Fire Sales and Liquidity Requirements*

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Abstract

We study liquidity requirements in a model of fire sales that nests three common pricing mechanisms—cash-in-the-market, second-best-use, and adverse selection—and can produce the same observables under each of these mechanisms. We identify three forces that shape the optimal policy. Absent risk-sharing considerations, the equilibrium is efficient with cash-in-the-market pricing; a liquidity requirement is optimal with second-best-use pricing; and a liquidity ceiling (i.e., a cap on liquid assets) is optimal with adverse selection. Accounting for risk-sharing considerations, the optimal level of liquidity remains higher with second-best-use pricing relative to cash-in-the-market pricing, and a liquidity ceiling remains optimal with adverse selection. Our results (i) provide a unifying theory of liquidity requirements applicable to a broad range of markets and asset classes, (ii) suggest that the mere possibility of fire sales is insufficient to justify liquidity requirements, and (iii) offer practical guidance for designing regulations governing intermediaries' liquidity holdings.

1 Introduction

Fire sales are common phenomena in periods of financial distress. These episodes are characterized by large sales of financial assets and a reduction in their prices, despite little to no change in the fundamentals, and they occur when investors are forced to sell their assets for various reasons. Examples abound across markets and asset classes, ranging from assets held by distressed banks

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(Granja, Matvos, and Seru, 2017) to asset-backed securities (Merrill et al., 2021) and highly rated corporate bonds (Falato, Goldstein, and Hortaçsu, 2021; Ma, Xiao, and Zeng, 2022).

To mitigate the risks posed by fire sales, policymakers have increasingly relied on liquidity requirements, which have become a cornerstone of financial regulation over the past 10–15 years. Following the 2008 financial crisis, liquidity requirements were imposed on banks and money market mutual funds. The financial distress caused by the COVID-19 crisis further spurred action, with the Securities and Exchange Commission (SEC) proposing liquidity requirements for open-end mutual funds.

Designing effective liquidity regulations, however, requires a thorough understanding of the root causes of fire sales. The literature provides several theories to explain fire sales using very different mechanisms that lead to low asset prices. Some theories are based on the assumption that buyers have limited cash available to purchase assets (Allen and Gale, 1998). Others assume that buyers have a low willingness to pay because they can collect lower cash flows than sellers (i.e., the so-called second-best-use assumption; Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997; Lorenzoni, 2008; Dávila and Korinek, 2018). A third set of theories is based on asymmetric information and adverse selection (Guerrieri and Shimer, 2014; Kurlat, 2016; Chang, 2018; Dow and Han, 2018).

Despite these theoretical advances, relatively little is known about how to optimally regulate intermediaries' liquidity holdings to mitigate the adverse effects of fire sales. While some papers study this topic (see Section 1.1), they typically focus on a single pricing mechanism, leaving open the question of how robust the policy prescriptions are to the possibility that fire sales may arise from a different pricing mechanism.

We fill the gap in the literature by analyzing liquidity requirements using a unifying framework that nests three main pricing mechanisms proposed by the fire-sale literature, namely, cash-in-the-market pricing, second-best-use pricing, and adverse-selection pricing. By offering a singly unifying framework, we are not only able to compare the optimal policies that arise under the different mechanisms, but we also identify key forces that shape the optimal policy and operate under all the mechanisms we consider.

Our first main result is that the optimal regulation of intermediaries' liquidity holdings might differ dramatically depending on the underlying pricing mechanism responsible for fire sales. Depending on the mechanism and the impact of market incompleteness on risk sharing, the optimal policy can be a liquidity requirement (i.e., a lower bound on liquidity holdings) or a liquidity ceiling (i.e., an upper bound on liquidity holdings). And in some cases, the equilibrium is efficient, and no regulation is needed.

Our second main result is that we identify three main elements that operate under all the pricing mechanisms we consider to shape the optimal policy: (i) the difference between the sellers and the buyers' ability to collect cash flow from the *marginal* unit traded, (ii) the sensitivity of the fire-sale price to the sellers' liquidity holdings, and (iii) how market incompleteness affects the investors' ability to share risk.[1] Importantly, even if investors are able to collect the same cash flow from any given asset (Kurlat 2021), they might trade assets of different quality on the margin, opening up a role for policy interventions.

We first use a simple baseline framework that allows us to abstract from risk-sharing considerations to deliver stark results. The model has two assets (i.e., a short-term liquid asset and a long-term asset) and, similar to the fire-sale literature, two sets of agents (which we label the sellers and the buyers). An exogenous shock that increases the sellers' liquidity needs triggers a fire sale, forcing them to sell long-term assets to the buyers. In the efficiency and policy analysis, we focus on the composition of the sellers' portfolios in terms of their liquid and long-term assets before the possible realization of fire sales, aiming to determine whether and how the mix of the two assets should be regulated. The optimal policy is no intervention, a liquidity requirement, or a liquidity ceiling when fire sales are driven by cash in the market, second-best use, and asymmetric information, respectively.

We design our framework such that different pricing mechanisms, under appropriate restrictions, give rise to observationally equivalent models of fire sales. That is, our framework can produce the same portfolio choices, trading volumes, prices, and buyers' demand under cash-in-the-market, second-best-use, and adverse-selection pricing. As a result, "anything goes," because a given equilibrium outcome can feature too much, too little, or the right amount of liquidity, depending on the pricing mechanism. We are thus able to highlight the role played by the pricing mechanism in shaping the optimal policy after "controlling" for variables that can be observed or estimated by empirical researchers.

With cash-in-the-market pricing and no risk-sharing considerations, the equilibrium is (Pareto) efficient and, thus, no liquidity regulation is necessary. This is because the occurrence of a fire sales—relative to an economy with no shocks that induce fire sales—simply redistribute resources

[1]Some of these results build on the insights of Dávila and Schaab (2023) regarding the general effects of policy interventions in incomplete-markets heterogeneous agent models.

from the sellers to the buyers because buyers, who are able to buy assets at a low price. Hence, fire sales generate no aggregate welfare losses.

With second-best-use pricing, the buyers collect less cash flow from their long-term assets than the sellers do. Thus, a fire sale reduces aggregate efficiency because long-term assets end up in the hands of the buyers, who are less efficient at collecting cash flow. Abstracting from risk-sharing considerations, the optimal policy is a liquidity requirement, which reduces the depth of a fire sale.

With adverse-selection pricing, the equilibrium is again inefficient. Even if all of the investors can collect the same cash flow from any given asset, the cash flow collected from the *marginal* unit traded is different between the buyers and the sellers. In the version of the model with adverse selection, there are high- and low-quality long-term assets, and the sellers have private information about the quality of each unit they hold. On the margin, the sellers sell high-quality assets because all of the low-quality ones are sold as infra-marginal units. But for the buyers, the marginal unit traded is the average asset in the market, which includes both high- and low-quality ones. Thus, similar to the second-best-use version of the model, the sellers collect more cash flow from the marginal unit traded than the buyers do. This gives rise to an inefficiency: because of the low price at which sellers trade (high-quality) assets on the margin, they invest too little in such assets ex-ante. While this result is similar to Kurlat (2021), we go one-step forward by identifying the regulation that avoids a too-low price during a fire sale. This regulation is a liquidity *ceiling* in the adverse-selection model, as opposed to a liquidity *requirement* as in the second-best-use model. The logic of this result is similar to that in Malherbe (2014). A ceiling is required because if the sellers enter a fire-sale episode with less liquidity, a larger fraction of the sales will be due to fundamental reasons and a smaller fraction to private information, reducing the extent of the adverse-information problem. Imposing a liquidity requirement when fire sales are driven by adverse selection would *amplify* a fire sale and reduce welfare.

We then extend our model to include risk-sharing considerations. The forces that arise in the baseline model continue to operate, but the optimal policy is also affected by the agents' inability to efficiently share risk, due to market incompleteness. The equilibrium can be inefficient even with cash-in-the-market pricing, similar to the cash-in-the-market banking model of Allen and Gale (2004). We establish that if a liquidity requirement is optimal with cash-in-the-market pricing, then the optimal requirement is stricter in an observationally equivalent model that is based on second-best-use pricing. And if a liquidity ceiling is optimal with cash-in-the-market pricing, then in an equivalent model with second-best-use pricing the optimal regulation is a lower ceiling or even a liquidity requirement. With adverse-selection pricing, the optimal regulation remains a liquidity ceiling under regularity conditions that are likely to hold in practice.

Our analysis focuses on the inefficiencies driven by market incompleteness, which the literature has labeled *distributive externalities* (Dávila and Korinek, 2018; Lanteri and Rampini, 2023). The effects of these externalities are often difficult to sign unambiguously. Our results make progress by distinguishing two forces that affect distributive externalities: the gap in the cash flow that the sellers and buyers are able to collect from the marginal unit they trade, and the role of imperfect risk sharing. While the first force typically points in one direction—in nearly all models in the literature, the sellers can collect the same or more cash flow than the buyers can—the second one is ambiguous and depends on whether imperfect risk sharing has higher impacts on the buyers or the sellers. We thus establish that the inability to unambiguously sign the effects of the distributive externalities is due solely to imperfect risk sharing. While this result is derived in the context of liquidity requirements, the logic behind it can be extended to more general settings. Note that we abstract from the so-called *collateral externalities* that are driven by binding collateral constraints, which are typically easier to sign and have been studied in greater detail (Bianchi, 2011; Dávila and Korinek, 2018).

A direct policy implication is related to the debate about the introduction of liquidity requirements for open-ended mutual funds, proposed by the SEC and motivated by the March 2020 "dash for cash." This event was a fire sale of high-quality corporate bonds and, thus, was likely unrelated to second-best-use considerations as the investors should easily collect cash flow from corporate bonds. There is also no evidence that this event was driven by adverse selection (Haddad, Moreira, and Muir, 2021). If the fire-sale prices in this event are driven by cash-in-the-market pricing, our analysis suggests that the impact of market incompleteness on risk sharing should have first-order importance in determining the optimal policy.

More generally, our results provide a warning to policymakers. The possibility of fire sales alone does not justify liquidity requirements, and the optimal policy depends crucially on the elements that drive buyers' low willingness to pay in a fire sales, as well as the effects of market incompleteness on risk sharing. In this respect, our results provide guidance for policymakers on how to identify the key forces that shape the optimal liquidity policy in practice. While we conduct our analysis using a simple framework, these forces are likely to remain valid even in richer environments.

1.1 Additional comparisons with the literature

Among the papers that study optimal policies to mitigate fire sales of financial assets, several focus on regulating ex-ante borrowing and total investments (e.g., Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021). Our paper complements these studies, as we focus on the composition of investors' portfolios and the share invested in liquid assets, abstracting from the size of investors' borrowing and investments.

Our work is closely related to Dávila and Korinek (2018). They identify, in a second-best-use model, collateral externalities and distributive externalities—the latter are driven by incomplete markets—and provide sufficient statistics to guide policy interventions. While our policy analysis builds on their approach, there are important distinctions. First, Dávila and Korinek (2018) focus on the size of investors' borrowing and investments, whereas we focus on the composition of their portfolios, in terms of liquid and illiquid assets, to study liquidity requirements. Second, we show that the sufficient statistics identified by Dávila and Korinek (2018) can be used not only with second-best-use pricing but also with cash-in-the-market and asymmetric-information pricing—overturning the conjecture of Kurlat (2021) about the inability to use the approach of Dávila and Korinek (2018) with asymmetric information. Third, we use the insights of Dávila and Schaab (2023) to further distinguish two forces that affect distributive externalities (i.e., the cash flow collected from the marginal unit traded, and imperfect risk sharing), allowing us to make progress in understanding the effects of distributive externalities. While Dávila and Korinek (2018) show that distributive externalities can lead to choices that are either too high or too low relative to those preferred by the regulator, we establish that the inability to unambiguously sign these effects to study liquidity requirements is due only to imperfect risk sharing.

Another closely related paper is Kurlat (2021), which compares the optimal size of ex-ante investments, using second-best-use and adverse-selection pricing. While the spirit of our exercise is similar, there are important differences. First, we also consider cash-in-the-market pricing. Second, Kurlat (2021) focuses on the size of ex-ante investments, whereas we focus on the composition in terms of liquid and illiquid assets. Third, in Kurlat (2021), investors have linear utility, whereas we extend our analysis to a setting with general utility to study the impact of risk-sharing considerations. Fourth, even though Kurlat (2021) states that "[t]he result of Dávila and Korinek (2018) that there are measurable statistics that suffice to determine the direction of the externality [...] does not extend to the asymmetric-information model," we show that the sufficient statistics identified in Dávila and Korinek (2018) can actually be used with asymmetric-information pricing (and with cash-in-the-market pricing too) to perform policy analysis. Fifth, we show that the optimal regulatory stance with asymmetric-information pricing is related to how asset prices respond to the sellers' liquidity holdings (consistent with Malherbe, 2014) and to the difference in the cash flow collected by the buyers and sellers from the *marginal* unit traded. Whether or not the buyers and sellers can collect the same cash flow from any given assets—a point the literature has often focused on (Dow and Han, 2018; Kurlat, 2021)—matters only insofar as it affects the cash flow collected from the marginal unit traded.

Our paper is also related to Allen and Gale (2004). In their banking model with cash-in-the-market pricing and incomplete markets, the equilibrium is generically inefficient and liquidity requirements increase welfare only when agents are sufficiently risk averse. Our results generalize their findings by showing that the equilibrium under cash-in-the-market pricing is efficient when we shut down any risk-sharing considerations, and that risk-sharing considerations affect the benefits of liquidity requirements not just under cash-in-the-market pricing but also under other pricing mechanisms.

Several other papers study liquidity regulation for financial intermediaries but focus on other aspects. Farhi, Golosov, and Tsyvinski (2009) show that liquidity requirements can mitigate the problem of hidden trades in a Diamond-Dybvig framework. Calomiris, Heider, and Hoerova (2015) show that regulating banks' liquidity holdings is beneficial because such assets are easily observable and do not suffer from asymmetric-information problems. Kara and Ozsoy (2020) and Kashyap, Tsomocos, and Vardoulakis (2024) show that capital requirements and liquidity requirements should be used together to improve welfare. Hachem and Song (2021) show that liquidity regulation can trigger credit booms, focusing on China from 2007 to 2014. Robatto (2023) studies the interaction between liquidity requirements and central bank liquidity injections.[2]

Our paper is also related to a large empirical literature on fire sales.[3] Coval and Stafford (2007) and Jotikasthira, Lundblad, and Ramadorai (2012) document fire sales in equity securities. Ellul, Jotikasthira, and Lundblad (2011), Falato et al. (2021), Falato, Goldstein, and Hortaçsu (2021), and Manconi, Massa, and Yasuda (2012) provide evidence of fire sales in corporate bond markets both

[2]A closely related strand of the banking literature uses fire-sale pricing mechanisms in models of runs and banking crises; see, for instance, Diamond and Dybvig (1983); Acharya and Yorulmazer (2008); Gale and Yorulmazer (2013); Gertler and Kiyotaki (2015); Robatto (2019); Goldstein et al. (2022).

[3]Some other closely related papers use structural models to study issues similar to ours. Kargar, Passadore, and Silva (2023) use a search model with aggregate risk to study transaction costs, trading volumes, and asset prices, with a focus on the 2020 "dash for cash." Geromichalos and Herrenbrueck (2016) focus on assets with different maturities, as we do, but abstract from fire sales.

in normal times and during crises (2007-2008 and COVID-19 crises), although the evidence about normal times is challenged by Ambrose, Cai, and Helwege (2012) and Choi et al. (2020). Li and Schürhoff (2019) focus on fire sales in municipal bond markets.

2 General model framework

We begin by describing the general model framework. Following a standard approach in the fire-sale literature (e.g., Dávila and Korinek, 2018), we consider an economy populated by two sets of investors—the sellers (s) and the buyers (b)—and the economy lasts for three periods, $t = 0, 1, 2$. At $t = 0$, the sellers make their portfolio choices by choosing their investments in a liquid asset and a long-term asset. At $t = 1$, the buyers are born, and a fire sale can occur depending on the realization of an exogenous shock that forces the sellers to sell some of their holdings of the long-term asset. At $t = 2$, the payoff of the long-term assets are realized.

2.1 Environment

We begin by describing the preferences of sellers and buyers. Sellers have linear utility from consumption, c_2^s, at $t = 2$. Buyers' utility is $u\left(c_1^b\right)+c_2^b$, where c_1^b and c_2^b denote buyers' consumption at $t = 1$ and $t = 2$, respectively. The linearity of the buyers and sellers' utility functions at $t = 2$ allows us to abstract from risk-sharing consideration and derive stark results. We extend the analysis in Section 4.1 to a framework with a general utility function at $t = 2$.

At $t = 0$, the sellers have an endowment, e^s, and issue debt, d_0^s, and allocate their resources, $e^s + d_0^s$, to liquid and long-term assets, denoted by l_0^s and k_0^s, subject to the budget constraint

$$l_0^s + k_0^s \leq e^s + d_0^s. \tag{1}$$

We assume that the debt, d_0^s, is exogenously given by $d_0^s = d^s$, and we focus on the choices of $\{l_0^s,\ k_0^s\}$.[4] This allows us to take the size of the sellers' portfolio as given (i.e., $e^s + d^s$) and focus on whether the allocation of these resources to long-term and liquid assets is efficient or the sellers' liquidity holdings should be regulated. Our analysis complements that of several other fire-sales

[4]Regarding d_0^s, one can assume that there is a mass of external agents that may deposit their endowments with the sellers. Assuming the external agents are risk neutral and that they can only deposit with the sellers or use a storage technology, and that the sellers can make a take-it-or-leave-it offer, the sellers will offer a zero return on deposits, and d_0^s will be equal to the external agents' total endowment.

papers, which often focus on the inefficiencies that lead to overborrowing (Lorenzoni 2008; Stein 2012; Dávila and Korinek 2018; Kurlat 2021).

Buyers are born at $t = 1$ with an endowment of liquid assets only, similar to e.g. Stein (2012). We normalize such an endowment to one.

The liquid asset is standard; for each unit invested at time $t = 0$, there is one unit available at $t = 1$. The liquid asset technology is also available at $t = 1$, so that for each unit invested at $t = 1$, there is one unit available at $t = 2$.

The long-term asset works as follow. For each unit invested by sellers at $t = 0$, the asset produces no output at $t = 1$, and its productivity at $t = 2$ depends on two elements: (i) a quality shock realized at $t = 1$ and (ii) whether the asset is held, at $t = 2$, by sellers or buyers.

- *Quality shock:* At $t = 1$, a fraction $1 - \theta$ of the long-term assets held by sellers becomes of high quality, and a fraction θ becomes of low-quality. The fraction θ of low-quality assets is a random variable realized at $t = 1$, and we will focus our analysis on the case in which θ can take values $\{0, \bar{\theta}\}$, with $\bar{\theta} \in (0, 1)$. (We will describe the exact process for θ later.) While the realization of θ is common knowledge to all agents, sellers have private information, at $t = 1$, about the quality of the long-term assets they hold.

- *Sellers and buyers productivity at $t = 2$:* If a seller s enters $t = 2$ with high-quality long-term assets, they collect a payoff R from each unit of such assets. However, if a buyer b enters $t = 2$ with an amount k of high-quality long-term assets, they collect a payoff that depends on k. Specifically, the total output collected by the buyer is $f(k)$, where $f(\cdot)$ is a strictly increasing and concave function that satisfies $f(0) = 0$ and $f'(0) = R$. Thus, for $k > 0$, we have $f(k) \leq R\,k$.

At $t = 1$, the sellers have to repay a fraction γ of debt d_0^s, while the remaining fraction $1 - \gamma$ will be due at $t = 2$. We assume that γ is an aggregate shock that can take values $\gamma \in \{0, \bar{\gamma}\}$, with $\bar{\gamma} \in (0, 1)$. We refer to $\gamma = 0$ as the low-withdrawal state and $\gamma = \bar{\gamma}$ as the high-withdrawal state. We can thus interpret sellers as banks, money market mutual funds, or mutual funds that may experience withdrawals or outflows or, more generally, acute liquidity needs.

We assume that the realization of the withdrawal shock γ and the quality shock θ are correlated. This generates a link between the sellers' liquidity needs and the degree of adverse selection.

Specifically, at $t = 1$, there are two possible states:[5]

$$(\gamma, \theta) = \begin{cases} (0,0) & \text{with probability } 1-\pi, \\ (\bar{\gamma}, \bar{\theta}) & \text{with probability } \pi. \end{cases} \tag{2}$$

In equilibrium, a fire sale occurs at $t = 1$ in the latter state.

At $t = 1$, there is a centralized market in which the investors can trade the liquid and long-term assets. We denote q_1 as the price of the long-term asset and normalize the price of the liquid asset to one. We assume that short selling is not allowed, although the analysis can be extended without altering the logic of the results. For instance, we can allow for short selling that is subject to some costs or limits. We note that at $t = 1$, the buyers and sellers are able to adjust their portfolio holdings of the liquid and long-term assets by trading in the centralized market. However, at the economy-wide level, it is not possible to change the overall supply of the two assets at $t = 1$; the overall supply is given by $1 + l_0^s$ and k_0^s, respectively (i.e., the amounts that the buyers and sellers have at the beginning of $t = 1$; recall that buyers start $t = 1$ with one unit of the liquid asset).

We impose restrictions on three parameters. First, we assume that the probability π of the high-withdrawal state is sufficiently large,

$$\pi > \frac{(R-1)(1-\bar{\gamma}d^s)}{\bar{\gamma}d^s}, \tag{3}$$

which guarantees that the possibility of fire sales at $t = 1$ is not negligible and, thus, the sellers want to have positive holdings of the liquid assets at $t = 0$. Second, we assume both $\bar{\gamma}$ and e^s sufficiently large to ensure that the sellers' investments in liquidity and long-term assets at $t = 0$ are both strictly positive and their time-2 consumption is also strictly positive, which allows us to sidestep the potential issue of the sellers' default.

2.2 How the environment nests commonly used pricing mechanisms

The general framework described in Section 2.1 nests three pricing mechanisms commonly employed in the literature: cash-in-the-market pricing, second-best-use pricing, and asymmetric-information pricing. To obtain each pricing mechanism, one can impose some assumptions on

[5]When making their time-0 choices, the sellers know the probability distribution over the withdrawal shock γ and quality shock θ in (2).

three elements of the model: buyers' ability to extract cash flow from the long-term assets, that is, $f(\cdot)$; buyers' utility at $t = 1$, that is, $u(c_1^b)$; and the degree of asymmetric information in the high-withdrawal state, that is, $\bar{\theta}$. Specifically:

- Cash-in-the-market pricing: $f(k) = Rk,\ \ u(c_1^b) = \log c_1^b,\ \ \bar{\theta} = 0.$
- Second-best-use pricing: $f(k) < Rk,\ \ u(c_1^b) = 0,\ \ \bar{\theta} = 0.$
- Asymmetric-information pricing: $f(k) = Rk,\ \ u(c_1^b) = 0,\ \ \bar{\theta} > 0.$

First, the productivity of buyers, $f(\cdot)$, is the same as that of sellers under cash-in-the-market pricing and asymmetric-information pricing, but lower with second-best-use pricing. Second, buyers derive utility $u(c_1^b)$ from consumption at $t = 1$ under cash-in-the-market pricing, but no utility under the other pricing mechanisms. The utility from consumption under cash-in-the-market provides a reason for buyers to use liquidity at $t = 1$ other than purchasing assets sold by sellers, thereby limiting the amount of cash in the market and giving rise to a downward sloping demand even if buyers have the same technology and information as sellers. Third, there are no low-quality assets under cash-in-the-market and second-best-use pricing, and thus, no informational asymmetries in those cases. Only with asymmetric-information pricing, a fraction of the long-term assets $\bar{\theta} > 0$ is of low quality, and sellers' private information is relevant.

As we show in details in the next sections, the elements $f(\cdot)$, $u(\cdot)$, and $\bar{\theta}$ that affect the pricing mechanism affect only the problem of buyers and have no impact on sellers' choices. This allows us to show, in Section 2.7, that the equilibrium is observationally equivalent under the three pricing mechanism, when we impose some appropriate restrictions on $f(\cdot)$, $u(\cdot)$, and $\bar{\theta}$.

We observe that, with cash-in-the-market pricing, the environment is isomorphic to the one in Stein (2012). While buyers in Stein (2012) (who are labeled "patient investors") can use liquidity at $t = 1$ to invest in a productive project, we assume here that they can use liquidity to finance consumption expenditures. But both the investment in Stein (2012) and the consumption here represent possible uses of the buyers' time-1 endowments, and both generate benefits according to a strictly concave function (i.e., the time-1 utility function here and the production function in Stein (2012). Hence, the two environments are essentially the same.

2.3 Sellers' choices at $t = 1$

We now present the sellers' choices at time 1. Recall that: sellers enter $t = 1$ with l_0^s units of the liquid asset and k_0^s units of the long-term asset; a fraction θ of the long-term asset holdings are low quality (i.e., they will produce no output at $t = 0$); and sellers have private information about the quality of their long-term asset holdings.

When $\theta > 0$, sellers' private information gives rise to an adverse selection problem, and we consider a pooling equilibrium. Unlike the classic lemons problem (Akerlof, 1970), in which only lemons are traded and the equilibrium price is zero, here sellers will also sell some high-quality assets to meet their liquidity needs, resulting in a positive price for such assets.

To solve sellers' time-1 problem, we focus on the relevant case in which sellers sell all of their holdings of low-quality assets (if any), and the marginal asset that they trade is a high-quality one. Let k_1^s denote the holdings of the high-quality assets that sellers retain. Their problem is to maximize consumption c_2^s at $t = 2$, which is the sum of the cash flow $R\,k_1^s$ from the amount k_1^s of retained high-quality long-term asset, and from their liquidity holdings chosen at, l_1^s, minus the repayment $(1 - \gamma)d^s$ owed to the debt holders at $t = 2$:

$$\max_{k_1^s, l_1^s} \; R\,k_1^s + l_1^s - (1 - \bar{\gamma})d^s$$

subject to $l_1^s \geq 0$ and the budget constraint

$$l_1^s + \bar{\gamma}d^s \leq l_0^s + q_1\theta k_0^s + q_1\left[(1 - \theta)k_0^s - k_1^s\right]. \tag{4}$$

The budget constraint (4) says that the sellers finance their holdings l_1^s of liquidity and their withdrawals γd^s by using the liquidity l_0^s carried from $t = 0$, selling their holdings of low-quality long-term assets θk_0^s at price q_1, and selling an amount $(1 - \theta)k_0^s - k_1^s$ of their high-quality long-term assets, also at price q_1.

We restrict our attention to the relevant equilibrium cases in which $q_1 \leq R$.[6] If $q_1 = R$, the liquid and long-term assets have the same returns, so the sellers are indifferent between the two. The outcome $q_1 = R$ will arise in the low-withdrawal state (i.e., when $\gamma = 0$ and $\theta = 0$), and without loss of generality, we will focus on the case in which the sellers do not engage in any trade,

[6] If $q_1 > R$, then the expected return of the long-term asset is negative, but because the return of the liquid asset is zero, no agent would invest in the long-term asset. This cannot be an equilibrium because the market-clearing condition for the long-term asset would not hold.

so that their holdings will be $l_1^s = l_0^s$ and $k_1^s = k_0^s$.

When $q_1 < R$, the high-quality long-term asset has a higher return than the liquid asset. The outcome $q_1 < R$ will arise when a fire sale occurs, that is, in the high-withdrawal state (i.e., $\gamma = \bar{\gamma}$ and $\theta = \bar{\theta}$). Sellers will use all their liquidity and sell all their low-quality assets, if any, to pay withdrawals, but will also be forced to sell some of their high-quality assets. Any wealth left after repaying the time-1 withdrawals will be invested only in the long-term assets, which have higher return than liquidity. That is, $k_1^s > 0$ and $l_1^s = 0$. Specifically, the amount of high-quality assets they retain, k_1^s, is residually determined by the budget constraint (4) and given by

$$k_1^s = \frac{q_1 k_0^s - (\gamma d^s - l_0^s)}{q_1}. \tag{5}$$

Note that this does not depend on the quality shock θ that gives rise to the asymmetric information friction.

2.4 Buyers' choices at $t = 1$

We now turn to the buyers' problem at time 1. We first state the buyers' problem in the general model, but we then analyze buyers' optimal choices separately under each pricing mechanisms. The buyers' problem varies across different pricing mechanisms, as the assumptions about buyers' preferences, technology, and information differ, and focusing on each pricing mechanism separately simplifies the exposition.

The buyers choose their holdings of the liquid and long-term assets l_1^b and k_1^b purchased at $t = 1$, and their consumption at $t = 1$ and $t = 2$, to solve the following problem:

$$\max_{l_1^b,\, k_1^b, c_1^b, c_2^b} \; u\left(c_1^b\right) + \mathbb{E}_1^b\{c_2^b\}, \tag{6}$$

subject to non-negativity constraints and to the budget costraint

$$c_1^b + l_1^b + q_1 k_1^b \leq 1. \tag{7}$$

Note that the resources available to the buyers (i.e., the right-hand side of (7)) are equal to one because the buyers enter $t = 1$ with a unit of the liquid asset and no holdings of the long-term asset; see Section 2.1.

The time-2 consumption is given by

$$\mathbb{E}_1^b\{c_2^b\} = l_1^b + \mathbb{E}_1^b\{f(\alpha\, k_1^b)\}, \tag{8}$$

and the term $\mathbb{E}_1^b$ denote the buyers' expectations over the fraction $\alpha \in [0,1]$ of high-quality assets that are traded in the market. That is, in general, only a fraction α of the k_1^b units purchased at $t = 1$ are high-quality assets—and the cash flow collected by the sellers on these assets is $f(\alpha\, k_1^b)$—and the remaining $1 - \alpha$ fraction produces zero. And when the asymmetric information problem is relevant (i.e., when $\theta = \bar{\theta}$ and $\bar{\theta} > 0$), buyers do not observe α and, thus, need to form beliefs about that.

We begin by analyzing the optimal choice of the buyers under cash-in-the-market pricing, that is, when $f(k) = Rk$, $u(c) = \log c$, and $\bar{\theta} = 0$. In this case, there is no uncertainty about the quality of the long-term asset, and thus, buyers' belief are simply given by the productivity of such assets: $\mathbb{E}^b\{f(\alpha\, k_1^b)\} = Rk_1^b$. The maximization in (6) implies the standard asset pricing condition

$$q_1 = \frac{1}{u'\left(c_1^b\right)} \times R, \tag{9}$$

where ${}^1\!/_{u'(c_1^b)}$ is the ratio of the marginal utility at $t = 2$ (i.e., one) and the marginal utility at $t = 1$ (i.e., $u'\left(c_1^b\right)$). Note that the time-1 consumption choice satisfies $u'\left(c_1^b\right) \geq 1$ because the buyers will never choose to consume more than one unit at $t = 1$. This result arises because the $u'(1) = 1$, since $u(c) = \log c$, and because the buyers can carry resources to $t = 2$ using the storage technology, which are then valued according to the linear utility at $t = 2$. Focusing again on the relevant case in which $q_1 \leq R$, and using $u(c) = \log c$, the buyers' optimal choices are

$$\left\{c_1^b, l_1^b, k_1^b\right\} = \begin{cases} \{1, 0, 0\} & \text{if } q_1 = R \\ \left\{\frac{q_1}{R}, 0, \frac{1}{q_1} - \frac{1}{R}\right\} & \text{if } q_1 < R \end{cases} \tag{10}$$

To preview some of the results, we note that in the low-withdrawal state $\gamma = 0$ (i.e., when no fire sales occur), the buyers consume $c_1^b = 1$ so that their marginal utility is $u'(c_1^b) = 1$, resulting in a time-1 price of $q_1 = R$ for the long-term asset. Hence, q_1 is equal to the cash flow that the asset produces at $t = 2$. In contrast, in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when a fire sale occurs), the buyers consume $c_1^b < 1$, so that their marginal utility is $u'\left(c_1^b\right) > 1$. Hence, the time-1 price of the long-term asset is $q_1 < R$, and thus, lower than the cash flow R.

Next, we turn to second-best-use pricing, that is, $f(k) < Rk$, $u(c_1^b) = 0$, and $\bar{\theta} = 0$. Similar to the case with cash-in-the-market pricing, there is no asymmetric information in equilibrium, and thus, buyers' belief are $\mathbb{E}_1^b\{f(\alpha\, k_1^b)\} = f(k_1^b)$. The maximization in (6) now imply

$$q_1 = f'(k_1^b) \leq R. \tag{11}$$

In particular, $q_1 < R$ when $k_1^b > 0$ because of the assumptions about $f(\cdot)$ introduced in Section 2.1. That is, the buyers are willing to purchase long-term assets at a low price because they are able to collect a lower cash flow than the sellers. Buyers' liquidity holdings, l_1^b, are residually determined from the budget constraint.

Finally, with asymmetric-information pricing, $f(k) = Rk$, $u(c_1^b) = 0$, and $\bar{\theta} > 0$. Thus, $\mathbb{E}_1^b\left\{f(\alpha\, k_1^b)\right\} = \mathbb{E}_1^b\{\alpha\} R\, k_1^b$, and the buyers' first-order condition is

$$q_1 = \mathbb{E}^b\left\{\alpha\right\} R. \tag{12}$$

Thus, buyers are willing to purchase any amount, provided that the price equals their belief about the output produced by the average asset traded. When deriving the equilibrium, we assume that buyers' beliefs are rational, and thus, they form their expectation $\mathbb{E}_1^b\left\{\alpha\right\}$ based on the ratio of high-quality assets sold by sellers:

$$\mathbb{E}_1^b\left\{\alpha\right\} = \frac{(1-\theta)\, k_0^s - k_1^s}{k_0^s - k_1^s}. \tag{13}$$

That is, the total amount of assets sold by the sellers is $k_0^s - k_1^s$, but only $(1-\theta)\, k_0^s - k_1^s$ are high-quality ones. Using (13), the buyers' first-oder condition (12) becomes

$$q_1 = \frac{R\left[(1-\theta)\, k_0^s - k_1^s\right]}{k_0^s - k_1^s} \leq R. \tag{14}$$

Finally, similar to the case with cash-in-the-market pricing, l_1^b is residually determined from the budget constraint.

2.5 Sellers' choices at $t = 0$

We now turn to the analysis at $t = 0$, when the sellers decide how to allocate their resources between the liquid and long-term assets. Then, in Section 3, we ask whether the sellers' choices at $t = 0$ are efficient and whether liquidity regulation interventions can improve the equilibrium outcome. Recall that buyers are born at $t = 1$, and thus, the time-0 analysis involves only sellers.

To determine the sellers' portfolio choices at $t = 0$, we proceed along the lines of Dávila and Korinek (2018) and derive the sellers' time-0 choices that maximize their time-1 indirect utility function at $t = 1$. This approach is very convenient because the analysis is independent of many features of the model, and it will make the comparison with the regulator's problem and solutions very transparent.

The sellers' indirect utility function at $t = 1$ is

$$V_1^s\left(l_0^s, k_0^s\right) = c_2^s + \lambda_1^s\left[l_0^s + q_1 k_0^s - \left(l_1^s + q_1 k_1^s + \gamma\, d^s\right)\right] + \mu_1^s l_1^s. \tag{15}$$

The first term on the right-hand side is the sellers' time-2 utility, which is linear in consumption, c_2^s. The second term is the Lagrange multiplier λ_1^s of the sellers' time-1 budget constraint (equation (4)) times the budget constraint itself. The last term is the Lagrange multiplier μ_1^s of the non-negative constraint on liquidity holdings, times such holdings, l_1^s. (The term $\mu_1^s l_1^s$ in (15) does not affect the analysis, but we include it because the non-negativity constraint $l_1^s \geq 0$ is binding in some cases in equilibrium.)

At $t = 0$, the sellers choose liquidity l_0^s and long-term asset holdings k_0^s to maximize their expected indirect utility function

$$\max_{l_0^s, k_0^s} \mathbb{E}_0\left\{V_1^s\left(l_0^s, k_0^s\right)\right\}, \tag{16}$$

subject to the budget constraint (1) and to non-negativity constraints on l_0^s and k_0^s. The problem in (16) is easy to analyze because we can exploit the envelope theorem to obtain

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}. \tag{17}$$

Recall that λ_1^s is the Lagrange multiplier of the sellers' budget constraint at $t = 1$ and, thus, it represents the sellers' marginal value of wealth. Equation (17) states that the sellers choose their time-0 portfolio so that the time-1 marginal value of holding one additional unit of the long-term asset, represented by the left-hand side, is equal to the time-1 marginal value of holding one additional unit of liquidity, on the right-hand side. That is, a marginal dollar of investments at $t = 0$ could be used to invest in the long-term asset or in liquidity, which have market values of q_1 and one at $t = 1$, respectively, and which the sellers value according to their time-1 marginal utility of wealth λ_1^s.

The marginal utility of the sellers' wealth, λ_1^s, (and the equivalent object for the buyers, λ_1^b) is a crucial object for our analysis, and plays a key role in the efficiency and policy analysis of Section 3. Because λ_1^s is formally defined as the Lagrange multiplier of (4), the analysis in Section 2.3 implies

$$\lambda_1^s = \frac{R}{q_1}. \tag{18}$$

That is, a marginal unit of wealth available to sellers at $t = 1$ can be used to purchase $1/q_1$ units of the long-term assets. Each unit of the asset will then produce a payoff R, which is evaluated according to the linear marginal utility of wealth. Note that λ_1^s corresponds to the "marginal value of a dollar", as the cash flow collected from the marginal unit traded is given by R and the marginal utility of the sellers' consumption at $t = 2$ is one.

2.6 Equilibrium definition

An equilibrium is a collection of the sellers' portfolio choice at $t = 0$ (i.e., $\{l_0^s, k_0^s\}$); and given a realization of the shocks $(\gamma, \theta) \in \big\{(0,0), (\overline{\gamma}, \overline{\theta})\big\}$, the sellers' and buyers' portfolio choices at $t = 1$ (i.e., $\{l_1^s, k_1^s\}$ and $\{l_1^b, k_1^b\}$); the buyers' beliefs about the fraction α of high-quality assets traded at $t = 1$; the buyers' consumption choices at $t = 1$ and $t = 2$ (i.e., c_1^b and c_2^b); the sellers' consumption choices at $t = 2$ (i.e., c_2^s), and a time-1 price for the long-term asset (i.e., q_1), such that the buyers and sellers maximize their utilities, the buyers' beliefs are rational, and the time-1 market clears. Specifically, the market-clearing condition for liquidity at $t = 1$ is

$$c_1^b + l_1^b + l_1^s + \gamma d_0^s = 1 + l_0^s, \tag{19}$$

where the right-hand side uses the assumption that the buyers are endowed with one unit of liquidity (see Section 2.1). That is, the liquid asset available in the economy, $1 + l_0^s$, is allocated between the buyers' consumption, c_1^b, their liquidity holdings, l_1^b, and the sellers' liquidity holdings, l_1^s, carried to $t = 2$, and the resources, γd_0^s, that are used to repay the sellers' debt holders at $t = 1$. The other market-clearing condition—for the long-term asset—holds by Walras' law, but we also state it for completeness:

$$k_1^b + k_1^s = k_0^s, \tag{20}$$

where the right-hand side uses the assumption that the buyers have no endowment of the long-term assets.

2.7 Equilibrium and equivalence under the three pricing mechanisms

We now characterize the equilibrium. Because the sellers' problem is independent of the pricing mechanisms (i.e., independent of the microfoundation of the buyers' demand), there are several features of the equilibrium that are independent of the pricing mechanism, and thus, arise under any pricing mechanism.

Observe that for the low-withdrawal state $\gamma = 0$, for all three models, the sellers do not need to sell any debt and the market-clearing condition in (19) implies that all of the liquidity remains in the hands of the buyers. No fire sales arise when $\gamma = 0$ as there is no trade, and the price $q_1 = R$ in such an equilibrium.[7] Given this, we can use equations (17) and (18) to pin down the value of q_1 in the high-withdrawal state (i.e., in the fire-sale state), which we denote by $q_1(\bar{\gamma}, \bar{\theta})$:

$$q_1(\bar{\gamma}, \bar{\theta}) = R\,\frac{\pi}{(R-1)+\pi} < 1, \tag{21}$$

where the inequality follows from $R > 1$. Note that (17) pins down the excess return of the long-term asset relative to the short-term one, and given $q_1 = R$ in the low-withdrawal state, we can easily solve for q_1 in the high-withdrawal state because sellers' marginal utility λ_1^s is independent of sellers' level of time-2 consumption—thanks to sellers' linear utility. Therefore, the fire-sale price is given by (21) *in any equilibrium*.[8]

In addition, because the sellers' problem at $t = 1$ is independent of the buyers' building-block of the model, the trading volume will also be the same independently of the microfoundation that drives the fire-sale price that buyers are willing to pay. In the low-withdrawal state, trading volume is zero. In the high-withdrawal state, we can use the sellers' budget constraint (5) to compute the trading volume, which is equal to $k_0^s - k_1^s$, or, given l_0^s, $\frac{(\gamma d^s - l_0^s)(R-1+\pi)}{\pi R}$.

[7]The idea is that when $\gamma = 0$, $q_1 = R$ supports the no-trade equilibrium, and any $q_1 < R$ or $q_1 > R$ cannot clear the market.

[8]More precisely, (21) holds in any equilibrium in which the time-0 choices of sellers are not constrained by $l_0^s \geq 0$ and $k_0^s \geq 0$.

Based on the above results, the next proposition offers a simple characterization of the equilibrium, showing that the model generates a fire sale in the low withdrawal state—not only when a pricing mechanism operates in isolation, but also when multiple pricing mechanisms play an active role in equilibrium.

Proposition 2.1. ***(Equilibrium)*** *If the time-0 non-negativity constraints of sellers $l_0^s \geq 0$ and $k_0^s \geq 0$ are not binding, the equilibrium can be characterized by the following.*

- *In the low withdrawal state, the price $q_1 = R$; the trading volume is 0; the sellers' time-1 portfolio choice is $k_1^s = k_0^s$ and $l_1^s = l_0^s$; the buyers' time-1 choices is $k_1^b = 0$ and $l_1^b + c_1^b = 1$.*
- *In the high-withdrawal state, the price $q_1 = R\,\frac{\pi}{(R-1)+\pi}$; the trading volume is $\frac{(\gamma d^s - l_0^s)(R-1+\pi)}{\pi R}$; the sellers' time-1 portfolio choice is $k_1^s < k_0^s$ and $l_1^s = 0$; the buyers' time-1 choices is $k_1^b > 0$ and $l_1^b + c_1^b < 1$.*

Note that, in Proposition 2.1, we focus on the combined term $l_1^b + c_1^b$ to provide a result that holds under any combination of pricing mechanisms. This is because, under cash-in-the-market pricing, buyers use liquidity for consumption, but with second-best-use and asymmetric-information pricing, buyers hold liquidity in their portfolios until $t = 2$ and there is no consumption at $t = 1$. Thus, the exact time-1 allocation of liquidity among l_1^b and c_1^b depends on the pricing mechanism(s).

Finally, we show that not only prices and trading volumes are identical across pricing mechanisms, but under certain parameter restrictions, the entire equilibrium is also the same. We establish this result through an equivalence proposition. That is, taking as given an equilibrium under cash-in-the-market pricing, we show that the equilibrium under second-best-use pricing or asymmetric-information pricing is the same provided that some parameter restrictions hold. This result establishes that the equilibrium under the three pricing mechanisms is observationally equivalent. Hence, if we look at a given episodes of fire sales in practice through the lenses of the model, we cannot identify the pricing mechanism without knowing the microfoundation of buyers' demand. The proposition shows that not only equilibrium variables but also the demand elasticity is the same under the three mechanisms.

Proposition 2.2. ***(Equivalence of the three models.)*** *Consider the equilibrium of the model with cash-in-the-market pricing (i.e., $f(k) = Rk$, $u(c) = \log c$, and $\bar{\theta} = 0$). Then:*

- *In the version of the model with second-best use pricing (that is, $f(k) < Rk$, $u(c) = \bar{\theta} = 0$), if $f'\left(\frac{R-1}{\pi R}\right) = \frac{\pi R}{R-1+\pi}$, the equilibrium has the same sellers' portfolio choices at $t = 0$*

and $t = 1$ as in the cash-in-the-market model, the same buyers' holdings of the long-term asset at $t = 1$ as well as the combined liquidity holdings and consumption $l_1^b + c_1^b$ at $t = 1$, the same price and trading volume at $t = 1$ in all states. Additionally, if f satisfies $f''(f^{-1}(\frac{\pi R}{R-1+\pi}))f^{-1}(\frac{\pi R}{R-1+\pi}) = -\frac{\pi R(R-1)}{(R-1+\pi)^2}$, the buyers' time-1 demand elasticity will also match that of the cash-in-the-market model.

- *In the version of the model with asymmetric information pricing (that is, $f(k) = Rk$, $u(c_1^b) = 0$, and $\bar{\theta} > 0$), if $\bar{\theta} = \bar{\gamma}d^s - \frac{(R\pi e^s-(R-1))(R-1)}{(R-1+\pi)R\pi e^s}$, the equilibrium has the same sellers' portfolio choices at $t = 0$ and $t = 1$ as in the cash-in-the-market model, the same buyers' holdings of the long-term asset at $t = 1$ as well as the combined liquidity holdings and consumption $l_1^b + c_1^b$ at $t = 1$, and the same price and trading volume at $t = 1$ in all states. Furthermore, if $R^2\pi^2 e^s = (R-1)^2$, the buyers' time-1 demand elasticity will match that of the cash-in-the-market model.*

3 Efficiency and policy analysis

We now study whether the equilibrium is efficient; that is, whether the equilibrium allocation—and, in particular, the sellers' time-0 portfolio choice—corresponds to that of a planner or regulator (hereinafter simply referred to as the "regulator"). Under the assumption that the buyers and sellers have linear utility at $t = 2$, we show in Section 3.2 that the equilibrium with the cash-in-the-market pricing is efficient and, thus, no liquidity regulation should be imposed on the sellers' time-0 choices. In Sections 3.3 and 3.4, we show that the equilibrium is, instead, inefficient under second-best-use and the asymmetric-information pricing, requiring liquidity regulation in those cases. Crucially, the optimal regulation is a liquidity requirement under second-best-use pricing but a liquidity ceiling under asymmetric-information pricing.

We use a standard approach employed in the fire-sale literature. Various papers, such as Lorenzoni (2008), Dávila and Korinek (2018), and Kurlat (2021), consider a regulator that makes the initial portfolio choices at $t = 0$ but has no influence on the trading and choices that occur in the subsequent time periods (i.e., at $t = 1$ and $t = 2$). The regulator internalizes the effects of the time-0 portfolio choices on the time-1 price, q_1, which is different from the individual sellers that take the time-1 price, q_1, as given. By following the same approach, our results are easily comparable with the literature. In addition, this approach has a good fit with the analysis of the actual liquidity requirements that are imposed, in practice, before the possible realization of fire sales.

To define efficiency, we rely on the concept of Pareto optimality because our model—like several others in the fire-sale literature—has two sets of agents (i.e., buyers and sellers). Thus, an equilibrium is constrained efficient if no regulatory intervention at $t = 0$ can improve the welfare of the buyers (keeping sellers' welfare unchanged) , the welfare sellers (keeping buyers' welfare unchanged), or both.

3.1 Regulator's problem and first-order conditions

We consider the problem of a regulator aiming to maximize the sellers' welfare while ensuring that the buyers' welfare is at least as high as it would be in the unregulated equilibrium. At $t = 0$, the regulator chooses their investments in the sellers' liquidity and long-term assets, l_0^s and k_0^s, that will maximize the sellers' utility. In addition, the regulator chooses a transfer, T, from the seller to the buyers to make sure that the buyers achieve the same level of utility as that in the unregulated equilibrium. Because the buyers are born at $t = 1$, we assume that the transfer from the sellers to the buyers involves an amount T of the liquid asset.[9] Thus, the sellers will enter $t = 1$ with liquidity $l_0^s - T$ and the buyers with liquidity $1 + T$. The regulator's problem is

$$\max_{l_0^s, k_0^s, T} \mathbb{E}_0 \left\{ V_1^s \left(l_0^s - T, k_0^s; \, q_1 \right) \right\} \tag{22}$$

where $V_1^s(\cdot)$ is the sellers' indirect utility functions, defined in (15), in which we have highlighted the sellers' dependence on the price, q_1. The maximization is subject to the sellers' time-0 budget constraint, (1) evaluated at $d_0^s = d^s$,

$$l_0^s + k_0^s \leq e^s + d^s \tag{23}$$

and to the constraint that the buyers' time-1 indirect utility $V_1^b(T; \, q_1)$ should be no less than the level $\overline{V}$ they achieved in the unregulated equilibrium:

$$V_1^b(T; q_1) \geq \overline{V}. \tag{24}$$

[9] As in the literature, the transfer cannot be contingent on the state of the economy at $t = 1$, otherwise it would violate the assumption that the regulator can affect only the time-0 choices.

Specifically, the buyers' time-1 indirect utility is defined analogously to that of the sellers:[10]

$$V_1^b\left(T; q_1\right) = u(c_1^b) + c_2^b + \lambda_1^b\left[1 + T - \left(l_1^b + q_1 k_1^b + c_1^b\right)\right] + \mu_1^b l_1^b. \tag{25}$$

The term λ_1^b is the Lagrange multiplier of the buyers' time-1 budget constraint and, thus, represents the buyers' marginal utility of wealth. The term μ_1^b is the Lagrange multiplier on the non-negativity constraint $l_1^b \geq 0$.

Next, we derive the regulator's first-order conditions. The key difference, compared to the sellers' individual problem analyzed in Section 2.5, is that the regulator accounts for the effects of its choices on the price q_1 of the long-term asset at $t = 1$.

Denoting ξ as the Lagrange multiplier of the buyers' utility constraint (24), the regulator's first-order conditions for the choice of the sellers' holdings of liquidity, l_0^s, and long-term assets, k_0^s, imply:

$$\mathbb{E}_0\left\{\lambda_1^s q_1\right\} = \mathbb{E}_0\left\{\lambda_1^s + \frac{\partial q_1}{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)\right\}. \tag{26}$$

The first-order condition for the choice of transfers, T, is

$$\mathbb{E}_0\left\{\frac{\partial q_1}{\partial T}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right) + \xi\lambda_1^b\right\} = \mathbb{E}_0\left\{\lambda_1^s\right\}. \tag{27}$$

The regulator's optimality condition (26) that pins down the optimal choice of the liquidity and long-term assets differs from that of the individual sellers in (17) because the regulator internalizes the effects of its choices on the time-1 price, q_1, as noted before. The difference between (17) and (26) can introduce a wedge between the regulator's choices and those of the private agents, which is captured by the second term on the right-hand side of (26). This wedge is affected by three elements, which are similar to those identified by Dávila and Korinek (2018) in regard to what they refer to as distributive externalities (i.e., externalities due to incomplete markets):

1. The sensitivity of the time-1 price q_1 with respect to the sellers' $t = 0$ choice of liquidity holdings, l_0^s, that is, ${}^{\partial q_1}/_{\partial l_0^s}$;

2. The sellers' purchases of the long-term assets at $t = 1$, $k_1^s - k_0^s$ (or sales, if negative);

3. The difference between the buyers' marginal utility of wealth, λ_1^b, and that of the sellers, λ_1^s, adjusted by the Lagrange multiplier ξ: $\xi\lambda_1^b - \lambda_1^s$.

[10] Recall from Section 2.1 that the buyers have no holdings of the long-term asset at the beginning of $t = 1$.

Liquidity requirements are optimal when the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)\}$ in (26), evaluated at the unregulated equilibrium, is positive. This is because the right-hand side of (26) represents the regulator's marginal value of investing in liquidity at $t = 0$. Hence, a positive value for the term $\mathbb{E}\frac{\partial q_1}{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)\}$ means that, at the unregulated equilibrium, the regulator's value of investing in liquidity exceeds that of the private agents.

3.2 Efficiency and liquidity requirements with cash-in-the-market pricing

We now characterize the efficiency under cash-in-the market pricing. The key result is that the equilibrium is efficient, and thus, no liquidity regulation is needed.

Under cash-in-the-market pricing, and given the key assumption that sellers and buyers have linear utility at $t = 2$, the next proposition shows that the term $\mathbb{E}\{\frac{\partial q_1}{\partial l_0^s}\left(k_1^s - k_0^s\right)\left(\xi\lambda_1^b - \lambda_1^s\right)\}$ in the regulator's first-order conditions (26) is zero. Thus, the planner's first-order condition (26) coincides with that of the private agents in (17), and the equilibrium is efficient. As a result, no liquidity regulation is required with cash-in-the-market pricing when all of the agents have linear utility at $t = 2$.[11]

Proposition 3.1. ***(Efficiency in the cash-in-the-market model)*** *The unregulated equilibrium is constrained efficient.*

In this model with cash-in-the-market pricing and linear utility of consumption at $t = 2$, λ_1^s and λ_1^b are equalized,

$$\lambda_1^s = \lambda_1^b = \frac{R}{q_1}, \tag{28}$$

because the buyers and sellers both collect the same cash flow, R, from any unit traded—including the marginal units—and they both have constant linear utility at $t = 1$. The linear utility at $t = 2$ also prevents any wealth effect that could arise from the planner's transfers, T. This implies that the time-1 price, q_1, is unresponsive to the transfers, T, and, thus, the term ${\partial q_1}/{\partial T}$ in the regulator's first-order condition (27) is zero as well. All of these results together imply that the Lagrange multiplier ξ of the regulator's constraint (24) is equal to one. That is, the sellers and buyers are effectively "symmetric"—not just at the unregulated equilibrium but also as we change the sellers and buyers'

[11] Dávila and Korinek (2018) show that when markets between $t = 0$ and $t = 1$ are complete, the equilibrium is efficient. Our model has two assets at $t = 0$ (i.e., long-term asset and liquidity) and two states at $t = 1$ (i.e., two possible realizations of γ), but the markets are not complete here because the buyers cannot invest in the long-term asset at $t = 0$. Hence, the efficiency result in Proposition 3.1 arises *despite* market incompleteness.

wealth, using the transfers, T. In other words, the realization of the high-withdrawal shock $\gamma = \bar{\gamma}$ that triggers a fire sale simply entail a redistribution from the sellers to the buyers and create no inefficiencies, relative to the allocation that arises under the low-withdrawal shock $\gamma = 0$. Formally, under $\xi = 1$ and $\lambda_1^s = \lambda_1^b$, the first-order condition (26) simplifies to $\mathbb{E}_0 \{\lambda_1^s q_1\} = \mathbb{E}_0 \{\lambda_1^s\}$ and is, thus, identical to that of the individual sellers, that is, to equation (17).

3.3 Inefficiency and liquidity requirements with second-best-use pricing

We now analyze efficiency and policy interventions under second-best-use pricing. The equilibrium is now inefficient, and the optimal regulation is a liquidity requirement. That is, the regulator should force sellers to hold more liquidity, relative to the unregulated equilibrium.

The problem of the regulator and the first-order conditions described in Section 3.1 continue to apply. The key difference relative to the analysis of efficiency and policy under cash-in-the-market pricing (in Section 3.2) is in the buyers' marginal utilities of wealth. In all states at $t = 1$, the buyers' marginal utility of wealth λ_1^b is now given by

$$\lambda_1^b = 1 \tag{29}$$

and, thus, is independent of the price, q_1, of the long-term asset—compare (29) with the expression for the cash-in-the-market model in (28). Equation (29) follows from the fact that λ_1^b is the Lagrange multiplier of the buyers' time-1 budget constraint (7). In contrast, the sellers' marginal utility of wealth, λ_1^s, is the same as in the baseline model because their problem is the same. That is, λ_1^s is still given by (18), namely, $\lambda_1^s = {}^{R}\!/_{q_1}$.

Comparing the sellers and buyers' marginal utilities in (18) and (29) shows that the two sets of agents have the same marginal utility of wealth in the low-withdrawal state $\gamma = 0$ (i.e., when $q_1 = R$) but different marginal utilities in the high-withdrawal state $\gamma = \bar{\gamma}$ (i.e., when $q_1 < R$ and a fire sale occurs). That is, a gap between the two marginal utilities opens up when a fire sale occurs; specifically, $\lambda_1^s > \lambda_1^b$.

The gap that opens up between the buyers' and sellers' marginal utilities of wealth is due to the buyers' lower ability to extract cash flow from the marginal unit traded. Because of this gap, when assets are transferred to buyers in fire sales, the total output available in the economy at $t = 2$ is lower, compared to the non-fire-sales state. Similar to Section 3.2, the linearity of the time-2 utility of both the buyers and the sellers implies that the considerations about risk sharing do not affect the policy analysis. These considerations will play a role when considering a more general time-2 utility in Section 4.

The planner can improve welfare by forcing the sellers to invest more in liquidity at $t = 0$. With more liquidity available at $t = 1$, each seller needs to sell fewer assets, resulting in a higher price, q_1, during a fire sale. The higher price, in turn, implies that the sellers need to sell even less, increasing the quantity of the long-term assets that remain in their hands and, thus, increasing the total output available in the economy at $t = 2$. The next proposition formalizes this result.

Proposition 3.2. ***(Inefficiency and liquidity requirements in the second-best-use model)*** *In the second-best-use model, the unregulated equilibrium is not constrained efficient and the sellers' time-0 liquidity holdings are lower than the socially optimal level.*

3.4 Inefficiency and liquidity ceiling in the asymmetric-information model

As a last step in the policy analysis, we focus on the asymmetric-information pricing mechanism. The equilibrium is inefficient, as in the case of second-best-use pricing. However, the optimal policy is a liquidity ceiling, as opposed to a liquidity requirement as with second-best-use pricing. The problem of the regulator and the first-order conditions described in Section 3.1 continue to apply.

As discussed in Sections 3.2, and 3.3, the efficiency is crucially affected by the sellers' and buyers' marginal utilities of wealth; that is, λ_1^s and λ_1^b. The sellers' marginal utility, λ_1^s, is again given by $\lambda_1^s = R/q_1$, as with cash-in-the-market and second-best-use pricing (see (18)). Here, the logic is that with an additional dollar available, the sellers can *avoid* selling an amount $1/q_1$ of *high-quality assets*, which are the assets with payoff R at $t = 2$. For the buyers, however, the marginal utility of wealth is $\lambda_s^b = 1$, independently of the realization of $\{\gamma, \theta\}$; this result follows from the buyers' problem in Section 2.4.

Why are the buyers and sellers' marginal utilities of wealth different, even if they have the same ability to collect cash flow at $t = 2$? As emphasized in the previous sections, it is important to understand the cash flow produced by the *marginal* unit that is traded. For the sellers, this marginal unit consists of high-quality assets, because all of the low-quality holdings are sold as inframarginal units. But for the buyers, the marginal unit traded is the average asset in the market, whose cash flow is the average of high- and low-quality assets.

The gap between sellers' and buyers' marginal utilities generates an inefficiency, similar to the case with second-best use pricing analyzed in Section 3.3. There is, however, a key difference. With second-best-use pricing, the gap in marginal utilities is directly related to the buyers' lower ability to collect cash flow from long-term assets, at $t = 2$, which reduces aggregate efficiency in a fire sale (i.e., total consumption at $t = 2$), relative to the case without fire sale. With asymmetric information, and conditional on the time-0 portfolio of sellers, total consumption at $t = 2$ is not affected by a fire sale because the buyers and sellers are able to collect the same cash flow. However, the gap between the marginal utilities of buyers and sellers affect the time-0 portfolio allocation of sellers. Specifically, the sellers invest too little in the (highly productive) long-term asset, anticipating that they will have to sell the marginal high-quality asset at an adverse-selection discount during a fire sale. This intuition is similar to the one discussed by Kurlat (2021) but, here, it affects the composition of the investments between long-term and liquid assets, as opposed to the total size of the portfolio, as in Kurlat (2021). Relative to Kurlat (2021), we also note that the inefficiency shows up as gap between the marginal utilities of wealth of buyers and sellers—a key object that Dávila and Korinek (2018) use to characterize the optimal policy intervention.

The optimal policy intervention supports the time-1 asset price to induce higher investments in the long-term asset at $t = 0$. While optimal regulation aims to also generate higher asset prices in the second-best-use model, it is achieved here by forcing the sellers to invest *less* in liquidity. This regulation is the opposite to that in the second-best-use model. With asymmetric information, reducing liquidity results in a higher time-1 price, q_1, during a fire sale, due to the same logic discussed in Malherbe (2014). That is, if the sellers hold less liquidity, a larger fraction of the assets traded are sold to meet their liquidity needs and, thus, consists of high-quality assets. Consequently, the share of lemons in the market is lower, mitigating the adverse-selection problem. Formally, with adverse-selection pricing, the term $\partial q_1/\partial l_0^s$ in the regulator's first-order condition (17) is negative, in contrast to the positive term found with cash-in-the-market and second-best-use pricing in Sections 3.2 and 3.3.

Proposition 3.3. ***(Inefficiency and liquidity ceiling in the asymmetric-information model)*** *In the asymmetric-information model, the unregulated equilibrium is not constrained efficient. Specifically, the sellers' time-0 liquidity holdings exceed the socially optimal level.*

4 General time-2 utility and risk-sharing considerations

The models of Sections 2-3 yield a simple and stark result: Three observationally equivalent pricing mechanisms that are commonly used to study fire sales have very different implications regarding the liquidity requirements that should be imposed on financial intermediaries. With cash-in-the-market pricing, the equilibrium is efficient and no regulation is needed. With a second-best-use assumption, a liquidity requirement is optimal. And with asymmetric information, the opposite regulation (i.e., a ceiling on liquidity) is optimal.

This section extends the framework used in Sections 2-3 by incorporating the impact of market incompleteness on the investors' inability to fully share risk in financial markets. To this end, we relax the assumption that the investors have linear utility at $t = 2$ and instead allow for an arbitrary concave utility function. Under this assumption, the forces discussed in Sections 2-3 continue to operate. However, the optimal policy stance is also influenced by the possibility that the unregulated equilibrium may not achieve full risk sharing.

The equilibrium in the general-utility model is generically inefficient in the cash-in-the-market model, and the optimal policy could be either a liquidity requirement or a liquidity ceiling, depending on the exact parameterization. However, if a liquidity requirement is optimal in the cash-in-the-market model, we show that it must be tighter in an observationally equivalent second-best-use model. And if a liquidity ceiling is optimal in the cash-in-the-market model, we show that the optimal regulation in the second-best-use model is either a lower ceiling or a liquidity requirement. The optimal regulation in the asymmetric-information model remains a liquidity ceiling, under regularity conditions.

4.1 Model with general time-2 utility

We assume that the sellers' time-2 utility from consuming c_2^s is $u_2^s(c_2^s)$ and the buyers' time-2 utility from consuming c_2^b is $u_2^b(c_2^b)$, where $u_2^s(\cdot)$ and $u_2^b(\cdot)$ are strictly increasing and weakly concave functions and at least one of them is strictly concave. For the cash-in-the-market model, we relabel the time-1 utility function as $u_1^b(\cdot)$ to avoid confusion. All the other features of the environment described in Section 2.1 are unchanged.

We derive the policy analysis (in the next section) under the assumption that there exists an equilibrium with the same features as in the baseline (i.e., as in Proposition 2.7): an interior portfolio choice for liquidity and long-term asset holdings of sellers at $t = 0$, no trading at $t = 1$ in the low-withdrawal state $\gamma = 0$, a positive trading volume and a fire sale at $t = 1$ in the high-withdrawal state $\gamma = \bar{\gamma}$, and a buyers' demand that is downward sloping in the trading volume.[12] The remainder of this section provides some remarks to show that the environment can generate an equilibrium with these features.

Without loss of generality, we normalize the sellers' time-2 utility function so that their time-1 marginal utility of wealth $\lambda_1^s(\gamma)$ is one in the low-withdrawal state. That is, when $\gamma = 0$ (i.e., when no fire sales occur), $\lambda_1^s(0) = 1$.

In addition, to generate the same observationally equivalent equilibrium under the three pricing mechanisms, the time-2 utility function of the buyers needs to be different. This is because the buyers consume at $t = 1$ and $t = 2$ in the cash-in-the-market model but only at $t = 2$ in the other two models. We normalize $u_2^b(\cdot)$ *under each pricing mechanism* so that the buyers' marginal utility is one at the equilibrium with low withdrawals (i.e., with no fire sales). That is, $\partial u_2^b(0)/\partial c_b^2 = 1$ with cash-in-the-market pricing and $\partial u_2^b(1)/\partial c_b^2 = 1$ with the second-best-use and asymmetric-information pricing.

Remark #1: Sellers and buyers' choices. Because the sellers' utility depends only on their time-2 consumption and because there is no uncertainty between $t = 1$ and $t = 2$, their objective at $t = 1$ is unchanged (i.e., maximizing time-2 consumption) and their choices are, thus, the same as in the baseline analyses of Sections 2-3. For the buyers, the time-1 first-order condition (9) under cash-in-the-market pricing is replaced by

$$q_1 = \frac{(u_2^b)'(c_2^b)}{(u_1^b)'(c_1^b)} R. \tag{30}$$

Under second-best-use and asymmetric-information pricing, because the buyers' utility depends only on time-2 consumption, the first-order conditions (11) and (14) are unchanged—the logic is identical to that discussed for the sellers' problem.

Remark #2: Sellers and regulator's problem at $t = 0$. The formulation of the sellers and regulator's problem in (16) and (22), respectively, is unchanged. While the indirect utility functions are slightly different because of the general time-2 utility, this difference does not affect the time-0 first-order conditions because of the envelope theorem (see discussion in Section 3). Thus, the

[12] As in Dávila and Korinek (2018), we sidestep the issue of the existence of the equilibrium, given the generality of the utility functions we consider.

sellers' time-0 first-order condition is still given by (17) and the regulator's first-order conditions are given by (26) and (27).

Remark #3: Equilibrium in the low-withdrawal state $\gamma = 0$. Under the normalizations regarding the buyers' marginal utility, which we introduced before, and taking as given time-0 choices, the price in the low-withdrawal state, $\gamma = 0$, is $q_1 = R$ under each of the three pricing mechanism, as in the baseline.

Remark #4: Equilibrium in the high-withdrawal state $\gamma = \bar{\gamma}$. Similar to the baseline, we can use (17) to pin down the price, $q_1(\bar{\gamma})$, in the fire-sale state.[13] To do so, we note that the time-1 marginal utility of the sellers' wealth, λ_1^s, which is given by (18) in the baseline, is now given by

$$\lambda_1^s = \frac{R}{q_1} \frac{\partial u_2^s\left(c_2^s\right)}{\partial c_2^s} \tag{31}$$

under all three pricing mechanisms. That is, an additional unit of wealth at $t = 1$ allows the sellers to reduce their sales by $1/q_1$ units of the long-term asset, obtaining a payoff, R, per unit of asset, which is then valued according to their time-2 marginal utility of consumption, ${}^{\partial u_2^s\left(c_2^s\right)}/_{\partial c_2^s}$. Combining (2), (17), (31) and the assumption that the sellers' marginal utility of wealth, λ_1^s, is normalized to one in the low-withdrawal state, we can solve for the price, $q_1(\bar{\gamma})$, of the long-term asset in the fire-sale state:

$$q_1(\bar{\gamma}) = \frac{\pi R \frac{\partial u_2^s\left(c_2^s(\bar{\gamma})\right)}{\partial c_2^s}}{(1-\pi)(R-1) + \pi R \frac{\partial u_2^s\left(c_2^s(\bar{\gamma})\right)}{\partial c_2^s}} < 1. \tag{32}$$

While (32) expresses $q_1(\bar{\gamma})$ as a function of the endogenous level of time-2 consumption, c_2^s, it shows that $q_1(\bar{\gamma}) < 1$ because $R > 1$ and $\pi < 1$. Thus, an equilibrium with high withdrawals, $\bar{\gamma}$, is characterized by a drop in the long-term asset price. In addition, because the sellers make the same choices as in the baseline (see Remark #1), the high-withdrawal state is again characterized by a higher trading volume relative to normal times, similar to the baseline of Sections 2-3.

[13] Because there are only two states, we simplify the notation and use only γ to refer to the state γ, θ.

4.2 Inefficiency and liquidity regulation with general utility

We are now ready to state our main results in the model with general utility. We begin by comparing efficiency and regulation under cash-in-the-market and second-best-use pricing, and then we turn to the model with asymmetric information.

Proposition 4.1. *(**General utility: cash-in-the-market and second-best-use pricing**) Consider two observationally equivalent equilibria derived under cash-in-the-market and a second-best-use pricing, respectively (i.e., the sellers make the same time-0 choices in the two models and for any γ, the time-1 price q_1, the time-1 trading volume k_1^b, and the sensitivity of the price q_1 to the trading volume k_1^b, are the same in the two models). Assume also that the sensitivity of the price q_1 to the sellers' time-0 liquidity holdings, $\partial q_1/\partial l_0^s$, is the same. Then,*

(i) If the sellers' time-0 liquidity holdings are higher than the socially optimal level under cash-in-the-market pricing (i.e., the optimal policy is a liquidity ceiling), the optimal policy under second-best-use pricing is either a lower liquidity ceiling or a liquidity requirement.

(ii) If the sellers' time-0 liquidity holdings are lower than the socially optimal level under cash-in-the-market pricing (i.e., the optimal policy is a liquidity requirement), then the optimal policy under second-best-use pricing is a tighter liquidity requirement.

In the general model with cash-in-the-market pricing, the equilibrium is generically inefficient and it is not possible to establish the general direction of the inefficiency, similar to Dávila and Korinek (2018). However, we can compare the efficiency and regulation with that of an equivalent model with second-best-use pricing.

Proposition 4.1 states that the socially optimal level of liquidity is always higher under second-best-use pricing, in comparison to cash-in-the-market pricing. To understand this result, note that in the model with general utility, during a fire sale, a gap can open up between the sellers and the buyers' marginal utilities of wealth (i.e., between λ_1^s and λ_1^b), so that the regulator's first-order condition (17) does not necessarily coincide with those of the individual sellers in (17). Importantly, with cash-in-the-market pricing, this gap is small—or even negative—because both the sellers and the buyers' marginal utilities, λ_1^s and λ_1^b, increase in a fire sale, relative to normal times. The sellers' marginal utility of wealth increases because they lose some wealth in a fire sale. The buyers' marginal utility of wealth increases because the increase in the demand for liquidity (due to the shock $\bar{\gamma}$) drains liquidity from the market and reduces the buyers' ability to consume at

$t = 1$. Differently, with second-best-use pricing, only the sellers' marginal utility of wealth increases, whereas the buyers' marginal utility of wealth decreases because these buyers gain on the inframarginal units they purchase. That is, for such inframarginal units, the cash flow collected is greater than the price paid—the cash flow is equal to the price only for the marginal unit.

Next, we analyze efficiency and regulation with asymmetric information. The forces that operate in the baseline model with linear utility continue to operate. The additional element that arises with general utility is related to the term $\partial q_1 / \partial T$ in the planner's first-order condition (27). In the baseline model, the linearity of the sellers' time-2 utility implies that a transfer, T, that reduces the sellers' wealth (and transfers such wealth to the buyers) has no effect on the equilibrium price, q_1, and, thus, $\partial q_1 / \partial T = 0$. With general utility, a reduction in the sellers' wealth generates a "wealth effect," resulting in a change in the sellers time-0 portfolio and, with it, the equilibrium price.[14] Hence, the term $\partial q_1 / \partial T$ is not necessarily zero with general utility. However, the next proposition provides a sufficient condition under which the optimality of the liquidity ceiling in the asymmetric-information model—derived in the baseline with linear utility—extends to the case with general utility.

Proposition 4.2. ***(General utility: asymmetric-information pricing)*** *Consider the model with asymmetric-information pricing augmented with a transfer, T, of liquidity from the sellers to the buyers, at the beginning of $t = 1$, with the transfer, T, announced at the beginning of $t = 0$. Define the sellers' $fundamental\ liquidity\ needs$ as*

$$(fundamental\ liquidity\ needs) = \frac{\bar{\gamma} d^s - (l_0^s - T)}{\theta k_0^s}, \tag{33}$$

which is the ratio of the liquidity the sellers raise in the market to finance their time-1 withdrawals in the high-withdrawal state, $\bar{\gamma} d^s - (l_0^s - T)$, to the quantity of lemons in that state, θk_0^s. If

$$\left. \frac{\partial \left(fundamental\ liquidity\ needs\right)}{\partial T} \right|_{T=0} \leq 0, \tag{34}$$

then the sellers' time-0 liquidity holdings are higher than the socially optimal level, so that the optimal policy is a liquidity ceiling.

[14] Formally, in the baseline model, the linearity of the time-2 sellers' utility implies that the sellers' time-0 liquidity choices are infinitely elastic and, thus, their time-0 first-order condition pins down the price, $q_1(\bar{\gamma})$. Because the tax, T, is a lump sum, it does not affect the time-0 first-order or the price, $q_1(\bar{\gamma})$. In the version of the model with general utility, the time-0 first-order condition depends on the time-2 marginal utility of consumption and, thus, $\partial q_1(\bar{\gamma}) / \partial T$ is not necessarily zero.

We argue that the sufficient condition (34) has a natural interpretation and, thus, we expect it to be satisfied. To understand (34), fix the price $q_1(\bar{\gamma})$ in a fire sale. Condition (34) says that when the sellers face a higher tax, T, that reduces their wealth, they tilt their time-0 portfolio in a way that reduces their *fundamental liquidity needs*; that is, the amount of liquidity they need to raise through selling long-term assets in a fire sale, relative to the stock of such assets. Because, on the margin, the sellers sell high-quality long-term assets to finance their liquidity needs (i.e., the assets with payoff $R > 1$), there is a cost to finance this liquidity need, given by the fire-sale discount. Under (34), an increase in the tax, T, that makes the sellers poorer creates an incentive for them to reduce the losses related to the fire-sale discount. Thus, (34) seems natural in the sense that the loss of wealth due to the tax creates an incentive for the seller to hedge against further losses they would face in a fire sale.

5 Conclusions

This paper analyzes liquidity requirements—a policy that has attracted growing attention over time from policymakers and academics—in a model in which financial intermediaries are forced to sell some assets to meet high liquidity needs. The model nests three mechanisms commonly employed in the literature to generate low fire-sale prices: cash-in-the-market pricing, second-best-use pricing, and adverse-selection pricing.

The optimal liquidity policy involves a liquidity requirement, or a liquidity ceiling, or no intervention, depending on the pricing mechanism and the effects of market incompleteness on investors' ability to share risk. More generally, we identified three main forces that determine the direction of the optimal policy and that are common to all three pricing mechanisms: (i) the cash flow that the buyers and sellers collect from the marginal unit traded, (ii) the sensitivity of the fire-sale price to the investors' liquidity holdings, and (iii) how market incompleteness affects the investors' ability to share risk.

For policymakers—such as the SEC, which is considering this policy for mutual funds, and banking regulators, which have introduced liquidity requirements in the last decade—our results suggest that the possibility of fire sales alone, such as those that took place during the March 2020 dash for cash, do not necessarily justify liquidity requirements. Instead, our results suggest that policymakers should consider whether liquidity requirements are warranted by the pricing mechanism that applies to the situation they are considering, whether market incompleteness gives rise to imperfect risk sharing, and whether the buyers or the sellers would suffer more, in a fire sale, from such imperfect risk sharing.

We derived our results using a standard fire-sale framework in which trades take place in centralized markets. In practice, some assets that have experienced fire sales—such as asset-backed securities and corporate bonds—are traded in decentralized over-the-counter (OTC) markets. While the forces we identified are likely to be important even in OTC markets, future research could study whether such forces interact with other possible distortions that are driven by the lack of centralized trading venues.

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