Uncertain Uncertainty

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Abstract

Total (multi-year) returns and average annual returns provide equivalent financial information, yet when asked about one versus the other, the same individual's implied variance ratios differ by a factor of 25. We hypothesize that most individuals have limited understanding of how return distributions evolve with horizon, causing them to compress near- and long-term uncertainty estimates. This compression produces implausibly strong negative risk-horizon relations when asked about total returns and implausibly strong positive relations when asked about average annual returns. Evidence from five datasets supports the hypothesis and helps explain puzzling results across multiple literatures.

JEL classifications: C11, G11, G12, G14, G17, G51, G53

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1 Introduction

Total returns and average annual returns are mathematically equivalent ways of expressing the same information about long-horizon uncertainty. Yet when the same person is asked about each, their implied variance ratios differ by a factor of 25. This is not a statistical artifact—the results hold over five datasets spanning business school students, retail investors, CFOs, and representative samples of Americans. The culprit is a single design choice: whether the long-horizon return question is framed in terms of total cumulative returns or average annual returns. When asked about cumulative returns—where variance increases with horizon—respondents report beliefs that imply an implausibly strong *negative* relation between risk and horizon. When asked about average annual returns—where variance decreases with horizon—the same respondents report beliefs that imply an implausibly strong *positive* relation. The resulting implied optimal equity allocations are, respectively, 255% and 5.7% of wealth for an investor with a 10-year horizon versus the 37.5% the standard model (Merton (1969)) prescribes. Investor beliefs are highly implausible in opposing directions relative to the standard model, and generated by respondents answering what is, mathematically, the same question.[1]

This unit-dependency (cumulative versus average annual returns) cuts to the heart of a long-standing debate in finance and economics about how economic agents should, and do, perceive the relation between risk and horizon. The interpretation of both theory and empirical evidence that considers economic agents' perceptions of uncertainty relies, explicitly or implicitly, on the assumption that agents' near- and long-term beliefs are linked by a consistent distributional logic. Our central finding is that they are not. Moreover, this failure helps explain a puzzling set of empirical results across multiple literatures that have resisted a common interpretation. First, CFOs appear to perceive risk as increasing with horizon (Pástor and Stambaugh (2012)) while individuals appear to perceive risk as decreasing with horizon (Sias, Starks, and Turtle (2026)). These estimates diverge in opposite directions and both are individually implausible by any reasonable theoretical benchmark. The median CFO variance ratio of 6.6 is six times the Pástor and Stambaugh (2012) estimate (of 1.1) that accounts for parameter uncertainty using "realistic" parameters. The median individual's variance ratio of 0.147 implies effectively zero probability of a negative 10-year

[1] These values are based on a continuous annual equity risk premium of 6%, an annual market volatility of 20% (both approximate historical values), and a coefficient of relative risk aversion (γ) of 4 (following Giglio, Maggiori, Stroebel, and Utkus (2021)). Thus, an investor with a 1-year horizon holds 37.5% of their wealth in equities (i.e., $(E(r_k) - r_{f,k})/(\gamma\sigma^2(r_k)) = 0.06/(4 \times 0.20^2)$). An investor with a 10-year horizon holds $0.06/(4 \times 0.20^2 \times VR_{10,i})$, where the 10-year variance ratio ($VR_{10,i}$) is 1.0 (when returns are *iid* and there is no parameter uncertainty), 0.147 (the median variance ratio for American Life Panel respondents answering their total return questions), or 6.61 (the median variance ratio for CFOs answering the Duke survey's average return questions).

market return, inconsistent with even the most extreme mean-reversion estimates in the empirical literature. Second, CFOs exhibit severely miscalibrated near-term uncertainty estimates but appear much less miscalibrated at long horizons (Ben-David, Graham, and Harvey (2013)). The mechanism driving this asymmetry, however, remains unclear. Third, the empirical relation between perceived subjective uncertainty and portfolio allocation is less than one-tenth the theoretical value—the "attenuation puzzle" (Amromin and Sharpe (2014)). Fourth, qualitative measures of perceived risk better explain investor behavior than quantitative model primitives (Nosić and Weber (2010), Kapteyn and Teppa (2011)). Our evidence suggests a single cognitive failure explains all four empirical regularities.

We propose that cognitive uncertainty is the central mechanism. Specifically, we hypothesize that (1) most economic agents have limited knowledge of uncertainty in the near-term return distribution, uncertainty in the long-term return distribution, or the cognitively challenging relation between volatility and horizon, (2) as a result of this cognitive uncertainty, near- and long-term subjective uncertainty estimates are compressed relative to objective values, and (3) because *total* return uncertainty increases with horizon while *average* return uncertainty decreases with horizon, the direction of the bias induced by this compression depends on whether respondents are asked about uncertainty in total or average annual long-horizon returns.

Using five datasets—the American Life Panel (ALP), the Health and Retirement Study (HRS), the Duke CFO surveys, the Understanding America Study (UAS), and a series of surveys to undergraduate business students over five semesters—we find strong and consistent support for our hypothesis. When ALP and HRS respondents are asked about total return uncertainty over the next decade, their median implied variance ratios are 0.147 and 0.145, respectively. When CFOs are asked about average annual return uncertainty over the same horizon, the median implied variance ratio is 6.6. When the same business school students are asked both question formats two weeks apart, 95% generate a larger variance ratio from the average return questions, with a median difference of 8.71—versus a value of zero for any agent with a consistent distributional model of risk.

Additional tests support the hypothesis that cognitive uncertainty plays a central role in driving these variance ratio patterns. First, the compression pattern is equally strong when students are asked about *historical* rather than forward-looking return distributions, ruling out overconfidence in near-term predictive ability as the primary driver. Second, when we reframe the CFO-style quantile elicitation questions in terms of cumulative returns—and the ALP-style probability elicitation questions in terms of average annual returns—the compression pattern flips accordingly:

variance ratios are always implausibly large when the question involves average annual returns and implausibly small when it involves cumulative returns, regardless of elicitation style. Third, the pattern worsens with horizon, consistent with the prediction that cognitive compression becomes more severe as the underlying calculation grows more complex. Fourth, perceived uncertainty itself appears fragile and poorly anchored. Business school students answering the ALP-style probability elicitation questions imply a median 1-year standard deviation of 23.7%; the same students answering the CFO-style confidence interval questions imply a median of 4.1%. CFOs tell a similar story: their median implied 1-year standard deviation from their confidence interval estimates is only 3.7%, yet their median direct volatility estimate in the same survey nearly triples to 11%—far closer to an erroneous seed value (an anchoring value presented to respondents) of 12.6% than to either the historical average or their own simultaneously reported confidence interval implied estimates. ALP respondents answering the bins-and-balls version of the probability elicitation questions imply median 1-year and 10-year standard deviations of 7.2% and 9.6%—far below both the standard ALP estimates and historical values—consistent with elicitation structure rather than underlying beliefs driving the estimates. Using data from Hartzmark and Sussman (2026), we find that respondents shown historical return data allocate balls to better match the bin structure rather than the underlying data, with inferred standard deviations more closely tracking the bin calibration rather than the data actually viewed. Finally, when we vary the probability level in CFO-style quantile elicitation questions (1-in-5, 1-in-10, 1-in-20), students' implied return thresholds barely move, and when we vary the return threshold in ALP-style probability elicitation questions (±10%, ±20%, ±30%), students' implied probabilities are nearly invariant. Taken together, the evidence consistently points to the same conclusion: respondents are constructing answers on the fly from available cues rather than drawing on stable, well-anchored uncertainty beliefs.

We also exploit heterogeneity across individuals to further distinguish cognitive uncertainty from alternative explanations. Cognitive uncertainty and parameter uncertainty make opposite predictions about the relation between financial sophistication and (total return) variance ratios: greater sophistication should reduce cognitive uncertainty, pushing variance ratios up, but also reduce parameter uncertainty, pushing variance ratios down. Consistent with cognitive uncertainty playing the dominant role, more financially sophisticated ALP respondents exhibit larger variance ratios. Consistent with the hypothesis that perceived mean-reversion contributes to heterogeneity, respondents whose beliefs reflect greater mean-reversion exhibit lower (total return) variance ratios. Although these effects are both statistically and economically meaningful, they are dwarfed by the effects of question structure. The largest characteristic effect we estimate (the difference in variance ratios between market participants and non-participants) is 0.132. The difference in median variance ratios by asking the same students about total versus average annual returns is 8.71, an effect size 66 times larger. In short, *who* is asked matters far less than *how* they are asked. We find no support for the two previously proposed explanations—overconfidence and expectations of increases in future volatility—for why, when asked about return confidence intervals, near-term miscalibration is so much larger than long-term miscalibration among both students and CFOs. Finally, consistent with evidence that individuals' levels of cognitive uncertainty are correlated across domains, the extent to which ALP respondents compress their near- and long-term forecasts for eight unrelated events, such as the likelihood of a car accident in the next year versus five years, is positively related to their equity market variance ratio compression.

Our analysis helps resolve all four puzzles discussed above. The apparent contradiction between CFOs' and individuals' variance ratios dissolves—both groups exhibit the same compression, but in opposite directions because they are asked about different return units. The implausible magnitudes follow from the same logic: neither group has a fundamental understanding of how uncertainty evolves with horizon and, as a result, their near- and long-term forecasts are compressed toward each other regardless of the return unit asked about. Similarly, the asymmetry between CFOs' near- and long-term miscalibration follows directly from compression: whatever the anchor, near- and long-term estimates are pulled toward each other, mechanically narrowing the apparent miscalibration gap. The results also help explain the attenuation puzzle. Most investors have holding periods longer than one year, yet empirical tests almost always use near-term uncertainty as the relevant risk measure. Our evidence suggests this creates two compounding problems: agents have cognitive uncertainty about how risk evolves with horizon, making near-term uncertainty a poor proxy for the uncertainty governing long-horizon allocation decisions; and even near-term uncertainty estimates are highly format-sensitive—the same students imply a 1-year standard deviation of 23.7% from one elicitation format and 4.1% from another. If the empirical measure of perceived risk is both the wrong horizon and poorly measured, cognitive uncertainty likely contributes to the attenuation of the empirical relation between perceived uncertainty and portfolio allocation. Related, qualitative risk measures may outperform quantitative model primitives precisely because they sidestep both problems: a qualitative assessment of riskiness is less sensitive to return unit or horizon framing and may better capture the stable component of risk perception that actually influences decisions.

Our work is related to a growing literature demonstrating that survey responses about financial expectations depend on how questions are framed. Hartzmark and Sussman (2026) find that distributional elicitation tends to provide more accurate estimates than confidence interval elicitation for near-horizon beliefs. Glaser, Iliewa, and Weber (2019) show that asking about prices versus returns changes near-term expected return estimates. Merkoulova and Veld (2022) find that more than 70% of Americans exhibit ignorance of the distribution of equity returns. Our contribution is distinct from these important studies in a fundamental respect. Prior work documents framing effects on beliefs—a parameter shifts when the question changes. Our paper documents that the *entire relation* between near- and long-term beliefs is frame-dependent. This is not miscalibration of a parameter—it is a failure of the cognitive architecture that connects horizon to uncertainty. The same person cannot hold simultaneously that stocks are extremely safe over a 10-year horizon *and* that the average annual return over 10 years is highly uncertain, yet that is precisely what 95% of our student respondents imply when answering both question formats. In sum, our study documents a fundamentally different issue from prior work—a cognitive limitation rather than a bias—with direct implications for how we interpret survey evidence about risk perceptions, how we inform investors about long-horizon risk, and how financial information is regulated and presented.

The implications are broad. For lifecycle investing, if investors' quantitative beliefs about long-horizon risk are frame-dependent outputs rather than stable underlying beliefs, the risk profiling that underlies target-date fund glide path construction may be measuring noise rather than genuine risk tolerance. For corporate finance, if the typical CFO cannot reliably scale risk across horizons, the well-documented preference for simple short-horizon decision rules such as payback period (Graham (2022)) may reflect a rational adaptation to cognitive noise rather than unsophisticated practice. More broadly, our results suggest that the format choice between cumulative and average annual returns influences whether long-run risk appears implausibly low or implausibly high—shaping the investment decisions of millions of Americans through the target-date funds they select and the capital budgeting decisions their CFOs make. If economic agents lack stable, well-anchored views of long-horizon uncertainty, the connection between elicited risk perceptions and actual financial decisions is more tenuous than standard models assume. Incorporating this fragility into models of financial decision-making—and into the design of surveys, disclosures, and regulations used to measure and communicate risk—may be an important step toward understanding the choices economic agents actually make.

2 Background

Theory and empirical work often rely (implicitly or explicitly) on the assumption that economic agents understand the relation between horizon and uncertainty. For instance, both the classic

Merton (1969) model and the growing literature that focuses on the relation between horizon and uncertainty solves for the optimal risky share for an investor who understands how uncertainty evolves with horizon.[2] Similarly, the interpretation of empirical evidence often relies on this assumption. Pástor and Stambaugh (2012) conclude, based on the Duke surveys, that CFOs "tend to exhibit" a view that stocks are "more volatile at long horizons" and both explanations Ben-David, Graham, and Harvey (2013) offer for CFOs' near- versus long-term miscalibration asymmetry implicitly assume CFOs understand how uncertainty evolves with horizon.[3] Correspondingly, labeling the weak empirical relation between perceived uncertainty and portfolio allocation the attenuation "puzzle" implicitly assumes investors know and use variance (or something highly correlated) and, assuming many have expected holding periods other than one year, understand the relation between horizon and risk.[4]

2.1 Measuring perceived distributions

Most work takes one of three approaches to measuring economic agents' perceptions of a return distribution. The first approach, quantile elicitation, gives respondents a probability and asks them to report the corresponding return. For example, the Duke CFO surveys ask respondents for their 10^{th} and 90^{th} return percentiles, generating an 80% confidence interval. The second approach, probability elicitation, gives respondents a return threshold and asks them to report the corresponding likelihood. The ALP and HRS surveys, for instance, ask respondents to estimate the likelihood that markets will gain more than 20% over the next year. The third approach, distributional elicitation, gives respondents a set of return bins and asks them to allocate balls across bins in proportion to their perceived likelihoods. Although the focus of our study is on the relation between near- and long-horizon perceptions holding the elicitation approach constant, a growing literature demonstrates that elicitation approach impacts responses. For instance, respondents tend to report similar confidence intervals regardless of the probability level asked about (Hartzmark

[2] A partial list of the studies that focus on the important question of how predictive risk varies with horizon for a forward looking investor includes Samuelson (1969), Merton (1969), Barberis (2000), Campbell and Viceira (2002), Pettenuzzo and Timmermann (2011), Pástor and Stambaugh (2012), Hoevenaars, Molenaar, Schotman, and Steenkamp (2014), Johannes, Korteweg, and Polson (2014), Siegel (2014), Müller and Watson (2016), Avramov, Cederburg, and Lučivjanská (2018), Carvalho, Lopes, and McCulloch (2018), Zhang, Zhu, and Zhu (2020), and Jondeau, Zhang, and Zhu (2021).

[3] Barrero (2022) and Boutros, Ben-David, Graham, Harvey, and Payne (2025) also find corporate managers exhibit near-term miscalibration.

[4] For instance, Jay Clayton (former Chairperson Securities Exchange Commission) claims, "Put simply, they are investing for the long term and it's their money that's in our markets as they save for retirement and other life events. We think about them every day at the Commission and they're not investing for next week, they're not investing for next year. Predominately they're investing for something 10, 20, 30 years away and they expect their money to be managed that way."

and Sussman (2026)).

Regardless of elicitation approach, surveys of long-horizon beliefs must also choose between asking about cumulative or average annual returns. For instance, the Duke CFO surveys use quantile elicitation framed in average annual returns (e.g., there is a 1-in-10 chance annual returns over the next decade will average less than ___), while the ALP surveys use probability elicitation framed in cumulative returns (e.g., what are the chances equity markets will have fallen in value by more than 20% in 10 years?). Crucially, all three elicitation approaches can be framed in either cumulative or average annual long-horizon returns—a choice that, as we demonstrate, has first-order consequences for the variance ratios implied by respondents' answers.

2.2 Cognitive uncertainty and hypothesis

Understanding why elicitation format generates such dramatically different variance ratio estimates requires first recognizing the mathematical complexity of the relation between horizon and uncertainty—given the variance of k-year returns is k^2 times the variance of the average annual return over k years (i.e., $\sigma^2(r_k) = \sigma^2(k\bar{r}_k) = k^2\sigma^2(\bar{r}_k)$), the k-period variance ratio (denoted VR_k), regardless of the level of parameter uncertainty or return predictability, satisfies:

$$VR_k = \frac{\sigma^2(r_k)}{k\sigma^2(r_1)} = \frac{k\sigma^2(\bar{r}_k)}{\sigma^2(r_1)}. \tag{1}$$

Even in the simplest case where there is no parameter uncertainty and returns are *iid* (the "base" case where $VR_k = 1 \ \forall \ k$), the variance of 10-year total returns is 10 times the 1-year return variance, the variance of average annual returns over 10 years is $1/10^{\text{th}}$ the 1-year return variance, and the 10-year total return variance is 100 times the 10-year average return variance.[5] Adding uncertainty regarding the return generating process and expectations of mean-reversion or other forms of predictability only further complicates the estimation of these values (e.g., see equation (12) in Pástor and Stambaugh (2012)).

A growing literature demonstrates that economic agents' cognitive uncertainty—that is, their recognition that they are uncertain of the correct answer—impacts their valuations, beliefs, forecasts, and actions. Specifically, as a result of this cognitive uncertainty, respondents' answers are compressed toward a cognitive default value.[6] This compression can explain probability weighting

[5] Equivalently, the standard deviation of 10-year returns is $\sqrt{10}$ times the standard deviation of 1-year returns, the standard deviation of the average return over 10 years is $1/\sqrt{10}$ times the standard deviation of 1-year returns, and the standard deviation of 10-year returns is 10 times the standard deviation of the average return over 10 years in the base case.

[6] As Enke and Graeber (2023) point out, the compression toward a cognitive default is related to the Tversky

in lotteries, base rate insensitivity and conservatism in updating beliefs, why individuals overestimate the likelihood of low probability events, fourfold risk patterns, loss aversion, and why investors exhibit only a weak transmission from beliefs to behaviors (see Enke and Graeber (2023), Charles, Frydman, and Kilic (2024), Oprea (2024)).[7]

A recent extension of this literature (see Enke, Graeber, and Oprea (2025), Enke and Graeber (2021), Gabaix and Laibson (2022), Gershman and Bhui (2020)) focuses on compression in estimates over different horizons. This work posits that the cognitive uncertainty associated with exponential discounting in traditional models (see Cohen, Ericson, Laibson, and White (2020)) causes compression of estimates over different horizons, such that the appearance of hyperbolic discounting arises from natural compression effects rather than non-standard preferences. For example, Enke, Graeber, and Oprea (2025) document that "hyperbolic discounting" is equally apparent when asking the same discounting questions in a manner that removes the time dimension entirely.[8]

The key result in this literature is that when respondents are uncertain about values over two horizons, their subjective estimates are compressed relative to objective values. Applied to equity return distributions, this compression has opposite implications depending on elicitation format. Because total return uncertainty increases with horizon, compression pulls long-term total return estimates insufficiently far from near-term values. specifically, if a respondent's estimates are compressed, their inferred long-term variance ($\hat{\sigma}^2(r_k)$) will be too small relative to their near-term estimate when asked about cumulative returns:[9]

$$\hat{\sigma}^2(r_1) < \hat{\sigma}^2(r_k) < (k \times \hat{\sigma}^2(r_1)). \tag{2}$$

Correspondingly, because average annual return uncertainty decreases with horizon, compression pulls long-term average return estimates insufficiently far from near-term values in the other direction and, as a result, the inferred long-term average annual variance ($\hat{\sigma}^2(\bar{r}_k)$) will be too large

and Kahneman (1974) anchoring and adjustment heuristic in which individuals form estimates with an insufficient adjustment from their cognitive anchor. See Woodford (2020) and Gabaix (2019) for models and reviews of the role of cognitive uncertainty.

[7] Enke and Graeber (2023) note that there is not a "general theory" for what serves as the cognitive default. As discussed in greater detail in the Internet Appendix, our study also suggests that the cognitive default value varies with question structure.

[8] Specicially the authors reframe the question as an "atemporal mirror," e.g., asking a person to value "$50 shrunk 12 times, each time by 4%."

[9] To reduce notational clutter, equations (2) and (3) are base case scenarios where the k-period variance ratio is 1. In the more general case, the last term in both equations is multiplied by the k-period varaince ratio (VR_k).

relative to their near-term estimate:[10]

$$\hat{\sigma}^2(r_1) > \hat{\sigma}^2(\bar{r}_k) > (\hat{\sigma}^2(r_1)/k). \tag{3}$$

In short, cognitive uncertainty implies the perceived long-term variance will be too close to the perceived near-term variance regardless of whether respondents are asked about cumulative returns (resulting in variance ratios that are too small; equation (2)) or average annual returns (resulting in variance ratios that are too large; equation (3)).[11]

3 Data

We use five datasets to examine respondents' near- and long-term perceived return distributions—the American Life Panel, the Health and Retirement Study, the Duke CFO surveys, the Understanding America Study, and a series of surveys given to undergraduate business students at two public universities.

3.1 The American Life Panel

The ALP is an ongoing nationally representative panel of Americans that has grown from approximately 2,000 participants in 2003 to more than 6,000 today. Between November 2008 and January 2016, ALP executed 61 "effects of the financial crisis" surveys in either long-form (29 surveys) or short-form (32 surveys) formats. The long-form surveys asked respondents six questions regarding both near- and long-term perceived stock market return distributions:

We are interested in how well you think the economy will do in the future. By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

By next year at this time, what is the percent chance that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have

-gained in value by more than 20 percent compared to what they are worth today?

-fallen in value by more than 20 percent compared to what they are worth today?

[10] Consider a simple example: If the individual perceives the standard deviation of annual returns is 20% (i.e., the approximate historical value), then, in the base case, the objective standard deviation of total 10-year returns is 63% (i.e., $(0.2^2 \times 10)^{0.5}$) and the objective standard deviation of the average annual return over the 10 years is 6.3% (i.e., $(0.2^2/10)^{0.5}$).

[11] As detailed in Section 2.1 and the Internet Appendix, different elicitation approaches may produce different cognitive anchors. Regardless, our hypothesis only requires compression in estimated values relative to objective values, holding the elicitation approach constant.

Now please think about how the stock market will change over the next 10 years: What are the chances that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more in 10 years than they are today?

What are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have
-increased in value by more than 20 percent in 10 years compared to what they are worth today?
-fallen in value by more than 20 percent in 10 years compared to what they are worth today?

The 32 short-form surveys asked the first four questions but not the final two. Because estimation of the long-term variance requires the final two questions, we limit our sample to the 29 long-form surveys.[12]

Following previous work (Dominitz and Manski (2007), Kézdi and Willis (2011), Ben-David, Graham, and Harvey (2013), Boutros, Ben-David, Graham, Harvey, and Payne (2025)), we infer the near-term (long-term) perceived return variance for each respondent at each point in time by assuming continuously compounded returns are normally distributed and solving for the parameters that fit the respondent's reported likelihoods that markets rise or fall 20% in the next year (decade).[13] Estimation requires that respondents' beliefs about the sum of the tail probabilities does not exceed 100% and that neither tail probability is zero.[14] We are able to infer variances for 69% of surveys where respondents report near-term tail probabilities and 62% where respondents report long-term tail probabilities. Because our variable of interest is the variance ratio, we limit the sample to observations with both near- and long-term variance estimates, and winsorize variance ratios and inferred volatility estimates at the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentiles. Our final sample consists of 22,745 pooled cross-sectional time-series observations from 3,022 individuals, with the average respondent participating in more than 7.5 of the 29 long-form surveys.

The panel structure of the ALP enables tests that are unavailable in most traditional survey datasets. Because we observe the same individuals across multiple waves, we can estimate

[12]In 18 of the 29 survey waves, approximately half of respondents were randomly assigned to answer a distributional elicitation version of the market expectations questions rather than the standard probability elicitation format. Our primary tests exclude these respondents; we examine their variance ratios using two alternative methods in the Internet Appendix. Twenty-three observations with valid responses to both question formats in the same wave are excluded from both samples. In addition, wave 44 was split into two portions and only respondents to wave 44.1 were asked the long-horizon tail probability questions.

[13]Following the variance ratio literature (e.g., Poterba and Summers (1987), Lo and MacKinlay (1988)), we focus on continuously compounded returns. Because we focus on continuously compounded returns, we convert arithmetic returns to continuously compounded returns via a log transformation before estimating perceived distributions.

[14]The substantial number of observations that violate probability laws is consistent with previous work (e.g., de Bruin, Fischhoff, Millstein, and Halpern-Felsher (2000), Merkoulova and Veld (2022)). See Internet Appendix for additional detail.

individual-level beliefs about mean-reversion in equity markets—allowing us to examine heterogeneity in mean-reversion beliefs in a way that most surveys (e.g., those where the sample changes each period or respondents are not uniquely identified) cannot. More broadly, the panel structure allows us to construct a rich set of individual-level measures including expected future economic conditions, overconfidence, financial literacy, and numeracy, which we exploit in Section 5 to investigate the sources of variance ratio heterogeneity. The Internet Appendix reports descriptive statistics of respondent characteristics and details of these additional ALP variables.

3.2 The Health and Retirement Study

The Health and Retirement Study (HRS) is a longitudinal panel of individuals ages 50 and older. In 2009, HRS executed an off-year internet survey to a random subset of participants asking the same six probability elicitation questions as the ALP long-form survey. We apply the same sample restrictions and variance inference procedure as for the ALP, yielding 1,252 respondents with implied variance ratios. Variance ratios and volatility estimates are winsorized at the 5^{th} and 95^{th} percentiles.

3.3 CFO survey data

The Duke CFO surveys use quantile elicitation, asking respondents to fill in the blanks:

Over the next 10 years, I expect the average annual S&P 500 return will be:

There is a 1-in-10 chance it will be less than ___

Expected return ___

There is a 1-in-10 chance it will be greater than ___

Over the next year, I expect the S&P 500 return will be:

There is a 1-in-10 chance it will be less than ___

Expected return ___

There is a 1-in-10 chance it will be greater than ___

For 63 quarters between 2004 and 2019, we gather the mean and median answers published at the Duke CFO survey website (cfosurvey.fuqua.duke.edu), computed from an average of approximately 400 CFOs per quarter (ranging from 186 to 781).[15] Following Pástor and Stambaugh (2012), we infer variance from the reported median and mean 10^{th} and 90^{th} return percentiles for the S&P 500 over the next year and next decade, based on continuously compounded returns.

[15] Data were not available for the first quarter of 2019.

3.4 Student surveys

In spring and fall 2024, undergraduate business students at two large public universities were given both the ALP probability elicitation questions (Section 3.1) and the CFO quantile elicitation questions (Section 3.3), framed in total and average annual returns respectively, two weeks apart.[16] We limit the sample to students whose answers follow probability laws and infer near- and long-term variances assuming normally distributed continuously compounded returns. Our spring and fall 2024 sample consists of 322 students with variance ratios based on the probability elicitation total return questions, 673 based on the quantile elicitation average return questions, and 241 with variance ratios based on both.

The fall 2024 surveys also included a historical returns component designed to distinguish the cognitive uncertainty explanation from greater overconfidence in near-term predictive ability explanation. Specifically, fall 2024 students were first given the forward-looking ALP probability elicitation questions and a backward-looking version of the CFO quantile elicitation questions reframed around historical returns. Two weeks later, the same students were given the forward-looking CFO quantile elicitation questions and a backward-looking version of the ALP probability elicitation questions reframed around historical returns. This design yields 251 students with variance ratios based on the historical probability elicitation questions and 290 based on the historical quantile elicitation questions.

In spring 2025, fall 2025, and spring 2026, we administered additional surveys to new groups of introductory finance students at both universities, varying elicitation approach (probability versus quantile), return unit (cumulative versus average annual), horizon (5, 10, or 15 years), and whether respondents were asked about historical or expected future returns. This sample consists of 1,368 students. Variance ratios and volatility estimates are winsorized at the 5^{th} and 95^{th} percentiles throughout. The Internet Appendix provides additional details regarding all student surveys.

3.5 Understanding America Study

The Understanding America Study (UAS) is a national panel of approximately 14,700 US households managed by the USC Dornsife Center for Economic and Social Research. In January 2025, UAS administered the six ALP probability elicitation total return questions to 9,455 respondents; we limit the sample to the 3,644 whose answers do not violate probability laws. In May–June 2019, UAS administered quantile elicitation questions about 10-year average annual returns and histor-

[16] The order of the two surveys was varied across sections. As detailed in the Internet Appendix, survey order did not materially influence the results.

ical annual return distributions to 2,461 respondents; after applying probability law restrictions, 1,439 remain.[17] Inferred standard deviations and variance ratios are winsorized at the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentiles.

4 The Perceived Relation between Horizon and Uncertainty

4.1 ALP respondents' variance ratios

Table I reports descriptive statistics for ALP respondents' perceived return distributions. Consistent with previous work, respondents tend to hold pessimistic near-term beliefs as the median respondent reports only a 40% likelihood markets rise in the next year despite markets rising approximately three out of every four years historically.[18] The bottom row of Panel A, however, reveals that the median ALP respondent's inferred 1-year standard deviation of 20.7% is almost identical to the historical value of 20%.

[Insert Table I about here]

Panel B shows the typical individual severely underestimates, relative to historical values, long-term expected returns when responding to the ALP survey questions—the median respondent's inferred 10-year expected return is only 4.3% (0.43% annually). More importantly, for our purposes, the median respondent's beliefs imply a 10-year standard deviation of only 29%—less than half the 65% value implied by their own near-term uncertainty estimates under serial independence (i.e., the 20.7% 1-year median standard deviation times $\sqrt{10}$; see equation 2). Nearly three out of four respondents report beliefs that generate a long-term variance less than the historical average.[19]

Because near- and long-term variance estimates are compressed toward each other, the implied variance ratios are implausibly small. Panel C reports the distribution of implied variance ratios. The historical 10-year variance ratio for the CRSP value-weighted index is 0.542, reflecting the mean reversion in equity markets. The median ALP respondent's implied variance ratio of 0.147 is far below even this historically mean-reverting value—a value only possible if equity markets

[17] The UAS CFO-style 1-year questions differ from the Duke CFO survey in that they ask about historical rather than expected returns, and rephrase the 10th and 90th percentile question as the six lowest and six highest years over the past 60 years. Note that the ALP-style and CFO-style UAS surveys were administered six years apart, a limitation we discuss further when presenting the UAS results. See Internet Appendix for details.

[18] Other studies suggesting respondents tend to hold pessimistic near-term beliefs include Hurd (2009), Hurd, Van Rooij, and Winter (2011), Kuhnen and Miu (2017), Das, Kuhnen, and Nagel (2020), Giglio, Maggiori, Stroebel, and Utkus (2021), and Sias, Starks, and Turtle (2023).

[19] The 1- and 10-year return distributions implied by median ALP respondent beliefs are reported in Figure IA-1 of the Internet Appendix.

exhibit unprecedented levels of mean reversion and ALP respondents have essentially no parameter uncertainty. Moreover, 89% of respondent-survey observations imply variance ratios below unity and 82% fall below the historical value of 0.542.

Figure 1 illustrates the compression pattern by plotting the implied distributions for near-term and long-term beliefs, recentered at zero to remove mean beliefs. The figure reports the distribution for the median ALP respondent's near-term uncertainty (red line), long-term uncertainty (blue line), and the long-term uncertainty implied by their near-term beliefs under serial independence (orange line) described in equation (2). The ALP total return long-term uncertainty estimate (blue) lies between the near-term estimate (red) and the independence-implied value (orange). In fact, the long-term estimate is far closer to the near-term estimate than the estimate under independence, generating variance ratios well below one. The results are consistent with cognitive uncertainty compressing ALP respondents' near- and long-term uncertainty estimates toward each other.

[Insert Figure 1 about here]

4.2 HRS respondents' variance ratios

Panels D, E, and F of Table I report analogous statistics for HRS respondents. The results are nearly identical to corresponding values for the ALP sample. Specifically, HRS respondents exhibit both near- and long-term bearishness. HRS respondents' near-term uncertainty is close to the historical value but their long-term uncertainty estimates tend to be substantially smaller than the historical value. As a result, 88% of the HRS participants' imputed variance ratios are less than unity and 80% are less than the historical CRSP value-weighted index's variance ratio. The median variance ratios for the ALP and HRS samples are nearly identical at 0.147 and 0.145, respectively.

4.3 CFO variance ratios

Table II reports the time-series descriptive statistics of the quarterly CFO surveys between 2004 and 2019 (n=63 quarters) for the cross-sectional mean (columns 2-5) and median (columns 6-9) CFO beliefs. Panel A reports CFO expectations for 1-year horizons, Panel B reports corresponding values for 10-year horizons, and Panel C reports variance ratios.

[Insert Table II about here]

Consistent with Ben-David, Graham, and Harvey (2013) and Boutros, Ben-David, Graham, Harvey, and Payne (2025), CFOs exhibit substantial near-term miscalibration—the mean (median)

CFO's inferred 1-year standard deviation averages 4.7% (3.3%) compared to the historical value of 20%, and in every quarter the inferred mean and median CFO near-term standard deviations fall below the historical value. In sharp contrast to ALP respondents, CFOs are severely miscalibrated in the near-term.

Panel B reveals that CFOs' long-term beliefs are also miscalibrated, but far less so than their near-term beliefs. In the base case, the standard deviation of average annual returns over 10 years should be only 32% ($1/\sqrt{10}$) of the annual standard deviation. Given the mean CFO's 1-year standard deviation of 4.7%, the implied base case average annual 10-year standard deviation is only 1.5%. Empirically, however, CFOs' average annual 10-year standard deviation is more than twice that value at 3.3%—still miscalibrated, but far less so than at the 1-year horizon. To allow direct comparison to ALP respondents' long-term beliefs (which are expressed in total return units), Panel B also reports the total return standard deviation over 10 years—simply 10 times the average annual return standard deviation. The mean (median) CFO's implied 10-year total return standard deviation is 33.3% (26.7%), compared to the historical value of 46.5%.[20]

Because long-term beliefs are less miscalibrated than near-term beliefs, the implied variance ratios are implausibly large. Panel C reports that the time-series average variance ratio is 6.6 based on median CFO beliefs and 5.1 based on mean CFO beliefs—more than five times the value implied by serial independence and nearly 10 times the historical value of 0.542. As pointed out by Pástor and Stambaugh (2012) and Ben-David, Graham, and Harvey (2013), a variance ratio greater than one is consistent with CFOs perceiving that parameter uncertainty increases per period risk at longer horizons. The magnitude, however, is far greater than that suggested by Pástor and Stambaugh (2012) model, who report a 10-year forward-looking variance ratio of approximately 1.1 with "realistic" parameters.[21] A variance ratio of 5 to 6 would require the impact of parameter uncertainty to be four to five times larger than that implied by the Pástor and Stambaugh (2012) model. Moreover, the results are hard to reconcile with parameter uncertainty as the sole explanation because CFOs' expected long-term variance remains below the historical value.

Figure 2 plots the median CFO's implied distribution for near-term uncertainty (red line), uncertainty in average annual long-term returns (blue line), and the average annual long-term return uncertainty implied by their near-term beliefs under serial independence (orange line), all

[20]The 1-year and 10-year return distributions implied by median CFO beliefs are reported in Figure IA-2 of the Internet Appendix.

[21]The authors' Figure 6 reports 1- and 10-year annualized variances of approximately 2.9% and 3.2%, respectively, implying a variance ratio of 1.1 (i.e., $\frac{\sigma^2(r_{10})/10}{\sigma^2(r_1)} = 0.032/0.029$).

recentered at zero. Because CFOs are asked about average annual long-term returns, all values are expressed in average annual return units. Consistent with equation (3), the long-term estimate (blue) lies between the near-term estimate (red) and the independence-implied value (orange)—but far closer to the near-term estimate, generating variance ratios well above one. The pattern is the mirror image of that for ALP respondents in Figure 1.

[Insert Figure 2 about here]

4.4 Business students' variance ratios

Although the analysis suggests that both CFOs and individuals struggle in their understanding of the relation between horizon and uncertainty, the dramatic difference in variance ratios could reflect differences in samples rather than differences in elicitation formats. CFOs, for instance, are presumably more financially sophisticated than the typical individual and are therefore simply more overconfident in their estimates and, for some reason, their overconfidence is especially strong for near-term forecasts. To rule this out, we give both the probability elicitation total return questions and the quantile elicitation average return questions to the same undergraduate business students at two large public universities two weeks apart.

Panels A, B, and C of Table III report results for students answering the probability elicitation total return questions. The results mirror the ALP findings as students hold bearish near- and long-term expected returns, show no systematic near-term overprecision (only 28% of forecasts imply a 1-year standard deviation below the historical value), and generate implausibly small variance ratios. Specifically, the median student variance ratio of 0.238 is low, 81% of student variance ratios are below unity, and 71% are below the historical value of 0.542. Panel A of Figure 3 confirms the compression pattern: identical to the pattern for ALP respondents and consistent with equation 2, the long-term uncertainty estimate (blue line) lies between the near-term estimate (red line) and the independence-implied value (orange line), closely tracking the near-term estimate.

[Insert Table III and Figure 3 about here]

Panels D, E, and F in Table III report results for students answering the quantile elicitation average return questions. The same students now exhibit severe near-term overprecision with a median implied 1-year standard deviation of 4.3% (nearly identical to the CFO mean of 4.7%) and generate implausibly large variance ratios. Specifically, the median student variance ratio is 9.32 and 90% of student estimates imply a variance ratio greater than one. Panel B of Figure 3 confirms the mirror-image compression pattern: identical to the pattern for CFOs and consistent with equation 3, the long-term estimate (blue line) lies between the near-term estimate (red line) and the independence implied value (orange line), but far closer to the near-term estimate.

Panel G delivers the key within-person result. For the 241 students who answer both sets of questions, the median student's quantile elicitation average return variance ratio is 25 times their probability elicitation total return variance ratio, the median difference is 8.71 (versus a value of zero for any agent with a consistent distributional model of risk), and 95% of students generate a larger variance ratios from the average return questions. The results leave little room for a sample-based explanation for these patterns: the same students, answering mathematically equivalent questions two weeks apart, generate variance ratios that are orders of magnitude apart depending solely on whether the question asks about cumulative or average annual returns.

4.5 Understanding America Study

The UAS data allow us to investigate the compression pattern in a nationally representative sample and, for the subset of respondents who completed both surveys, to replicate the within-person comparison from the student analysis. The UAS data differ from the student sample, however, in three respects: (1) the probability elicitation total return questions and the quantile elicitation average annual return questions were administered six years (rather than two weeks) apart, (2) the quantile elicitation 10-year questions are phrased slightly differently than the Duke CFO survey questions, and (3) the quantile elicitation 1-year questions ask about historical returns rather than expected returns. Despite these differences, the results are fully consistent with our hypothesis. The median probability elicitation variance ratio for the 3,644 UAS respondents who answer the total return questions in 2025 is 0.187—comparable to the ALP median of 0.147. Correspondingly, the median quantile elicitation variance ratio for the 1,439 respondents who answer the average annual return questions in 2019 is 2.60. For the 469 respondents who complete both surveys, 79% generate a larger variance ratio from the average annual return questions, with a median CFO-style average return variance ratio nearly ten times the median ALP-style total return variance ratio. The Internet Appendix provides complete details of the UAS analysis.

4.6 Overconfidence or misunderstanding? Historical estimates

Miscalibration implies that either respondents overestimate their ability to predict market returns or underestimate market risk (Ben-David, Graham, and Harvey (2013)). Our cognitive uncertainty hypothesis suggests that neither the unrealistically large variance ratios associated with CFO-style quantile elicitation questions nor the unrealistically small variance ratios associated with ALP-style probability elicitation questions reflect overconfidence in forecasting ability—instead, both reflect systematic misestimation of uncertainty. To test this directly, in our fall 2024 surveys we asked students about their perceptions of the *historical* 1- and 10-year return distributions using both question styles. In the first fall 2024 survey, students answered ALP-style probability elicitation questions about expected future returns followed by CFO-style quantile elicitation questions about historical return distributions. In the second fall 2024 survey (two weeks later), students answered CFO-style quantile elicitation questions about expected future returns followed by ALP-style probability elicitation questions about historical return distributions. For example, the CFO-style historical question asked, "Now we will ask you some questions about the historical average annual return over 10-year periods. Historically, over a 10-year period, I think the average S&P 500 return has had a 1-in-10 chance of being less than ___%." Correspondingly, the ALP-style historical question asked, "What do you think the chances are, historically, that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average have increased in value by more than 20 percent over a 10-year period?"

The logic of the test is straightforward. If students' miscalibration results from overconfidence in their ability to predict market returns, variance ratios computed from perceptions of historical distributions should be reasonable—there is nothing to be overconfident about when reporting historical facts. In contrast, if the extreme variance ratios in Tables I-III reflect a failure to understand the relation between horizon and uncertainty, the same patterns should emerge when asking about historical returns. The results strongly favor the latter interpretation. Panel A in Table IV reports summary statistics for inferred 1- and 10-year standard deviations and resulting variance ratios for 251 students responding to the historical ALP-style probability elicitation questions (the likelihood of a 20% gain or loss over 1 or 10 years). Panel B reports analogous statistics from 290 students for the historical CFO-style quantile elicitation questions (the 10^{th} and 90^{th} percentiles of 1- or 10-year returns). Panel C reports descriptive statistics for the 139 students with sufficient data to compute variance ratios from both question styles.[22] The results closely mirror those in Table III: the historical ALP-style probability elicitation questions generate unrealistically small variance ratios (median of 0.199 in Panel A), the historical CFO-style quantile elicitation questions generate unrealistically large variance ratios (median of 11.454 in Panel B), and the median student's variance ratio from the CFO-style questions is more than 88 times that from the ALP-style questions

[22]The Internet Appendix provides complete descriptive statistics for the historical questions.

(Panel C). In short, overconfidence in forecasting ability plays little role in driving the differences in variance ratios across question styles—the same patterns emerge even when there is no future return to forecast.

[Insert Table IV about here]

4.7 Average annual versus cumulative returns

The results in Tables I-IV demonstrate the CFO survey's average return questions yield substantially different variance ratio estimates than the ALP survey's total return questions. However, the two question structures differ not only with respect to asking about average annual versus cumulative returns, but also in question style—the CFO survey gives a quantile and elicits a return (e.g., 1-in-10 chance of an average annual return less than __%) while the ALP survey gives a return and elicits a probability (e.g., the likelihood markets have increased in value by more than 20%). To investigate the role of question style (quantile versus probability elicitation) versus average or cumulative returns, in spring 2025, fall 2025, and spring 2026 we fielded new surveys to new sets of introductory finance students at both universities.[23] Specifically, for half the respondents, the CFO-style quantile elicitation surveys reframed the average annual return questions into cumulative returns (e.g., there is a 1-in-10 chance the total (cumulative) return over the next 10 years will be less than __%) and reframed the ALP probability elicitation survey's total return questions as average annual returns (e.g., what are the chances that the US stock market will average an annual return of at least 10% per year over the next 10 years?). The other half of student respondents answered average annual return questions for the CFO-style quantile elicitation surveys and cumulative return questions for the ALP-style probability elicitation survey (i.e., the original structure of both surveys). Our central hypothesis is that variance ratios computed from estimated uncertainty in cumulative returns will be unreasonably small while variance ratios computed from estimated uncertainty in average annual returns will be unreasonably large, regardless of whether we use quantile elicitation (i.e., the CFO-style surveys) or probability elicitation (i.e., the ALP-style surveys).

Panel A in Table V summarizes the results for the ALP-style probability elicitation questions. The first six columns provide information on survey question structure. The final column in rows 1–4 report median variance ratios when asking students the likelihood of an average annual return while rows 5–8 report median variance ratios when asking students about the likelihood of a cumulative

[23] See Section 3 for a description of the two universities.

return. Panel C reports corresponding values for the CFO-style quantile elicitation surveys that ask respondents for average annual (rows 14–17) or cumulative (rows 18–21) return confidence intervals. Panels B and D in Table V report Z-statistics associated with tests of difference in medians (for Panels A and C, respectively) between the groups identified as “Group 1” and “Group 2”.

[Insert Table V about here]

The results in Table V are easily summarized. Most important, regardless of question style (ALP-style probability elicitation in Panel A or CFO-style quantile elicitation in Panel C), the typical (median) respondent always generates estimates that lead to variance ratios greater than one (rows 1–4 and 14–17) when answering questions about average annual returns and less than one (rows 5–8 and 18–21) when answering questions about cumulative returns. Moreover, regardless of question style or other parameters, these differences are always statistically meaningful at the 1% level for both the ALP-style probability elicitation questions (rows 9–11) and the CFO-style quantile elicitation questions (rows 22–24).

The cognitive uncertainty hypothesis also predicts that the bias will become more severe as the forecast horizon increases, since compressing near- and long-term forecasts leads to greater distortion when the underlying calculations are more complex (see Enke, Graeber, and Oprea (2025)). Consistent with our prediction, extending the forecast horizon from 5 to 15 years magnifies the bias in both directions and across both question styles. For the ALP-style probability elicitation questions, the variance ratio associated with average annual returns is larger at the 15-year horizon (row 3 versus 4; row 12) and the variance ratio associated with cumulative returns is smaller at the 15-year horizon (row 7 versus 8; row 13). For the CFO-style quantile elicitation questions, the same pattern holds: the variance ratio associated with average annual returns increases with horizon (row 16 versus 17; row 25) and the variance ratio associated with cumulative returns decreases with horizon (row 20 versus 21; row 26). All four horizon comparisons are statistically significant at the 1% level.

4.8 The fragility of perceived uncertainty

The cognitive uncertainty hypothesis predicts that respondents lack stable, well-anchored estimates of uncertainty—instead constructing their answers from whatever cues the elicitation provides rather than drawing on a coherent internal view of the return distribution. The evidence in this section directly supports this prediction: uncertainty estimates are highly sensitive to seemingly irrelevant features of the elicitation—the specific question format, the bin structure used to collect responses, the confidence interval level, and the return threshold used to elicit probabilities—and respondents readily anchor to uninformative and even factually incorrect reference points.[24]

The 2011 Q1 Duke CFO survey offers a striking illustration. In addition to the standard 80% confidence interval questions, CFOs were asked, "In 2010, the volatility of S&P 500 returns was 12.6%. What do you think the volatility of S&P 500 returns will be: __% in 2011; __% in 2012." Using the median 80% confidence intervals (from the same survey) to infer CFOs' 1-year return standard deviation (i.e., the same 2011 volatility) yields a value of 3.7%—nearly identical to the 3.8% median reported by Ben-David, Graham, and Harvey (2013) and the 3.3% median in our Table II. In contrast, when asked directly for their 2011 volatility belief in the same survey, the median CFO response nearly triples to 11%.[25] The results are inconsistent with the hypothesis that the typical CFO holds stable and internally consistent estimates of near-term uncertainty.

The anchoring evident in the 2011 Q1 CFO survey responses reinforces this conclusion. The survey's statement that "In 2010, the volatility of S&P 500 returns was 12.6%" appears to be an error—while the S&P 500 index *returned* 12.6% in 2010, annualized daily (monthly) volatility was 18.1% (19.3%) and the VIX averaged 22.55%. Yet CFOs reported values clustered around this erroneous seed (median CFO response of 11% with a cross-sectional standard deviation 3.9%), far from the true 2010 volatility (18–23%), the long-run historical average (20.1%), or the value implied by their own confidence interval responses (3.7%). This pattern is consistent with the cognitive uncertainty hypothesis (see also Tversky and Kahneman (1974)): lacking a reliable internal estimate of volatility, CFOs anchor on whatever reference point is provided—even an incorrect one.

The same sensitivity to elicitation method emerges for ALP respondents, again consistent with fragile uncertainty estimates. As noted in Section 3.1 (see footnote 12), in 18 of the 29 long-form ALP survey waves, approximately half of respondents were randomly assigned to answer a bins-and-balls version of the probability elicitation questions; despite holding the return unit (total returns) constant, the results reveal substantial sensitivity to elicitation format. Specifically, using

[24]The evidence in this section focuses primarily on 1-year uncertainty estimates, which are the focus of most survey questions. If respondents cannot form stable estimates of near-term uncertainty, then (1) cognitive uncertainty and the resulting compression at longer horizons follow directly, and (2) it seems unlikely they hold stable estimates of long-term uncertainty, which requires understanding the more complex relation between time and uncertainty.

[25]The mean CFO answer to the 2011 volatility belief question is 12.1%, with a cross-sectional standard deviation of 3.9%. Assuming approximately normally distributed responses, 97% of CFOs report a 2011 volatility estimate exceeding the 3.7% inferred from their own confidence interval responses. The implied 10-year variance ratio from the standard CFO questions in 2011 Q1 is 6.27—nearly identical to the 6.61 reported in Table II—but falls to 0.72 when the directly stated volatility estimate replaces the confidence-interval-inferred 1-year standard deviation. Note that the mean and median estimates for 2012 volatility were similarly anchored to the 2010 seed value (the mean and median were 11.5% and 10%, respectively). One possibility is that the difference arises because although the typical CFO has a good understanding of the historical 10^{th} and 90^{th} return percentiles (and strong overconfidence in their ability to forecast returns), they either do not know what standard deviation means or do not understand that "volatility" refers to standard deviation.

the distributional approach of Hartzmark and Sussman (2026) to infer volatility from the full distribution of ball allocations, the median 1-year standard deviation is 7.2% and the median 10-year standard deviation is 9.6%—far below both the corresponding estimates from the standard ALP probability elicitation questions in Table I (20.7% and 29.1%) and historical values (20.0% and 46.5%, respectively).[26] Because both the 1-year and 10-year inferred uncertainty estimates are distorted in the same direction by the bins-and-balls format, the median variance ratio (0.180) closely mirrors that from the standard ALP questions—even as the underlying estimates are substantially miscalibrated relative to both the standard questions and historical values. The Internet Appendix reports complete details (Table IA-IX and Figure IA-5).

Further consistent with the hypothesis that perceived uncertainty is fragile and sensitive to elicitation cues, Hartzmark and Sussman (2026) design an experiment in which respondents sequentially view 100 years of either historical US or Japanese returns, then allocate balls across bins to reflect what they observed. Critically, the bin cutoffs are calibrated to either US or Japanese historical data—and the two differ substantially, with Japanese bins spanning a much wider range (e.g., $< -42\%$ for the lowest Japanese bin versus $< -23\%$ for the lowest US bin). As detailed in the Internet Appendix (see Figure IA-6), the inferred return distribution is driven primarily by the bin structure rather than the data the respondent actually viewed. For instance, after viewing historical Japanese data (historical $\sigma = 27.24\%$), the median respondent's inferred standard deviation was nearly identical at 28.1%. However, when viewing the *same* Japanese data but placing balls in bins formed using the US historical distribution, the median inferred standard deviation fell to 19.66%—nearly identical to the US historical value of 18.35%. In short, respondents appear to match the distribution of bins rather than the underlying data.[27]

Further consistent with the view that respondents lack stable uncertainty estimates, Hartzmark and Sussman (2026) also document that CFO-style quantile elicitation responses are nearly invariant to the probability level used. We find the same pattern in our student sample: as detailed in the Internet Appendix (see Figure IA-7), when we vary the probability level (1-in-5, 1-in-10, and 1-in-20), students' median implied return thresholds barely move—for example, at the 1-year horizon the implied downside threshold shifts only from -0.050 (1-in-5) to -0.049 (1-in-10) to -0.040

[26] We also infer volatility using a tail probability approach, which yields median 1-year and 10-year standard deviations of 15.8% and 17.5%, respectively—closer to the standard ALP estimates. However, because most respondents allocate no balls to the extreme bins, this approach is limited to the small subset of respondents who do so (see Table IA-IX).

[27] The same pattern holds—albeit somewhat less starkly—for respondents who view US historical data: inferred standard deviations shift substantially depending on whether bins are calibrated to US or Japanese historical data. See also Appendix A10 of Hartzmark and Sussman (2026).

(1-in-20), while historically the corresponding thresholds shift substantially from -0.044 to -0.129 to -0.230.

The same pattern holds for ALP-style probability elicitation questions. The Internet Appendix (see Figure IA-8) reports the implied probability that returns fall within symmetric bands of ±10%, ±20%, and ±30% for students and for the historical CRSP distribution, at both the 1- and 10-year horizon. For the historical distribution, the probability of a return falling within the band rises sharply as the band widens—for example, at the 1-year horizon, the historical probability rises from 0.28 (±10%) to 0.60 (±20%) to 0.80 (±30%). In contrast, students' implied probabilities are nearly invariant to the band width: the corresponding 1-year medians are 0.55, 0.55, and 0.60. The 10-year results tell the same story. Taken together, the evidence across CFO-style quantile elicitation, ALP-style probability elicitation, and bins-and-balls elicitation consistently points to the same conclusion: perceived uncertainty is fragile, poorly anchored, and highly sensitive to seemingly irrelevant features of the elicitation.[28]

5 Cross-sectional Variance Ratio Heterogeneity

The results in the previous section demonstrate that cognitive uncertainty and question format drive most variance ratios far from theoretical and historical values. Despite these implausible levels, cross-sectional variation in both near- and long-term perceived volatility (and therefore variance ratios) appears to matter. Ben-David, Graham, and Harvey (2013) find that both near- and long-term miscalibration play important roles in explaining heterogeneity in firms' financial decisions. Sias, Starks, and Turtle (2026) find that near- and long-term expectations both play meaningful roles in individuals' equity market participation and risky share decisions. In this section, we therefore exploit heterogeneity in the ALP dataset to consider whether, holding question format constant, variation in cognitive uncertainty can help explain heterogeneity in variance ratios. In addition, we investigate five other (non-mutually exclusive) factors that have been proposed to explain heterogeneity in subjective forward-looking variance ratios: (1) respondents believe that

[28] An interesting open question is why the fragility of uncertainty estimates manifests differently across elicitation methods. For instance, students generate substantially miscalibrated near-term volatility estimates when asked CFO-style quantile elicitation questions (closely matching the pattern for CFOs) but reasonable near-term volatility estimates when asked ALP-style probability elicitation questions. Similarly, ALP respondents generate reasonable near-term volatility estimates when answering the standard probability elicitation questions but exhibit substantially greater miscalibration when answering the mathematically identical question using the bins-and-balls format. Some of the evidence in this section speaks to this question—for example, the near-invariance of confidence interval estimates to the probability level suggests that respondents anchor on a single uncertainty estimate rather than constructing a coherent distribution—but a full explanation of why elicitation method drives near-term miscalibration in some cases but long-term miscalibration in others remains an important direction for future research.

long-term returns are mean-reverting, (2) respondents believe markets are riskier or safer at longer horizons, (3) respondents recognize that parameter uncertainty increases long-term uncertainty relative to near-term uncertainty, (4) respondents expect market returns to be more (or less) volatile in the future, and (5) respondents' near-term overconfidence is, for some reason, greater than their long-term overconfidence.[29]

We begin by considering how cognitive uncertainty and parameter uncertainty generate opposite predictions for the relation between financial sophistication and variance ratios. The cognitive uncertainty hypothesis predicts that, for ALP-style probability elicitation questions where compression generates unrealistically small variance ratios, more financially sophisticated respondents will have greater confidence in their near- and long-term total return perceptions, exhibit less compression, and therefore generate larger (i.e., more realistic) variance ratios.[30] The parameter uncertainty hypothesis predicts the opposite: more financially sophisticated respondents will have less uncertainty regarding the parameters of the return generating process and therefore exhibit *relatively* lower long-term perceived uncertainty, resulting in smaller variance ratios.[31] We use education, income, equity market participation, self-rated understanding of equity markets, numeracy, and financial literacy as financial sophistication measures, and also consider overconfidence, expected changes in future market volatility, and mean-reversion beliefs.[32]

To examine how these characteristics relate to heterogeneity in variance ratios, we estimate panel regressions of variance ratios on wave fixed effects, respondent demographic controls (age, and indicators for gender, White ethnicity, married, working, and retired), and each of the remain-

[29]Bodie (2021) and Choi (2022) find that both financial experts and individuals often believe markets are less risky in the long run, even absent a formal understanding of mean-reversion. We therefore treat (1) and (2) as distinct: a respondent may believe long-term investing is safer without understanding or invoking mean-reversion as the reason. Pástor and Stambaugh (2012) argue that parameter uncertainty increases long-horizon uncertainty, providing the theoretical motivation for (3). Ben-David, Graham, and Harvey (2013) hypothesize that CFOs' long-term miscalibration is smaller than their near-term miscalibration because CFOs expect volatility to rise in the future. Finally, (5) follows directly from the evidence of greater near-term miscalibration in Ben-David, Graham, and Harvey (2013).

[30]Correspondingly, the prediction reverses for CFO-style quantile elicitation questions, where compression generates unrealistically large variance ratios—more financially sophisticated respondents would therefore be expected to exhibit smaller variance ratios.

[31]The parameter uncertainty hypothesis implicitly assumes respondents understand that parameter uncertainty increases long-run risk—an assumption that may be reasonable for CFOs (as in Pástor and Stambaugh (2012) and Ben-David, Graham, and Harvey (2013)) but is a stronger assumption for the general population respondents in the ALP sample.

[32]In the Internet Appendix, we present results for respondent sorts into high and low groups by these characteristics as well as sorts for gender, age, race, marital status, employment status, and retirement status. Consistent with our regression results and the hypothesis that cognitive uncertainty plays a larger role than parameter uncertainty in explaining variance ratio heterogeneity, we find that respondents with greater financial sophistication perceive more accurate variance ratios. In addition, Enke and Graeber (2023) find that women and younger individuals exhibit greater cognitive uncertainty. Consistent with their evidence, we also find that, relative to men and older individuals, women and younger individuals tend to exhibit smaller ALP variance ratios.

ing characteristics—individually and collectively—education, income, stock market participation, self-rated understanding of equity markets, numeracy, financial literacy, overconfidence, perceptions of expected changes in financial position, and mean-reversion beliefs. Except for the binary stock market participation indicator and the expected changes in financial position variable (which take the values of 1, 0, or -1), the characteristics are standardized to allow direct comparison of coefficients. A larger value for mean reversion indicates a greater belief in mean reversion. Table VI reports the estimates (standard errors are clustered at the respondent level).

[Insert Table VI about here]

For the first six characteristics, the cognitive uncertainty hypothesis predicts a positive coefficient while the parameter uncertainty hypothesis predicts a negative coefficient. The results uniformly support the hypothesis that the cognitive uncertainty effect dominates any parameter uncertainty effect, even when controlling for respondent demographics. For instance, the results in column (4) suggest that a one standard deviation higher perceived understanding of markets is associated with a 0.094 larger variance ratio (statistically significant at the 1% level).[33]

We find no support for the hypotheses that variation in overconfidence or economic expectations contributes to heterogeneity in variance ratios (columns (7) and (8))—overconfidence is not materially related to variance ratios once controlling for demographics, and the coefficient associated with economic expectations has the "wrong" sign.[34] The results in column (9) support the hypothesis that mean-reversion beliefs result in smaller variance ratios: a one standard deviation higher belief in mean-reversion is associated with a 0.037 lower variance ratio (statistically significant at the 1% level). Columns (10) and (11) report joint regression results including most or all variables; in column (10) we exclude numeracy, financial literacy, and overconfidence because these variables substantially limit sample sizes. In both columns, the results continue to support the hypothesis that heterogeneity in cognitive uncertainty and mean-reversion beliefs help explain heterogeneity in variance ratios.[35]

[33] An alternative interpretation is that these characteristics capture the degree to which respondents understand parameter uncertainty—e.g., more financially literate ALP respondents better understand that parameter uncertainty increases long-run risk, inducing a positive relation between variance ratios and financial literacy. This alternative, however, is hard to reconcile with the unrealistically small variance ratios throughout the ALP sample.

[34] Because beliefs of an improving financial situation are positively correlated with variables such as education, income, understanding of markets, numeracy, and financial literacy, which in turn appear inversely related to cognitive uncertainty, a likely explanation is that beliefs of an improving financial situation are associated with less cognitive uncertainty and therefore a higher variance ratio.

[35] Collinearity between many of the respondent characteristics likely has an impact on the weakened relations between variance ratios and some of the reported respondent characteristics in the joint panel regressions (see Table IA-VII for correlations between the explanatory variables).

Although the regressions suggest that heterogeneity in cognitive uncertainty and mean-reversion expectations can help explain heterogeneity in variance ratios when holding question format constant, comparison to the results in the previous section suggests that question format has an order of magnitude greater effect. For instance, the largest effect in Table VI (column (3)) shows that ALP stock market participants exhibit a 0.132 larger variance ratio than non-participants. In contrast, Panel G of Table III shows the median student variance ratio inferred from the CFO-style quantile elicitation questions exceeds the median student variance ratio inferred from the ALP-style probability elicitation questions by 8.71—an effect size 66 times greater than the largest effect in Table VI.

5.1 Long-term risk perceptions and variance ratios

In our fall 2024 surveys, we also asked students how they viewed the relation between the riskiness of investing in the stock market and their expected holding period, offering four choices ranging from long-term investing being more risky to less risky than near-term investing.[36] Of the 455 responses, 62% viewed markets as less risky in the long run, 21% as more risky, 13% as independent of horizon, and 4% had no idea. Despite the majority view that long-term investing is safer, we find no evidence that variance ratios differed between students who viewed markets as safer versus riskier in the long run. Specifically, the median CFO-style quantile elicitation variance ratios were 9.66 and 10.00 for students who view long-horizon investing as less and more risky, respectively, and the corresponding ALP-style probability elicitation variance ratios were 0.27 and 0.16. In neither case can we reject the hypothesis that the medians are equal (Z-statistics of 0.52 and -1.75, respectively).[37]

5.2 Cognitive uncertainty in non-financial estimates

In an ALP survey completed between August 2006 and November 2007, respondents were asked to estimate the likelihood of eight events (such as getting into a car accident, dying, or having something stolen) over both the next year and the next five years.[38] This survey provides a unique opportunity to test two predictions of the cognitive uncertainty hypothesis. First, if cognitive uncertainty causes compression across horizons, respondents' 5-year forecasts should be systemat-

[36] Specifically, students selected one of: (a) investing for the long-term (e.g., 10 years) is more risky than investing for the near-term (e.g., 1 year); (b) investing for the long-term is less risky; (c) riskiness does not depend on holding period; or (d) no idea. The Internet Appendix provides complete details.

[37] This result need not be inconsistent with our earlier finding that mean-reversion beliefs drive more reasonable variance ratios: a respondent may believe long-term investing is safer without understanding or invoking mean-reversion as the reason, and it is the latter that matters for variance ratios (see footnote 29).

[38] The complete list of questions is provided in the Internet Appendix.

ically too low relative to what their 1-year forecasts imply under independence. Second, Enke and Graeber (2023) and Enke, Graeber, and Oprea (2025) find that cognitive uncertainty is correlated across domains, so respondents who exhibit greater compression in these non-financial estimates should also exhibit lower ALP total return variance ratios.

The evidence strongly supports the first prediction. For instance, the median respondent reports a 20% chance of a car accident in the next year, implying a 5-year likelihood of 67.2% under independence—yet the typical respondent estimates only a 30% 5-year likelihood. Across all eight questions, 87% of respondents' 5-year estimates fall below the value implied by their 1-year estimate under independence.[39] For each respondent and question, we compute a standardized "compression error" as the difference between the implied 5-year likelihood under independence and the respondent's actual 5-year estimate; higher values indicate greater compression. Because ALP total return questions generate smaller variance ratios when there is greater compression, the predicted relation between compression errors and variance ratios is negative. Table VII reports the results.

[Insert Table VII about here]

The evidence also supports the second prediction. The coefficient associated with compression errors is negative and statistically meaningful at the 5% level or better for each of the eight questions. For instance, an individual with a one standard deviation higher compression error for the car accident question has a 4.9% lower variance ratio—nearly a 10% standard deviation change given the standard deviation of ALP variance ratios is 0.586 (Table I, Panel C). Taken together, the results in this section suggest that heterogeneity in cognitive uncertainty—whether measured through financial sophistication, beliefs about market risk, or compression errors in non-financial domains—plays a substantial role in explaining cross-sectional variation in ALP total return variance ratios.

6 Conclusions

Even in the simplest possible setting—where returns are *iid* and there is no parameter uncertainty or return predictability—understanding how uncertainty evolves with horizon requires recognizing that 10-year total return variance is 10 times 1-year variance while 10-year average annual return variance is only $1/10^{th}$ the annual variance. These relations are cognitively challenging for most—even financially sophisticated individuals. The additional complexity introduced by return

[39]We exclude observations where individuals report zero or 100% likelihoods for either horizon, as such forecasts imply no uncertainty. For instance, 90% of respondents report a 100% likelihood of visiting a dentist in the next five years.

predictability, parameter uncertainty, and the return generating process only compounds the difficulty. Traditional models implicitly assume economic agents have a reasonable understanding of this relation. We find little evidence to support this assumption. Across representative samples of Americans, CFOs, and business school students, perceived variance ratios are far from theoretical and historical benchmarks—and in opposite directions depending on how uncertainty is elicited. When asked about total return uncertainty, respondents generate implausibly small variance ratios, as if markets become nearly risk-free over time. When asked about average annual return uncertainty, respondents generate implausibly large variance ratios, as if per-year risk surges at longer horizons.

Our empirical tests suggest that cognitive uncertainty plays a central role in driving this compression. First, perceived uncertainty appears fragile and poorly anchored: estimates are highly sensitive to seemingly irrelevant features of the elicitation—the bin structure, the confidence interval level, and the return threshold—consistent with respondents constructing judgments on the fly from available cues rather than drawing on stable underlying beliefs. Second, the same compression pattern emerges when students are asked about historical rather than forward-looking return distributions, ruling out overconfidence in forecasting ability as the primary driver. Third, the pattern is robust to question style and worsens with horizon, consistent with the prediction that cognitive compression becomes more severe as the underlying calculation becomes more complex. Fourth, holding question format constant, more financially sophisticated ALP respondents generate more reasonable variance ratios. Finally, compression errors in non-financial domains—such as estimating the likelihood of a car accident over one versus five years—are meaningfully correlated with equity market variance ratios, suggesting that cognitive uncertainty is a general feature of how individuals reason about horizon and risk, not a phenomenon specific to financial markets.

The implications are broad. For household finance, our results suggest that the perceived riskiness of long-horizon equity investing—and therefore decisions about stock market participation, risky share allocation, and target-date fund choice—are heavily influenced by how uncertainty is elicited rather than by stable underlying beliefs. For corporate finance, the sensitivity of perceived uncertainty to question format may help explain why qualitative risk measures outperform quantitative ones in predicting firm decisions. More broadly, if economic agents lack stable, well-anchored views of long-horizon uncertainty, the connection between subjective risk perceptions and portfolio decisions is more tenuous than standard models assume—with implications for asset pricing, financial regulation, and the design of surveys used to elicit beliefs.

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Table I – Individuals' Beliefs Regarding Near- and Long-Term Stock Returns
Panels A, B, and C report descriptive statistics for the pooled cross-sectional time-series of American Life Panel Survey data between 2008 and 2016. Panels D, E, and F report cross-sectional descriptive statistics from the 2009 Health and Retirement Study Internet survey. Both surveys use probability elicitation: respondents report the likelihood that markets will rise, rise by at least 20%, and fall by at least 20% over the next year (Panels A and D) and next decade (Panels B and E). Expected returns and standard deviations reported in the bottom two rows of Panels A, B, D, and E are inferred from the reported likelihoods markets rise or fall at least 20% assuming normally distributed continuously compounded returns. The penultimate column reports the historical average (computed from the CRSP value-weighted index between 1926 and 2024). The final column reports the fraction of observations below the historical average.

Description	N	Mean	25th	Median	75th	Std Dev.	Hist.	%<Hist
Panel A: American Life Panel stock market expectations over next year								
P(market>0)	22,722	0.406	0.200	0.400	0.600	0.248	0.747	0.863
P(market>20%)	22,745	0.201	0.100	0.150	0.300	0.155	0.336	0.822
P(market<-20%)	22,745	0.183	0.100	0.150	0.250	0.142	0.060	0.219
$E_{i,t}(r_{1year})$	22,745	-0.010	-0.046	-0.020	0.027	0.092	0.095	0.887
$\sigma_{i,t}(r_{1year})$	22,745	0.274	0.158	0.207	0.301	0.191	0.200	0.491
Panel B: American Life Panel stock market expectations over next decade								
P(market>0)	22,733	0.515	0.250	0.500	0.750	0.287	0.960	0.976
P(market>20%)	22,745	0.347	0.150	0.300	0.500	0.236	0.932	0.996
P(market<-20%)	22,745	0.169	0.060	0.100	0.250	0.132	0.013	0.030
$E_{i,t}(r_{10years})$	22,745	0.134	-0.020	0.043	0.182	0.253	0.969	0.970
$\sigma_{i,t}(r_{10years})$	22,745	0.414	0.179	0.291	0.505	0.353	0.465	0.721
Panel C: American Life Panel variance ratios								
Variance ratios	22,745	0.391	0.100	0.147	0.373	0.586	0.542	0.819
%Variance ratio<1	22,745	0.892						
Panel D: Health and Retirement Study stock market expectations over next year								
P(market>0)	1,250	0.370	0.200	0.300	0.500	0.231	0.747	0.897
P(market>20%)	1,252	0.241	0.100	0.200	0.300	0.181	0.336	0.758
P(market<-20%)	1,252	0.176	0.100	0.100	0.250	0.134	0.060	0.181
$E_{i,t}(r_{1year})$	1,252	0.016	-0.020	-0.020	0.065	0.117	0.095	0.800
$\sigma_{i,t}(r_{1year})$	1,252	0.284	0.158	0.225	0.316	0.180	0.200	0.423
Panel E: Health and Retirement Study stock market expectations over next decade								
P(market>0)	1,249	0.525	0.300	0.500	0.750	0.261	0.960	0.990
P(market>20%)	1,252	0.386	0.200	0.300	0.600	0.246	0.932	0.999
P(market<-20%)	1,252	0.158	0.100	0.100	0.200	0.115	0.013	0.014
$E_{i,t}(r_{10years})$	1,252	0.193	-0.020	0.065	0.256	0.302	0.969	1.000
$\sigma_{i,t}(r_{10years})$	1,252	0.424	0.191	0.301	0.521	0.320	0.465	0.698
Panel F: Health and Retirement Study variance ratios								
Variance ratios	1,252	0.434	0.100	0.145	0.400	0.664	0.542	0.795
%Variance ratio<1	1,252	0.875						

Table II – CFOs' Beliefs Regarding Near- and Long-Term Stock Returns

The table reports summary statistics for the time series of the mean and median Duke CFO survey over the sample period from March 2004 to December 2019 (n=63 quarters). The survey uses quantile elicitation: respondents report the 10^{th} and 90^{th} percentile returns (P10 and P90) and their expected return for the next year (Panel A) and average annual return over the next decade (Panel B). Standard deviations are inferred from the reported percentiles assuming normally distributed continuously compounded returns. Column (1) reports historical results for the CRSP value-weighted index between 1926 and 2024. Columns (2) through (5) report results based on mean CFO beliefs for the next year (Panel A), the next decade (Panel B), and the associated variance ratios (Panel C). Columns (6) to (9) report analogous results for median CFO beliefs. Panel B reports two measures of long-term uncertainty: the standard deviation of average annual returns ($\sigma_t(\bar{r}_{10years})$), which is the quantity directly elicited by the quantile elicitation questions, and the standard deviation of total returns ($\sigma_t(r_{10years})$), which equals 10 times the former and is directly comparable in units to the long-term standard deviation reported for ALP and HRS respondents in Table I.

		Cross-sectional mean CFO beliefs				Cross-sectional median CFO beliefs			
Description	Historical (1)	Mean (2)	Minimum (3)	Maximum (4)	%<Hist (5)	Mean (6)	Minimum (7)	Maximum (8)	%<Hist (9)
Panel A: CFOs' stock market expectations over the next year									
$P90(r_{1year})$	0.302	0.098	0.069	0.129	1	0.089	0.058	0.113	1
$P10(r_{1year})$	-0.138	-0.023	-0.090	0.008	0	0.003	-0.051	0.030	0
$E_t(r_{1year})$	0.095	0.052	0.022	0.075	1	0.048	0.020	0.070	1
$\sigma_t(r_{1year})$	0.200	0.047	0.037	0.073	1	0.033	0.023	0.046	1
Panel B: CFOs' stock market expectations over the next decade									
$P90(\bar{r}_{10years})$	0.152	0.103	0.084	0.125	1	0.093	0.077	0.113	1
$P10(\bar{r}_{10years})$	0.035	0.018	-0.004	0.041	0.952	0.025	0.010	0.039	0.857
$E_t(r_{10years})$	0.969	0.682	0.554	0.861	1	0.616	0.488	0.770	1
$\sigma_t(\bar{r}_{10years})$	0.047	0.033	0.025	0.041	1	0.027	0.022	0.033	1
$\sigma_t(r_{10years})$	0.465	0.333	0.246	0.413	1	0.267	0.219	0.326	1
Panel C: CFOs' variance ratios									
Variance ratios	0.542	5.113	2.675	9.003	0	6.606	3.895	11.783	0
%Variance ratio<1		0				0			

Table III – Students' Beliefs Regarding Near- and Long-Term Stock Returns

Panels A, B, and C report descriptive statistics for 322 undergraduate students based on probability elicitation cumulative return questions: respondents report the likelihood markets rise, rise by at least 20%, and fall by at least 20% over the next year and decade. Panels D, E, and F report descriptive statistics for 673 students based on quantile elicitation average annual return questions: respondents report the 10^{th} and 90^{th} percentile returns and expected return for the next year and average annual return over the next decade. Standard deviations are inferred from the reported percentiles or likelihoods assuming normally distributed continuously compounded returns. Panel E reports two measures of long-term uncertainty: the standard deviation of average annual returns ($\sigma_{i,t}(\bar{r}_{10years})$), which is the quantity directly elicited, and the standard deviation of total returns ($\sigma_{i,t}(r_{10years})$), which equals 10 times the former and is directly comparable to the long-term standard deviation in Panel B. Panel G summarizes variance ratio differences for the 241 students with sufficient data to compute variance ratios based on both sets of questions. Students were surveyed during the spring and fall 2024 semesters. The Internet Appendix provides details of the student surveys.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Students' stock market expectations over next year (probability elicitation, cumulative returns)								
P(market>0)	322	0.659	0.500	0.700	0.800	0.189	0.747	0.581
P(market>20%)	322	0.298	0.150	0.250	0.400	0.185	0.336	0.646
P(market<-20%)	322	0.216	0.100	0.200	0.300	0.149	0.060	0.118
$E_{i,t}(r_{1year})$	322	0.049	-0.041	0.001	0.107	0.184	0.095	0.736
$\sigma_{i,t}(r_{1year})$	322	0.433	0.191	0.297	0.521	0.361	0.200	0.276
Panel B: Students' stock market expectations over next decade (probability elicitation, cumulative returns)								
P(market>0)	322	0.757	0.600	0.850	0.950	0.244	0.960	0.814
P(market>20%)	322	0.544	0.360	0.600	0.750	0.241	0.932	0.994
P(market<-20%)	322	0.169	0.050	0.100	0.250	0.143	0.013	0.068
$E_{i,t}(r_{10years})$	322	0.428	0.050	0.282	0.680	0.475	0.969	0.907
$\sigma_{i,t}(r_{10years})$	322	0.673	0.297	0.505	0.922	0.500	0.465	0.432
Panel C: Students' variance ratios (probability elicitation, cumulative returns)								
"ALP" variance ratio	322	0.712	0.086	0.238	0.639	1.154	0.542	0.696
%Variance ratio<1	322	0.807						
Panel D: Students' stock market expectations over next year (quantile elicitation, average annual returns)								
$P90(r_{1year})$	673	0.273	0.113	0.182	0.405	0.254	0.302	0.697
$P10(r_{1year})$	673	0.092	0.030	0.049	0.095	0.109	-0.138	0.001
$E_{i,t}(r_{1year})$	673	0.202	0.077	0.113	0.262	0.198	0.095	0.315
$\sigma_{i,t}(r_{1year})$	673	0.065	0.023	0.043	0.087	0.057	0.200	0.942

Table III –
Students' Reported Beliefs Regarding Near- and Long-Term Stock Returns (Cont.)

Panel E: Students' stock market expectations over next decade (quantile elicitation, average annual returns)								
$P90(\bar{r}_{10years})$	673	0.205	0.095	0.140	0.262	0.169	0.152	0.554
$P10(\bar{r}_{10years})$	673	0.067	0.020	0.049	0.095	0.102	0.035	0.407
$E_{i,t}(r_{10years})$	673	1.439	0.583	0.953	1.823	1.350	0.969	0.574
$\sigma_{i,t}(\bar{r}_{10years})$	673	0.051	0.019	0.035	0.068	0.042	0.047	0.575
$\sigma_{i,t}(r_{10years})$	673	0.512	0.187	0.355	0.680	0.417	0.465	0.575
Panel F: Students' variance ratios (quantile elicitation, average annual returns)								
"CFO" variance ratio	673	13.570	2.988	9.318	13.596	17.694	0.542	0.067
%Variance ratio<1	673	0.098						
Panel G: Students' differences in variance ratios								
"ALP" variance ratio	241	0.789	0.100	0.241	0.745	1.250	0.542	0.680
"CFO" variance ratio	241	13.541	3.365	9.462	13.742	17.148	0.542	0.066
("CFO")/("ALP")	241	119.765	6.552	25.235	117.004	275.326		
"CFO" - "ALP"	241	12.752	2.712	8.706	13.483	16.954		
("CFO" - "ALP")> 0	241	0.946						

Table IV – Students' Perceptions of Historical Return Distributions

Panel A reports descriptive statistics for 251 undergraduate students based on their perceived likelihood that, historically, markets have risen or fallen by at least 20% over 1- and 10-year periods (i.e., the probability elicitation questions reframed as perceived historical return distributions). Panel B reports descriptive statistics for 290 students based on their perceived 10^{th} and 90^{th} percentiles of historical 1- and 10-year returns (i.e., the quantile elicitation questions reframed as perceived historical return distributions). Panel B also reports two measures of long-term uncertainty: the standard deviation of average annual returns ($\sigma_{i,t}(\bar{r}_{10years})$), which is the quantity directly elicited by the quantile elicitation questions, and the standard deviation of total returns ($\sigma_{i,t}(r_{10years})$), which equals 10 times the former and is directly comparable in units to the long-term standard deviation reported in Panel A. Panel C summarizes the variance ratio information for the 139 students with sufficient data to compute variance ratios based on both sets of questions. Students were surveyed during the fall 2024 semester. The Internet Appendix provides complete summary information (analogous to Table III).

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Students' variance ratios based on historical return distributions (probability elicitation, cumulative returns)								
$\sigma_{i,t}(r_{1year})$	251	0.325	0.158	0.237	0.387	0.249	0.200	0.430
$\sigma_{i,t}(r_{10years})$	251	0.508	0.225	0.362	0.601	0.418	0.465	0.614
"ALP" variance ratio	251	0.599	0.086	0.199	0.618	0.939	0.542	0.729
%Variance ratio<1	251	0.825						
Panel B: Students' variance ratios based on historical return distributions (quantile elicitation, average annual returns)								
$\sigma_{i,t}(r_{1year})$	290	0.054	0.021	0.041	0.076	0.042	0.200	1.000
$\sigma_{i,t}(\bar{r}_{10years})$	290	0.074	0.026	0.048	0.094	0.079	0.047	0.483
$\sigma_{i,t}(r_{10years})$	290	0.675	0.257	0.481	0.941	0.555	0.465	0.483
"CFO" variance ratio	290	50.836	5.811	11.454	40.831	86.681	0.542	0.034
%Variance ratio<1	290	0.066						
Panel C: Students' differences in variance ratios (historical return distributions)								
"ALP" variance ratio	139	0.681	0.086	0.199	0.624	1.044	0.542	0.712
"CFO" variance ratio	139	43.510	7.432	11.527	38.928	71.989	0.542	0.007
("CFO")/("ALP")	139	435.357	16.662	88.294	210.344	1593.260		
"CFO" - "ALP"	139	42.829	6.363	10.844	38.791	71.954		
("CFO" - "ALP")> 0	139	0.978						

Table V – The Importance of Average Annual versus Cumulative Returns

Rows 1–4 of Panel A report the median inferred variance ratio for samples of students answering ALP-style probability elicitation questions framed in average annual returns. Rows 5–8 report median variance ratios for samples of students answering ALP-style probability elicitation questions framed in cumulative returns. Rows 14–17 of Panel C report the median inferred variance ratio for samples of students answering CFO-style quantile elicitation questions framed in average annual returns. Rows 18–21 report median variance ratios for samples of students answering CFO-style quantile elicitation questions framed in cumulative returns. Panels B and D report Z-statistics for comparisons across the indicated groups. The sample is based on surveys given to undergraduate business students in the spring 2025, fall 2025, and spring 2026 semesters.

Panel A: ALP-style probability elicitation questions							
Row no.	Returns	Horizon	Past or future	Return above	Return below	N	Variance ratio
1	Average annual	10	Future	≥10%	≤-5%	133	5.813
2	Average annual	10	Future	≥10%	≤-5%	129	4.555
3	Average annual	5	Past	≥10%	≤-5%	144	4.531
4	Average annual	15	Past	≥10%	≤-5%	134	11.164
5	Cumulative	10	Future	≥10%	≤-10%	77	0.367
6	Cumulative	10	Future	≥30%	≤-30%	143	0.358
7	Cumulative	5	Past	≥20%	≤-20%	168	0.466
8	Cumulative	15	Past	≥20%	≤-20%	113	0.205
Panel B: ALP-style probability elicitation questions—Test statistics							
	Compare	Sample	Group 1 rows	Group 2 rows	Z-statistic		
9	Ave. annual vs. cumul.	Future and past	1, 2, 3, 4	5, 6, 7, 8	20.369***		
10	Ave. annual vs. cumul.	Future only	1, 2	5, 6	13.155***		
11	Ave. annual vs. cumul.	Past only	3, 4	7, 8	15.621***		
12	5 year vs. 15 yr horizon	Ave. annual	3	4	−4.313***		
13	5 year vs. 15 yr horizon	Cumulative	7	8	3.230***		
Panel C: CFO-style quantile elicitation questions							
Row no.	Returns	Horizon	Past or future	Likelihood		N	Variance ratio
14	Average annual	10	Past	1 in 20		157	18.403
15	Average annual	10	Past	1 in 5		169	11.437
16	Average annual	5	Future	1 in 10		248	5.963
17	Average annual	15	Future	1 in 10		256	16.893
18	Cumulative	10	Past	1 in 5		162	0.135
19	Cumulative	10	Past	1 in 20		170	0.165
20	Cumulative	5	Future	1 in 10		236	0.409
21	Cumulative	15	Future	1 in 10		272	0.205
Panel D: CFO-style quantile elicitation questions—Test statistics							
	Compare	Sample	Group 1 rows	Group 2 rows	Z-statistic		
22	Ave. annual vs. cumul.	Future and past	14, 15, 16, 17	18, 19, 20, 21	29.943***		
23	Ave. annual vs. cumul.	Future only	16, 17	20, 21	23.250***		
24	Ave. annual vs. cumul.	Past only	14, 15	18, 19	18.699***		
25	5 year vs. 15 year horizon	Ave. annual	16	17	−8.189***		
26	5 year vs. 15 year horizon	Cumulative	20	21	4.088***		

Table VI – Panel Regressions of Variance Ratios on Characteristics

This table reports coefficients from panel regressions of ALP respondents' variance ratios on their characteristics. Control variables include wave fixed effects, respondent age, and indicators for gender, White race, married, working, and retired. All reported explanatory variables, except the indicator variables for holding equity and the expected change in financial position (better next year) are standardized (rescaled to zero mean and unit variance). Larger mean reversion values indicate stronger beliefs about mean reversion. Standard errors are clustered at the respondent level. Significance at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively.

Description	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Years education	0.053***									0.020***	0.012
Income		0.054***								0.017**	-0.005
Holds equity indicator			0.132***							0.066***	0.052*
Understands market				0.094***						0.069***	0.069***
Numeracy					0.096***						0.054***
Financial literacy						0.130***					0.048***
Overconfidence							0.013				0.015
Better next year								0.034***		0.021**	0.016
Mean reversion									-0.037***	-0.031***	-0.047**
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	22,690	22,617	22,681	22,626	11,465	8,164	8,143	22,542	21,506	21,308	6,900

Table VII – Panel Regressions of Variance Ratios on Respondents' Compression Errors

This table reports coefficients from panel regressions of ALP respondents' perceived variance ratios on the difference between the implied 5-year likelihood (assuming independence given their reported 1-year likelihood) and their reported 5-year event likelihood (i.e., their compression error). Control variables include wave fixed effects, respondent age, and indicators for gender, White race, married, working, and retired. All variables are standardized (rescaled to zero mean and unit variance) and standard errors are clustered at the respondent level. Significance at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
P(car accident)	-0.049***							
P(cavity filled)		-0.068***						
P(die)			-0.048***					
P(theft victim)				-0.038**				
P(move)					-0.060***			
P(die terrorism)						-0.060***		
P(break in victim)							-0.071***	
P(visit dentist)								-0.047**
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	7,851	5,927	7,773	7,583	3,910	5,764	7,470	1,704

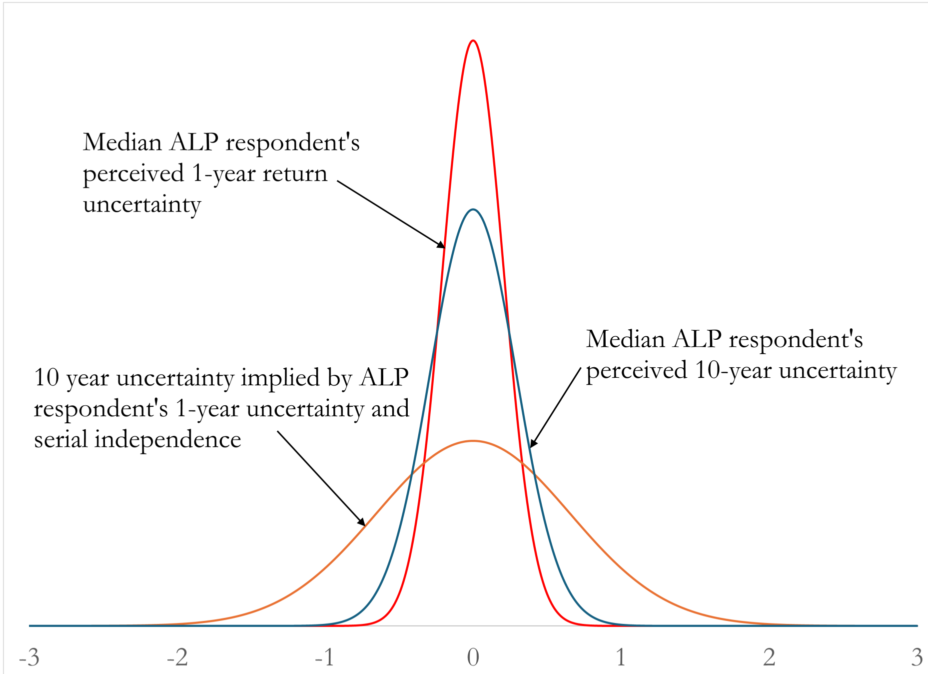


Figure 1 – Near- and long-term uncertainty for ALP respondents. The figure presents the perceived return distributions for the median ALP respondent for three measures of uncertainty after recentering at zero. The distributions are described as follows: 1-year imputed uncertainty (solid red line), 10-year total return imputed uncertainty (solid blue line), and the 10-year total return uncertainty implied by the median respondent's 1-year beliefs under serial independence (solid orange line). Uncertainty estimates are based on ALP-style probability elicitation questions: respondents report the likelihood markets rise, rise by at least 20%, and fall by at least 20% over the next year and decade. All distributions are based on continuously compounded returns. The uncentered 1-year and 10-year return distributions with the accompanying historical distributions are reported in Figure IA-1 of the Internet Appendix.

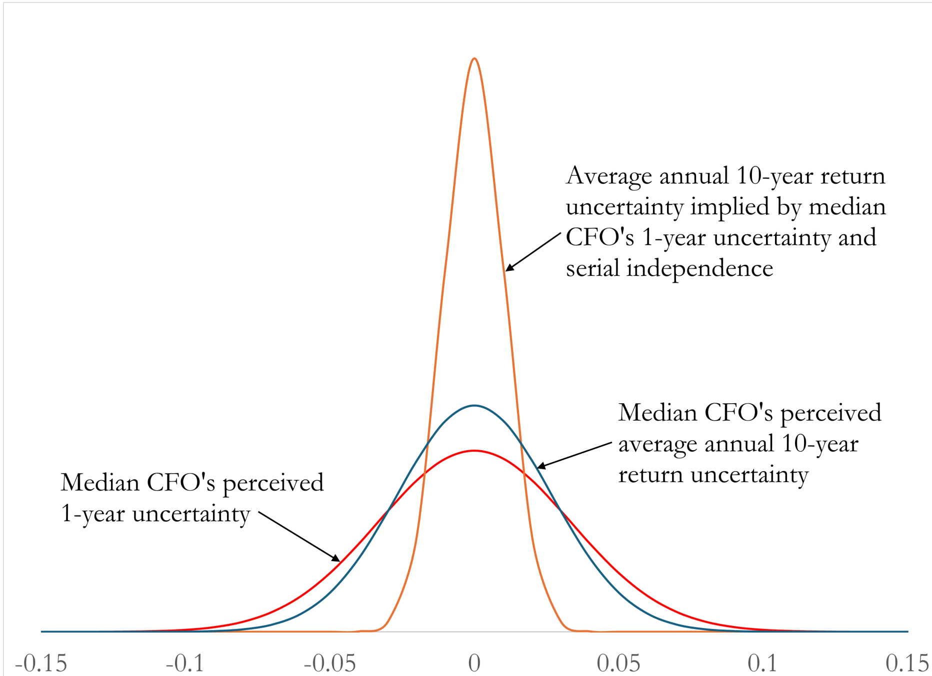
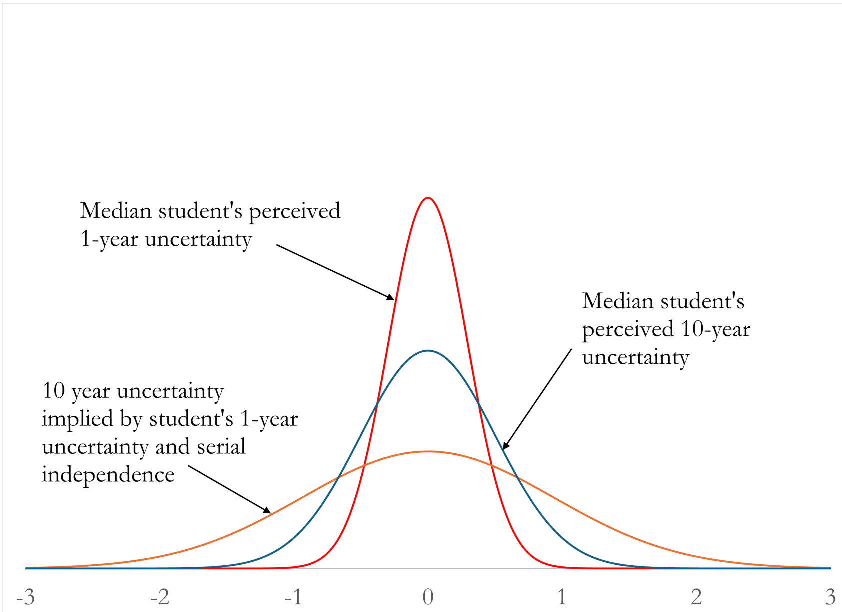
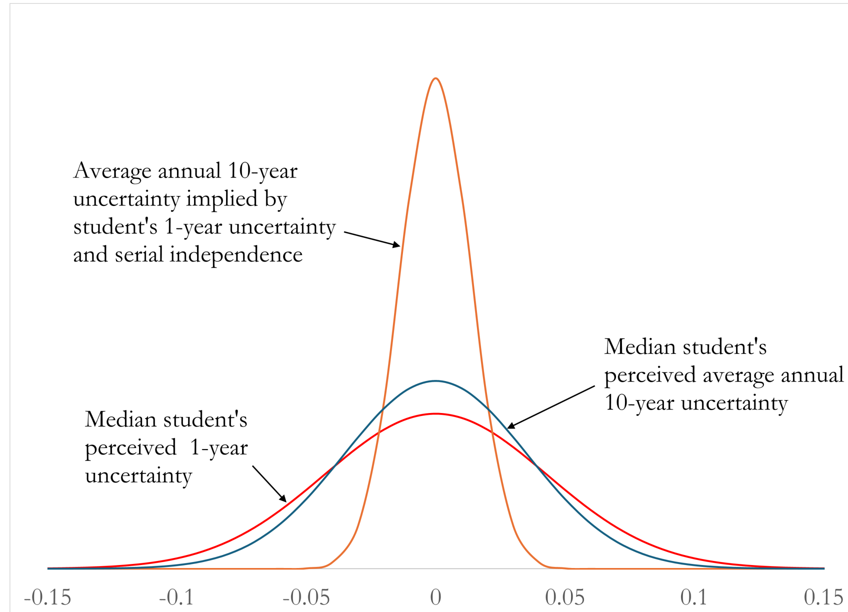


Figure 2 – Near- and long-term uncertainty for CFOs. The figure presents the perceived return distributions for the median CFO respondent for three measures of uncertainty after recentering at zero. The distributions are described as follows: 1-year imputed uncertainty (solid red line), imputed uncertainty in average annual returns over 10 years (solid blue line), and the average annual 10-year return uncertainty implied by the median CFO's 1-year beliefs under serial independence (solid orange line). Uncertainty estimates are based on CFO-style quantile elicitation questions: respondents report the 10^{th} and 90^{th} percentile returns and expected return for the next year and average annual return over the next decade. All distributions are based on continuously compounded returns. Because CFOs are asked about average annual long-term returns, the blue and orange lines are expressed in terms of average annual returns rather than total returns. The uncentered 1-year and 10-year return distributions with the accompanying historical distributions are reported in Figure IA-2 of the Internet Appendix.



(A) Median student's 1- and 10-year uncertainty based on probability elicitation total return questions



(B) Median student's 1- and 10-year uncertainty based on quantile elicitation average annual return questions

FIGURE 3 – NEAR- AND LONG-TERM UNCERTAINTY FOR STUDENTS. Panel A presents the perceived return distributions for the median student respondent for three measures of uncertainty after recentering at zero. The distributions are described as follows: 1-year imputed uncertainty (solid red line), 10-year total return imputed uncertainty (solid blue line), and the 10-year total return uncertainty implied by the median student's 1-year beliefs under serial independence (solid orange line). Uncertainty estimates are based on ALP-style probability elicitation questions: respondents report the likelihood markets rise, rise by at least 20%, and fall by at least 20% over the next year and decade. Panel B presents analogous distributions based on CFO-style quantile elicitation questions: respondents report the 10^{th} and 90^{th} percentile returns for the next year and for the average annual return over the next decade. The blue and orange lines in Panel B are expressed in terms of average annual returns rather than total returns. All distributions are based on continuously compounded returns. The uncentered 1-year and 10-year return distributions with the accompanying historical distributions are reported in Figures IA-3 and IA-4 of the Internet Appendix.

Internet Appendix for
"Uncertain Uncertainty"

Contents

IA-1 American Life Panel—additional detail

We use all 61 (short- and long-form) ALP surveys—not just the 29 long-form surveys used in our main analysis—to estimate each individual's mean-reversion beliefs. Specifically, for each respondent, we estimate a time-series regression of their perceived likelihood markets rise in the next decade (which is asked in all 61 surveys) on the market return over the previous 12 months. For ease of interpretation, we standardize lagged returns by the standard deviation of the annual CRSP value-weighted market return, so the coefficient reflects the change in the perceived likelihood markets rise in the next decade associated with a one standard deviation higher lagged annual return. To facilitate reporting, we define the mean reversion variable as the estimated regression coefficient x -1, so that larger positive values suggest greater mean reversion. We require that respondents have at least 10 observations (from the 61 surveys) to estimate their mean-reversion beliefs. To ensure outliers do not drive the results, we winsorize the estimated coefficients at the 5^{th} and 95^{th} percentiles.

We hypothesize that individuals who expect their personal financial situation to worsen also perceive greater future economic uncertainty and riskier equity markets. Based on this hypothesis, we use the answer to the question *Now looking ahead – do you think a year from now you will be better off financially, worse off, or about the same as now?* as a proxy for expected future changes in market volatility. Consistent with our hypothesis, in two ALP surveys not in our main sample (ALP surveys 17 and 61; fielded in December 2007–April 2008 and January–March 2009, respectively), respondents were asked this question alongside their expectations for business conditions (*And how about a year from now, do you expect that in the country as a whole business conditions will be better, or worse than they are at present, or just about the same?*) and unemployment (*How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same or less?*). We reverse score the unemployment variable (such that higher values indicate expected lower unemployment) so expected correlations are all positive. Sample sizes for these questions range from 501 to 555 per survey. The six correlations between the three metrics are statistically significant at the 1% level and range from 0.22 to 0.47, consistent with the hypothesis that expected changes in personal financial situation proxy for expected changes in market volatility (e.g., Hamilton and Lin (1996)).

The ALP surveys also collect data regarding respondents' gender, race, marital status, employment status, retirement status, age, years of education, income, stock market participation, as well as measures of numeracy, financial literacy, and overconfidence, which we use to proxy for financial sophistication and cognitive uncertainty in Section 5 (see Section IA-14 for variable construction details).

Table IA-I reports descriptive statistics of ALP respondent characteristics for our pooled cross-sectional time-series of 22,745 observations from 3,022 individuals (the average respondent participates in more than 7.5 long-form surveys). Our ALP sample is 58% female, 88% White race, 66% married, 61% working, and 23% retired. As shown in the bottom row of Table IA-I, approximately two-thirds of observations reveal a belief in mean-reversion, indicating that the typical respondent perceives a higher likelihood of a positive market return over the next decade when recent returns have been lower.

[Insert Table IA-I about here]

IA-2 ALP, CFO, and Student near- and long-term distributions

Figure IA-1 displays the inferred and historical near- and long-term return distributions for ALP respondents (these inferred distributions underlie Figure 1). In Panel A, the dashed red line reports the historical distribution for continuously compounded 1-year U.S. equity returns, based on the mean and standard deviation of the CRSP value-weighted index. The solid red line reflects the median estimated ALP respondent's perceived near-term distribution, based on their inferred expected return and standard deviation. Although the typical ALP respondent holds a bearish near-term view, their inferred near-term uncertainty is close to the historical average. Panel B presents the corresponding long-term distributions.

Figure IA-2 displays the analogous distributions for CFO respondents (these inferred distributions underlie Figure 2). In Panel A, the dashed red line reflects the historical 1-year return distribution from the CRSP value-weighted index, while the solid red line reflects the inferred 1-year distribution based on the mean CFO's expected return and variance. Similar to ALP and HRS respondents, CFOs tend to exhibit near-term expected returns below the historical market average. Unlike ALP and HRS respondents, however, CFO beliefs imply 1-year standard deviations substantially below historical values. Panel B presents the corresponding long-term distributions, with the solid and dashed blue lines reflecting mean CFO beliefs and the CRSP value-weighted index, respectively.

Figures IA-3 and IA-4 present the inferred and historical near- and long-term return distributions for students answering the ALP-style and CFO-style questions, respectively (these inferred distributions underlie Figure 3). In each figure, Panel A presents the 1-year distributions and Panel B presents the 10-year distributions, with solid lines reflecting mean student beliefs and dashed lines reflecting the historical CRSP value-weighted index. The patterns mirror those of their survey-design counterparts. When answering the ALP-style total return question, the median student's inferred 1-year standard deviation is close to the historical value. When answering the same question reframed in the CFO style, students exhibit, similar to CFOs, substantial miscalibration—implying a 1-year standard deviation well below the historical average.

IA-3 Spring and fall 2024 student surveys—additional detail

The results in Table III and Figure 3 are based on surveys given in the spring and fall of 2024. A total of 828 students completed all four probability elicitation questions required to estimate variance ratios (i.e., chance market rises at least 20% in next year or decade; chance market falls at least 20% in next year or decade). To estimate variances, we require: (1) the sum of the perceived likelihoods that markets rise and fall at least 20% in the next year (decade) must be less than 100%, and (2) none of the perceived likelihoods can be zero. Of the 828 students, 238 report values that sum to exactly 100%, 193 report values that sum to greater than 100%, and 75 report a 0% likelihood for at least one of the four required probabilities. Panels A and B of Table IA-II report descriptive statistics for the perceived return likelihoods for the complete sample (n=828), analogous to the first three rows of Panels A and B in Table III. The results, based on raw (i.e., unwinsorized) data, exhibit the same patterns as reported in Table III.

[Insert Table IA-II about here]

A total of 824 students completed all four quantile elicitation questions required to estimate near- and long-term variances (i.e., 10^{th} and 90^{th} return percentiles for the next year and average annual return percentiles for the next decade). Estimation requires that the 10^{th} percentile return is less than the 90^{th} percentile return; 151 students violate this constraint. Panels C and D of Table IA-II report descriptive statistics for the perceived returns for the complete sample (n=824), analogous to the first three rows of Panels D and E in Table III. The results, based on raw (i.e., unwinsorized) data, exhibit the same patterns as reported in Table III.

IA-4 Student surveys—question order

Both the probability elicitation and quantile elicitation survey questions were given to undergraduate business students enrolled in introductory finance courses at two large public universities. In the fall 2024 semester, one section of students at University A were first given the quantile elicitation questions then two weeks later given the probability elicitation questions. Students in the second section at University A were first given the probability elicitation questions then two weeks later given the quantile elicitation questions. Table IA-III reports the distribution of inferred variance ratios for each section (analogous to Panels C and F in Table III). The results reveal the same pattern in both sections. Specifically, median variance ratios based on the probability elicitation questions were 0.198 and 0.140 in the two sections; a difference in medians test reveals no evidence the difference was statistically significant (p-value=0.82). Correspondingly, the median variance ratios based on the quantile elicitation questions were 9.81 and 7.15, and the difference was not statistically significant (p-value=0.55).

[Insert Table IA-III about here]

IA-5 Students' perceived historical return distributions

The results in Table IV are based on the fall 2024 surveys. Table IA-IV reports descriptive statistics for students' estimates of the historical 1- and 10-year return distributions used to generate the values reported in Table IV, based on the probability elicitation (total return) and quantile elicitation (average return) questions. Consistent with the forward-looking results in Table III, the patterns are nearly identical regardless of whether students are asked about historical or expected future returns—the probability elicitation questions generate unrealistically small variance ratios while the quantile elicitation questions generate unrealistically large variance ratios.

[Insert Table IA-IV about here]

IA-6 Spring 2025, fall 2025, and spring 2026 student surveys—additional detail

The results in Table V are based on student surveys given in spring 2025, fall 2025, and spring 2026. Students were randomly assigned to one of four survey versions, with each student completing two surveys two weeks apart. In one survey, students were assigned probability elicitation questions about either average annual or cumulative returns (reported in rows 1, 2, 5, and 6 of Table V) paired with quantile elicitation questions about either average annual or cumulative historical returns (reported in rows 14, 15, 18, and 19). In the other survey, students were assigned quantile elicitation questions about average annual or cumulative future returns (reported in rows 16, 17, 20, and 21) paired with probability elicitation questions about average annual or cumulative historical returns (reported in rows 3, 4, 7, and 8).

The four survey pairings correspond to the ordered pairs reported in rows (1, 18), (2, 19), (5, 14), and (6, 15) of Table V for one survey, and rows (16, 8), (17, 7), (20, 4), and (21, 3) for the other. Note that the surveys in rows 1 and 2 use identical question formats but represent two distinct randomly assigned groups. In each survey, between the two sets of questions, students were asked several financial literacy questions as a palate cleanser to reduce carry-over effects.

IA-7 Understanding America Survey

Our UAS sample is based on data from two surveys executed six years apart. In January 2025 (UAS survey 685), 9,455 individuals completed the six probability elicitation total return questions (see Section 3.1). We limit the sample to the 3,644 individuals whose answers allow us to calculate perceived variances (i.e., the sum of the probability of a return less than 20% and a return greater than 20% is less than 100%, and neither probability is zero). Panels A, B, and C of Table IA-V (directly analogous to Panels A, B, and C of Table III) report summary statistics for UAS respondents' answers, inferred expected returns, inferred standard deviations, and variance ratios based on the probability elicitation total return questions. The results are nearly identical to the corresponding results for ALP participants (Table I) and students (Table III). For example, the median UAS probability elicitation variance ratio of 0.187 is between the median value for ALP participants (0.147; Panel C of Table I) and students (0.238; Panel C of Table III).

[Insert Table IA-V about here]

In UAS survey 184—in the field between May and June 2019—UAS respondents were asked about average annual returns using quantile elicitation. Specifically, respondents were first asked about their expectations for returns over the next decade:

Please answer the next questions based on your best guess on how the stock market will perform over the next 10 years. Here is an example of what we mean by average annual (yearly) return: It is a simple average of each year's return. So if in the first year the stock market goes up by 2% and in the second year market goes up by 4%, the average annual return over two years is 3% - we want your own best guess, so please do not look anything up.

I expect the average annual return over the next 10 years will be ___

I believe that there is a small (1-in-10, or 10 percent) chance the actual return over 10 years will be less than ___

I believe that there is a small (1-in-10 or 10 percent) chance the actual return over 10 years will be greater than ___

UAS respondents were then asked (in the same survey) about their perceptions of annual returns over the previous 60 years:

We are interested in finding out what people know and do not know about the history of stock market returns. The next question asks about what you think about historical stock market returns over the past 60 years in the S&P 500. Over that time, there were some years, or periods of years, where returns were much higher, or much lower, than the average over the whole time period.

Please answer the next questions about the last 60 years of the stock market history (from 1959 to 2018), as measured by S&P 500 index. If you are not sure, we would like your best guess - please do not look up any answers.

I believe that the average annual return over the past 60 years was ___

During the 6 lowest years of the market over that time, returns were less than ___

During the 6 highest years of the market over that time, returns were greater than ___

The 1-year horizon questions differ from the Duke CFO survey questions in two respects: (1) they ask about historical rather than expected returns, and (2) they rephrase the 10^{th} and 90^{th} percentile question as the six lowest and six highest years over the past 60 years. A total of 2,461 UAS respondents answered all six questions from the May–June 2019 survey; after applying probability law restrictions (e.g., excluding respondents who report a 90^{th} percentile equal to their 10^{th} percentile), 1,439 remain. Panels D, E, and F of Table IA-V (directly analogous to Panels D, E, and F of Table III) report summary statistics for UAS respondents' answers, inferred expected returns, inferred standard deviations, and variance ratios based on the quantile elicitation average annual return questions. The results are fully consistent with the patterns for both CFOs and students. For instance, the median inferred UAS 1-year standard deviation, based on estimates of

10^{th} and 90^{th} percentile returns, is 0.036 versus 0.033 for CFOs (Table II) and 0.043 for students (Table III).[40] Similarly, based on the quantile elicitation average annual return questions, the vast majority of variance ratios for UAS participants (82%, see Table IA-V), students (93%, see Table III), and the median CFO (100%, see Table II) exceed the historical market variance ratio of 0.56.

Panel G of Table IA-V (directly analogous to Panel G of Table III) reports descriptive statistics for the 472 UAS respondents who completed both the 2019 and 2025 surveys. For this subset, the vast majority of both UAS respondents (79%) and students (95%) generate a larger variance ratio when asked about average annual returns (denoted "CFO variance ratio") than total returns (denoted "ALP variance ratio"). In sum, despite differences in question wording, the use of historical rather than forward-looking 1-year returns, and the six-year gap between the two surveys, the UAS results are fully consistent with our hypothesis.

IA-8 Respondent characteristic sorts and variance ratio heterogeneity

We sort ALP respondents into two groups by each of the 15 characteristics listed in the panel descriptions of Table IA-VI. In each of the 29 ALP survey waves, we compute the cross-sectional median variance ratio as well as the fraction of variance ratios less than 1 and the fraction less than the historical market variance ratio (0.559). Finally, for each group and wave, we test whether the fraction of variance ratios less than 1, or less than 0.559, differs meaningfully from 50%. In total, we examine 1,740 tests (i.e., 15 characteristics $\times$ 2 groups $\times$ 29 waves $\times$ 2 tests) of whether the variance ratio is less than 1, or less than 0.559. The first two rows of each panel in Table IA-VI report the time-series averages of the cross-sectional median variance ratios and cross-sectional average fraction of variance ratios less than 1, or 0.559.

[Insert Table IA-VI about here]

The results in Table IA-VI reveal that unrealistically small variance ratios in the ALP sample are ubiquitous. The largest reported (cross-sectional average of time-series median) variance ratio in Table IA-VI is 0.229 for high numeracy individuals in Panel E. In addition, the fraction of variance ratios less than 0.559, and less than 1, are always meaningfully greater than 50%. That is, all 1,740 tests summarized in the last two columns are statistically significant at the 1% level.

Panels A-F in Table IA-VI report the variance ratios for the financial sophistication proxies (in each case, the top row represents the 'more' sophisticated group). The third row reports the difference in variance ratios and indicates whether the value differs meaningfully from zero (based on a paired t-test of the time-series of the 29 cross-sectional medians). Consistent with the hypothesis that variation in cognitive uncertainty plays a larger role than variation in perceived parameter uncertainty in explaining variance ratio heterogeneity, the differences in Panels A-F are all positive—consistent with cognitive uncertainty playing the dominant role—and statistically significant at the 1% level. Further consistent with variation in cognitive uncertainty contributing to variance ratio heterogeneity (as well as the Enke and Graeber (2023) evidence), Panels G and H reveal that men and older respondents tend to exhibit larger variance ratios than women and younger respondents, respectively. In both cases, the results are statistically meaningful at the 1% level. Panels I-L also reveal meaningful differences in variance ratios based on race, marital status, and retirement status. The results are consistent with the hypothesis that non-Whites, single, and

[40] UAS respondents appear more miscalibrated than either students or CFOs about long-term returns—the median 10-year 90^{th} percentile average annual return for UAS respondents, students, and CFOs are, respectively, 9.5%, 14%, and 13% (versus a historical value of 15%). As a result, long-term inferred variances are lower for UAS participants relative to CFOs and students. In all three cases, however, long-term inferred variances are too large relative to near-term inferred values, generating implausibly large variance ratios when asked the quantile elicitation average annual return questions.

non-retired respondents tend to exhibit greater cognitive uncertainty with respect to the near- and long-term total return distributions.

The results in Panels M and N reveal no support for the hypotheses that individuals with greater overconfidence or individuals who expect economic risk to increase in the future exhibit larger variance ratios (i.e., the differences in Panel M and N have the "wrong" sign). The results in Panel O, however, support the hypothesis that heterogeneity in variance ratios results, in part, from heterogeneity in mean-reversion beliefs as individuals who exhibit mean-reversion beliefs also exhibit significantly (at the 1% level) smaller variance ratios relative to those who do not exhibit mean-reversion beliefs.

IA-9 Correlations between explanatory variables

Table IA-VII reports correlations between the explanatory variables for the Table VII panel regressions including Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Mean reversion, and Better next year for our pooled cross-sectional time-series of 22,745 observations (from 3,022 individuals).

[Insert Table IA-VII about here]

IA-10 Cognitive uncertainty in other domains

As detailed in the paper, ALP respondents were asked a series of questions regarding the likelihood of an event in the next year. Specifically, respondents were asked, "What is the percent chance that ___ during the next year?" where the blank was filled with:

. . . you will get into a car accident. . .
. . . you will have a cavity filled. . .
. . . you will die (from any cause – crime, illness, accident, and so on). . .
. . . someone will steal something from you. . .
. . . you will move your permanent address to another state some time. . .
. . . you will die in a terrorist attack. . .
. . . someone will break into your home and steal something from you. . .
. . . you will visit a dentist, for any reason,. . .

Respondents were then asked the same set of questions regarding the next five years, i.e., "What is the percent chance that ___ during the next 5 years?"

Columns (2) and (3) in Table IA-VIII report the median reported likelihood for each question over the next year and the corresponding implied 5-year likelihood calculated assuming independence. Column (4) reports the median respondent-reported 5-year likelihood and column (5) reports the difference in medians between the 5-year likelihood assuming independence and the reported 5-year likelihood. Columns (6)-(9) report analogous mean values.

[Insert Table IA-VIII about here]

If cognitive uncertainty causes respondents to compress their annual and 5-year forecasts, the differences between their 5-year likelihoods implied by their 1-year forecasts and their elicited 5-year likelihoods will be positive (columns 5 and 9). The final column reveals the fraction of respondents with a positive difference averages 87% across the eight questions and ranges from 68% (for the likelihood one moves out of state) to 94% (for the likelihood one is a theft victim). For instance, the median respondent reports a 20% chance of a car accident in the next year. Assuming that, on average, respondents' car crash risk is approximately independent over the next five years, the 5-year likelihood should be 67.2% (i.e., $1-(0.8^5)$). Yet the typical respondent estimates only a 30%

5-year likelihood—remarkably, less than the 36% implied 2-year likelihood (i.e., $1-(0.8^2)$), let alone the 67.2% implied 5-year value. In short, the results are uniformly consistent with the hypothesis that most individuals experience cognitive uncertainty regarding the relation between horizon and uncertainty. Specifically, as shown in the final column of Table IA-VIII, we can reject the hypothesis that the difference between implied 5-year likelihoods and estimated 5-year likelihoods is as likely to be positive as negative at the 1% level in every case.

IA-11 Variance ratios with ball and bin approach

As discussed in Section 2.1, a third approach to generating estimates of perceived distributions is to ask respondents to distribute "balls" into "bins" representing the distribution. Hartzmark and Sussman (2026) find that such an approach has advantages relative to the CFO style confidence interval question when evaluating near-horizon beliefs. Fortunately, we can investigate how the ball and bin approach influences the perceived relation between time and uncertainty as ALP also asked their total return questions in the ball and bin format for approximately 50% of respondents (and the other 50% comprise our main sample that answer the "standard" questions) in 18 of the 29 surveys used in our primary analysis. Specifically, the ALP ball and bin questions were framed as:

> *In the next question we will ask you about your expectations with respect to the one-year change in the U.S. stock market. By how much do you expect U.S. stock prices to be higher or lower one year from now?*
>
> *Please put the 20 balls in the 6 bins to reflect your expectations about the one-year change in the U.S. stock market. The more likely you think that the change will be in a range represented by one of the bins, the more balls you should put in that bin. To move balls into a bin, click on the + and - buttons under each bin. You can also "drag and drop" the balls with your mouse.*
>
> *Next we would like to ask you about your expectations with respect to the 10-year change in the U.S. stock market. By how much do you expect U.S. stock prices to be higher or lower 10 years from now?*
>
> *Please put the 20 balls in the 6 bins to reflect your expectations about the 10-year change in the U.S. stock market. The more likely you think that the change will be in a range represented by one of the bins, the more balls you should put in that bin. To move balls into a bin, click on the + and - buttons under each bin. You can also "drag and drop" the balls with your mouse.*

Directly analogous to the standard questions (which ask the likelihood markets rise or fall by at least 20% over the next year or decade), the bins are identical for the 1- and 10-year horizon. Specifically, the six bins (for both 1- and 10-year returns) are <-20%, -20% to -10%, -10% to 0%, 0% to 10%, 10% to 20% and >20%.

We take two approaches to estimating respondents' inferred standard deviations from the ball and bin approach.[41] First, because the exterior bins reflect the likelihood of a 20% or greater gain or loss, we follow the approach for the standard questions and estimate perceived variance by assuming normality and inferring the standard deviation. Second, we follow Hartzmark and Sussman (2026) and estimate the inferred standard deviation directly from the distribution of balls. For the interior bins, we use the bin midpoints as expected values. For the exterior bins (<-20% and >20%), we use the historical CRSP return distribution to estimate the conditional expected value of the bin (e.g., we use the historical average annual return conditional on the return being greater than 20% over a one year period as the expected value for the 1-year >20% bin). Specifically, for the 1- (10-)

[41]For consistency with our earlier tests, we work in log returns.

year distribution, the first bin, <-20%, has an expected value of -34.87% (-24.96%) and the last bin, >20%, has an expected value of 38.87% (130.8%). Note that the first approach requires an individual place at least one ball in each of the exterior bins for both 1- and 10-year horizons. The second approach only requires respondents do not place all 20 balls in a single bin.

Table IA-IX reports the estimates from the ball and bin approach when estimated via the tail probability approach (Panels A, B, and C) and the distributional approach (Panels D, E, and F). Note that if respondents' perceptions of return uncertainty are well grounded, the estimates in the table should be similar to each other as well as the Table I estimates based on the standard ALP questions. The results reveal three key insights. First, the sample sizes differ greatly between the two ball and bin estimation approaches. Specifically, as shown in Panels D-F, the ALP data provides 25,504 observations where respondents answer the ball and bin question. As shown in Panels A-C, however, because 93% of respondents place no balls in at least one of the four exterior bins, only 7% of those observations (1,765/25,504) allow for estimation of standard deviation based on tail probabilities. In contrast, when asked the direct probability question (e.g., what is the chance markets are 20% higher in a decade), only 20% of respondents answer a zero likelihood to at least one of the four questions. Moreover, although 49% of respondents place no balls in *any* of the four extreme bins, only 3% of respondents report a zero likelihood for all four of the standard questions.[42]

[Insert Table IA-IX about here]

Second, the inferred standard deviations exhibit substantial differences across the methods. Specifically, near- and long-term inferred standard deviation estimates reported in Table IA-IX are substantially reduced relative to those reported in Table I. For instance, relative to the values based on the standard questions, the median 1- and 10-year inferred standard deviation computed from the distributional approach (Panels D and E in Table IA-IX) are 65% and 67% smaller, respectively. When limited to the tail probability sample (Panels A and B in Table IA-IX), the reductions in medians are 24% for the 1-year inferred standard deviation and 40% for the 10-year. Because the median 1-year standard deviation estimate from the direct elicitation method (Table I) is nearly identical to the historical value, the ball and bin approach, at least for the specific construction of both approaches in the ALP data, leads to estimates further away from historical values. For example, approximately half of the Table I 1-year standard deviation estimates are above and half are below the historical value. In contrast, 100% of the (winsorized) 1-year ball and bin estimates using the distributional method are less than the historical values.

Third, despite these differences, the ball and bin variance ratios exhibit the identical pattern as those inferred from the standard question format. Specifically, the median variance ratio is 0.147 when estimated from the standard ALP total return questions (Panel C of Table I), 0.112 when estimated using the tail probabilities in the ball and bin questions (Panel C of Table IA-IX), and 0.175 when estimated using the distributional approach for the ball and bin questions (Panel F of Table IA-IX).

Analogous to Figure 1, Figure IA-5A combines and centers at zero ALP near-term perceived uncertainty (blue line), long-term total return perceived uncertainty (green line), and long-term total return uncertainty, assuming returns are independent, given ALP respondents' near-term uncertainty (orange line) when estimated from the tail probabilities in the ball and bin approach. Figure IA-5B reports analogous values based on the ball and bin distributional estimates. Although, relative to 1, the scales in Figure IA-5 are compressed, the patterns are identical. Specifically,

[42]The minimum non-zero ball allocation in the bins format is one ball, equivalent to a 5% probability. Using this threshold for comparability, 30% of standard question respondents report less than 5% for at least one of the four tail beliefs, and 4% report less than 5% for all four—compared to 93% and 49% respectively for bins respondents who place zero balls in at least one or all four extreme bins.

consistent with compression of near- and long-term uncertainty forecasts (and equation (2)), ALP respondents' long-term total return uncertainty (green distribution) is only slightly greater than their near-term uncertainty (blue) line and far away from the uncertainty implied by their one-year beliefs if returns were serially independent (orange distribution).

[Insert Figure IA-5 about here]

IA-12 Anchoring in the ball and bin approach—evidence from Hartzmark and Sussman (2026)

Consistent with the hypothesis that most CFOs have little understanding of near-term return uncertainty, the results in Section 4.8 demonstrate that when told that 2010 S&P 500 volatility was 12.6%, CFOs nearly echoed back that value with a median of 11%. This is notable for two reasons: their simultaneously measured confidence intervals implied a standard deviation of only 3.7%, and the 12.6% seed value was itself an error—true 2010 volatility was nearly 20%. The experiment reported by Hartzmark and Sussman (2026) offers an opportunity to examine such anchoring effects in a ball and bin framework. Specifically, for their distribution elicitation experiment, respondents viewed either 100 years of US stock returns (with μ_{US}=7.97% and σ_{US}=18.35%) or 100 years of Japanese returns (with μ_{Japan}=10.21% and σ_{Japan}=27.24%). Regardless of which set of returns viewed, respondents were then asked to place 100 balls in nine bins where the "bin distribution" was based on either historical Japanese or US returns (bin width was based on 0.5 standard deviations). Specifically, the bin distribution based on US and Japanese data were, respectively:

US bins: $<-23\%$, -23% to -15%, -14% to -6%, -5% to 3%, 4% to 12%, 13% to 21%, 22% to 30%, 31% to 39%, $> 39\%$

Japan bins: $<-42\%$, -42% to -28%, -27% to -13%, -12% to 2%, 3% to 17%, 18% to 32%, 33% to 47%, 48% to 62%, $> 62\%$

Thus, a given individual viewing Japanese historical data may face either "Japanese bins" or "US bins." Correspondingly, an individual viewing US historical data may face either US bins or Japanese bins. If, just after viewing 100 years of historical data, individuals have a good understanding of the distribution, then the standard deviation implied by their ball and bin experiment should largely be invariant to whether balls are placed in Japanese or US bins. Alternatively, if respondents have little understanding of the distribution they just viewed, then we hypothesize they will place balls to largely mimic a normal distribution of the bins (US or Japanese) they view.

The solid lines in Figure IA-6A report implied distribution (recentered at zero for ease of comparison) based on historical Japanese (solid red line) or US (solid blue line) volatility.[43] The broken red line reports the implied distribution based on respondents who viewed Japanese data and Japanese bins. The broken blue line reports the implied distribution based on respondents who viewed *the same Japanese data* but US bins. Consistent with the typical CFO providing an estimate closely anchored to the seed value (Section 4.8), the average respondent in the Hartzmark and Sussman (2026) sample essentially distributed the balls to approximate a normal distribution centered on whichever bin structure they faced. That is, despite viewing the same Japanese stock returns, respondents using Japanese bins essentially returned the Japanese distribution (i.e., solid and broken red lines closely track) while those using US bins essentially returned the US distribution (i.e., the solid and broken blue lines closely track).

[43] Values in Figure IA-6 are based on the authors' Table A10 for bins centered at the historical means. Nonetheless, as shown in the authors' Table A10, the same pattern occurs when returns are centered at zero.

[Insert Figure IA-6 about here]

Figure IA-6B reports analogous results for respondents who viewed US data. When faced with US bins, respondents effectively replicated the US distribution (the solid blue line is based on σ_{US}=18.35% while the dashed blue line, which is largely covered by the solid blue line, reflects the average inferred standard deviation of 18.32%). When faced with the Japanese bins, however, the average estimated standard deviation (23.06%) is closer to the Japanese historical value (27.24%) than the US historical value (18.35%). In short, the results suggest that the average respondent largely places balls to generate something close to a normal distribution *of the bins* regardless of the distribution of the underlying data.

IA-13 Question format and near-term miscalibration

Our results support the hypothesis that most economic agents have, at best, only the most rudimentary understanding of the relation between horizon and equity market risk. As a result, individuals' near- and long-term forecasts are compressed which can explain why asking about average annual returns generates implausibly large variance ratios while asking about cumulative long-term returns generates implausibly small variance ratios.

Our study also demonstrates that the compression of near- and long-horizon estimates occurs regardless of the approach used to measure perceived distributions. Specifically, the results in Table V demonstrate that questions framed in average annual long-horizon returns consistently generate variance ratios greater than 1 and questions framed as cumulative long-horizon returns consistently generate variance ratios less than 1, regardless of whether respondents are asked about confidence intervals (i.e., the CFO survey style) or the likelihood of a return larger or smaller than some value (i.e., the ALP survey style).

Our focus is on the compression of near- and long-horizon estimates rather than on why CFO-style questions generate severely miscalibrated near-term forecasts while ALP-style questions generate reasonable ones. Our student surveys, the UAS survey, and the Duke 2011 Q1 survey all suggest, however, that question format is a primary driver of miscalibration differences across respondent groups, though overconfidence may still contribute to heterogeneity within groups. Both students and UAS respondents exhibit severely miscalibrated near-term beliefs when asked CFO-style 80% confidence interval questions, but overestimate near-term uncertainty when asked ALP-style questions about the likelihood of a 20% gain or loss. Moreover, the 2011 Q1 Duke survey demonstrates that the typical CFO's near-term uncertainty estimate is sensitive to question format and anchors on an erroneous seed value.

Our focus is on the compression of near- and long-horizon estimates and does not focus on why the CFO survey style questions generate near-term forecasts are severely miscalibrated (e.g., Panel A of Table II, Panel D of Table III, and Panel D of Table IA-V) but ALP style questions generate reasonable forecasts of near-term uncertainty (Panels A and D of Table I, Panel A of Table III, and Panel A of Table IA-V). As noted by Ben-David, Graham, and Harvey (2013), miscalibrated beliefs, "happen because either most overestimate their ability to predict the future or because they underestimate the volatility of random events." Thus, one potential explanation for differences between ALP respondents' and CFOs' near-term volatility estimates is that CFOs are simply more overconfident of their ability to forecast market returns than a representative sample of Americans. Our student surveys, the UAS survey, and the Duke 2011 Q1 survey, however, all suggest that the extent of miscalibration in equity market distribution forecasts also depends on question format. First, similar to CFOs, both students and UAS respondents exhibit severely miscalibrated near-term beliefs when asked the CFO survey style 80% confidence interval questions (Panel D of Tables III and IA-V) but, if anything, tend to overestimate near-term uncertainty (relative to historical values) when asked the ALP survey style questions regarding the likelihood of a 20% gain or fall in values over the next year (Panel A of Tables III and IA-V). That is, both a broad sample of Americans (UAS participants) and students exhibit severely overprecise estimates of near-term volatility when answering the CFO survey questions, but underprecise estimates of near-term volatility when answering the ALP survey questions. In addition, as detailed in our study, the 2011 Q1 Duke survey demonstrates that (1) the typical CFO's estimate of near-term uncertainty is sensitive to question format and (2) the typical CFO anchors their 2011 and 2012 volatility estimate on an erroneous seed value for 2010.

Consistent with these patterns, work suggests that the extent of miscalibration depends, in part, on question structure. For example, Juslin, Wennerholm, and Olsson (1999) evaluate three approaches to examining confidence in beliefs: confidence interval estimation, full range questions, and half range questions. The authors find severe overprecision in interval estimation, but only weak overprecision in full range questions, and underprecision in half range questions.[44] A number of factors appear to contribute to the miscalibration in confidence interval estimation. For example, work (Teigen and Jorgensen (2005)) demonstrates that respondents report nearly identical (statistically indistinguishable) intervals when asked for 90% confidence intervals, 70% confidence intervals, or 50% confidence intervals (see Hartzmark and Sussman (2026) for similar results in a finance context). As a result, miscalibration tends to be larger when the confidence interval widens. A popular explanation (e.g., Yaniv and Foster (1995), Yaniv and Foster (1997), and Cesarini, Sandewall, and Johannesson (2006)) for the severe miscalibration in confidence interval estimates is that respondents trade off accuracy (i.e., a properly calibrated confidence interval) and informativeness (i.e., a narrow confidence interval). Moreover, consistent with the accuracy-informativeness trade-off, evidence suggests that respondents recognize their confidence intervals are overprecise. For instance, Cesarini, Sandewall, and Johannesson (2006) ask respondents 90% confidence intervals for 10 questions and then ask respondents to guess how many of their 90% confidence intervals contain the correct answer. The average individual responds six. Individuals not only recognize their own confidence intervals are too precise, but also recognize this pattern is pervasive—the average respondent estimates that other respondents' 90% confidence intervals will only contain six of 10 correct answers.

Consistent with previous evidence (e.g., Teigen and Jorgensen (2005), Hartzmark and Sussman (2026)), we also find that students' estimates are relatively insensitive to the size of the confidence interval. Specifically, using the data from Tables III and V, the red lines in Figure IA-7A report 90%, 80%, and 60% confidence intervals for historical annual market returns, while the blue lines report the corresponding confidence intervals based on the median student 1-year return perceptions. The green line reports the median CFO 1-year return perceptions (based on the medians in Table II Panel A). For example, the middle red line shows that, historically, there is 80% chance the market's annual return is between -13% and 35%, but students, when asked the CFO style questions, estimate there is an 80% chance returns in the next year will between 5% and 18%, and CFOs estimate there is an 80% chance returns in the next year will between 0% and 9%. Nearly identical to Figure 1 in Hartzmark and Sussman (2026), the range in market returns (i.e., the red lines) greatly shrink as the confidence interval falls, but student estimates (the blue lines) move little. As a result, the extent of miscalibration is much greater when confidence intervals are wider.

[44]The authors give examples of each format. Specifically interval estimation is given by "Assess the (smallest) interval within which you are 80% certain that the population of Norway lies: Between ___ million and ___ million inhabitants." The full range question format is, "Norway has less than 6 million inhabitants. What is the probability that this statement is true?" Respondents then answer on an 11 point scale from a probability of 0% (certainly false) to 100% (certainly true). The half-range question format is "Does the population of Norway lie above or below 6 million?" The respondent selects above or below and then a probability from six alternatives ranging from 50% (just guessing) to 100% (certain).

[Insert Figure IA-7 about here]

Figure IA-7B reports market values (red lines), median student estimates (blue lines), and median CFO estimates (green line) for average annual returns over the next decade. Similar to the annual results in Figure IA-7A, the 10-year horizon results in Figure IA-7B demonstrate that the range in average annual market returns over a decade (red lines) shrinks as the confidence interval narrows, but student estimates (blue lines) are much less sensitive to changes in the confidence interval, and therefore the level of miscalibration increases as the confidence interval widens.

We find a similar pattern for ALP style total return questions. Specifically, Figure IA-8A reports the historical likelihood (red bars) annual returns fall within a range ($-30\% < r_{1year} < 30\%$; $-20\% < r_{1year} < 20\%$; $-10\% < r_{1year} < 10\%$) and the likelihood based on median student beliefs (blue bars). Analogous to the pattern for variation in confidence intervals (i.e., Figure IA-7), the likelihood of a 1-year market return within a given range falls systematically as the range decreases (i.e., the red bars shrink), while student estimates (blue bars) are largely immune to changes in the range. That is, the results are consistent with evidence that respondents tend to assign a significant likelihood to any event which is deemed important enough to be asked about (e.g., Clemen and Ulu (2008)). Figure IA-8B reports the corresponding values for cumulative 10-year returns. The red bars are short because, for example, historically, markets have earned more than 30% in 94% of 10-year periods (i.e., there are few 10-year periods where markets earned, cumulatively, less than 30%, less than 20%, or less than 10%). Although the median student's estimate is poor (i.e., students severely overestimate their ability to predict long-term total returns when answering the ALP style questions), the pattern goes in the "correct" direction as the blue bars shrink as the interval shrinks. In short, relative to historical values, students overestimate the likelihood a low 10-year return (e.g., the median student estimates a 10% likelihood of a 10-year market return less than -20% versus a historical likelihood of 1%; see Panel B of Table II), but greatly underestimate the likelihood of a high 10-year return (e.g., the median student estimates a 60% likelihood of a 10-year market return greater than 20% versus a historical likelihood of 93%; see Panel B of Table II).

[Insert Figure IA-8 about here]

IA-14 Variable details

Variable construction follows Sias, Starks, and Turtle (2024) and Sias, Starks, and Turtle (2026). As a result, this appendix contains identical descriptions to their appendices. For many questions, respondents are prompted if they do not initially respond to a question. For instance, if respondents do not respond to the question regarding direct stock holdings, they are prompted, "[You did not answer. Your answers are important to us. Please answer the question to the best of your ability.] In the next set of questions we will ask you about stock holdings besides those that you may have already told us about. Do [you (and/or your husband/wife/partner)] have any shares of stock or stock mutual funds besides stock holdings that are part of an IRA, 401(k), Keogh or similar retirement accounts?" These responses generally show up as a second variable (e.g., no value for ST001, but a value for ST001_NR_DK) that we include in our analysis.

Variable	Description
Female	Gender is identified in the pre-loaded demographic data for each "effects of the financial crisis" survey.
White race	Ethnicity is identified in the pre-loaded demographic data for each "effects of the financial crisis" survey.
Married	Current living situation is asked in each survey. Those who respond, "Married or living with a partner" are classified as married. All others (e.g., separated, divorced, widowed, never married) are classified as non-married.
Working	Current job status is identified in each survey. Respondents who report, "working now" are classified as working.
Retired	Current job status is identified in each survey. Respondents who report "retired" are classified as retired.
Age	Respondent age is reported in each survey. Respondents 50 and younger are classified as young while those older than 50 are classified as old.
Years Education	Respondents report 16 possible answers for "What is the highest level of school you have completed or the highest degree you have received?" We assign the following years of education for each answer (1) less than 1st grade=0, (2) 1st, 2nd, 3rd or 4th grade=2.5, (3) 5th or 6th grade=5.5, (4) 7th or 8th grade=7.5, (5) 9th grade=9, (6) 10th grade=10, (7) 11th grade=11, (8) 12 grade no diploma=12, (9) high school graduation=12, (10) some college but no degree=13, (11) associate degree in college occupational/vocational program=14, (12) associate degree in college academic program=14, (13) bachelor's degree=16, (14) master's degree=18, (15) professional school degree (e.g., MD, DDS, DVM, LLB, JD)=22, (16) Doctorate degree (e.g., PhD EdD)=22. We classify respondents with at least 13 years of education as "more" education.

Income	Respondents report values for family income questions. The first question, "family income" reports 14 possible income buckets—with the final bucket indicating income greater than \$75,000. "Family income part 2" asks those who report family income greater than \$75,000 to report income in four additional buckets. The 14 family income buckets are: inc $<$ \$5k, \$5k $\leq$ inc $<$ \$7.499k, \$7.5k $\leq$ inc $<$ \$9.999k, \$10k $\leq$ inc $<$ \$12.499k, \$12.5k $\leq$ inc $<$ \$14.999k, \$15k $\leq$ inc $<$ \$19.999k, \$20k $\leq$ inc $<$ \$24.999k, \$25k $\leq$ inc $<$ \$29.999k, \$30k $\leq$ inc $<$ \$34.999k, \$35k $\leq$ inc $<$ \$39.999k, \$40k $\leq$ inc $<$ \$49.999k, \$50k $\leq$ inc $<$ \$59.999k, \$60k $\leq$ inc $<$ \$74.999k, \$75k $\leq$ inc. The family income part 2 groupings are: \$75k $\leq$ inc $<$ \$99.999k, \$100k $\leq$ inc $<$ \$124.999k, \$125k $\leq$ inc $<$ \$199.999k, \$200k $\leq$ inc. For respondents who report income less than \$75K, we use the bucket midpoint. For respondents who report income of at least \$75k, but less than \$200k, we use the bucket midpoint of family income part 2. For respondents who report income greater than \$200k, we assume income is \$250k. We classify respondents with income of at least \$75,000 as "higher" income.
Numeracy	ALP survey 32 (in the field from 5-27-2008 to 6-30-2008) included a series of 17 questions used to compare measures of numeracy in Weller, Dieckmann, Tusler, Mertz, Burns, and Peters (2013). The authors conclude that the eight-item Rasch-based numeracy measure generally performed better than other measures. The Rasch metric consists of a series of increasingly difficult problems to better discriminate numeracy. The authors provide (via the ALP website for registered users; scores.xlsx), the Rasch based score for ALP survey 32 participants. We classify respondents with Rasch numeracy score greater than 4 as high numeracy and those with score of 4 or less as low numeracy.
Understand market	In 14 of the financial crisis waves (1, 2, 11, 14, 24, 38, 41, 50, 54, 55, 58, 59, 60, and 61) respondents are asked "How would you rate your understanding of the stock market with scores from 1 (extremely good) to 6 (extremely poor). We reverse score the variable (so higher values indicate greater understanding) and average the score of all waves for which the respondent answers this question. Those with an average score of at least 3 (understands the market extremely good, very good, somewhat good) are classified as high self-rated understanding of the market. Those with average scores less than 3 (somewhat poor, very poor, extremely poor) are classified as low self-rated understanding of the market.
Financial literacy	ALP survey 5 (in the field from 5-8-2006 to 11-1-2007) asked respondents a series of 13 financial literacy questions. The survey is described in Parker, de Bruin, Yoong, and Willis (2012). We compute the fraction of the 13 questions correctly answered and classify respondents who correctly answered more than 75% (i.e., at least 10) of the questions as high financial literacy.
Overconf.	ALP survey 6 (in the field from 8-14-2006 to 11-20-2007) asked respondents 14 true/false general knowledge questions (e.g., alcohol causes dehydration). Respondents are also asked their confidence in each answer ranging from 50% (just guessing) to 100% (absolutely sure). Following Parker, de Bruin, Yoong, and Willis (2012), overconfidence is computed as the difference between the average confidence in their answer and fraction of correct answers. We classify respondents with positive differences as overconfident and those with negative (or no) differences as not overconfident.

Holds equities	In the first two waves of the financial crisis surveys (both long-form), respondents (ST001) are asked, "Do [you (or your husband/wife/partner)] have any shares of stock or stock mutual funds? Please include stocks that [you (or your husband/wife/partner)] hold in an employer pension account." Thus, in the first two waves, respondents are classified as equity market participants based on this question. In all subsequent waves, respondents are asked (ST001), "In the next set of questions we will ask you about stock holdings. Please, do not include stock holdings that are part of an IRA, 401(k), Keogh or similar retirement accounts. Do [you(and/or your husband/wife/partner)] have any shares of stock or stock mutual funds?" In short-form wave 3 and all subsequent long-form waves, respondents who report having retirement account (RA001) are asked (RA006), "Are any of these retirement accounts invested in stocks or stock mutual funds, either fully or partially?" In these waves, we classify respondents who report owning stocks (either directly or in retirement account) as equity holders. Respondents who answer they do not have money in stock directly and either report no retirement stock or do not have a retirement account are classified as non-participants.
Better next year	In the 29 long-form surveys, respondents are asked, "Now looking ahead - do you think that a year from now you will be better off financially, worse off, or about the same as now?" We code respondents as 1 if they believe they will be better off in a year, -1 if they believe they will be worse off, and 0 if they believe they will be about the same. For the sorts, those who believe they will be better off are compared to those who believe they will be worse off.
Mean-reversion	In all 61 effects of the financial crisis surveys, respondents are asked "What are the chances that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more in 10 years than they are today?" For respondents who answer this question in at least 10 surveys, we regress the time-series of respondents' perceived likelihood markets rise in the next decade on scaled lagged 12 month returns. Specifically, to ease in interpretation, we divide raw lagged 12 month return by the standard deviation of CRSP 12-month returns such that the coefficient reflects the expected change in the likelihood markets rise in the next decade given a one standard deviation higher lagged 12-month return. For the sorts, we classify those with a negative relation between lag returns and future expected long-term returns as the mean-reversion sample and those with non-negative coefficients as the no mean-reversion sample. To facilitate reporting, we define the mean reversion variable as the estimated regression coefficient x -1, so that larger positive values suggest greater mean reversion.

Table IA-I – Descriptive Statistics for ALP

This table reports descriptive statistics for American Life Panel respondents' characteristics including Female, White race, Married, Working, Retired, Age, Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Better next year, Mean reversion, and Mean reversion>0. Characteristics are based on the pooled cross-sectional time-series of 22,745 observations (from 3,022 individuals; the average respondent participates in more than 7.5 long-form surveys) that include any individual-survey wave observation where the respondent's variance ratio can be estimated. Detailed variable descriptions are provided in Section IA-14.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.
Female	22,695	0.580	0.000	1.000	1.000	0.493
White race	22,745	0.878	1.000	1.000	1.000	0.327
Married	22,745	0.662	0.000	1.000	1.000	0.473
Working	22,744	0.609	0.000	1.000	1.000	0.488
Retired	22,744	0.229	0.000	0.000	0.000	0.420
Age	22,745	51.658	40.000	54.000	62.000	14.782
Years education	22,693	14.963	13.000	14.000	16.000	2.678
Income	22,620	71,511	37,500	55,000	87,500	51,672
Holds equity	22,732	0.589	0.000	1.000	1.000	0.492
Understand markets	22,677	3.194	2.444	3.286	4.000	1.076
Numeracy	11,467	4.496	3.000	5.000	6.000	1.764
Financial literacy	8,164	0.775	0.654	0.846	0.923	0.199
Overconfidence	8,143	-0.056	-0.114	-0.057	-0.007	0.095
Better next year	22,592	0.159	0.000	0.000	1.000	0.591
Mean reversion	21,546	1.451	-0.971	1.348	4.042	4.747
Mean reversion> 0	21,546	0.657				

Table IA-II – Additional Detail on Students' Beliefs

The table reports descriptive statistics for all respondents who provide estimates including those whose estimates violate probability laws for the spring and fall 2024 semesters (i.e., the sample for Table III). Panels A and B report, based on raw (i.e., unwinsorized) data, descriptive statistics for all 828 student respondents who answered the four ALP-style probability elicitation questions necessary to compute estimated variances over the next year or decade (i.e., chance market rises at least 20% in next year or decade; chance market falls at least 20% in next year or decade). Panels C and D report, based on raw (i.e., unwinsorized) data, beliefs for the 824 students who complete all four CFO-style quantile elicitation questions required to estimate near- and long-term variances (i.e., 10^{th} and 90^{th} return percentiles for the next year and average annual return percentiles for the next decade).

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Probability elicitation questions, next year								
P(market>0)	828	0.658	0.500	0.700	0.800	0.198	0.747	0.574
P(market>20%)	828	0.388	0.200	0.400	0.500	0.226	0.336	0.459
P(market<-20%)	828	0.317	0.130	0.300	0.500	0.220	0.060	0.110
Panel B: Probability elicitation questions, next decade								
P(market>0)	828	0.774	0.650	0.850	0.970	0.234	0.960	0.746
P(market>20%)	828	0.645	0.500	0.700	0.800	0.244	0.932	0.885
P(market<-20%)	828	0.273	0.100	0.200	0.400	0.225	0.013	0.094
Panel C: Quantile elicitation questions, next year								
$E_t(r_{1year})$	824	0.201	0.077	0.113	0.262	0.193	0.095	0.326
$P90(r_{1year})$	824	0.256	0.095	0.170	0.372	0.242	0.302	0.712
$P10(r_{1year})$	824	0.114	0.030	0.055	0.140	0.137	-0.138	0.001
Panel D: Quantile elicitation questions, next decade								
$E_t(r_{10years})$	824	1.493	0.583	0.953	1.823	1.407	0.969	0.562
$P90(\bar{r}_{10years})$	824	0.198	0.086	0.140	0.262	0.167	0.152	0.567
$P10(\bar{r}_{10years})$	824	0.088	0.020	0.049	0.095	0.124	0.035	0.351

Table IA-III – Survey Order Robustness Tests

The table reports results for two samples of undergraduate business students enrolled in an introductory finance course at a large public university in spring 2024. Students in Section 1 were given the CFO-style quantile elicitation average annual return questions and then the ALP-style probability elicitation cumulative return questions two weeks later. Students in Section 2 were given the ALP-style probability elicitation questions initially, and the CFO-style quantile elicitation questions two weeks later. Panels A and B report summary statistics for the distribution of probability elicitation and quantile elicitation variance ratios, respectively, for each section. Difference in medians tests are reported in the final row of each panel.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Probability elicitation variance ratio results								
Section 1								
"ALP" variance ratio	37	0.898	0.079	0.198	0.789	1.379	0.542	0.622
%ALP variance ratio<1	37	0.757						
Section 2								
"ALP" variance ratio	37	0.531	0.061	0.140	0.400	1.044	0.542	0.784
%ALP variance ratio<1	37	0.892						
	Z	p-value						
Diff in median test	0.231	0.817						
Panel B: Quantile elicitation variance ratio results								
Section 1								
"CFO" variance ratio	67	14.495	4.071	9.814	14.249	19.289	0.542	0.060
%CFO variance ratio<1	67	0.104						
Section 2								
"CFO" variance ratio	66	12.242	1.920	7.154	13.087	15.838	0.542	0.076
%CFO variance ratio<1	66	0.136						
	Z	p-value						
Diff in median test	-0.605	0.545						

Table IA-IV – Students' Perceptions of Historical Return Distributions

Panels A, B, and C report descriptive statistics for 251 undergraduate students who have sufficient data to compute their inferred variance ratio based on their perceived likelihood that, historically, markets have risen or fallen by at least 20% over 1- and 10-year periods (i.e., the probability elicitation questions reframed as perceived historical return distributions). Panels D, E, and F report descriptive statistics for 290 students who have sufficient data to compute their inferred variance ratio based on their perceived 10^{th} and 90^{th} percentiles of historical 1-year returns and historical average annual 10-year returns (i.e., the quantile elicitation questions reframed as perceived historical return distributions). Panel E reports two measures of long-term uncertainty: the standard deviation of average annual returns ($\sigma_{i,t}(\bar{r}_{10years})$), which is the quantity directly elicited by the quantile elicitation questions, and the standard deviation of total returns ($\sigma_{i,t}(r_{10years})$), which equals 10 times the former and is directly comparable in units to the long-term standard deviation reported in Panel B. Panel G summarizes the variance ratio information and presents differences for students with sufficient data to compute variance ratios based on both sets of questions. The sample is based on students surveyed in the fall 2024 semester.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Student perceptions of historical 1-year stock market return (ALP questions)								
P(market>0)	251	0.643	0.500	0.700	0.800	0.251	0.747	0.562
P(market>20%)	251	0.246	0.100	0.200	0.350	0.179	0.336	0.737
P(market<-20%)	251	0.185	0.100	0.150	0.280	0.147	0.060	0.227
$E_{i,t}(r_{1year})$	251	0.029	-0.020	0.001	0.050	0.147	0.095	0.821
$\sigma_{i,t}(r_{1year})$	251	0.325	0.158	0.237	0.387	0.249	0.200	0.430
Panel B: Student perceptions of historical 10-year stock market return (ALP questions)								
P(market>0)	251	0.678	0.500	0.800	0.900	0.295	0.960	0.841
P(market>20%)	251	0.414	0.200	0.400	0.600	0.254	0.932	0.972
P(market<-20%)	251	0.183	0.050	0.150	0.260	0.142	0.013	0.072
$E_{i,t}(r_{10years})$	251	0.216	-0.020	0.065	0.357	0.355	0.969	0.932
$\sigma_{i,t}(r_{10years})$	251	0.508	0.225	0.362	0.601	0.418	0.465	0.614
Panel C: Students' variance ratios (based on reframed ALP questions)								
"ALP" variance ratio	251	0.599	0.086	0.199	0.618	0.939	0.542	0.729
%Variance ratio<1	251	0.825						
Panel D: Student perceptions of historical 1-year stock market return (CFO questions)								
$P90(r_{1year})$	290	0.231	0.104	0.182	0.262	0.184	0.302	0.783
$P10(r_{1year})$	290	0.085	0.020	0.049	0.095	0.111	-0.138	0.003
$E_{i,t}(r_{1year})$	290	0.148	0.068	0.095	0.182	0.148	0.095	0.486
$\sigma_{i,t}(r_{1year})$	290	0.054	0.021	0.041	0.076	0.042	0.200	1.000

Table IA-IV – Students Perceptions of Historical Return Distributions (Cont.)

Panel E: Student perceptions of historical 10-year stock market return (CFO questions)								
$P90(\bar{r}_{10years})$	290	0.329	0.140	0.223	0.470	0.286	0.152	0.359
$P10(\bar{r}_{10years})$	290	0.140	0.049	0.095	0.182	0.146	0.035	0.117
$E_{i,t}(r_{10years})$	290	2.583	0.953	1.398	3.716	2.560	0.969	0.383
$\sigma_{i,t}(\bar{r}_{10years})$	290	0.074	0.026	0.048	0.094	0.079	0.047	0.483
$\sigma_{i,t}(r_{10years})$	290	0.675	0.257	0.481	0.941	0.555	0.465	0.483
Panel F: Students' variance ratios (based on reframed CFO questions)								
"CFO" variance ratio	290	50.836	5.811	11.454	40.831	86.681	0.542	0.034
%Variance ratio<1	290	0.066						
Panel G: Students' differences in variance ratios (historical return distributions)								
"ALP" variance ratio	139	0.681	0.086	0.199	0.624	1.044	0.542	0.712
"CFO" variance ratio	139	43.510	7.432	11.527	38.928	71.989	0.542	0.007
("CFO")/("ALP")	139	435.357	16.662	88.294	210.344	1593.260		
"CFO" - "ALP"	139	42.829	6.363	10.844	38.791	71.954		
("CFO" - "ALP")> 0	139	0.978						

Table IA-V – Understanding America Surveys

Panels A, B, and C report descriptive statistics for 3,644 individuals from the Understanding America Survey (UAS) who have sufficient data to compute their inferred variance ratio in January 2025 (UAS survey 685) based on their perceived likelihood markets rise or fall 20% over the next year or decade (i.e., the probability elicitation total return questions). Panels D, E, and F report descriptive statistics for 1,439 individuals from the 2019 UAS survey (UAS survey 184) who have sufficient data to compute their inferred variance ratio based on their perceived *historical* 10^{th} and 90^{th} percentiles of annual returns over the previous 60 years, and their perceived *forward-looking* 10^{th} and 90^{th} percentiles of average annual returns over the next decade (i.e., the quantile elicitation average annual return questions). Panel E reports two measures of long-term uncertainty: the standard deviation of average annual returns ($\sigma_{i,t}(\bar{r}_{10years})$), which is the quantity directly elicited by the quantile elicitation questions, and the standard deviation of total returns ($\sigma_{i,t}(r_{10years})$), which equals 10 times the former and is directly comparable in units to the long-term standard deviation reported in Panel B. Panel G summarizes the variance ratio information and presents differences for individuals with sufficient data to compute variance ratios based on both the 2019 quantile elicitation average annual return questions and the 2025 probability elicitation total return questions.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: UAS participants' stock market expectations over next year (ALP total return questions, 2025 survey)								
P(market>0)	3,644	0.442	0.230	0.470	0.610	0.254	0.747	0.837
P(market>20%)	3,644	0.272	0.100	0.240	0.400	0.197	0.336	0.667
P(market<-20%)	3,644	0.294	0.110	0.250	0.440	0.212	0.060	0.111
$E_{i,t}(r_{1year})$	3,644	-0.062	-0.126	-0.020	0.072	0.718	0.095	0.780
$\sigma_{i,t}(r_{1year})$	3,644	0.746	0.204	0.316	0.601	1.647	0.200	0.243
Panel B: UAS participants' stock market expectations over next decade (ALP total return questions, 2025 survey)								
P(market>0)	3,644	0.564	0.350	0.520	0.800	0.281	0.960	0.911
P(market>20%)	3,644	0.436	0.250	0.430	0.600	0.242	0.932	0.984
P(market<-20%)	3,644	0.249	0.100	0.200	0.370	0.179	0.013	0.027
$E_{i,t}(r_{10years})$	3,644	0.285	-0.034	0.065	0.381	1.110	0.969	0.904
$\sigma_{i,t}(r_{10years})$	3,644	1.172	0.287	0.505	1.040	2.177	0.465	0.460
Panel C: UAS participants' variance ratios (ALP total return questions, 2025 survey)								
"ALP" variance ratio	3,644	1.003	0.076	0.187	0.711	1.983	0.542	0.709
%Variance ratio<1	3,644	0.837						

Table IA-V – Understanding America Surveys (Cont.)

Panel D: UAS participants' historical perceived annual return distributions (CFO average annual return questions, 2019 survey)								
$P90(r_{1year})$	1,439	0.165	0.095	0.122	0.182	0.161	0.302	0.904
$P10(r_{1year})$	1,439	0.031	0.010	0.020	0.039	0.070	-0.138	0.008
$E_{i,t}(r_{1year})$	1,439	0.132	0.049	0.077	0.113	0.266	0.095	0.570
$\sigma_{i,t}(r_{1year})$	1,439	0.048	0.025	0.036	0.063	0.034	0.200	1.000
Panel E: UAS participants' stock market expectations over next decade (CFO average annual return questions, 2019 survey)								
$P90(\bar{r}_{10years})$	1,439	0.120	0.058	0.095	0.122	0.124	0.152	0.831
$P10(\bar{r}_{10years})$	1,439	0.045	0.020	0.030	0.049	0.061	0.035	0.525
$E_{i,t}(r_{10years})$	1,439	0.765	0.392	0.583	0.770	1.304	0.969	0.883
$\sigma_{i,t}(\bar{r}_{10years})$	1,439	0.026	0.011	0.018	0.032	0.024	0.047	0.856
$\sigma_{i,t}(r_{10years})$	1,439	0.259	0.111	0.181	0.318	0.240	0.465	0.856
Panel F: UAS participants' variance ratios (CFO average annual return questions, 2019 survey)								
"CFO" variance ratio	1,439	6.805	0.867	2.595	7.277	10.476	0.542	0.180
%Variance ratio<1	1,439	0.270						
Panel G: UAS participants' differences in variance ratios								
"ALP" variance ratio	469	1.195	0.100	0.255	0.908	2.162	0.542	0.674
"CFO" variance ratio	469	19.843	0.892	2.546	7.146	98.559	0.542	0.171
("CFO")/("ALP")	469	375.177	1.474	7.654	51.415	4401.810		
"CFO" - "ALP"	469	18.648	0.078	1.905	6.214	98.677		
("CFO" - "ALP")> 0	469	0.785						

Table IA-VI – Variance Ratio by Respondent Characteristics

We sort respondents by characteristic (education level, income, stock market participation, understanding of the stock market, numeracy, financial literacy, gender, age, race, marital status, employment status, retirement status, overconfidence, economic expectations, and beliefs about mean-reversion) for each of 29 long-form waves. The initial two rows of the first column report the time-series mean of the 29 cross-sectional median variance ratio for respondents in that group. Columns 2 and 3 present the time-series average (across the 29 waves) of the cross-sectional mean percentage of variance ratios less than 1 and the historical average for US equity markets (0.542), respectively. For each wave and sample, we test the hypothesis that the fraction of variance ratios less than 1 or 0.542, respectively, does not differ from 50%. † indicates that we can reject the hypothesis at the 1% level for all 29 waves. The third row in each panel reports the difference in median variance ratios and associated statistical significance (1%, 5%, and 10% levels are indicated by ***, **, and *, respectively) associated with a paired t-test (N=29) of the null hypothesis that the difference is zero.

Variable	Variance Ratio (median)	%Variance Ratio<1	%Variance Ratio<0.542
Panel A: Education			
More education	0.169	0.874†	0.790†
Less education	0.125	0.921†	0.865†
Difference	0.045***		
Panel B: Income			
High income	0.183	0.868†	0.777†
Low income	0.134	0.906†	0.843†
Difference	0.049***		
Panel C: Stock market participation			
Holds equity	0.176	0.870†	0.782†
No equity	0.119	0.925†	0.874†
Difference	0.057***		
Panel D: Self-rated understanding of stock market			
High understanding	0.172	0.869†	0.783†
Low understanding	0.120	0.931†	0.880†
Difference	0.052***		
Panel E: Numeracy			
High numeracy	0.229	0.836†	0.728†
Low numeracy	0.131	0.914†	0.853†
Difference	0.098***		
Panel F: Financial literacy			
High financial literacy	0.218	0.835†	0.731†
Low financial literacy	0.114	0.939†	0.893†
Difference	0.104***		

Table IA-VI – Variance Ratio Tests by Respondent Characteristics (Cont.)

Panel G: Gender			
Male	0.173	0.866†	0.779†
Female	0.135	0.912†	0.850†
Difference	0.037***		
Panel H: Age			
Old	0.154	0.886†	0.809†
Young	0.141	0.903†	0.837†
Difference	0.013***		
Panel I: Race			
White	0.151	0.889†	0.814†
Non-white	0.130	0.919†	0.862†
Difference	0.022***		
Panel J: Marital status			
Married	0.153	0.889†	0.813†
Not married	0.140	0.900†	0.835†
Difference	0.013***		
Panel K: Employment status			
Working	0.150	0.895†	0.822†
Not working	0.146	0.889†	0.817†
Difference	0.004		
Panel L: Retirement status			
Retired	0.162	0.872†	0.791†
Not retired	0.145	0.899†	0.829†
Difference	0.017***		
Panel M: Overconfidence			
Overconfident	0.149	0.881†	0.810†
Not overconfident	0.173	0.868†	0.779†
Difference	-0.024***		
Panel N: Expectations for a year from now			
Better	0.155	0.883†	0.804†
Worse	0.142	0.896†	0.825†
Difference	0.013**		
Panel O: Mean-reversion beliefs			
Mean-reversion belief	0.144	0.898†	0.828†
No mean-reversion	0.162	0.884†	0.804†
Difference	-0.019***		

Table IA-VII – Correlation Matrix of Panel Regression Explanatory variables

The table reports correlations between the panel regression (see Table VI) explanatory variables including Years education, Income, Holds equity, Understands markets, Numeracy, Financial literacy, Overconfidence, Mean reversion, and Better next year. Data includes the pooled cross-sectional time-series of 22,745 observations (from 3,022 individuals) that include any individual-survey wave observation with an estimable variance ratio. Variable descriptions are provided in the Internet Appendix, Section IA-14. Significance at the 1, 5, and 10% levels are indicated by ***, **, and *, respectively.

	Years education	Income	Holds equity	Understands markets	Numeracy	Financial literacy	Overconfidence	Mean reversion	Better next year
Years education	1.000								
Income	0.404***	1.000							
Holds equity	0.321***	0.397***	1.000						
Understands markets	0.332***	0.329***	0.410***	1.000					
Numeracy	0.405***	0.313***	0.328***	0.409***	1.000				
Financial literacy	0.340***	0.337***	0.428***	0.583***	0.521***	1.000			
Overconfidence	-0.014	-0.024**	-0.033***	0.019*	-0.031***	0.015	1.000		
Mean reversion	-0.029***	-0.056***	-0.026***	-0.034***	-0.080***	-0.102***	0.004	1.000	
Better next year	0.058***	0.036***	-0.017***	0.015**	0.024**	0.037***	0.032***	-0.002	1.000

Table IA-VIII – Uncertainty and Time

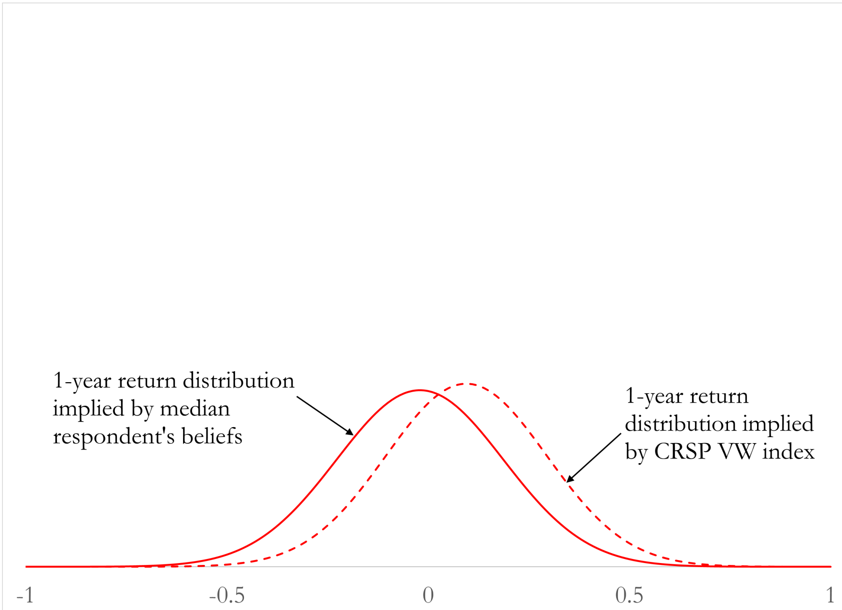
This table reports descriptive statistics for ALP survey results between August 2006 and November 2007 regarding respondents' uncertainty beliefs around different events in the next year or 5 years. Events considered include the likelihood of a car accident, having a cavity filled, dying, being a victim of theft, moving, dying from terrorism, having their home broken into, or visiting the dentist. Sample sizes for each event are reported in column (1). Median respondents' beliefs about event risk are reported in columns (2) through (5). Beliefs for the next year and 5 years are given in columns (2) and (4), respectively. Column (3) reports the implied 5-year probability given independence and the reported 1-year probability response. Column (5) reports the difference between the median implied and median reported 5-year probabilities. Columns (6) through (9) report analogous results for mean beliefs. Column (10) reports the fraction of respondents with a positive difference between the implied and reported 5-year probabilities. Significance at the 1% level is indicated by *** for the test that the fraction reported in column (10) does not differ from 50%.

		Median				Mean				%>0
	N	P(1-year)	Implied P(5-year)	P(5-year)	Difference	P(1-year)	Implied P(5-year)	P(5-year)	Difference	
Description	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
P(car accident)	859	0.200	0.672	0.300	0.372	0.278	0.673	0.352	0.321	0.934***
P(cavity filled)	642	0.200	0.672	0.500	0.172	0.302	0.665	0.492	0.173	0.815***
P(die)	838	0.100	0.410	0.200	0.210	0.207	0.523	0.279	0.244	0.870***
P(theft victim)	818	0.250	0.763	0.300	0.463	0.306	0.676	0.355	0.321	0.941***
P(move)	390	0.100	0.410	0.200	0.210	0.153	0.393	0.312	0.081	0.679***
P(die terrorism)	610	0.050	0.226	0.100	0.126	0.107	0.326	0.146	0.180	0.931***
P(break in victim)	813	0.100	0.410	0.150	0.260	0.154	0.443	0.220	0.223	0.888***
P(visit dentist)	190	0.500	0.969	0.750	0.219	0.560	0.851	0.670	0.180	0.932***

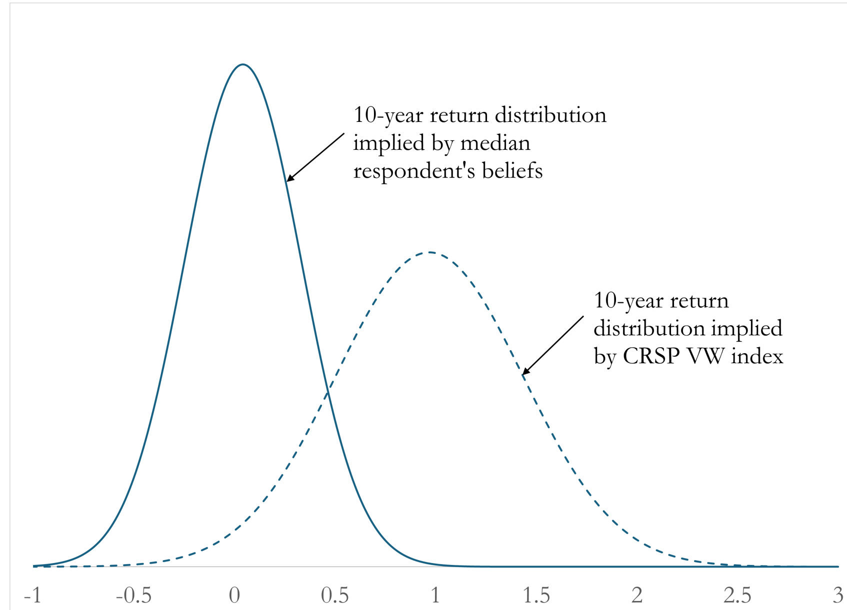
Table IA-IX – Beliefs from the ALP Bins-and-Balls Format

Panels A, B, and C report descriptive statistics for ALP respondents assigned to the bins-and-balls elicitation format, using the tail probability approach, which derives tail probabilities directly from ball allocations in the extreme bins and feeds them into the same lognormal identification as the standard questions. Because this approach requires respondents to allocate balls to both extreme bins, only 1,765 of the 25,504 respondents qualify. Therefore, Panels A–C are the subsample of individuals in Panels D–F who place at least one ball in each of the exterior bins. Panels D, E, and F report results for all respondents using the distributional approach, which computes mean and standard deviation from the full distribution of ball allocations across all bins, using CRSP-calibrated conditional expectations for the open-ended tails. The penultimate column reports the historical average (computed from the CRSP value-weighted index between 1926 and 2024). The final column reports the fraction of observations below the historical average. See Table I for comparison with standard question respondents.

Description	N	Mean	25^{th}	Median	75^{th}	Std Dev.	Hist.	%<Hist
Panel A: Tail probability approach — stock market expectations over next year								
P(market>0)	1,765	0.528	0.450	0.500	0.600	0.139	0.747	0.923
P(market>20%)	1,765	0.115	0.050	0.100	0.150	0.078	0.336	0.982
P(market<-20%)	1,765	0.115	0.050	0.100	0.150	0.080	0.060	0.397
$E_{i,t}(r_{1year})$	1,765	-0.020	-0.042	-0.020	0.001	0.038	0.095	1.000
$\sigma_{i,t}(r_{1year})$	1,765	0.165	0.139	0.158	0.191	0.036	0.200	0.841
Panel B: Tail probability approach — stock market expectations over next decade								
P(market>0)	1,765	0.600	0.500	0.600	0.700	0.161	0.960	1.000
P(market>20%)	1,765	0.179	0.100	0.150	0.250	0.133	0.932	1.000
P(market<-20%)	1,765	0.107	0.050	0.100	0.150	0.080	0.013	0.000
$E_{i,t}(r_{10years})$	1,765	0.011	-0.020	0.001	0.045	0.059	0.969	1.000
$\sigma_{i,t}(r_{10years})$	1,765	0.181	0.151	0.175	0.200	0.044	0.465	1.000
Panel C: Tail probability approach — variance ratios								
Variance ratios	1,765	0.129	0.100	0.112	0.152	0.055	0.542	1.000
%Variance ratio<1	1,765	1.000						
Panel D: Distributional approach — stock market expectations over next year								
P(market>0)	25,504	0.591	0.500	0.550	0.750	0.252	0.747	0.668
P(market>20%)	25,504	0.041	0.000	0.000	0.000	0.101	0.336	0.971
P(market<-20%)	25,504	0.029	0.000	0.000	0.000	0.087	0.060	0.881
$E_{i,t}(r_{1year})$	25,504	0.016	-0.008	0.007	0.051	0.059	0.095	0.906
$\sigma_{i,t}(r_{1year})$	25,504	0.082	0.050	0.072	0.105	0.037	0.200	1.000
Panel E: Distributional approach — stock market expectations over next decade								
P(market>0)	25,504	0.706	0.500	0.750	0.950	0.262	0.960	0.752
P(market>20%)	25,504	0.112	0.000	0.000	0.150	0.188	0.932	0.999
P(market<-20%)	25,504	0.028	0.000	0.000	0.000	0.091	0.013	0.837
$E_{i,t}(r_{10years})$	25,504	0.130	-0.001	0.066	0.202	0.185	0.969	1.000
$\sigma_{i,t}(r_{10years})$	25,504	0.193	0.050	0.096	0.371	0.160	0.465	1.000
Panel F: Distributional approach — variance ratios								
Variance ratios	25,504	0.895	0.094	0.175	1.069	1.385	0.542	0.645
%Variance ratio<1	25,504	0.738						

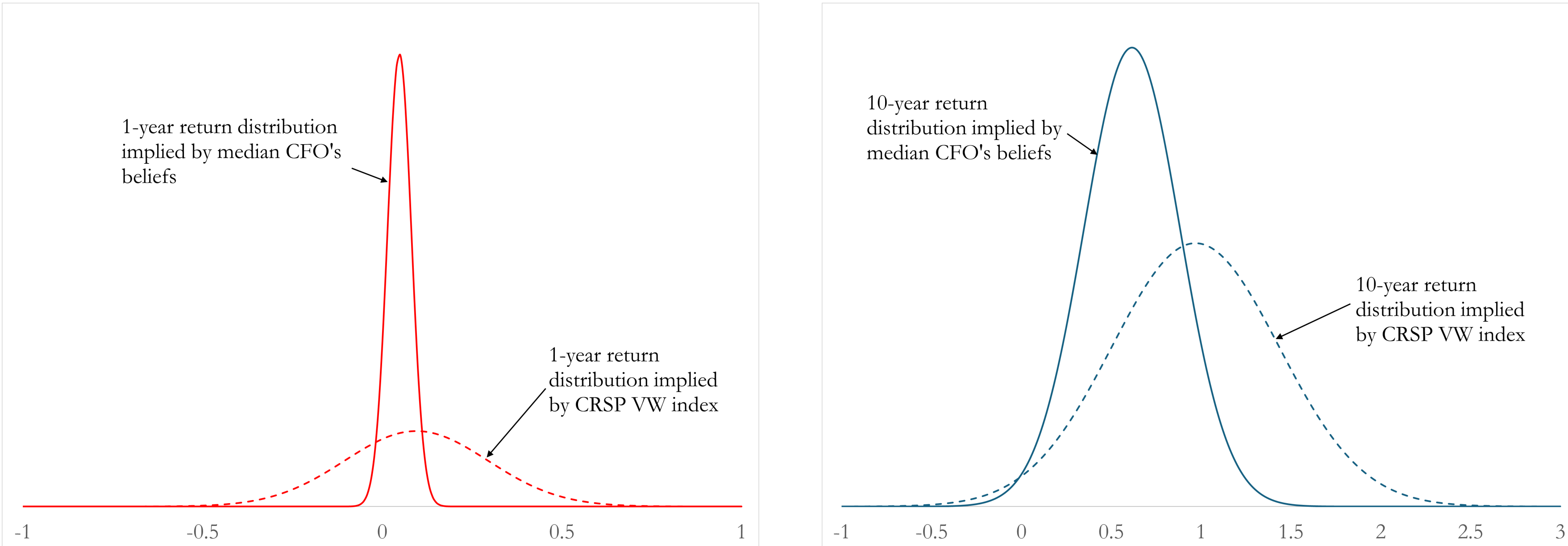


(A) ALP median respondent's 1-year return distribution



(B) ALP median respondent's 10-year total return distribution

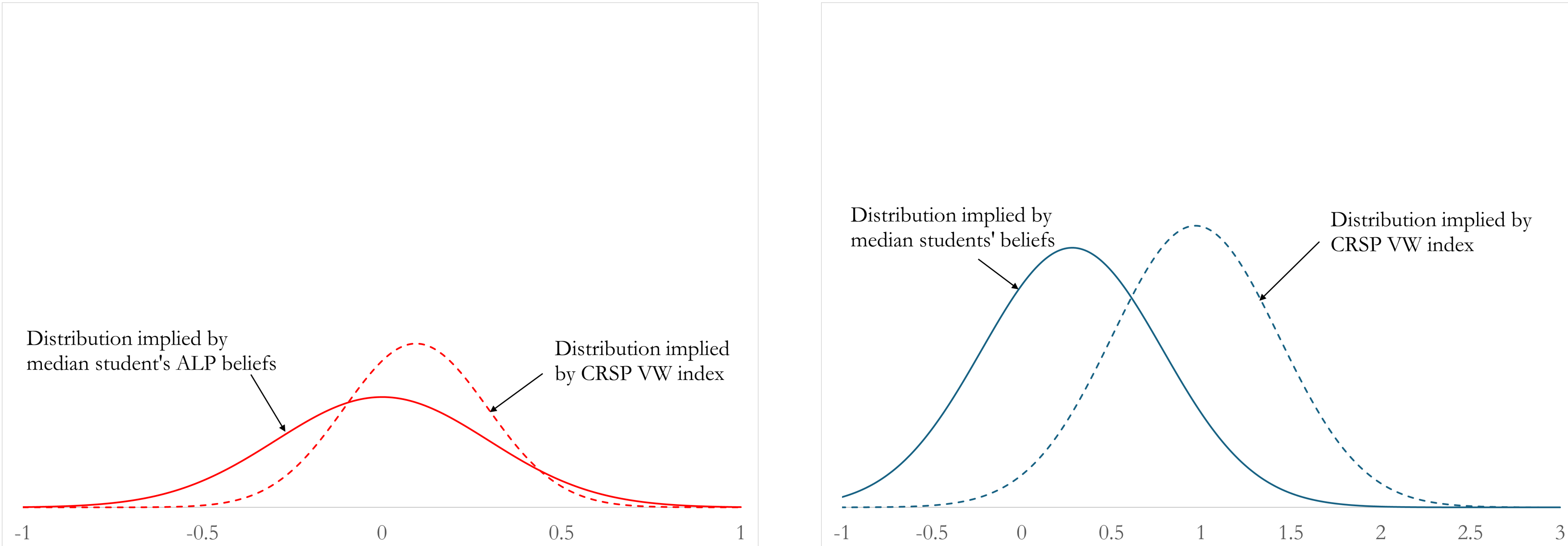
Figure IA-1 – Historical and perceived return distributions for ALP respondents. Panel A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926–2024 period (dashed red line) and the imputed 1-year distribution based on the median ALP respondent's beliefs over the 2008–2016 period (solid red line). Panel B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926–2024 period (dashed blue line) and the implied 10-year total return distribution based on the median ALP respondent's beliefs over the 2008–2016 period (solid blue line). The compression of near- and long-term uncertainty implied by these distributions is summarized in Figure 1 of the main text.



(A) Median CFO 1-year return distribution

(B) Median CFO 10-year total return distribution

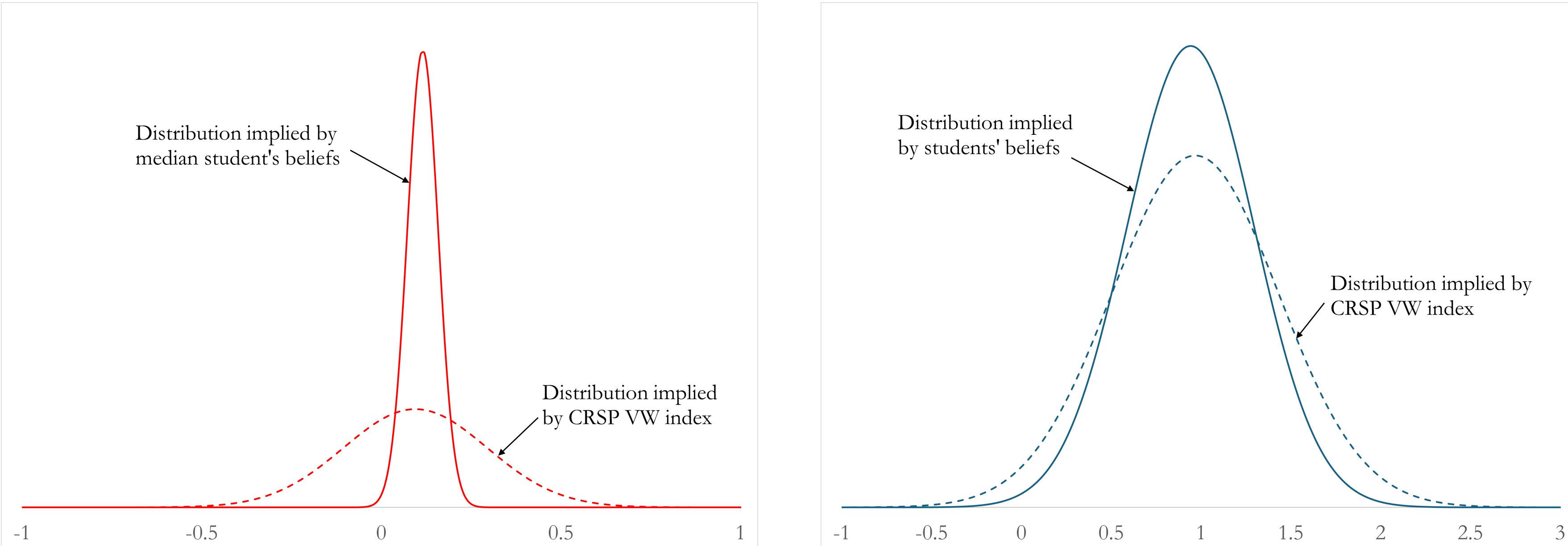
Figure IA-2 – Historical and perceived return distributions for CFOs. Panel A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926–2024 period (dashed red line) and the imputed 1-year distribution based on the median CFO's beliefs over 2004–2019 (solid red line). Panel B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926–2024 period (dashed blue line) and the implied 10-year total return distribution based on the median CFO's beliefs over 2004–2019 (solid blue line). The compression of near- and long-term uncertainty implied by these distributions is summarized in Figure 2 of the main text.



(A) Median students' 1-year return distribution

(B) Median students' 10-year total return distribution

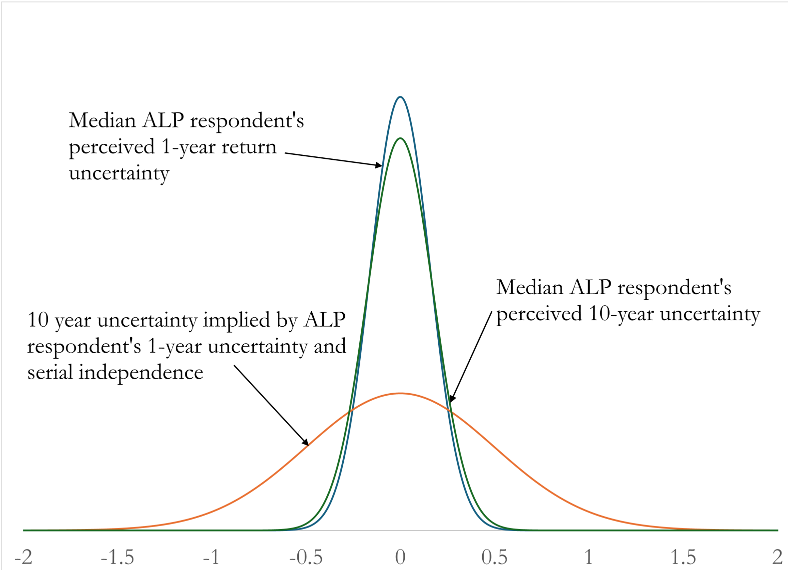
Figure IA-3 – Historical and perceived return distributions for students based on probability elicitation total return questions. Panel A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926–2024 period (dashed red line) and the imputed 1-year distribution based on the median students' beliefs over spring–fall 2024 (solid red line). Panel B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926–2024 period (dashed blue line) and the implied 10-year total return distribution based on the median students' beliefs over spring–fall 2024 (solid blue line). The compression of near- and long-term uncertainty implied by these distributions is summarized in Panel A of Figure 3 of the main text.



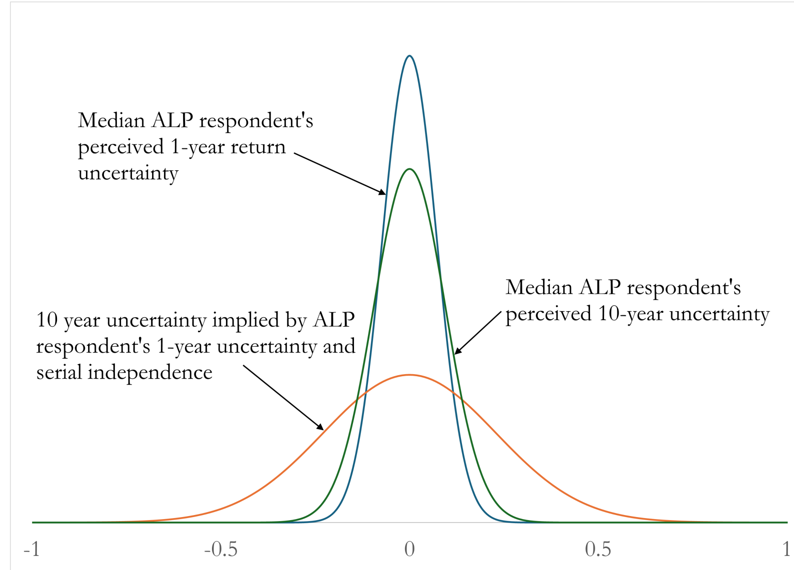
(A) Median student's 1-year return distribution based on quantile elicitation average annual return questions

(B) Median student's 10-year total return distribution based on quantile elicitation average annual return questions

Figure IA-4 – Historical and perceived return distributions for students based on quantile elicitation average annual return questions. Panel A reports the distribution of 1-year returns implied by the first two moments of the CRSP value-weighted annual return over the 1926–2024 period (dashed red line) and the imputed 1-year distribution based on the median student's beliefs (solid red line). Panel B reports the distribution of 10-year total returns implied by the first two moments of the CRSP value-weighted index 10-year return over the 1926–2024 period (dashed blue line) and the imputed 10-year total return distribution based on the median student's beliefs (solid blue line). The compression of near- and long-term uncertainty implied by these distributions is summarized in Panel B of Figure 3 of the main text.

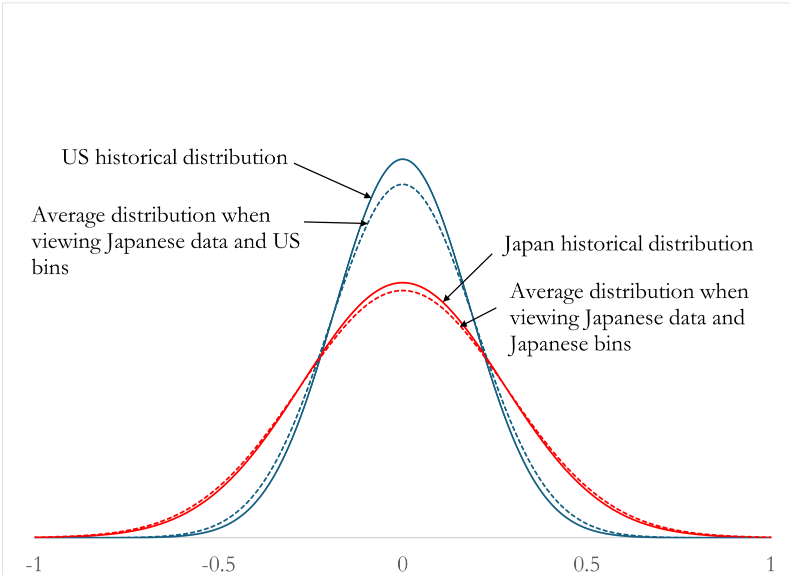


(A) Median ALP respondent's 1-year and 10-year return distributions, tail probability approach

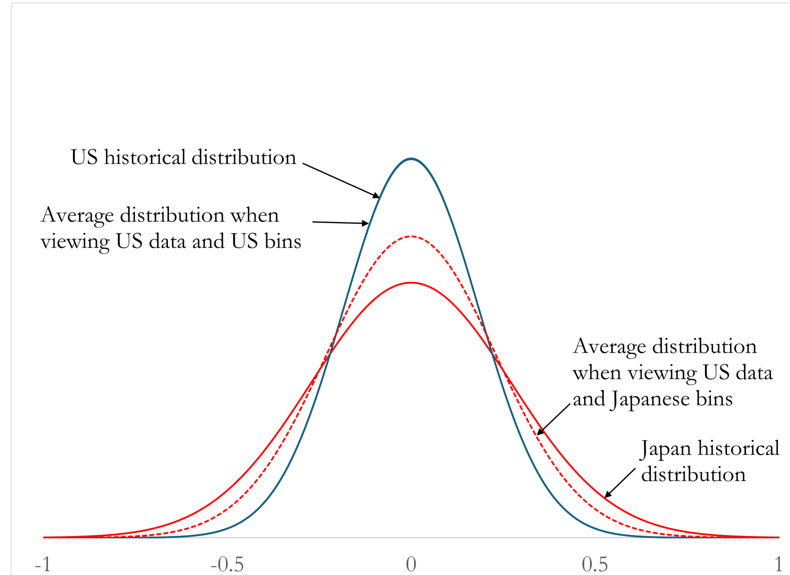


(B) Median ALP respondent's 1-year and 10-year return distributions, distributional approach

Figure IA-5 – Historical and perceived return distributions for ALP respondents based on the bins-and-balls format. Figures IA-5A and IA-5B report the median ALP respondent's imputed 1-year return distribution (blue solid line), the 10-year total return distribution implied by the 1-year distribution under serial independence (orange solid line), and the inferred 10-year total return distribution based on the median ALP respondent's long-run beliefs (green solid line). In Panel A, standard deviations are inferred using the tail probability approach, which derives tail probabilities directly from ball allocations in the extreme bins. In Panel B, standard deviations are inferred using the distributional approach following Hartzmark and Sussman (2026), which computes mean and standard deviation from the full distribution of ball allocations across all bins.

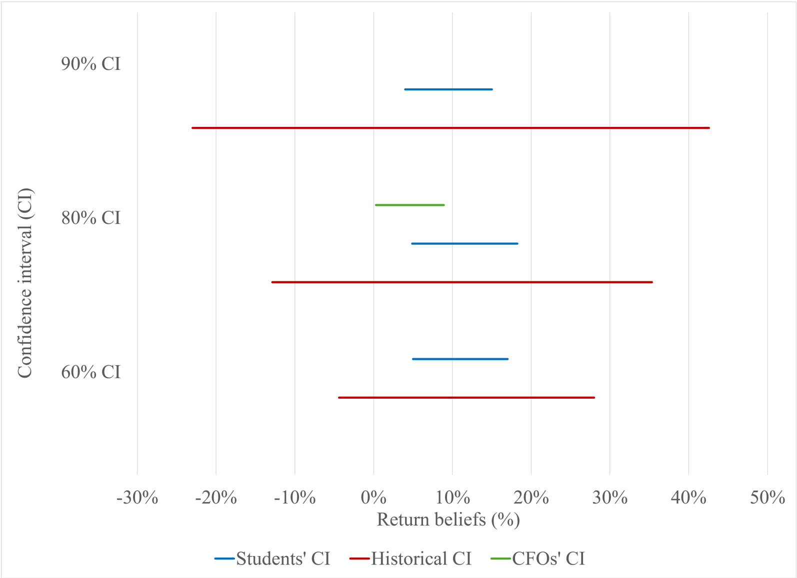


(A) Inferred return distributions for respondents who view Japanese historical returns, by bin definition

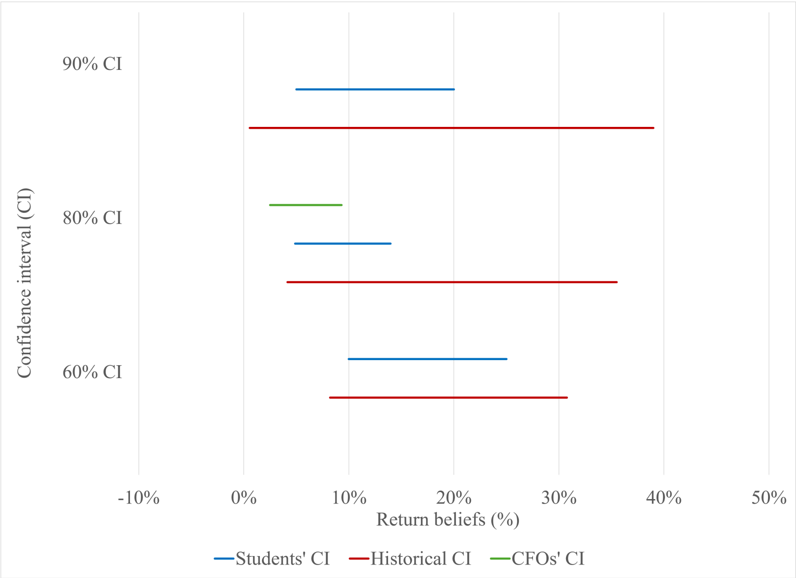


(B) Inferred return distributions for respondents who view US historical returns, by bin definition

Figure IA-6 – Sensitivity of inferred standard deviation to bin definitions. Respondents in Hartzmark and Sussman (2026) sequentially view 100 years of either historical US or Japanese returns in random order, then allocate balls across bins to reflect what they observed. Panel A reports results for respondents who view Japanese historical returns; Panel B reports results for respondents who view US historical returns. In each panel, the solid line reports the inferred distribution when bins are defined to match the historical data the respondent actually viewed, and the dashed line reports the inferred distribution when bins are instead defined to match the other country's historical data. The red (blue) lines correspond to bins formed to match historical Japanese (US) data. For example, the Japanese bins are defined as <-42%, -42 to -28%, . . . , >62%, while the US bins are defined as <-23%, -23 to -15%, . . . , >39%. Historical standard deviations are as reported in Hartzmark and Sussman (2026) and mean inferred standard deviations are based on the authors' Appendix A10.

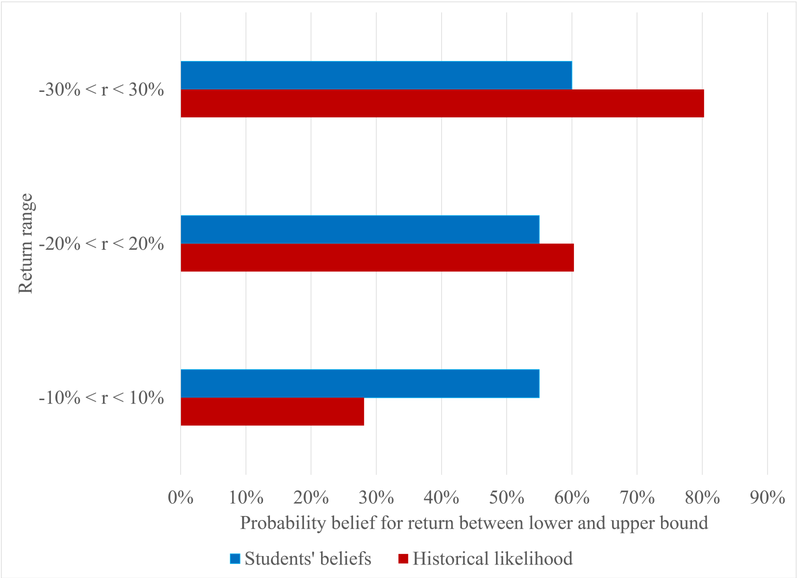


(A) 1-year return confidence intervals for the median student (60%, 80%, and 90%), median CFO (80%), and historical returns

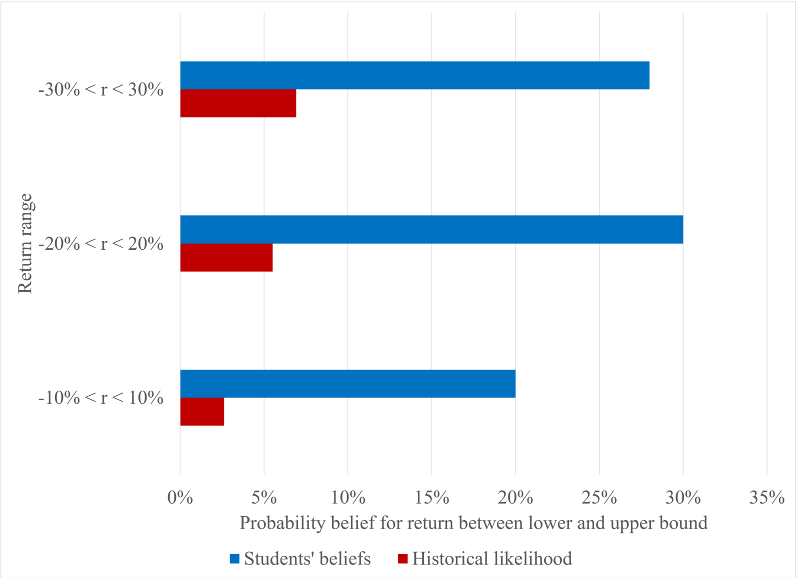


(B) 10-year return confidence intervals for the median student (60%, 80%, and 90%), median CFO (80%), and historical returns

FIGURE IA-7 – STUDENT AND CFO CONFIDENCE INTERVALS, CFO-STYLE QUESTIONS Panel A reports 1-year return confidence intervals and Panel B reports 10-year return confidence intervals, each for the median student (blue), median CFO (green), and historical returns (red). For students, confidence intervals are reported at the 60%, 80%, and 90% levels. For CFOs, only the 80% confidence interval is reported, as CFO-style questions ask respondents to report the 10th and 90th percentile of their forward-looking return distribution. Historical confidence intervals are based on the CRSP value-weighted index over the 1926–2024 period. The student sample is based on surveys conducted in the spring 2025, fall 2025, and spring 2026 semesters.



(A) 1-year implied probability of a return within ±30%, ±20%, and ±10% for students (blue) and historical returns (red)



(B) 10-year implied probability of a return within ±30%, ±20%, and ±10% for students (blue) and historical returns (red)

Figure IA-8 – Student probability beliefs and historical likelihoods, ALP-style questions. Panel A reports 1-year results and Panel B reports 10-year results. In each panel, blue bars report the implied probability that market returns fall within ±30% (top cluster), ±20% (middle cluster), and ±10% (bottom cluster), derived from students' reported likelihoods of a gain or loss exceeding each threshold. Red bars report the corresponding historical frequencies based on the CRSP value-weighted index over the 1926–2024 period. ALP-style questions ask respondents to report the likelihood that the market rises or falls by at least a given percentage. The student sample is based on surveys conducted in the spring 2025, fall 2025, and spring 2026 semesters.