Fire Sales and Liquidity Requirements*

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June 16, 2025

Abstract

We study liquidity requirements in a model of fire sales that nests three common pricing mechanisms—cash-in-the-market, second-best-use, and adverse selection—and can produce the same observables under these mechanisms. We identify a novel externality that arises under adverse selection and operates through the average quality of the assets traded, and three additional forces that shape the optimal policy under all pricing mechanisms: (i) the difference between the sellers' and buyers' ability to collect cash flow from the marginal unit traded, (ii) the sensitivity of the fire-sale price to the sellers' liquidity holdings, and (iii) incomplete risk sharing. Absent risk-sharing considerations and collateral constraints, the equilibrium is (Pareto) efficient under cash-in-the-market pricing; a liquidity requirement is optimal under second-best-use pricing; and a liquidity ceiling (i.e., a cap on liquid assets) is optimal under adverse selection. With inefficient risk sharing and collateral constraints, the socially optimal level of liquidity remains higher with second-best-use pricing compared to cash-in-the-market pricing, and a liquidity ceiling remains optimal with adverse selection.

1 Introduction

Fire sales are common phenomena in periods of financial distress. These episodes are characterized by large sales of financial assets and a reduction in their prices, despite little to no change in the fundamentals, and they occur when investors are forced to sell their assets for various reasons. Examples abound across markets and asset classes, ranging from assets held by distressed banks

*Cheng: ycheng@bank-banque-canada.ca. Robatto: robatto@wisc.edu. A previous version of this paper circulated under the title "Financial Intermediation and Fire Sales with Liquidity Risk Pricing." We thank Briana Chang, Eduardo Dávila, and Pablo Kurlat, and seminar participants at SED Meeting and Winter Meeting 2024, the Liquidity in Macroeconomic Workshop at the Federal Reserve Bank of St. Louis, the Bundesbank conference "Challenges for Financial Intermediaries and their Supervisors," and the EEA Meetings for valuable conversations and suggestions. The views expressed herein are those of the authors and not necessarily those of the Bank of Canada.

(Granja, Matvos, and Seru, 2017) to asset-backed securities (Merrill et al., 2021) and highly rated corporate bonds (Falato, Goldstein, and Hortaçsu, 2021; Ma, Xiao, and Zeng, 2022).

To mitigate the risks posed by fire sales, policymakers have increasingly relied on liquidity requirements, which have become a cornerstone of financial regulation over the past two decades. Following the 2008 financial crisis, liquidity requirements were imposed on banks and money market mutual funds. The financial distress caused by the COVID-19 crisis further spurred action, with the Securities and Exchange Commission (SEC) tightening liquidity requirements on money market mutual funds and proposing liquidity requirements for open-end mutual funds.

The literature provides several theories that use different pricing mechanisms to explain low asset prices in a fire sale. Some theories are based on the assumption that buyers have limited cash available to purchase assets (Allen and Gale, 1998). Others assume that buyers have a low willingness to pay because they can collect lower cash flows than sellers (i.e., the so-called second-best-use assumption; Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Lorenzoni, 2008; Dávila and Korinek, 2018). A third set of theories is based on asymmetric information and adverse selection (Guerrieri and Shimer, 2014; Kurlat, 2016; Chang, 2018; Dow and Han, 2018).

Despite a consensus on the main theories to explain fire sales, there is strong disagreement in the literature on the role and effects of liquidity requirements. Some theoretical analyses argue that liquidity requirements are beneficial (Goldstein et al., 2022; Kara and Ozsoy, 2020; Kashyap, Tsomocos, and Vardoulakis, 2024), others suggest that liquidity requirements can be counterproductive and reduce welfare (Malherbe, 2014), and others find positive or negative welfare effects depending on some model parameters (Allen and Gale, 2004). While these studies adopt different fire-sale pricing mechanisms, the lack of comparability among models leaves open the question of whether the pricing mechanism itself or other economic forces drive the divergent policy implications. More broadly, the literature still lacks a comprehensive analysis of the full set of forces that determine the optimal design of liquidity regulation.

In this paper, we study fire-sale inefficiencies and liquidity requirements using a unifying framework that nests the three main pricing mechanisms proposed by the fire-sale literature. That is, our framework includes, as special cases, a model with cash-in-the-market pricing, one with second-best-use pricing, and a third one with adverse-selection pricing.

We provide two main contributions. First, the socially optimal level of intermediaries' liquidity holdings and the optimal regulatory stance differs dramatically depending on the underlying pricing mechanism responsible for fire sales. Second, our analysis sheds light on the forces that shape the optimal liquidity regulation. Specifically, we identify a novel externality that arises under adverse selection and operates through the average quality of the assets traded, and three additional forces that shape the optimal policy under all pricing mechanisms: (i) the difference between the sellers and the buyers' ability to collect cash flow from the marginal unit traded, (ii) the sensitivity of the fire-sale price to the sellers' liquidity holdings, and (iii) if and how market incompleteness prevents full and efficient risk sharing. Depending on the mechanism and the impact of market incompleteness on risk sharing, the optimal policy may involve no regulation, a liquidity requirement (i.e., a lower bound on liquidity holdings), or a liquidity ceiling (i.e., an upper bound).

Because of the generality of our framework, our analysis provides a unifying theory of liquidity regulation applicable to a broad range of markets and asset classes. Our results imply that the mere possibility of fire sales is insufficient to justify liquidity requirements, and offer practical guidance for designing regulations governing intermediaries' liquidity holdings.

Our framework yields observationally equivalent equilibria under the three pricing mechanisms we consider, under appropriate parameter restrictions. This means that our framework can produce identical outcomes in portfolio choices, trading volumes, prices, and buyers' demand elasticity under cash-in-the-market, second-best-use, and adverse-selection pricing. Consequently, determining the socially optimal level of liquidity requires understanding the microfoundations, as identical equilibrium outcomes could mask either excess, insufficient, or optimal liquidity levels.

We first use a simple baseline framework that allows us to abstract from risk-sharing considerations and the effects of collateral constraints, delivering stark results. The model has two assets (i.e., a short-term liquid asset and a long-term asset) and, similar to the fire-sale literature, two sets of agents (which we label the *sellers* and the *buyers*). An exogenous shock that increases the sellers' liquidity needs triggers a fire sale, forcing them to sell long-term assets to the buyers. In the efficiency and policy analysis, we focus on the composition of the sellers' portfolios in terms of their liquid and long-term assets before the possible realization of fire sales, aiming to determine whether and how the mix of the two assets should be regulated.

With cash-in-the-market pricing, the equilibrium is (Pareto) efficient and, thus, no liquidity regulation is necessary. This is because the occurrence of a fire sales—relative to a scenario with no shocks that induce fire sales—simply redistribute resources from the sellers to the buyers, as buyers are able to buy assets at a low price. Hence, fire sales generate no aggregate welfare losses because buyers can extract the same cash flow as sellers from the long-term assets. Policy interventions would either move the equilibrium along the Pareto frontier or create aggregate welfare losses.

With second-best-use pricing, the buyers collect less cash flow from long-term assets than the sellers do. Thus, a fire sale reduces aggregate efficiency because long-term assets end up in the hands of the buyers, who are less efficient at collecting cash flow. The optimal policy is a liquidity requirement, which reduces the depth of a fire sale and results in more long-term assets being retained by sellers, increasing the economy-wide cash flow collected from such assets.

With adverse-selection pricing, the equilibrium is again inefficient, and two forces affect the optimal policy. First, there is a standard distributive externality (Dávila and Korinek, 2018; Lanteri and Rampini, 2023) because buyers collect less cash flow from the *marginal* unit traded, relative to sellers. This gap in cash-flow collected arises even if buyers and sellers can collect the same cash flow from any given asset. Indeed, sellers have private information about the asset quality under adverse-selection pricing, and on the margin, trade high-quality assets—all of the low-quality ones are sold as infra-marginal units. But for buyers, the marginal unit traded is the average asset in the market, which includes both high- and low-quality ones. Second we highlight a novel externality relative to the literature that studies fire sales inefficiencies, which we label *market quality externality*. This externality arises because, when sellers make their initial portfolio choices, they do not internalize that such choices affect the average quality of the assets traded in a fire sales, and with it, the cash flow that buyers can collect from such assets—higher quality translates into higher cash flow, and thus, higher welfare for buyers. The two externalities partially offset each other, and to see why, consider a regulatory intervention that results in an increase in the average asset quality, and with it, higher asset prices. Buyers lose as they have to pay more for any amount they purchase, but gains because the higher quality translates into higher consumption for them. These two effects exactly offset each other, and ultimately, only sellers' welfare is relevant to design the optimal regulation. Thus, the optimal policy resembles the choice of a "monopolistic seller" that makes its ex-ante portfolio choices with no regards for buyers' welfare.

Importantly, with adverse-selection pricing, the optimal policy is a liquidity ceiling, rather than a liquidity requirement as with second-best-use pricing. Under both second-best-use and asymmetric-information pricing, the regulator wants to reduce the depth of a fire sale. But with asymmetric information, this objective is achieved with a ceiling on sellers' liquidity holdings, rather than a lower bound. The logic is similar to that in Malherbe (2014). That is, if the sellers enter a fire-sale episode with less liquidity, a larger fraction of the sales will be due to fundamental reasons and a smaller fraction to private information, reducing the extent of the adverse-information problem. This result highlight the importance of the sensitivity of asset prices to sellers' liquidity holdings in the design of the optimal regulation.

We then provide two extensions. First, we relax some assumptions about buyers' and sellers' utility so that market incompleteness prevents full and efficient risk sharing. Second, we add a collateral constraint for sellers—a common feature in fire-sale models in the literature. Under both extensions, the equilibrium is generically inefficient under all pricing mechanism—even cash-in-the-market pricing. However, the policy analysis mirrors the results of the baseline model. First, when comparing two observationally equivalent equilibria under cash-in-the-market and second-best-use pricing, the socially optimal level of liquidity is higher under second-best-use pricing. Thus, regulation should be "tighter" if the buyers' low willing to pay is driven by second-use considerations. Second, under asymmetric information, the baseline result is qualitatively unchanged, and the optimal policy remains a liquidity ceiling akin to the choice of a "monopolistic seller."

Our analysis shows that the socially optimal level of liquidity and the optimal policy are crucially affected by distributive externalities as well as a novel externality in the context of fire sales, that we label *market quality externality*. Relative to the existing literature that focuses on distributive externalities, our results make progress by building on the insight of Dávila and Schaab (2023) to distinguish two forces that affect such externalities: the gap in the cash flow that the sellers and buyers are able to collect from the marginal unit they trade, and the role of imperfect risk sharing. While the first force typically points in one direction—in nearly all fire-sale models in the literature, the sellers can collect the same or more cash flow than the buyers can—the second one is ambiguous and depends on whether imperfect risk sharing has a higher impact on the buyers or the sellers. We thus establish that the inability to unambiguously sign the effects of the distributive externalities is due solely to imperfect risk sharing. While this result is derived in the context of liquidity requirements, the logic behind it seems robust to other policy analyses.

Our analysis also shows that externalities linked to collateral constraints—which are widely analyzed in the literature—are not central in understanding our key findings. That is, even though collateral externalities can be important quantitatively, they do not affect the basic result that the socially optimal level of liquidity is higher under second-best-use relative to cash-in-the-market, and that the optimal policy is a liquidity ceiling under asymmetric information.

A direct policy implication is related to the debate about the introduction of liquidity requirements for open-ended mutual funds, proposed by the SEC and motivated by the March 2020 "dash for cash." This event was a fire sale of high-quality corporate bonds and, thus, was likely unrelated to second-best-use considerations as the investors should easily collect cash flow from corporate bonds. There is also no evidence of adverse selection (Haddad, Moreira, and Muir, 2021). If the fire-sale prices in this event were driven by cash-in-the-market pricing, and if the concerns are about similar events in the future, our analysis suggests that the impact of market incompleteness on risk sharing should have first-order importance in determining the optimal policy.

More generally, our analysis indicates that policymakers should be cautious about the link between fire sales and liquidity requirements. The mere possibility of fire sales does not, by itself, justify imposing liquidity requirements, and there is no one-size-fits-all approach to regulating intermediaries' liquid-asset holdings. The optimal policy hinges on the micro-foundations of fire sales and on how market incompleteness shapes risk sharing, which likely differs depending on the characteristics of the assets, intermediaries, and potential buyers. Our results isolate the key forces on which regulators should focus when tailoring liquidity rules, and although we derive these insights in a pared-down framework, the mechanisms are likely to remain relevant in richer environments.

1.1 Additional comparisons with the literature

Among the papers that study optimal policies to mitigate fire sales of financial assets, several focus on regulating ex-ante borrowing and total investments (e.g., Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021). Our paper complements these studies, as we focus on the composition of investors' portfolios and the share invested in liquid assets, abstracting from the size of investors' borrowing and investments.

Our work is closely related to Dávila and Korinek (2018). Using second-best-use pricing, they identify collateral externalities and distributive externalities—the latter are driven by incomplete markets—and provide sufficient statistics to guide policy interventions. While our policy analysis builds on their approach, there are important distinctions. First, Dávila and Korinek (2018) focus on the size of investors' borrowing and investments, whereas we focus on the composition of their portfolios, in terms of liquid and illiquid assets, to study liquidity requirements. Second, we show that the sufficient statistics identified by Dávila and Korinek (2018) can be used not only with second-best-use pricing but also with cash-in-the-market and asymmetric-information pricing—overturning the conjecture of Kurlat (2021) about the inability to use the approach of Dávila and Korinek (2018) with asymmetric information. We also highlight the novel market quality externality, that arises under asymmetric information and interacts with the distributive externality. Third, we use the insights of Dávila and Schaab (2023) to further distinguish two forces that affect distributive externalities (i.e., the cash flow collected from the marginal unit traded, and imperfect risk sharing), allowing us to make progress in understanding the effects of distributive externalities. While Dávila and Korinek (2018) show that distributive externalities can lead to choices that are either too high or too low relative to those preferred by the regulator, our results show that the inability to unambiguously sign these effects to study liquidity requirements is due only to imperfect risk sharing.

Another closely related paper is Kurlat (2021), which compares the optimal size of ex-ante investments, using second-best-use and adverse-selection pricing. While the spirit of our exercise is similar, there are important differences. First, we also consider cash-in-the-market pricing. Second, Kurlat (2021) focuses on the size of ex-ante investments, whereas we focus on the composition in terms of liquid and illiquid assets. Third, in Kurlat (2021), investors have linear utility, whereas we extend our analysis to a setting with general utility to study the impact of risk-sharing considerations. Fourth, even though Kurlat (2021) states that "[t]he result of Dávila and Korinek (2018) that there are measurable statistics that suffice to determine the direction of the externality [...] does not extend to the asymmetric-information pricing," we show that the sufficient statistics identified in Dávila and Korinek (2018) can actually be used with asymmetric-information pricing (and with cash-in-the-market pricing too) to perform policy analysis. Fifth, we identify the novel market quality externality, which affects the optimal policy stance. Sixth, we show that the optimal regulatory stance with asymmetric-information pricing is related to how asset prices respond to the sellers' liquidity holdings (consistent with Malherbe, 2014), in combination with the distributive and market quality externality. Whether or not the buyers and sellers can collect the same cash flow from any given assets—a point the literature has often focused on (Dow and Han, 2018; Kurlat, 2021)—matters only insofar as it affects the cash flow collected from the marginal unit traded.

A third closely related paper is Malherbe (2014). In both Malherbe (2014) and the version of our model with asymmetric information, an increase in the sales of high-quality (long-term) assets increases the market price, as the average asset quality in the market increases. The key result of Malherbe (2014) is that this mechanism contributes to the existence of multiple equilibria—a good equilibrium with large trading volume and a bad equilibrium with a liquidity dry-up. We show that the same mechanism creates an inefficiency even if one abstracts from multiplicity or the equilibrium is unique. And while Malherbe (2014) note that a liquidity requirement reduces welfare with asymmetric information, we establish that the unregulated equilibrium is inefficient when fire sales are driven by asymmetric information, we show that the inefficiency is related to distributive and market quality externalities, and we show that the optimal policy is a liquidity ceiling.

Other papers study fire sales and and intermediaries' liquidity holdings but focus on other aspects. Farhi, Golosov, and Tsyvinski (2009) show that liquidity requirements can mitigate the problem of hidden trades in a Diamond-Dybvig framework. Calomiris, Heider, and Hoerova (2015) show that regulating banks' cash holdings is beneficial because cash is easily observable and riskless, increasing a bank incentives to manage risk in the remaining, non-cash portfolio of risky asset. Hachem and Song (2021) show that liquidity regulation can trigger credit booms, focusing on China from 2007 to 2014. Robatto (2023) studies the interaction between liquidity requirements and central bank interventions. A separate literature that includes Diamond and Dybvig (1983), Acharya and Yorulmazer (2008) and Gertler and Kiyotaki (2015) focuses on fire sales in the context of bank runs, and other papers such as Bolton, Santos, and Scheinkman (2011), Gale and Yorulmazer (2013), Li (2023), and Robatto (2019) study central bank interventions and expansion of public liquidity during fire sales.

2 General model framework

This section presents a general model framework that nests three pricing mechanisms commonly used in the literature. That is, under some assumptions about the primitives of the model, the model produces fire sales driven by cash-in-the-market pricing, or second-best-use pricing, or asymmetric information.

Following a standard approach in the fire-sale literature (e.g., Dávila and Korinek, 2018), we consider an economy populated by two sets of investors—the sellers (s) and the buyers (b)—and the economy lasts for three periods, $t = 0, 1, 2$. At $t = 0$, the sellers make their portfolio choices by choosing their investments in a liquid asset and a long-term asset. At $t = 1$, the buyers are born, and a fire sale can occur depending on the realization of an exogenous shock that forces the sellers to sell some of their holdings of the long-term asset. At $t = 2$, the payoff of the long-term assets are realized.

2.1 Environment

We begin by describing the preferences of sellers and buyers. Sellers have linear utility from consumption, c_2^s, at $t = 2$. Buyers' utility is $u\left(c_1^b\right) + c_2^b$, where c_1^b and c_2^b denote buyers' consumption at $t = 1$ and $t = 2$, respectively, and $u(\cdot)$ is weakly increasing and weakly concave. The linearity of the buyers and sellers' utility functions at $t = 2$ allows us to abstract from inefficiencies driven by incomplete risk sharing. We extend the analysis in Section 4 to a framework with a general utility function at $t = 2$ to account for such inefficiencies.

At $t = 0$, the sellers have an endowment, e^s, and issue debt, d_0^s, where d_0^s represents the face value of the debt. We assume that the debt is issued at par, and we return below to the timing of the debt repayment. The sellers allocate their resources, $e^s + d_0^s$, to liquid and long-term assets, denoted by l_0^s and k_0^s, subject to the budget constraint

$$l_0^s + k_0^s \leq e^s + d_0^s. \tag{1}$$

We assume that the debt, d_0^s, is exogenously given by $d_0^s = d^s$, and we focus on the choices of $\{l_0^s,\ k_0^s\}$.[1] This allows us to take the size of the sellers' portfolio as given (i.e., $e^s + d^s$) and focus on whether the allocation of these resources to long-term and liquid assets is efficient or the sellers' liquidity holdings should be regulated. Our analysis complements that of several other fire-sales papers, which often focus on the inefficiencies that lead to overborrowing (Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021).

Buyers are born at $t = 1$ with an endowment of liquid assets only, similar to e.g. Stein (2012). We normalize such an endowment to one.[2]

The liquid asset is standard; for each unit invested at time $t = 0$, there is one unit available at $t = 1$. The liquid asset technology is also available at $t = 1$, so that for each unit invested at $t = 1$, there is one unit available at $t = 2$.

The long-term asset works as follow. For each unit invested by sellers at $t = 0$, the asset produces no output at $t = 1$, and its productivity at $t = 2$ depends on two elements: (i) a quality shock realized at $t = 1$ and (ii) whether the asset is held, at $t = 2$, by sellers or buyers.

- *Quality shock:* At the beginning of $t = 1$, a fraction $1 - \theta$ of the long-term assets held by each sellers becomes of high quality, and a fraction θ becomes of low-quality. The fraction θ of low-quality assets is a random variable realized at $t = 1$, and we will focus our analysis

[1] Regarding d_0^s, one can assume that there is a mass of external agents that may deposit their endowments with the sellers. Assuming the external agents are risk neutral and that they can only deposit with the sellers or use a storage technology, and that the sellers can make a take-it-or-leave-it offer, the sellers will offer a zero return on deposits, and d_0^s will be equal to the external agents' total endowment.

[2] To prove some of our results, we require the endowment of the buyers' liquid asset to be $1 + \varepsilon$ for a (small) $\varepsilon > 0$. This is just a technical assumption and, for simplicity, we focus the exposition on the limiting case $\varepsilon \to 0$.

on the case in which θ can take values $\{0, \bar{\theta}\}$, with $\bar{\theta} \in [0, 1)$. (We will describe the exact process for θ later.) While the realization of θ is common knowledge to all agents, sellers have private information, at $t = 1$, about the quality of each asset they hold. Low quality assets produce no output at $t = 2$, and the productivity of high-quality assets depend on whether they are hold by buyers or sellers, as described next.

- *Sellers and buyers productivity at $t = 2$:* Consider an agent that enters $t = 2$ with an amount $k \geq 0$ of high-quality assets. If the agent is a seller, she collects output $R\,k$ (i.e., the marginal productivity is R). If the agent is a buyer, she collects output $f(k)$, where $f(\cdot)$ is a strictly increasing and weakly concave function that satisfies $f(0) = 0$ and $f'(0) = R$ (i.e., the marginal productivity is $f'(k) \leq R$).

At $t = 1$, the sellers have to repay a fraction γ of debt d_0^s, while the remaining fraction $1 - \gamma$ will be due at $t = 2$. We assume that γ is an aggregate shock realized at $t = 1$ that can take values $\gamma \in \{0, \bar{\gamma}\}$, with $\bar{\gamma} \in (0, 1)$. We refer to $\gamma = 0$ as the low-withdrawal state and $\gamma = \overline{\gamma}$ as the high-withdrawal state. We can thus interpret sellers as banks, money market mutual funds, or mutual funds that may experience withdrawals or outflows or, more generally, acute liquidity needs.

We assume that the realization of the two shocks at $t = 1$—the withdrawal shock γ and the quality shock θ—is correlated. Specifically, at $t = 1$, there are two possible states:

$$(\gamma, \theta) = \begin{cases} (0, 0) & \text{with probability } 1 - \pi \\ (\bar{\gamma}, \bar{\theta}) & \text{with probability } \pi, \end{cases} \tag{2}$$

where $\pi \in (0, 1)$. In equilibrium, a fire sale occurs at $t = 1$ in the latter state.

At $t = 1$, there is a centralized market in which the buyers and the sellers can trade the liquid and long-term assets. We denote q_1 as the price of the long-term asset and normalize the price of the liquid asset to one. We assume that short selling is not allowed. These assumptions imply that, at $t = 1$, the buyers and sellers are able to adjust their portfolio holdings of the liquid and long-term assets by trading in the centralized market, but at the economy-wide level, it is not possible to change the overall supply of the two assets. The overall supply is given by $1 + l_0^s$ and k_0^s for the liquid and long-term asset, respectively (i.e., the amounts that the buyers and sellers have at the beginning of $t = 1$; recall that buyers start $t = 1$ with one unit of the liquid asset).

We impose three sets of parameter restrictions. First, we assume that the long-term asset is on average more productive than liquidity, that is, $(1 - \pi)R + \pi R(1 - \bar{\theta}) > 1$, which implies $R > 1$.

Second, we assume that the probability π of the high-withdrawal state is sufficiently large,

$$\pi > \frac{(R-1)(1-\bar{\gamma}d^s)}{\bar{\gamma}d^s}, \tag{3}$$

which guarantees that the possibility of fire sales at $t = 1$ is not negligible and, thus, the sellers want to have positive holdings of the liquid assets at $t = 0$. Second, we assume that $\bar{\gamma}$ and e^s are sufficiently large to ensure that the sellers' investments in liquidity and long-term assets at $t = 0$ are both strictly positive and their time-2 consumption is also strictly positive, which allows us to sidestep the potential issue of the sellers' default.

2.2 How the environment nests commonly used pricing mechanisms

The general framework described in Section 2.1 nests three pricing mechanisms commonly employed in the literature: cash-in-the-market pricing, second-best-use pricing, and asymmetric-information pricing. To obtain each pricing mechanism, one can impose assumptions on three key elements of the model: buyers' utility at $t = 1$, that is, $u(c_1^b)$; buyers' ability to extract cash flow from the long-term assets, that is, $f(\cdot)$; and the degree of asymmetric information in the high-withdrawal state, that is, $\bar{\theta}$. Specifically:

- Cash-in-the-market pricing: $u(c_1^b) = \log c_1^b,\ f''(k) = 0$ for any $k \geq 0,\ \bar{\theta} = 0$.
- Second-best-use pricing: $u(c_1^b) = 0,\ f''(k) < 0$ for any $k \geq 0,\ \bar{\theta} = 0$.[3]
- Asymmetric-information pricing: $u(c_1^b) = 0,\ f''(k) = 0$ for any $k \geq 0,\ \bar{\theta} > 0$.

Buyers derive utility $u(c_1^b)$ from consumption at $t = 1$ under cash-in-the-market pricing, but no utility under the other pricing mechanisms. With cash-in-the-market pricing, we specialize to the log utility case for simplicity, but the results can be generalized to more general well-behaved utility functions. The utility from consumption under cash-in-the-market provides a reason for buyers to use liquidity at $t = 1$ other than purchasing assets sold by sellers, giving rise to a downward sloping demand even if buyers have the same technology and information as sellers.

The assumptions about the function $f(\cdot)$ imply that the productivity of buyers (i.e., $f'(k)$), is the same as that of sellers (i.e., R) under cash-in-the-market pricing and asymmetric-information

[3] For second-best use pricing, we also assume that $\lim_{k\to(e^s+d^s)} f'(k) < \pi R/(R-1+\pi)$, which guarantees that the price in a fire sale is less than one.

pricing, but strictly lower with second-best-use pricing. This lower productivity generates a fire-sale price under the second-best-use pricing.

The parameter that governs the fraction of low-quality assets, $\bar{\theta}$, is zero under cash-in-the-market and second-best-use pricing, and thus, no informational asymmetries in those cases. Only with asymmetric-information pricing, if $\theta = \bar{\theta}$, a fraction $\bar{\theta} > 0$ of the long-term assets is of low quality and sellers' private information is relevant.

Remark 1. To avoid possible confusion, we clarify the distinction between cash-in-the-market and second-best-use pricing. Our notion of cash-in-the-market pricing is one in which the buyers' opportunity cost to use liquidity to purchase assets from sellers at $t = 1$ depends on a strictly concave function that governs the payoff of alternative uses of liquidity. In our model, this function is the time-1 utility of buyers. An alternative and isomorphic formulation is the one in Stein (2012), in which buyers can invest in a project that produces at $t = 2$ according to a strictly concave function. The second-best-use formulation is instead one in which the purchase of long-term assets by buyers results in lower output extracted *from the long-term assets*. While our second-best-use formulation follows the tradition of Kiyotaki and Moore (1997), Lorenzoni (2008), and Shleifer and Vishny (1992), a similar result could be achieved with the formulation of Goldstein et al. (2022) in which buyers have a very limited endowment and can raise additional resources at a cost that results in a deadweight loss for the society.

Remark 2. As we show in details in the next sections, the elements $u(\cdot)$, $f(\cdot)$, and $\bar{\theta}$ that determine the pricing mechanism have no impact on sellers' choices, and they affect only the problem of buyers. This allows us to show, in Section 2.7, that the equilibrium is observationally equivalent under the three pricing mechanism, under appropriate parameter restrictions.

2.3 Sellers' choices at $t = 1$

We now present the sellers' choices at time 1. Recall that: sellers enter $t = 1$ with l_0^s units of the liquid asset and k_0^s units of the long-term asset; a fraction θ of the long-term asset holdings k_0^s are low quality (i.e., they will produce no output at $t = 0$); sellers have private information about the quality of their long-term asset holdings; and sellers have to repay a fraction γ of their debt d^s;.

When $\theta > 0$, sellers' private information gives rise to an adverse selection problem, and we consider a pooling equilibrium. Unlike the classic lemons problem (Akerlof, 1970), in which trade collapses, here sellers sell some high-quality long-term assets to meet their liquidity needs at $t = 1$, resulting in a positive price for long-term assets.

To solve sellers' time-1 problem, we focus on the relevant case in which sellers sell all of their holdings of long-term low-quality assets (if any), and on the margin, they trade a long-term high-quality asset. Let k_1^s denote the holdings of the long-term high-quality assets that the sellers retain. Their problem is to maximize consumption c_2^s at $t = 2$, which is the sum of the cash flow $R\,k_1^s$ from the amount k_1^s of retained high-quality long-term asset, and from their liquidity holdings chosen at $t = 1$, l_1^s, minus the repayment $(1-\gamma)d^s$ owed to the debt holders at $t = 2$:

$$\max_{k_1^s, l_1^s} \; R\,k_1^s + l_1^s - (1-\gamma)d^s \tag{4}$$

subject to $l_1^s \geq 0$ and the budget constraint

$$l_1^s + \gamma d^s \leq l_0^s + q_1 \theta k_0^s + q_1 \left[(1-\theta)k_0^s - k_1^s\right]. \tag{5}$$

The budget constraint (5) says that the sellers finance their holdings l_1^s of liquidity and the withdrawals γd^s by using the liquidity l_0^s carried from $t = 0$, selling their holdings of low-quality long-term assets θk_0^s at price q_1, and selling an amount $(1-\theta)k_0^s - k_1^s$ of their high-quality long-term assets, also at price q_1.

We restrict our attention to the relevant equilibrium cases in which $q_1 \leq R$.[4] If $q_1 = R$, the liquid and long-term assets have the same returns, so the sellers are indifferent between the two. The outcome $q_1 = R$ will arise in the low-withdrawal state (i.e., when $\gamma = 0$ and $\theta = 0$), and without loss of generality, we will focus on the case in which the sellers do not engage in any trade, so that their holdings will be $l_1^s = l_0^s$ and $k_1^s = k_0^s$.

When $q_1 < R$, the high-quality long-term asset has a higher return than the liquid asset. The outcome $q_1 < R$ will arise when a fire sale occurs, that is, in the high-withdrawal state (i.e., $\gamma = \bar{\gamma}$ and $\theta = \bar{\theta}$). Sellers will use all their liquidity l_0^s and sell all their low-quality assets, if any, to pay withdrawals, but will also be forced to sell some of their high-quality assets. Any wealth left after repaying the time-1 withdrawals will be invested only in the long-term assets, which have higher return than liquidity. That is, $k_1^s > 0$ and $l_1^s = 0$. Specifically, the amount of high-quality assets

[4]If $q_1 > R$, then the expected return of the long-term asset is negative, but because the return of the liquid asset is zero, no agent would invest in the long-term asset. This cannot be an equilibrium because the market-clearing condition for the long-term asset would not hold.

they retain, k_1^s, is residually determined by the budget constraint (5) and given by

$$k_1^s = \frac{q_1 k_0^s - (\bar{\gamma} d^s - l_0^s)}{q_1}, \tag{6}$$

where we have set $\gamma = \bar{\gamma}$ because we are focusing on the high-withdrawal state. Note that buyers' choices in the high-withdrawal state (i.e., $\gamma = \bar{\gamma}$ and $\theta = \bar{\theta}$) do not depend on value of $\bar{\theta}$ (i.e., the fraction of low-quality long-term assets). Thus, (6) holds for any $\bar{\theta}$, including $\bar{\theta} = 0$.

2.4 Buyers' choices at $t = 1$

We now turn to the buyers' problem at time 1. We first state the buyers' problem in the general model, but we then analyze buyers' optimal choices separately under each pricing mechanisms. The buyers' problem varies across different pricing mechanisms, as the assumptions about buyers' preferences, technology, and information differ, and focusing on each pricing mechanism separately simplifies the exposition.

To state the buyer problem in general, we need to specify buyers' beliefs about the fraction of high- and low-quality long-term assets that are traded at $t = 1$ because of the possible asymmetric information problem. Let $\alpha_1 \in [0, 1]$ be the (endogenously determined) fraction of high-quality long term assets traded at $t = 1$. We proceed under the assumption that buyers' beliefs are rational and, thus, the buyers' belief about the fraction of high-quality long-term assets traded at $t = 1$ is equal to α_1. Thus, if a buyer purchases an amount k of long-term assets at $t = 1$, the buyer anticipates collecting output $f(\alpha_1 k)$ at $t = 2$. The share α_1 of high-quality long-term assets traded at $t = 1$ is

$$\alpha_1 = \frac{(1-\theta)\, k_0^s - k_1^s}{k_0^s - k_1^s}. \tag{7}$$

That is, α_1 is the ratio of the high-quality long-term assets sold by sellers (i.e., the total amount of high-quality assets $(1-\theta)\, k_0^s$ minus the amount k_1^s that are retained by sellers) relative to the total amount of long-term assets sold by the sellers (i.e., $k_0^s - k_1^s$).

We can now state the buyers' problem. A buyer chooses their holdings of the liquid and long-term assets l_1^b and k_1^b purchased at $t = 1$, and their consumption at $t = 1$ and $t = 2$, to solve:

$$\max_{l_1^b,\, k_1^b, c_1^b, c_2^b(\alpha_1)} \; u\left(c_1^b\right) + c_2^b(\alpha_1), \tag{8}$$

where we have emphasized the dependence of the time-2 consumption c_2^b on the belief α_1:

$$c_2^b(\alpha_1) = l_1^b + f(\alpha_1\, k_1^b). \tag{9}$$

The problem is subject to non-negativity constraints and to the budget constraint

$$c_1^b + l_1^b + q_1 k_1^b \leq 1. \tag{10}$$

Note that the resources available to the buyer (i.e., the right-hand side of (10)) are equal to one because the buyers are born at $t = 1$ with a unit of the liquid asset and no holdings of the long-term asset, as discussed in Section 2.1.

We begin by analyzing the optimal choice of the buyers under cash-in-the-market pricing, that is, when $u(c) = \log c$, $f(k) = R\,k$, and $\bar{\theta} = 0$. In this case, there is no uncertainty about the quality of the long-term asset, and thus, the buyers' belief is simply $\alpha_1 = 1$. In addition, because $f(k) = Rk$, the maximization in (8) implies the standard asset pricing condition

$$q_1 = \frac{1}{u'\left(c_1^b\right)} \times R, \tag{11}$$

where $1/u'(c_1^b)$ is the ratio of the marginal utility at $t = 2$ (i.e., one) and the marginal utility at $t = 1$ (i.e., $u'\left(c_1^b\right)$). Note that the time-1 consumption choice satisfies $u'\left(c_1^b\right) \geq 1$ because the buyers will never choose to consume more than one unit at $t = 1$, given the quasi-linear preference structure. Focusing again on the relevant case in which $q_1 \leq R$, and using $u(c) = \log c$, the buyers' optimal choices are

$$\left\{c_1^b, l_1^b, k_1^b\right\} = \begin{cases} \{1, 0, 0\} & \text{if } q_1 = R \\ \left\{\frac{q_1}{R}, 0, \frac{1}{q_1} - \frac{1}{R}\right\} & \text{if } q_1 < R \end{cases} \tag{12}$$

To preview some of the results, we note that in the low-withdrawal state $\gamma = 0$ (i.e., when no fire sales occur), the buyers consume $c_1^b = 1$ so that their marginal utility is $u'(c_1^b) = 1$, and (11) implies a time-1 price of $q_1 = R$ for the long-term asset. Hence, q_1 is equal to the cash flow that the asset produces at $t = 2$. In contrast, in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when a fire sale occurs), the buyers consume $c_1^b < 1$, so that their marginal utility is $u'\left(c_1^b\right) > 1$. Hence, (11) implies that the time-1 price of the long-term asset is $q_1 < R$, and thus, lower than the cash flow R.

Next, we turn to second-best-use pricing, that is, $u(c_1^b) = 0$, $f(k) < Rk$ for any $k > 0$, and

$\bar{\theta} = 0$. Similar to the case with cash-in-the-market pricing, there is no asymmetric information in equilibrium, and thus, buyers' belief are $\alpha_1 = 1$. The maximization in (8) now imply

$$q_1 = f'(k_1^b) \leq R. \tag{13}$$

In particular, $q_1 < R$ when $k_1^b > 0$ because of the strict concavity of $f(\cdot)$ under second-best use pricing. That is, the buyers are willing to purchase long-term assets at a low price because they are able to collect a lower cash flow than the sellers. Buyers' liquidity holdings, l_1^b, are residually determined from the budget constraint, and consumption c_1^b at $t = 1$ is zero because buyers only value consumption at $t = 2$.

Finally, with asymmetric-information pricing, $u(c_1^b) = 0$, $f(k) = Rk$, and $\bar{\theta} > 0$. Thus, $f(\alpha_1 \, k_1^b) = \alpha_1 R \, k_1^b$, and the buyers' first-order condition is

$$q_1 = \alpha_1 R. \tag{14}$$

Thus, buyers are willing to purchase any amount, provided that the price equals their belief about the output produced by the average asset traded. Similar to the case with cash-in-the-market pricing, l_1^b is residually determined from the budget constraint, and consumption c_1^b at $t = 1$ is zero because buyers only value consumption at $t = 2$.

2.5 Sellers' choices at $t = 0$

We now turn to the analysis at $t = 0$, when the sellers decide how to allocate their resources between the liquid and long-term assets. Then, in Section 3, we ask whether the sellers' choices at $t = 0$ are efficient and whether liquidity regulation can improve the equilibrium outcome. Recall that buyers are born at $t = 1$, and thus, the time-0 analysis involves only sellers.

To determine the sellers' portfolio choices at $t = 0$, we proceed along the lines of Dávila and Korinek (2018) and derive the sellers' time-0 choices that maximize their indirect utility function at $t = 1$. This approach is very convenient because the analysis is independent of many features of the model, and it will make the comparison with the regulator's problem and solution very transparent.

The sellers' indirect utility function at $t = 1$ is

$$V_1^s \left(l_0^s, k_0^s\right) = c_2^s + \lambda_1^s \left[l_0^s + q_1 k_0^s - \left(l_1^s + q_1 k_1^s + \gamma \, d^s\right)\right] + \mu_1^s l_1^s. \tag{15}$$

The first term on the right-hand side is the sellers' time-2 utility, which is linear in consumption, c_2^s. The second term is the Lagrange multiplier λ_1^s of the sellers' time-1 budget constraint (equation (5)) times the budget constraint itself. The last term is the Lagrange multiplier μ_1^s of the non-negative constraint on liquidity holdings, times such holdings, l_1^s. (The term $\mu_1^s l_1^s$ in (15) does not affect the analysis, but we include it because the non-negativity constraint $l_1^s \geq 0$ is binding in some cases in equilibrium.)

At $t = 0$, the sellers choose liquidity l_0^s and long-term asset holdings k_0^s to maximize their expected indirect utility function

$$\max_{l_0^s, k_0^s} \mathbb{E}_0 \left\{ V_1^s \left(l_0^s, k_0^s \right) \right\}, \tag{16}$$

subject to the budget constraint (1) and to non-negativity constraints on l_0^s and k_0^s. The problem in (16) is easy to analyze because we can exploit the envelope theorem to obtain

$$\mathbb{E}_0 \left\{ \lambda_1^s q_1 \right\} = \mathbb{E}_0 \left\{ \lambda_1^s \right\}, \tag{17}$$

provided that the time-0 non-negativity constraints on l_0^s and k_0^s do not bind. Recall that λ_1^s is the Lagrange multiplier of the sellers' budget constraint at $t = 1$ and, thus, it represents the sellers' marginal value of wealth. Equation (17) states that the sellers choose their time-0 portfolio to equalize the time-1 marginal value of holding one additional unit of the long-term asset (i.e., the left-hand side) to the marginal value of holding one additional unit of liquidity (i.e., the right-hand side). That is, a marginal dollar of investments at $t = 0$ could be used to invest in the long-term asset or in liquidity, which have market values of q_1 and one at $t = 1$, respectively, and which the sellers value according to their time-1 marginal utility of wealth λ_1^s.

The marginal utility of the sellers' wealth, λ_1^s, (and the equivalent object for the buyers, λ_1^b) is a crucial object for our analysis, and plays a key role in the efficiency and policy analysis of Section 3. Because λ_1^s is formally defined as the Lagrange multiplier of (5), the analysis in Section 2.3 implies

$$\lambda_1^s = \frac{R}{q_1}. \tag{18}$$

That is, a marginal unit of wealth available to sellers at $t = 1$ can be used to purchase (or retain) $1/q_1$ units of the long-term assets. Each unit of the asset will then produce a payoff R, which is evaluated according to the linear marginal utility of wealth.

2.6 Equilibrium definition

An equilibrium is a collection of the sellers' portfolio choice at $t = 0$ (i.e., $\{l_0^s, k_0^s\}$); and given a realization of the shocks $(\gamma, \theta) \in \{(0,0), (\overline{\gamma}, \overline{\theta})\}$, the sellers' and buyers' portfolio choices at $t = 1$ (i.e., $\{l_1^s, k_1^s\}$ and $\{l_1^b, k_1^b\}$); the buyers' beliefs about the fraction α of high-quality assets traded at $t = 1$; the buyers' consumption choices at $t = 1$ and $t = 2$ (i.e., c_1^b and c_2^b); the sellers' consumption choices at $t = 2$ (i.e., c_2^s), and a time-1 price for the long-term asset (i.e., q_1), such that the buyers and sellers maximize their utilities, the buyers' beliefs are rational, and the time-1 market clears. Specifically, the market-clearing condition for liquidity at $t = 1$ is

$$c_1^b + l_1^b + l_1^s + \gamma d_0^s = 1 + l_0^s, \tag{19}$$

where the right-hand side uses the assumption that the buyers are endowed with one unit of liquidity (see Section 2.1). That is, the liquid assets available in the economy, $1 + l_0^s$, is allocated between the buyers' consumption, c_1^b, their liquidity holdings, l_1^b, and the sellers' liquidity holdings, l_1^s, carried to $t = 2$, and the resources γd_0^s that are used to repay the sellers' debt holders at $t = 1$. The other market-clearing condition—for the long-term asset—holds by Walras' law, but we also state it for completeness:

$$k_1^b + k_1^s = k_0^s, \tag{20}$$

where the right-hand side uses the assumption that only sellers enter $t = 1$ with some holdings of the long-term asset.

2.7 Equilibrium and equivalence under the three pricing mechanisms

We now characterize the equilibrium. Since the sellers' problem is independent of the pricing mechanism (i.e., independent of the microfoundation of buyers' demand), several equilibrium features emerge regardless of the specific pricing mechanism.

In the low-withdrawal state (i.e., $\gamma = 0$ and $\theta = 0$), the equilibrium price at $t = 1$ must be $q_1 = R$ so that the liquid and long-term assets have the same return. If $q_1 < R$ or $q_1 > R$, the two assets would have a different return, and because there is no uncertainty between $t = 1$ and $t = 2$, sellers (and possibly buyers) would demand the asset with the higher return, and markets would not clear.[5]

[5]Note also that in the low-withdrawal state $\gamma = 0$ and $\theta = 0$, and thus, there is never asymmetric information.

In the high-withdrawal state (i.e., $\gamma = \bar{\gamma}$ and $\theta = \bar{\theta}$), we can determine the time-1 price q_1 using the sellers' first-order condition at $t = 0$, equation (17), together with the expression in (18) for the marginal utility of wealth λ_1^s and the process for (γ, θ) in (2). The resulting price is

$$q_1 = R \, \frac{\pi}{(R-1)+\pi} < R, \tag{21}$$

where the inequality follows from $R > 1$.

These considerations show that the price q_1 is given by $q_1 = R$ in the low-withdrawal state and by (21) in the high-withdrawal state, *in any equilibrium*.[6] Note that this result holds not only when one pricing mechanism is at play, but even when multiple pricing mechanisms operate together to generate a fire-sale price.

The trading volume is also the same independently of the microfoundation of the fire-sale price, for any given amount of the sellers' liquidity holdings l_0^s at $t = 0$. In general, trading volume in our model is equal to the amount of long-term assets sold by sellers, that is, $k_0^s - k_1^s$. Given the results in Section 2.3, trading volume is zero in the low-withdrawal state (i.e., when $\gamma = 0$ and $\theta = 0$), and equal to $\frac{(\gamma d^s - l_0^s)(R-1+\pi)}{\pi R}$ in the high-withdrawal state (i.e., when $\gamma = \bar{\gamma}$ and $\theta = \bar{\theta}$).

Based on the above results, the next proposition offers a simple characterization of the equilibrium variable at $t = 1$ (i.e., when a fire sale can happen). Because we are not taking any stance on the pricing mechanism, the result shows that the model generates a fire sale in the low withdrawal state not only when a pricing mechanism operates in isolation, but also when multiple pricing mechanisms play an active role.[7] All proofs are in Appendix A, and a more comprehensive description of the full equilibrium under each pricing mechanism can be found in Appendix B.

Proposition 2.1. ***(Equilibrium at** $t = 1$ **in the general framework)*** *If the time-0 non-negativity constraints of sellers $l_0^s \geq 0$ and $k_0^s \geq 0$ are not binding, the equilibrium variables at $t = 1$ satisfy:*

- *In the low withdrawal state, the price is $q_1 = R$; the trading volume is zero; the sellers' portfolio choices are $k_1^s = k_0^s$ and $l_1^s = l_0^s$; and the buyers' choices are $k_1^b = 0$ and $l_1^b + c_1^b = 1$.*
- *In the high-withdrawal state, the price is $q_1 = \frac{R\pi}{(R-1)+\pi}$; the trading volume is $\frac{(\gamma d^s - l_0^s)(R-1+\pi)}{\pi R}$; the sellers' portfolio choices are $k_1^s < k_0^s$ and $l_1^s = 0$; and the buyers' choices are $k_1^b > 0$ and $l_1^b + c_1^b < 1$.*

[6] More precisely, (21) holds in any equilibrium in which sellers are not constrained by $l_0^s \geq 0$ and $k_0^s \geq 0$.

[7] Proposition 2.1 focuses on the combined term $l_1^b + c_1^b$ to provide a result that holds under any combination of pricing mechanisms. This is because, depending on buyers' preference at $t = 1$, buyers might use liquidity for consumption or for investments in the liquid asset.

Next, we compare the equilibrium under the three pricing mechanism we consider. We show that not only prices and trading volumes are independent of the mechanism, but under certain parameter restrictions, the entire equilibrium is the same too. We establish this result through an equivalence proposition. That is, taking as given an equilibrium under cash-in-the-market pricing, we show that the equilibrium under second-best-use pricing or asymmetric-information pricing is the same under appropriate parameter restrictions. Hence, the equilibrium under the three pricing mechanisms is observationally equivalent. Thus, if we look at a given episodes of fire sales in practice through the lenses of the model, we cannot identify the pricing mechanism without knowing the microfoundation of buyers' demand.

While not necessary for our results, the proposition also shows that the demand elasticity during a fire sale—an object which can be identified in practice—is the same under the three mechanisms, under appropriate parameter restrictions. We define the demand elasticity based on a comparative static exercise in which we vary the sellers' supply of long-term assets by changing the parameter $\bar{\gamma}$ that governs the time-1 withdrawals. The idea is to mimic how the elasticity can be estimated in practice, that is, by finding exogenous variations to the supply curve.[8]

Proposition 2.2. ***(Observational equivalence of the three models.)*** *Consider the equilibrium under cash-in-the-market pricing (i.e., $u(c_1^b) = \log c_1^b$, $f(k) = R\,k$, and $\bar{\theta} = 0$). Then:*

(i) Under second-best use pricing (i.e., $u(c) = 0$, $f(k) < Rk$ for any $k > 0$, and $\bar{\theta} = 0$), if $f'\left(\frac{R-1}{\pi R}\right) = \frac{\pi R}{R-1+\pi}$, the equilibrium has the same sellers' portfolio choices at $t = 0$ as under cash-in-the-market pricing, and in each state (γ, θ), the same sellers' choices at $t = 1, 2$, the same buyers' holdings of k_1^b and the same combined liquidity holdings and consumption $l_1^b + c_1^b$ at $t = 1$, and the same price q_1 and trading volume at $t = 1$. If, in addition, $f''\left(\frac{R-1}{\pi R}\right) = -\frac{(\pi R)^2}{(R-1+\pi)^2}$, the buyers' time-1 demand elasticity in state $(\bar{\gamma}, \bar{\theta})$ is the same as under cash-in-the-market pricing.

(ii) Under asymmetric-information pricing (i.e., $u(c_1^b) = 0$, $f(k) = Rk$, and $\bar{\theta} > 0$), if $e^s = \frac{(R-1)^2}{(R-1)R\pi-(R-1+\pi)R\pi(\bar{\gamma}d^s-\bar{\theta})}$, the equilibrium has the same sellers' portfolio choices at $t = 0$

[8] While not central to our main results, we sidestep a common issue in standard asymmetric information models. Specifically, these models often predict that an increase in sellers' liquidity needs—that raises trading volume during fire sales—leads to higher asset prices. This prediction contradicts both cash-in-the-market and second-best-use models, as well as observed fire-sale dynamics (see, e.g., the discussion in Eisfeldt (2004) and Uhlig (2010)). Proposition 2.2 avoids this counterfactual implication by assuming that adverse selection (i.e., the fraction of lemons) intensifies as sellers' liquidity needs increase, ensuring that greater liquidity needs are associated with a larger share of low-quality assets traded in the market, and with it, lower asset prices. For alternative approaches with deeper microfoundations, see Kurlat (2016) and Chang (2018).

as under cash-in-the-market pricing, and in each state (γ, θ)*, the same sellers' choices at* $t = 1, 2$*, the same buyers' holdings of* k_1^b *and the same combined liquidity holdings and consumption* $l_1^b + c_1^b$ *at* $t = 1$*, and the same price* q_1 *and trading volume at* $t = 1$*. If, in addition,* θ *and* γ *are such that* $\theta = g(\gamma)$ *for some function* $g(\cdot)$ *with* $g(0) = 0$ *and* $g'(\bar{\gamma}) = \frac{\pi^2 \frac{d^s}{R}(A+\bar{\theta}BR)^2 + B(R-1+\pi)^2 d^s \bar{\theta}}{B(R-1+\pi)^2 A - \pi^2 B(A+\bar{\theta}BR)^2}$ *where* $A = \frac{R-1}{R-1+\pi}$ *and* $B = e^s + (1-\bar{\gamma})d^s + A$*, the buyers' time-1 demand elasticity in state* $(\bar{\gamma}, \bar{\theta})$ *is the same as under cash-in-the-market pricing.*

Regarding the equivalence between the equilibrium under cash-in-the-market and second-best-use pricing (i.e., Item (i) in Proposition 2.2), note such equivalence is established when $f(k) = \log(1 + R\,k)$.

3 Efficiency and policy analysis

We now study whether the equilibrium is efficient; that is, whether the equilibrium allocation—and, in particular, the sellers' time-0 portfolio choice—corresponds to that of a planner or regulator (hereinafter simply referred to as the "regulator"). Under the assumption that the buyers and sellers have linear utility at $t = 2$, we show in Section 3.2 that the equilibrium with cash-in-the-market pricing is efficient and, thus, no liquidity regulation should be imposed on the sellers' time-0 choices. In Sections 3.3 and 3.4, we show that the equilibrium is, instead, inefficient under second-best-use and the asymmetric-information pricing, requiring liquidity regulation in those cases. Crucially, the optimal regulation is a liquidity requirement under second-best-use pricing but a liquidity ceiling under asymmetric-information pricing.

We use a standard approach employed in the fire-sale literature. Various papers, such as Lorenzoni (2008), Dávila and Korinek (2018), and Kurlat (2021), consider a regulator that makes the initial portfolio choices at $t = 0$ but has no influence on the trading and choices that occur in the subsequent time periods (i.e., at $t = 1$ and $t = 2$). Besides following a common approach in the literature, the methodology is in line with our objective of studying liquidity requirements because this regulation is imposed, in practice, before the possible realization of fire sales.

There are two key differences between the regulator's problem and that of individual agents. First, similar to the literature, the regulator internalizes the effects of the time-0 portfolio choices on the time-1 price q_1, whereas private agents take the price as given. Second, the regulator internalizes the effects of its choices on the average quality of the assets that are traded at $t = 1$ (i.e., the share α_1 of high-quality long-term assets that are traded), which is also taken as given by private agents.

This second effect is novel in the literature that formalizes fire sales externalities.

To define efficiency, we rely on the concept of Pareto optimality because our model—like several others in the fire-sale literature—has two sets of agents (i.e., buyers and sellers). Thus, an equilibrium is constrained efficient if no regulatory intervention at $t = 0$ can improve the welfare of the buyers (keeping sellers' welfare unchanged), the welfare of the sellers (keeping buyers' welfare unchanged), or both.

3.1 Regulator's problem and first-order conditions

We consider the problem of a regulator aiming to maximize the sellers' welfare while ensuring that the buyers' welfare is at least as high as it would be in the unregulated equilibrium. At $t = 0$, the regulator chooses investments in the sellers' liquidity and long-term assets, l_0^s and k_0^s, that will maximize the sellers' utility. In addition, the regulator chooses a transfer, T, from the sellers to the buyers to make sure that the buyers achieve the same level of utility as that in the unregulated equilibrium. Because the buyers are born at $t = 1$, we assume that the transfer from the sellers to the buyers involves an amount T of the liquid asset.[9] Thus, the sellers will enter $t = 1$ with liquidity $l_0^s - T$ and the buyers with liquidity $1 + T$—recall that buyers are endowed with one unit of the liquid asset at $t = 1$. The regulator's problem is

$$\max_{l_0^s, k_0^s, T} \mathbb{E}_0 \left\{ V_1^s \left(l_0^s - T, k_0^s;\, q_1, \alpha_1 \right) \right\} \tag{22}$$

where $V_1^s(\cdot)$ is the sellers' indirect utility functions, defined in (15), in which we have highlighted the dependence on the time-1 price q_1 and the fraction of high-quality assets α_1 that are traded at $t = 1$. The maximization is subject to the sellers' time-0 budget constraint, (1) evaluated at $d_0^s = d^s$,

$$l_0^s + k_0^s \leq e^s + d^s \tag{23}$$

and to the constraint that the buyers' time-1 indirect utility $V_1^b(T;\, q_1)$ should be no less than the level $\overline{V}$ they achieve in the unregulated equilibrium:

$$V_1^b(T; q_1, \alpha_1) \geq \overline{V}. \tag{24}$$

[9] As in the literature, the transfer cannot be contingent on the state of the economy at $t = 1$, otherwise it would violate the assumption that the regulator can affect only the time-0 choices.

Specifically, the buyers' time-1 indirect utility is defined analogously to that of the sellers:[10]

$$V_1^b\left(T; q_1, \alpha_1\right) = u(c_1^b) + c_2^b(\alpha_1) + \lambda_1^b\left[1 + T - \left(l_1^b + q_1 k_1^b + c_1^b\right)\right] + \mu_1^b l_1^b + \eta_1^b c_1^b, \tag{25}$$

where we have emphasized the dependence of the time-2 consumption $c_2^b(\alpha_1)$ on the average quality of the assets that are traded at $t = 1$ (i.e., the share of high-quality long-term assets α_1, defined in (7)). The term λ_1^b is the Lagrange multiplier of the buyers' time-1 budget constraint and, thus, represents the buyers' marginal utility of wealth. The terms μ_1^b and η_1^b are the Lagrange multiplier on the non-negativity constraints $l_1^b \geq 0$ and $c_1^b \geq 0$, respectively.

Next, we derive the regulator's first-order conditions. Denoting ξ as the Lagrange multiplier of the buyers' utility constraint (24), the regulator's first-order conditions for the choice of the sellers' holdings of liquidity, l_0^s, and long-term assets, k_0^s, imply:

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right) + \frac{\partial \alpha_1}{\partial l_0^s}\frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1}\xi\right\}. \tag{26}$$

When comparing the regulator's optimality condition (26) at $t = 0$ with that of individual sellers in (17), there are two key differences.

First, the regulator considers the impact of the time-0 choices on the time-1 price q_1 of long-term assets—this is a typical element of policy analyses in the fire-sale literature. This gives rise, in our context, to a *distributive externality* (as in Dávila and Korinek 2018). In our model, this effect is captured by the term $\frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right)$ on the right-hand side of (26). The term ${\partial q_1}/{\partial l_0^s}$ is the sensitivity of the time-1 price to sellers' time-0 liquidity holdings; the term $k_0^s - k_1^s > 0$ denotes the quantity of long-term assets sold by sellers (i.e., the trading volume); and the term $\lambda_1^s - \xi\lambda_1^b$ measures the gap between sellers' and buyers' marginal utilities of wealth, adjusted by the tightness of the of the constraint (24)—as measured by its Lagrange multiplier ξ. If, for instance, ${\partial q_1}/{\partial l_0^s} > 0$, a liquidity requirement that increases l_0^s produces a higher q_1, and buyers have to transfer more resources to sellers to purchase any of the $k_0^s - k_1^s$ assets that are traded in a fire sale. Whether this additional transfer from buyers to sellers is beneficial from the regulator's perspective depends on the gap $\lambda_1^s - \xi\lambda_1^b$ between sellers' and buyers' marginal utilities of wealth.

Second, the regulator considers how a change in sellers' time-0 liquidity holdings affects the quality α_1 of assets traded at $t = 1$, and with it, buyers' time-2 consumption c_2^b. This effect is novel

[10]Recall from Section 2.1 that the buyers have no holdings of the long-term asset at the beginning of $t = 1$.

in the literature that formalizes fire sale externalities, and we label it *market quality externality*, along the lines of the *distributive* and *collateral* externalities identified by Dávila and Korinek (2018). The market quality externality is captured by the term $\frac{\partial \alpha_1}{\partial l_0^s} \frac{\partial c_2^b(\alpha)}{\partial \alpha_1} \xi$ on the right-hand side of (26). To understand this externality, consider a change in sellers' liquidity holdings l_0^s that, for instance, increases the average quality α_1 at $t = 1$. The higher quality of assets in the market increase buyers' time-2 consumption, thereby relaxing the constraint (24) that requires buyers' to achieve at least the same utility as in the unregulated equilibrium. Thus, the regulator can reduce the transfers T to buyers for any given regulatory intervention, and the utility value of this lower transfer is measured by the Lagrange multiplier ξ of the constraint (24).

Liquidity requirements are optimal when the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s} \left(k_1^s - k_0^s\right) \left(\xi \lambda_1^b - \lambda_1^s\right) + \frac{\partial \alpha}{\partial l_0^s} \frac{\partial c_2^b(\alpha)}{\partial \alpha} \xi\}$ in (26), evaluated at the unregulated equilibrium, is positive. This is because the right-hand side of (26) represents the regulator's marginal value of investing in liquidity at $t = 0$. Hence, a positive value for the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s} \left(k_1^s - k_0^s\right) \left(\xi \lambda_1^b - \lambda_1^s\right) + \frac{\partial \alpha}{\partial l_0^s} \frac{\partial c_2^b(\alpha)}{\partial \alpha} \xi\}$ means that, at the unregulated equilibrium, the regulator's value of investing in liquidity exceeds that of the private agents. Vice versa, a negative sign for such a term implies that a liquidity ceiling is optimal.

The first-order condition for the choice of transfers, T, is

$$\mathbb{E}_0 \left\{\lambda_1^s\right\} = \mathbb{E}_0 \left\{ \xi \lambda_1^b + \frac{\partial q_1}{\partial T} \left(k_0^s - k_1^s\right) \left(\lambda_1^s - \xi \lambda_1^b\right) + \frac{\partial \alpha_1}{\partial T} \frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1} \xi \right\}. \tag{27}$$

The welfare effect of a transfer T depends on how the marginal utility of wealth for sellers λ_1^s (on the left-hand side of (27)) compares with that of buyers λ_1^b (on the right-hand side of (27), adjusted by the Lagrange multiplier ξ). In addition, and similar to (26), the regulator accounts for the impact of T on the time-1 price q_1 and on the quality α_1 of the assets that are traded at $t = 1$.

3.2 Efficiency and liquidity requirements with cash-in-the-market pricing

We now specialize the analysis to the case with cash-in-the market pricing. The key result is that the equilibrium is efficient, and thus, no liquidity regulation is needed.

Under cash-in-the-market pricing, and given the key assumption that sellers and buyers have linear utility at $t = 2$, the next proposition shows that the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s} \left(k_0^s - k_1^s\right) \left(\lambda_1^s - \xi \lambda_1^b\right) + \frac{\partial \alpha_1}{\partial l_0^s} \frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1} \xi\}$ in the regulator's first-order conditions (26) is zero. Thus, the regulator's first-order condition (26) coincides with that of the private agents in (17), and the equilibrium is efficient. As a result, no liquidity regulation is required under cash-in-the-market pricing in our baseline model.[11]

Proposition 3.1. ***(Efficiency in the cash-in-the-market pricing)*** *Under cash-in-the-market pricing (i.e.,* $u(c_1^b) = \log c_1^b$*,* $f(k) = R\,k$*, and* $\bar{\theta} = 0$*), the unregulated equilibrium is constrained efficient.*

Under cash-in-the-market pricing, there is no informational asymmetry. Hence, the market quality externality does not operate because α_1 is constant at one—formally, $\partial\alpha_1/\partial l_0^s = 0$. In addition, given the linear utility of consumption at $t = 2$, the marginal utility of wealth of buyers and sellers, λ_1^s and λ_1^b, are equalized:

$$\lambda_1^s = \lambda_1^b = \frac{R}{q_1}. \tag{28}$$

This result arises because the buyers and sellers collect the same cash flow, R, from any unit traded—including the marginal unit—and they both have constant linear utility at $t = 2$. The linear utility at $t = 2$ also prevents any wealth effect that could arise from the planner's transfers, T. This implies that the time-1 price, q_1, is unresponsive to the transfers, T, and, thus, the term $\partial q_1/\partial T$ in the regulator's first-order condition (27) is zero as well. All of these results together imply that the Lagrange multiplier ξ of the regulator's constraint (24) is equal to one. That is, the sellers and buyers are effectively "symmetric"—not just at the unregulated equilibrium but also as we change the sellers and buyers' wealth, using the transfers, T. In other words, a fire sale simply entails a redistribution from the sellers to the buyers and create no inefficiencies. Formally, because $\xi = 1$ and $\lambda_1^s = \lambda_1^b$, the first-order condition (26) simplifies to $\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}$ and is, thus, identical to that of the individual sellers, that is, to equation (17).

3.3 Inefficiency and liquidity requirements with second-best-use pricing

We now turn to the case with second-best-use pricing. The equilibrium is now inefficient, and the optimal regulation is a liquidity requirement. That is, the regulator should force sellers to hold more liquidity, relative to the unregulated equilibrium. The next proposition formalizes this result.

Proposition 3.2. ***(Inefficiency under second-best-use pricing)*** *Under second-best pricing (i.e.,* $u(c_1^b) = 0$*,* $f(k) < Rk$ *for any* $k > 0$*, and* $\bar{\theta} = 0$*), the unregulated equilibrium is not constrained efficient, and the sellers' time-0 liquidity holdings are lower than the socially optimal level.*

[11] Dávila and Korinek (2018) show that when markets between $t = 0$ and $t = 1$ are complete, the equilibrium is efficient. Our model has two assets at $t = 0$ (i.e., long-term asset and liquidity) and two states at $t = 1$ (i.e., two possible realizations of γ), but the markets are not complete here because the buyers are born at $t = 1$ and, thus, have essentially no market access at $t = 0$. Thus, efficiency under cash-in-the-market pricing arises despite markets are incomplete.

The problem of the regulator and the first-order conditions described in Section 3.1 continue to apply. Similar to cash-in-the-market pricing, the market quality externality does not operate because there is no asymmetric information, and thus, ${}^{\partial \alpha_1}/_{\partial l_0^s} = 0$. The key difference relative to the cash-in-the-market is in the buyers' marginal utilities of wealth. In all states at $t = 1$, the buyers' marginal utility of wealth λ_1^b is now given by

$$\lambda_1^b = 1 \tag{29}$$

and, thus, is independent of the price q_1 of the long-term asset—compare (29) with the corresponding expression under the cash-in-the-market pricing in (28). Equation (29) arises under second-best-use pricing because buyers are indifferent, on the margin, between investing in liquidity or long-term assets at $t = 1$, and the marginal value of liquidity is always one. In contrast, the sellers' marginal utility of wealth, λ_1^s, is the same as under cash-in-the-market pricing (because their problem is the same), that is, $\lambda_1^s = {}^{R}/_{q_1}$; see (18).

Comparing the sellers' and buyers' marginal utilities in (18) and (29) shows that the two sets of agents have the same marginal utility of wealth in the low-withdrawal state $\gamma = 0$ (i.e., when $q_1 = R$) but different marginal utilities in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when $q_1 < R$ and a fire sale occurs). That is, a gap between the two marginal utilities opens up when a fire sale occurs; specifically, $\lambda_1^s > \lambda_1^b$. The proof of Proposition 3.2 shows that even after accounting for the adjustment required by the Lagrange multiplier ξ of (24), we still obtain that $\lambda_1^s > \xi \lambda_1^b$.

The gap that opens up between the buyers' and sellers' marginal utilities of wealth is due to the buyers' lower ability to extract cash flow from the marginal unit traded. Because of this gap, when assets are transferred to buyers in fire sales, the economy-wide output at $t = 2$ is lower, compared to the non-fire-sales state.

The regulator can improve welfare by forcing the sellers to invest more in liquidity at $t = 0$. With more liquidity available at $t = 1$, each seller needs to sell fewer assets, resulting in a higher price q_1 during a fire sale. The higher price, in turn, implies that the sellers need to sell even less, increasing the quantity of the long-term assets that remain in their hands and, thus, increasing the total output available in the economy at $t = 2$.

3.4 Inefficiency and liquidity ceiling in the asymmetric-information pricing

As a last step in the policy analysis, we focus on the asymmetric-information pricing mechanism. The equilibrium is inefficient, as in the case of second-best-use pricing. However, the optimal policy is a liquidity ceiling, as opposed to a liquidity requirement as with second-best-use pricing.

Proposition 3.3. ***(Inefficiency under asymmetric-information pricing)*** *Under asymmetric-information pricing (i.e., $u(c_1^b) = 0$, $f(k) = R\,k$, and $\bar{\theta} > 0$) the unregulated equilibrium is not constrained efficient, and the sellers' time-0 liquidity holdings exceed the socially optimal level.*

The problem of the regulator and the first-order conditions described in Section 3.1 continue to apply. Under asymmetric information pricing, both the distributive and market quality externalities operate under asymmetric information. Crucially, the distributive externality is partially offset by the market quality externality. This is because the time-1 price q_1 of the long-term asset is affected by both the distributive externality (directly through the effect of l_0^s on q_1) and the market quality externality (indirectly through the effect that l_0^s has on the average quality α_1, which is then transmitted to the price q_1 because of the buyers' first-order condition (14)). To understand this result, consider a change in regulation that increases the average quality α_1 of the assets traded at $t = 1$. On the one hand, the higher quality results in a higher price q_1, so that buyers have to transfer more resources to purchase long-term assets at $t = 1$; this effect reduces buyers' consumption. On the other hand, for any given trading volume, the higher quality of the assets traded allows buyers to increase their consumption. These two effects offset each other exactly, and the regulator's first-order condition (26) simplifies to[12]

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\lambda_1^s\right\}. \tag{30}$$

Note that this is the same first-order condition that would be obtained if the regulator chooses sellers' time-0 portfolio to maximize the utility of sellers, without any consideration for buyers' utility (i.e., without the constraint (24)). Indeed, under asymmetric information, buyers always attain the same level of consumption (and thus, the same overall utility) for any feasible time-0 portfolio choice of sellers.[13] This is the case because, for buyers, the average and marginal unit

[12]Equation (30) is obtained by differentiating (14) with respect to l_0^s to obtain ${}^{\partial q_1}/_{\partial l_0^s} = R\left({}^{\partial \alpha_1}/_{\partial l_0^s}\right)$, which implies $\left({}^{\partial q_1}/_{\partial l_0^s}\right)\left(k_0^s - k_1^s\right)\left(-\xi\lambda_1^b\right) + \xi\left({}^{\partial \alpha_1}/_{\partial l_0^s}\right)\left({}^{\partial c_2^b(\alpha_1)}/_{\partial \alpha_1}\right) = 0$ using $k_0^s - k_1^s = k_1^b$ from the market clearing condition for capital (20) and $\lambda_1^b = 1$ from the problem of buyers (8) under the assumption of asymmetric information pricing (i.e., $u(c_1^b) = 0$, $f(k) = R\,k$, and $\bar{\theta} > 0$).

[13]This follow from combining (9) and (10) with the first-order condition (14), which implies that c_2^b is always equal

traded at $t = 1$ are identical, and thus, they not only break even on the trade of the marginal unit but also on the trades of all the inframarginal units. Hence, as the regulator alters the time-0 portfolio of sellers, any choice made by the regulator does not affect buyers' utility, and the constraint (24) is satisfied with no transfers: $T = 0$. Thus, the full problem (22) of the regulator has the same solution as the problem of a regulator that focuses only on sellers' utility, without any consideration for buyers' welfare. In other words, under asymmetric information, the solution to the regulator's problem (22) is the same as that of a "monopolist seller" that tries to increase the price and reduce the quantity traded relative to the unregulated equilibrium in which each seller is a price taker. This logic is very robust to all the extensions we consider.

Because of the equivalence with a monopolist problem, the regulator's solution involves restricting the time-1 trading volume and increasing the price, relative to the unregulated equilibrium. This objective is similar to that of the regulator in the second-best-use version of the model, but it is achieved with the opposite regulation, that is, a liquidity ceiling (as opposed to a liquidity requirement, which is optimal under second-best-use pricing). With asymmetric information, reducing liquidity results in a higher time-1 price q_1 during a fire sale, due to the same logic discussed in Malherbe (2014). That is, if the sellers hold less liquidity, a larger fraction of the assets traded are sold to meet their liquidity needs and, thus, consists of high-quality assets. Consequently, the share of lemons in the market is lower, mitigating the adverse-selection problem. Formally, with adverse-selection pricing, the term ${}^{\partial q_1}\!/\!{}_{\partial l_0^s}$ in the regulator's first-order condition (30) is negative, in contrast to the positive sign that arises for the same term under second-best-use pricing.

4 Extensions: General time-2 utility and collateral constraints

The model of Sections 2-3 yield a simple and stark result: Three observationally equivalent pricing mechanisms that are commonly used to study fire sales have very different implications regarding the liquidity regulation that should be imposed on financial intermediaries. With cash-in-the-market pricing, the equilibrium is efficient and no regulation is needed. With second-best-use pricing, a liquidity requirement is optimal. And with asymmetric information, the opposite regulation (i.e., a ceiling on liquidity) is optimal.

We now provide two extensions to our baseline model. In the first extension (Section 4.1), we relax the assumption that the investors have linear utility at $t = 2$, and instead allow for an arbitrary

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to one.

concave utility function (possibly strictly concave). This extension allows for the possibility that market incompleteness prevent full and efficient risk sharing in financial markets, giving rise to an additional force that affects the optimal policy. In the second extension (Section 4.2), we augment the baseline by introducing a collateral constraint for sellers at $t = 1$.[14]

The equilibrium in these extensions is generically inefficient—even under cash-in-the-market pricing. However, the result we derive mirrors what we obtained in the baseline model. First, liquidity regulation should be "tighter" under second-best-use pricing, relative to cash-in-the-market pricing. More precisely, in the general utility model, the optimal policy could be a liquidity requirement or a liquidity ceiling under cash-in-the-marker pricing, but the socially optimal level of liquidity is higher in an observationally equivalent second-best-use setting. In the model with collateral constraints, the optimal policy is always a liquidity requirement under both cash-in-the-market and second-best-use pricing, but the requirement is tighter under second-best-use pricing. And second, in both extensions, the optimal policy is always a liquidity ceiling under asymmetric-information pricing.

4.1 General time-2 utility

In our first extension, we consider a version of the baseline model in which both buyers and sellers have a more general utility at $t = 2$. The key difference with the baseline model is that the optimal policy can be a liquidity requirement or a liquidity ceiling under both cash-in-the-market and second-best use pricing. However, similar to the baseline model, we obtain that (i) the socially optimal level of liquidity is higher under second-best use pricing relative to cash-in-the market pricing, and (ii) the optimal regulation is a liquidity ceiling under asymmetric-information pricing.

4.1.1 General time-2 utility: Model

The sellers' time-2 utility from consuming c_2^s is $u_2^s(c_2^s)$, and the buyers' time-2 utility from consuming c_2^b is $u_2^b(c_2^b)$, where $u_2^s(\cdot)$ and $u_2^b(\cdot)$ are strictly increasing and weakly concave functions and at least one of them is strictly concave. We relabel the time-1 utility function of buyers as $u_1^b(\cdot)$ to avoid confusion. All the other features of the environment described in Section 2.1 are unchanged.

Without loss of generality, we impose a normalization on the sellers' and the buyers' time-2

[14] In principle, we could also add a collateral constraint for seller at $t = 0$. However, because we take the total size of sellers' portfolio as given, adding a collateral constraint at $t = 0$ would not change the analysis, provided that liquidity and long-term assets are treated symmetrically in the constraint.

utility function. For sellers, we normalize $u_2^s(\cdot)$ so that their time-1 marginal utility of wealth λ_1^s is equal to one in the low-withdrawal state $(\gamma, \theta) = (0, 0)$.[15] For buyers, we normalize $u_2^b(\cdot)$ so that the buyers' marginal utility is one if the low-withdrawal state $(\gamma, \theta) = (0, 0)$ is realized. This requires ${\partial u_2^b(0)}/{\partial c_b^2} = 1$ with cash-in-the-market pricing and ${\partial u_2^b(1)}/{\partial c_b^2} = 1$ with the second-best-use and asymmetric-information pricing.

We derive the policy analysis (in Section 4.1.2) under the assumption that there exists an equilibrium with the same features as in the baseline (i.e., as in Proposition 2.7): an interior portfolio choice for liquidity and long-term asset holdings of sellers at $t = 0$, no trading at $t = 1$ in the low-withdrawal state $(\gamma, \theta) = (0, 0)$, a positive trading volume and a fire sale at $t = 1$ in the high-withdrawal state $(\gamma, \theta) = (\bar{\gamma}, \bar{\theta})$, and a buyers' demand that is downward sloping in the trading volume. Appendix B.3 provide some examples, focusing on cash-in-the-market pricing.[16] The remainder of this section provides some remarks to show that an equilibrium with these features is consistent with the environment we consider.

Remark #1: Sellers and buyers' choices at $t = 1$. Because the sellers' utility depends only on their time-2 consumption and because there is no uncertainty between $t = 1$ and $t = 2$, the sellers' objective function at $t = 1$ is the same as i the baseline model (i.e., maximizing time-2 consumption). Thus, the sellers' time-1 choices are the same as those described in Section 2.3. For the buyers, the problem and solution are also the same, under second-best-use and asymmetric-information pricing—the buyers' utility depends only on time-2 consumption and there is no uncertainty between $t = 1$ and $t = 2$—and thus the first-order conditions (13) and (14) are unchanged. Under cash-in-the-market pricing, the time-1 first-order condition (11) is replaced by

$$q_1 = \frac{(u_2^b)'(c_2^b)}{(u_1^b)'(c_1^b)} R. \tag{31}$$

Remark #2: Sellers' and regulator's problems at $t = 0$. The formulation of the sellers and regulator's problem in (16) and (22), respectively, is unchanged. While the sellers' first-order condition (17) is unchanged, the regulator's first-order conditions (26) and (27) are slightly different

[15]This simply require scaling the utility function by a constant and, thus, has no effect on the equilibrium allocation.

[16]We sidestep the issue of the conditions under which the equilibrium exists, given the generality of the utility functions we consider. Nonetheless, the concavity assumptions on the utility functions are in line with standard assumptions that guarantee existence in general equilibrium models.

because of the general time-2 utility:

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right) + \frac{\partial \alpha_1}{\partial l_0^s}\frac{\partial u_2^b\left(c_2^b(\alpha_1)\right)}{\partial c_2^b(\alpha_1)}\frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1}\xi\right\}, \tag{32}$$

$$\mathbb{E}_0\left\{\lambda_1^s\right\} = \mathbb{E}_0\left\{\xi\lambda_1^b + \frac{\partial q_1}{\partial T}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right) + \frac{\partial \alpha_1}{\partial T}\frac{\partial u_2^b\left(c_2^b(\alpha_1)\right)}{\partial c_2^b(\alpha_1)}\frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1}\xi\right\}. \tag{33}$$

That is, the last term on the right-hand side includes the marginal utility of buyers, $\partial u_2^b\left(c_2^b(\alpha_1)\right)/\partial c_2^b(\alpha_1)$. This term is equal to one in the baseline model because of the linear time-2 utility in that framework.

Remark #3: Asset prices in the low-withdrawal state $(\gamma, \theta) = (0, 0)$. Under the normalizations regarding the buyers' marginal utility, which we introduced before, and taking as given time-0 choices, the price in the low-withdrawal state is $q_1 = R$ under each of the three pricing mechanism, as in the baseline. The argument is the same as the one discussed in Section 2.7.

Remark #4: Asset prices and trading volume in the high-withdrawal state $(\gamma, \theta) = (\bar{\gamma}, \bar{\theta})$. Similar to the baseline, we can use (17) to pin down the price q_1 of the long-term asset in the high-withdrawal state. To do so, we note that the time-1 marginal utility of the sellers' wealth, λ_1^s, which is given by (18) in the baseline, is now given by

$$\lambda_1^s = \frac{R}{q_1}\frac{\partial u_2^s\left(c_2^s\right)}{\partial c_2^s} \tag{34}$$

under all three pricing mechanisms. That is, an additional unit of wealth at $t = 1$ allows the sellers to reduce their sales by $1/q_1$ units of the long-term asset, obtaining a payoff, R, per unit of asset, which is then valued according to their time-2 marginal utility of consumption, $\partial u_2^s\left(c_2^s\right)/\partial c_2^s$. Combining (2), (17), (34) and the assumption that the sellers' marginal utility of wealth, λ_1^s, is normalized to one in the low-withdrawal state, we can solve for the price q_1 of the long-term asset in high-withdrawal state:

$$q_1 = \frac{\pi R\frac{\partial u_2^s\left(c_2^s(\bar{\gamma},\bar{\theta})\right)}{\partial c_2^s}}{(1-\pi)(R-1) + \pi R\frac{\partial u_2^s\left(c_2^s(\bar{\gamma},\bar{\theta})\right)}{\partial c_2^s}} < R, \tag{35}$$

where $c_2^s(\bar{\gamma}, \bar{\theta})$ is sellers' consumption at $t = 2$ in the high-withdrawal state. Even though (35) expresses q_1 as a function of $c_2^s(\bar{\gamma}, \bar{\theta})$ (i.e., as a function of another endogenous variable), it shows that $q_1 < R$, using the assumptions $R > 1$ and $\pi < 1$. Thus, in the high-withdrawal state, the price of the long-term asset drops relative to the low-withdrawal state. In addition, because the sellers make the same choices as in the baseline (see Remark #1), the high-withdrawal state is again characterized by a higher trading volume relative to normal times, similar to the baseline of Sections 2-3. That is, a fire sale occurs in the high-withdrawal state.

4.1.2 General time-2 utility: Policy analysis

We are now ready to state our main results in the model with general utility. We begin by comparing efficiency and regulation under cash-in-the-market and second-best-use pricing, and then we turn to the model with asymmetric information.

Under cash-in-the-market pricing, the equilibrium is generically inefficient, and the optimal policy could be a liquidity requirement or a liquidity ceiling. We provide two examples in Appendix B.3 to show the general inefficiency and that the sign of the optimal regulation depends on the parameterization. However, the next proposition implies that if the optimal policy is a liquidity ceiling under cash-in-the-market pricing, the optimal policy under second-best-use pricing is either a lower liquidity ceiling or a liquidity requirement; and if the optimal policy under cash-in-the-market pricing is a liquidity requirement, the optimal policy under second-best-use pricing is a tighter liquidity requirement.

Proposition 4.1. ***(General utility: cash-in-the-market and second-best-use pricing)*** *Consider two observationally equivalent equilibria derived under cash-in-the-market and a second-best-use pricing, respectively (i.e., the sellers make the same time-0 choices in the two models and for any (γ, θ), the time-1 price q_1, the time-1 trading volume k_1^b, and the sensitivity of the price q_1 to the trading volume k_1^b, are the same in the two models).*

Then, the socially optimal level of liquidity is higher under second-best-use pricing, in comparison to cash-in-the-market pricing.

To understand this result, note that in the model with general utility, during a fire sale, a gap can open up between the sellers and the buyers' marginal utilities of wealth (i.e., between λ_1^s and λ_1^b), so that the regulator's first-order condition does not necessarily coincide with those of the individual sellers.[17] Importantly, with cash-in-the-market pricing, this gap is smaller in comparison to second-best-use pricing. Under cash-in-the-market pricing, both the sellers and the buyers' marginal util-

[17]Formally, the gap is given by $\lambda_1^s - \xi\lambda_1^b$, where ξ is the Lagrange multiplier of (24).

ities, λ_1^s and λ_1^b, increase in a fire sale, relative to normal times—for both sellers and buyers, the marginal unit of the long-term asset that is traded has a higher return, because of its lower price relative to normal times. Differently, with second-best-use pricing, only the sellers' marginal utility of wealth increases. The buyers' marginal utility of wealth decreases because buyers gain on the inframarginal units they purchase—for such inframarginal units, the cash flow collected is greater than the price paid because $f'' < 0$, and the cash flow is equal to the price only for the marginal unit. Thus, the gap in marginal utilities of wealth is greater under second-best use pricing, implying a higher socially optimal level of liquidity. The proof of Proposition 4.1 formalizes this result, accounting for the fact, in the regulator's first-order condition (32), the gap between λ_1^s and λ_1^b is adjusted using the Lagrange multiplier ξ of the constraint (24). In addition, the proof shows that the ${}^{\partial q_1}/_{\partial l_0^s}$ (i.e., the other key object that appears in the regulator's first-order condition (32)) is the same under cash-in-the-market pricing and second-best-use, under the assumption that the equilibria under both pricing mechanisms are observationally equivalent. Specifically, the assumption that q_1 has the same sensitivity to k_1^b under the two mechanisms (i.e., the demand elasticity is the same) implies that changes in l_0^s that shift the supply of long-term assets sold during a fire sale have the same impact on q_1.

As a last step, we analyze efficiency and regulation with asymmetric information. The forces that operate in the baseline model with linear utility continue to operate. That is, the regulator's optimal choice is the same as that of a "monopolistic seller" that maximizes sellers' joint utility without any regards for buyers' welfare, as discussed in Section 3.4. Hence, the regulator wants to reduce the depth of a fire sale (i.e., reduce trading volume and increase prices). In addition, as discussed in Section 3.4, a higher price q_1 is achieved with lower time-0 liquidity holdings, as this allocation reduces the adverse selection problem.

Proposition 4.2. ***(General utility: asymmetric-information pricing)*** *Consider a version of the model with asymmetric-information pricing (i.e., $u_1^b(c) = 0$ for all c, $f(k) = Rk$, and $\bar{\theta} > 0$). The sellers' time-0 liquidity holdings are higher than the socially optimal level, so that the optimal policy is a liquidity ceiling.*

4.2 Collateral constraints

We now extend the baseline model by introducing a collateral constraint on sellers at $t = 1$. The main results are qualitatively identical to those of the baseline model. While a liquidity requirement is now optimal with both cash-in-the-market and second-best-use pricing, the socially optimal amount of liquidity is higher under second-best-use pricing—as in the baseline—and thus, the liquidity requirement should be tighter under second-best-use pricing. Under asymmetric information, the optimal regulation continues to take the form of a liquidity ceiling.

4.2.1 Collateral constraints: Model

At $t = 1$, sellers are subject to the collateral constraint

$$q_1 k_0^s + l_0^s - d^s \geq \zeta\big(q_1 k_1^s + l_1^s\big). \tag{36}$$

The left-hand side of (36) represents the sellers' equity, which must be no less than a fraction ζ of the value of their total assets at time $t = 1$.

We also introduce the possibility that sellers' withdrawals can be adjusted endogenously, and we denote $w_1^s d^s \geq 0$ to be the withdrawals in addition to the baseline level γd^s (i.e., total withdrawals at $t = 1$ are $(\gamma + w_1^s)d^s$). We include this extension because the collateral constraints can trigger endogenous deleveraging of sellers to meet the collateral constraint. If this deleveraging occurs, some of the sellers' assets are sold, and their liabilities are also reduced so that their total assets equal their liabilities plus equity.

All the other features of the environment described in Section 2.1 are unchanged. Note that because the buyers' building block of the model is unchanged, the buyers' problem and choices are the same as those described in Section 2.4. In what follows, we focus on the problem of sellers and of the regulator.

We reformulate the time-1 problem of sellers to account for the collateral constraint (36) and the endogenous withdrawals $w_1^s d^s$. That is, sellers choose w_1^s in addition to the time-1 holdings of liquidity l_1^s and high-quality long-term assets k_1^s. Thus, the problem in (4) is replaced by

$$\max_{k_1^s, w_1^s, l_1^s} \; R k_1^s + l_1^s - (1 - \gamma - w_1^s) d, \tag{37}$$

subject to the budget constraint

$$l_1^s + (\gamma + w_1^s)\, d^s \leq l_0^s + q_1 \theta k_0^s + q_1 \left[(1-\theta) k_0^s - k_1^s\right]. \tag{38}$$

and to the collateral constraint (36). Any wealth available after repaying the time-1 withdrawals will be invested only in the long-term assets, which have higher return than liquidity, implying $l_1^s = 0$. When the sellers' collateral constraint is binding, we can use (36) together with the budget constraint (38) to solve for the sellers' choices of w_1^s and k_1^s:

$$w_1^s = \frac{(q_1 k_0^s + l_0^s)\left(1 - \frac{1}{\zeta}\right) + \frac{d^s}{\zeta} - \gamma\, d^s}{d^s}, \quad k_1^s = \frac{q_1 k_0^s + l_0^s - d^s}{\zeta q_1}.$$

To solve for the sellers' problem at $t = 0$, we augment the indirect utility (15) to include the collateral constraint at $t = 1$:

$$V_1^s\left(l_0^s, k_0^s\right) = c_2^s + \lambda_1^s\left[l_0^s + q_1 k_0^s - (l_1^s + q_1 k_1^s + \gamma\, d^s + w_1^s d)\right] + \eta_1^s[q_1 k_0^s + l_0^s - d^s - \zeta(q_1 k_1^s + l_1^s)] + \mu_1^s l_1^s,$$

where η_1^s is the Lagrange multiplier of the time-1 collateral constraint. Thus, the first-order condition for the time-0 choices of the sellers is

$$\mathbb{E}_0\left\{(\lambda_1^s + \eta_1^s) q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \eta_1^s\right\}. \tag{39}$$

The regulator's optimality condition for the time-0 portfolio choice of sellers, (26), becomes

$$\begin{aligned} &\mathbb{E}_0\left\{(\lambda_1^s + \eta_1^s) q_1\right\} \\ &\quad = \mathbb{E}_0\left\{(\lambda_1^s + \eta_1^s) + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi \lambda_1^b\right) + \frac{\partial \alpha_1}{\partial l_0^s}\frac{\partial c_2^b(\alpha_1)}{\partial \alpha_1}\xi + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - \zeta k_1^s\right)\eta_1^s\right\}. \end{aligned} \tag{40}$$

The main difference relative to (26) is the last term on the right-hand side, that is, $\left(\partial q_1 / \partial l_0^s\right)\left(k_0^s - \zeta k_1^s\right)\eta_1^s$. This term captures the fact that the regulator internalizes the effects of the time-0 choices on the price q_1 at $t = 1$, and with it, the tightness of the sellers' collateral constraint (36). This is a standard effect in models of fire sales.

4.2.2 Collateral constraints: Policy analysis

We begin by analyzing efficiency and regulation under cash-in-the-market and second-best-use pricing. Because of the introduction of the collateral constraint, the allocation of liquidity becomes inefficient even under cash-in-the-market pricing, in contrast to the baseline model. This is because of the standard logic, according to which an individual seller does not internalize that their time-0 choices impact the time-1 price q_1. Specifically, increasing the liquidity holdings at $t = 0$ would increase the price q_1 and with that, relax the collateral constraint of other sellers. The same force operates under second-best-use pricing, thereby increasing the socially optimal level of liquidity relative to the baseline model.

Crucially, the socially optimal level of liquidity is higher under second-best-use pricing than under a comparable case with cash-in-the-market pricing. This is because the effects that operate through the collateral constraint are the same under both pricing mechanism, and the force identified in the baseline model that makes liquidity holdings more socially desirable under second-best-use pricing still operates.

Proposition 4.3. ***(Cash-in-the-market and second-best-use pricing with collateral constraints)***

(i) Under cash-in-the-market and second-best-use pricing, the unregulated equilibrium is not constrained efficient, and the optimal policy is a liquidity requirement.

(ii) Consider two observationally equivalent equilibria derived under cash-in-the-market and second-best-use pricing (i.e., the sellers make the same time-0 choices in the two models and for any (γ, θ), the time-1 price q_1, the time-1 trading volume k_1^b, and the sensitivity of the price q_1 to the trading volume k_1^b are the same in the two models). Then, the socially optimal level of liquidity is higher under second-best-use pricing than under cash-in-the-market pricing.

Finally, we turn to asymmetric-information pricing. Under asymmetric information, the equilibrium is inefficient and the optimal regulation is a liquidity ceiling, as in the baseline model. The logic is the same as the one discussed in Section 3.4. That is, the regulator's solution is the same as that of a "monopolistic seller," and even though the tightness of the liquidity constraint affect quantitatively the optimal regulatory stance, the optimal policy is qualitatively identical.

Proposition 4.4. ***(Asymmetric-information pricing with collateral constraints)*** *The sellers' time-0 liquidity holdings are higher than the socially optimal level, so that the optimal policy is a liquidity ceiling.*

5 Conclusions

This paper analyzes liquidity requirements—a policy that has attracted growing attention over time from policymakers and academics—in a model in which financial intermediaries are forced to sell some assets to meet high liquidity needs. The model nests three mechanisms commonly employed in the literature to generate low fire-sale prices: cash-in-the-market pricing, second-best-use pricing, and adverse-selection pricing.

The optimal regulation of intermediaries' liquidity holdings is a liquidity requirement, or a liquidity ceiling, or no intervention, depending on the pricing mechanism and the effects of market incompleteness on investors' ability to efficiently share risk. More generally, we have highlight four forces that determine the optimal policy: (i) the cash flow that the buyers and sellers collect from the marginal unit traded, (ii) the sensitivity of the fire-sale price to the investors' liquidity holdings, (iii) a novel externality that depends on the sensitivity of the average quality of the assets traded to the sellers' liquidity holdings, and (iv) if and how market incompleteness prevents full and efficient risk sharing.

We have derived our results using a standard fire-sale framework in which trades take place in centralized markets, in line with a common approach used in the literature. In practice, however, some assets that have experienced fire sales—such as asset-backed securities and corporate bonds—are traded in decentralized over-the-counter (OTC) markets. While the forces we identified are likely to be important even in OTC markets, future research could study whether such forces interact with other possible distortions that are driven by the lack of centralized trading venues.

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APPENDIX

A Proofs

Proof of Proposition 2.1 In the low withdrawal state, $q_1 = R$, and thus, the liquid and long-term assets have the same returns. As a result, both the sellers and the buyers are indifferent between the two. Consequently, a no-trade allocation constitutes an equilibrium. In the high withdrawal state, the price and the trading volume are derived in Section 2.7. Because $q_1 < R$, the return on the long-term asset is less than one, and thus, lower than the return on liquid assets. Then, we argue that $l_0^s < \bar{\gamma} d^s$ must holds (i.e., buyers do not have enough liquidity to finance their withdrawals). By contradiction, assume that $l_0^s \geq \bar{\gamma} d^s$. If $\bar{\theta} = 0$, sellers sell no long-term assets, and no trade takes place; however, the only way to sustain a no-trade equilibrium is with $q_1 = R$, which is a contradiction. If $\bar{\theta} > 0$, sellers sell only their low-quality assets, but the price for such assets would be $q_1 = 0$, which is again a contradiction—the price q_1 is given by (21) and satisfies $q_1 > 0$. Finally, because $l_0^s < \bar{\gamma} d^s$, sellers need to sell some of their long-term assets to finance their withdrawals, and thus, $k_1^s < k_0^s$. Then, the market-clearing condition (20) implies $k_1^b > 0$, and the buyers' budget constraint (10) implies $l_1^b + c_1^b < 1$.

Proof of Proposition 2.2 The equivalence result regarding portfolio choice, prices, and trading volume follows from using the value of the equilibrium objects derived in Appendix B and plugging in the parameter restrictions stated in the proposition.

Regarding the buyers' demand elasticity, we write k_1^s as function of q_1, and evaluate the elasticity

$$\epsilon \equiv \frac{dk_1^b(q_1)}{dq_1} * \frac{q_1}{k_1^b(q_1)}$$

at the equilibrium. Under cash-in-the-market pricing, we have $k_1^b = \frac{1}{q_1} - \frac{1}{R}$ (see Section 2.4), hence the elasticity at equilibrium price is

$$\epsilon_{\text{cash-in-the-market}} = -\frac{1}{q_1^2} \frac{q_1}{\frac{1}{q_1} - \frac{1}{R}}\bigg|_{q_1 = \frac{\pi R}{R-1+\pi}} = -\frac{R-1+\pi}{R-1}. \tag{41}$$

In the second-best-use pricing, the assumption of the proposition about f' implies $k_1^b = \frac{R-1}{\pi R}$, and (13) implies $\frac{dk_1^b}{dq_1} = \frac{1}{f''(k_1^b)}$. Hence, using $q_1 = \frac{\pi R}{R-1+\pi}$, we have

$$\epsilon_{\text{second-best-use}} = \frac{1}{f''(\frac{R-1}{\pi R})} \frac{\pi R}{(R-1+\pi)\frac{R-1}{\pi R}}, \tag{42}$$

and $\epsilon_{\text{second-best-use}} = \epsilon_{\text{cash-in-the-market}}$ using the assumption about f'' stated in the proposition.

Under asymmetric information pricing, the sellers' budget constraint in fire sales times, (6), together with (7), (14), and the market clearing condition (20) evaluated at $\theta = g(\gamma)$ imply

$$k_1^b = \frac{\gamma d^s - l_0^s}{q_1}, \quad q_1 = \frac{(\gamma d^s - l_0^s)R}{\gamma d^s - l_0^s + g(\gamma)k_0^s R}.$$

We can then solve for $k_1^b(\gamma)$ and $q_1(\gamma)$ (where we have emphasized the dependence of these equilibrium object on γ), totally differentiate to compute $dk_1^b/d\gamma$ and $dq_1/d\gamma$, and rearrange to obtain

$$\frac{dk_1^b}{dq_1} = \frac{(\frac{d^s}{R} + g'(\gamma)k_0^s)(\gamma d^s - l_0^s + g(\gamma)k_0^s R)^2}{R^2 k_0^s(d^s g(\gamma) - (\gamma d^s - l_0^s)g'(\gamma))}.$$

The elasticity, evaluated at equilibrium items (q_1, k_1^b, l_0^s and k_0^s), is thus

$$\begin{aligned}\epsilon_{\text{asymmetric-information}} &= \frac{(\frac{d^s}{R} + g'(\gamma)k_0^s)(\gamma d^s - l_0^s + g(\gamma)k_0^s R)^2}{R^2 k_0^s(d^s g(\gamma) - (\gamma d^s - l_0^s)g'(\gamma))} * \frac{q_1}{k_1^b}\Big|_{\text{equilibrium}} \\ &= \frac{(\frac{d^s}{R} + g'(\gamma)B)(A + g(\gamma)BR)^2}{B(d^s g(\gamma) - Ag'(\gamma))} \frac{\pi^2}{(R-1)(R-1+\pi)}\end{aligned}$$

where $A = \frac{R-1}{R-1+\pi}$ and $B = e^s + (1-\gamma)d^s + \frac{R-1}{R-1+\pi}$. To make it equal to $\epsilon_{\text{cash-in-the-market}}$, let $\gamma = \bar{\gamma}$, and $\bar{\theta} = g(\bar{\gamma})$, we hence need

$$-\frac{\pi^2(\frac{d^s}{R} + g'(\bar{\gamma})B)(A + \bar{\theta}BR)^2}{B(R-1+\pi)^2(d^s\bar{\theta} - Ag'(\bar{\gamma}))} = 1, \tag{43}$$

or

$$g'(\bar{\gamma}) = \frac{\pi^2 \frac{d^s}{R}(A + \bar{\theta}BR)^2 + B(R-1+\pi)^2 d^s \bar{\theta}}{B(R-1+\pi)^2 A - \pi^2 B(A + \bar{\theta}BR)^2}. \tag{44}$$

The result follows.

Proof of Proposition 3.1. First note that $\frac{\partial \alpha}{\partial l_0^s} = 0$ in this case. The two FOCs of the regulator's problem are simplified to

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right)\right\}. \tag{45}$$

and

$$\mathbb{E}_0\left\{\frac{\partial q_1}{\partial T}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right) + \xi\lambda_1^b\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}. \tag{46}$$

We then evaluate the first-order condition of the regulator at the unregulated equilibrium. Specifically, we rederive the equilibrium in a version of the model in which the regulator announces a transfer, T, close to zero before the sellers make their time-0 decisions, so that we can compute the expression $\partial q_1/\partial T$ that appears in the regulator's first-order condition (46). We then evaluate this equilibrium at $T = 0$.

In the version of the model with the transfer T close to zero, the sellers' time-0 problem (16) becomes

$$\max_{l_0^s, k_0^s} \mathbb{E}_0 \left\{ V_1^s(l_0^s - T, k_0^s) \right\},$$

subject to the budget constraint (1) evaluated at $d_0^s = d^s$. The first-order condition (17), however, is unchanged. The expression for q_1 in a fire sale in Proposition 2.1 is also unchanged because the time-1 market-clearing condition for liquidity (19) is independent of T. This is the case because the buyers enter $t = 1$ with $1 + T$ units of liquidity and the sellers enter $l_0^s - T$ and, thus, the total liquidity available in the economy is unchanged at $1 + l_0^s$. Hence, the price q_1 in (21) that follows from (17) and (18) is also unchanged. Note that q_1 in (21) does not depend on T and, thus, ${}^{\partial q_1}/_{\partial T} = 0$ in the high-withdrawal state. Turning to the price q_1 in the low-withdrawal state, and under the assumption that the buyers' endowment is $1 + \varepsilon$ (see footnote 2), a marginal change in T away from $T = 0$ does not affect the buyers' optimal choice $c_1^b = 1$. Thus, because (11) implies the price q_1 depends only on c_1^b, we have $\partial q_1/\partial T = 0$ in the low-withdrawal state too.

Using the result $\partial q_1/\partial T = 0$ in all states for a T close to zero, the first-order condition (46) of the regulator simplifies to

$$\xi = \frac{\mathbb{E}\left\{\lambda_1^s\right\}}{\mathbb{E}\left\{\lambda_1^b\right\}}. \tag{47}$$

Then, using (28), we obtain $\xi = 1$. Thus, using $\xi = 1$ and (28), the regulator's optimality condition (45) becomes identical to that of the sellers in (17). In other words, the regulator's first-order condition holds when evaluated at the unregulated equilibrium and, thus, such an equilibrium is constrained efficient.

Proof of Proposition 3.2. We proceed as in the proof of Proposition 3.1 by rederiving the equilibrium with a transfer, T, that is close to zero, evaluating such an equilibrium at $T = 0$, and showing that ${}^{\partial q_1}/_{\partial T} = 0$ in all states at $t = 1$, when evaluating this derivative at $T = 0$. And $\frac{\partial \alpha}{\partial l_0^s} = 0$ also hold in this case, hence (45) and (46) are the simplified FOCs of regulators' problem.

In the high-withdrawal state, the result ${}^{\partial q_1}/_{\partial T} = 0$ can be shown as in the proof of Proposition

3.1. In the low-withdrawal state, we have $k_1^b = 0$ for any T close to zero. Thus, because q_1 is pinned down by (13) evaluated at $k_1^b = 0$, we have ${}^{\partial q_1}/_{\partial T} = 0$ when evaluated at $T = 0$.

Thus, as in the proof of Proposition 3.1, the value of ξ is given by (47). However, the expression for λ_1^b is different here, relative to the proof of Proposition 3.1, and in particular, $\lambda_1^b = 1$ in all states under second-best-use pricing. Thus, $\xi = \mathbb{E}\{\lambda_1^s\}$.

Next, again using $\lambda_1^b = 1$ and $\lambda_1^s = R/q_1$ together with the last result $\xi = \mathbb{E}\{\lambda_1^s\}$, the regulator's first-order condition (45) becomes

$$\begin{aligned} \mathbb{E}\left\{\lambda_1^s\left(q_1-1\right)\right\} &= \mathbb{E}\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\mathbb{E}\left\{\lambda_1^s\right\}-\lambda_1^s\right)\right\} \\ &= (1-\pi)\times 0+\pi(1-\pi)\frac{\partial q_1(\bar{\gamma})}{\partial l_0^s}\left(k_1^s(\bar{\gamma})-k_0^s\right)\left(1-\frac{R}{q_1(\bar{\gamma})}\right), \end{aligned} \tag{48}$$

where $q_1(\bar{\gamma})$ and $k_1^s(\bar{\gamma})$ denote the price and the sellers' end-of-period holdings of the long-term assets in the high-withdrawal state, respectively. The second line uses the assumption that fire sales happen with probability π (according to (2)), and the result $\frac{\partial q_1}{\partial l_0^s} = 0$ in the low-withdrawal state (which holds because the sellers enter $t = 1$ with $l_0^s > 0$, where the inequality follows from (3)). As a last step, we show that the right-hand side of (48) is not zero and, thus, the equilibrium is not efficient. Specifically, the right-hand side is positive and, thus, the right-hand side of the regulator's first-order condition (45) evaluated at the unregulated equilibrium is higher than the marginal value of the sellers' wealth λ_1^s. Because the right-hand side of (45) is the marginal social value of investing in liquidity, such a value is higher for the regulator than for the individual agents and, thus, the sellers' time-0 liquidity holdings are lower than the socially optimal level.

To establish that $\frac{\partial q_1(\bar{\gamma})}{\partial l_0^s}\left(k_1^s(\bar{\gamma})-k_0^s\right)\left(1-\frac{R}{q_1(\bar{\gamma})}\right) > 0$, we begin by noting that $k_1^s(\bar{\gamma}) - k_0^s < 0$ because the sellers sell some of their long-term asset holdings in a fire sale and that $1 - {}^{R}/_{q_1(\bar{\gamma})} < 0$ because $q_1(\bar{\gamma}) < R$ in a fire sale. Thus, we need to show that ${}^{\partial q_1(\bar{\gamma})}/_{\partial l_0^s} > 0$. To establish this result, Figure 1 plots the left- and right-hand sides of the buyers' first-order condition (13) evaluated at the equilibrium value of k_0^b; that is, using the time-1 market-clearing condition for capital, (20), and the budget constraint of the sellers in times of fire sales (6):

$$q_1(\bar{\gamma}) = f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right) \tag{49}$$

as a function of $q_1(\bar{\gamma})$. The left-hand side is given by $q_1(\bar{\gamma})$ and, thus, is represented by the 45-degree line (solid line). The right-hand side $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$ is represented by the dotted line and

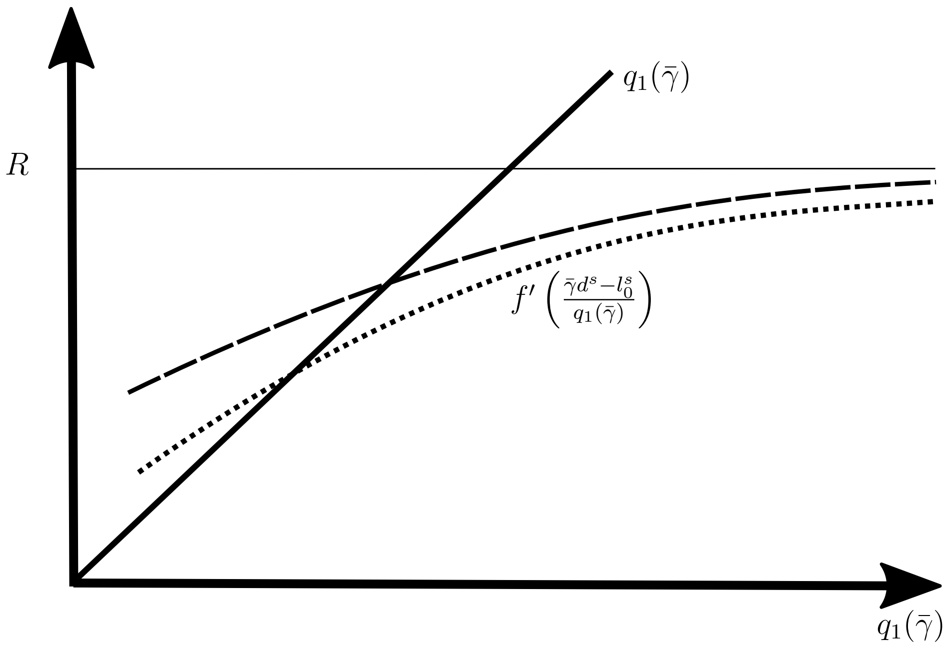

Figure 1: Establishing the sign of $\partial q_1(\bar{\gamma})/\partial l_0^s$ in the second-best-use pricing. The figure plots $q_1(\bar{\gamma})$ (solid line) and $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$ (dotted and dashed lines). An increase in l_0^s causes an increase in $f'\left(\frac{\bar{\gamma}d^s - l_0^s}{q_1(\bar{\gamma})}\right)$, represented by the shift from the dotted to the dashed line, and thus, an increase in the equilibrium value of $q_1(\bar{\gamma})$.

depends on $q_1(\bar{\gamma})$ through $f'(\cdot)$. As $q_1(\bar{\gamma}) \to \infty$, the argument of $f'(\cdot)$ goes to zero and, thus, $f'(\cdot)$ converges to R, given the assumptions in Section 2.1. And as $q_1(\bar{\gamma})$ decreases, the argument of $f'(\cdot)$ increases, and $f'(\cdot)$ decreases because $f'' < 0$. In addition, the intersection with the 45-degree line is somewhere at a point where $q_1(\bar{\gamma}) < R$. If l_0^s increases, the argument of $f'(\cdot)$ decreases and, again, because $f'' < 0$, the value of $f'(\cdot)$ increases for any $q_1(\bar{\gamma})$. Thus, an increase in l_0^s causes an upward shift in Figure 1 (i.e., the shift from the dotted line to the dashed line). In other words, an increase in l_0^s generates an increase in $q_1(\bar{\gamma})$, and vice versa, establishing $\partial q_1(\bar{\gamma})/\partial l_0^s > 0$.

Proof of Proposition 3.3. In the asymmetric-information pricing, $c_2 = l_1^b + \alpha R k_1^b$, hence $\frac{\partial c_2}{\partial \alpha} = k_1^b R$, and the regulator's FOC w.r.t. l_0^s becomes

$$\mathbb{E}_0 \left\{ \lambda_1^s (q_1 - 1) \right\} = \mathbb{E}_0 \left\{ \frac{\partial q_1}{\partial l_0^s} \left(k_0^s - k_1^s \right) \left(\lambda_1^s - \xi \lambda_1^b \right) + \xi k_1^b R \frac{\partial \alpha}{\partial l_0^s} \right\}. \tag{50}$$

Note that $q_1 = \alpha R$, we have

$$R \frac{\partial \alpha}{\partial l_0^s} = \frac{\partial q_1}{\partial l_0^s}.$$

Using this condition, alongside the fact that $\lambda_1^b = 1$, and the market clearing condition $k_1^b = k_0^s - k_1^s$, (50) can be rearranged as

$$\mathbb{E}_0 \left\{ \lambda_1^s (q_1 - 1) \right\} = \mathbb{E}_0 \left\{ \frac{\partial q_1}{\partial l_0^s} \left(k_0^s - k_1^s \right) \lambda_1^s \right\}. \tag{51}$$

In the fire sale state, $k_0^s - k_1^s > 0$. We also have $\lambda_1^s > 0$, and we argue that the sign of $\partial q_1 / \partial l_0^s$ is negative here (and thus different from the positive sign in the proof of Proposition 3.2). To see this, use the sellers' optimal choice for k_1^s, which is given by (6), together with (7) and (14), and time-0 budget constraint (1) to get

$$q_1 = \frac{R}{\left(1 + R \frac{\bar{\theta}(e^s + d^s - l_0^s)}{\bar{\gamma} d^s - l_0^s}\right)}. \tag{52}$$

It follows that $\partial q_1 / \partial l_0^s < 0$ in the fire-sale state.[18] As a result, the right-hand side of (51) is negative in the asymmetric-information pricing, rather than positive as in the second-best-use pricing. Therefore, the sellers' holdings of the liquidity assets at $t = 0$ in the unregulated equilibrium are higher than the socially optimal level, and a liquidity ceiling is required.

Proof of Proposition 4.1. Because the proposition focuses on cash-in-the-market and second-best use pricing, and because there is no asymmetric information under these pricing mechanisms (i.e., $\bar{\theta} = 0$), we simply use γ rather than (γ, θ) to refer to identify the state at $t = 1$.

We begin by establishing the intermediate results that $\partial q_1 / \partial T$ evaluated at $T = 0$ is equal to zero in the low-withdrawal state (i.e., when $\gamma = 0$), both under cash-in-the-market pricing and second-best-use pricing, similar to the baseline. This follows from the same logic used in the baseline. That is, under cash-in-the-market pricing, the buyers' first order conditions when $\gamma = 0$ are

$$(u_1^b)'(c_1^b) = \frac{R}{q_1} (u_2^b)'(c_2^b), \tag{53}$$

$$(u_1^b)'(c_1^b) = (u_2^b)'(c_2^b), \tag{54}$$

using the assumption that the buyers are endowed with $1 + \varepsilon$ units of liquidity (see footnote 2) and that the trading volume is zero in equilibrium (so that $k_1^b = 0$). These equations imply that $q_1 = R$, independently of the level of the buyers' consumption and, thus, independently of T. In the second-best-use pricing, the result can be shown as in the proof of Proposition 3.2.

[18]More precisely, when allowing for transfers, T, the price q_1 is given by $q_1 \left(\bar{\gamma}, \bar{\theta}\right) = R / \left(1 + R \frac{\bar{\theta}(e^s + d^s - l_0^s)}{\bar{\gamma} d^s - (l_0^s - T)}\right)$. However, the dependence on T does not affect the sign of $\partial q_1 / \partial l_0^s$.

The term $\partial q_1/\partial T$ in the high-withdrawal state $\gamma = \bar{\gamma}$, however, is not zero, in general, because the price q_1 depends on T; see (35) and note that the time-2 consumption c_2^s is, in general, a function of T. However, because q_1 and c_2^s are the same under the cash-in-the-market and second-best-use pricing (see Remarks #1 and #4 in Section 4.1), $\partial q_1/\partial T$ will also be the same in both models.

Next, we establish another intermediate result. That is, we show that

$$\lambda_1^b = 1 \tag{55}$$

in the low-withdrawal state (i.e., when $\gamma = 0$), under both pricing mechanisms. Under the cash-in-the-market pricing, the buyers' marginal utility of wealth is $\lambda_1^b = 1/c_1^b$. This can be obtained by differentiating the buyers' time-1 Lagrangian with respect to c_1^b and using the functional form $(u_1^b)(c) = \log c$ (see Section 2.2). Then, the normalization $(u_2^b)'(0) = 1$ and the first-order conditions (53) and (54), together with the buyers' budget constraint (10), imply that $c_1^b = 1$ and, thus, $\lambda_1^b = (u_1^b)'(1) = 1$. Under the second-best-use pricing, the buyers' marginal utility of wealth is $\lambda_1^b = (u_2^b)'(c_2^b)$; this can be obtained by differentiating the buyers' time-1 Lagrangian with respect to l_1^b. The result, $\lambda_1^b = 1$, under second-best-use pricing, follows from the fact that, in the low-withdrawal state (in which $\gamma = 0$, and $q_1 = R$), no trading takes place and the buyers' time-2 consumption is equal to their endowment of liquidity, $c_2^b = 1$, so that $(u_2^b)'(1) = 1$ because of the normalization introduced in Section 4.1.

Next, we turn to the regulator's first-order conditions (32) and (33). First, we note that there is no asymmetric information under both cash-in-the-market and second-best-use pricing, and thus, $\partial \alpha_1/\partial l_0^s = \partial \alpha_1/\partial T = 0$. Second, for both models, we can rewrite the regulator's first-order condition (33) using (i) the result $\partial q_1/\partial T = 0$ in the low-withdrawal state (i.e., when $\gamma = 0$), (ii) the normalization $\lambda_1^s = 1$ in the low-withdrawal state (i.e., when $\gamma = 0$) introduced in Section 4.1, and (iii) the fact that $\lambda_1^b = 1$ in the low-withdrawal state (i.e., when $\gamma = 0$), from (55). Thus, using "$(\bar{\gamma})$" to denote the variables in the high-withdrawal state, $\gamma = \bar{\gamma}$, (33) becomes

$$\pi \frac{\partial q_1(\bar{\gamma})}{\partial T} \left(k_1^s(\bar{\gamma}) - k_0^s\right) \left[\xi \lambda_1^b(\bar{\gamma}) - \lambda_1^s(\bar{\gamma})\right] + \mathbb{E}_0 \left\{\xi \lambda_1^b - \lambda_1^s\right\} = 0$$

and rearranging

$$\xi = \frac{1 - \pi + \lambda_1^s(\bar{\gamma})\pi \frac{\partial q_1(\bar{\gamma})}{\partial T} \left[k_1^s(\bar{\gamma}) - k_0^s(\bar{\gamma})\right] + \pi \lambda_1^s(\bar{\gamma})}{1 - \pi + \lambda_1^b(\bar{\gamma})\pi \left(1 + \frac{\partial q_1(\bar{\gamma})}{\partial T} \left[k_1^s(\bar{\gamma}) - k_0^s(\bar{\gamma})\right]\right)}. \tag{56}$$

Note that (56) holds in both the cash-in-the-market and second-best-use pricings.

The last step is to compare the expression $\mathbb{E}_0\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\xi\lambda_1^b-\lambda_1^s\right)\right\}$ in the other regulator's first-order condition, (32), under cash-in-the-market and second-best-use pricing. In both models, ${}^{\partial q_1}/_{\partial l_0^s}=0$ in the low-withdrawal state $\gamma=0$, which can be established similarly to the result ${}^{\partial q_1}/_{\partial T}=0$ for that state derived before. In the high-withdrawal state, we can establish that ${}^{\partial q_1}/_{\partial l_0^s}$ is the same under both cash-in-the-market and second-best-use pricing using the assumption that the sensitivity of the price q_1 to the trading volume k_1^b (i.e., the demand elasticity) is the same. To do so, we combine the sellers' budget constraint (6) with the long-term asset market-clearing condition (20) to obtain

$$k_1^b(\gamma)=\frac{\bar{\gamma}d^s-l_0^s}{q_1(\bar{\gamma})},$$

where we have emphasized that we are focusing on the time-1 variables in the high-withdrawal state $\bar{\gamma}$. Totally differentiating with respect to l_0^s and rearranging, we obtain

$$\frac{\partial q_1(\bar{\gamma})}{\partial l_0^s}=\frac{-1/q_1(\bar{\gamma})}{\frac{\partial k_1^b(\bar{\gamma})}{\partial q_1(\gamma)}+\frac{k_1^b(\bar{\gamma})}{q_1(\bar{\gamma})}}.$$

Because the proposition assumes that the equilibria under cash-in-the-market and second-best-use are equivalent (i.e., $q_1(\bar{\gamma})$, $k_1^b(\bar{\gamma})$, and ${}^{\partial k_1^b(\bar{\gamma})}/_{\partial q_1(\gamma)}$ are the same), the term ${}^{\partial q_1(\bar{\gamma})}/_{\partial l_0^s}$ is also the same under the two pricing mechanisms. Thus, because the trading volume $k_1^s-k_0^s$ is also the same under the assumption that the equilibria are equivalent, we only need to show

$$\left[\xi\lambda_1^b(\bar{\gamma})-\lambda_1^s(\bar{\gamma})\right]_{\text{cash-in-the-market pricing}}>\left[\xi\lambda_1^b(\bar{\gamma})-\lambda_1^s(\bar{\gamma})\right]_{\text{second-best-use pricing}}. \tag{57}$$

To see why this is the case, note that if $\xi\lambda_1^b(\bar{\gamma})-\lambda_1^s(\bar{\gamma})>0$ under cash-in-the-market pricing, the expression ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\xi\lambda_1^b-\lambda_1^s\right)<0$ in the high-withdrawal state $\gamma=\bar{\gamma}$, using ${}^{\partial q_1}/_{\partial l_0^s}>0$ and $k_1^s-k_0^s<0$ in that state. Hence, the sellers' liquidity holdings are higher than the socially optimal level (and the optimal policy is a liquidity ceiling), as discussed in the proof of Proposition 3.3. Therefore, if the term $\xi\lambda_1^b(\bar{\gamma})-\lambda_1^s(\bar{\gamma})$ is smaller under second-best-use pricing, the expression ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\xi\lambda_1^b-\lambda_1^s\right)$ is closer to zero or positive, implying that the optimal policy under second-best-use pricing is a lower ceiling (if ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\xi\lambda_1^b-\lambda_1^s\right)<0$) or a liquidity requirement (if ${}^{\partial q_1}/_{\partial l_0^s}\left(k_1^s-k_0^s\right)\left(\xi\lambda_1^b-\lambda_1^s\right)>0$).

As $\lambda_1^s(\bar{\gamma})$ is the same under both pricing mechanisms, establishing (57) is equivalent to showing

$$\left[\xi\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market pricing}}>\left[\xi\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use pricing}} \tag{58}$$

or, using (56),

$$\left[\frac{1-\pi+\lambda_1^s(\bar{\gamma})\pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]+\pi\lambda_1^s(\bar{\gamma})}{1-\pi+\lambda_1^b(\bar{\gamma})\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)}\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market pricing}} > \left[\frac{1-\pi+\lambda_1^s(\bar{\gamma})\pi\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]+\pi\lambda_1^s(\bar{\gamma})}{1-\pi+\lambda_1^b(\bar{\gamma})\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)}\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use pricing}}.$$

The numerator is the same under both pricing mechanisms and, thus, we need to show that

$$\left[(1-\pi)\frac{1}{\lambda_1^b(\bar{\gamma})}+\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)\right]_{\text{cash-in-the-market pricing}} < \left[(1-\pi)\frac{1}{\lambda_1^b(\bar{\gamma})}+\pi\left(1+\frac{\partial q_1(\bar{\gamma})}{\partial T}\left[k_1^s(\bar{\gamma})-k_0^s\right]\right)\right]_{\text{second-best-use pricing}}.$$

The only term that is different under the two pricing mechanisms is the buyers' marginal utility of wealth $\lambda_1^b(\bar{\gamma})$. Thus, we need to show that

$$\left[\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market pricing}} > \left[\lambda_1^b(\bar{\gamma})\right]_{\text{second-best-use pricing}},$$

and we do so by showing that

$$\left[\lambda_1^b(\bar{\gamma})\right]_{\text{cash-in-the-market pricing}} > 1 \geq \left[\lambda_1^b(\bar{\gamma})\right]_{\text{second-best- use pricing}}.$$

To establish this result, we show that, in the high-withdrawal state, $\gamma=\bar{\gamma}$ (i.e., when fire sales occur), the buyers' marginal utility increases under cash-in-the-market pricing, relative to the low-withdrawal state, whereas it decreases under second-best-use pricing.

Under cash-in-the-market pricing, the equilibrium in the low-withdrawal state is the same as in the baseline; that is, $c_1^b=1$ and $c_2^b=0$. With this allocation, the buyers' first-order condition (31) holds, given the normalization $(u_2^b)'(0)=1$ and the fact that sellers behave as in the baseline, and the market-clearing condition for liquidity, which is still given by (19), holds as well. Then, as in the baseline, the market-clearing condition evaluated at $l_1^b=0$ and $l_1^s=0$ implies that $c_1^b<1$ in the high-withdrawal state $\gamma=\bar{\gamma}$ and, thus, $\lambda_1^b(\bar{\gamma})>1$ because $\lambda_1^b=1/c_1^b$, as established before.

Under second-best-use pricing, the buyers' time-2 consumption is

$$\begin{aligned} c_2^b &= l_1^b + f\left(k_1^b\right) \\ &= l_0^b - q_1 k_1^b + f\left(k_1^b\right), \end{aligned}$$

where the last line uses the time-1 budget constraint. Differentiating with respect to k_1^b,

$$\begin{aligned} \frac{\partial c_2^b}{\partial k_1^b} &= -\frac{\partial q_1}{\partial k_1^b} k_1^b - q_1 + f'\left(k_1^b\right) \\ &= -f''\left(k_1^b\right) k_1^b > 0, \end{aligned}$$

where the second line uses $q_1 = f'(k_1^b)$ from (13), which continues to hold in the model with general utility, as noted in Remark #1 in Section 4.1. Because k_1^b increases in the high-withdrawal state $\gamma = \bar{\gamma}$ relative to the low-withdrawal state $\gamma = 0$ (i.e., trading increases in a fire sale and, thus, buyers acquire assets in a fire sale relative to the non-fire sale state), the buyers' time-2 consumption under second-best-use pricing also increases. As a result, the marginal utility of wealth $\lambda_1^b = (u_2^b)'(c_2^b)(\bar{\gamma})$ is weakly lower than in the low-withdrawal state $\gamma = 0$; that is, it is less than or equal to one, because $u_2^b(\cdot)$ is weakly concave (and possibly strictly concave).

Proof of Proposition 4.2. In the general model, the regulator's FOC w.r.t. l_0^s writes as

$$\mathbb{E}_0\left\{\lambda_1^s(q_1-1)\right\} = \mathbb{E}_0\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\left(\lambda_1^s - \xi\lambda_1^b\right) + \xi k_1^s R \frac{\partial \alpha}{\partial l_0^s}(u_2^b)'(c_2^b)\right\}. \tag{59}$$

Because on the margin, the buyers can purchase one unit of liquidity at $t = 1$, which allows them to increase consumption by one unit at $t = 2$, we can express their marginal utility of wealth as $\lambda_1^b = (u_2^b)'(c_2^b)$. Consequently, (59) can also be rearranged as

$$\mathbb{E}_0\left\{\lambda_1^s(q_1-1)\right\} = \mathbb{E}_0\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\lambda_1^s\right\},$$

which is the same as (51). The right-hand side is negative, implying the sellers' holdings of the liquidity assets at $t = 0$ in the unregulated equilibrium are higher than the socially optimal level. Note the result ${\partial q_1}/{\partial l_0^s} < 0$ can be derived as in baseline model; see the proof of Proposition 3.3.

Proof of Proposition 4.3. We first prove the inefficiency with cash-in-the-market pricing by showing that the sum of the last two items on the right-hand side of (40) is positive. The first-order condition for the planner's choice of transfers, T, is now

$$\mathbb{E}_0\left\{\frac{\partial q_1}{\partial T}\left(k_0^s-k_1^s\right)\left(\lambda_1^s-\xi\lambda_1^b\right)+\xi\lambda_1^b+\frac{\partial q_1}{\partial T}\left(k_0^s-\zeta k_1^s\right)\eta_1^s\right\}=\mathbb{E}_0\left\{\lambda_1^s+\eta_1^s\right\}. \tag{60}$$

As the time-1 price, q_1, is unresponsive to T, (60) implies

$$\xi=\frac{\mathbb{E}_0(\lambda_1^s+\eta_1^s)}{\mathbb{E}_0(\lambda_1^b)}=\frac{(1-\pi)+\pi(\frac{R}{\zeta q_1}+1-\frac{1}{\zeta})}{(1-\pi)+\pi\frac{R}{q_1}}.$$

Hence

$$\begin{aligned}
&\mathbb{E}_0\left\{\frac{\partial q_1}{\partial l_0^s}\left(k_0^s-k_1^s\right)\left(\lambda_1^s-\xi\lambda_1^b\right)+\frac{\partial q_1}{\partial l_0^s}\left(k_0^s-\zeta k_1^s\right)\eta_1^s\right\}\\
=&\,\pi\frac{\partial q_1(\bar\gamma)}{\partial l_0^s}\left[\left(k_0^s-k_1^s(\bar\gamma)\right)\left(\lambda_1^s(\bar\gamma)-\xi\lambda_1^b(\bar\gamma)\right)+\left(k_0^s-\zeta k_1^s(\bar\gamma)\right)\eta_1^s(\bar\gamma)\right]\\
>&\,\pi\frac{\partial q_1(\bar\gamma)}{\partial l_0^s}\left[\left(k_0^s-k_1^s(\bar\gamma)\right)\left(\lambda_1^s(\bar\gamma)-\xi\lambda_1^b(\bar\gamma)+\eta_1^s(\bar\gamma)\right)\right]\\
=&\,\pi\frac{\partial q_1(\bar\gamma)}{\partial l_0^s}\left[\left(k_0^s-k_1^s(\bar\gamma)\right)\left(\frac{R}{\zeta q_1}+1-\frac{1}{\zeta}-\frac{R}{q_1}\frac{(1-\pi)+\pi(\frac{R}{\zeta q_1}+1-\frac{1}{\zeta})}{(1-\pi)+\pi\frac{R}{q_1}}\right)\right]\\
=&\,\pi\frac{\partial q_1(\bar\gamma)}{\partial l_0^s}\left(k_0^s-k_1^s(\bar\gamma)\right)\left[\frac{(1-\zeta)(1-\pi)(\frac{R}{q_1}-1)}{\zeta(1+\pi(\frac{R}{q_1}-1))}\right]\\
&>0.
\end{aligned}$$

The first inequality comes from the fact that $\zeta<1$.

In the second-best-use pricing, we show that the wedge is positive by showing the second term on the RHS of (40) is positive (as the third term is clearly positive). To see this, note that $\lambda_1^b=1$ in the second-best-use pricing. And we only need to show $\lambda_1^s-\xi>0$. In the second-best-use pricing,

$$\xi=1-\pi+\pi\left(\frac{R}{q_1}+1-\frac{1}{\zeta}\right),$$

and λ_1^s is the same as in the cash-in-the-market pricing, the result follows.

We then show that the socially optimal level of liquidity is higher under second-best-use pricing, in comparison to cash-in-the-market pricing. That is, we show that the wedge in the second-best use is larger than that in the cash-in-the market. When the two equilibria are observational equivalent, we can show that $\partial q_1/\partial l_0^s$ is the same under both pricing mechanisms following the same steps used in the proof of Proposition 4.1. Thus, because the observational equivalence also implies that $(k_0^s - k_1^s(\bar{\gamma}))$ is the same under both pricing mechanisms, the proof boils down to showing

$$[\lambda_1^s(\bar{\gamma}) - \xi\lambda_1^b(\bar{\gamma})]_{\text{Cash-in-the-market}} < [\lambda_1^s(\bar{\gamma}) - \xi\lambda_1^b(\bar{\gamma})]_{\text{Second-best use}},$$

or

$$[\xi\lambda_1^b(\bar{\gamma})]_{\text{Cash-in-the-market}} > [\xi\lambda_1^b(\bar{\gamma})]_{\text{Second-best use}},$$

or

$$\frac{(1-\pi) + \pi(\frac{R}{\zeta q_1} + 1 - \frac{1}{\zeta})}{(1-\pi) + \pi\frac{R}{q_1}}\frac{R}{q_1} > 1 - \pi + \pi\left(\frac{R}{q_1} + 1 - \frac{1}{\zeta}\right),$$

which is true by $R > q_1$ in the fire-sale state.

Proof of Proposition 4.4. Using $q_1 = \alpha R$, $\lambda_1^b = 1$ and the market clearing condition $k_1^b = k_0^s - k_1^s$, (40) becomes

$$\mathbb{E}_0\left\{(\lambda_1^s + \eta_1^s)q_1\right\} = \mathbb{E}_0\left\{(\lambda_1^s + \eta_1^s) + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - k_1^s\right)\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_0^s - \zeta k_1^s\right)\eta_1^s\right\}.$$

The wedge is negative, as $\frac{\partial q_1}{\partial l_0^s} < 0$ in this case, and $k_0^s - \zeta k_1^s > k_0^s - k_1^s > 0$ in the fire-sale state.

B Full equilibrium characterization

In this appendix, we describe the full equilibrium under each pricing mechanism.

B.1 Equilibrium in the cash-in-the-market pricing

The equilibrium at $t = 0, 1, 2$ under cash-in-the-market pricing is the following:

- At $t = 0$, the sellers invest an amount $l_0^s = \frac{\pi}{R-1+\pi} + \bar{\gamma}d^s - 1$ in liquidity and $k_0^s = e^s + d^s(1 - \bar{\gamma}) - \frac{\pi R}{R-1+\pi} + 1$ in the long-term asset.
- At $t = 1$

- If $\gamma = 0$, the price of the long-term asset is $q_1 = R$, the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 0$ for the buyers), and the buyers' consumption is $c_1^b = 1$;
- If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = \frac{\pi R}{R-1+\pi} < 1$, the sellers' portfolio choices are

$$k_1^s = \frac{\pi^2 R\left(d^s(1-\bar{\gamma}) + e^s\right) + \pi(R-1)\left[R\left(d^s(1-\bar{\gamma}) + e^s\right) + R - 1\right] - (R-1)^2}{\pi R\left(R - 1 + \pi\right)}$$

and $l_1^s = 0$, the buyers' portfolio choices are $k_1^b = \frac{R-1}{\pi R}$ and $l_1^b = 0$, and the buyers' consumption is $c_1^b = \frac{\pi}{R-1+\pi}$.

- At $t = 2$
  - If $\gamma = 0$, the sellers consume $c_2^s = Re^s + (R-1)\left[d^s\left(1-\bar{\gamma}\right) + \frac{R-1}{R-1+\pi}\right]$ and the buyers consume $c_2^b = 0$;
  - If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = \frac{\pi(R-1+\pi)[d(R-1)(1-\bar{\gamma})+e^s R]-(1-\pi)(R-1)^2}{\pi(R-1+\pi)}$ and the buyers consume $c_2^b = \frac{R-1}{\pi}$.

B.2 Equilibrium in the second-best-use pricing

The equilibrium at $t = 0, 1, 2$ under second-best-use pricing is the following:

- At $t = 0$, the sellers invest an amount $l_0^s = \bar{\gamma}d^s - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ in liquidity and $k_0^s = e^s + d^s - \frac{\pi R}{R-1+\pi}\left[\bar{\gamma}d^s - (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right]$ in the long-term asset.
- At $t = 1$
  - If $\gamma = 0$, the price of the long-term asset is $q_1 = R$, the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 1$ for the buyers), and the buyers' consumption c_1^b is zero;
  - If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = \frac{\pi R}{R-1+\pi} < 1$ and the sellers' portfolio choices are

$$k_1^s = e^s + d^s(1-\bar{\gamma}) + (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right) \times \left(\frac{\pi R}{R-1+\pi} - 1\right)$$

and $l_1^s = 0$, the buyers' portfolio choices are $k_1^b = (f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ and $l_1^b = 1 - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(R\frac{\pi}{R-1+\pi}\right)$, and the buyers' consumption c_1^b is zero;

- At $t = 2$
  - If $\gamma = 0$, the sellers consume $c_2^s = Re^s + (R-1)\left[d^s(1-\bar{\gamma}) + \left(\frac{\pi R}{(R-1)+\pi}\right)(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right]$ and the buyers consume $c_2^b = 1$;
  - If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = Re^s + (R-1)d^s(1-\bar{\gamma}) + R\left(\frac{\pi R}{R-1+\pi} - 1\right)(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)$ and the buyers consume $c_2^b = f\left[(f')^{-1}\left(\frac{\pi R}{R-1+\pi}\right)\right] + 1 - \frac{\pi R}{R-1+\pi}(f')^{-1}\left(R\frac{\pi}{R-1+\pi}\right)$.

B.3 Equilibrium in the asymmetric-information pricing

The equilibrium at $t = 0, 1, 2$ in the asymmetric-information pricing is the following:

- At $t = 0$, the sellers invest an amount $l_0^s = \frac{R\pi\bar{\theta}e^s - (R-1)\bar{\gamma}d^s}{R\pi e^s - (R-1)}$ in liquidity and

$$k_0^s = \frac{(R-1+R\pi\bar{\theta})e^s - R\pi(e^s)^2 + (R-1)(1-\gamma)d_0^s - R\pi e^s d_0^s}{(R-1) - R\pi e^s}$$

  in the long-term asset.

- At $t = 1$
  - If $\gamma = 0$, the price of the long-term asset is $q_1 = R$, the trading volume is zero (i.e., $k_1^s = k_0^s$ and $l_1^s = l_0^s$ for the sellers, and $k_1^b = 0$ and $l_1^b = 1$ for the buyers), and the buyers' consumption c_1^b is zero;
  - If $\gamma = \bar{\gamma}$, the price of the long-term asset is $q_1 = \frac{\pi R}{R-1+\pi} < 1$, the sellers' portfolio choices are $k_1^s = \frac{d^s((R-1)(-1+\gamma)+e^s(\pi R+\gamma-(\pi+R)\gamma))+e^s\left(e^s\pi R-(R-1)(1+(\pi-1)\bar{\theta})\right)}{1+(e^s\pi-1)R}$ and $l_1^s = 0$, the buyers' portfolio choices are $k_1^b = \frac{\bar{\gamma}d^s - \frac{(R-1)\bar{\gamma}d^s - R\pi\bar{\theta}e^s}{(R-1)-R\pi e^s}}{R\pi}(R-1+\pi)$ and $l_1^b = 1 + \frac{(R-1)\bar{\gamma}d^s - R\pi\bar{\theta}e^s}{(R-1)-R\pi e^s} - \bar{\gamma}d^s$, and the buyers' consumption c_1^b is zero;

- At $t = 2$
  - If $\gamma = 0$, the sellers consume $c_2^s = Re^s + d^s(R-1) - (R-1)\left[\frac{\pi}{R-1+\pi} + \bar{\gamma}d^s - 1\right]$ and the buyers consume $c_2^b = 1$;
  - If $\gamma = \bar{\gamma}$, the sellers consume $c_2^s = \frac{d^s((R-1)(-1+\gamma)+e^s(\pi R+\gamma-(\pi+R)\gamma))+e^s\left(e^s\pi R-(R-1)(1+(\pi-1)\bar{\theta})\right)R}{1+(e^s\pi-1)R}$ and the buyers consume $c_2^b = 1$.

Appendix C

Numerical example under cash-in-the-market pricing

This appendix provides a numerical example that illustrates the results of Section 4.1. We show that, depending on parameter values, the unregulated equilibrium can feature either excessive or insufficient liquidity relative to the social optimum.

Utility. Buyers have

$$u^b(c_1^b, c_2^b) \;=\; \log(c_1^b) \;+\; \log\big(1 + c_2^b\big),$$

which satisfies the condition $\partial u_2^b(0)/\partial c_2^b = 1$ stated in Section 4.1. Sellers' utility is identical to that in the baseline model.

Time 1 portfolios. In the low-withdrawal state $(\gamma = 0)$ the portfolios are

$$k_1^s\big|\gamma = 0 = k_0^s, \qquad l_1^s\big|\gamma = 0 = l_0^s, \tag{61a}$$

$$k_1^b\big|\gamma = 0 = 0, \qquad l_1^b\big|\gamma = 0 = 0, \qquad c_1^b\big|\gamma = 0 = 1. \tag{61b}$$

In the high-withdrawal state $(\gamma = \bar{\gamma})$ the buyer's first-order condition (31) implies

$$q_1\big|\gamma = \bar{\gamma} \;=\; \frac{c_1^b}{1 + c_2^b}\, R.$$

Combining $c_2^b = l_1^b + Rk_1^b$ with the time-1 budget $1 = c_1^b + l_1^b + q_1 k_1^b$ yields

$$k_1^s\big|\gamma = \bar{\gamma} = \frac{q_1 k_0^s - \big(\bar{\gamma} d - l_0^s\big)}{q_1}, \qquad l_1^s\big|\gamma = \bar{\gamma} = 0, \tag{62a}$$

$$k_1^b\big|\gamma = \bar{\gamma} = \frac{R - q_1}{2Rq_1}, \qquad l_1^b\big|\gamma = \bar{\gamma} = 0, \quad c_1^b\big|\gamma = \bar{\gamma} = \frac{q_1 + R}{2R}. \tag{62b}$$

Time 0 price and portfolios. With $\lambda_1^s = R/q_1$, the equilibrium price in the high-withdrawal state remains

$$q_1\big|\gamma = \bar{\gamma} \;=\; \frac{\pi R}{R - 1 + \pi}. \tag{63}$$

Using the time-0 constraint $k_0^s = e^s + d - l_0^s$ we obtain

$$l_0^s = \frac{1 + 2d(\pi - 1)\bar{\gamma} + R\big[-1 + 2d\bar{\gamma}\big]}{2\big[-1 + \pi + R\big]}, \tag{64a}$$

$$k_0^s = d(1 - \bar{\gamma}) + \frac{-1 + R + 2e^s\big[-1 + \pi + R\big]}{2\big[-1 + \pi + R\big]}. \tag{64b}$$

(The remaining time-1 allocations follow mechanically; we reproduce them for completeness.)

Planner's first-order condition. Under cash-in-the-market pricing the planner's condition (32) reduces to

$$\mathbb{E}_0[\lambda_1^s q_1] \;=\; \mathbb{E}_0\Big[\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\big(k_0^s - k_1^s\big)\big(\lambda_1^s - \xi\lambda_1^b\big)\Big].$$

In the high-withdrawal state:

$$\frac{\partial q_1}{\partial l_0^s} \;=\; 2R \;>\; 0, \quad \lambda_1^s - \xi\lambda_1^b \;=\; \frac{(-1 + \pi + R)\big[-1 - \pi + 2\pi^2 + R - \pi R\big]}{\pi\big[-1 - \pi + 2\pi^2 + R + \pi R\big]}.$$

Numerical calibration. Fix $R = 1.1$, $e^s = 10$, $d = 3$, and $\bar{\gamma} = 0.3$. We vary π to show that equilibrium liquidity can exceed or fall short of the social optimum.

Case 1: $\pi = 0.10$

$$q_1 = 0.55, \quad l_0^s = 0.35, \qquad \lambda_1^s - \xi\lambda_1^b = -1.3846.$$

Because the wedge $\lambda_1^s - \xi\lambda_1^b$ is negative—and $\frac{\partial q_1}{\partial l_0^s} > 0$ and $k_0^s - k_1^s > 0$—the sellers' liquidity is **above** the planner's optimum.

Case 2: $\pi = 0.04$

$$q_1 = 0.314286, \quad l_0^s = 0.242857, \qquad \lambda_1^s - \xi\lambda_1^b = 0.626866.$$

The positive wedge $\lambda_1^s - \xi\lambda_1^b$ implies that equilibrium liquidity is **below** the planner's optimum.

These two cases confirm that, even under the same cash-in-the-market pricing mechanism, the sign of the regulatory wedge—and hence whether liquidity is excessive or insufficient—is ambiguous and depends on the parameterization of the model.