduces bankruptcy-induced deadweight losses by 52% (row 4), while a relaxation by 1% causes deadweight losses to rise by 38.5%. These changes in bankruptcy losses affect consumption in an intuitive way – lower losses mean higher consumption; see rows 2 and 3 of the table. However, since baseline losses are small, so are the corresponding changes in consumption. The benefit of a tighter capital requirement is thus higher consumption, which is traded off against lower liquidity provision. Row 6 displays this effect on utility from liquidity services, which declines by 1.5% with a 2% tighter capital requirement. In row 5, which shows the dispersion in the marginal product of lending, we can see how capital requirements affect the efficiency of capital allocation in the banking sector. In a frictionless model, the marginal product of capital should be equalized across banks. Row 5 shows that dispersion in MPK declines with a tighter capital requirement in the model, which is an additional source of welfare gain from tighter regulation that is not directly linked to avoiding bankruptcy losses.[13]

Row 7 shows that overall welfare is roughly maximized at the baseline value of θ at 88%, implying a 12% risk-weighted capital charge on loans. A tighter capital requirement increases consumption in both periods but also lowers liquidity services, with the second effect dominating. Looser capital requirements, in turn, cause an increase in liquidity utility but simultaneously reduce consumption through greater defaults and misallocation, with the latter effect dominating. As a result, welfare decreases almost symmetrically as θ deviates from its baseline value. Note that our model only captures the micro-prudential effects of bank capital requirements and that macro-prudential considerations such as larger consumption losses during a systemic crisis may warrant a tighter capital charge on loans.

Liquidity requirements. Liquidity requirements are meant to ensure that banks have liquid assets which can be sold without loss in case of large deposit withdrawals or runs. Our model captures both runs and the heterogeneous exposure of banks to run risk through their endogenous choice of uninsured deposits. A natural policy to mitigate the risk stemming from runs are liquidity requirements tied to uninsured deposits. We implement this policy in the model through a modified liquidity constraint (see equation (13)):

$$\theta^D D_i^I + (\theta^D + \theta^U) D_i^U \leq B_i, \tag{20}$$

where θ^U is a new parameter that requires banks to hold bonds in proportion to their uninsured deposits. In our baseline model, large banks hold portfolios such that runs only play a minor quantitative role. Therefore, to give heightened liquidity requirements the best shot at reducing run risk, we use the model with TBTF (bailout) guarantees as baseline in this section.

Table 3 shows how the same aggregate outcomes studies in Table 2 respond to an increase in θ^U relative to its baseline value of 0% in the first column. The run risk of the largest banks is listed in row 8 of the table, and one can see that the additional liquidity requirements work as intended: they reduce this risk by almost 50% at the 8% value for θ^U. It follows that

[13] Better loan allocation measured by lower dispersion in MPK materializes in higher consumption, same as lower bankruptcy losses.

Table 3: Varying liquidity requirements for uninsured deposits

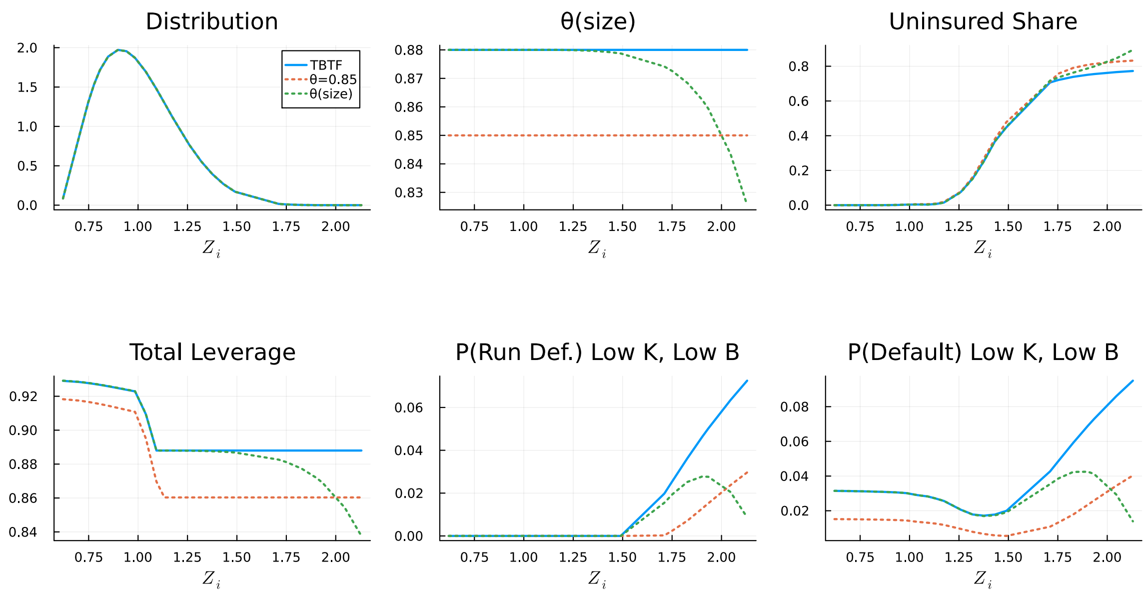
Outcome	θ^U				
	0%	2%	4%	6%	8%
1. Loans	1.613	-0.137	-0.275	-0.416	-0.557
2. C_0	2.072	-0.006	-0.012	-0.018	-0.025
3. E[C_1]	2.201	0.006	0.011	0.017	0.023
4. E[DWL]	0.004	-4.496	-8.660	-12.427	-15.731
5. SD[MPK]	0.003	1.592	3.271	5.041	6.900
6. Liquidity	1.020	-0.033	-0.069	-0.106	-0.145
7. Welfare	1.518	-0.002	-0.005	-0.007	-0.010
8. Run Def. top 0.1%	0.027	-11.295	-22.782	-34.347	-45.811

Notes: This table shows changes in aggregate model outcomes as the additional liquidity requirement for uninsured deposits θ^U is raised from its baseline value of 0%. The *levels* of listed outcomes for the baseline model with TBTF guarantees are shown in the $\theta^U = 0\%$ column of the table. The columns to the right show *percentage changes* of the same moments relative to baseline. Moments: 1. Loans – aggregate lending $E_i[K]$, 2. C_0 – time-0 consumption, 3. E[C_1] – time-1 expected consumption, 4. E[DWL] – time-1 aggregate expected DWL, including default bankruptcy costs and firesale losses during runs, 5. SD[MPK] – expected standard deviation in marginal product of capital across banks at time 1, 6. Liquidity – liquidity utility H, 7. Welfare – total time-0 expected utility, 8. Run Def. top 0.1% – Run-induced default rate of top 0.1% largest banks in worst aggregate payoff state.

deadweight losses from defaults are reduced considerably as the liquidity requirement is raised, as can be seen in row 4. At the same time, these higher bond holding requirements only reduce liquidity provision (row 6) and lending (row 1) slightly. However, the policy fails to achieve a welfare improvement, with aggregate welfare almost unchanged across columns of the table. The reason for this missing welfare improvement is increased dispersion in the marginal product of capital listed in row 5. While the liquidity requirement makes large banks less vulnerable to runs, it also distorts their asset portfolio away from loans and towards bonds. This causes capital misallocation and offsets the benefits of lower deadweight losses from defaults. Higher liquidity requirements are effective at reducing run risk, but they do this at the expense of lending efficiency.

Size-dependent capital requirements. Size-dependent capital requirements that explicitly condition on assets are designed to account for the systemic importance of large banks. Since the model captures the cross-sectional distribution of banks, we can use it to evaluate size-dependent policies.

Figure 14: Size-dependent Capital Requirements



Notes: Effects of size dependent capital requirements in an economy with bailout guarantees.

Figure 14 compares the baseline model with TBTF guarantees, to the same model with a tighter unconditional capital requirement (red line), and a third model in which the capital requirement is increasing in bank size (green line). The resulting level of maximum leverage θ is plotted in the middle panel of the top row. The bottom row displays how this policy affects leverage, run-driven defaults, and total defaults. The size-dependent requirement is as effective in curbing run-driven defaults among the largest banks as the unconditionally tighter requirement.

However, as we can see in Table 4, the unconditional increase in θ in the 2nd column causes a decline in welfare, since it reduces bank leverage throughout the whole distribution, and causes a decline in liquidity supply by 2.3%. The size-dependent requirement in column 3, on the other hand, only reduces leverage of the largest banks and leads to a much smaller reduction in liquidity supply. The overall welfare impact of an unconditional tightening of θ is negative, while the size-dependent requirement is roughly welfare-neutral. Thus, like liquidity requirements, size-dependent capital requirements are an effective tool to reduce run exposure of large banks. At the same time, they have a minimal negative impact on other aspects of

Table 4: Size-dependent capital requirements

Outcome	TBTF	$\theta = 85\%$	θ(size)
1. Loans	1.613	-1.493	-0.062
2. C_0	2.072	0.012	0.004
3. $E[C_1]$	2.201	0.134	0.006
4. E[DWL]	0.004	-67.696	-6.953
5. SD[MPK]	0.003	-17.938	0.261
6. Liquidity	1.02	-2.296	-0.069
7. Welfare	1.518	-0.053	0.002

Notes: This table shows the effects of size-dependent capital requirements. The baseline economy for the capital requirement experiment is the case with too-big-to-fail (TBTF) guarantees from Figure 11. The *levels* of listed outcomes are shown in the baseline column of the table. The columns to the right show *percentage changes* of the same moments relative to baseline. Moments: 1. Loans – aggregate lending $E_i[K]$, 2. C_0 – time-0 consumption, 3. $E[C_1]$ – time-1 expected consumption, 4. E[DWL] – time-1 aggregate expected DWL, including default bankruptcy costs and firesale losses during runs, 5. SD[MPK] – expected standard deviation in marginal product of capital across banks at time 1, 6. Liquidity – liquidity utility H, 7. Welfare – total time-0 expected utility.

bank portfolios. We conclude that size-contingent capital requirements are an effective targeted regulation to address run risk caused by concentration of uninsured deposits.

6 Conclusion

In this study, we developed a model to investigate the complex interplay between banks' portfolio and funding choices–particularly the reliance on uninsured deposits and investment in interest rate sensitive securities–and the resulting in cross-sectional differences between banks. We then examine their impact on financial stability in the face of interest rate- and run risk. Our analysis reveals that large banks' reliance on uninsured deposit funding exposes them to greater financial instability, especially during periods of rising interest rates. The model shows that large banks–while benefiting from greater loan and deposit productivity–face heightened run-risk, while small banks are more vulnerable to solvency risk.

Our model underscores the importance of considering the heterogeneous impact of regulatory policies. Uniform regulations may not adequately address the unique challenges faced by banks of varying sizes and risk profiles. Therefore, a more nuanced regulatory approach that differentiates between the risks posed by smaller versus larger institutions is essential for maintaining financial stability in a dynamic economic environment.

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A Calibration

Section 4 discusses the calibration of the main parameters. Table A1 lists all parameters used in the model.

Table A1: Model Calibration Parameters

Name	Description	Value
κ	Production function curvature	0.123
r	Bond return	0.0123
β	Discount factor	0.99
μ_{RK}	Mean capital return	1.07
σ_{RK}	Deviation of capital return	0.05
π_K	Probability of high capital return	0.5
μ_{RB}	Mean bond return	1
σ_{RB}	Deviation of bond return	0.035
π_B	Probability of high bond return	0.5
ψ	Liquidity preference	0.12
α	Insured deposit share	0.8
ρ	Elasticity of substitution between insured deposits	0.855
ρ_U	Elasticity of substitution between uninsured deposits	0.96
θ	Leverage constraint	0.88
θ_B	Leverage constraint for bonds	0.97
θ_D	Liquidity constraint ($\theta_D \times DU \leq B$)	0.1
θ_{DU}	Liquidity constraint ($\theta_D \times DU \leq B$)	0
π	Run probability	0.25
ϕ	Uninsured deposit haircut in runs	0.1
δ	1 - Firesale costs of capital	0.5
δ_Z	Firesale value = (1-δ_Z) $\times \delta + \delta_Z \times \delta \times Z$	0
γ	Risk aversion	2
$\sigma_{\epsilon \ln}$	Volatility of idiosyncratic capital shocks	0.12
Z_{mean}	Mean loan productivity	1
Z_{loc}	Minimum loan productivity	0.6
Z_{scale}	Max - min loan productivity	1.55
Z_{shape}	Shape of loan productivity distribution std = shape $\times$ scale	0.13
A_{high}	Deposit productivity high type	1
π_{high}	Deposit productivity high type probability	1
Z_{high}	Increase in mean loan productivity for high deposit types	0
ξ	Share lost in recovery	0.25
scaleAnd_{-}	Slope of Deposit Productivity $-$	1.1
scaleAnd_{+}	Slope of Deposit Productivity +	0.5
W_0	Initial wealth set to ensure $K_{agg} = 1$	4

B Derivations

B.1 Household Problem

Denoting equity shares of bank i that are in unit supply as S_i, households solve

$$\max_{\{S_i\},\{D_i^I\},\{D_i^U\},C_0,C_1} \log(C_0) + \psi\log\left(H\left(\{D_i^I\},\{D_i^U\}\right)\right) + \beta\log\left(\mathrm{E}\left[C_1^{1-\varphi}\right]^{1/(1-\varphi)}\right) \tag{21}$$

subject to

$$C_0 = W_0 + \int_i \Pi_i di - T - \int_i p_i S_i di - \int_i q_i^I D_i^I di - \int_i q_i^U D_i^U di, \tag{22}$$

$$C_1 = \int_i D_i^I di + \int_i \left(\mathbb{I}_{\epsilon_i \geq \bar{\epsilon}_i} + \mathbb{I}_{\bar{\epsilon}_i \geq \epsilon_i \geq \underline{\epsilon}_i}\mathbb{I}_{\varsigma_i=0}\right) S_i \mathrm{Div}_i di + \int_i D_i^U \mathcal{P}_i^U di. \tag{23}$$

At time 0, households purchase all securities issued by banks: debt of both types and equity. Household funds consist of initial wealth W_0 and bank profits, net of taxes raised to cover expected deposit insurance payouts. At time 1, households receive insured deposit payouts. They also receive the dividend payouts of non-defaulting banks, with the survival rate being

$$1 - \hat{F}(\underline{\epsilon}_i, \bar{\epsilon}_i) \equiv \mathrm{E}\left[\mathbb{I}_{\epsilon_i \geq \bar{\epsilon}_i} + \mathbb{I}_{\bar{\epsilon}_i \geq \epsilon_i \geq \underline{\epsilon}_i}\mathbb{I}_{\varsigma_i=0}\right] = 1 - F(\bar{\epsilon}_i) - \pi\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right).$$

The payoff on uninsured deposits depends on the realization of banks' idiosyncratic productivity shocks ϵ_i. As explained in the main text, the payoff to uninsured deposits is given in (18):

$$\begin{aligned}\mathcal{P}_i^U =& 1 - F(\bar{\epsilon}_i) + F(\underline{\epsilon}_i)\mathrm{E}[r_i^{nr}|\epsilon_i < \underline{\epsilon}_i] \\ &+ (1-\pi)\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right) + \pi\left(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i)\right)\left(1 - \phi + \phi\mathrm{E}[r_i^r|\underline{\epsilon}_i \leq \epsilon_i \leq \bar{\epsilon}_i]\right).\end{aligned}$$

Marginal liquidity value. First, we compute marginal value of liquidity of each type.

$$\mathcal{H}_i^U = \frac{\partial\log\left(H\left(\{D_i^I\},\{D^U\}\right)\right)}{\partial D_i^U} = (1-\alpha)\left(\frac{A_i^D D_i^U}{D^U}\right)^{\rho_U}\left(\frac{D^U}{H}\right)^{\eta}\frac{1}{D_i^U} \tag{24}$$

$$\mathcal{H}_i^I = \frac{\partial\log\left(H\left(\{D_i^I\},\{D^U\}\right)\right)}{\partial D_i^I} = \alpha\left(\frac{A_i^D D_i^I}{D^I}\right)^{\rho_I}\left(\frac{D_I}{H}\right)^{\eta}\frac{1}{D_i^I} \tag{25}$$

where $D^j = \left(\int_i (A_i^D D_i^j)^{\rho_j} di\right)^{1/\rho_j}$, for $j = I, U$.

Consumption-savings choice. Denote total household wealth at time 0 as $W = W_0 + \int_i \Pi_i di - T$, and the total value of the household portfolio for time 0 (=savings) as $Z = \int_i p_i S_i di + \int_i q_i^I D_i^I di + \int_i q_i^U D_i^U di$. The return on this portfolio is

$$R_1 = C_1/Z = \int_i d_i^I di + \int_i \mathbb{I}_{\mathrm{nd}_i} s_i \mathrm{Div}_i di + \int_i d_i^U \mathcal{P}_i^U di,$$

where $d_i^I = D_i^I/Z$, $d_i^U = D_i^U/Z$, and $s_i = S_i/Z$. We can rewrite the HH problem as

$$\max_{\{s_i\},\{d_i^I\},\{d_i^U\},Z} \log(W - Z) + \psi \log\left(ZH\left(\{d_i^I\},\{d_i^U\}\right)\right) + \beta \log\left(\mathrm{E}\left[(R_1 Z)^{1-\varphi}\right]^{1/(1-\varphi)}\right).$$

The FOC for Z is

$$\frac{1}{W-Z} = \frac{\psi}{Z} + \frac{\beta}{\mathrm{E}\left[(R_1 Z)^{1-\varphi}\right]^{1/(1-\varphi)}} \frac{1}{1-\varphi} \left(\mathrm{E}\left[(R_1 Z)^{1-\varphi}\right]^{1/(1-\varphi)-1}\right) \mathrm{E}\left[R_1^{1-\varphi}(1-\varphi) Z^{-\varphi}\right]$$

which reduces to

$$Z = \frac{\beta + \psi}{1 + \beta + \psi} W,$$

and therefore

$$C_0 = \frac{1}{1 + \beta + \psi} W.$$

SDF. To derive the representative household's stochastic discount factor, we consider the first-order condition for a hypothetical riskfree bond without any liquidity benefits, with price $\tilde{q}$. The FOC would be

$$\frac{\tilde{q}}{C_0} = \frac{\beta}{\mathrm{E}\left[C_1^{1-\varphi}\right]^{1/(1-\varphi)}} \frac{1}{1-\varphi} \left(\mathrm{E}\left[C_1^{1-\varphi}\right]^{1/(1-\varphi)-1}\right) \mathrm{E}\left[(1-\varphi) C_1^{-\varphi}\right].$$

Canceling and multiplying by C_0

$$\tilde{q} = \beta \frac{C_0^{1-\varphi}}{\mathrm{E}\left[C_1^{1-\varphi}\right]} \mathrm{E}\left[\left(\frac{C_1}{C_0}\right)^{-\varphi}\right].$$

We can thus define

$$M = \beta \frac{C_0^{1-\varphi}}{\mathrm{E}\left[C_1^{1-\varphi}\right]} \left(\frac{C_1}{C_0}\right)^{-\varphi}, \tag{26}$$

such that $\tilde{q} = \mathrm{E}[M]$.

Deposits. Then HH FOCs for deposits are

$$q_i^I = \psi \mathcal{H}_i^I C_0 + \mathrm{E}[M], \tag{27}$$

$$q_i^U = \psi \mathcal{H}_i^U C_0 + \mathrm{E}\left[M \mathcal{P}_i^U\right]. \tag{28}$$

Bank equity. The FOC for bank equity shares is

$$p_i = \mathrm{E}\left\{M\left((1 - F(\bar{\epsilon}_i))\mathrm{E}_\epsilon[\mathrm{Div}_i | \epsilon_i > \bar{\epsilon}_i] + (1-\pi)(F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i))\mathrm{E}_\epsilon[\mathrm{Div}_i | \bar{\epsilon}_i > \epsilon_i > \underline{\epsilon}_i]\right)\right\}. \tag{29}$$

B.2 Bank Problem

Stage 1a. We start with the optimization problem at the run stage 1a. Given Assumptions 1 and 2, banks will sell bonds at market price R_B until they have either paid out all running depositors, or they have liquidated all bonds

$$\hat{B}_i = \min\left\{B_i, \frac{(1-\phi)D_i^U}{R_B}\right\}.$$

Banks do not want to sell more capital than necessary at fire sale prices, and the constraint (7) is always binding. Thus capital fire sales are

$$\hat{K}_i = \frac{(1-\phi)D_i^U - R_B\hat{B}_i}{R_K\delta}.$$

We can write the post-run dividend more compactly defining the function

$$x(B_i, D_i^U) = \min\left\{1, \frac{R_B B_i}{(1-\phi)D_i^U}\right\}. \tag{30}$$

We can express parts of the dividend using this function, namely

$$R_B(B_i - \hat{B}_i) - (1-\phi)D_i^U = R_B B_i - (1-\phi)D_i^U x(B_i, D_i^U),$$

and

$$(1-\phi)D_i^U - R_B\hat{B}_i = (1-\phi)D_i^U(1 - x(B_i, D_i^U)).$$

This in turn allows us to write the upper default threshold as

$$\bar{\epsilon}_i = \frac{D_i^I + \phi D_i^U + (1-\phi)D_i^U x(B_i, D_i^U) - R_B B_i}{A_i^K R_K\left(K_i - \frac{(1-\phi)D_i^U(1-x(B_i,D_i^U))}{R_K\delta}\right)^{1-\kappa}}. \tag{31}$$

Recall that the lower default threshold is

$$\underline{\epsilon}_i = \frac{D_i^I + D_i^U - R_B B_i}{A_i^K R_K K_i^{1-\kappa}}. \tag{32}$$

Note that for $R_B B_i \geq (1-\phi)D_i^U$, we have $x(B_i, D_i^U) = 1$ and the default thresholds become identical.

Stage 0. The dividend for the time-0 problem becomes

$$\mathrm{E}[M\max\{0,\mathrm{Div}_i^*\}] = \mathrm{E}\left[M\left(\mathbb{I}_{\epsilon_i\geq\bar{\epsilon}_i}+\mathbb{I}_{\bar{\epsilon}_i\geq\epsilon_i\geq\underline{\epsilon}_i}\mathbb{I}_{\varsigma_i=0}\right)\left(A_i^K R_K\epsilon_i K_i^{1-\kappa}+B_i-D_i^I-D_i^U\right)\right],$$

which we can write as

$$\begin{aligned}&\mathrm{E}\left[M(1-F(\bar{\epsilon}_i))\left(A_i^K R_K\bar{\epsilon}_i^+ K_i^{1-\kappa}+B_i-D_i^I-D_i^U\right)\right]\\&+\mathrm{E}\left[M(1-\pi)(F(\bar{\epsilon}_i)-F(\underline{\epsilon}_i))\left(A_i^K R_K\underline{\epsilon}_i^+ K_i^{1-\kappa}+B_i-D_i^I-D_i^U\right)\right]\end{aligned}$$

with the the conditional expectations

$$\begin{aligned}\bar{\epsilon}_i^+ &= \mathrm{E}[\epsilon_i|\epsilon_i>\bar{\epsilon}_i],\\ \underline{\epsilon}_i^+ &= \mathrm{E}[\epsilon_i|\bar{\epsilon}_i>\epsilon_i>\underline{\epsilon}_i].\end{aligned}$$

Given this definition of the bank dividend, the optimization problem in (10) only needs to be solved at time 0, with the function (30) reflecting optimal choices at the run stage, and the default thresholds (31) and (32) capturing the optimal default decision.

In problem (10), $x(B_i, D_i^U)$ is a non-differentiable function of B_i and D_i^U. For the numerical implementation, we define

$$Q(z)=\int\min\{\nu,z\}\,dF(\nu)=F_\nu(z)\,\mathrm{E}\left[\nu\,|\,\nu<z\right]+(1-F_\nu(z))\,z.$$

where ν is a random variable with positive support, c.d.f F_ν, and $\mathrm{E}[\nu]=1$. We then define the function

$$\tilde{x}(B_i,D_i^U)=Q\left(\frac{R_B B_i}{(1-\phi)D_i^U}\right)$$

and approximate

$$x(B_i,D_i^U)\approx\tilde{x}(B_i,D_i^U).$$

First-order conditions. We attach multiplier μ_i to the leverage constraint (12) and λ_i to the liquidity constraint (13). The bank FOC for capital is

$$1=\mu_i\theta^K+\frac{\partial q_i^U(\mathcal{A}_i)}{\partial K_i}D_i^U+\mathrm{E}\left[M\left((1-F(\bar{\epsilon}_i))\,\bar{\epsilon}_i^+ +(1-\pi)(F(\bar{\epsilon}_i)-F(\underline{\epsilon}_i))\underline{\epsilon}_i^+\right)(1-\kappa)R_K A_i^K K_i^{-\kappa}\right]. \tag{33}$$

The bank FOC for bonds B_i is

$$p = \mu_i \theta^B + \lambda_i + \frac{\partial q^U(\mathcal{A}_i)}{\partial B_i} D_i^U + \mathrm{E}\left[M(1 - \hat{F}(\underline{\epsilon}_i, \overline{\epsilon}_i)) R_B\right]. \tag{34}$$

The FOCs for deposits are

$$q_i^I = \mu_i + \lambda_i \theta^D - \frac{\partial q^I(D_i^I)}{\partial D_i^I} D_i^I - \frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^I} D_i^U + \mathrm{E}\left[M(1 - \hat{F}(\underline{\epsilon}_i, \overline{\epsilon}_i))\right], \tag{35}$$

$$q_i^U = \mu_i + \lambda_i \theta^D - \frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^U} D_i^U + \mathrm{E}\left[M(1 - \hat{F}(\underline{\epsilon}_i, \overline{\epsilon}_i))\right]. \tag{36}$$

The FOCs in (33) – (36) contain derivatives of the deposit demand functions from households in (16) and (17). In the next section, we provide expressions for these terms.

B.3 Derivatives of q^I and q^U

B.3.1 Insured

To compute the derivative of $q^I(D_i^I)$ given by (16) with respect to D_i^I, we apply the same assumptions as in the usual monopolistic competition setup. Banks internalize the effect on D_i^I in household demand, but not the effect on the aggregate D^I.

$$\frac{\partial q^I(D_i^I)}{\partial D_i^I} = C_0 \psi (\rho_I - 1) \alpha \frac{(A_i^D)^{\rho_I}}{(D_i^I)^2} \left(\frac{D^I}{D_i^I}\right)^{-\rho_I} \left(\frac{H}{D^I}\right)^{-\eta} = -C_0 \psi \frac{1 - \rho_I}{D_i^I} \mathcal{H}_i^I. \tag{37}$$

B.3.2 Uninsured

Recall the household FOC for uninsured deposits

$$q^U(\mathcal{A}_i) = \psi \mathcal{H}_i^U C_0 + \mathrm{E}\left[M \mathcal{P}^U(\mathcal{A}_i)\right],$$

with the payoff $\mathcal{P}^U(\mathcal{A}_i)$ provided in (18),

$$\begin{aligned}\mathcal{P}_i^U = & 1 - F(\overline{\epsilon}_i) + F(\underline{\epsilon}_i) \mathrm{E}[r_i^{nr} | \epsilon_i < \underline{\epsilon}_i] \\ & + (1 - \pi)\left(F(\overline{\epsilon}_i) - F(\underline{\epsilon}_i)\right) + \pi \left(F(\overline{\epsilon}_i) - F(\underline{\epsilon}_i)\right)\left(1 - \phi + \phi \mathrm{E}[r_i^r | \underline{\epsilon}_i \leq \epsilon_i \leq \overline{\epsilon}_i]\right).\end{aligned}$$

We want to calculate the derivative of this function with respect to bank choices in $\mathcal{A}_i$. The derivatives of the deposit demand function are

$$\begin{aligned}
\frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^U} &= -\psi \frac{1-\rho_U}{D_i^U}\mathcal{H}_i^U + \mathrm{E}\left[M\frac{\partial \mathcal{P}^U(\mathcal{A}_i)}{\partial D_i^U}\right],\\
\frac{\partial q^U(\mathcal{A}_i)}{\partial D_i^I} &= \mathrm{E}\left[M\frac{\partial \mathcal{P}^U(\mathcal{A}_i)}{\partial D_i^I}\right],\\
\frac{\partial q^U(\mathcal{A}_i)}{\partial B_i} &= \mathrm{E}\left[M\frac{\partial \mathcal{P}^U(\mathcal{A}_i)}{\partial B_i}\right],\\
\frac{\partial q^U(\mathcal{A}_i)}{\partial K_i} &= \mathrm{E}\left[M\frac{\partial \mathcal{P}^U(\mathcal{A}_i)}{\partial K_i}\right].
\end{aligned}$$

Computing the derivatives of q^U therefore comes down to computing the derivatives of $\mathcal{P}^U$ for each $Z_i \in \{D_i^I, D_i^U, K_i, B_i\}$. In what follows, denote the derivatives of the default probability weighted recovery values as

$$\mathcal{R}_i^{Z,nr} = \frac{\partial(F(\underline{\epsilon}_i)\mathrm{E}[r_i^{nr}|\epsilon_i < \underline{\epsilon}_i])}{\partial Z_i}$$

for the no-run states, and

$$\mathcal{R}_i^{Z,r} = \frac{\partial((F(\bar{\epsilon}_i) - F(\underline{\epsilon}_i))\,\mathrm{E}[r_i^r|\underline{\epsilon}_i \leq \epsilon_i \leq \bar{\epsilon}_i])}{\partial Z_i}$$

for run states. The derivative of the payoff $\mathcal{P}^U(\mathcal{A}_i)$ takes the general form

$$\frac{\partial \mathcal{P}^U(\mathcal{A}_i)}{\partial Z_i} = -\left(\pi\phi f(\bar{\epsilon}_i)\frac{\partial \bar{\epsilon}_i}{\partial Z_i} + (1-\pi\phi)f(\underline{\epsilon}_i)\frac{\partial \underline{\epsilon}_i}{\partial Z_i}\right) + \mathcal{R}_i^{Z,nr} + \pi\phi\mathcal{R}_i^{Z,r}.$$

We thus need to calculate the derivatives of both default thresholds with respect to all bank choices. To do so, we define the marginal product of capital for bank i, conditional on the outcome of the run game as

$$\mathrm{MPK}_i^{nr} = (1-\kappa)R_K A_i^K K_i^{-\kappa}, \tag{38}$$

$$\mathrm{MPK}_i^{r} = (1-\kappa)R_K A_i^K \left(K_i - \frac{(1-\phi)D_i^U(1-\tilde{x}_i)}{\delta R_K}\right)^{-\kappa}. \tag{39}$$

Solvency default threshold. First, for $\underline{\epsilon}_i$ given in (32) we get

$$\frac{\partial \underline{\epsilon}_i}{\partial D_i^U} = \frac{1}{\underline{\mathcal{D}}_i},\quad \frac{\partial \underline{\epsilon}_i}{\partial D_i^I} = \frac{1}{\underline{\mathcal{D}}_i},\quad \frac{\partial \underline{\epsilon}_i}{\partial K_i} = -\frac{\mathrm{MPK}_i^{nr}\underline{\epsilon}_i}{\underline{\mathcal{D}}_i},\quad \frac{\partial \underline{\epsilon}_i}{\partial B_i} = -\frac{R_B}{\underline{\mathcal{D}}_i}$$

where

$$\underline{\mathcal{D}}_i = R_K A_i^K K_i^{1-\kappa}.$$

Run default threshold. For the threshold $\bar{\epsilon}_i$ provided in (31), we obtain similar expressions for capital and insured deposits

$$\frac{\partial \bar{\epsilon}_i}{\partial D_i^I} = \frac{1}{\overline{\mathcal{D}}_i}, \quad \frac{\partial \bar{\epsilon}_i}{\partial K_i} = -\frac{\mathrm{MPK}_i^r \bar{\epsilon}_i}{\overline{\mathcal{D}}_i},$$

where

$$\overline{\mathcal{D}}_i = A_i^K R_K \left(K_i - \frac{(1-\phi) D_i^U (1-x_i)}{\delta R_K} \right)^{1-\kappa}.$$

For the derivative w.r.t. D_i^U, we get

$$\frac{\partial \bar{\epsilon}_i}{\partial D_i^U} = \frac{(\phi + (1-\phi) F_{\nu,i} \nu_i^-) \overline{\mathcal{D}}_i + \frac{1-\kappa}{\delta} A_i^K \left(K_i - \frac{(1-\phi) D_i^U (1-\tilde{x}_i)}{\delta R_K} \right)^{-\kappa} (1 - F_{\nu,i} \nu_i^-) \mathcal{N}_i}{(\overline{\mathcal{D}}_i)^2}$$

$$\mathcal{N}_i = D_i^I + \tilde{x}_i D_i^U - B_i,$$

and we have further defined the conditional expectations $\nu_i^- = \mathrm{E}\left[\nu \,|\, \nu < \frac{R_B B_i}{(1-\phi) D_i^U} \right]$. To calculate the expression above we have used the fact that

$$\frac{\partial \tilde{x}(B_i, D_i^U)}{\partial D_i^U} = -\frac{(1 - F_{\nu,i}) R_B B_i}{(1-\phi)(D_i^U)^2}.$$

We can simplify the expression to get

$$\frac{\partial \bar{\epsilon}_i}{\partial D_i^U} = \frac{\phi + (1-\phi) \left(F_{\nu,i} \nu_i^- + (1 - F_{\nu,i} \nu_i^-) \frac{\mathrm{MPK}_i^r}{\delta R_K} \bar{\epsilon}_i \right)}{\overline{\mathcal{D}}_i}. \tag{40}$$

Similarly, we take the derivative w.r.t. B_i

$$\frac{\partial \bar{\epsilon}_i}{\partial B_i} = \frac{\left(-R_B + R_B(1 - F_\nu, i)\right) \overline{\mathcal{D}}_i - \frac{1-\kappa}{\delta} A_i^K \left(K_i - \frac{(1-\phi) D_i^U (1-\tilde{x}_i)}{\delta R_K} \right)^{-\kappa} R_B (1 - F_{\nu,i}) \mathcal{N}_i}{(\overline{\mathcal{D}}_i)^2}$$

where we use the fact that

$$\frac{\partial x(B_i, D_i^U)}{\partial B_i} = \frac{1 - F_{\nu,i}}{\bar{\phi} D_i^U}.$$

This reduces to

$$\frac{\partial \bar{\epsilon}_i}{\partial B_i} = -\frac{R_B \left(F_{\nu,i} + (1 - F_{\nu,i}) \frac{\mathrm{MPK}_i^r}{\delta R_K} \bar{\epsilon}_i \right)}{\overline{\mathcal{D}}_i}. \tag{41}$$

Combining. We can now combine the expressions above to obtain the complete derivatives. First, for D_i^U

$$\frac{\partial q_i^U}{\partial D_i^U} = -\psi \frac{1-\rho_U}{D_i^U}\mathcal{H}_i^U - \mathrm{E}\left[M\left((1-\pi\phi)f(\underline{\epsilon}_i)\frac{1}{\underline{\mathcal{D}}_i} - \mathcal{R}_i^{D^U,nr}\right)\right]$$
$$-\pi\phi\mathrm{E}\left[M\left(\frac{\phi+(1-\phi)\left(F_{\nu,i}\nu_i^- + (1-F_{\nu,i}\nu_i^-)\frac{\mathrm{MPK}_i^r}{\delta R_K}\overline{\epsilon}_i\right)}{\overline{\mathcal{D}}_i} - \mathcal{R}_i^{D^U,r}\right)\right]. \tag{42}$$

For insured deposits we get

$$\frac{\partial q_i^U}{\partial D_i^I} = -\mathrm{E}\left[M\left((1-\pi\phi)f(\underline{\epsilon}_i)\frac{1}{\underline{\mathcal{D}}_i} - \mathcal{R}_i^{D^I,nr} + \pi\phi f(\overline{\epsilon}_i)\frac{1}{\overline{\mathcal{D}}_i} - \pi\phi\mathcal{R}_i^{D^I,r}\right)\right]. \tag{43}$$

For capital, the derivative is

$$\frac{\partial q_i^U}{\partial K_i} = \mathrm{E}\left[M\left((1-\pi\phi)f(\underline{\epsilon}_i)\frac{\mathrm{MPK}_i^{nr}\underline{\epsilon}_i}{\underline{\mathcal{D}}_i} + \mathcal{R}_i^{K,nr} + \pi\phi f(\overline{\epsilon}_i)\frac{\mathrm{MPK}_i^r\overline{\epsilon}_i}{\overline{\mathcal{D}}_i} + \pi\phi\mathcal{R}_i^{K,r}\right)\right]. \tag{44}$$

Finally, for bonds we calculate

$$\frac{\partial q_i^U}{\partial B_i} = \mathrm{E}\left[M\left((1-\pi\phi)f(\underline{\epsilon}_i)\frac{R_B}{\underline{\mathcal{D}}_i} + \mathcal{R}_i^{B,nr}\right)\right]$$
$$+\pi\phi\mathrm{E}\left[M\left(f(\overline{\epsilon}_i)\frac{R_B\left(F_{\nu,i} + (1-F_{\nu,i})\frac{\mathrm{MPK}_i^r}{\delta R_K}\overline{\epsilon}_i\right)}{\overline{\mathcal{D}}_i} + \mathcal{R}_i^{B,r}\right)\right]. \tag{45}$$

Inserting the derivatives in (42) – (45) into the first-order conditions in (33) – (36) completes the bank's optimality conditions. Note that the expressions above contain unresolved derivatives of the recovery values, $\mathcal{R}_i^{Z_i,j}$, for $Z_i \in \{D_i^U, D_i^I, K_i, B_i\}$ and $j \in \{nr, r\}$. Calculating these derivatives explicitly requires substantial algebra, but provides little additional insight. The derivatives can be signed unambiguously, with

$$\mathcal{R}_i^{D_i^U,j} < 0, \quad \mathcal{R}_i^{D_i^I,j} < 0$$

and

$$\mathcal{R}_i^{K_i,j} > 0, \quad \mathcal{R}_i^{B_i,j} > 0,$$

for $j \in \{nr, r\}$. This in turn implies that we can sign the derivatives as

$$\frac{\partial q_i^U}{\partial D_i^U} < 0, \quad \frac{\partial q_i^U}{\partial D_i^I} < 0, \quad \frac{\partial q_i^U}{\partial K_i} > 0, \quad \frac{\partial q_i^U}{\partial B_i} > 0.$$

These signs are intuitive: on the margin, issuing more deposits of either kind increases the bank's default risk and thus the required interest paid to households, equivalent to a lower issuance price at time 0. By the same logic, holding more assets either in the shape of loans or bonds reduces the bank's default risk and the interest rate required on uninsured deposits.

B.4 Bank Profit

[TO DO]

Starting from the bank budget constraint

$$p_i + q_i^I(D_i^I)D_i^I + q_i^U(K_i, B_i, D_i^U, D_i^I)D_i^U = K_i + pB_i + \Pi_i + Q(D_i^I, D_i^U, A_i),$$

we can solve for profit

$$\Pi_i = p_i + q_i^I(D_i^I)D_i^I + q_i^U(K_i, D_i^U, D_i^I)D_i^U - K_i - pB_i - Q(D_i^I, D_i^U, A_i).$$

First, define the payoff function of the bank

$$\mathcal{P}(K_i, B_i, D_i^I, D_i^U) = \mathrm{E}[M \max\{0, \mathrm{Div}_i\}].$$

This function is homogeneous of degree 1:

$$\begin{aligned}\mathcal{P}(K_i, B_i, D_i^I, D_i^U) = (1-\pi)\mathrm{E}\left[M^{nr}\left(1 - F(\hat{\epsilon}_i^{nr})\right)\left(\epsilon_i^{+,nr} R_K K_i + B_i - D_i^I - D_i^U\right)\right] \\ +\pi\mathrm{E}\left[M^r\left(1 - F(\hat{\epsilon}_i^r)\right)\left(\epsilon_i^{+,r} R_K\left(K_i - \frac{\bar{\phi} D_i^U(1-x_i)}{\delta}\right) + B_i - D_i^I - (1-\bar{\phi}(1-x_i))D_i^U\right)\right].\end{aligned}$$

Note: the default thresholds $\hat{\epsilon}_i^j$ and the run function x_i are homogeneous of degree 0. Thus we have

$$\mathcal{P}(K_i, B_i, D_i^I, D_i^U) = K_i \mathcal{P}_K() + B_i \mathcal{P}_B() + D_i^I \mathcal{P}_I() + D_i^U \mathcal{P}_U().$$

Next, we write the bank's FOC in general form

$$\begin{aligned} q_i^I + \frac{\partial q_i^I}{\partial D_i^I} D_i^I + \frac{\partial q_i^U}{\partial D_i^I} D_i^U =& \mu_i - \mathcal{P}_I(K_i, B_i, D_i^I, D_i^U) + A_i(D_i^I + D_i^U), \\ q_i^U + \frac{\partial q_i^U}{\partial D_i^U} D_i^U =& \mu_i - \mathcal{P}_U(K_i, B_i, D_i^I, D_i^U) + A_i(D_i^I + D_i^U) \\ 1 - \frac{\partial q_i^U}{\partial K_i} D_i^U =& \mu_i \theta - \mathcal{P}_K(K_i, B_i, D_i^I, D_i^U) \\ p - \frac{\partial q_i^U}{\partial B_i} D_i^U =& \mu_i + \lambda_i^B - \mathcal{P}_B(K_i, B_i, D_i^I, D_i^U). \end{aligned}$$

Further note that the HH FOC for bank equity implies

$$p_i = \mathcal{P}(K_i, B_i, D_i^I, D_i^U).$$

Inserting these FOC into the definition of bank profit gives, after cancelling,

$$\Pi_i = -(D_i^I)^2 \frac{\partial q_i^I}{\partial D_i^I} - D_i^U \left(D_i^I \frac{\partial q_i^U}{\partial D_i^I} + D_i^U \frac{\partial q_i^U}{\partial D_i^U} + K_i \frac{\partial q_i^U}{\partial K_i} + B_i \frac{\partial q_i^U}{\partial B_i} \right) - \frac{A_i}{2}(D_i^U + D_i^I)^2 + A_i(D_i^U + D_i^I)^2.$$

The HH FOC for uninsured deposits is

$$q_i^U(K_i, B_i, D_i^I, D_i^U) = \psi \mathcal{H}_i^U C_0 + \mathrm{E}\left[M \left(\phi + (1-\phi)\left(1 - F(\hat{\epsilon}_i) + F(\hat{\epsilon}_i)\pi^B(K_i) + F(\hat{\epsilon}_i)(1 - \pi^B(K_i)) r_i^U \right)\right)\right].$$

We break the RHS into three terms

$$q_i^U(K_i, B_i, D_i^I, D_i^U) = \psi \mathcal{H}_i^U C_0 + \pi^B(K_i)\mathrm{E}\left[M(1-\phi)F(\hat{\epsilon}_i)(1 - r_i^U)\right] + \tilde{q}_i^U(K_i, B_i, D_i^I, D_i^U),$$

where

$$\tilde{q}_i^U(K_i, B_i, D_i^I, D_i^U) = \mathrm{E}\left[M \left(\phi + (1-\phi)\left(1 - F(\hat{\epsilon}_i) + F(\hat{\epsilon}_i) r_i^U \right)\right)\right]$$

is the part of $q_i^U(K_i, B_i, D_i^I, D_i^U)$ that is homogeneous of degree 0 in all arguments. Further note that $\mathrm{E}\left[M(1-\phi)F(\hat{\epsilon}_i)(1 - r_i^U)\right]$ is also homogeneous of degree zero.