Uncertain Uncertainty

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Abstract

We hypothesize that most individuals have little understanding of how the distribution of equity returns evolves over horizons. As a result, individuals compress their estimates of near- (1-year) and long-term (10-year) uncertainty. Consistent with our hypothesis, individuals report estimates that imply an implausibly strong negative relation between risk and horizon when asked about *total* returns but estimates that imply an implausibly strong positive relation between risk and horizon when asked about *average* returns. The analysis helps explain puzzling results across several literatures and has implications for household finance, corporate finance, and asset pricing.

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1 Introduction

The real and perceived relations between equity market risk and horizon is a central question in finance. For instance, the relation between horizon and the optimal fraction of wealth invested in equities (and therefore the efficacy of target date funds) depends critically on whether *per period* equity risk for a forward-looking investor is constant, increasing with horizon due to parameter uncertainty and estimation risk, or decreasing with horizon due to historical mean-reversion and predictability.[1] The perceived relations between equity market risk and horizon can also impact corporate decisions—both in aggregate and cross-sectionally. For example, if most CFOs perceive that (per period) equity market uncertainty increases with horizon (Pástor and Stambaugh (2012a), Ben-David, Graham, and Harvey (2013)), then firms will favor short-term projects (Graham (2022)).[2] Consistent with the hypothesis that economic agents perceive differences in the shape of near- and long-horizon return distributions, recent work suggests that both appear to be critical inputs for retail investors' asset allocation decisions (Sias, Starks, and Turtle (2024b)), CFOs' corporate finance decisions (Ben-David, Graham, and Harvey (2013)), and analysts' security valuations (Décaire and Graham (2024)).

The interpretation of both theory and empirical evidence (detailed in the next section) that considers economic agents' perceptions of uncertainty largely relies on an (explicit or implicit) assumption that economic agents understand the relation between horizon and uncertainty. Although empirical evidence is limited, what we do know, however, is both puzzling and troublesome. Specifically, surveys suggest that CFOs exhibit severely miscalibrated (i.e., overconfident) estimates of near-horizon (1-year) stock market uncertainty, but less miscalibrated long-horizon (10-year) uncertainty. In contrast, the typical individual appears to hold a reasonable estimate of near-term stock market uncertainty (nearly identical to historical volatility) but miscalibrated long-term uncertainty. At face value, the patterns suggest CFOs perceive that per period risk dramatically increases with horizon while individuals perceive that per period risk sharply declines with horizon. Moreover, the empirical relation between perceived uncertainty and investor behavior is much weaker than that implied by theory (known as the "attenuation puzzle"), qualitative risk per-

[1] A partial list of the studies that focus on the important question of how predictive risk varies with horizon for a forward looking investor includes Samuelson (1969), Merton (1969), Barberis (2000), Campbell and Viceira (2002), Pettenuzzo and Timmermann (2011), Pástor and Stambaugh (2012a), Hoevenaars, Molenaar, Schotman, and Steenkamp (2014), Johannes, Korteweg, and Polson (2014), Siegel (2014), Müller and Watson (2016), Avramov, Cederburg, and Lučivjanská (2018), Carvalho, Lopes, and McCulloch (2018), Zhang, Zhu, and Zhu (2020), and Jondeau, Zhang, and Zhu (2021).

[2] Ben-David, Graham, and Harvey (2013) find that CFOs' miscalibration of equity market returns is correlated with their miscalibration of own firm IRRs as well as their own firm's investment intensity and debt levels.

ceptions appear to better explain investors' behavior than quantitative model primitives (i.e., the variance inferred from individuals' perceived return distributions), and (as we show in this study) CFOs' uncertainty forecasts are sensitive to how the question is framed.

This study considers whether most economic agents fundamentally understand the relation between horizon and risk and whether systematic biases in the relation between near- and long-run uncertainty forecasts can help explain this disparate set of empirical results. Our central hypothesis is that (1) most individuals have limited knowledge of uncertainty in the near-term return distribution, uncertainty in the long-term return distribution, or the cognitively challenging relation between volatility and horizon, (2) as a result of this cognitive uncertainty, near- and long-term subjective uncertainty estimates will be compressed (relative to objective values), and (3) because *total* return uncertainty increases with horizon while *average* return uncertainty decreases with horizon, the direction of the bias induced by cognitive uncertainty will depend on whether individuals are asked about uncertainty in long-horizon total or average annual returns.

To begin, consider that regardless of the level of parameter uncertainty or predictability in equity returns, the relation between near- and long-term risk is complex—given the variance of k-year returns is k^2 times the variance of the average annual return over k years (i.e., $\sigma^2(r_k) = \sigma^2(k\bar{r}_k) = k^2\sigma^2(\bar{r}_k)$), the k-period variance ratio (denoted VR_k) is, equivalently, (1) $1/k$ times the ratio of the variance of k-year returns to the variance of 1-year returns, and (2) k times the ratio of the variance of the average annual return over k years to the variance of 1-year returns:

$$VR_k = \frac{\sigma^2(r_k)}{k\sigma^2(r_1)} = \frac{k\sigma^2(\bar{r}_k)}{\sigma^2(r_1)}. \tag{1}$$

For example, even in the simplest case where an individual knows, with certainty, the return generating process and returns are *iid* (henceforth the "base" case where $VR_k = 1 \ \forall \ k$), the variance of 10-year (henceforth, long-term) total returns is 10 times the 1-year (henceforth, near-term) return variance, the variance of average annual returns over a 10-year horizon is $1/10^{\text{th}}$ the 1-year return variance, and the 10-year return variance is 100 times the 10-year average return variance.[3] Adding uncertainty regarding the return generating process and expectations of mean-reversion or other forms of predictability only further complicates the estimation of these values (e.g., see equation (12) in Pástor and Stambaugh (2012a)).

[3]Equivalently, the standard deviation of 10-year returns is $\sqrt{10}$ times the standard deviation of 1-year returns, the standard deviation of the average return over 10 years is $1/\sqrt{10}$ times the standard deviation of 1-year returns, and the standard deviation of 10-year returns is 10 times the standard deviation of the average return over 10 years in the base case.

A growing literature (detailed in the next section) suggests that when forming beliefs and forecasts individuals are self-aware of their lack of certainty (i.e., they display "cognitive noise").[4] As a result, individuals only partially adjust their perceptions in response to signals, resulting in anchoring-like "compression" effects in observed beliefs relative to objective values. In this framework, the effect of compression of near- and long-term estimates on variance ratios will depend on whether respondents are asked about total or average annual long-term returns. If, for instance, a respondent's near- and long-term estimates of the confidence intervals or return likelihoods are compressed, then their inferred long-term variance ($\hat{\sigma}^2(r_k)$) will be, relative to their near-term estimate, too small when asked about k period total returns because they will insufficiently adjust their long-term estimate away from the "smaller" annual estimate:[5]

$$\hat{\sigma}^2(r_1) < \hat{\sigma}^2(r_k) < (k \times \hat{\sigma}^2(r_1)). \tag{2}$$

Correspondingly, our hypothesis predicts that when asked about about average annual returns, the inferred long-term average annual variance ($\hat{\sigma}^2(\bar{r}_k)$) will be too large relative to their near-term estimate as respondents' long-term estimate will be insufficiently adjusted away from their "larger" annual value:[6]

$$\hat{\sigma}^2(r_1) > \hat{\sigma}^2(\bar{r}_k) > (\hat{\sigma}^2(r_1)/k). \tag{3}$$

In short, if respondents have difficulty understanding that total return variance is proportional to horizon (in the base case), and at the same time, average return variance is inversely proportional to horizon, then cognitive uncertainty implies the perceived long-term variance will be too close to the perceived near-term variance regardless of whether respondents are asked about total returns (resulting in low variance ratios; equation (2)) or average annual returns (resulting in high variance ratios; equation (3)).

Using five different datasets—the American Life Panel (ALP), the Health and Retirement Study (HRS), the Duke CFO surveys, the Understanding America Study (UAS), and a series of surveys to undergraduate business students at two large public universities—we find strong support for our

[4]For example, Woodford (2020) provides a discussion of how economic models could benefit by introducing imprecision in people's beliefs, and Gabaix (2019) provides a Bayesian development of how cognitive noise in signals results in the compression of beliefs and behaviors.

[5]To reduce notational clutter, equations (2) and (3) are base case scenarios. In the more general case, the last term in both equations are multiplied by VR_k.

[6]Consider a simple example: If the individual perceives the standard deviation of annual returns is 20% (i.e., the approximate historical value), then, in the base case, the objective standard deviation of total 10-year returns is 63% (i.e., $(0.2^2 \times 10)^{0.5}$) and the objective standard deviation of the average annual return over the 10 years is 6.3% (i.e., $(0.2^2/10)^{0.5}$).

hypothesis. When ALP and HRS respondents are asked about total return uncertainty over the next decade, individuals tend to estimate long-term uncertainty only slightly greater than their annual uncertainty estimate resulting in unreasonably small variance ratios. For instance, ALP and HRS respondents' median implied 10-year variance ratios are 0.147 and 0.145, respectively (versus 1.0 in the base case). In contrast, when asked about average annual return uncertainty over the next decade, the typical CFO estimates long-term average return uncertainty only slightly smaller than their near-term uncertainty estimate resulting in unreasonably large variance ratios. Specifically, the median CFO estimate generates a 10-year variance ratio of 6.61 (versus the 1.0 base case estimate). When business school students are asked (two weeks apart) about uncertainty in both total returns and average annual returns, they generate the identical pattern—the median student variance ratio based on the total return questions is 0.238 versus 9.318 for the average return questions. When the same student answers both sets of questions, 95% report values that generate an "average return" variance ratio that is larger than their "total return" variance ratio and the median difference in their inferred variance ratios is an astonishing 8.706 (versus a value of zero for an economic agent who understands the relation between horizon and uncertainty). Similarly, when a representative sample of Americans (UAS participants) are asked about uncertainty in both total and average annual returns, their estimates generate the same pattern—the median UAS "total return" variance ratio of 0.187 is 1/14 the "average return" variance ratio of 2.595.

The variance ratios for both the typical CFO and individual are so far from historical and theoretical values that they appear highly implausible—but in opposite directions. Consider, for example, what the CFO and ALP respondents' median variance ratio estimates imply about the 10-year stock return distribution relative to historical values for the 1-year return distribution.[7] Specifically, assume the annual (continuously compounded) expected return is 9.3%, the risk-free rate is 3.3%, and the standard deviation of annual returns is 20% (i.e., the approximate historical averages). In the base case (i.e., variance ratio=1), the likelihood of a negative 10-year return is 7.1%. In contrast, the likelihood of a negative 10-year return implied by the median CFO and ALP variance ratio estimates are, respectively, 28.4% and, effectively, zero (i.e., $< 0.01\%$).[8] Further, if

[7]We frame this discussion around the implied 10-year return distribution and the historical 1-year return distribution. The discussion can be equivalently framed around the implied 1-year return distribution and the historical 10-year return distribution. That is, the variance ratios lead to seemingly implausible—but in the opposite directions—implied long-term return distributions given the historical near-term return distribution, or equivalently, implausible near-term return distributions given historical long-term return distribution.

[8]We calculate these values by combining median variance ratios and historical annual market data. Specifically, the values are computed based on a 10-year expected return of 93% (i.e., 10 × the 1-year continuous expected return) and standard deviations of $\sqrt{0.20^2 \times 10 \times VR_i}$, where VR_i is the variance ratio for the base case (1.0), median ALP respondent (0.147), or median CFO (6.606). Because the market has exhibited mean-reversion, the historical average likelihood of a 10-year loss is 4.2%. As noted above, the forward looking variance increases with parameter uncertainty

one assumes the risky share is determined by the standard Merton (1969) model for power-utility investors:

$$\%Equity = \frac{E(R_t) - R_{f,t}}{\gamma\sigma^2(R_t)}, \tag{4}$$

and the coefficient of relative risk aversion (γ) is 4 (following the example in Giglio, Maggiori, Stroebel, and Utkus (2021)), then an investor with a 1-year horizon holds 37.5% of their wealth in equities (i.e., $(0.093 - 0.033)/(4 \times 0.20^2)$ and, in the base case, so does an investor with a 10-year horizon. In contrast, the implied optimal risky share for the median CFO and ALP investor with a 10-year horizon are, respectively, 5.7% and 255% of their wealth.[9] Both values are orders of magnitude different than the values reported in the theoretical literature.[10]

Additional tests provide further support for our hypothesis. First, consistent with the hypothesis that most CFOs have relatively little understanding of stock market uncertainty, we demonstrate that the typical CFOs' forecast of equity market risk varies dramatically based on question structure. Second, additional student surveys demonstrate the patterns in variance ratios we document remain nearly identical when students are asked about historical return distributions rather than expected future returns inconsistent with the hypothesis that differences in the total versus average annual return variance ratios are driven by differences in overconfidence in one's predictive abilities. Moreover, regardless of the approach used (return confidence intervals or assigning probabilities to return quantiles) to ascertain respondents' perceived return distributions, the typical respondents' "average return variance ratio" is implausibly large while their "total return variance ratio" is implausibly small. Finally, consistent with the hypothesis that cognitive uncertainty plays a substantial role in explaining the unreasonable variance ratios, the bias worsens with longer horizons.

Despite the implausibility of the implied variance ratios, extant work (detailed below) demonstrates that variation in both subjective near- and long-term uncertainty are informative in that both play an important role in explaining household and corporate decisions. Therefore, the balance of the study uses the ALP and student survey data to investigate variance ratio heterogeneity with a focus on understanding the roles of cognitive uncertainty as well as previously proffered explanations for why respondents' variance ratios differ from unity including parameter uncer-

and declines with expectations of mean-reversion.

[9]These values are computed as $(10 \times (0.093 - 0.033))/(4 \times 0.20^2 \times 10 \times VR_i)$, where VR_i is the variance ratio for the base case (1.0), median ALP respondent (0.147), or median CFO (6.606).

[10]For example, using "realistic" parameters that incorporate both mean-reversion and parameter uncertainty, Pástor and Stambaugh (2012a) estimate the 10-year variance ratio is 1.1 and the optimal equity share for an investor with a 10-year horizon is 50-60%.

tainty, serial correlation beliefs, perceptions of the relation between investment horizon and risk, overconfidence, and expectations of changes in volatility. First, we exploit the fact that cognitive uncertainty and parameter uncertainty hypotheses predict opposite relations between financial sophistication and variance ratio heterogeneity.[11] Consistent with the hypothesis that cognitive uncertainty plays a more important role than uncertainty regarding the parameters of the return generating process in explaining variance ratio heterogeneity, more financially sophisticated ALP respondents exhibit larger (total return) variance ratios. Second, consistent with the hypothesis that variation in perceived serial correlation helps explain heterogeneity in variance ratios, ALP respondents who perceive greater mean-reversion exhibit lower variance ratios. We also find, however, that although most students report they believe markets are safer for long-term investors, heterogeneity in students' variance ratios appears largely independent of their beliefs regarding the relation between investment horizon and risk. Third, the ALP data provides no empirical support for the two previously proposed explanations—overconfidence and beliefs that market volatility will rise in the future—for why CFOs' forward-looking variance ratios differ from unity. Finally, consistent with compression of near- and long-term forecasts, we find strong evidence that ALP participants compress near- and long-term forecasts for eight events (e.g., the likelihood of experiencing a car accident in the near year or five years) that have nothing to do with equity market uncertainty. Moreover, consistent with the hypothesis that individuals' levels of cognitive uncertainty are positively correlated across domains (Enke and Graeber (2023)), the extent that an ALP respondent compresses their near- and long-term stock market forecast is positively related to the extent they compress their perceived near- and long-term forecasts for these completely unrelated (e.g., car accident likelihood) events.

Our work contributes to multiple literatures. First, the results suggest that representative samples of Americans, CFOs, and business school students have, at best, only the most rudimentary understanding of the relation between horizon and risk as all three groups appear to make only small adjustments from their near-term distribution perceptions when reporting their long-term distribution perceptions regardless of whether they are asked about uncertainty in total or average annual long-term returns. Second, our work provides a unifying explanation the disparate set of empirical findings discussed above. For instance, consistent with their view that "...investors should view stocks as more volatile over long horizons than over short horizons," Pástor and Stambaugh (2012a) conclude that, "...the typical CFO views the annualized variance of 10-year returns to be

[11]As detailed below, financial sophistication should reduce cognitive uncertainty resulting in larger (total return) variance ratios. In contrast, financial sophistication should result in lower variance ratios by reducing parameter uncertainty. Which effect dominates is an empirical question.

at least twice the 1-year variance." Our evidence suggests, however, that the large variance ratios inferred from CFOs' responses largely arise because CFOs are asked about average annual returns. Specifically, both students and representative samples of Americans (UAS) exhibit the identical pattern as CFOs when asked about average annual returns but the opposite pattern as CFOs when asked about total returns. Moreover, the CFO variance estimates appears very sensitive to the manner in which they are asked. In short, our results suggest that because respondents' long-term and near-term estimates are compressed, one cannot infer whether most economic agents view risk as increasing or decreasing with horizon from their perceived near- and long-term (either total or average annual) return distributions.

Our analysis can also help explain the attenuation puzzle—the fact that although the risky share decreases with subjective uncertainty and increases with subjective expected returns, the empirical relation is much weaker than the theoretical relation. For example, Amromin and Sharpe (2014) find that the coefficient associated with perceived uncertainty is less than $1/10^{\text{th}}$ the theoretical value. Our results suggest that the attenuation results, at least in part, because (1) most investors likely have expected holding periods other than a single year, but have cognitive uncertainty about the relation between horizon and risk, and (2) cognitive uncertainty leads to attenuation (e.g., Enke and Graeber (2023), Charles, Frydman, and Kilic (2024)).[12] The results also help explain why qualitative measures of uncertainty do a better job than quantitative model primitives in explaining risky share or participation and provide support for the "Risk as Feelings" literature (Lowenstein, Hsee, Weber, and Welch (2001), Shiller (2017)).[13]

Our analysis also helps explain the puzzling result that CFOs' near-term miscalibration is much greater than their long-term miscalibration. The cognitive uncertainty explanation is arguably more satisfying than either of the previously proffered explanations for the difference in near- and long-term miscalibration—which require either CFOs always expect risk to rise in the future or that CFOs recognize parameter uncertainty increases per period long-term uncertainty—given the facts that the typical CFO's variance ratios are an order of magnitude larger than that suggested by models of parameter uncertainty, that CFOs underestimate both near- and long-term uncertainty, and that students and UAS participants exhibit the identical pattern when asked about average annual returns but the opposite pattern when queried about total returns.

[12] Our results directly speak to the attenuation of the coefficient associated with perceived return variance. The effect on the coefficient associate with long-term expected returns is more complex (e.g., Abel (2018)).

[13] Although a theory should be judged on how well it explains outcomes rather than the validity of its assumptions (Friedman (1953)), the attenuation puzzle is, essentially, the fact that the theory does a poor job of explaining the risky share outcome.

Our results also provide insights into the source of miscalibration when CFOs, students, or a representative sets of Americans are asked for their near-term return confidence intervals (i.e., the Duke CFO survey questions). Specficially, Ben-David, Graham, and Harvey (2013) point out that miscalibration, "happens because either most overestimate their ability to predict the future or because they underestimate the volatility of random events." Our results suggests the latter explanation plays an important role.[14] First, students generate strongly miscalibrated near-term beliefs when asked the Duke average annual return confidence interval questions, but provide tend to overestimate near-term uncertainty when asked the ALP total return questions. Second, inconsistent with the explanation that overconfidence in their ability to predict the future primarily drives the results, both students and representative samples of Americans (UAS participants) generate nearly identical estimates of near-term miscalibration when asked about historical returns rather than future expected returns.

Given the evidence that most individuals have little understanding of the relation between horizon and uncertainty, the increased importance of defined contribution plans (Poterba (2014)), the central role of long-term beliefs in portfolio formation choices (Sias, Starks, and Turtle (2024b)), and the importance of equity market participation in wealth inequality (Favilukis (2013), Bhamra and Uppal (2019), Zumbrun (2023)), our work also calls for greater emphasis in describing the relation between uncertainty and horizon to economic agents.[15] For instance, our results suggest that respondents' ignorance may mean that most individuals' subjective utility-maximizing portfolio equity allocation could differ from the objective value. Assume, for example, that per period uncertainty either increases with horizon (Pástor and Stambaugh (2012a)) or is constant (Merton (1969)). If, however, the individual has little understanding of the relation between horizon and uncertainty and clings to the (Bodie (2021)), "dangerous fallacy" that risk declines with time, the individual may select the objectively suboptimal target date fund based on the individuals' (erroneous) subjective beliefs.

Our analysis also points to a promising new direction for future theoretical models of household

[14]Our study focuses on the relation between near- and long-term uncertainty estimates. We do not claim cognitive uncertainty (1) fully explains why when asked the Duke survey questions, CFOs, business school students, and UAS respondents are near-term miscalibrated, (2) that our results necessarily fully explain why CFOs', students', and UAS respondents' near-term returns are more miscalibrated than their long-term returns, or (3) that the Duke CFO survey questions do not capture cross-sectional variation in CFO overconfidence. The Internet Appendix provides additional discussion and tests that help in understanding why CFOs, students, and UAS respondents exhibit such severe near-term miscalibration when asked the Duke survey questions.

[15]For example, the EU appears to recognize that most retail investors have little idea of how to interpret a standard deviation and, as such, requires every retail investment vehicle report a "risk score" (on a scale of 1 to 7) based on the annual return standard deviation but not report the standard deviation itself. Thus, the EU approach requires a measure of risk that is correlated with annual standard deviation, but does not provide insights into how that may translate into long-horizon risk.

finance, asset pricing, and corporate finance. Specifically, one of the central tenants of cognitive uncertainty is that because decision makers recognize they have doubts about their beliefs, there is a muted response between beliefs and action. Thus, for example, cognitive uncertainty associated with CFOs' perceptions of long-term risk could help explain (Graham (2022)) why managers continue to use simple short-term decisions rules (such as payback period) which may have less cognitive uncertainty. At the same time, our results may present new challenges for financial theory. For instance, Hartzmark and Sussman (2024) point out that one of the "central assumptions" of economic and asset pricing theory is that individuals "have something like a distribution in mind" when making decisions. Our results suggest that although individuals may have a distribution in mind, most have little idea of how that distribution evolves with horizon. Finally, our results contribute to the growing literature on how survey structure can impact inferences (e.g., Fox and Clemen (2005), Clemen and Ulu (2008), Glaser, Iliewa, and Weber (2019), Hartzmark and Sussman (2024)).

2 Background

Theory and empirical work often rely (implicitly or explicitly) on the assumption that economic agents understand the relation between horizon and uncertainty. For instance, implicitly, both the classic Merton (1969) model and the growing literature (see footnote 1) that focuses on the relation between horizon and uncertainty solves for the optimal risky share for an investor who understands the relation between horizon and return uncertainty. Similarly, the interpretation of empirical evidence often relies on the assumption that economic agents understand the relation between horizon and uncertainty. For instance, Pástor and Stambaugh (2012a) conclude, based on the Duke surveys of CFOs' near- and long-horizon uncertainty estimates, that CFOs "tend to exhibit" a view that stocks are "more volatile at long horizons."[16] Similarly, both the potential explanations offered by Ben-David, Graham, and Harvey (2013) to explain why CFOs' long-term miscalibration is less than their near-horizon miscalibration implicitly assume CFOs understand how uncertainty evolves with horizon.[17] Correspondingly, in labeling the observation that the relation between the fraction invested in equity and heterogeneity in beliefs about the perceived near-term expected

[16] Pástor and Stambaugh (2012b) note that, "The reason we arrive at a different conclusion is that we take the investor's perspective."

[17] Specifically, the authors propose that the difference in near- and long-term term miscalibration arises because: (1) CFOs understand parameter uncertainty increases per period volatility and therefore allow for larger errors over longer horizons, or (2) CFOs expect volatility to rise in the future. Barrero (2022) and Boutros, Ben-David, Graham, Harvey, and Payne (2025) also find corporate managers exhibit near-term miscalibration.

return distribution is much weaker than that suggested by theory as the attenuation "puzzle," the literature implicitly assumes that investors (1) know and use variance (or something closely associated with variance) and, assuming many have expected holding periods other than one year, (2) understand the relation between horizon and risk.[18]

Although a growing body of work recognizes that economic agents' perceptions of the forward looking return distribution is central to understanding the choices they make, asset pricing, corporate finance, and macroeconomic outcomes, the empirical evidence of near- versus long-term return distribution perceptions is puzzling.[19] First, although CFOs exhibit "severely miscalibrated" (Ben-David, Graham, and Harvey (2013)) perceptions of the near-term return distribution, most individuals' perceptions of the near-term return distribution appear well-calibrated. For instance, relatively sophisticated and wealthy Vanguard investors (Ameriks, Kézdi, Lee, and Shapiro (2020), Giglio, Maggiori, Stroebel, and Utkus (2021)), members of the American Association of Individual Investors (Jiang, Peng, and Yan (2024)) with a median wealth of \$3.5M, and a representative sample of Americans (Sias, Starks, and Turtle (2024b)) exhibit estimates of 1-year stock return uncertainty nearly identical to historical values.[20] These patterns, however, do not hold at longer horizons as CFOs' long-horizon forecasts exhibit less miscalibration than their near-horizon forecasts (Ben-David, Graham, and Harvey (2013), Pástor and Stambaugh (2012a)), while individuals long-horizon forecasts exhibit greater miscalibration than their near-horizon forecasts (Sias, Starks, and Turtle (2024b)). As discussed in the introduction, at face value, the result implies that CFOs perceive an implausibly strong positive relation between horizon and risk while individuals perceive an implausibly strong negative relation between horizon and risk.

In addition, the CFO evidence is puzzling in the sense that the inferred variance ratios are orders of magnitude larger than theoretical values (see discussion above) and inconsistent with (1) the consensus that, "It is widely believed that although stocks are very risky in the short run, in the long run they are far less risky . . . " (Bodie (2021)), (2) the popular press financial experts'

[18]For instance, Jay Clayton (former Chairperson Securities Exchange Commission) claims, "Put simply, they are investing for the long term and it's their money that's in our markets as they save for retirement and other life events. We think about them every day at the Commission and they're not investing for next week, they're not investing for next year. Predominately they're investing for something 10, 20, 30 years away and they expect their money to be managed that way."

[19]For discussions of the importance of investors' return distribution beliefs, see, for example, Manski (2004), Barberis (2018), Beshears, Choi, Laibson, and Madrian (2018), Malmendier (2018), Manski (2018), Brunnermeier, Farhi, Koijen, Krishnamurthy, Ludvigson, Lustig, Nagel, and Piazzesi (2021), Adam and Nagel (2023), and D'Acunto and Weber (2024).

[20]For example, investors in the Jiang, Peng, and Yan (2024) sample have an implied annual standard deviation of 23% (based on the median reported likelihoods of a 20% rise or fall in prices over the next year) versus the 20.1% historical value. In contrast, Ben-David, Graham, and Harvey (2013) report that median CFO beliefs imply an annual standard deviation of only 3.8%.

view that stocks are less risky in the long-run (Choi (2022)), and (3) the fact that, by a ratio of 10 to 1, individuals report they would increase (rather than decrease) their equity share if they decided to work an additional 10 years (Choi and Robertson (2020)).

A further puzzling result is that although nearly all models (e.g., Merton (1969)) assume investors' decisions are based on quantitative risk measures (and implicitly assume investors understand the relation between time and uncertainty), work (e.g., Weber and Hsee (1998), Nosić and Weber (2010)) finds that qualitative measures of perceived risk (e.g., a Likert scale where equity risk ranges from "no risk at all" to "very high risk") better explain behaviors than the quantitative theoretical model primitive (i.e., perceived return variance). Last, as detailed in the introduction, although heterogeneity in perceived uncertainty is materially related to variation in risky share, the relation accounts for only a small portion of the theoretical relation (the attenuation puzzle). In sum, CFOs exhibit greater miscalibration in near-term than long-term returns (which leads to large variance ratios), individuals exhibit little miscalibration near-term returns but strong miscalibration of long-term returns (which leads to small variance ratios), qualitative risk perceptions appear to better explain behaviors than quantitative model primitives, and the empirical relation between perceived risk and behavior is much weaker than that implied by theory.

Despite this puzzling set of empirical results, heterogeneity in perceived long-term return uncertainty appears to matter for both firms and individuals. For instance, Ben-David, Graham, and Harvey (2013) report that cross-sectional variation in CFOs' long-term uncertainty perceptions is more important than cross-sectional variation in their near-term uncertainty perceptions in explaining heterogeneity in firms' investment intensity.[21] Sias, Starks, and Turtle (2024b) find that perceptions of long-horizon uncertainty are at least as important as perceptions of near-term uncertainty in explaining stock market participation and the fraction of wealth invested in risky assets. Décaire and Graham (2024) report that approximately 75% of analysts' valuation arises from cash flows forecasted beyond 5 years. Thus, perceptions of long-term uncertainty should be of central importance to security analysts. In addition, Sias, Starks, and Turtle (2024a) find that although most individuals extrapolate near-term returns, their long-term return expectations are counter-cyclical.

[21] Lochstoer and Muir (2022) use the CFO near-term return distribution perceptions to examine the time-series of respondents' volatility perceptions and associated asset pricing implications.

2.1 Measuring perceived distributions

Most work takes one of three approaches to measuring economic agents' perceptions of a return distribution. First, respondents are asked for a confidence interval. For example, the Duke surveys ask CFOs for their 10th and 90th return percentile estimates resulting in an 80% confidence interval. Second, respondents may be given a reference value in the distribution and are then asked to estimate the likelihood of a value greater or less than that value. The ALP and HRS surveys, for instance, ask respondents to estimate the likelihood of a 20% or greater gain over the next year. Third, respondents are given "bins" in the distribution and asked to distribute balls in proportion to the likelihood. Although the focus of our study is on the relation between near- and long-horizon perceptions (holding the survey approach constant), as detailed in the Internet Appendix, a growing literature demonstrates that survey structure impacts participants' responses. For instance, respondents tend to report similar answers regardless of the confidence interval (see, for example, Hartzmark and Sussman (2024)) and are unlikely to assign a negligible probability to any outcome deemed important enough to be asked about (e.g., Fox and Clemen (2005), Clemen and Ulu (2008)).

Regardless of the approach, once moving beyond a single period, surveys must also make a choice between asking about expected cumulative or average annual long-horizon return distributions. For instance, the Duke CFO surveys ask respondents for their 80% confidence interval for average annual long-horizon returns (e.g., there is a 1-in-10 chance annual returns over the next decade will average less than ___). In contrast, the ALP surveys ask respondents for their perceptions of cumulative long-horizon returns (e.g., what are the chances equity markets will have fallen in value by more than 20% in 10 years compared to what they are worth today?). All three approaches, however, can be framed in cumulative or average annual long-horizon returns (e.g., there is a 1-in-10 chance cumulative returns over the next decade be less than ___).

2.2 Cognitive uncertainty

A growing literature demonstrates that economic agents' cognitive uncertainty—that is, their recognition that they are uncertain of the correct answer—impacts their valuations, beliefs, forecasts, and actions. Specifically, as a result of this cognitive uncertainty, respondents' answers are compressed toward a cognitive default value.[22] This compression can explain, for instance, probability

[22] As Enke and Graeber (2023) point out, the compression toward a cognitive default is related to the Tversky and Kahneman (1974) anchoring and adjustment heuristic in which individuals form estimates with an insufficient adjustment from their cognitive anchor. See Woodford (2020) and Gabaix (2019) for models and reviews of the role

weighting in lotteries, base rate insensitivity and conservatism in updating beliefs, why individuals overestimate the likelihood of low probability events and overweight the likelihood of high probability events, fourfold risk patterns, loss aversion, and why investors exhibit only a weak transmission from beliefs to behaviors (see Enke and Graeber (2023), Charles, Frydman, and Kilic (2024), Oprea (2024)).[23]

A recent extension of this literature (see, Enke, Graeber, and Oprea (2025), Enke and Graeber (2021), Gabaix and Laibson (2022), Gershman and Bhui (2020)) focuses on compression in estimates over different horizons. Specifically this work posits that the cognitive uncertainty associated with the exponential discounting in traditional models (see Cohen, Ericson, Laibson, and White (2020)) causes compression of estimates over different horizons. As a result, the *appearance* of hyperbolic discounting arises from these natural compression effects rather than the traditional interpretation of non-standard preferences. For example, Enke, Graeber, and Oprea (2025) document “hyperbolic discounting” is equally apparent when asking the same discounting questions but in an manner which removes the time dimension (denoted as an atemporal mirror).[24]

The key result in this literature is that when respondents are uncertain about values over two horizons, the typical respondent’s subjective estimates of those values are compressed relative to the objective values. Our hypothesis—that most economic agents have less than perfect certainty of near- and long-term return distributions—will result in a corresponding compression of their subjective estimates. In the case of average annual long-horizon returns, such compression will lead to variance ratios that are too large relative to objective values (see equation 3) and in the case of cumulative long-horizon returns, such compression will lead to variance ratios that are too small relative to objective values (see equation 2).[25]

2.3 Other explanations

Ben-David, Graham, and Harvey (2013) offer two potential explanations for why CFOs’ overprecision in near-term returns is much greater than in long-term returns: (1) CFOs expect near-term volatility to rise in the future, and (2) CFOs recognize that parameter uncertainty increases the

of cognitive uncertainty.

[23] Enke and Graeber (2023) note that there is not a “general theory” for what serves as the cognitive default. As discussed in greater detail in the Internet Appendix, our study also suggests that the cognitive default value varies with question structure (e.g., CFOs, students, and UAS respondents generate remarkably different estimates depending on how they are asked about future return uncertainty).

[24] An example of a atemporal mirror is asking a person to value “$50 shrunk 12 times, each time by 4%.”

[25] As detail in the Section 2.1 and discussed in greater detail in the Internet Appendix, different approaches (e.g., confidence intervals versus perceived likelihoods of a given return) may lead to different biases. Thus, for example, the cognitive anchor will likely differ across approaches. Regardless of these differences across approaches, our hypothesis only requires compression in estimated values relative to objective values (holding the approach constant).

forward-looking variance for long-horizon returns (e.g., Pástor and Stambaugh (2012a)).[26] Consistent with the former, the authors find a positive, albeit not statistically meaningful, relation between cross-sectional variation in 2011 Q1 forecasts of 2012 volatility and cross-sectional variation in the difference between CFOs' 2011 Q1 long- and near-term volatility inferred from their confidence interval estimates. To test the second explanation, the authors propose that parameter uncertainty in the return generating process is time-varying and that when cross-sectional heterogeneity in CFOs' expected returns is higher, the average CFOs' parameter uncertainty is higher. Consistent with parameter uncertainty contributing to the difference in overprecision, the gap between average long-term and near-term return volatilities tends to be greater when the dispersion in long-term expected returns is greater. A third potential explanation is that CFOs tend to be overconfident (Malmendier and Tate (2015)) and for some reason their overconfidence is stronger for near-term versus long-term forecasts.

3 Data

We use five datasets to examine respondents' near- and long-term perceived return distributions—the American Life Panel, the Health and Retirement Study, the Duke CFO surveys, the Understanding America Study, and a series of surveys given to undergraduate business students at two public universities.

3.1 The American Life Panel

The ALP is an ongoing nationally representative panel of Americans that has grown from approximately 2,000 participants in 2003 to more than 6,000 today. Because respondents are compensated, the panel has much higher completion rates (70-80%) and lower attrition rates (6-13%) than most surveys (see Pollard and Baird (2017) for additional ALP details). Between November 2008 and January 2016, ALP executed 61 "effects of the financial crisis" surveys in either long-form (29 surveys) or short-form (32 surveys) formats. The long-form surveys asked respondents six questions regarding both near- and long-term perceived stock market return distributions:

[26] For instance, the authors find that the average CFO reports an 80% confidence interval (i.e., the difference between the 90^{th} and 10^{th} percentile) for near-term returns of 14.5% compared to the historical value for the S&P 500 of 42.2%. The average CFO's 80% confidence interval for annualized 10-year returns is 9.4%—while still miscalibrated, these longer term beliefs are much closer to the historical annualized 10-year value for the S&P 500 of 12.4%. In a recent update that only focuses on near-term uncertainty, Boutros, Ben-David, Graham, Harvey, and Payne (2025) confirm that CFOs' near-term views remain miscalibrated, the trait is highly persistent across individual CFOs, and that there is little evidence of learning by CFOs.

We are interested in how well you think the economy will do in the future. By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

By next year at this time, what is the percent chance that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have
-gained in value by more than 20 percent compared to what they are worth today?
-fallen in value by more than 20 percent compared to what they are worth today?

Now please think about how the stock market will change over the next 10 years: What are the chances that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more in 10 years than they are today?

What are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have
-increased in value by more than 20 percent in 10 years compared to what they are worth today?
-fallen in value by more than 20 percent in 10 years compared to what they are worth today?

The 32 short-form surveys asked the first four questions but not the final two questions. Because estimation of the long-term variance is computed from the final two questions, we limit our sample to the 29 long-form surveys.[27]

Following previous work (Dominitz and Manski (2007), Kézdi and Willis (2011), Ben-David, Graham, and Harvey (2013), Boutros, Ben-David, Graham, Harvey, and Payne (2025)), we infer the near-term (long-term) perceived return variance, for each respondent at each point in time, by assuming continuous returns are normally distributed and solve for the parameters that fit the respondent's reported likelihoods that markets rise or fall 20% in the next year (decade).[28] The estimation therefore requires that a respondents' beliefs follow probability laws, e.g., the sum of the likelihoods cannot exceed 100%.[29] We are able to infer variances for 69% of the surveys where

[27] Wave 44 of the survey was split into two portions (44.1 and 44.2) and only respondents to 44.1 were asked the likelihood that stock returns over the next decade would exceed 20% or fall by more than 20%.

[28] Following the variance ratio literature (e.g., Poterba and Summers (1987), Lo and MacKinlay (1988)), we focus on continuously compounded returns. As detailed in prior work (e.g., Fama and French (2018)) long-horizon returns more closely resemble a log-normal, relative to normal, distribution. Because we focus on continuously compounded returns, we convert respondents' estimates of the likelihood of discrete returns to continuously compounded returns when estimating the respondent's perceived distribution of near- or long-term expected returns.

[29] Specifically, we required the sum of the probability of a return less than 20% and a return greater than 20% is less than 100% and that neither probability is zero. As pointed out by Sias, Starks, and Turtle (2024b), the substantial number of observations that violate probability laws is consistent with previous work (e.g., de Bruin, Fischhoff, Millstein, and Halpern-Felsher (2000), Merkoulova and Veld (2022)). See Internet Appendix for additional detail.

respondents report values for both the likelihood markets rise or fall by 20% in the next year, and for 62% of the surveys where respondents report values for the likelihood markets rise or fall by at least 20% over the next decade. Because our variable of interest is the variance ratio, we limit the sample to respondent-survey observations with both near- and long-term variance estimates. To ensure that outliers from the variance inference process do not drive the results, we winsorize variance ratios and both near- and long-term volatility estimates at the 5^{th} and 95^{th} percentiles.

The 29 long-form ALP surveys also ask respondents about expectations for their financial condition in a year: "Now looking ahead - do you think that a year from now you will be better off financially, worse off, or about the same as now?" We use this question to proxy for expected future changes in volatility based on the hypothesis that, on average, those who expect greater risk to their personal financial situation also perceive greater future economic uncertainty and more volatile equity markets. Consistent with this hypothesis, in other ALP surveys (that do not ask about return expectations and therefore are not in our sample), respondents' answers to the question of expected changes in their personal financial situation are strongly positively correlated with their expected changes in the broader economy (business conditions "for the country as a whole") and strongly negatively correlated with their expected changes in unemployment (see Internet Appendix for details). Nonetheless, although the horizon approximately matches that used by Ben-David, Graham, and Harvey (2013) to capture CFOs' expected second year ahead near-term volatility (see Section 2), our measure is an indirect proxy for ALP respondents' future expected near-term equity volatility.

We use all 61 ALP effects of the financial crisis surveys (i.e., both short- and long-form) to generate an estimate of each respondent's beliefs regarding mean-reversion (or momentum) in equity markets. Specifically, for each respondent, we estimate a time-series regression of each individual's perceived likelihood markets rise in the next decade (which is asked in all 61 surveys) on the market return over the previous 12 months. For ease of interpretation, we standardize lag annual market returns by dividing lag returns by the standard deviation of the annual Center for Research in Security Prices (CRSP) value-weighted market return such that the coefficient reflects the change in the perceived likelihood markets rise in the next decade associated with a one standard deviation higher lag annual return. We require respondents have at least 10 observations (from the 61 surveys) to estimate their mean-reversion beliefs. To ensure outliers do not drive the results, we winsorize the estimated coefficients at the 5^{th} and 95^{th} percentiles.

The ALP surveys also collect data regarding respondents' gender, race, marital status, employment status, retirement status, age, years of education, income, stock market participation, as well as measures of numeracy, financial literacy, and overconfidence (see Internet Appendix for details). The Internet Appendix also reports descriptive statistics of respondent characteristics for our pooled cross-sectional time-series of 22,748 observations (from 3,023 individuals; the average respondent participates in more than 7.5 of the 29 long-form surveys) that include any individual-survey wave observation where the respondent has adequate data to estimate their variance ratio. Our sample is 58% female, 88% White race, 66% married, 61% working, and 23% retired. In addition, consistent with Sias, Starks, and Turtle (2024a), the typical (median) respondent appears to believe markets are mean-reverting as a one standard deviation higher lag return is associated with a 1.37% decline the perceived likelihood markets rise in the next decade.

3.2 The Health and Retirement Study

The Health and Retirement Study (HRS) is a longitudinal panel of individuals ages 50 and older (and their spouses). The interviews, which began in 1992 are biennial, and the sample has grown to approximately 20,000 Americans. In 2009, HRS executed an "off-year" Internet survey to a random subset of HRS participants which asked the same six questions as the ALP long-form survey regarding 1- and 10-year return expectations. As above, we limit the sample to respondents whose perceived likelihoods do not violate probability laws and infer both their near- and long-term variances from the perceived likelihood markets rise or fall at least 20% in the next year or decade. In total, we calculate implied variance ratios based on near- and long-term return distribution beliefs for 1,252 HRS respondents who participated in the 2009 HRS Internet survey. As with the ALP data, we winsorize variance ratios and both near- and long-term volatility estimates at the 5^{th} and 95^{th} percentiles.

3.3 CFO survey data

The Duke CFO surveys ask a series of six questions:

Over the next 10 years, I expect the average annual S&P 500 return will be:

There is a 1-in-10 chance it will be less than ___

Expected return ___

There is a 1-in-10 chance it will be greater than ___

Over the next year, I expect the S&P 500 return will be:

There is a 1-in-10 chance it will be less than ___

Expected return ___

There is a 1-in-10 chance it will be greater than ___

For 63 quarters between 2004 and 2019 (data were not available for the first quarter of 2019), we gather the mean and median answers published at the Duke CFO survey website (cfosurvey.fuqua.duke.edu). On average, the medians and means are computed from samples of approximately 400 CFOs (ranging from 186 to 781). Following Pástor and Stambaugh (2012a), and directly analogous to the procedure described above, we infer variance (as well as a second estimate of expected returns), at each point in time, from the reported CFO median or mean 10^{th} and 90^{th} return percentiles for the S&P 500 over the next year and the next decade. As with ALP and HRS values, estimates are based on continuously compounded returns.

3.4 Student surveys

In spring of 2024, business students at two large public universities were given both the ALP survey's total expected return questions (see Section 3.1) and CFO survey's average expected return questions (see Section 3.3) for beliefs over the next year and decade. Specifically, university A juniors enrolled in one section of an introductory finance course were first given the ALP survey's total return questions. Two weeks later, the same set of students were given the CFO survey's average return questions. At university B, juniors in two sections of an introductory finance course were similarly given both surveys two weeks apart. In one section, students were first given the CFO survey then, two weeks later, the ALP survey. In the second section, students were first given the ALP survey followed by the CFO survey two weeks later. As detailed in Internet Appendix, the order of the surveys did not appear to influence students' answers. In fall of 2024, we gave both the ALP and CFO surveys (again, two weeks apart) as well as additional questions (discussed below) to a new group of introductory finance students at both universities.[30]

Analogous to the ALP surveys, we limit the sample to students whose answers follow probability laws and infer near- and long-term variances by assuming continuous returns are normally distributed at both the 1- and 10-year horizons. Our final sample (from the spring and fall 2024 semesters) consists of 322 students with sufficient data to compute variance ratios based on the ALP survey's total return expectations questions, 673 students with sufficient data to compute variance ratios based on the CFO survey's average return expectations questions, and 241 students with sufficient data to compute variance ratios based on both their total return expectations and

[30] As detailed below, in spring 2025 we ran a third set of two surveys to introductory finance students at both universities to further investigate how question structure influences respondents' inferred variance ratios.

their average return expectations.[31] As with the ALP and HRS data, we winsorize variance ratios, and both near- and long-term volatility estimates at the 5th and 95th percentiles. The Internet Appendix provides additional details regarding the student sample.

3.5 Understanding America Study

The Understanding America Study (UAS) is a national panel of approximately 14,700 US households.[32] In January 2025, the UAS asked respondents the six ALP survey style questions (see Section 3.1). In May-June 2019, UAS asked the three CFO survey style questions about 10-year horizon returns (see Section 3.3). In the same survey, UAS also asked respondents for their perceptions of the historical average annual market return, as well as their 10th and 90th percentiles of historical annual returns (over the past 60 years). As above, we limit the sample to respondents whose perceived likelihoods do not violate probability laws and compute variance ratios based on the ALP- (CFO-) style total (average annual) return questions for 3,644 (1,439) respondents in 2025 (2019). Inferred standard deviations and variance ratios are winsorized at the 5th and 95th percentiles. The Internet Appendix provides details of the UAS surveys.

4 The Perceived Relation between Horizon and Uncertainty

4.1 ALP respondents' variance ratios

The top three rows in Panel A of Table I reports descriptive statistics for ALP respondents' perceived likelihood markets rise, rise at least 20%, and fall at least 20% in the next year. The bottom two rows report, respectively, the inferred 1-year expected return and standard deviation. The penultimate column reports the historical average based on the CRSP value-weighted index. The final column reports the fraction of respondents who report a value less than the historical average. Consistent with previous work using the ALP data (Sias, Starks, and Turtle (2023)) as well as work based on other datasets (Hurd (2009), Hurd, Van Rooij, and Winter (2011), Kuhnen and Miu (2017), Das, Kuhnen, and Nagel (2020), Giglio, Maggiori, Stroebel, and Utkus (2021)), respondents tend to hold pessimistic near-term beliefs. For instance, despite the fact that, historically, markets have experienced a positive return in approximately three out of every four years, the median

[31] The ALP survey questions require that none of the four probabilities (the chance markets are up by at least 20% in 1 or 10 years; the chance markets are down at least 20% in 1 or 10 years) be zero and that the sum of the tail probabilities (chance of $\leq$ -20% and chance $\geq$ 20%) is less than 100%. In the case of the CFO questions, the only constraint is that the left tail 1-year (10-year) 10th percentile return is less than the right tail 1-year (10-year) 90th percentile return. The Internet Appendix provides complete descriptive statistics of the raw data.

[32] UAS is managed by the USC Dornsife Center for Economic and Social Research.

respondent reports only a 40% likelihood markets will rise in the next year.

[Insert Table I about here]

The bottom row of Panel A reveals that the perceived 1-year standard deviation imputed from the typical ALP respondents' beliefs (median of 20.7%) is close to the historical value (20.1%). Figure 1A summarizes the results in Panel A. The dashed red line reports the distribution for continuously compounded 1-year US equity returns based on the mean and standard deviation of the CRSP value-weighted index (i.e., the values in the bottom two rows of the penultimate column in Panel A). The solid red line is computed from the median estimated ALP respondent's perceived expected return and standard deviation (i.e., based on the values in the bottom two rows of the fourth column in Panel A). In short, although the typical ALP response suggests respondents hold a bearish near-term view, their inferred perceived near-term uncertainty is close to the historical average volatility.

[Insert Figure 1 about here]

Panel B in Table I reports analogous statistics for ALP respondents' perceptions of the distribution of expected equity returns over the next decade and shows the typical individual severely underestimates long-term expected returns. For example, the median respondent's inferred 10-year expected return is only 4.3% (i.e., 0.43% annually). Recall that in the base case, the standard deviation of 10-year total returns is 3.2 ($\sqrt{10}$) times the standard deviation of 1-year returns implying that if respondents linearly extrapolate variance, the median respondent's long-term uncertainty should be 65% (i.e., the 20.7% 1-year ALP median standard deviation times $\sqrt{10}$; see equation 2). Adding uncertainty regarding the return generating process suggests the value should even be larger. In contrast, both cognitive uncertainty and respondents' perceptions of mean-reversion imply a lower value. Empirically, the median respondent's beliefs imply a 10-year standard deviation of only 29%—less than half the value under independence and, as shown in the final column, nearly three out of four respondents report beliefs that generate a long-term variance less than the historical average. Thus, in contrast to the CFO evidence in Ben-David, Graham, and Harvey (2013), ALP respondents exhibit miscalibration of long-term returns but not near-term returns. Figure 1B summarizes the results in Panel B—the solid blue line reports the 10-year perceived distribution based on median ALP inferred beliefs (bottom two rows of fourth column in Panel B) and the dashed blue line reports the distribution based on historical 10-year mean and variance of the CRSP value-weighted index (bottom two rows of penultimate column in Panel B). The median

ALP respondent's long-term beliefs are far too bearish and far too confident relative to historical values.

Panel C in Table I reports the distribution of implied variance ratios for ALP participants (i.e., $(\hat{\sigma}^2_{i,t}(r_{10year})/10)/\hat{\sigma}^2_{i,t}(r_{1year})$). Consistent with previous work (e.g., Poterba and Summers (1987), Pástor and Stambaugh (2012a)), the penultimate column in Panel C shows that, historically, long-term US equity returns have exhibited mean reversion—as the 10-year variance ratio for the continuously-compounded CRSP value-weighted index is 0.559 (versus 1.0 if returns were serially independent). As shown in Panel C, however, the median ALP respondent's perceived near- and long-term return distributions imply a variance ratio of 0.147—a value only possible if equity markets exhibit unprecedented levels of mean reversion and ALP respondents have very limited uncertainty regarding the return generating process. Moreover, 89% of respondent-survey observations report beliefs that imply variance ratios less than unity (bottom row) and 82% report beliefs that imply variance ratios less than the CRSP historical value (last column of top row in Panel C).

Figure 1C summarizes the near- and long-term volatility for ALP respondents after removing the impact of mean beliefs. Specifically, Figure 1C combines and recenters (at zero to easily compare differences in uncertainty) ALP respondents' near-term perceived uncertainty (solid red line; distribution is identical to Figure 1A), long-term perceived uncertainty (solid blue line; distribution is identical to Figure 1B), and long-term uncertainty given ALP respondents' near-term uncertainty and assuming returns were serially independent (solid orange line). Consistent with our hypothesis (see equation (2)), ALP respondents' long-term uncertainty estimate (blue distribution) is between their near-term uncertainty estimate (red distribution), and the uncertainty implied by their 1-year beliefs if returns were serially independent (orange distribution). Because ALP respondents' inferred long-term uncertainty is much smaller than that implied by independence (i.e., the blue distribution is much less disperse than the orange distribution), the variance ratio implied by the median ALP respondent's beliefs is much smaller than one. In sum, the results support the hypothesis cognitive uncertainty compresses ALP respondents' estimates of near- and long-term uncertainty.

4.2 HRS respondents' variance ratios

Panels D, E, and F of Table I report analogous statistics for HRS respondents. The results are nearly identical to corresponding values for the ALP sample. Specifically, HRS respondents exhibit both near- and long-term bearishness. HRS respondents' near-term uncertainty is close to the historical value but their long-term uncertainty estimates tend to be substantially smaller than the historical value. As a result, 88% of the HRS participants' imputed variance ratios are less than unity and 80% are less than the historical CRSP value-weighted index's variance ratio. The median variance ratios for the ALP and HRS samples are nearly identical at 0.147 and 0.145, respectively.

4.3 CFO variance ratios

Table II reports the time-series descriptive statistics of the quarterly CFO surveys between 2004 and 2019 (n=63 quarters) for the cross-sectional mean (columns 2-5) and median (columns 6-9) CFO beliefs. The first column reports the historical values (from the CRSP value-weighted index). Panel A reports CFO expectations for 1-year horizons and Panel B reports corresponding values for 10-year horizons. Panel C reports variance ratios. Columns (5) and (9) report the fraction of time-series mean and medians, respectively, that are less than the historical value. Correspondingly, the bottom row in Panel C reports the fraction of variance ratios less than 1.

[Insert Table II about here]

Consistent with Ben-David, Graham, and Harvey (2013) and Boutros, Ben-David, Graham, Harvey, and Payne (2025), the results in Panel A reveal that CFOs exhibit substantial miscalibration in near-term beliefs as the mean (median) CFO's inferred 1-year standard deviation averages 4.7% (3.3%) compared to the market's historical value of 20.1% and, in every case (see columns 5 and 9; n=63 quarters), the inferred mean and median CFO near-term standard deviations are less than the historical standard deviation of market returns. To allow further comparison to the ALP and HRS samples, we report a second estimate of expected returns (i.e., in addition to the CFOs' reported expected return value in the top row) inferred from the 10$^{\text{th}}$ and 90$^{\text{th}}$ percentiles. Reported (top row of Panel A) and inferred (fourth row of Panel A) expected returns, however, are nearly identical. Figure 2A summarizes the results—the solid red line reflects the inferred 1-year return distribution computed from the CFOs' average expected return and variance (i.e., the values in the top and bottom rows of Panel A column 2) while the dashed red line reflects the 1-year return distribution computed from the mean and standard deviation of the CRSP value-weighted index (i.e., the values in the top and bottom row of Panel A column 1). Similar to ALP and HRS respondents, CFOs tend to exhibit near-term expected returns lower than the historical market average. Unlike ALP and HRS respondents, however, CFO beliefs generate inferred 1-year standard deviations much less than historical values (i.e., severely miscalibrated).

[Insert Figure 2 about here]

The top row in Panel B reports *total* 10-year expected returns (to allow direct comparison to Table I) while the second and third rows report the 10^{th} and 90^{th} percentiles for average annual returns over the next decade. Similar to ALP and HRS respondents (although to a lesser degree), CFOs exhibit 10-year expected returns that are less than the historical average. The bottom two rows of Panel B report descriptive statistics for both average annual return standard deviation for the next decade (i.e., the variable CFOs are asked about) and total return standard deviation for the next decade (i.e., directly comparable to the values in Panels B and E of Table I).[33] In the base case, the standard deviation of the average annual return over the next 10 years, should be only 32% ($1/\sqrt{10}$) of the annual standard deviation. For instance, given the time-series mean of the average CFO's 1-year standard deviation is 4.7% (column 2, bottom row of Panel A), the implied (base case) average annual return 10-year standard deviation is 1.5% (i.e., $0.047/\sqrt{10}$). Empirically, however, CFOs' average annual return 10-year standard deviation is more than twice that value at 3.3% (column 2, penultimate row of Panel B). CFOs' long-term variance estimates, while much greater than that implied by their near-term variance estimates under serial independence and no parameter uncertainty, remain below historical values—a result consistent with Ben-David, Graham, and Harvey (2013). That is, both near- and long-term beliefs are miscalibrated, but near-term are much more miscalibrated, resulting in large variance ratios. To allow direct comparions to ALP respondents' long-term beliefs, Figure 2B summarizes CFO long-term total return beliefs. The solid blue line is based on CFOs' mean beliefs (i.e., the values in the top and bottom rows of Panel B column 2) while the dashed blue line is based on the CRSP value-weighted index (i.e., the values in the top and borrom rows of Panel B column 1).[34]

Panel C of Table II reports that the time-series average variance ratio is 6.6 based on median CFO beliefs (Panel C, column 6) and 5.1 based on mean CFO beliefs (Panel C, column 2) implying that the average CFOs' beliefs are that the long-term variance is more than five times larger than that implied by a world with serial independence, and nearly 10 times greater than the historical

[33] Without making any assumptions regarding the relation between risk and horizon, long-term variances can be equivalently expressed in three ways: (1) the variance of long-term total returns, (2) the variance of long-term average annual returns, or (3) the annualized variance of long-term total returns. That is, because $\sigma^2(r_k) = \sigma^2(k\bar{r}_k) = k^2\sigma^2(\bar{r}_k)$, the variance of total returns over the next decade is 100 times the variance of average annual returns over the same period. Equivalently, the standard deviation of total returns (bottom row of Panel B) is 10 times the standard deviation of average annual returns (penultimate row of Panel B). Note that the 10-year "Imputed individual volatility" reported in Table I of Ben-David, Graham, and Harvey (2013) are annualized standard deviations, i.e., equivalent to the values in the penultimate row of our Panel B, Table II multiplied by $\sqrt{10}$ or the bottom row of our Panel B divided by $\sqrt{10}$.

[34] We follow Pástor and Stambaugh (2012a) and focus on average CFO beliefs. As show in Table II, however, median beliefs yield nearly identical results.

variance ratio (i.e., 0.559). At the same time, CFOs' long-term return uncertainty is less than the historical value. As pointed out by both Pástor and Stambaugh (2012a) and Ben-David, Graham, and Harvey (2013), the fact that the typical CFO's implied variance ratio is larger than one is consistent with the hypothesis that CFOs perceive that parameter uncertainty increases (per period) risk at longer horizons. The magnitude, however, is much greater than that suggested by the Pástor and Stambaugh (2012a) model as the authors estimate that the 10-year forward-looking variance ratio implied by their model is approximately 1.1 with "realistic" parameters.[35] A variance ratio of 5 would require that the impact of parameter uncertainty is more than four times the expected variance impact if CFOs' beliefs were consistent with serially independent returns and no parameter uncertainty (see Pástor and Stambaugh (2012a) Equation (12)). The results are also hard to reconcile with parameter uncertainty as the only explanation for the large variance ratios as CFOs' expected long-term variance is lower than the historical value (columns (5) and (9) of Panel B).

Figure 2C summarizes the CFO beliefs by combining and centering CFOs' implied 1- and 10-year distributions at zero to facilitate comparison. Specifically, Figure 2C displays CFOs' near-term perceived uncertainty (solid red line; from Figure 2A), perceived uncertainty in *average annual* long-term returns (solid blue line; i.e., the *total return* standard deviation used for the blue line in Figure 2B divided by 10), and average annual long-term return uncertainty given CFOs' near-term uncertainty and assuming returns are serially independent (solid orange line). Because CFOs are asked about average annual long-term returns, Figure 2C reports values in terms of average annual returns (whereas Figure 1C reports values in terms of long-term total returns which ALP respondents are asked about). Further consistent with our hypothesis (and equation (3)), CFOs' long-term uncertainty (blue distribution) is between their near-term uncertainty (red distribution), and the uncertainty implied by their 1-year beliefs if returns were serially independent (orange distribution). In this case, because estimated annual average long-term return uncertainty is much greater than that implied by independence (i.e., the blue distribution is much more disperse than the orange distribution), variance ratios are larger than one.

4.4 Business students' variance ratios

Although the analysis suggest that both CFOs and individuals struggle in their understanding of the relation between horizon and uncertainty and, as a result, their perceptions of near- and long-term

[35]The authors' Figure 6 reports 1- and 10-year annualized variances (i.e., total variance/k) of approximately 2.9% and 3.2%, respectively, implying a variance ratio of 1.1 (i.e., $\frac{\sigma^2(r_{10})/10}{\sigma^2(r_1)} = 0.032/0.029$).

uncertainty are compressed, it is possible that the dramatic difference in variance ratios (e.g., the median CFO's variance ratio is 45 times the median ALP participant's variance ratio; 6.606/0.147) primarily arises from differences in samples rather than because the effects of cognitive uncertainty depend on whether one is asked about long-term average annual or cumulative returns. Perhaps, for instance, CFOs, who no doubt are more financially sophisticated than the typical individual, are simply more overconfident in their estimates and, for some reason, their overconfidence is especially strong for near-term forecasts.

To rule out the possibility that differences in samples drive our results, we provide both the ALP and CFO survey questions to undergraduate business students at two large public universities. Panels A, B, and C of Table III report descriptive statistics for students when asked the ALP total return questions (e.g., the likelihood that in 10 years markets will have fallen by at least 20%). The results in Panels A and B are consistent with the ALP results in Table I—the median student's 1- and 10-year expected returns are bearish relative to historical averages. Students tend to exhibit greater uncertainty than ALP respondents for both near- and long-horizon returns. Students' responses to the ALP total return questions also reveal no evidence of systematic near-term overprecision, e.g., only 28% of students' forecasts imply a 1-year variance less than the CRSP historical value. Figures 3A and 3B, directly analogous to Figures 1A and 1B, summarize the 1- and 10-year total return distributions implied by the median student's beliefs (based on the values in the bottom two rows of the fourth column of Panels A and B) relative to the CRSP historical values (based on the values in the bottom two rows of the penultimate column of Panels A and B).

[Insert Table III and Figure 3 about here]

Panel C in Table III reports descriptive statistics for the variance ratios implied by students' responses to the ALP survey's total return questions. Similar to ALP and HRS respondents, the median student's variance ratio is 0.238, 81% of students' variance ratios are less than 1, and 71% are less than the historical value of 0.559. Directly analogous to Figure 1C, Figure 3C combines and centers at zero students' near-term perceived uncertainty (solid red line; from Figure 3A), long-term total return perceived uncertainty (solid blue line; from Figure 3B), and long-term total return uncertainty, assuming returns are independent, given students' near-term uncertainty (solid orange line). The pattern is identical to that for ALP respondents (i.e., compare Figures 1C and 3C). Specifically, consistent with compression of near- and long-term uncertainty forecasts (and equation (2)), students' long-term uncertainty (blue distribution) is between their near-term uncertainty (red distribution), and the uncertainty implied by their one-year beliefs if returns were serially independent (orange distribution).

Panels D and E in Table III report descriptive statistics for business students' answers to the CFO survey's average return questions. Figures 4A and 4B (directly analogous to Figures 2A and 2B), based on the values in the top and bottom rows of the fourth and penultimate columns of Panels D and E in Table III, summarize the results for the median student. The patterns are identical to those for CFOs—students appear overconfident regarding near- and long-term total returns but the apparent miscalibration is much greater for near-term returns. In fact, the median student's inferred standard deviation of 1-year returns, at 4.3%, is between the mean (4.7%) and median (3.3%) for CFOs (see Table II Panel A).

[Insert Figure 4 about here]

Panel F in Table III summarizes students' variance ratios based on the CFO average return questions. The median student variance ratio is 9.32 and 91% of the student estimates imply a variance ratio greater than 1. Figure 4C summarizes the cognitive uncertainty hypothesis for students answering the CFO average return questions. The pattern is identical to that for CFOs (i.e., compare Figures 2C and 4C) and consistent with our hypothesis (see equation (3)) as students' average annual long-term return uncertainty (blue distribution) is between their near-term uncertainty (red distribution), and the uncertainty in average annual long-term returns implied by their 1-year beliefs if returns were serially independent (orange distribution).

Panel G in Table III summarizes the difference in each student's variance ratio implied by their answers to the ALP survey's total return questions and their variance ratio implied by their answers to the CFO survey's average return questions for the 241 students who answer both sets of questions and whose answers to all questions follow probability laws. The third row demonstrates that the median student's variance ratio based on the CFOs survey's average return questions is 25 times their variance ratio inferred from their answers to the ALP survey's total return questions. Correspondingly, the fourth and fifth rows reveal the difference in variance ratios for the median student is 8.71 and 95% of students have a larger variance ratio when answering the CFO survey's average return questions relative to the ALP survey's total return questions. In sum, the results in Table III support the explanation that only a rudimentary understanding of how total and average return uncertainty evolve with horizon (rather than differences in perceptions of the importance of parameter uncertainty, mean reversion, or overconfidence) plays a central role in explaining the difference in variance ratios between the CFO and ALP/HRS samples.

4.5 The sensitivity of CFOs estimates to question format

Ben-David, Graham, and Harvey (2013) point out that the miscalibration implies that either CFOs overestimate their ability to predict market returns or underestimate market risk. Our hypothesis implies that the latter factor plays a substantial role in explaining the miscalibration, i.e., that the typical CFO does not know reasonable estimates of the 10^{th} and 90^{th} return percentiles. In contrast, if a CFO had perfect understanding (and no cognitive uncertainty) of the near-term return distribution (including the standard deviation of returns), then their answer to questions about near-term risk should be invariant to question structure. In contrast, if the typical CFO has little idea of the near-term return distribution (and therefore substantial cognitive uncertainty), their answers may be sensitive to question format.

The 2011 Q1 Duke survey (completed prior to March 3, 2011) offers an opportunity to examine the sensitivity of CFOs' forecasts to question structure. Specifically, in addition to the standard Duke survey 80% 1-year return confidence interval questions (see Section 2.1), CFOs were asked, "In 2010, the volatility of S&P 500 returns was 12.6%. What do you think the volatility of S&P 500 returns will be: __% in 2011; __% in 2012." Using the median 80% confidence intervals in 2011 Q1 to infer CFOs' 1-year return standard deviation belief yields a value of 3.7% nearly identical to the 3.8% median (for 2001-2011) reported by Ben-David, Graham, and Harvey (2013) and the 3.3% median reported in our Table II (for 2004-2019). In contrast, when asked, in the same survey, their volatility belief for 2011, the median CFO 1-year return standard deviation belief mean nearly triples to 11% with a cross-sectional standard deviation of 3.9%.[36] If we assume the distribution of answers are approximately normal, then 97% of CFOs answer that their expected volatility in 2011 is greater than the median volatility (3.7%) inferred from the usual CFO survey style questions regarding the 10^{th} and 90^{th} percentile near-term returns. Given the inferred 10-year CFO standard deviation (based on median 10^{th} and 90^{th} percentile 10-year return forecasts in the 2011 Q1 survey) was 29.45%, the 10-year variance ratio (in Q1 2011) based on the standard CFO questions is 6.27 (nearly identical to the 6.61 value in Table II). However, if we compute the variance ratio based on CFO's 10-year inferred variance (0.2945^2) and 1-year directly estimated volatility (0.11^2), the CFO variance ratio falls to 0.72. The results are inconsistent with the hypothesis that the typical CFO holds reasonable estimates of the standard deviation, 10^{th} percentile, and 90^{th} percentile of

[36]The mean CFO answer to the 2011 "volatility belief" question is 12.1%. The mean and median 2012 volatility belief estimates are, respectively, 11.5% and 10%. One possibility is that difference in estimates arises because although the typical CFO has a good understanding of the historical 10^{th} and 90^{th} return percentiles (and therefore the miscalibration results from their overestimation of their ability to predict market returns), they either do not know standard deviation or understand that by "volatility" the Duke survey is asking about standard deviation.

1-year return distribution.

The cognitive uncertainty hypothesis also suggests that respondents' forecasts are weighted toward a cognitive anchor. Given the evidence (e.g., Tversky and Kahneman (1974)) that individuals who are uncertain will use an anchor when it is given—even if the anchor is uninformative—the 2011 Q1 Duke survey also provides an opportunity to test for anchoring. Specifically, the statement, "In 2010, the volatility of S&P 500 returns was 12.6%" (noted in the previous paragraph) appears to be an error. Although the S&P 500 index rose 12.6% (based on 2010 open to 2010 close) in 2010, annualized daily (monthly) volatility was 18.1% (19.4%) and daily VIX averaged 22.55%. The fact that CFOs reported values (median of 11% with a standard deviation of 3.9%) close to the erroneous "seed" value of 12.6% rather than the true 2010 value (18-23%), the historical average (20.1%), or the value inferred from their 80% confidence interval expectations (3.7%) is consistent with the explanation that because the typical CFO has significant cognitive uncertainty regarding volatility, most anchored their estimate on the erronous seed value.

4.6 Perceptions of historical return distributions

As noted above, miscalibration implies that either respondents overestimate their ability to predict market returns or underestimate market risk (Ben-David, Graham, and Harvey (2013)). Our cognitive uncertainty hypothesis suggests that the miscalibration in students' views when asked the CFO survey questions results from their systematic misestimation of uncertainty rather than overestimation of their forecasting ability. Correspondingly, our hypothesis suggests the unrealistically low variance ratios associated with the ALP questions result from systematic misestimation rather than overestimating their ability to predict long-term returns. Thus, to differentiate overconfidence in the ability to predict returns from misestimation of volatility, we also asked students about their perceptions of the *historical* 1- and 10-year return distributions in our fall 2024 survey. For example, we reframed the CFO survey questions as, "Now we will ask you some questions about the historical average annual return over 10-year periods. Historically, over a 10-year period, I think the average S&P 500 return has had a 1-in-10 chance of being less than ___%." Correspondingly, in the second spring 2025 survey (two weeks later), we reframed the ALP survey questions to ask about historical values (e.g., "What do you think the chances are, historically, that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average have increased in value by more than 20 percent over a 10-year period?").[37]

[37]As detailed in the Internet Appendix, in the first fall 2024 survey students were asked ALP style questions about their expected (future) 1- and 10-year returns followed about CFO style questions about the historical 1- and 10-year

Our hypothesis is straightforward—if students' miscalibration results from a systematic bias in their perceived ability to predict market returns then variance ratios computed from perceptions of historical distributions should be reasonable. In contrast, if extreme variance ratios reported in Tables I-III result from a failure to understand the relation between horizon and uncertainty, then we should find the same patterns when asking about historical returns. Our results are fully consistent with the latter. Specifically, Panel A in Table IV reports summary statistics for inferred 1- and 10-year standard deviations and resulting variance ratios for the 251 students' responses to the historical likelihood of a 20% gain or loss over 1 or 10 years (the reframed ALP survey questions). Panel B reports analogous statistics from 290 students for the historical 10^{th} and 90^{th} percentiles for 1 or 10 year returns (the reframed CFO survey questions). Panel C reports descriptive statistics for the 139 students whose responses allowed us to compute variance ratios based on both the reframed CFO survey historical average annual return questions and reframed ALP historical total return questions.[38] The results in Table IV are nearly identical to those in Table III as the reframed ALP survey's historical total return questions generate unrealistically small variance ratios (the median variance ratio in Panel A of Table IV is 0.199), the reframed CFO survey's historical average return questions generate unrealistically large variance ratios (the median variance ratio in Panel B of Table IV is 11.454), and, as shown in Panel C, the median student's variance ratio inferred from the reframed CFO survey's average return questions is more than 88 times their variance ratio inferred from the reframed ALP survey's total return questions. In short, the results in Table IV suggest that over- (or under-) confidence in one's ability to forecast returns plays little role in driving the differences in variance ratios inferred from the ALP survey's total return questions versus those inferred from the CFO survey's average annual return questions.

[Insert Table IV about here]

4.7 Understanding America Study

The UAS data allow us to investigate how question structure impacts the inferred variance ratios for a broad sample of Americans. The UAS data, however, differ slightly from the student sample in that (1) there is a six year gap between UAS participants answering the CFO-style average return questions and the ALP-style total return questions, (2) the CFO-style expected 10-year average annual returns questions are phrased slightly differently than the Duke CFO surveys (see

return distributions. Correspondingly, in the second fall 2024 survey (two weeks later) students were asked CFO style questions about 1- and 10-year expected returns followed by ALP style questions about the historical 1- and 10-year return distributions.

[38]The Internet Appendix provides complete descriptive statistics for the "historical" likelihood questions.

Internet Appendix for details), and (3) the CFO-style 1-year return distribution questions ask about historical returns (analogous to the student results in Section 4.6). Nonetheless, despite these differences, the results for the UAS sample also support our hypotheses. For example, the median variance ratio for the 3,644 individuals who answer the ALP-style total return questions in 2025 is 0.187 (comparable to the median value of 0.147 for ALP respondents reported in Table I) and the median variance ratio for the 1,439 UAS respondents who answer the CFO-style average annual return questions in 2019 is 2.60. For the 469 UAS respondents who complete both sets of questions, 79% have a larger variance ratio based on their responses to the CFO-style average return questions versus the ALP-style total return questions, and the median CFO-survey style average annual return variance ratio is nearly ten times the median ALP-survey style total return variance ratio. The Internet Appendix provides complete details of the UAS analysis.

4.8 Average annual versus cumulative returns

The results in Tables I-IV demonstrate the CFO survey's average return questions yield substantially different variance ratio estimates than the ALP survey's total return questions. However, the two question structures differ not only with respect to asking about average annual versus cumulative returns, but also in question style—the CFO survey gives a probability and asks for a return (e.g., 1-in-10 chance of a average annual return less than ___%) while the ALP survey gives a return and asks for a probability (e.g., the likelihood markets have increased in value by more than 20 percent). To investigate the role of question style versus average or cumulative returns, in spring 2025 we executed a new pair of surveys to a new set of introductory finance students at both universities. Specifically, for half the respondents, the CFO surveys reframed the average annual return questions into cumulative returns (e.g., there is a 1-in-10 chance the total (cumulative) return over the next 10 years will be less than ___%) and reframed the ALP survey's total return questions as average annual returns (e.g., what are the chances that the US stock market will average an annual return of at least 10% per year over the next 10 years?). The other half of student respondents answered average annual return questions for the CFO surveys and cumulative return questions for the ALP survey (i.e., the original structure of both surveys). Our central hypothesis is variance ratios computed from estimated uncertainty in cumulative returns will be tend to be unreasonably small while variance ratios computed from estimated uncertainty in average annual returns will tend to be unreasonably large regardless of whether we ask respondents for average annual or cumulative return confidence intervals (i.e., the CFO survey structure) or the likelihood of a given average annual or cumulative return (i.e., the ALP survey structure).

Panel A in Table V summarizes the results for the ALP style questions. The first six columns provide information on survey question structure. The final column in rows 1-4 report median variance ratios when asking students the likelihood of an average annual return while rows 5-8 report median variance ratios when asking students about the likelihood of a cumulative return. Panel C reports corresponding values for the CFO style surveys that ask respondents for average annual (rows 14-17) or cumulative (rows 18-21) return confidence intervals. Panels B and D in Table V report Z-statistics associated with tests of difference in medians (for Panels A and C, respectively) between the groups identified as "Group 1" and "Group 2".

[Insert Table V about here]

The results in Table V are easily summarized. Most important, regardless of question style (ALP in Panel A or CFO in Panel C), the typical (median) respondent always generates estimates that lead to variance ratios greater than one (rows 1-4 and 14-17) when answering questions about average annual returns and less than one (rows 5-8 and 18-21) when when answering questions about cumulative returns. Moreover, regardless of question style or other parameters, these differences are always statistically meaningful at the 1% level for both the ALP survey style questions (rows 9-11) and the CFO survey style questions (row 22-24).

Although our primary focus is on understanding the role of asking respondents to consider average annual versus cumulative returns on the resulting inferred variance ratio, our spring 2025 surveys also varied horizon (e.g., 5 versus 15 years).[39] If respondents compress their near- and long-term forecasts, the bias will become more severe as horizon increases (i.e., the cognitive uncertainty will increase as the calculations become more complicated at longer horizons; see Enke, Graeber, and Oprea (2025)). Consistent with our hypothesis, the variance ratio associated with average annual returns is larger (row 3 versus 4; row 16 versus 17) when extending the forecast horizon and the differences are statistically meaningful at the 10% (row 12) and 1% level (row 25). Correspondingly, the variance ratio associated with cumulative returns becomes even smaller (row 7 versus 8; row 20 versus 21) when extending the forecast horizon and the difference is statistically significant (at the 5% level, see row 13) for the ALP style questions.

[39] In addition, we varied the return parameter in the ALP questions (e.g., the likelihood of a return greater than 30% versus 20%) and the confidence interval parameter for the CFO survey questions (e.g., asking about a 1-in-5 chance versus 1-in-10 chance). Although not the focus of this study, in the Internet Appendix we use these permutations to help understand differences in miscalibration in the ALP and CFO style surveys.

5 Respondent Characteristics and Variance Ratio Heterogeneity

The results in the previous section support the hypothesis that cognitive uncertainty regarding the complex relations between horizon, total return uncertainty, and average return uncertainty, primarily drives respondent variance ratios that are far from theoretical and historical values. In this section, we exploit heterogeneity in the ALP dataset to consider whether, holding question format constant, variation in cognitive uncertainty can help explain heterogeneity in variance ratios. In addition, we investigate five other (non-mutually exclusive) factors that have been proposed to explain subjective forward-looking variance ratios: (1) respondents believe that long-term returns are mean-reverting, (2) respondents believe markets are riskier or safer at longer horizons, (3) respondents recognize that parameter uncertainty increases long-term uncertainty relative to near-term uncertainty, (4) respondents expect market returns to be more (or less) volatile in the future, and (5) respondents' near-term overconfidence is, for some reason, greater than their long-term overconfidence.[40]

We begin by considering how cognitive uncertainty and parameter uncertainty influence the relation between respondent financial sophistication and variance ratios. Specifically, the cognitive uncertainty explanation suggests ALP respondents who know more about markets will have greater confidence in their near- and long-term total return perceptions, and therefore exhibit less compression resulting in larger (i.e., more realistic) variance ratios. In contrast, the parameter uncertainty explanation suggests respondents who know more about markets will have less uncertainty regarding the parameters of the return generating process and therefore *relatively* lower long-term perceived uncertainty resulting in smaller variance ratios. We use education, income, equity market participation, self-rated understanding of equity markets, numeracy, and financial literacy as financial sophistication measures.[41]

The final set of characteristics we consider test other proffered explanations. Specifically, if overconfidence impacts near-term beliefs to a greater degree than long-term beliefs (as suggested by the CFO evidence), then more overconfident respondents should exhibit larger variance ratios.

[40]Bodie (2021) and Choi (2022) posit that both financial experts and individuals view markets as less risky in the long run for reasons other than the mean-reversion argument in the academic literature. As a result, we investigate both possibilities.

[41]In the Internet Appendix, we present results for respondent sorts into high and low groups by these characteristics as well as sorts for gender, age, race, marital status, employment status, and retirement status. Consistent with our regression results (see Section 5.1) and the hypothesis that cognitive uncertainty plays a larger role than parameter uncertainty in explaining variance ratio heterogeneity, we find that respondents with greater financial sophistication perceive more accurate variance ratios. In addition, Enke and Graeber (2023) find that women and younger individuals exhibit greater cognitive uncertainty. Consistent with their evidence, we also find that, relative to men and older individuals, women and younger individuals will tend to exhibit smaller ALP variance ratios.

Analogously, if expected changes in future market volatility contribute to variance ratio heterogeneity, then respondents who believe markets will become more (less) risky in the future should exhibit larger (smaller) variance ratios. Finally, if respondents' views regarding return serial correlation impact their estimates, then respondents who hold stronger beliefs that market returns are mean reverting should exhibit lower variance ratios.

5.1 Regressions

To examine how characteristics relate to heterogeneity in variance ratios, we estimate panel regressions of variance ratios on wave (i.e., time) fixed effects, respondent demographic controls (respondent age, and indicators for gender, White ethnicity, married, working, and retired), and each of the remaining characteristics—individually and collectively—education, income, stock market participation, self-rated understanding of equity markets, numeracy, financial literacy, overconfidence, perceptions of expected changes in financial position, and mean-reversion beliefs. Except for the binary stock market participation indicator and the expected changes in financial position variable (which take the values of 1, 0, or -1), the characteristics are standardized (rescaled to unit variance and zero mean) so that the coefficients can be directly compared. We multiply time-series serial correlation beliefs by -1 so that a higher value indicates a stronger belief in mean-reversion. Table VI reports the estimates (standard errors are clustered at the respondent level).

[Insert Table VI about here]

For the first six characteristics, the cognitive uncertainty hypothesis predicts a positive coefficient (e.g., respondents who perceive themselves as better understanding the market should exhibit less compression generating larger variance ratios) while the parameter uncertainty hypothesis predicts a negative coefficient (e.g., respondents who perceive themselves as better understanding the market will have lower parameter uncertainty and therefore exhibit a lower variance ratio). The results in the first six columns uniformly support the hypothesis that the cognitive uncertainty effect dominates any parameter uncertainty effect even when controlling for respondent demographics. For instance, the results in the fourth column suggest that a one standard deviation higher perceived understanding of markets is associated with a 0.094 larger variance ratio (statistically significant at the 1% level).[42]

[42]Our tests are based on the assumption that if heterogeneity in parameter uncertainty drives heterogeneity in variance ratios, then respondents must recognize (1) there is parameter uncertainty in the return generating process and (2) that the parameter uncertainty increases long-term risk. An alternative interpretation is that these characteristics capture the degree to which respondents understand parameter uncertainty, e.g., more financially literate ALP

We continue to find no support for the hypotheses that variation in overconfidence or economic expectations contributes to heterogeneity in variance ratios (columns (7) and (8))—overconfidence is not materially related to variance ratios once controlling for demographics and the coefficient associated with economic expectations has the "wrong" sign.[43] The results in column (9) continue to support the hypothesis that mean-reversion beliefs results in smaller variance ratios as a one standard deviation higher belief in mean reversion is associated with a 0.037 lower variance ratio (statistically significant at the 1% level). Columns (10) and (11) report the regression results jointly including most, or all, of the variables. In the case of column (10), we exclude numeracy, financial literacy, and overconfidence because these variables substantially limit sample sizes. In both columns, however, the results continue to support the hypothesis that heterogeneity in cognitive uncertainty and mean-reversion beliefs help explain the heterogeneity in variance ratios.[44]

Although the tests in this section demonstrate that heterogeneity in cognitive uncertainty and mean-reversion expectations can help explain heterogeneity in variance ratios when holding question format constant, comparison to the results in the previous section suggests that question format (i.e., asking about uncertainty in total or average annual long-term returns) has an order of magnitude greater effect. For instance, the largest value in Table VI (column (3)) shows that ALP stock market participants exhibit a 0.132 larger variance ratio than non-participants. In contrast, Panel G of Table III shows the median student variance ratio inferred from the CFO survey style questions exceeds the median student variance ratio inferred from the ALP survey style questions by 8.71, i.e., an effect size 66 times greater than the largest effect size in Table VI (8.71/0.132).

5.2 Students' variance ratios and perceived riskiness of long-term investing

In our fall 2024 surveys, we also asked students how they viewed the "relation between the riskiness of investing in the stock market and your expected holding period?" and select one of four choices: (a) Investing for the long-term (e.g., 10 years) is more risky than investing for the near-term (e.g., 1 year); (b) Investing for the long-term (e.g., 10 years) is less risky than investing for the near-term

respondents better understand the parameter uncertainty in the return generating process inducing a positive relation between variance ratios and financial literacy. This alternative, however, is hard to reconcile with the unrealistically small variance ratios throughout the ALP sample.

[43] Because beliefs of an improving financial situation are positively correlated with variables such as education, income, understanding of markets, numeracy, and financial literacy (see Internet Appendix for details), which in turn appear inversely related to cognitive uncertainty (i.e., the results in columns (1)-(5)), a likely explanation for this result is that beliefs of an improving financial situation are associated with less cognitive uncertainty (and therefore a higher variance ratio with less anchoring).

[44] Collinearity between many of the respondent characteristics likely has an impact on the weakened relations between variance ratios and some of the reported respondent characteristics in the joint panel regressions (see Table IA-VII for correlations between the explanatory variables).

(e.g., 1 year); (c) The riskiness of investing in the stock market does not depend on how long you plan to be invested; and (d) I have no idea if the stock market is riskier in the long-term or near-term. Of the 455 responses, 62% viewed markets as less risky in the long run, 21% viewed markets as more risky in the long run, 13% viewed risk as independent of horizon, and 4% had no idea of the relation between horizon and uncertainty. We find no evidence, however, that variance ratios differed between students who viewed markets as safer in the long run versus those who viewed markets as riskier in the long run.[45]

5.3 Cognitive uncertainty in non-financial estimates

In an ALP survey completed between August 2006 and November 2007, respondents were asked a series of questions regarding the likelihood of an event in the next year. Specifically, respondents were asked, "What is the percent chance that ___ during the next year?" where the blank was filled with:

...you will get into a car accident...

...you will have a cavity filled...

...you will die (from any cause – crime, illness, accident, and so on)...

...someone will steal something from you...

...you will move your permanent address to another state some time...

...you will die in a terrorist attack...

...someone will break into your home and steal something from you...

...you will visit a dentist, for any reason,...

Respondents were then asked the same set of questions regarding the next five years, i.e., "What is the percent chance that ___ during the next 5 years?"

This survey provides a unique opportunity to examine two issues. First, we can examine whether, similar to uncertainty estimates at different horizons, respondents compress their perceived likelihood of these events over different horizons. Specifically, for most individuals, there is little reason to expect that the true likelihood of the events asked about will systematically decline with time. As a result (if the likelihood is constant over time), an individual's 5-year likelihood can be computed from their 1-year likelihood. For example, if a person's likelihood of getting into a car accident does not change over time, then their 5-year likelihood is $1-(1-PerceivedProbability_{1year})^5$.

[45]Specifically, the median CFO average annual return (ALP total return) survey question variance ratios were 9.66 and 10.00 (0.27 and 0.16), respectively, for respondents who view market risk as lower at long horizons and those who view market risk as higher at long horizons. In both cases, we cannot reject the hypothesis that the medians are different (the Z-statistic for the CFO (ALP) survey questions was 0.52 (-1.75)).

Because the objective 5-year likelihood is larger than the objective 1-year likelihood, cognitive uncertainty will result in compression of subjective 1- and 5-year likelihoods. That is, if cognitive uncertainty causes compression across horizons, then relative to the respondent's perceived 1-year likelihood, their perceived 5-year forecast will be too small.

Second, work (Enke and Graeber (2023), Enke, Graeber, and Oprea (2025)) finds that individuals' levels of cognitive uncertainty are correlated across domains. Thus, we hypothesize that the extent an individual compresses their perceived likelihood of these measures that involve cognitively challenging considerations of horizon and uncertainty—which have nothing to do with equity markets—will be related to variance ratios if heterogeneity in cognitive uncertainty helps explain the variance ratio patterns.

Our empirical analysis yields strong evidence that respondents' 5-year forecasts of these events (e.g., car accident) are compressed to their near-term forecast.[46] For instance, the median respondent reports a 20% chance of a car accident in the next year. Assuming that, on average, respondents' car crash risk is approximately independent over the next five years, the 5-year likelihood should be 67.2% (i.e., $1 - (0.8^5)$) and the 2-year likelihood is 36% (i.e., $1 - (0.8^2)$). Yet the typical respondent estimates only a 30% 5-year likelihood. As detailed in the Internet Appendix, on average (across the eight questions), the results reveal that 87% of respondents' 5-year estimate is lower than the 5-year estimate implied by their 1-year estimate under independence. In short, the results are uniformly consistent with compression of estimates over horizons due to cognitive uncertainty and suggest that most respondents do not understand the relation between horizon and uncertainty.

We next consider whether the magnitude of the cognitive compression errors in answering this set of eight general questions regarding likelihoods and horizon are related to variance ratios. Specifically, for each respondent and question, we compute the "compression error" as the difference between their implied 5-year likelihood assuming independence and the respondent's estimated 5-year likelihood (e.g., the 67.2% implied likelihood less the 30% respondent estimated 5-year likelihood of a car accident). Higher values (i.e., a larger compression error) imply greater compression of 1- and 5-year estimates. We then standardize (rescaled to unit variance and zero mean) the compression errors to allow for ease in interpretation and estimate regressions of variance ratios on the same set of control variables used earlier (respondent age, and indicators for gender, White ethnicity, married, working, and retired) and standardized compression errors. Because the ALP

[46] We exclude observations where individuals report zero or 100% likelihoods for either near- or long-term likelihoods as such forecasts imply the individual has no uncertainty regarding the judgment. For instance, 90% of respondents who answer the question of the likelihood of visiting a dentist in the next 5 years report 100%.

total return questions generate smaller variance ratios when there is greater compression of near- and long-term variances, the predicted relation between variance ratios and compression errors is negative. The results are reported in Table VII (as before, standard errors are clustered at the respondent level).

[Insert Table VII about here]

Consistent with the hypotheses that (1) cognitive uncertainty causes compression of near- and long-term forecasts, and (2) this effect varies across individuals and is correlated across domains, the coefficient associated with compression errors for each of the eight questions are negative and statistically meaningful at the 5% level or better. For instance, the first cell suggests that an individual with a one standard deviation higher compression error (with respect to an auto accident in the next year versus 5 years) has a 4.9% lower variance ratio. Given the standard deviation of ALP variance ratios is 0.586 (see Table I, Panel C), this reflects an almost 10% standard deviation change in the variance ratio.

6 Conclusions

Traditional theory, where variance serves as a model primitive, implicitly requires that economic agents have a reasonable view of the expected variance of equity markets and, if the horizon is anything other than a single period, also understand how *iid* uncertainty, uncertainty regarding the return generating process, and return predictability impact the relation between horizon and uncertainty. Although there is a substantial academic debate regarding how economic agents *should* view the relation between horizon and uncertainty, we have relatively little understanding of how these individuals *actually* view the relation. Moreover, although previous work documents that long-term uncertainty appears to play an important role in the decisions of both corporations and individuals, previous evidence of economic agents understanding of risk, taken as a whole, is puzzling and contradictory.

We propose that because the relation between horizon and uncertainty is complex and cognitively challenging, most individuals struggle to understand how uncertainty evolves with horizon. As a result, this cognitive uncertainty causes individuals to compress their near- and long-term uncertainty estimates. Consistent with the hypothesis that cognitive uncertainty plays substantial role in explaining uncertainty perceptions, CFOs, business school students, and UAS participants report perceptions that generate unreasonably large variance ratios when asked about long-term average annual return uncertainty, while representative sets of Americans and business school students report perceptions that generate unreasonably small variance ratios when asked about total return uncertainty. Moreover, our evidence suggests the effects are pervasive, worse for less financially literate respondents, women, younger respondents, non-White race respondents, and are correlated across domains. As detailed in the introduction, our evidence has important implications for understanding household finance, asset pricing, corporate finance, and cognitive uncertainty.

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Table I – Individuals' Beliefs Regarding Near- and Long-Term Stock Returns
Panels A, B, and C report descriptive statistics for the pooled cross-sectional time-series of American Life Panel Survey data between 2008 and 2016. Panels D, E, and F report cross-sectional descriptive statistics from the 2009 Health and Retirement Study Internet survey. The penultimate column reports the historical average (computed from the CRSP value-weighted index between 1926 and 2020). The final column reports the fraction of observations below the historical average.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: American Life Panel stock market expectations over next year								
P(market>0)	22,725	0.406	0.200	0.400	0.600	0.248	0.747	0.863
P(market>20%)	22,748	0.201	0.100	0.150	0.300	0.155	0.330	0.822
P(market<-20%)	22,748	0.183	0.100	0.150	0.250	0.142	0.063	0.219
$E_{i,t}(r_{1year})$	22,748	-0.010	-0.046	-0.020	0.027	0.092	0.093	0.887
$\sigma_{i,t}(r_{1year})$	22,748	0.274	0.158	0.207	0.301	0.191	0.201	0.491
Panel B: American Life Panel stock market expectations over next decade								
P(market>0)	22,736	0.515	0.250	0.500	0.750	0.287	0.958	0.976
P(market>20%)	22,748	0.347	0.150	0.300	0.500	0.236	0.929	0.996
P(market<-20%)	22,748	0.169	0.060	0.100	0.250	0.132	0.014	0.030
$E_{i,t}(r_{10years})$	22,748	0.134	-0.020	0.043	0.182	0.253	0.962	0.970
$\sigma_{i,t}(r_{10years})$	22,748	0.414	0.179	0.291	0.505	0.353	0.474	0.722
Panel C: American Life Panel variance ratios								
Variance ratios	22,748	0.391	0.100	0.147	0.373	0.586	0.559	0.822
%Variance ratio<1	22,748	0.892						
Panel D: Health and Retirement Study stock market expectations over next year								
P(market>0)	1,250	0.370	0.200	0.300	0.500	0.231	0.747	0.897
P(market>20%)	1,252	0.241	0.100	0.200	0.300	0.181	0.330	0.758
P(market<-20%)	1,252	0.176	0.100	0.100	0.250	0.134	0.063	0.181
$E_{i,t}(r_{1year})$	1,252	0.016	-0.020	-0.020	0.065	0.117	0.093	0.800
$\sigma_{i,t}(r_{1year})$	1,252	0.284	0.158	0.225	0.316	0.180	0.201	0.423
Panel E: Health and Retirement Study stock market expectations over next decade								
P(market>0)	1,249	0.525	0.300	0.500	0.750	0.261	0.958	0.990
P(market>20%)	1,252	0.386	0.200	0.300	0.600	0.246	0.929	0.999
P(market<-20%)	1,252	0.158	0.100	0.100	0.200	0.115	0.014	0.014
$E_{i,t}(r_{10years})$	1,252	0.193	-0.020	0.065	0.256	0.302	0.962	1.000
$\sigma_{i,t}(r_{10years})$	1,252	0.424	0.191	0.301	0.521	0.320	0.474	0.699
Panel F: Health and Retirement Study variance ratios								
Variance ratios	1,252	0.434	0.100	0.145	0.400	0.664	0.559	0.797
%Variance ratio<1	1,252	0.875						

Table II – CFOs' Beliefs Regarding Near- and Long-Term Stock Returns

The table reports summary statistics for the time series of the mean and median Duke CFO survey over the sample period from March 2004 to December 2019 (n=63 quarters). Column (1) reports historical results for the CRSP value-weighted index. Columns (2) through (5) report results based on mean CFO beliefs for the next year (Panel A), the next decade (Panel B), and the associated variance ratios (Panel C). Columns (6) to (9) report analogous results for median CFO beliefs. The expected return inferred from the 10^{th} and 90^{th} percentile values is denoted with the superscript ‡.

		Cross-sectional mean CFO beliefs				Cross-sectional median CFO beliefs			
Description	Historical (1)	Mean (2)	Minimum (3)	Maximum (4)	%<Hist (5)	Mean (6)	Minimum (7)	Maximum (8)	%<Hist (9)
Panel A: CFOs' stock market expectations over the next year									
$E_t(r_{1year})$	0.093	0.052	0.022	0.075	1	0.048	0.020	0.070	1
$P90(r_{1year})$	0.301	0.098	0.069	0.129	1	0.089	0.058	0.113	1
$P10(r_{1year})$	-0.138	-0.023	-0.090	0.008	0	0.003	-0.051	0.030	0
$E_t(r_{1year})^{\ddagger}$	0.093	0.038	0.004	0.060	1	0.046	0.008	0.067	1
$\sigma_t(r_{1year})$	0.201	0.047	0.037	0.073	1	0.033	0.023	0.046	1
Panel B: CFOs' stock market expectations over the next decade									
$E_t(r_{10years})$	0.962	0.682	0.554	0.861	1	0.616	0.488	0.770	1
$P90(\bar{r}_{10years})$	0.152	0.103	0.084	0.125	1	0.093	0.077	0.113	1
$P10(\bar{r}_{10years})$	0.034	0.018	-0.004	0.041	0.952	0.025	0.010	0.039	0.841
$E_t(r_{10years})^{\ddagger}$	0.962	0.606	0.440	0.826	1	0.588	0.484	0.739	1
$\sigma_t(\bar{r}_{10years})$	0.047	0.033	0.025	0.041	1	0.027	0.022	0.033	1
$\sigma_t(r_{10years})$	0.474	0.333	0.246	0.413	1	0.267	0.219	0.326	1
Panel C: CFOs' variance ratios									
Variance ratios	0.559	5.113	2.675	9.003	0	6.606	3.895	11.783	0
%Variance ratio<1		0				0			

Table III – Students' Beliefs Regarding Near- and Long-Term Stock Returns

Panels A, B, and C reports descriptive statistics for 322 undergraduate students who have sufficient data to compute their inferred variance ratio based on their perceived likelihood markets rise or fall 20% over the next year or decade (i.e., the ALP survey questions). Panels D, E, and F report descriptive statistics for 673 students who have sufficient data to compute their inferred variance ratio based on their perceived 10^{th} and 90^{th} percentiles of returns over the next year and average annual returns over the next decade (i.e., the CFO survey questions). Panel G summarizes the variance ratio information and presents differences for students with sufficient data to compute variance ratios based on both sets of questions. The expected return inferred from the 10^{th} and 90^{th} percentile values is denoted with the superscript ‡. The Internet Appendix provides details of the student surveys.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Students' stock market expectations over next year (ALP total return questions)								
P(market>0)	322	0.659	0.500	0.700	0.800	0.189	0.747	0.581
P(market>20%)	322	0.298	0.150	0.250	0.400	0.185	0.330	0.646
P(market<-20%)	322	0.216	0.100	0.200	0.300	0.149	0.063	0.118
$E_{i,t}(r_{1year})$	322	0.049	-0.041	0.001	0.107	0.184	0.093	0.733
$\sigma_{i,t}(r_{1year})$	322	0.433	0.191	0.297	0.521	0.361	0.201	0.276
Panel B: Students' stock market expectations over next decade (ALP total return questions)								
P(market>0)	322	0.757	0.600	0.850	0.950	0.244	0.958	0.814
P(market>20%)	322	0.544	0.360	0.600	0.750	0.241	0.929	0.988
P(market<-20%)	322	0.169	0.050	0.100	0.250	0.143	0.014	0.068
$E_{i,t}(r_{10years})$	322	0.428	0.050	0.282	0.680	0.475	0.962	0.907
$\sigma_{i,t}(r_{10years})$	322	0.673	0.297	0.505	0.922	0.500	0.474	0.435
Panel C: Students' variance ratios (ALP total return questions)								
"ALP" variance ratio	322	0.712	0.086	0.238	0.639	1.154	0.559	0.708
%Variance ratio<1	322	0.807						
Panel D: Students' stock market expectations over next year (CFO average return questions)								
$E_{i,t}(r_{1year})$	673	0.202	0.077	0.113	0.262	0.198	0.093	0.314
$P90(r_{1year})$	673	0.273	0.113	0.182	0.405	0.254	0.301	0.697
$P10(r_{1year})$	673	0.092	0.030	0.049	0.095	0.109	-0.138	0.001
$E_{i,t}(r_{1year})^{\ddagger}$	673	0.174	0.072	0.116	0.250	0.135	0.093	0.376
$\sigma_{i,t}(r_{1year})$	673	0.065	0.023	0.043	0.087	0.057	0.201	0.942
Panel E: Students' stock market expectations over next decade (CFO average return questions)								
$E_{i,t}(r_{10years})$	673	1.439	0.583	0.953	1.823	1.350	0.962	0.574
$P90(\bar{r}_{10years})$	673	0.205	0.095	0.140	0.262	0.169	0.152	0.554
$P10(\bar{r}_{10years})$	673	0.067	0.020	0.049	0.095	0.102	0.034	0.407
$E_{i,t}(r_{10years})^{\ddagger}$	673	1.323	0.576	0.943	1.788	1.084	0.962	0.532
$\sigma_{i,t}(\bar{r}_{10years})$	673	0.051	0.019	0.035	0.068	0.042	0.047	0.585
$\sigma_{i,t}(r_{10years})$	673	0.512	0.187	0.355	0.680	0.417	0.474	0.585

Table III –
Students' Reported Beliefs Regarding Near- and Long-Term Stock Returns (Cont.)

Panel F: Students' variance ratios (CFO average return questions)								
"CFO" variance ratio	673	13.570	2.988	9.318	13.596	17.694	0.559	0.067
%Variance ratio<1	673	0.098						
Panel G: Students' differences in variance ratios								
ALP total ret. VR	241	0.789	0.100	0.241	0.745	1.250	0.559	0.693
CFO average ret. VR	241	13.541	3.365	9.462	13.742	17.148	0.559	0.066
(CFO VR)/(ALP VR)	241	119.765	6.552	25.235	117.004	275.326		
CFO VR - ALP VR	241	12.752	2.712	8.706	13.483	16.954		
(CFO VR - ALP VR)> 0	241	0.946						

Table IV – Students' Perceptions of Historical Return Distributions

Panel A reports descriptive statistics for 251 undergraduate students who have sufficient data to compute their inferred variance ratio based on their perceived likelihood that, historically, markets have risen or fallen by at least 20% over 1- and 10-year periods (i.e., the ALP survey questions reframed as perceived historical return distributions). Panel B reports descriptive statistics for 290 students who have sufficient data to compute their inferred variance ratio based on their perceived 10^{th} and 90^{th} percentiles of historical 1- and 10-year returns (i.e., the CFO survey questions reframed as perceived historical return distributions). Panel C summarizes the variance ratio information and presents differences for students with sufficient data to compute variance ratios based on both sets of questions. The Internet Appendix provides complete summary information (analogous to Table III).

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Students' variance ratios based on historical total return (ALP) questions								
$\sigma_{i,t}(r_{1year})$	251	0.325	0.158	0.237	0.387	0.249	0.201	0.430
$\sigma_{i,t}(r_{10years})$	251	0.508	0.225	0.362	0.601	0.418	0.474	0.618
"ALP" variance ratio	251	0.599	0.086	0.199	0.618	0.939	0.559	0.737
%Variance ratio<1	251	0.825						
Panel B: Students' variance ratios based on historical average return (CFO) questions								
$\sigma_{i,t}(r_{1year})$	290	0.054	0.021	0.041	0.076	0.042	0.201	1.000
$\sigma_{i,t}(\bar{r}_{10years})$	290	0.074	0.026	0.048	0.094	0.079	0.047	0.490
$\sigma_{i,t}(r_{10years})$	290	0.675	0.257	0.481	0.941	0.555	0.474	0.490
"CFO" variance ratio	290	50.836	5.811	11.454	40.831	86.681	0.559	0.034
%Variance ratio<1	290	0.066						
Panel C: Students' differences in variance ratios based on historical return questions								
ALP total ret. VR	139	0.681	0.086	0.199	0.624	1.044	0.559	0.727
CFO average ret. VR	139	43.510	7.432	11.527	38.928	71.989	0.559	0.007
(CFO VR)/(ALP VR)	139	435.357	16.662	88.294	210.344	1593.260		
CFO VR - ALP VR	139	42.829	6.363	10.844	38.791	71.954		
(CFO VR - ALP VR)> 0	139	0.978						

Table V – The Importance of Average Annual versus Cumulative Returns

The first four rows of Panel A reports the median inferred variance ratio for samples of students answering the ALP survey questions reframed as average annual returns. In a similar manner, the initial four rows of Panel C reports the median inferred variance ratio for samples of students answering the CFO average annual return questions. The bottom four rows of Panels A and C report variance ratios for samples of students answering the ALP total return questions and the CFO survey questions, reframed as cumulative returns, respectively. Panels B and D report Z-statistics for comparisons across the indicated groups.

Panel A: ALP survey's return likelihood question structure							
Row no.	Returns	Horizon	Past or future	Return above	Return below	N	Variance ratio
1	Average annual	10	Future	≥10%	≤-5%	40	6.385
2	Average annual	10	Future	≥10%	≤-5%	33	2.389
3	Average annual	5	Past	≥10%	≤-5%	47	7.149
4	Average annual	15	Past	≥10%	≤-5%	44	11.164
5	Cumulative	10	Future	≥10%	≤-10%	14	0.403
6	Cumulative	10	Future	≥30%	≤-30%	34	0.289
7	Cumulative	5	Past	≥20%	≤-20%	41	0.466
8	Cumulative	15	Past	≥20%	≤-20%	26	0.245
Panel B: ALP survey's return likelihood question structure—Test statistics							
	Compare	Sample	Group 1 rows	Group 2 rows	Z-statistic		
9	Ave. annual vs. cumul.	Future and past	1, 2, 3, 4	5, 6, 7, 8	11.727***		
10	Ave. annual vs. cumul.	Future only	1, 2	5, 6	6.589***		
11	Ave. annual vs. cumul.	Past only	3, 4	7, 8	9.789***		
12	5 year vs. 15 yr horizon	Ave. annual	3	4	-1.770*		
13	5 year vs. 15 yr horizon	Cumulative	7	8	2.392**		
Panel C: CFO survey confidence interval question structure							
Row no.	Returns	Horizon	Past or future	Likelihood		N	Variance ratio
14	Average annual	10	Past	1 in 20		48	18.818
15	Average annual	10	Past	1 in 5		46	9.326
16	Average annual	5	Future	1 in 10		71	5.000
17	Average annual	15	Future	1 in 10		73	18.657
18	Cumulative	10	Past	1 in 5		48	0.130
19	Cumulative	10	Past	1 in 20		56	0.164
20	Cumulative	5	Future	1 in 10		69	0.489
21	Cumulative	15	Future	1 in 10		75	0.264
Panel D: CFO survey confidence interval question structure—Test statistics							
	Compare	Sample	Group 1 rows	Group 2 rows	Z-statistic		
22	Ave. annual vs. cumul.	Future and past	15, 16, 17, 18	19, 20, 21, 22	16.317***		
23	Ave. annual vs. cumul.	Future only	17, 18	21, 22	12.000***		
24	Ave. annual vs. cumul.	Past only	15, 16	19, 20	10.789***		
25	5 year vs. 15 year horizon	Ave. annual	17	18	-5.149***		
26	5 year vs. 15 year horizon	Cumulative	21	22	1.496		

Table VI – Panel Regressions of Variance Ratios on Characteristics

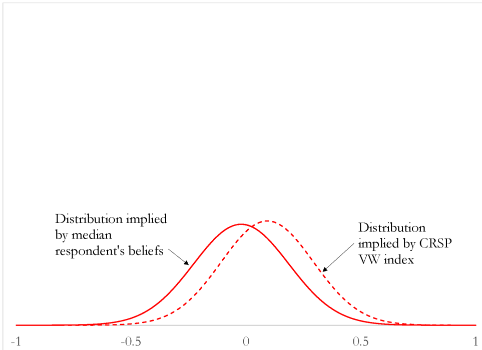
This table reports coefficients from panel regressions of ALP respondents' variance ratios on their characteristics. Control variables include wave fixed effects, respondent age, and indicators for gender, White race, married, working, and retired. All reported explanatory variables, except the indicator variables for holding equity and the expected change in financial position (better next year) are standardized (rescaled to zero mean and unit variance). Standard errors are clustered at the respondent level. The mean reversion variable is multiplied by -1, thus, larger values indicate stronger mean reversion beliefs. Significance at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively.

Description	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Years education	0.053***									0.020***	0.012
Income		0.054***								0.017**	-0.005
Holds equity indicator			0.132***							0.066***	0.051*
Understands market				0.094***						0.069***	0.069***
Numeracy					0.096***						0.054***
Financial literacy						0.130***					0.048***
Overconfidence							0.013				0.015
Better next year								0.034***		0.021**	0.016
Mean reversion ($\times$-1)									-0.037***	-0.031***	-0.047**
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	22,693	22,620	22,684	22,630	11,466	8,165	8,144	22,545	21,508	21,310	6,901

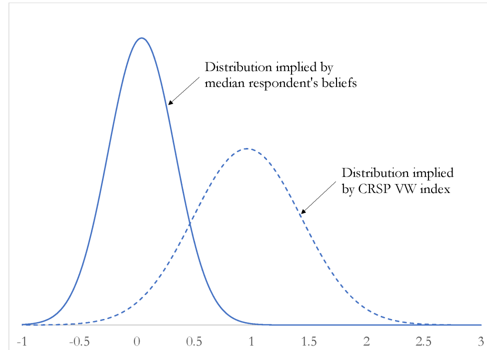
Table VII – Panel Regressions of Variance Ratios on Respondents' Compression Errors

This table reports coefficients from panel regressions of ALP respondents' perceived variance ratios on the difference between implied (assuming independence) 5-year likelihood given their reported 1-year likelihood and their reported 5-year event likelihood (i.e., their compression error). Control variables include wave fixed effects, respondent age, and indicators for gender, White race, married, working, and retired. All variables are standardized (rescaled to zero mean and unit variance) and standard errors are clustered at the respondent level. Significance at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively.

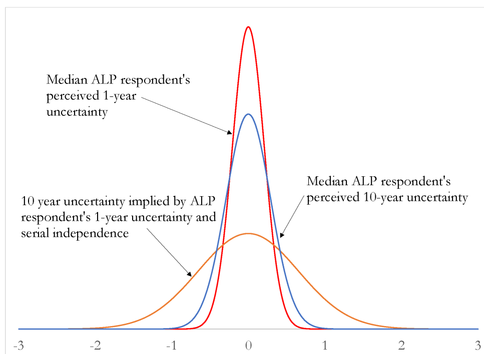
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
P(car accident)	-0.049***							
P(cavity filled)		-0.068***						
P(die)			-0.048***					
P(theft victim)				-0.038**				
P(move)					-0.060***			
P(die terrorism)						-0.060***		
P(break in victim)							-0.071***	
P(visit dentist)								-0.047**
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	7,852	5,927	7,774	7,584	3,911	5,764	7,471	1,704



(A) ALP median respondent's 1-year return distribution

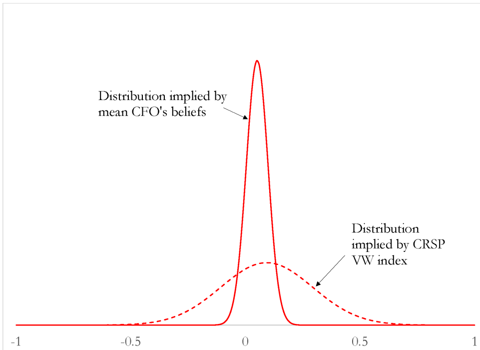


(B) ALP median respondent's 10-year total return distribution

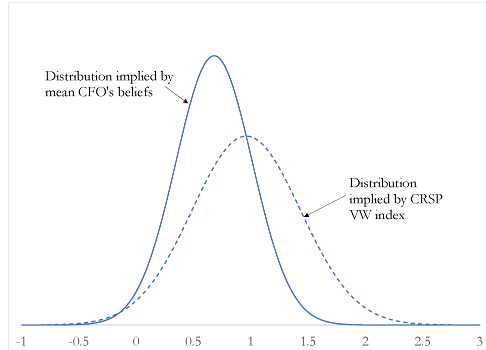


(C) ALP median respondent's 1- and 10-year uncertainty, and implied 10-year uncertainty

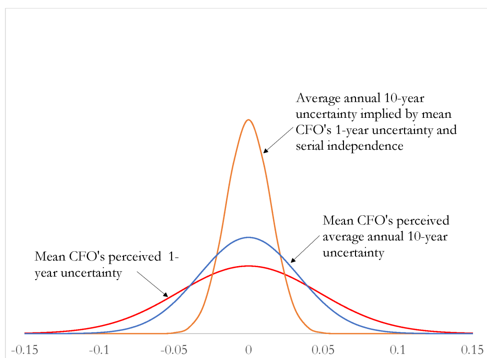
FIGURE 1 – HISTORICAL AND PERCEIVED RETURN DISTRIBUTIONS FOR ALP RESPONDENTS. Figure 1A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926-2020 period (dashed red line) and the imputed 1-year distribution of the median ALP respondent beliefs over 2008-2016 period (solid red line). Figure 1B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926-2020 period (dashed blue line) and the imputed 10-year total return distribution based on the median ALP respondent beliefs (solid blue line). Figure 1C rescales and centers (at zero) the ALP respondent's 1-year imputed distribution (red solid line), 10-year total return imputed distribution (blue solid line), and implied 10-year total return distribution given the ALP respondent's 1-year distribution and serial independence (orange solid line).



(A) Average CFO 1-year return distribution

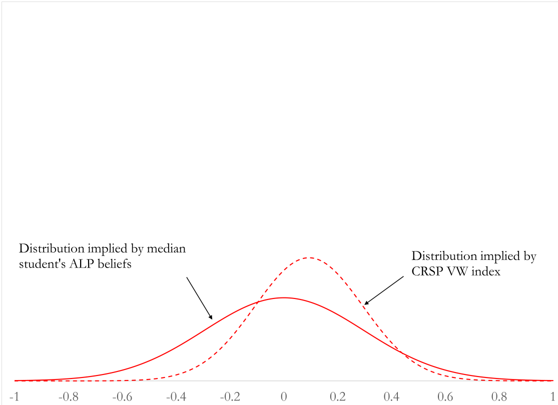


(B) Average CFO 10-year total return distribution

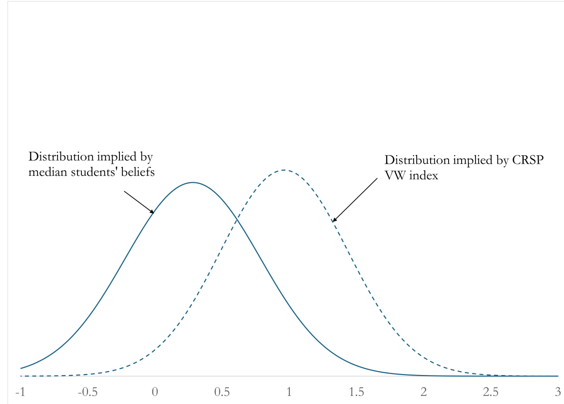


(C) Average CFO 1- and 10-year uncertainty, and implied 10-year uncertainty

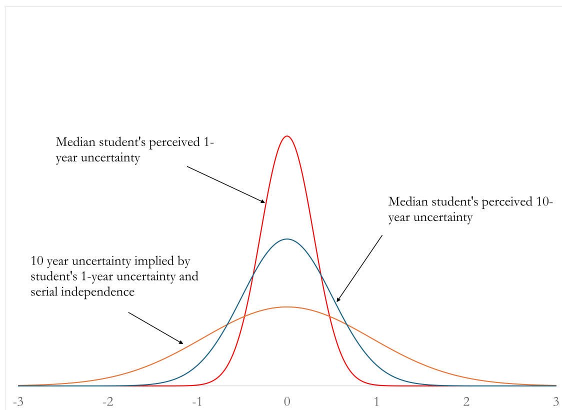
Figure 2 – Historical and perceived return distributions for CFOs. Figure 2A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926-2020 period (dashed red line) and the imputed 1-year distribution based on the mean CFO's surveys over 2004-2019 (solid red line). Figure 2B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926-2020 period (dashed blue line) and the imputed 10-year return distribution based on the mean CFO's beliefs (solid blue line). Figure 2C rescales and centers (at zero) the mean CFO's 1-year imputed distribution (solid red line), average annual return over 10 years imputed distribution (solid blue line), and implied average annual return over 10 years distribution given the CFO's 1-year distribution and serial independence (solid orange line).



(A) Median student's 1-year return distribution based on ALP questions

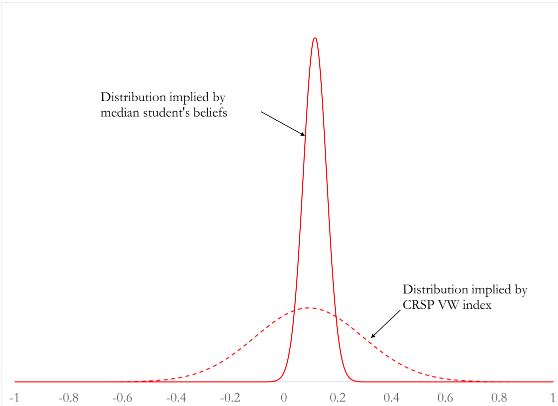


(B) Median student's 10-year total return distribution based on ALP questions

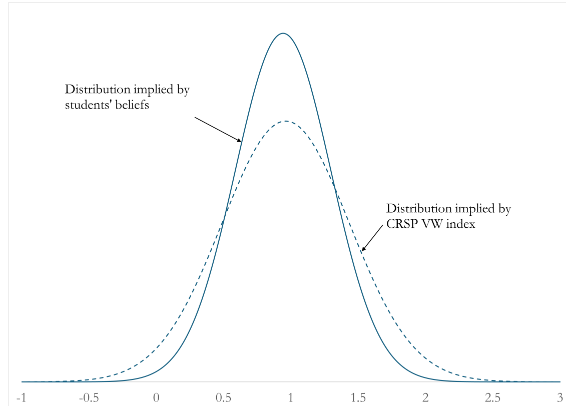


(C) Median student's 1- and 10-year uncertainty, and implied 10-year uncertainty based on ALP questions

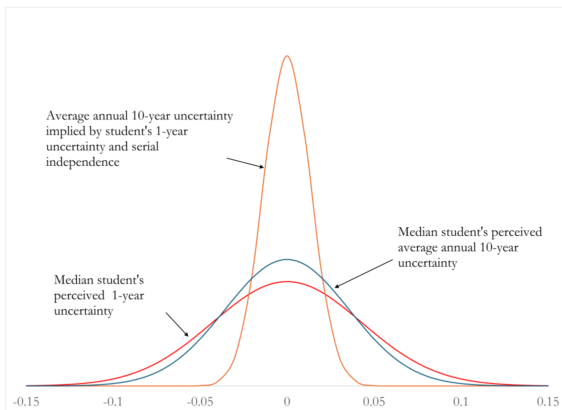
Figure 3 – Historical and perceived return distributions for students based on ALP total return question format. Figure 3A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926-2020 period (dashed red line) and the imputed 1-year distribution of median student beliefs based on the American Life Panel survey questions (solid red line). Figure 3B reports the distribution of 10-year returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926-2020 period (dashed blue line) and the imputed 10-year return distribution based on the median student beliefs (solid blue line). Figure 3C rescales and centers (at zero) the student's 1-year imputed return distribution (red solid line), 10-year imputed total return distribution (blue solid line), and implied 10-year total return distribution given the student's 1-year distribution and serial independence (orange solid line).



(A) Median student's 1-year return distribution based on CFO questions



(B) Median student's 10-year total return distribution based on CFO questions



(C) Median student's 1- and 10-year uncertainty, and implied 10-year uncertainty based on CFO questions

Figure 4 – Historical and perceived return distributions for students based on CFO average annual return question format. Figure 4A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926-2020 period (dashed red line) and the imputed 1-year distribution of median student beliefs based on the Duke CFO survey question (solid red line). Figure 4B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926-2020 period (dashed blue line) and the imputed 10-year total return distribution based on the median student beliefs (solid blue line). Figure 4C rescales and centers (at zero) the median student's 1-year imputed distribution (solid red line), average annual return over 10 years imputed distribution (solid blue line), and the implied average annual return over 10 years distribution given the student's 1-year distribution and serial independence (solid orange line).

Internet Appendix for
"Uncertain Uncertainty"

Contents

IA-1 Descriptive statistics

Table IA-I reports descriptive statistics of ALP respondent characteristics for our pooled cross-sectional time-series of 22,748 observations (from 3,023 individuals; the average respondent participates in more than 7.5 long-form surveys) that include any individual-survey wave observation where the respondent has adequate data to estimate their variance ratio. Our ALP sample is 58% female, 88% White race, 66% married, 61% working, and 23% retired. As shown in the bottom row, approximately two-thirds of the observations have negative mean-reversion coefficient estimates (i.e., perceive higher likelihood of a positive return over the next decade when lag returns are lower).

[Insert Table IA-I about here]

IA-2 Perceived personal and general economic changes

We use the answer to the question *Now looking ahead – do you think a year from now you will be better off financially, worse off, or about the same as now?* as a proxy for changes in expected risk based on the hypothesis that, on average, those who expect their personal financial situation to worsen also perceive greater future economic uncertainty and riskier equity markets. In two ALP surveys not in our sample—ALP survey 17 (in the field between December 2007 and April 2008) and ALP survey 61 (in the field between January-March 2009)—respondents were asked this question and also their expectations for business (*And how about a year from now, do you expect that in the country as a whole business conditions will be better, or worse than they are at present, or just about the same?*) and unemployment (*How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same or less?*). We reverse score unemployment so expected correlations are all positive (e.g., lower unemployment associated with stronger business conditions). Sample sizes for these questions for each survey range from 501 to 555. The six correlations between the three metrics (for two surveys) are statistically significant at the 1% level and range from 0.22 (ALP survey 61 correlation between expected changes in personal financial situation and expected changes in employment) to 0.47 (ALP survey 61 correlation between expected changes in personal financial situation and expected changes in business conditions). Given the negative relation between stock market volatility and the real economy (e.g., Hamilton and Lin (1996)), the results are consistent with the hypothesis that changes in personal financial situation proxy for expected changes in market volatility.

IA-3 Student surveys—question order

Both the ALP and CFO survey questions were given to undergraduate business students enrolled in introductory finance courses at two large public universities. In the fall 2024 semester, one section of students at University A were first given the CFO survey questions then two weeks later given the ALP survey questions. Students in the second section at University A were first given the ALP survey questions then two weeks later given the CFO survey questions. Table IA-II reports the distribution of inferred variance ratios for each section (analogous to Panel C and F in Table III). The results reveal the same pattern in both sections. Specifically, median variance ratios based on the ALP survey questions were 0.198 and 0.140 in the two sections. A difference in medians test reveals no evidence the difference was statistically significant (p-value=0.82). Correspondingly, the median variance ratios based on the CFO survey questions were 9.81 and 7.15, and as with the ALP variance ratios, the difference was not statistically meaningful (p-value=0.55).

[Insert Table IA-II about here]

IA-4 Spring and fall 2024 student surveys—additional detail

A total of 828 students completed all four ALP questions required to estimate variance ratios (i.e., chance market rises at least 20% in next year or decade; chance market falls at least 20% in next year or decade). As detailed in the study, these forecasts must follow probability laws to estimate variances. Specifically, (1) the sum of the perceived likelihoods market rise at least 20% in the next year (decade) and markets fall at least 20% in then next year (decade), must sum to less than 100%, and (2) none of the perceived likelihoods can be zero. Of the 828 students, 238 report values that sum to exactly 100% (e.g., 50% chance market rises by 20% in the next year and 50% chance markets fall by at least 20% in the next year), 193 report values that sum to greater than 100% (e.g., 80% chance market rises by at least 20% in the next decade and 25% chance it falls by at least 20% in the next decade), and 75 students report a 0% likelihood for at least one of the four required probabilities. Panels A and B of Table IA-III report the descriptive statistics for the perceived return likelihoods (i.e., analogous to the first three rows of Panels A and B in Table III) for the complete sample ($n = 828$) of students. The results, based on raw (i.e., unwinsorized) data, exhibit the same patterns as reported in Table III.

[Insert Table IA-III about here]

A total of 824 students complete all four CFO questions required to estimate near- and long-term variances (i.e., 10^{th} and 90^{th} return percentiles for the next year and average annual return percentiles for the next decade). To estimate variances requires that the 10^{th} percentile return is less than the 90^{th} percentile return. A total of 151 students violate this constraint (e.g., report a 1-in-10 chance market returns are lower than 50% in the next year (decade) and a 1-in-10 chance market returns are greater than 50% in the next year (decade)). Panels C and D of Table IA-III report the descriptive statistics for the perceived returns (i.e., analogous to the first three rows of Panels D and E in Table III) for the complete sample ($n = 824$) of students. Once again, the results, based on raw (i.e., unwinsorized) data, exhibit the same patterns as reported in Table III.

IA-5 Spring 2025 student surveys—additional detail

Each spring 2025 student was randomly assigned to one of four versions of our survey. In the first survey, students were randomly assigned one of four sets of survey questions asking about the forward looking likelihood (i.e., the ALP format) of either an average annual or cumulative return and the historical 10^{th} and 90^{th} percentiles (i.e., the CFO format) for average annual or cumulative returns. In the second survey, two weeks later, students were asked about forward looking 10^{th} and 90^{th} percentiles (i.e., the CFO format) for average annual or cumulative returns and the historical likelihood (i.e., the ALP format) of either an average annual or cumulative return. Thus, in Table V, each student is assigned one ALP survey about future returns (reported in rows 1, 2, 5, 6) and and one CFO survey about historical values (reported in rows 14, 15, 18, 19). In the second wave of the survey, two weeks later, each student is assigned a CFO survey about future returns (reported in rows 16, 17, 20, 21) and an ALP survey about past returns (reported in rows 3, 4, 7, 8).

The four initial surveys are given by the ordered pairs reported in rows (1, 18), (2, 19), (5, 14), and (6, 15), respectively. In a similar manner, two weeks later, the second wave surveys are given by the ordered pairs reported in rows (16, 8), (17, 7), (20, 4), and (21, 3). Note that each survey is different except for rows 1 and 2 (which are identical but represent two distinct first set of questions in the initial wave survey). In each wave, and between each set of CFO or ALP style questions, as a palette cleanser, we ask several financial literacy questions.

IA-6 Understanding America Survey

Our UAS sample is based on data from two surveys executed six years apart. In January of 2025 (ALP survey 685), 9,455 individuals completed the six ALP-style survey questions (see Section 3.1). We limit the sample to the 3,644 individuals whose answers do not violate probability laws (i.e., the sum of the probability of a return less than 20% and a return greater than 20% is less than 100% and that neither probability is zero). Panels A, B, and C of Table IA-IV (directly analogous to Panels A, B, and C of Table III) reports summary statistics for UAS respondents' answers, inferred expected returns, inferred standard deviations, and variance ratios based on the ALP style total return questions. The results are nearly identical to the corresponding results for ALP participants (Table I) and students (Table III). For example, the median UAS ALP-survey style variance ratio of 0.187 is between the median value for ALP participants (0.147; Panel C of Table I) and students (0.238; Panel C of Table III).

[Insert Table IA-IV about here]

In UAS survey 184—in the field between May and June 2019—UAS respondents were asked about average annual returns (i.e., CFO survey style questions). Specifically, respondents were first asked about their expectations for returns over the next decade:

Please answer the next questions based on your best guess on how the stock market will perform over the next 10 years. Here is an example of what we mean by average annual (yearly) return: It is a simple average of each year's return. So if in the first year the stock market goes up by 2% and in the second year market goes up by 4%, the average annual return over two years is 3% - we want your own best guess, so please do not look anything up.

I expect the average annual return over the next 10 years will be ___

I believe that there is a small (1-in-10, or 10 percent) chance the actual return over 10 years will be less than ___

I believe that there is a small (1-in-10 or 10 percent) chance the actual return over 10 years will be greater than ___

UAS respondents were then asked (in the same survey) about their perceptions of annual returns over the previous 60 years:

We are interested in finding out what people know and do not know about the history of stock market returns. The next question asks about what you think about historical stock market returns over the past 60 years in the S&P 500. Over that time, there were some years, or periods of years, where returns were much higher, or much lower, than the average over the whole time period.

Please answer the next questions about the last 60 years of the stock market history (from 1959 to 2018), as measured by S&P 500 index. If you are not sure, we would like your best guess - please do not look up any answers.

I believe that the average annual return over the past 60 years was ___

During the 6 lowest years of the market over that time, returns were less than ___

During the 6 highest years of the market over that time, returns were greater than ___

Thus, the 1-year horizon questions differ from the CFO survey questions in that (1) it asks about historical returns, and (2) it rephrases the 10^{th} and 90^{th} percentile question from a 1-in-10 chance to the six lowest (or highest) years over the past 60 years. A total of 2,461 UAS respondents answered all six questions from the May-June 2019 survey. A total of 41% of respondents' answers violate probability laws (e.g., report a 90^{th} percentile equal to their 10^{th} percentile) leaving a sample of 1,439 UAS respondents. Panels D, E, and F of Table IA-IV (directly analogous to Panels D, E, and F of Table III) report summary statistics for UAS respondents' answers, inferred expected returns, inferred standard deviations, and variance ratios based on the CFO-style average annual return questions. The results are fully consistent with the patterns for both CFOs and students. For instance, the median inferred UAS 1-year standard deviation, based on estimates of 10^{th} and 90^{th} percentile returns, is 0.036 versus 0.033 for CFOs (Table II) and 0.043 for students (Table III).[47] Similarly, based on the CFO-style average annual return questions, the vast majority of variance ratios for UAS participants (82%, see Table IA-IV), students (93%, see Table III), and the median CFO (100%, see Table II) are greater than historical market variance ratio (0.56).

Panel G in Table IA-IV (directly analogous to Panel G of Table III) reports descriptive statistics for the 472 UAS respondents who completed both the 2019 and 2025 UAS surveys. Once again, the results are consistent with our student surveys, as the vast majority of both UAS respondents (79%) and students (95%) generate a larger variance ration when asked about average annual returns (denoted "CFO variance ratio") than total returns (denoted "ALP variance ratio). In sum, despite the differences in the wording of the UAS questions, the fact that UAS CFO-style annual return questions are based on historical values, and the fact the UAS ALP- and CFO-style surveys are six years apart, the results in Table IA-IV are fully consistent with our hypothesis.

IA-7 Perceived historical return distributions

Table IA-V reports distributions for students' estimates of the historical 1- and 10-year return distributions used to generated the values reported in Table IV, based on both the ALP survey's style total return questions and the CFO's survey's style average return questions.

[Insert Table IA-V about here]

IA-8 Respondent Characteristic Sorts and Variance Ratio Heterogeneity

We sort ALP respondents into two groups by each of the 15 characteristics listed in the panel descriptions of Table IA-VI. In each of the 29 ALP survey waves, we compute the cross-sectional median variance ratio as well as the fraction of variance ratios less than 1 and the fraction less than the historical market variance ratio (0.559). Finally, for each group and wave, we test whether the fraction of variance ratios less than 1, or less than 0.559, differs meaningfully from 50%. In total, we examine 1,740 tests (i.e., 15 characteristics x 2 groups x 29 waves x 2 tests) of whether the variance ratio is less than 1, or less than 0.559. The first two rows of each panel in Table IA-VI report the time-series averages of the cross-sectional median variance ratios and cross-sectional average fraction of variance ratios less than 1, or 0.559.

[Insert Table IA-VI about here]

The results in Table IA-VI reveal that unrealistically small variance ratios in the ALP sample are ubiquitous. The largest reported (cross-sectional average of time-series median) variance ratio in Table IA-VI is 0.229 for high numeracy individuals in Panel E. In addition, the fraction of variance ratios less than 0.559, and less than 1, are always meaningfully greater than 50%. That is, all 1,740 tests summarized in the last two columns are statistically significant at the 1% level.

[47] UAS respondents appear more miscalibrated than either students or CFOs about long-term returns, i.e., the median 10-year 90^{th} percentile average annual return for UAS respondents, students, and CFOs are, respectively, 9.5%, 14%, and 13% (versus a historical value of 15%). As a result, long-term inferred variances are lower for UAS participants relative to CFOs and students. In all three cases, however, long-term inferred variances are too large relative to near-term inferred values generating implausibly large variance ratios in all three samples when asking the CFO style average annual return questions.

Panels A-F in Table IA-VI report the variance ratios for the financial sophistication proxies (in each case, the top row represents the 'more' sophisticated group). The third row reports the difference in variance ratios and indicates whether the value differs meaningfully from zero (based on a pair t-test of the time-series of the 29 cross-sectional medians). Consistent with the hypothesis that variation in cognitive uncertainty plays a larger role than variation in perceived parameter uncertainty in explaining variance ratio heterogeneity, the differences in Panels A-F are all positive and statistically significant at the 1% level. Further consistent with variation in cognitive uncertainty contributing to variance ratio heterogeneity (as well as the Enke and Graeber (2023) evidence), Panels G and H reveal that men and older respondents tend to exhibit larger variance ratios than women and younger respondents, respectively. In both cases, the results are statistically meaningful at the 1% level. Panels I-L also reveal meaningful differences in variance ratios based on race, marital status, and retirement status. The results are consistent with the hypothesis that non-Whites, single, and non-retired respondents tend to exhibit greater cognitive uncertainty with respect to the near- and long-term total return distributions.

The results in Panels M and N reveal no support for the hypotheses that individuals with greater overconfidence or individuals who expect economic risk to increase in the future exhibit larger variance ratios (i.e., the differences in Panel M and N have the "wrong" sign). The results in Panel O, however, support the hypothesis that heterogeneity in variance ratios results, in part, from heterogeneity in mean-reversion beliefs as individuals who exhibit mean-reversion beliefs also exhibit significantly (at the 1% level) smaller variance ratios relative those who do not exhibit mean-reversion beliefs.

IA-9 Correlations between explanatory variables

Table IA-VII reports correlations between the explanatory variables for the Table VII panel regressions including Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Mean reversion (times -1), and Better next year for our pooled cross-sectional time-series of 22,748 observations (from 3,023 individuals).

[Insert Table IA-VII about here]

IA-10 Miscalibration and survey structure

Our results support the hypothesis that most economic agents have, at best, only the most rudimentary understanding of the relation between horizon and equity market risk. As a result, individuals' near- and long-term forecasts are compressed which can explain why asking about average annual returns generates implausibly large variance ratios while asking about cumulative long-term returns generates implausibly small variance ratios.

Our study also demonstrates that the compression of near- and long-horizon estimates occurs regardless of the approach used to measure perceived distributions. Specifically, the results in Table V demonstrate that questions framed in average annual long-horizon returns consistently generate variance ratios greater than 1 and questions framed as cumulative long-horizon returns consistently generate varaince ratios less than 1, regardless of whether respondents are asked about confidence intervals (i.e., the CFO survey style) or the likelihood of a return larger or smaller than some value (i.e., the ALP survey style).

Our focus in on the compression of near- and long-horizon estimates and does not focus on why the CFO survey style questions generate near-term forecasts are severely miscalibrated (e.g., Panel A of Table II, Panel D of Table III, and Panel D of Table IA-IV) but ALP style questions generate reasonable forecasts of near-term uncertainty (Panels A and D of Table I, Panel A of Table III, and Panel A of Table IA-IV). As noted by Ben-David, Graham, and Harvey (2013), miscalibrated beliefs, "happen because either most overestimate their ability to predict the future or because they underestimate the volatility of random events." Thus, one potential explanation for differences between ALP respondents' and CFOs' near-term volatility estimates is that CFOs are simply more overconfident of their ability to forecast market returns than a representative sample of Americans. Our student surveys, the UAS survey, and the Duke 2011 Q1 survey, however, all suggests that the extent of miscalibration in equity market distribution forecasts also depends on question format. First, similar to CFOs, both students and UAS respondents exhibit severely miscalibrated near-term beliefs when asked the CFO survey style 80% confidence interval questions (Panel D of Tables III and IA-IV) but, if anything, tend to overestimate near-term uncertainty (relative to historical values) when asked the ALP survey style questions regarding the likelihood of a 20% gain or fall in values over the next year (Panel A of Tables III and IA-IV). That is, both a broad sample of Americans (UAS participants) and students exhibit severely overprecise estimates of near-term volatility when answering the CFO survey questions, but underprecise estimates of near-term volatility when answering the ALP survey questions. In addition, as detailed in our study, the 2011 Q1 Duke survey demonstrates that (1) the typical CFO's estimate of near-term uncertainty is sensitive to question format and (2) the typical CFO anchors their 2011 and 2012 volatility estimate on an erroneous seed value for 2010.

Consistent with these patterns, work suggests that the extent of miscalibration depends, in part, on question structure.[48] For example, Juslin, Wennerholm, and Olsson (1999) evaluate three approaches to examining confidence in beliefs: confidence interval estimation, full range questions, and half range questions. The authors find severe overprecision in interval estimation, but only weak overprecision in full range questions, and underprecision in half range questions.[49] A number of factors appear to contribute to the miscalibration in confidence interval estimation. For example, work (Teigen and Jorgensen (2005)) demonstrates that respondents report nearly identical (statistically indistinguishable) intervals when asked for 90% confidence intervals, 70% confidence intervals, or 50% confidence intervals (see Hartzmark and Sussman (2024) for similar results in a finance context). As a result, miscalibration tends to be larger when the confidence interval widens. A popular explanation (e.g., Yaniv and Foster (1995), Yaniv and Foster (1997), and Cesarini, Sandewall, and Johannesson (2006)) for the severe miscalibration in confidence interval estimates is that respondents trade off accuracy (i.e., a properly calibrated confidence interval) and informativeness (i.e., a narrow confidence interval). Moreover, consistent with the accuracy-informativeness tradeoff, evidence suggests that respondents recognize their confidence intervals are overprecise. For instance, Cesarini, Sandewall, and Johannesson (2006) ask respondents 90% confidence intervals for 10 questions and then ask respondents to guess how many of their 90% confidence intervals contain the correct answer. The average individual responds six. Individuals not only recognize their own confidence intervals are too precise, but also recognize this pattern is pervasive—the average respondent estimates that other respondents' 90% confidence intervals will only contain six of 10 correct answers.

Consistent with previous evidence (e.g., Teigen and Jorgensen (2005), Hartzmark and Sussman (2024)), we also find that students' estimates are relatively insensitive to the size of the confidence interval. Specifically, using the data from Tables III and V, the red lines in Figure IA-1A report 90%, 80%, and 60% confidence intervals for historical annual market returns, while the blue lines report

[48]The literature examining overprecision is vast. We limit our discussion to closely related work.

[49]The authors give examples of each format. Specifically interval estimation is given by "Assess the (smallest) interval within which you are 80% certain that the population of Norway lies: Between ___ million and ___ million inhabitants." The full range question format is, "Norway has less than 6 million inhabitants. What is the probability that this statement is true?" Respondents then answer on an 11 point scale from a probability of 0% (certainly false) to 100% (certainly true). The half-range question format is "Does the population of Norway lie above or below 6 million?" The respondent selects above or below and then a probability from six alternatives ranging from 50% (just guessing) to 100% (certain).

the corresponding confidence intervals based on the median student 1-year return perceptions. The green line reports the median CFO 1-year return perceptions (based on the medians in Table II Panel A). For example, the middle red line shows that, historically, there is 80% chance the market's annual return is between -13% and 35%, but students, when asked the CFO style questions, estimate there is an 80% chance returns in the next year will between 5% and 18%, and CFOs estimate there is an 80% chance returns in the next year will between between 0% and 9%. Nearly identical to Figure 1 in Hartzmark and Sussman (2024), the range in market returns (i.e., the red lines) greatly shrink as the confidence interval falls, but student estimates (the blue lines) move little. As a result, the extent of miscalibration is much greater when confidence intervals are wider.

[Insert Figure IA-1 about here]

Figure IA-1B reports market values (red lines), median student estimates (blue lines), and median CFO estimates (green line) for average annual returns over the next decade. Similar to the annual results in Figure IA-1A, the 10-year horizon results in Figure IA-1B demonstrate that the range in average annual market returns over a decade (red lines) shrinks as the confidence interval narrows, but student estimates (blue lines) are much less sensitive to changes in the confidence interval, and therefore the level of miscalibration increases as the confidence interval widens.

We find a similar pattern for ALP style total return questions. Specifically, IA-2A reports the historical likelihood (red bars) annual returns fall within a range ($-30\% < r_{1year} < 30\%$; $-20\% < r_{1year} < 20\%$; $-10\% < r_{1year} < 10\%$) and the likelihood based on median student beliefs (blue bars). Analogous to the pattern for variation in confidence intervals (i.e., Figure IA-1), the likelihood of a 1-year market return within a given range falls systematically as the range decreases (i.e., the red bars shrink), while student estimates (blue bars) are largely immune to changes in the range. That is, the results are consistent with evidence that respondents tend to assign a significant likelihood to any event which is deemed important enough to be asked about (e.g., Clemen and Ulu (2008)). Figure IA-2B reports the corresponding values for cumulative 10-year returns. The red bars are short because, for example, historically, markets have earned more than 30% in 94% of 10-year periods (i.e., there are few 10-year periods where markets earned, cumulatively, less than 30%, less than 20%, or less than 10%). Although the median student's estimate is poor (i.e., students severely overestimate their ability to predict long-term total returns when answering the ALP style questions), the pattern goes in the "correct" direction as the blue bars shrink as the interval shrinks. In short, relative to historical values, students overestimate the likelihood a low 10-year return (e.g., the median student estimates a 10% likelihood of a 10-year market return less than -20% versus a historical likelihood of 1%; see Panel B of Table II), but greatly underestimate the likelihood of a high 10-year return (e.g., the median student estimates a 60% likelihood of a 10-year market return greater than than 20% versus a historical likelihood of 93%; see Panel B of Table II).

[Insert Figure IA-2 about here]

IA-11 Cognitive uncertainty in other domains

As detailed in the paper, ALP respondents were asked a series of questions regarding the likelihood of an event in the next year. Specifically, respondents were asked, "What is the percent chance that ___ during the next year?" where the blank was filled with:

...you will get into a car accident...
...you will have a cavity filled...
...you will die (from any cause – crime, illness, accident, and so on)...
...someone will steal something from you...
...you will move your permanent address to another state some time...
...you will die in a terrorist attack...
...someone will break into your home and steal something from you...
...you will visit a dentist, for any reason,...

Respondents were then asked the same set of questions regarding the next five years, i.e., "What is the percent chance that ___ during the next 5 years?"

Columns (2) and (3) in Table IA-VIII report the median reported likelihood for each question over the next year and the corresponding implied 5-year likelihood calculated assuming independence. Column (4) reports the median respondent-reported 5-year likelihood and column (5) reports the difference in medians between the 5-year likelihood assuming independence and the reported 5-year likelihood. Columns (6)-(9) report analogous mean values.

[Insert Table IA-VIII about here]

If cognitive uncertainty causes respondents to compress their annual and 5-year forecasts, the differences (columns 5 and 9) between their 5-year likelihoods implied by their 1-year forecasts and their elicited 5-year likelihoods will be positive. The final column reveals the fraction of respondents with a positive difference averages 87% across the eight questions and ranges from 68% (for the likelihood one moves out of state) to 94% (for the likelihood one is a theft victim). For instance, the median respondent reports a 20% chance of a car accident in the next year. Assuming that, on average, respondents' car crash risk is approximately independent over the next five years, the 5-year likelihood should be 67.2% (i.e., $1-(0.8^5)$) and the 2-year likelihood is 36% (i.e., $1-(0.8^2)$). Yet the typical respondent estimates only a 30% 5-year likelihood. In short, the results are uniformly consistent with the hypothesis that most individuals experience cognitive uncertainty regarding the relation between horizon and uncertainty. Specifically, as shown in the final column of Table IA-VIII, we can reject the hypothesis that the difference between implied 5-year likelihoods and estimated 5-year likelihoods are as likely to be positive as negative at the 1% level in every case.

IA-12 Variable details

Variable	Description
Female	Gender is identified in the pre-loaded demographic data for each "effects of the financial crisis" survey.
White race	Ethnicity is identified in the pre-loaded demographic data for each "effects of the financial crisis" survey.
Married	Current living situation is asked in each survey. Those who respond, "Married or living with a partner" are classified as married. All others (e.g., separated, divorced, widowed, never married) are classified as non-married.
Working	Current job status is identified in each survey. Respondents who report, "working now" are classified as working.
Retired	Current job status is identified in each survey. Respondents who report "retired" are classified as retired.
Age	Respondent age is reported in each survey. Respondents 50 and younger are classified as young while those older than 50 are classified as old.
Years Education	Respondents report 16 possible answers for "What is the highest level of school you have completed or the highest degree you have received?" We assign the following years of education for each answer (1) less than 1st grade=0, (2) 1st, 2nd, 3rd or 4th grade=2.5, (3) 5th or 6th grade=5.5, (4) 7th or 8th grade=7.5, (5) 9th grade=9, (6) 10th grade=10, (7) 11th grade=11, (8) 12 grade no diploma=12, (9) high school graduation=12, (10) some college but no degree=13, (11) associate degree in college occupational/vocational program=14, (12) associate degree in college academic program=14, (13) bachelor's degree=16, (14) master's degree=18, (15) professional school degree (e.g., MD, DDS, DVM, LLB, JD)=22, (16) Doctorate degree (e.g., PhD EdD)=22. We classify respondents with at least 13 years of education as "more" education.
Income	Respondents report values for family income questions. The first question, "family income" reports 14 possible income buckets—with the final bucket indicating income greater than \$75,000. "Family income part 2" asks those who report family income greater than \$75,000 to report income in four additional buckets. The 14 family income buckets are: inc $<$ \$5k, \$5k $\leq$ inc $<$ \$7.499k, \$7.5k $\leq$ inc $<$ \$9.999k, \$10k $\leq$ inc $<$ \$12.499k, \$12.5k $\leq$ inc $<$ \$14.999k, \$15k $\leq$ inc $<$ \$19.999k, \$20k $\leq$ inc $<$ \$24.999k, \$25k $\leq$ inc $<$ \$29.999k, \$30k $\leq$ inc $<$ \$34.999k, \$35k $\leq$ inc $<$ \$39.999k, \$40k $\leq$ inc $<$ \$49.999k, \$50k $\leq$ inc $<$ \$59.999k, \$60k $\leq$ inc $<$ \$74.999k, \$75k $\leq$ inc. The family income part 2 groupings are: \$75k $\leq$ inc $<$ \$99.999k, \$100k $\leq$ inc $<$ \$124.999k, \$125k $\leq$ inc $<$ \$199.999k, \$200k $\leq$ inc. For respondents who report income less than \$75K, we use the bucket midpoint. For respondents who report income of at least \$75k, but less than \$200k, we use the bucket midpoint of family income part 2. For respondents who report income greater than \$200k, we assume income is \$250k. We classify respondents with income of at least \$75,000 as "higher" income.

Numeracy	ALP survey 32 (in the field from 5-27-2008 to 6-30-2008) included a series of 17 questions used to compare measures of numeracy in Weller, Dieckmann, Tusler, Mertz, Burns, and Peters (2013). The authors conclude that the eight-item Rasch-based numeracy measure generally performed better than other measures. The Rasch metric consists of a series of increasingly difficult problems to better discriminate numeracy. The authors provide (via the ALP website for registered users; scores.xlsx), the Rasch based score for ALP survey 32 participants. We classify respondents with Rasch numeracy score greater than 4 as high numeracy and those with score of 4 or less as low numeracy.
Understand market	In 14 of the financial crisis waves (1, 2, 11, 14, 24, 38, 41, 50, 54, 55, 58, 59, 60, and 61) respondents are asked "How would you rate your understanding of the stock market with scores from 1 (extremely good) to 6 (extremely poor). We reverse score the variable (so higher values indicate greater understanding) and average the score of all waves for which the respondent answers this question. Those with an average score of at least 3 (understands the market extremely good, very good, somewhat good) are classified as high self-rated understanding of the market. Those with average scores less than 3 (somewhat poor, very poor, extremely poor) are classified as low self-rated understanding of the market.
Financial literacy	ALP survey 5 (in the field from 5-8-2006 to 11-1-2007) asked respondents a series of 13 financial literacy questions. The survey is describe in Parker, de Bruin, Yoong, and Willis (2012). We compute the fraction of the 13 questions correctly answered and classify respondents who correctly more than 75% (i.e., at least 10) of the questions as high financial literacy.
Overconf.	ALP survey 6 (in the field from 8-14-2006 to 11-20-2007) asked respondents 14 true/false general knowledge questions (e.g., alcohol causes dehydration). Respondents are also asked their confidence in each answer ranging from 50% (just guessing) to 100% (absolutely sure). Following Parker, de Bruin, Yoong, and Willis (2012), overconfidence is computed as the difference the average confidence in their answer and fraction of correct answers. We classify respondents with positive differences as overconfident and those with negative (or no) differences as not overconfident.
Holds equities	In the first two waves of the financial crisis surveys (both long-form), respondents (ST001) are asked, "Do [you (or your husband/wife/partner)] have any shares of stock or stock mutual funds? Please include stocks that [you (or your husband/wife/partner)] hold in an employer pension account." Thus, in the first two waves, respondents are classified as equity market participants based on this question. In all subsequent waves, respondents are asked (ST001), "In the next set of questions we will ask you about stock holdings. Please, do not include stock holdings that are part of an IRA, 401(k), Keogh or similar retirement accounts. Do [you(and/or your husband/wife/partner)] have any shares of stock or stock mutual funds?" In short-form wave 3 and all subsequent long-form waves, respondents who report having retirement account (RA001) are asked (RA006), "Are any of these retirement accounts invested in stocks or stock mutual funds, either fully or partially?" In these waves, we classify respondents who report owning stocks (either directly or in retirement account) as equity holders. Respondents who answer they do not have money in stock directly and either report no retirement stock or do not have a retirement account are classified as non-participants.

Better next year	In the 29 long-form surveys, respondents are asked, "Now looking ahead - do you think that a year from now you will be better off financially, worse off, or about the same as now?" We code respondents as 1 if they believe they will be better off in a year, -1 if they believe they will be worse off, and 0 if they believe they will be about the same. For the sorts, those who believe they will be better off are compared to those who believe they will be worse off.
Mean-reversion	In all 61 effects of the financial crisis surveys, respondents are asked "What are the chances that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more in 10 years than they are today?" For respondents who answer this question in at least 10 surveys, we regress the time-series of respondents' perceived likelihood markets rise in the next decade on scaled lag 12 month returns. Specifically, to ease in interpretation, we divide raw lag 12 month return by the standard deviation of CRSP 12-month returns such that the coefficient reflects the expected change in the likelihood markets rise in the next decade given a one standard deviation higher lag 12-month return. For the sorts, we classify those with a negative relation between lag returns and future expected long-term returns as the mean-reversion sample and those with non-negative coefficient as the no mean-reversion sample.
General comments	For many questions, respondents are prompted if they do not initially respond to a question. For instance, if respondents do not respond to the question regarding direct stock holdings, they are prompted, "[You did not answer. Your answers are important to us. Please answer the question to the best of your ability.] In the next set of questions we will ask you about stock holdings besides those that you may have already told us about. Do [you (and/or your husband/wife/partner)] have any shares of stock or stock mutual funds besides stock holdings that are part of an IRA, 401(k), Keogh or similar retirement accounts?" These responses generally show up as a second variable (e.g., no value for ST001, but a value for ST001_NR_DK) that we include in our analysis.

Table IA-I – Respondents' Characteristics Descriptive Statistics

This table reports descriptive statistics for ALP respondents' characteristics including Female, White race, Married, Working, Retired, Age, Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Better next year, Mean reversion, and Mean reversion<0. Characteristics are based on the pooled cross-sectional time-series of 22,748 observations (from 3,023 individuals; the average respondent participates in more than 7.5 long-form surveys) that include any individual-survey wave observation where the respondent has adequate data to estimate their variance ratio. Variable descriptions are provided in the Internet Appendix, Section IA-12.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.
Female	22,698	0.580	0.000	1.000	1.000	0.493
White race	22,748	0.878	1.000	1.000	1.000	0.327
Married	22,748	0.662	0.000	1.000	1.000	0.473
Working	22,747	0.609	0.000	1.000	1.000	0.488
Retired	22,747	0.229	0.000	0.000	0.000	0.420
Age	22,748	51.660	40.000	54.000	62.000	14.782
Years education	22,696	14.963	13.000	14.000	16.000	2.678
Income	22,623	71,505	37,500	55,000	87,500	51,672
Holds equity	22,735	0.589	0.000	1.000	1.000	0.492
Understand markets	22,681	3.194	2.444	3.286	4.000	1.076
Numeracy	11,468	4.496	3.000	5.000	6.000	1.764
Financial literacy	8,165	0.775	0.692	0.846	0.923	0.199
Overconfidence	8,144	-0.056	-0.114	-0.057	-0.007	0.095
Better next year	22,595	0.159	0.000	0.000	1.000	0.591
Mean reversion	21,548	-1.454	-4.060	-1.367	0.973	4.753
Mean reversion< 0	21,548	0.658				

Table IA-II – Survey Order Robustness Tests

The table reports results for two samples of undergraduate business students enrolled in an introductory finance course at a large public university. Students in Section 1 were given the CFO average annual return questions and then the ALP total return questions two weeks later. Students in Section 2 were given the ALP questions initially, and the CFO questions two weeks later. Panels A and B report summary statistics for the distribution of ALP and CFO variance ratios, respectively, for each section. Difference in medians tests are reported in the final row of each panel.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: ALP-style survey variance ratio results								
Section 1								
"ALP" variance ratio	37	0.980	0.079	0.198	0.789	1.613	0.559	0.649
%ALP variance ratio<1	37	0.757						
Section 2								
"ALP" variance ratio	37	0.566	0.061	0.140	0.400	1.186	0.559	0.784
%ALP variance ratio<1	37	0.892						
	Z	*p*-value						
Diff in median test	0.231	0.817						
Panel B: CFO-style survey variance ratio results								
Section 1								
"CFO" variance ratio	67	12.003	4.071	9.814	14.249	12.456	0.559	0.060
%CFO variance ratio<1	67	0.104						
Section 2								
"CFO" variance ratio	66	11.036	1.920	7.154	13.087	12.177	0.559	0.076
%CFO variance ratio<1	66	0.136						
	Z	*p*-value						
Diff in median test	-0.605	0.545						

Table IA-III – Additional Detail on Students' Beliefs

The table reports descriptive statistics for all respondents who provide estimates including those whose estimates violate probability laws. Panels A and B report, based on raw (i.e., unwinsorized) data, descriptive statistics for all 828 student respondents who answered the four ALP questions necessary to compute estimated variances over the next year or decade (i.e., chance market rises at least 20% in next year or decade; chance market falls at least 20% in next year or decade). Panels C and D report, based on raw (i.e., unwinsorized) data, beliefs for the 824 students who complete all four CFO questions required to estimate near- and long-term variances (i.e., 10th and 90th return percentiles for the next year and average annual return percentiles for the next decade).

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: ALP-style survey students' responses over next year								
P(market>0)	828	0.658	0.500	0.700	0.800	0.198	0.747	0.574
P(market>20%)	828	0.388	0.200	0.400	0.500	0.226	0.330	0.459
P(market<-20%)	828	0.317	0.130	0.300	0.500	0.220	0.063	0.110
Panel B: ALP-style survey students' responses over next decade								
P(market>0)	828	0.774	0.650	0.850	0.970	0.234	.958	0.746
P(market>20%)	828	0.645	0.500	0.700	0.800	0.244	0.929	0.882
P(market<-20%)	828	0.273	0.100	0.200	0.400	0.225	0.014	0.094
Panel C: CFO-style survey students' responses over next year								
$E_t(r_{1year})$	824	0.201	0.077	0.113	0.262	0.193	0.093	0.325
$P90(r_{1year})$	824	0.256	0.095	0.170	0.372	0.242	0.301	0.712
$P10(r_{1year})$	824	0.114	0.030	0.055	0.140	0.137	-0.138	0.001
Panel D: CFO-style survey students' responses over next decade								
$E_t(r_{10years})$	824	1.493	0.583	0.953	1.823	1.407	0.962	0.562
$P90(\bar{r}_{10years})$	824	0.198	0.086	0.140	0.262	0.167	0.152	0.567
$P10(\bar{r}_{10years})$	824	0.088	0.020	0.049	0.095	0.124	0.034	0.351

TABLE IA-IV – UNDERSTANDING AMERICA SURVEYS

Panels A, B, and C reports descriptive statistics for 3,644 individuals from the Understanding America Survey (UAS) who have sufficient data to compute their inferred variance ratio in January 2025 (UAS survey 685) based on their perceived likelihood markets rise or fall 20% over the next year or decade (i.e., the ALP style survey questions). Panels D, E, and F report descriptive statistics for 1,439 individuals from the 2019 UAS surveys (UAS survey 184) who have sufficient data to compute their inferred variance ratio based on their perceived *historical* 10^{th} and 90^{th} percentiles of annual returns over the previous 60 years, and their perceived *forward-looking* 10^{th} and 90^{th} percentiles of average annual returns over the next decade (i.e., the CFO style survey questions). Panel G summarizes the variance ratio information and presents differences for individuals with sufficient data to compute variance ratios based on both the 2019 CFO survey style average annual return questions and the 2025 ALP survey style total return questions. The expected return inferred from the 10^{th} and 90^{th} percentile values is denoted with the superscript ‡.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: UAS participants' stock market expectations over next year (ALP total return questions, 2025 survey)								
P(market>0)	3,644	0.442	0.230	0.470	0.610	0.254	0.747	0.837
P(market>20%)	3,644	0.272	0.100	0.240	0.400	0.197	0.330	0.667
P(market<-20%)	3,644	0.294	0.110	0.250	0.440	0.212	0.063	0.111
$E_{i,t}(r_{1year})$	3,644	-0.062	-0.126	-0.020	0.072	0.718	0.093	0.779
$\sigma_{i,t}(r_{1year})$	3,644	0.746	0.204	0.316	0.601	1.647	0.201	0.244
Panel B: UAS participants' stock market expectations over next decade (ALP total return questions, 2025 survey)								
P(market>0)	3,644	0.564	0.350	0.520	0.800	0.281	0.958	0.911
P(market>20%)	3,644	0.436	0.250	0.430	0.600	0.242	0.929	0.982
P(market<-20%)	3,644	0.249	0.100	0.200	0.370	0.179	0.014	0.027
$E_{i,t}(r_{10years})$	3,644	0.285	-0.034	0.065	0.381	1.110	0.962	0.904
$\sigma_{i,t}(r_{10years})$	3,644	1.172	0.287	0.505	1.040	2.177	0.474	0.465
Panel C: UAS participants' variance ratios (ALP total return questions, 2025 survey)								
"ALP" variance ratio	3,644	1.003	0.076	0.187	0.711	1.983	0.559	0.714
%Variance ratio<1	3,644	0.837						
Panel D: UAS participants' historical perceived annual return distributions (CFO average annual return questions, 2019 survey)								
$E_{i,t}(r_{1year})$	1,439	0.132	0.049	0.077	0.113	0.266	0.093	0.570
$P90(r_{1year})$	1,439	0.165	0.095	0.122	0.182	0.161	0.301	0.904
$P10(r_{1year})$	1,439	0.031	0.010	0.020	0.039	0.070	-0.138	0.008
$E_{i,t}(r_{1year})^{\ddagger}$	1,439	0.092	0.053	0.072	0.106	0.063	0.093	0.664
$\sigma_{i,t}(r_{1year})$	1,439	0.048	0.025	0.036	0.063	0.034	0.201	1.000

Table IA-IV – Understanding America Surveys (Cont.)

Panel E: UAS participants' stock market expectations over next decade (CFO average annual return questions, 2019 survey)								
$E_{i,t}(r_{10years})$	1,439	0.765	0.392	0.583	0.770	1.304	0.962	0.883
$P90(\bar{r}_{10years})$	1,439	0.120	0.058	0.095	0.122	0.124	0.152	0.831
$P10(\bar{r}_{10years})$	1,439	0.045	0.020	0.030	0.049	0.061	0.034	0.524
$E_{i,t}(r_{10years})^{\ddagger}$	1,439	0.770	0.392	0.582	0.858	0.562	0.962	0.806
$\sigma_{i,t}(\bar{r}_{10years})$	1,439	0.026	0.011	0.018	0.032	0.024	0.047	0.864
$\sigma_{i,t}(r_{10years})$	1,439	0.259	0.111	0.181	0.318	0.240	0.474	0.864
Panel F: UAS participants' variance ratios (CFO average annual return questions, 2019 survey)								
"CFO" variance ratio	1,439	6.805	0.867	2.595	7.277	10.476	0.559	0.180
%Variance ratio<1	1,439	0.270						
Panel G: UAS participants' differences in variance ratios								
ALP total ret. VR	469	1.195	0.100	0.255	0.908	2.162	0.559	0.674
CFO average ret. VR	469	19.843	0.892	2.546	7.146	98.559	0.559	0.171
(CFO VR)/(ALP VR)	469	375.177	1.474	7.654	51.415	4401.810		
CFO VR - ALP VR	469	18.648	0.078	1.905	6.214	98.677		
(CFO VR - ALP VR)> 0	469	0.785						

Table IA-V – Students Perceptions of Historical Return Distributions

Panels A, B, and C reports descriptive statistics for 251 undergraduate students who have sufficient data to compute their inferred variance ratio based on their perceived likelihood that, historically, markets have risen of fallen by at least 20% over 1- and 10-year periods (i.e., the ALP survey questions reframed as perceived historical distributions). Panels D, E, and F report descriptive statistics for 290 students who have sufficient data to compute their inferred variance ratio based on their perceived 10^{th} and 90^{th} percentiles of historical 1-year returns and historical average annual 10-year returns (i.e., the CFO survey questions reframed as perceived historical return distributions). Panel G summarizes the variance ratio information and presents differences for students with sufficient data to compute variance ratios based on both sets of questions. The perceived average historical return inferred from the 10^{th} and 90^{th} percentile values is denoted with the superscript ‡.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Student perceptions of historical 1-year stock market return (ALP questions)								
P(market>0)	251	0.643	0.500	0.700	0.800	0.251	0.747	0.562
P(market>20%)	251	0.246	0.100	0.200	0.350	0.179	0.330	0.737
P(market<-20%)	251	0.185	0.100	0.150	0.280	0.147	0.063	0.227
$E_{i,t}(r_{1year})$	251	0.029	-0.020	0.001	0.050	0.147	0.093	0.821
$\sigma_{i,t}(r_{1year})$	251	0.325	0.158	0.237	0.387	0.249	0.201	0.430
Panel B: Student perceptions of historical 10-year stock market return (ALP questions)								
P(market>0)	251	0.678	0.500	0.800	0.900	0.295	0.958	0.841
P(market>20%)	251	0.414	0.200	0.400	0.600	0.254	0.929	0.972
P(market<-20%)	251	0.183	0.050	0.150	0.260	0.142	0.014	0.072
$E_{i,t}(r_{10years})$	251	0.216	-0.020	0.065	0.357	0.355	0.962	0.932
$\sigma_{i,t}(r_{10years})$	251	0.508	0.225	0.362	0.601	0.418	0.474	0.618
Panel C: Students' variance ratios (based on reframed ALP questions)								
"ALP" variance ratio	251	0.599	0.086	0.199	0.618	0.939	0.559	0.737
%Variance ratio<1	251	0.825						
Panel D: Student perceptions of historical 1-year stock market return (CFO questions)								
$E_{i,t}(r_{1year})$	290	0.148	0.068	0.095	0.182	0.148	0.093	0.486
$P90(r_{1year})$	290	0.231	0.104	0.182	0.262	0.184	0.301	0.783
$P10(r_{1year})$	290	0.085	0.020	0.049	0.095	0.111	-0.138	0.003
$E_{i,t}(r_{1year})^{\ddagger}$	290	0.155	0.070	0.112	0.179	0.124	0.093	0.400
$\sigma_{i,t}(r_{1year})$	290	0.054	0.021	0.041	0.076	0.042	0.201	1.000

Table IA-V – Students Perceptions of Historical Return Distributions (Cont.)

Panel E: Student perceptions of historical 10-year stock market return (CFO questions)								
$E_{i,t}(r_{10years})$	290	2.583	0.953	1.398	3.716	2.560	0.962	0.383
$P90(\bar{r}_{10years})$	290	0.329	0.140	0.223	0.470	0.286	0.152	0.359
$P10(\bar{r}_{10years})$	290	0.140	0.049	0.095	0.182	0.146	0.034	0.117
$E_{i,t}(r_{10years})^{\ddagger}$	290	2.222	0.943	1.592	3.275	1.568	0.962	0.286
$\sigma_{i,t}(\bar{r}_{10years})$	290	0.074	0.026	0.048	0.094	0.079	0.047	0.490
$\sigma_{i,t}(r_{10years})$	290	0.675	0.257	0.481	0.941	0.555	0.474	0.490
Panel F: Students' variance ratios (based on reframed CFO questions)								
"CFO" variance ratio	290	50.836	5.811	11.454	40.831	86.681	0.559	0.034
%Variance ratio<1	290	0.066						
Panel G: Students' differences in variance ratios (historical return perceptions)								
ALP total ret. VR	139	0.681	0.086	0.199	0.624	1.044	0.559	0.727
CFO average ret. VR	139	43.510	7.432	11.527	38.928	71.989	0.559	0.007
(CFO VR)/(ALP VR)	139	435.357	16.662	88.294	210.344	1593.260		
CFO VR - ALP VR	139	42.829	6.363	10.844	38.791	71.954		
(CFO VR - ALP VR)> 0	139	0.978						

Table IA-VI – Variance Ratio by Respondent Characteristics

We sort respondents by characteristic (education level, income, stock market participation, understanding of the stock market, numeracy, financial literacy, gender, age, race, marital status, employment status, retirement status, overconfidence, economic expectations, and beliefs about mean-reversion) for each of 29 long-form waves. The initial two rows of the first column report the time-series mean of the 29 cross-sectional median variance ratio for respondents in that group. Columns 2 and 3 present the time-series average (across the 29 waves) of the cross-sectional mean percentage of variance ratios less than 1 and the historical average for US equity markets (0.559), respectively. For each wave and sample, we test the hypothesis that the fraction of variance ratios less than 1 or 0.559, respectively, does not differ from 50%. † indicates that we can reject the hypothesis at the 1% level for all 29 waves. The third row in each panel reports the difference in median variance ratios and associated statistical significance (1%, 5%, and 10% levels are indicated by ***, **, and *, respectively) associated a with paired t-test (N=29) of the null hypothesis that the difference is zero.

Variable	Variance Ratio (median)	%Variance Ratio<1	%Variance Ratio<0.559
Panel A: Education			
More education	0.169	0.874†	0.795†
Less education	0.125	0.921†	0.868†
Difference	0.044***		
Panel B: Income			
High income	0.183	0.868†	0.782†
Low income	0.134	0.906†	0.847†
Difference	0.049***		
Panel C: Stock market participation			
Holds equity	0.176	0.870†	0.786†
No equity	0.119	0.925†	0.877†
Difference	0.057***		
Panel D: Self-rated understanding of stock market			
High understanding	0.172	0.869†	0.787†
Low understanding	0.120	0.931†	0.883†
Difference	0.052***		
Panel E: Numeracy			
High numeracy	0.229	0.836†	0.735†
Low numeracy	0.131	0.914†	0.856†
Difference	0.098***		
Panel F: Financial literacy			
High financial literacy	0.218	0.835†	0.738†
Low financial literacy	0.114	0.939†	0.894†
Difference	0.104***		

Table IA-VI – Variance Ratio Tests by Respondent Characteristics (Cont.)

Panel G: Gender			
Male	0.173	0.866†	0.783†
Female	0.135	0.912†	0.854†
Difference	0.037***		
Panel H: Age			
Old	0.154	0.886†	0.812†
Young	0.141	0.903†	0.841†
Difference	0.013***		
Panel I: Race			
White	0.151	0.889†	0.818†
Non-white	0.130	0.919†	0.866†
Difference	0.022***		
Panel J: Marital status			
Married	0.152	0.889†	0.816†
Not married	0.140	0.900†	0.838†
Difference	0.013***		
Panel K: Employment status			
Working	0.150	0.895†	0.826†
Not working	0.146	0.889†	0.821†
Difference	0.004		
Panel L: Retirement status			
Retired	0.162	0.872†	0.794†
Not retired	0.145	0.899†	0.833†
Difference	0.017***		
Panel M: Overconfidence			
Overconfident	0.149	0.881†	0.813†
Not overconfident	0.173	0.868†	0.784†
Difference	-0.024***		
Panel N: Expectations for a year from now			
Better	0.155	0.883†	0.808†
Worse	0.142	0.896†	0.829†
Difference	0.013**		
Panel O: Mean-reversion beliefs			
Mean-reversion belief	0.144	0.898†	0.832†
No mean-reversion	0.162	0.883†	0.808†
Difference	-0.019***		

Table IA-VII – Correlation Matrix of Panel Regression Explanatory variables

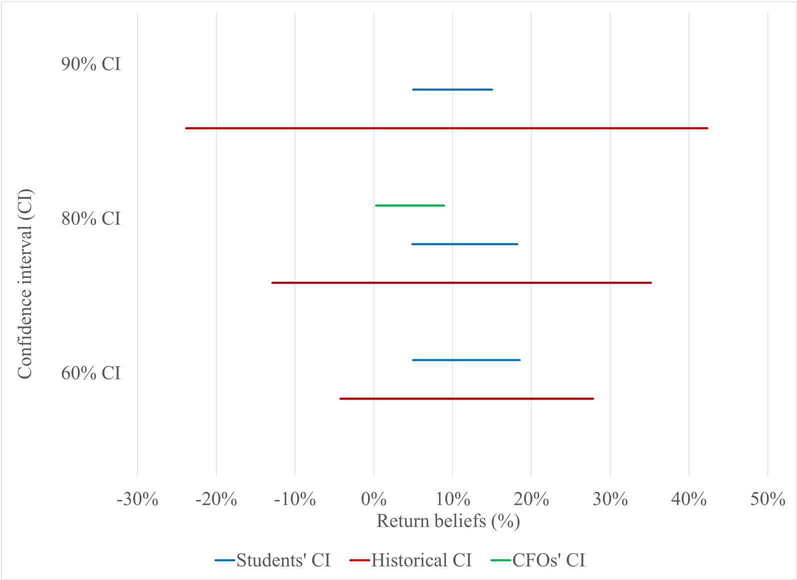
The table reports correlations between the panel regression (see Table VII) explanatory variables including Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Mean reversion (× -1), and Better next year. Data includes the pooled cross-sectional time-series of 22,748 observations (from 3,023 individuals) that include any individual-survey wave observation where the respondent has adequate data to estimate their variance ratio. Variable descriptions are provided in the Internet Appendix, Section IA-12. Significance at the 1, 5, and 10% levels are indicated by ***, **, and *, respectively.

	Years education	Income	Holds equity	Understands markets	Numeracy	Financial literacy	Overconfidence	$-1x$Mean reversion	Better next year
Years education	1.000								
Income	0.404***	1.000							
Holds equity	0.321***	0.397***	1.000						
Understands markets	0.332***	0.329***	0.409***	1.000					
Numeracy	0.405***	0.313***	0.328***	0.409***	1.000				
Financial literacy	0.340***	0.337***	0.428***	0.583***	0.520***	1.000			
Overconfidence	-0.014	-0.024**	-0.033***	0.019*	-0.031***	0.015	1.000		
Mean reversion (× -1)	-0.029***	-0.056***	-0.026***	-0.034***	-0.080***	-0.102***	0.004	1.000	
Better next year	0.058***	0.036***	-0.017***	0.015**	0.024**	0.037***	0.032***	-0.003	1.000

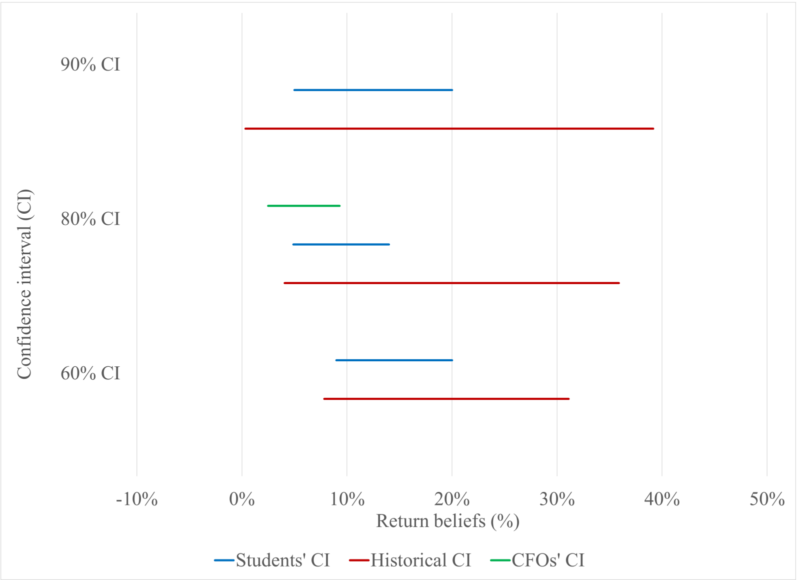
Table IA-VIII – Uncertainty and Time

This table reports descriptive statistics for ALP survey results over August 2006 and November 2007 regarding respondents' uncertainty beliefs around different events in the next year or 5 years. Events considered include the likelihood of a car accident, having a cavity filled, dying, being a victim of theft, moving, dying from terrorism, having their home broken into, or visiting the dentist. Sample sizes for each event are reported in column (1). Median respondents' beliefs about event risk are reported in columns (2) through (5). Beliefs for the next year and 5 years are given in columns (2) and (4), respectively. Column (3) reports the implied 5-year probability given independence and the reported 1-year probability response. Column (5) reports the difference between the median implied and median reported 5-year probabilities. Columns (6) through (9) report analogous results for mean beliefs. Column (10) reports the fraction of respondents with a positive difference between the implied and reported 5-year probabilities. Significance at the 1% level is indicated by *** for the test that the fraction reported in column (10) does not differ from 50%.

		Median				Mean				%>0
	N	P(1-year)	Implied P(5-year)	P(5-year)	Difference	P(1-year)	Implied P(5-year)	P(5-year)	Difference	
Description	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
P(car accident)	859	0.200	0.672	0.300	0.372	0.278	0.673	0.352	0.321	0.934***
P(cavity filled)	642	0.200	0.672	0.500	0.172	0.302	0.665	0.492	0.173	0.815***
P(die)	838	0.100	0.410	0.200	0.210	0.207	0.523	0.279	0.244	0.870***
P(theft victim)	818	0.250	0.763	0.300	0.463	0.306	0.676	0.355	0.321	0.941***
P(move)	390	0.100	0.410	0.200	0.210	0.153	0.393	0.312	0.081	0.679***
P(die terrorism)	610	0.050	0.226	0.100	0.126	0.107	0.326	0.146	0.180	0.931***
P(break in victim)	813	0.100	0.410	0.150	0.260	0.154	0.443	0.220	0.223	0.888***
P(visit dentist)	190	0.500	0.969	0.750	0.219	0.560	0.851	0.670	0.180	0.932***

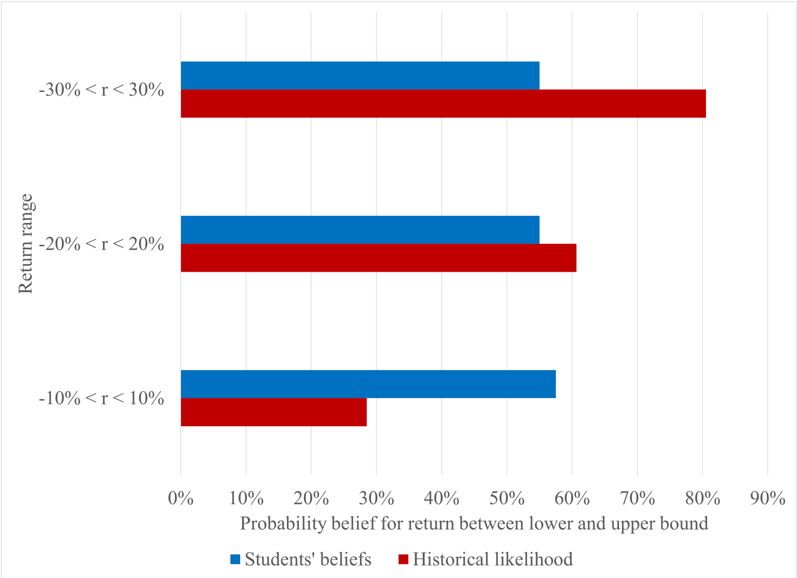


(A) 1-year return confidence intervals for the median student, median CFO, and for historical returns

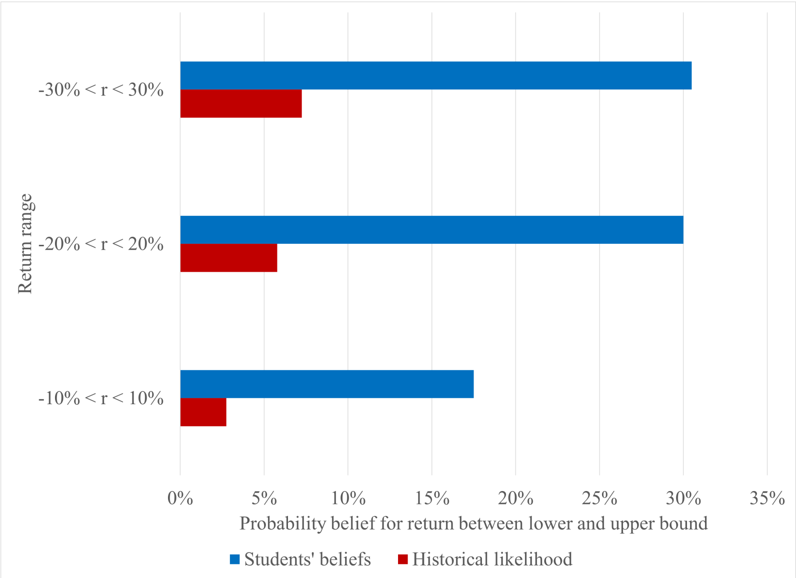


(B) 10-year return confidence intervals for the median student, median CFO, and historical returns

Figure IA-1 – Confidence intervals for the median student, median CFO, and historical returns for CFO style questions. Panel A reports the 1-year return confidence intervals for the median student (blue), median CFO (green), and for historical returns (red). Panel B reports the 10-year return confidence intervals for the median student (blue), median CFO (green), and for historical returns (red). Historical confidence intervals are based on returns for the CRSP value-weighted index 1- and 10-year return over the 1926-2020 period. CFO style questions prompt respondents for the forward looking percentile related return.



(A) 1-year student return probability beliefs for returns within the range provided. The historical likelihood of analogous market return ranges are provided in red bars.



(B) 10-year student return probability beliefs for returns within the range provided. The historical likelihood of analogous market return ranges are provided in red bars.

FIGURE IA-2 – STUDENT RETURN BELIEFS FOR RANGES OF RETURNS ALONGSIDE HISTORICAL LIKELIHOODS OF RETURNS FOR ALP STYLE QUESTIONS. Panel A provides 1-year return belief probabilities and Panel B reports 10-year results for ranges of returns that fall between -30% to 30% (top cluster), -20% to 20% (middle cluster), and -10% to 10% (bottom cluster). For each range of returns on the vertical axes, we report student beliefs in blue bars, and historical market likelihoods in red bars. Historical return likelihoods are based on returns for the CRSP value-weighted index 1- and 10-year return over the 1926-2020 period. ALP style questions prompt respondents for the likelihood of different return possibilities.