Building Voice in Socially Responsible Investing*

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Abstract

We develop a dynamic model of socially responsible investment where large households trade firm equity and vote on production decisions involving the depletion of a nonrenewable resource. Although accumulating wealth and exercising voice are intratemporal substitutes, they are dynamic complements because influence is tied to wealth. Socially responsible households delay implementing resource-preserving policies that reduce firm productivity to amass wealth for future influence, while financially-motivated households may accumulate wealth to block conservation efforts. The constrained efficient technological choice balances higher productivity with society's willingness to pay for conservation, and can be implemented through a voting protocol that assigns voice based on how depletion impacts welfare rather than shareholdings.

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1 Introduction

Advocacy groups and activist investors increasingly focus on bringing about social change by implementing socially responsible corporate policies. Rather than relying directly on political processes, these groups seek to, for example, reduce carbon output and promote more equitable labor practices through direct pressure on corporations. In fact, in April 2025 the Securities Exchange Commission approved the first U.S. sustainability-focused exchange, the GIX, to facilitate socially responsible investment. For public firms, these social goals may be pursued through shareholder voting. For such an approach, influence is determined by the fraction of shares that different blocs holds, which in turn depends on prior investment decisions and the associated wealth accumulation from such decisions. Thus, investors who wish to influence corporate policies away from value maximization and toward social goals face a dynamic tradeoff, both in terms of how to vote given their share ownership and what shares to invest in. Analyzing this dynamic also naturally raises questions about how firms should aggregate shareholder preferences when their activities impact the provision of public goods.[1]

In this paper, we attempt to address these issues by considering a dynamic deterministic economy in which production causes an externality that shareholders can mitigate through voting. To fix ideas, we focus on pollution, but our insights apply to other production externalities such as the loss of biodiversity and data privacy. Firms in the economy can employ either a clean or dirty technology at each date. The dirty technology is more productive than the clean technology, but it also depletes a scarce, nonrenewable social resource.[2] Two types of strategic households and a competitive fringe trade firm equity. One type of strategic household type is averse to resource depletion, while

[1]For instance, Hansmann and Pargendler [2014] argues that in the 1800s, firms that provided public goods, such as utilities, departed from the one-share-one-vote paradigm to protect consumers. We examine this insight through the lens of firms that have an impact on public goods through production externalities.

[2]For example, running generative Artificial Intelligence (AI) models requires maintaining data centers that consume vast amounts of water and threaten local water security. See https://www.lawfaremedia.org/article/ai-data-centers-threaten-global-water-security.

the other and the competitive fringe only care about financial returns. To separate preferences over production externalities from preferences over cash flows, we also allow households to trade debt, so that they have an alternative financial instrument for intertemporal consumption smoothing. At each date, households vote with their shares to determine whether firms should use the dirty technology. Because households accumulate wealth and trade firm equity at each date, the preferences of the marginal shareholder over resource depletion can change over time.

The model is dynamic and features oligopolistic financial markets. Without dynamics, we cannot capture the potential for households to accumulate wealth to obtain greater voice to influence corporate policies, and with only small households, no investor would view his vote, or the votes he could obtain, as having any impact on corporate policy. We therefore have strategic agents who internalize their impact on asset prices and firm behavior.[3] Such rich interactions, however, can render dynamic models intractable depending on how strategic interaction is modeled. To make progress, we use a Cournot-Walras framework with a competitive fringe that has log utility. This enforces no arbitrage and dynamic consistency in equity pricing. To tractably solve for a strategic agent's optimal policies with both trading and voting, we use a convex-dual characterization of a strategic agent's decision problem that assigns the shadow costs of wealth and the social resource constraints as co-states in the optimization problem.

Our analysis yields several key insights. First, although accumulating wealth and exercising voice are intratemporal substitutes, they are intertemporal complements. Within a period, a socially responsible household trades off exercising voice to mitigate resource depletion with earning a higher return by supporting the implementation of dirty technology. In fact, it is possible for a socially responsible household to have such a large a stake in firms such that she will vote for the dirty technology because not doing so would sacrifice too much of her wealth. Such a sacrifice has the obvious direct effect

[3]Limited liquidity as a deterrent to activist investing is a recurring theme in the literature on corporate governance (e.g., Maug, 1998, Aghion et al., 2004).

on consumption but also may result in a loss of influence in the future, when it may be more important to mitigate the externality. Further, both strategic households also trade off preserving wealth with exhausting it to buy shares from each other at a premium to garner control. However, across periods, accumulating wealth today facilitates the exercising of voice in the future. This is because a strategic household's wealth is the present discounted value of both future consumption and future votes less the price concessions garnered from trading with price impact.[4] Consequently, a socially responsible household may choose to delay the implementation of the clean technology because financing voice is part of her optimal dynamic portfolio choice problem. Exercising voice is then akin to exercising a real option whose payoff is increasing in her marginal cost of future resource depletion, which rises over time. This insight makes it difficult to gauge the behavior of socially responsible investors and asset managers because investing in more productive dirty technologies may be part of a long-term sustainable strategy.

Further, a financially-motivated household directly affects the timing of the transition to the clean technology because he will pay a premium to obtain control rights to implement the dirty technology compared to the competitive fringe. As a consequence, these households can buy shares from socially responsible households to delay the clean transition, waiting until the shadow cost of resource depletion is so large that they can extract a premium from those households to sell control to them. We show in an example that, compared to the case of only socially responsible households, the case with oligopolistic competition reaches the all-clean technology limit at a higher level of marginal disutility for resource depletion. Absent share trading, a socially responsible household makes the financially-motivated household worse off because the former makes the firm more likely to adopt the low productivity clean technology. However, when shares can be traded, the financially-motivated household can extract a premium from selling control. Because

[4]Interestingly, when socially responsible households want firms to use more of the clean technology, their price impact is lower than that of financially-motivated households. This is because asset prices reflect that economic growth will be lower if they acquire more shares.

fringe households cannot coordinate their trading behavior, and will sell equity without obtaining the premium that financially-motivated households can command, both the socially responsible and the financially-motivated households can benefit from departing from shareholder value maximization at the expense of these smaller investors.

Finally, we show that a constrained optimal voting protocol implements a modified Dréze rule that takes into account the production externality. In a first-best economy, a planner would make decisions based on initial shareholdings and the marginal willingness of socially responsible households to pay to abate resource depletion. Notably, the latter does not depend on the socially responsible household's wealth, whereas how she votes in the decentralized economy depends on her accumulated wealth at each date. If the planner is constrained to make production decisions that respect how shareholders vote, then it alters the voting rule to try to achieve the constrained efficient production plan that would be achieved under a modified Dréze rule: the planner weighs cash flow considerations by shareholdings among investors while also weighing the cost of resource depletion according to the socially responsible investor's shadow cost. Although all shareholders are exposed to the firms' dividends in proportion to their shareholdings, and consequently proportional control is optimal for assigning cash flow rights, the incidence of production externalities on shareholders is independent of their wealth, and consequently a planner aims to correct this dynamic portfolio distortion.

We then consider several extensions of our framework. First, we show that with aggregate risk, control rights give rise to a voting risk premium that depends on how the value of control varies with aggregate output. Second, if dirty and clean firms use a common factor of production, then the decline in factor costs as firms transition to the clean technology subsidizes the remaining dirty firms and increases the monetary benefit of keeping them dirty. Last, if the supply of firms is elastic, then socially responsible investors may exhaust all their wealth buying new firms for their control rights to ensure use of the clean technology. These last two extensions illustrate economic limits to socially responsible investing as compared to implementing policy through direct government intervention.

Our paper relates to the literature on the interaction between corporate governance and shareholder preferences.[5] Traditionally, this has been studied in the context of incomplete financial markets (e.g., Dréze, 1974, Geanakoplos et al., 1990) and with voting (e.g., Drèze [1985], DeMarzo [1993]) where disagreement over production plans arose from heterogeneous state-contingent valuations More recently, this question has taken on a social dimension in which firms may maximize stakeholder rather than shareholder value (e.g., Magill et al., 2015, Hart and Zingales, 2017, Gollier and Pouget, 2022). Admati et al. [1994] shows how shareholders' risk-sharing needs can induce a blockholder to monitor firm management. Morgan and Tumlinson [2019] studies how pro-social activist shareholders can induce managers to divert profits toward providing public goods. Levitt et al. [2021] shows how multiple equilibria can arise with a continuum of shareholders who trade stock and vote, while Levitt et al. [2022] explores how price impact affects a large blockholder's incentives to become the median voter. Piccolo et al. [2022] studies the pernicious effect on competition when socially responsible shareholders concentrate their holdings, while Dangl et al. [2023, 2024] examine how social preferences and hedging demand interact with corporate investment decisions. Dottling et al. [2024] studies the interplay between public good provision by the government and firms without share trading, and emphasizes that wealth inequality can distort firm behavior.

By contrast, we consider a dynamic general equilibrium setting with multiple strategic investors who have preferences over production externalities and trade in addition to voting on firm decisions. Although financial markets are complete, there is no market for compensating investors for resource depletion, and their concerns are expressed through their voting behavior. To our knowledge, we are among the first to study the dynamic feedback between wealth accumulation and strategic voting on firm policies.

[5] See Edmans [2014] for a review of this literature.

Our paper also contributes to the literature on whether financial markets can address production externalities. For instance, Broccardo et al. [2022] contrasts with Admati and Pfleiderer [2009] by showing voting is more effective than divestment or boycotting for promoting pro-social behavior, while Crès and Tvede [2013], Crèes and Tvede [2023] examine voting when shareholders internalize (1) production spillovers across firms and (2) imperfect competition among firms through common ownership, respectively. Heinkel et al. [2001] and Berk and van Binsbergen [forthcoming] examine the impact of divestment on firms' cost of capital, while Oehmke and Opp [2020, 2022] and Green and Roth [2020] explore investor-driven complementarities in funding socially responsible or harmful technologies. Gupta et al. [2023] explores a hold-up problem when gains from trade are making firms cleaner, while Albuquerque et al. [2025] shows that anticipation of regulation can paradoxically lead to more financing of dirty firms. Arnold [2023] argues that if firms maximize shareholder value in complete financial markets with competitive investors, then there is no need for socially responsible policies. By contrast, we study strategic investors who vote and trade shares while internalizing how firms' production plans impact asset prices and production externalities. We emphasize that, when viewed as part of a dynamic portfolio problem, socially responsible investors may optimally delay addressing production externalities because voice is linked to wealth.

Our equilibrium concept is a Cournot-Walras equilibrium in financial markets.[6] In this tradition, Basak [1997] studies asset pricing with a monopolistic agent in an Arrow-Debreu economy. Basak and Pavlova [2004] examines the time-consistency issues that arise when a monopolistic firm internalizes its impact on both output prices and equity valuation. Similar to our focus on wealth as a part of a strategic household's strategy, Chen et al. [forthcoming] shows how leverage dynamics and financial distress interact with the extent of collusion within an industry. Neuhann and Sockin [2023] studies

[6] A related approach is the sequential auction equilibrium concept based on Kyle [1985]. In this paradigm, Back et al. [2018] shows that higher liquidity need not coincide with higher economic efficiency when an activist who can affect firm value trades dynamically in a market with asymmetric information.

the dynamic feedback between market power, risk sharing, and wealth accumulation. Our focus is on how strategic interaction among large investors affects firms' production plans when they trade long-lived assets that convey both cash flow and control rights. A methodological contribution of our paper is to solve for a strategic household's optimal policies using Hamiltonian rather than dynamic programming methods, which does not require that they have a recursive, or even a Markov, representation.

2 Model

Consider a three-period economy with dates $t \in \{0, 1, 2\}$. At dates 0 and 1, strategic households vote on the production decisions of firms that take place at dates 1 and 2. Voting influences whether firms employ a dirty or a clean technology. The dirty technology depletes a social resource that is in limited supply, creating a disutility for a subset of strategic households whom we refer to as socially responsible (S) strategic households. Households that do not care about the depletion of the resource are referred to as financially motivated (F) strategic households. There is also a continuum of competitive households (f) of mass m_f who take prices as given and are also exclusively financially motivated. To focus on the impact of production decisions on voting and wealth accumulation, we do not consider other frictions associated with transitioning from one technology to another, such as fixed costs of transition and irreversibility of capital investments.

There is a continuum of firms of unit mass that have access to two technologies that produce output. A firm using clean technology can produce 1 unit of output. A firm using the dirty technology produces $z > 1$ units of output at date t. This productivity the gap, $z - 1$, can be viewed as the cost for clean firms to mitigate the externality. Each firm has an unit of stock outstanding that trades in centralized public markets at dates 0 and 1. We assume that a share of firm equity is a claim to the aggregate dividend produced by all firms. Firm equity is initially equally allocated across all households (i.e., each group owns $\frac{1}{2+m_f}$ shares) and firms pay an exogenous aggregate dividend, D_0, at date 0.[7]

At the beginning of dates 1 and 2, before production occurs, firms' shareholders vote on the extent to which firms use the dirty technology. Household i that acquires s_{it} shares of firms can cast up to s_{it} votes for whether firms should use the dirty or clean technology. Let x_{it} be the number of votes cast for the dirty technology and $s_{it} - x_{it}$ the number of votes cast for the clean technology. In aggregate, a fraction $\chi_t = x_{St} + x_{Ft} + m_f x_{ft}$ of votes by households are for the dirty technology, and we assume the same votes are cast at every firm in the continuum. That households vote at the beginning of the date after they trade reflects that voting by shareholders on proposals occurs at annual and specially-called shareholder meetings, while trading occurs throughout the year.

Firms adopt shareholder proposals with a probability based on the proportion of shareholders who vote for it according to a voting rule $G(\cdot) : [0,1] \to [0,1]$ that maps the number of votes in favor of the dirty technology, χ_t, into the probability that a firm employs it, $G(\chi_t)$.[8,9] Historically, firms have employed various voting rules for aggregating shareholder preferences (e.g., Hilt [2008], Hansmann and Pargendler [2014]), such as quadratic voting and dual-class shares. We restrict attention to aggregation rules, $G(\chi_t)$, that are $\mathcal{C}^1([0,1])$-differentiable, (weakly) increasing, and respect unanimity, i.e., $G(0) = 0$ and $G(1) = 1$. Because firms are part of a continuum, by the weak law of large numbers, exactly a fraction $G(\chi_t)$ use the dirty and $1 - G(\chi_t)$ use the clean technology.[10]

[7] Although not essential, this assumption allows us to avoid having to specify an initial insider or founder for firms. See, for instance, Grossman and Hart [1988] and Hart and Zingales [2017] for the nuanced tension between initial owners and subsequent shareholders.

[8] This avoids the need for mixed strategies because investors would have perfect foresight as to whether they are pivotal; as such, none can be pivotal unless they are very wealthy and have strong operational preferences. Otherwise, we would need mixed strategies, in the spirit of Burdett and Judd [1983], or super-majorities with veto power (e.g., Drèze [1985], DeMarzo [1993]) to construct voting equilibrium.

[9] In practice, corporate boards are not always bound to follow the operational recommendations of shareholders, even when the decisions of shareholders are unanimous (e.g., Cres and Tvede [2021]).

[10] We can allow for this voting rule to be arbitrarily steep such that it is a close to a step function that jumps at $\chi_t = \frac{1}{2}$ to approximate majority rule. For instance, we can choose for $m > 0$

$$G(\chi_t) = \frac{1}{2}\left(1 - \tanh\left(m \frac{2x-1}{2x(x-1)}\right)\right) \mathbf{1}_{x \in [0,1]}.$$

The aggregate dividend, D_t, at dates 1 and 2 is

$$D_{t+1} = 1 + G(\chi_t)(z-1). \tag{1}$$

Although more productive, the dirty technology is costly because of a production externality. There is a social resource with initial stock $\epsilon_0 > 0$ in the economy that firms consume when they use the dirty technology. S households have a preference for maintaining the social resource. This ϵ_t can represent trees, metals, energy, or usable land, which are directly consumed by production, or more general amenities like clean air or water that is made less valuable by pollution.

At dates 0 and 1, households can purchase two types of assets in financial markets. The first is a one-period bond in zero net supply (i.e., an inside asset) at price $\frac{1}{r_t}$ that pays one unit of the numeraire upon maturity. The second is the equity of firms with price p_t. This equity is a claim to the composite portfolio of all firms and pays as its dividend their aggregate dividend, D_{t+1}. Because there is no risk, financial markets are complete with respect to firms' cash flows, and production decisions are not mechanically chosen based on spanning concerns that arise with incomplete markets.

Because we allow for short-selling, we must specify how the additional asset float affects voting to ensure that only a unit mass of shares having voting rights. We assume an owner of a share of firms can lend it to a short-seller at a fee, ϕ_t, set by a large intermediary that strategic households take as given. A household who lends her shares forfeits her vote but does receive the dividend and capital gain when the stock is returned. This intermediary sets ϕ_t to be the lowest fee such that there is no excess demand in the share-lending market. This weaker requirement than supply must equal demand makes the supply curve for share-lending a correspondence rather than a step function.[11]

Although our model is three periods, all proofs are done in the general case of T pe-

[11] Because each household will want to lend all their shares if the lending fee is sufficiently high, the supply of total shares available in the lending market jumps discontinuously with the fee. This makes it difficult to clear both the equity and share-lending markets if we impose supply must equal demand.

riods, and we examine the limit as $T \to \infty$ to allow for an infinite horizon. Proofs of key results are relegated to the Appendix and of additional results to Online Appendix A.

2.1 Competitive Households

Competitive households are indexed by f. Their key function is to provide a pricing system for strategic households that satisfies no arbitrage and is dynamically consistent. They have log preferences over consumption and no non-consumption related preferences over the resource, although this can be relaxed. For most relevant cases, their preferences are immaterial for the positive implications of the model because these atomistic households do not believe their voting affects production outcomes; as such, they will immediately sell to a household seeking to influence corporate policy. This standard preference specification will be useful for tractability of asset prices (and price impact), and mutes any direct impact of depletion of the social resource on asset prices.

Household f has wealth W_{ft}. She purchases b_{ft} shares of the one-period bond at date t, and s_{ft} shares of the equity of firms, and can lend out her shares of firms to other households or short-sell shares at fee ϕ_t per share. Let $L_{ft} \leq s_t$ be the number of shares the household lends to short-sellers when she is long firm equity, and $x_{ft} \in [0, s_{ft} - L_{ft}]$ be the number of votes she casts for the dirty technology.

Household f faces the budget constraint at dates t

$$c_{ft} + \frac{1}{r_t} b_{ft} + \left(p_t - \phi_t \mathbf{1}_{\{s_{ft}<0\}}\right) s_{ft} \leq W_{ft} + \phi_t L_{ft}, \tag{2}$$

and realized wealth at dates 1 and 2 are given by

$$W_{ft+1} \leq b_{ft} + (D_{t+1} + p_{t+1})\, s_{ft}. \tag{3}$$

Household f is initially allocated a fraction $\frac{1}{2+m_f}$ of equity shares in each firm and of the firms' total initial dividend D_0, $W_{f0} = \frac{D_0 + p_{j0}}{2+m_f}$.

Household f has subjective discount rate β and chooses its consumption and investment decisions to maximize her lifetime utility

$$\begin{aligned} U_f &= \sup_{\{c_f, b_f, s_f, x_f\}} \sum_{t=0}^{2} \beta^t \log\left(c_{ft}\right) \\ s.t. &: (2), (3). \end{aligned} \tag{4}$$

Proposition 1 provides a characterization of competitive households' optimal policies. In what follows, we define $h(t) = \frac{1-\beta}{1-\beta^{3-t}}$. We also define $q_t = \beta \frac{h(t)}{h(t+1)} \frac{W_{ft}}{W_{ft+1}}$ to be the state price of competitive fringe over consumption at date $t+1$.

Proposition 1. *At date t, competitive household f:*

- *chooses an optimal consumption policy*

$$c_{ft} = h(t)\, W_{ft}; \tag{5}$$

- *votes for the dirty technology with the shares she does not lend out*

$$x_{fjt} = s_{ft} - L_{ft}; \tag{6}$$

- *chooses optimal asset positions in debt and firm equity that are linear in wealth, $b_{ft} = \hat{b}_{ft} W_{ft}$ and $s_{ft} = \hat{s}_{fjt} W_{ft}$,respectively, that satisfy*

$$\begin{aligned} \frac{1}{r_t} &= q_t, & (7) \\ p_t &= q_t \left(D_{t+1} + p_{t+1}\right) + \phi_t. & (8) \end{aligned}$$

Price impact in debt and equity markets for strategic household i is summarized by the 2×2 matrix J_{it} in equation (A.21), and is higher for the F than the S household because of the *price impact of a marginal change in control.*

As is standard, the price of debt, $\frac{1}{r_t}$, is the competitive fringe's state price, which reflects its average inter-temporal marginal rate of substitution. The price of firms' equity, p_t, is the present-value of its aggregate dividend and stock price next period along with the share lending fee. If the competitive fringe shorts the stock, it must pay this fee per share. If it lends shares, then it receives this fee. Because competitive households do not value their votes, they will be the first shareholders to sell and lend their shares. However, when they do vote, they vote for the dirty technology because it maximizes the excess return on firm equity, which they price competitively.

We can also derive the price impact function for strategic households using the residual demand curves of the fringe. In standard models, price impact is anonymous among strategic traders. Here, the price impact of a F household is different than that of a S household because prices also adjust based on how the strategic household will vote with the marginal shares she acquires. If the F household acquires additional equity of firms, then this makes firms more productive and raises their cash flows, magnifying the F household's price impact when acquiring shares. In contrast, if the S household votes for the clean technology when it acquires additional equity, then this makes firms less productive and lowers their cash flows, attenuating the S household's price impact when acquiring shares and enabling her to buy more.

To see this, we can write price impact in firm equity of the S household, $J_{St,22}$, from Proposition 1 as

$$j_{St,22} = \underbrace{q_t \left(D_{t+1} + \phi_{t+1}\right) \frac{D_{t+1} + p_{t+1}}{W_{ft+1}}}_{\text{Standard Price Impact}} + \underbrace{-\frac{q_t \left(z-1\right) b_{ft}}{b_{ft} + \left(D_{t+1} + \phi_{t+1}\right) s_{ft}} G'\left(\chi_t\right)}_{\text{Reduction in Price Impact from Voting}} .$$

If the slope of the voting rule, $G'\left(\chi_t\right)$, is sufficiently steep, or the difference in cash flows, $z-1$, is sufficiently large, then her price impact can approach zero or even be negative for a given set of parameters and holdings. In this case, it is easier for a S household to buy firm equity than the F household (for whom the second term is positive) because of rational expectations that she will reduce firm's cash flows tomorrow, further depressing prices today. The S household will acquire more shares in this situation until her price impact became positive. Note that this effect has the potential to push firms toward low cash flow policies, since strategic investors who favor such policies will have an advantage when trying to buy up shares. Steep voting rules, such as strict majority rule, can exacerbate this effect and potentially lead to even relatively small S households having an outsized impact on policy.

2.2 Firms and the Social Resource

Firms produce output using a single factor as an input, i.e., capital. A firm of type j has one unit of this factor that can be deployed either to the dirty or clean technology at date t. Firms pay out all output as dividends to shareholders and maintain zero savings.

At time t, firm j uses one unit of the factor to produce output y_{jt} according to

$$y_{jt+1} = 1 + (z-1)\,K_{jt}, \tag{9}$$

where K_{jt} is an indicator that is 1 if the firm chose to operate the dirty technology at time t. Although the clean technology does not consume any of the social resource, ϵ_t, the dirty technology depletes the social resource by ηK_{jt} for $\eta > 0$. We assume $z > 1$ so that the dirty technology is more productive than the clean technology.

The social resource ϵ_t consequently depletes according to the law of motion

$$\epsilon_{t+1} = \max\{\epsilon_t - \eta\int_0^1 K_{jt}dj, 0\} = \max\{\epsilon_t - \eta G\left(\chi_t\right), 0\}. \tag{10}$$

2.3 Strategic Households

There are two *types* of strategic households, indexed by $i \in \{F, S\}$. To tractably vary market concentration without affecting aggregate resources in the economy, we assume, within each type, there exist n symmetric households who each have mass $1/n$. As such, $1/n$ determines the share of a type's total wealth that is controlled by an individual household and the extent to which a strategic household internalizes her price impact.

A household of type i has wealth W_{it} and is indexed by its utility over consumption c and the social resource ϵ, $\log(c) + v_{i,t}(\epsilon)$, where $v_{i,t}(\epsilon) \equiv 0$ for financially-motivated (F) households and $v_{i,t}(\epsilon)$ is increasing and concave for socially responsible (S) households. This class of broad deontological preferences over production externalities is standard and captures that households have heterogeneous non-consumption related preferences over the resource. These could represent concerns about how environmental depletion or extraction of a non-renewable resource impact society, or how the scarcity of the resource affects her life or those of people and animals around her. This specification also covers non-production related goals implementable by firms at the cost of shareholder value.[12]

At each date, strategic household i chooses its consumption, c_{it}, its holdings of one-period debt, b_{it}, at a price $\frac{1}{r_t}$, and its holdings of firm equity, s_{it}, at price p_t. At the beginning of dates 1 and 2 before production has completed, households vote for the dirty or clean technology at that date. Let x_{it} be the number of votes she casts for the dirty technology at firms if she votes, and $s_{it} - x_{it}$ the number of votes she casts for the clean technology. Households can forfeit their voting rights and lend out their shares or short-sell firm equity at fee ϕ_t per share. Let $L_{it} \leq s_{it}$ be the number of shares the household lends to short-sellers if she is long firm equity. Because a household cannot vote with shares she has lent out, we have that $x_{it} \in [0, s_{it} - L_{it}]$.

[12]Strategic households' private benefit of control in our setting is endogenous, time-varying and based on aggregate outcomes rather than time-invariant and firm-specific (e.g., Grossman and Hart [1988]).

Household i faces the budget constraint at dates 0 and 1

$$c_{it} + \frac{1}{r_t} b_{it} + \left(p_t - \phi_t \mathbf{1}_{\{s_{it}<0\}} \right) s_{it} \leq \phi_t L_{it} + W_{it}, \tag{11}$$

and wealth has a law of motion

$$W_{it+1} \leq b_{it+1} + (D_{t+1} + p_{t+1}) s_{it}, \tag{12}$$

Strategic household types are each allocated a fraction $\frac{1}{2+m_f}$ of equity shares in firms and the initial dividend, D_0. Their initial wealth is consequently $W_{i0} = \frac{D_0+p_0}{2+m_f}$.

Household i for $i \in F, S$ has subjective discount rate $\beta \in (0,1)$ and chooses its consumption and investment decisions to maximize her lifetime utility

$$\begin{aligned} U_i &= \sup_{\{c_i,b_i,s_i,x_i\}} \sum_{t=0}^{2} \beta^t \left(\log(c_{it}) + v_{i,t}(\epsilon_t) \right) \\ s.t. &: (11), (12). \end{aligned} \tag{13}$$

Importantly, strategic household i internalizes how her trades impact asset prices and firm behavior. As such, she must forecast her price impact in asset markets, summarized by pricing functionals $\{\tilde{r}_i, \tilde{p}_i\}$, and on firm objectives and social resource depletion through voting. We assume she lacks commitment and follows Markov Perfect strategies, i.e., she cannot condition her actions based on non-payoff relevant past states.[13]

To characterize strategic households' optimal policies, we must define two objects in addition to the price impact matrix, J_{it}, from Proposition 1. The first, Θ_{it}, is the marginal change in the household's life-time utility at date t from increasing the fraction of firms that use the dirty technology

$$\Theta_{it} = \frac{z-1}{c_{it+1}} \partial_{D_{t+1}} W_{it+1} = \frac{z-1}{c_{it+1}} s_{it} \left(1 + \frac{(p_{t+1} - \phi_{t+1}) s_{ft}}{b_{ft} + (D_{t+1} + \phi_{t+1}) s_{ft}} \right). \tag{14}$$

[13]Commitment is encoded in backward-looking objects (e.g., Marcet and Marimon [2019]).

This captures both the marginal increase in both next period's dividend, D_{t+1}, and price, p_{t+1}, through how the change in production alters the fringe's marginal utility in the future. In addition, we define Ξ_{it} to be strategic household i's forward-looking private shadow cost of depleting the social resource at date t

$$\Xi_{it} = \sum_{s=t+1}^{2} \beta^{s-t} v'_{i,s}\left(\epsilon_s\right). \tag{15}$$

That is, Ξ_{it} is the marginal change in the household's life-time utility from addition depletion of the social resource ϵ_t. If she has a high marginal utility when the resource is low, i.e., high $v'_{i,t}\left(\epsilon_{t'}\right)$, then she will find it very costly to continue to deplete it.

We first discuss how a strategic household chooses its positions in debt and firm equity. Similar to Kyle [1989] and Vayanos [1999], who examine strategic trading in CARA-normal settings, strategic household i chooses its optimal asset positions by putting a wedge between asset prices and her marginal valuations based on her private stochastic discount factor (SDF), $\beta c_{it}/c_{it+1}$, which differs from the fringe's. This is to extract infra-marginal rents from their trades. This wedge not only takes into account the strategic household's price impact, but also her marginal value of control in the case of equity.

In the case of debt, strategic household i's optimal position b_{it} satisfies

$$\frac{\beta c_{it}}{c_{it+1}} = \frac{1}{r_t} + \frac{1}{nm_f} \boldsymbol{e}_1' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}, \tag{16}$$

where $\boldsymbol{e}_k$is the k^{th} 2×1 Euclidian basis vector. Because $\partial_{b_i}\left(\frac{1}{r_t}\right) > 0$, i.e., an increase in demand for debt increases its price and lowers its return, a strategic household that buys debt puts a positive wedge between her discounted growth rate in marginal utility, $\beta c_{it}/c_{it+1}$, and the inverse of the interest rate, $\frac{1}{r_t}$. In contrast, a seller puts a negative wedge and consequently has a lower growth rate in marginal utility compared to the inverse of the interest rate. This wedge is given by the price impact term that takes into account how a marginal change in her debt holdings impacts the cost of all her trades.

In the case of firm equity, strategic household i chooses an optimal asset position in the stock of firms, s_{it}, if she is long stock

$$p_t = \frac{\beta c_{it}}{c_{it+1}} \left(D_{t+1} + p_{t+1}\right) + \max\{\phi_t, \nu_{it}\} - \frac{1}{nm_f} \boldsymbol{e}_2' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}, \tag{17}$$

where ν_{it} is her marginal value of control, defined below. If she is short stock, the $\max\{\phi_t, \nu_{it}\}$ term is instead replaced by the lending fee, ϕ_t. Similar to her optimal choice of debt, strategic household i's optimal choice of equity puts a wedge between the price for equity, p_t, and her marginal valuation, which is the first-term on the left-hand side of equation (17). However, this wedge is not only her price impact; the second term takes into account how household i's voting impacts equity prices and the marginal value of her share for the control rights it provides, ν_{it}, or for lending to a short-seller, ϕ_t, whichever is higher.

Having characterized a strategic household's optimal asset positions, Proposition (2) characterizes her optimal voting policy and present-value of her wealth.

Proposition 2. *For strategic household i at dates* 0 *and* 1*:*

- *While $\varepsilon_t > 0$, her private value of control in firms is*

$$\nu_{it} = \frac{\beta}{n} \frac{|\Theta_{it} - \eta \Xi_{it}|}{1/c_{it}} G'\left(\chi_t\right) \left(1 - \mathbf{1}_{\{\Theta_{it} > \eta \Xi_{it} \wedge G(\chi_t) = 1\}}\right). \tag{18}$$

If this value is less than the lending fee, i.e., $\nu_{it} < \phi_t$, she lends her shares; otherwise, she votes according to the following rule:

$$x_{it} = \begin{cases} 1, & if\ \Theta_{it} > \eta \Xi_{it} \\ \in [0, s_{it} - L_{it}], & if\ \Theta_{it} = \eta \Xi_{it} \\ 0, & if\ \Theta_{it} < \eta \Xi_{it} \end{cases}; \tag{19}$$

- *Household i's present-value budget constraint satisfies*

$$W_{i0} = \underbrace{\left(1+\beta+\beta^2\right)c_{i0}}_{\text{PV of Future Consumption}} + \underbrace{\sum_{t=0}^{1}\beta^t \frac{c_{i0}}{c_{it}}\left(\nu_{it}\left(s_{it}-L_{it}\right)\mathbf{1}_{\{s_{it}>0\}} - PI_{it}\right)}_{\text{PV of Future Votes Less Price Impact Profits}}, \quad (20)$$

where PI_{it} are the profits from price impact at date t

$$PI_{it} = \frac{1}{nm_f}\boldsymbol{e}_1' J_{it}\begin{bmatrix} b_{it} \\ s_{it}-s_{it-1}\end{bmatrix} b_{it} + \frac{1}{nm_f}\boldsymbol{e}_2' J_{it}\begin{bmatrix} b_{it} \\ s_{it}-s_{it-1}\end{bmatrix} s_{it}. \quad (21)$$

Proposition 2 characterizes a strategic household's marginal value of control, ν_{it}, which is the value of marginally altering the fraction of firms that use the dirty technology, $G(\chi_t)$. It balances the discounted inframarginal value in utility terms of an increase in the use of the dirty technology, $\frac{\beta}{n}\Theta_{it}$, with the discounted pecuniary shadow cost of the marginal disutility from depletion of the social resource, $\frac{\beta}{n}\eta\Xi_{it}$. This value is scaled by $\frac{1}{n}$ to reflect that the size of each strategic household of type i, and also reflects a free-riding externality (e.g., Edmans and Manso [2011]): the more strategic households there are, the less each internalizes how it impacts the equilibrium and free-rides on other strategic households with the same trading motives when trading equity. Her value of control is related to the absolute value of this difference because the household changes the direction in which she votes depending on the sign. It is scaled by her state price deflator, β^t/c_{it}, to convert this marginal utility difference into a monetary value, and also modulated by the shape of the voting aggregation rule, $G(\chi_t)$. Because the marginal impact of control varies with the fraction that vote for the dirty technology, the incentives of strategic households to acquire additional votes is dampened in regions where the voting rule, $G(\chi_t)$ is flatter, i.e., $G'(\chi_t) < 1$, and amplified where it is steeper, i.e., $G'(\chi_t) > 1$.

Although a strategic household's marginal benefit to using the dirty technology (greater wealth) scales with her shares in firms, the marginal cost of additional resource depletion does not. In particular, if a S household's wealth has sufficient exposure to the firm's dividend, i.e., $\partial_{D_{t+1}} W_{it+1}$ is sufficiently large, then she votes for the dirty technology. Thus, there is a cutoff for share holdings of a S household, s_{it}, above which she will not vote for the clean technology because she is too exposed to firm dividends. In addition, because future consumption, c_{it+1}, is increasing in current wealth, W_{it}, wealthier S discounts the pecuniary benefit of the dirty technology more heavily and consequently have a higher marginal value of control for switching to the clean technology.

Now, suppose that strategic household i wishes to acquire additional equity shares of firms for voting rights, which is the only reason for trade in our model. To finance this purchase, she can either consume less (lower c_{it}) and/or issue debt ($b_{it} < 0$). Because a strategic household internalizes her price impact, however, she will take advantage of this fungibility between traded securities only imperfectly. A subtle feature of trading and voting with price impact is that a strategic household's marginal value of control, Θ_{it}, is based on her private valuations, i.e., $\beta c_{it}/c_{it+1}$, while the price impact of a marginal vote (encoded in J_{it}) is based on market valuations, i.e., q_t.

Finally, we derive the present value of wealth based on strategic household i's private valuation of its wealth in Equation (20), which has two uses. The first is to finance future consumption, its standard use. The second is to finance voting at firms, $\nu_{it}\left(s_{it} - L_{it}\right) \geq 0$, at dates 0 and 1, which is offset in part by rents from manipulating asset prices for price concessions, PI_{it}. The value of control is highest for households who have extreme discounted views, i.e., a large ν_{it}, over the net benefit (cost) of using the dirty technology.

2.4 Share-Lending Intermediary

A large intermediary chooses the lowest share lending fee, ϕ_t, such that there is no excess demand for share lending. The lending fee is determined by a pecking order: the competitive fringe, which does not value its votes, lends shares first, followed by those strategic households who value voting the least. As a result, the lending fee will equal the value of control for the marginal strategic household when the competitive fringe no longer holds any equity. This is summarized in Corollary 3.

Corollary 3. *If the competitive fringe has non-positive equity holdings, then the lending fee is*

$$\phi_t = \nu_{*t}, \tag{22}$$

for the marginal strategic household $*$ that lends; otherwise, it is zero.

This corollary has two implications. First, the strategic households who value their vote the least determine how voting is priced and lend shares first. These will be the S households if their disutility from resource depletion is in an intermediate range because their value of control (the benefit of dirty technology vs the cost of resource depletion) is smaller than the F household's value, which only includes the benefit. In contrast, when the S household's disutility is sufficiently high, the F households will determine how voting is priced. Second, firm equity only prices the value of control when the lending fee is nontrivial, in which case it equals the value of the marginal strategic lender.

2.5 Equilibrium

In what follows, we search for an equilibrium in which households trade the stocks of firms while voting on their production plans. This has historically been referred to as a stock market equilibrium (e.g., Dréze, 1974). However, because we have strategic households trading and voting in a dynamic economy, we must further specify a strategic equilibrium concept, to which we refer to as a Cournot-Walras Markov Perfect Equilibrium. In what follows, let $\mathbf{A_t}$ refer to the vector of all asset positions $\{b_i, s_i\}_{i\in F,S}$ of strategic agents.

Definition 4. *A Cournot-Walras Markov Perfect Equilibrium consists of a strategy σ_i for each strategic agent type, a strategy σ_f for the competitive fringe, and pricing functions $\tilde{r}_i(\mathbf{A})$ and $\tilde{p}_i(\mathbf{A})$ such that:*

1. *Household optimization: Given asset prices* $\{r, p\}$ *and perceived pricing functions* $\{\tilde{r}_i(\mathbf{A})\}$, $\tilde{p}_i(\mathbf{A})\}$, $\{c_i, b_i, s_i\}$ *solve decision problem* (13) *given other households' strategies and perceived pricing.*

2. *Market clearing in the consumption at date t requires*

$$c_{Ft} + c_{St} + m_f c_{ft} \leq D_t, \tag{23}$$

while in the securities markets

$$\begin{aligned} b_{Ft} + b_{St} + m_f b_{ft} &= 0, &(24)\\ s_{Ft} + s_{St} + m_f s_{ft} &= 1, &(25)\end{aligned}$$

and in the securities lending market

$$s_{Ft}\mathbf{1}_{\{s_{Ft}<0\}} + s_{St}\mathbf{1}_{\{s_{St}<0\}} + m_f s_{ft}\mathbf{1}_{\{s_{ft}<0\}} \leq L_{Ft} + L_{St} + m_f L_{ft}. \tag{26}$$

3. *Consistency: all agents have rational expectations, which requires for households* $i \in \{1, ..., N\}$ *that* $\tilde{r}_i(\mathbf{A}) = r(\mathbf{A})$ *and* $\tilde{p}_i(\mathbf{A}) = p(\mathbf{A})$ *for all* i.

The following proposition establishes existence of an equilibrium in our economy.

Proposition 5. *There exists a Cournot-Walras Markov Perfect equilibrium.*

In what follows, it will often be useful to contrast the Cournot-Walras equilibrium with a competitive benchmark economy.

Definition 6. *A Competitive Equilibrium is the Cournot-Walras equilibrium in the special case when* $n = \infty$.

Because there are no gains from trade when households behave competitively, the equilibrium is autarkic, with each household consuming an equal fraction of output y_t at each date. However, a S household continues to vote as it did as a large strategic household because of its deontological preferences. This is summarized in the Proposition 7.

Proposition 7. *The competitive equilibrium is autarky in which all households consume*

$$c_{it} = \frac{y_t}{2 + m_f}, \tag{27}$$

and S households vote according to the voting policy characterized in Proposition (2).

This benchmark reveals that there are no underlying sources of gains from trade in our model in the absence of strategic households who internalize the value of their vote when trading. However, if all strategic households were financially-motivated, then there would also be no trade because there is no value of control, and consequently no benefit to rationing traded quantities through internalizing price impact.

3 Dynamics of Socially Responsible Investment

In this section, we examine the dynamic portfolio choice problem faced by socially responsible investors. First, we discuss the static and dynamic trade-offs of socially responsible investment. Our key observation is that although wealth accumulation and exercising voice are intratemporal substitutes, because wealth buys voice they are intertemporal complements. We examine two special cases of our model to highlight this effect: one in which there are only S households and the competitive fringe, and one in which there are only S and F strategic households. For simplicity, we assume a linear voting aggregation rule, i.e., $G(\chi_t) = \chi_t$.[14] Finally, we discuss how the value of control arises from disagreement among households that cannot be remedied in financial markets.

[14]We provide numerical algorithms for both settings in Online Appendix B.

3.1 A Glidepath for Socially Responsible Investment

In this subsection, we discuss how a socially responsible household balances portfolio considerations with exercising voice over time. From Proposition 2, within a period, a socially responsible S household votes to balance two motives, the pecuniary benefit of dirty vs clean production, Θ_{St}, and the cost of depleting the social resource, $\eta\Xi_{St}$. These motives are in conflict because the value of control is based on the absolute value of their difference, $|\Theta_{St} - \eta\Xi_{St}|$. When $\Theta_{St} > \eta\Xi_{St}$, the S household votes for the dirty technology (achieving consensus). As her wealth increases and/or the resource depletes and Ξ_{St} rises, her marginal value of control falls as she becomes indifferent toward the technology choice, but then can rise enough that she both begins voting for the clean technology and acquires more shares (at a premium). In this sense, a S household intratemporally trades off wealth accumulation with exercising voice.

At the same time, from Equation (20), a socially responsible household's wealth from Equation (20) is equal to the present value of her future consumption and her exercise of control. She consequently finances future control by forfeiting consumption to investors from whom she buy shares in firms, mitigated by the price concessions she can extract from her price impact resulting from her, in effect, credible threat to lower the cash flows of the firm once she exercises some control. In this sense, a S household intertemporally accumulates wealth to exercise voice.

Our model suggests that socially responsible investors may forego reforming firms to first amass sufficient wealth to exercise control in the future when it is more valuable. Consistent with this view, Andersen et al. [2024] finds that expressing social responsibility in portfolio choice is akin to a luxury good for wealthy investors, while Edmans et al. [2024] provides survey evidence that even sustainable funds choose stocks, engagement, and voting primarily based on financial considerations. Further, Briere et al. [2024] documents that five universal investors historically supported climate resolutions less than similar fund families. Such behavior can be rationalized as these universal investors taking a long-term view on socially responsible investing. Consequently, our insights suggest that it is difficult to evaluate whether an investor or fund manager is acting in a socially responsible manner based on short-term voting behavior or portfolio composition. Such an evaluation requires a long-horizon window and a benchmark that accounts for the glidepath of socially responsible investment. Of course, this dynamic makes it challenging to separate behavior of long-term focused socially responsible investors from those who are effectively engaged in greenwashing and who actually lack socially focused preferences.

3.2 Only Socially Responsible Strategic Households

In this subsection, we consider a special case of our model in which there is only one type of strategic household, the socially responsible S household, and a competitive fringe of unit mass (i.e., $m_f = 1$). This example allows us to study how the S households behaves in the absence of strategic households with different preferences.

In what follows, we assume that $n > 1$, i.e., there are least 2 identical S households, and that the S household derives linear disutility from resource depletion only at date 2, i.e., $v_{S,1}(\epsilon_1) = 0$ and $v_{S,2}(\epsilon_2) = B\epsilon_2$. Since all strategic households have identical preferences, they will all vote for the same ideal mix between clean and dirty technology, thus effectively acting as a single agent who fully internalizes the externality when voting. Because there are only two types of households, there is no short-selling or share-lending, and equity positions are bounded from above by unity.

Proposition 8 characterizes the equilibrium into four regions depending on the magnitude of the S household's marginal utility to resource depletion, B.

Proposition 8. *The equilibrium can be divided into four regions:*

1. *If $B \leq \underline{B}$, where $\underline{B}$ is given by equation (OA.8), then the equilibrium is autarky and all firms use the dirty technology;*

2. *If $B \in [\underline{B}, B^*]$, where B^* is given by equation (OA.10), then the equilibrium is autarky, and the S household votes for the clean technology at date 1 such that a fraction $\chi_1(B)$ of firms use the dirty technology at date 2, where*

$$\chi_1(B) = \frac{1}{\eta B} - \frac{1}{z-1};$$

3. *For $B \in (B^*, \bar{B})$, where $\bar{B}$ is given by equation (OA.26), household S accumulates shares of equity and votes increasingly for the clean technology as B increases;*

4. *For $B \geq \bar{B}$, the S household purchases all firm equity at date 0 and votes for all clean technology at both dates.*

From Proposition 8, if a S household's disutility from resource depletion, parameterized by B, is sufficiently small, then a S household prefers to use the dirty technology and the equilibrium is autarky because there are no gains from trade. As B increases past a threshold, $\underline{B}$, a S household first begins to vote her date 1 shares increasingly for the clean technology. This is because she has higher disutility from depletion of the resource at date 1, and it is cheaper to change the vote on existing shares than to buy more shares. This activity continues until all her date 1 shares are converted, at which point a further increase in B forces her to buy more shares and/or convert her votes on her date 0 equity holdings. Finally, if B is so high that it exceeds a threshold, $\bar{B}$, then S households buy all shares and all firms operate the clean technology at both dates.

This special case reveals how a socially responsible investor would facilitate a transition to clean technology when she effectively internalizes all externalities when voting– there is no disagreement among S households on the ideal mix of clean and dirty technology. This scenario represents an upper bound on the willingness of S households to complete the transition. Because portfolio returns compound and disutility over resource depletion increases over time, the S household has incentive to delay mitigating production externalities at earlier dates because it entails sacrificing portfolio returns.

3.3 Strategic Limit with Duopolistic Households

In this subsection, we consider a special case of our model in which there is no competitive fringe and only the two types of strategic households. To pin down the price impact functional, we consider the limit in which all agents become arbitrarily small, i.e., $n \to \infty$ and $m_f \to 0$, but price impact remains nontrivial because $\frac{1}{nm_f} \to \kappa \in [0,1]$.[15] We further assume strategic households internalize that they can coordinate with a mass of κ households of their type so that voting remains valuable even with atomistic households.

Proposition 9 characterizes the equilibrium into four regions based on the magnitude of the S household's marginal utility to resource depletion, B. For comparability to Proposition 8, we set $\kappa = \frac{1}{n^*}$, where n^* is the value of n in the case with the competitive fringe.

Proposition 9. *The equilibrium can be divided into four regions:*

1. *If $B \leq \underline{B}$, where $\underline{B}$ is given by equation (OA.8), then the equilibrium is autarky and all firms use the dirty technology;*

2. *If $B_2 \in [\underline{B}, B^s]$, where B^s is given by equation (OA.29), then the F household acquires shares of equity from the S household to delay the transition to clean technology;*

3. *For $B \in \{B^s, \bar{B}_s\}$, where $\bar{B}_s$ is given by equation (OA.40), household S accumulates shares of equity and votes increasingly for the clean technology;*

4. *For $B \geq \bar{B}_s$, the S household purchases all firm equity at date 0 and votes for all clean technology at both dates. Compared to the case in Proposition 8, the S household consumes less and delays the transition to all clean technology to a higher $\bar{B}_s$.*

Proposition 9 shows that the S household prefers to use the dirty technology if the S household's disutility from resource depletion, parameterized by B, is sufficiently small, with the equilibrium being autarky. This result aligns with the case above with only the

[15] In this limit, the competitive fringe continues to prices the underlying state prices at each date, and we can use asset prices to recover their implied holdings of each asset.

strategic S household and a competitive fringe. In contrast to that case, the presence of a strategic F household will delay the clean transition; for intermediate values of B, such a household has a higher marginal value of control compared to an S household whose value balances both the profit and resource-preservation motives. In addition, because an F household prices equity based on his marginal valuation, it is more expensive for a S household to acquire shares of equity. As a consequence, an S household consumes less and delays the transition to clean technology to a higher critical value of B, $\bar{B}_s \geq \bar{B}$.

Our analysis with two strategic households highlights a limitation of using wealth to transition firms to using clean technologies. When there is strategic competition among large investors, the transition to clean technology is delayed compared to the case of a monopolist S household because financially-motivated households may value voting rights more on the margin and either hold out or buy firm equity to prevent the transition.

3.4 Value of Control

Although households can smooth consumption across dates in financial markets, control has value because they cannot insure against production externalities. This is because of the additive separability of the marginal utility of consumption from that of resource depletion. As a result, they can disagree about which technology firms should adopt.

If preferences were not additively-separable, such as in models with Constant Absolute Risk Aversion preferences in which they are log-additive (e.g., Pastor et al. [2021]), then households could insure monetarily against depletion through precautionary savings (or hedging demand with risk). In this case, production decisions would be unanimous and socially responsible households would consume more as the resource depleted to lower their marginal utility of consumption and align it with that of the financially-motivated and fringe households. A similar phenomenon would occur if resource depletion had a pecuniary impact on wealth rather than a non-pecuniary impact on utility.

4 Optimal Voting Protocols

In this section, we consider how a planner would address production externalities in the economy and aggregate the dispersed preferences of households. To begin our analysis, we first construct a first-best equilibrium benchmark for how a planner would choose consumption and production outcomes without any constraints on its decision-making. The Cournot-Walras Markov Perfect equilibrium may be inefficient for three reasons. First, as in Kyle [1989], strategic households voluntarily do not realize all gains from trade. As a result, state prices remain dispersed across households whereas with perfect competition they would be fully aligned. Second, the objective of firms is determined by voting among households who own and vote with their shares. This not only mutes the views of households who short the stock, but also aggregates preferences that are voluntarily misaligned because of price impact in financial markets. Third, strategic households vote on production externalities based on private rather than social marginal benefit, which is the key source of inefficiency we wish to highlight.

A subtle issue with characterizing the First-Best equilibrium is that when households vote on production externalities, initial wealth is not sufficient for recovering appropriate Pareto weights to decentralize the allocation. We consequently appeal to a refinement in which we impose that the consumption paths of all agents must be financed by their initial wealth (when evaluated at the shadow prices of reallocating resources across time). We refer to this as a First-Best Equilibrium without Initial Transfers.

Definition 10. *A First-Best Equilibrium without Initial Transfers is a Pareto efficient equilibrium in which the planner assigns allocations that are based on the initial wealth of all agents.*

We then have the following proposition that characterizes the First Best Equilibrium without Initial Transfers with these specially chosen (but intuitive) Pareto weights.

Proposition 11. *In the First Best Equilibrium without Initial Transfers:*

- *optimal household consumption is the same for all households and satisfies*

$$c_{it} = \frac{y_t}{2+m_f} \;\forall\; i = F, S \tag{28}$$

- *there exists an unique optimal share of firms, X_t, that employ the dirty technology. At an interior solution, this X_t satisfies*

$$\frac{1}{2+m_f}\left(\frac{\beta c_{Ft}}{c_{Ft+1}} + \frac{\beta c_{St}}{c_{St+1}} + m_f \frac{\beta c_{ft}}{c_{ft+1}}\right)(z-1) = \frac{\beta\eta\Xi_{St}}{1/c_{St}}, \tag{29}$$

  and is (weakly) decreasing over time. Because all households have $\frac{1}{2+m_f}$ shares of firm equity, this economy is equivalent to one in which control rights are allocated based on date 0 *initial shareholdings modified by the pecuniary disutility of the S household.*

As is standard, the planner in this First-Best equilibrium fully aligns the marginal utilities of consumption across agents who all have log utility. The planner faces a trade-off when depleting the social resource. If marginally more firms use the dirty technology, this relaxes the consumption resource constraint by increasing next period's output. However, this comes at the cost of increased marginal lifetime disutility among the S strategic households from incrementally depleting the social resource, the present monetary value of which is $\frac{\beta\eta\Xi_{St}}{1/c_{St}}$.

This benchmark highlights the fundamental coordination failure among households. While the Planner weighs social costs and benefits when depleting the resource, households vote based on private trade-offs and do not internalize their collective impact on production externalities. In addition, the planner weighs the marginal willingness of the S household to pay to mitigate resource depletion equally against that of all households to use the more productive, dirty technology according to their Pareto weights, which are time-invariant. Interestingly, this technology rule is equivalent to one in which control rights are determined based on initial shareholdings, which is the dynamically consistent

Grossman and Hart [1979] criterion modified by how much the S household is marginally willing to pay to mitigate resource depletion.

We now consider a constrained efficient rule for aggregating shareholder preferences. Suppose a planner can choose a voting rule $G^*(\cdot) : [0,1] \to [0,1]$ that maps the fraction of votes in favor of the dirty technology at firms, χ_t, into the fraction of firms that ultimately employ it, $G^*(\chi_t)$. We again restrict attention to aggregation rules, $G^*(\chi_t)$, that are $\mathcal{C}^1([0,1])$-differentiable, (weakly) increasing, and respect unanimity, i.e., $G^*(0) = 0$ and $G^*(1) = 1$. As is standard, we also allow for transfers at dates 0 and 1 across households to compensate for less desirable production outcomes.

Proposition 12 characterizes voting rules that are constrained efficient.

Proposition 12. *Define the social stochastic discount factor (SDF) for firms, Λ^*_{t+1}*

$$\Lambda^*_{t+1} = \frac{c_{Ft}}{c_{Ft+1}} s_{Ft} + \frac{c_{St}}{c_{St+1}} s_{St} + m_f \frac{c_{ft}}{c_{ft+1}} s_{ft}. \tag{30}$$

The constrained efficient technology choice is that the fraction of firms that use the dirty technology at an interior solution satisfies

$$\Lambda^*_{t+1}\left(1 + \partial_{D_{t+1}} p_{t+1}\right)(z-1) = \frac{\eta \Xi_{St}}{1/c_{St}}, \tag{31}$$

where $\partial_{D_{t+1}} p_{t+1}$ is given by equation (A.78). This constrained efficient choice can be implemented with a differentiable voting rule, $G^*(\chi_t)$, that satisfies

$$\left(\Lambda^*_{t+1}\left(1 + \partial_{D_{t+1}} p_{t+1}\right)(z-1) - c_{St}\eta \Xi_{St}\right) G^{*\prime}(\chi_t) = 0, \tag{32}$$

where $G'(\chi_t) = 0$ only when the argument in parentheses is positive at $G^*(\chi_t) = 1$ or negative at $G^*(\chi_t) = 0$. Compared to a linear rule $G(\chi_t) = \chi_t$, there is a χ^*_t such that $G^*(\chi_t) > \chi_t$ for $\chi > \chi^*_t$, and $G^*(\chi_t) \leq \chi_t$ otherwise.

Proposition 12 reveals that the Planner balances the average marginal pecuniary benefit to households from increasing the use of the dirty technology by firms (the left-hand side of equation 31) with the cost of depleting the resource for the S household. The marginal benefit is the sum of the marginal increase in the asset price and dividend multiplied by the average SDF across households weighted by their equity holdings. The marginal cost is the S household's shadow cost of depleting the resource, deflated into units of the numeraire (the right-hand side). In contrast to the social stochastic discount factor, the social marginal resource cost is not weighted by shareholdings because a household cannot choose its exposure to the production externality.

Importantly, the constrained efficient rule in equation (31) suggests a natural separation of cash flow and production externality rights. Cash flow votes are weighted according to shareholdings because each additional share exposes a household to a share more of the firm's dividends. By contrast, a household's shareholdings do not affect how she is impacted by the firm's production externalities. Indeed, the optimal rule calls for an equal weighting of households' disutilities from depleting the social resource, which is akin to a government that decides policies based on a utilitarian welfare criterion. Because the socially responsible household does not internalize the benefit of dirty production to the fringe and the F agent, the optimal rule downweights a socially responsible household's votes when concerns about the resource are mild. In contrast, it overweights her votes when her concerns are severe because, when this occurs, the F household blocks the clean transition by extracting rents in financial markets for firm shares.

Although equation (31) is reminiscent of the constrained efficient rule with multiple goods of Geanakoplos et al. [1990], there are several key differences. In Geanakoplos et al. [1990], household SDFs are dispersed because markets are incomplete and the planner modifies the Dréze [1974] rule to take into account that production alters goods spot prices, which affects households' wealth. In our setting, there is dispersion in SDFs even though markets are complete because of price impact. In addition, the planner takes into account how production depletes a social resource that impacts household utility. Consequently, the rule we have derived is useful in valuing non-pecuniary externalities.[16]

These insights generalize in a straightforward manner to the case in which the planner may care about depletion of the social resource for reasons beyond the S household's disutility. For instance, the planner may put weight on preservation of other inhabitants of the economy who cannot vote, such as wildlife and plant life. In this case, the social cost in Equation (32) would include the dollar value of these additional concerns.

5 Extensions

In this section, we explore several extensions of our framework. First, we introduce aggregate risk to discuss asset pricing implications of our model. Second, we modify the technology of firms so that they must acquire a flexible factor of production from a centralized rental market. This will allow us to highlight an externality that the transition to clean technology imposes on firms through the factor's rental rate. Finally, we discuss the limitations of universal ownership when the supply of dirty firms is elastic.

5.1 Asset Pricing Implications

In this subsection, we extend our framework to include aggregate risk to explore asset pricing implications.[17] Suppose production now entails correlated risk across firms. Our key observation based on Proposition 1 and Corollary 3 is that voting because of social concerns gives rise to risk premia when it is sufficiently contentious among strategic households in the future. If the competitive fringe owns equity, there is no premium in stock prices, i.e., $\phi_t = 0$, because they are determined solely by firms' cash flows–otherwise the fringe would have sold its shares. However, when $\phi_t > 0$, the equity price includes the minimum private value of control among strategic households.

[16] Such insights also contrast the one-stakeholder, one-vote approach of Crès and Tvede [2013], in which each agent that has a stake in firm production through cash flow rights and/or externalities gets one vote.

[17] There is an empirical literature that studies asset pricing implications of socially responsible investing (e.g., Luo and Balvers, 2017, Bolton and Kacperczyk, 2022). See Liang and Renneboog [2020] for a review.

In a static model, this implies an upward bias in stock prices (e.g., Zingales, 1995), but does not affect expected returns because the share lending fee is paid by the short-seller and recouped by the share lender. This is akin to date 1 in our model. However, in our dynamic setting, uncertainty over future share-lending fees introduces a source of systematic risk that the competitive fringe prices at date 0. We can then express the expected excess return to firm equity at date 0 as

$$\mathbb{E}\left[\frac{D_1 + r_0\phi_0 + p_1}{p_0}\right] - r_0 = \underbrace{-Cov\left(\frac{q_0(z_1)}{\mathbb{E}[q_0(z_1)]}, \frac{D_1 + \sum_{z_2 \in \{z_L, z_H\}} q_1(z_2) D_2}{p_0}\right)}_{\text{Standard Cash Flow Risk Premium}} + \underbrace{-Cov\left(\frac{q_0(z_1)}{\mathbb{E}[q_0(z_1)]}, \frac{\phi_1}{p_0}\right)}_{\text{Voting Rights Risk Premium}}. \tag{33}$$

The first term on the right-hand side of equation (33) is the standard risk premium, which arises because the cash flows of firms correlate with the competitive fringe's marginal utility of consumption. The second term, which is novel to our setting, is a risk premium that arises because the premium in the stock price when control has value also correlates with the fringe's marginal utility. This additional risk premium is absent when there is sufficient consensus among households over future production decisions. Without this consensus, the realizations of the productivity of the two technologies influence future production decisions, which is the source of risk that must be priced.

Our analysis suggests the additional risk premia that arises from socially responsible concerns are inherently time-varying, appearing in stocks once socially responsible concerns become severe enough. The sign of the risk premium also depends on how the value of control correlates with the state of the economy. This distinguishes our mechanism from that in Pastor et al. [2021] in which green stocks earn lower returns because arbitrageurs cannot correct the mispricing ESG preferences induce because of limited risk-bearing capacity. Here, the risk premium arises because strategic households would not lend their shares for free when control has value.[18]

5.2 Externalities in Rental Markets

In this subsection, we explore an externality by which a firm's choice of production technology affects its factor input demand through its impact on the equilibrium rental rate. We now assume firm j produces according to the decreasing-returns-to-scale technology

$$y_{jt+1} = \left(1 + (z-1)\,K_{jt}\right) f_{jt}^{\alpha}, \tag{34}$$

where f_{jt} is a flexible factor of production, such as rented capital or labor. Firms rent this factor of production from households, which in aggregate is in unit fixed supply, at an equilibrium rental rate R_t set by market clearing in the rental market. For simplicity, we follow Broccardo et al. [2022] and assume shareholders vote either to adopt the dirty technology or not, but not on the level of dirty or clean factor demand.

In this situation, a firm's factor demand, f_{jt}, satisfies the first-order condition

$$\alpha \left(1 + (z-1)\,K_{jt}\right) f_{jt}^{\alpha-1} \quad = \quad R_t, \tag{35}$$

Market clearing in the factor rental market then imposes an equilibrium wage

$$R_t = \alpha \left(1 + G\left(\chi_t\right)\left(z^{\frac{1}{1-\alpha}} - 1\right)\right)^{1-\alpha}, \tag{36}$$

which is increasing in $G\left(\chi_t\right)$ because $z > 1$.

Because the clean technology is less productive than dirty, clean firms employ less of the factor for the same equilibrium rental rate because they have a lower marginal

[18] It is also distinct from Pedersen et al. [2021] who derives an ESG-adjusted CAPM based on ESG scores, Baker et al. [2022] who argues that brown firms may have lower expected returns if they hedge against climate risk, Goldstein et al. [2021] who shows that disjoint preferences of investors over firm behavior can generate instability in asset prices when prices aggregate their private information, and Jagannathan et al. [2022] who highlights how realized returns can differ depending on the success of the clean transition.

product. This lower factor demand reduces the rental rate, which raises the optimal factor demand of the dirty firms. Through this channel, a transition toward clean technologies acts as a subsidy for remaining dirty firms, which makes voting for the dirty technology more valuable and undermines the transition to the clean technology.

5.3 Supply Response and Limits to Universal Ownership

One proposed panacea to global societal challenges is universal ownership, in which investors hold well-diversified portfolios that effectively make them residual claimants to the entire economy. As a result, their fortunes are intertwined with economic performance, and they should approach corporate governance with an interest in mitigating societal harms caused by production externalities. By internalizing societal harms, shareholders can successfully address them through corporate governance.

Our model, which takes into account wealth considerations, raises an important issue with this solution. If the supply of firms is elastic instead of fixed, which it is in practice, then universal owners with limited financial resources will have to effectively engage in a whack-a-mole game to acquire entrants that use dirty technologies. Because entrepreneurs cannot commit to only create new clean firms, there is always the threat of resource depletion in the future. When control has value, even if it costs entrepreneurs the present-value of all future cash flows to start a dirty firm, they can sell it at a premium to socially responsible households who have a higher valuation for the control rights.

5.4 Extensive Margin of Voice

Although we focus on the intensive margin of SRI voice through wealth accumulation, recent financial innovations, such as the creation of SRI funds, make it easier for retail and institutional investors to express socially responsible views through delegated portfolio management. In addition, efforts that raise awareness of societal issues like climate change and ethical supply chains may lead to more support for SRI initiatives. Both approaches ultimately aim to broaden the base of socially responsible investors.

Our analysis suggests that the commonality between increasing voice in the time-series (intensive margin) and in the cross-section (extensive margin) is that both shore up the capital deployed to reforming corporate governance. Regardless of which approach SRI activists take to garner control, our work suggests that limited wealth remains a key impediment to their success.

6 Conclusion

We present a dynamic model of strategic trading and production when households can vote based on their shareholdings to mitigate the depletion of a scarce social resource. Because an investor's voice is inextricably linked to her wealth, she must distort her portfolio to acquire more shares in the future if she values changing which technology is used. As a result, accumulating wealth is an intratemporal substitute but an intertemporal complement to exercising voice. We further show a constrained optimal voting protocol separates voting over production externalities from shareholder wealth because the incidence of the externality is not related to it. This voting rule is potentially applicable in other settings, such as aggregating investor preferences within asset management funds.

Perhaps surprising, both socially responsible and financially-motivated investors may prefer that social policy be implemented through socially responsible investing. Once investors who care about the social resource begin to acquire shares to exercise voice, large investors unconcerned with social outcomes can benefit as they obtain a premium for their shares. However, smaller retail investors cannot collectively hold-out on selling to extract a similar premium, and production decisions may not reflect their preferences. In such a world, exercising voice through wealth accumulation need not achieve an efficient outcome compared to fiscal policy with taxation and regulation.

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Appendix

Proof of Proposition 1:

In this proof, we solve for the T-period optimal solution to the household's problem, which will later coincide with the infinite-horizon problem if we appropriately adjust the transversality conditions.

Our goal is to first express the primal problem of the competitive fringe 13 as a Lagrangian in which the constraints are the household's budget constraint (2) and the law of motion of her wealth (3). This is a saddle-point problem and its solution solves the primal problem according to the Saddle Point Theorem, and the optimal values to the household coincide under the Weak Duality Theorem.

We first start with a finite horizon T (which is 2 in the three-period model), which will later allow us to take the limit as $T \to \infty$ to arrive at the appropriate transversality conditions to move to infinite-horizon. We then apply summation-by-parts to the constraint on wealth to arrive at a Hamiltonian with a law of motion for the co-state for the fringe's wealth Λ_{ft}. Finally, we apply the Minimax Theorem to first solve for the optimal choices of consumption and investment to transform the saddle-point problem into a convex dual problem.

Step 1: The Saddle-Point Problem and Hamiltonian

We first combine equations (2) and (3) by substituting for b_{ft}. Thus, we arrive at the law of motion of wealth at dates 1 to 2:

$$\begin{aligned} W_{ft+1} \quad \leq \quad & r_t \left(W_{ft} + \phi_t L_{ft} - c_{ft}\right) + D_{t+1} + p_{t+1} \\ & + r_t \left(\phi_t \mathbf{1}_{\{s_{ft}<0\}} - p_t\right) s_{ft}. \end{aligned} \tag{A.1}$$

This expression is the law of motion for the fringe's wealth that its optimization program must respect.

Let $\beta^t \Lambda_{ft}$ be the Lagrange multiplier on the law of motion of household f's wealth (A.1) and $\beta^t \vartheta_{ft}$ the multiplier on the lending constraint $L_{ft} \leq s_{ft}$ when $s_{ft} > 0$ (i.e., the fringe cannot lend shares it does not own). It follows we can express the primal problem of household f 13 using the Lagrangian as a saddle-point problem for a finite time T

$$\begin{aligned}
\mathcal{L}_{f0} \quad = \quad & \sup_{\{c_f s_f, x_f, L_f, W_f\}} \inf_{\Lambda_f} \sum_{t=0}^{T} \beta^t \left(\log c_{ft} - \Lambda_{ft+1} r_t c_{ft} + \beta \Lambda_{ft+1} r_t \phi_t L_{ft} \right) \\
& + \sum_{t=0}^{T} \beta^{t+1} \Lambda_{ft+1} \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{ft}<0\}} - p_t \right) \right) s_{ft} \\
& - \sum_{t=0}^{T} \beta^{t+1} \Lambda_{ft+1} \left(W_{ft+1} - r_t W_{ft} \right) - \sum_{t=0}^{T} \beta^t \vartheta_t \left(L_{ft} - s_{ft} \mathbf{1}_{\{s_{ft} \geq 0\}} \right),
\end{aligned} \tag{A.2}$$

and we can recover a_{ft} from the budget constraint (2).

By summation-by-parts,

$$\begin{aligned}
\sum_{t=0}^{T} \beta^{t+1} \Lambda_{ft+1} \left(W_{ft+1} - r_t W_{ft} \right) \quad = \quad & \beta^{T+1} \Lambda_{fT+1} W_{fT+1} - \Lambda_{f0} W_{f0} \\
& - \sum_{t=0}^{T} \beta^t W_{ft} \left(\beta \Lambda_{ft+1} r_t - \Lambda_{ft} \right),
\end{aligned} \tag{A.3}$$

and the (A.2) becomes the convex dual problem

$$\begin{aligned}
\mathcal{L}_{f0} \quad = \quad & \sup_{\{c_f, s_{fj}, x_{fj}, L_{fj}, W_f\}} \inf_{\Lambda_f} \sum_{t=0}^{T} \beta^t \left(\log c_{ft} - \beta \Lambda_{ft+1} r_t c_{ft} + \beta \Lambda_{ft+1} r_t \phi_t L_{ft} \right). \\
& + \sum_{t=0}^{T} \beta^{t+1} \Lambda_{ft+1} \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{ft}<0\}} - p_t \right) \right) s_{ft} \\
& - \beta^T \Lambda_{fT} W_{fT+1} + \Lambda_{f0} W_{f0} + \sum_{t=0}^{T} \beta^t W_{ft} \left(\beta \Lambda_{ft+1} r_t - \Lambda_{ft} \right) \\
& - \sum_{t=0}^{T} \beta^t \vartheta_t \left(L_{ft} - s_{ft} \mathbf{1}_{\{s_{ft} \geq 0\}} \right)
\end{aligned} \tag{A.4}$$

Complementary slackness for the state variable W_{ft} imposes

$$\frac{1}{r_t} = \beta \frac{\Lambda_{ft+1}}{\Lambda_{ft}}, \tag{A.5}$$

otherwise the household could achieve arbitrarily high value by making W_{ft} arbitrarily positive or negative. For share lending L_{ft}

$$\vartheta_t = \mathbb{E}_t \left[\beta \Lambda_{ft+1}\right] r_t \phi_t = \Lambda_{ft} \phi_t, \tag{A.6}$$

and for asset positions s_{ft}

$$\begin{aligned} p_t &= \beta \frac{\Lambda_{ft+1}}{\Lambda_{ft}} \left(D_{t+1} + p_{t+1}\right) + \phi_t \mathbf{1}_{\{s_{ft}<0\}} + \frac{1}{\Lambda_{ft}} \vartheta_t \mathbf{1}_{\{s_{ft}\geq 0\}} \\ &= \beta \frac{\Lambda_{ft+1}}{\Lambda_{ft}} \left(D_{t+1} + p_{t+1}\right) + \phi_t, \end{aligned} \tag{A.7}$$

by similar arguments that otherwise the household could achieve unbounded value.

We can also define the fringe's state price at date t referencing consumption at date $t+1$ as

$$q_t = \beta \frac{\Lambda_{ft+1}}{\Lambda_{ft}}. \tag{A.8}$$

Complementary slackness for the state variable W_{fT+1} from (A.2) imposes the boundary condition

$$\beta^{T+1} \Lambda_{fT+1} W_{fT+1} = 0, \tag{A.9}$$

which we satisfy by imposing

$$\Lambda_{fT+1} = 0, \tag{A.10}$$

so that no time $T+1$ objects are valued after the game ends.

Step 2: Optimal Consumption and Optimally Invested Wealth

Assuming the Minimax Theorem is valid, we can switch the order of the sup and inf operators to transform the Hamiltonian problem (A.4) into a convex dual problem. Maximizing over consumption, we derive the first-order necessary condition from the decision problem (A.4) for c_{ft}

$$\Lambda_{ft} = 1/c_{ft}. \tag{A.11}$$

Consider now the law of motion of the household's wealth (A.1). Multiplying it by Λ_{ft+1} and substituting with equations (A.5) and (A.17), we find that

$$\Lambda_{ft+1} W_{ft+1} = \Lambda_{ft} \frac{1}{r_t} b_{ft} + \Lambda_{ft} p_t s_{ft} = \Lambda_{ft} \left(W_{ft} - c_{ft} \right), \tag{A.12}$$

from which follows by iterating forward, imposing the transversality condition (A.25), and substituting with (2) and for Λ_{ft} with equation (A.11)

$$W_0 = \sum_{t=0}^{T} \beta^t \frac{\Lambda_{ft}}{\Lambda_{f0}} c_{ft} = \frac{1-\beta^{T+1}}{1-\beta} c_0. \tag{A.13}$$

Notice for the consumption bundle of the competitive fringe to be marketed, it must satisfy the present-value budget constraint (A.13) at each date, from which it follows that

$$c_{ft} = \frac{1-\beta}{1-\beta^{T+1-t}} W_{ft} = h(t) W_{ft}, \tag{A.14}$$

where $h(t) = \frac{1-\beta}{1-\beta^{T+1-t}}$.

Step 3: Asset Prices and Price Impact

From equations (A.11) and (A.14), $\Lambda_{ft} = \frac{1}{h(t) W_{ft}}$ and Arrow asset prices from equations (A.5)

$$\frac{1}{r_t} = \beta \frac{h(t)}{h(t+1)} \frac{W_{ft}}{W_{ft+1}}, \tag{A.15}$$

and from (A.8), state prices are given by

$$q_t = \beta \frac{h(t)}{h(t+1)} \frac{W_{ft}}{W_{ft+1}}. \tag{A.16}$$

Consequently, equation (A.17) reduces to

$$p_t = q_t \left(D_{t+1} + p_{t+1}\right) + \phi_t. \tag{A.17}$$

Because household f pins down asset prices from equations (A.15), (A.16), and (A.17), and household f's wealth at date $t+1$ satisfies the budget constraint (3), we can derive price impact in each asset market from the competitive fringe's marginal utility. Imposing market-clearing conditions (24) and (25), it is useful to define the impact of a strategic household demanding more of a claim to debt

$$\partial_{b_i} q_t = \beta \frac{h(t)}{h(t+1)} \frac{W_{ft}}{W_{ft+1}^2} = \frac{q_t}{W_{ft+1}}, \tag{A.18}$$

and stack this into the 2×2 diagonal matrix, Γ_t, with diagonal entries $\Gamma_{11} = \partial_{a_i} q_t$and $\Gamma_{22} = \partial_{a_i} q_t$. Iterating forward on equation (A.17), notice that

$$p_t = \sum_{\tau=t+1}^{T} \prod_{s=t}^{\tau-1} q_s D_{j\tau} = q_t D_{t+1} + \sum_{\tau=t+2}^{T} \prod_{s=t+1}^{\tau-1} q_s D_{j\tau}, \tag{A.19}$$

and from equation (A.15)

$$\frac{1}{r_t} = q_t. \tag{A.20}$$

Consequently, we need only consider the price impact of a strategic household's trading on the present-discounted value of the next period's dividend.

We then define the 2×2 price impact matrix

$$J_{it} = \begin{bmatrix} \partial_{b_i}\left(\frac{1}{r_t}\right) & \partial_{s_i}\left(\frac{1}{r_t}\right) \\ \partial_{b_i} p_t & \partial_{s_i} p_t \end{bmatrix} + \begin{bmatrix} 0 & h_{t,12} G'\left(\chi_t\right) \partial_{s_{it}} \chi_t \\ 0 & h_{t,22} G'\left(\chi_t\right) \partial_{s_{it}} \chi_t \end{bmatrix} = \frac{q_t}{W_{ft+1}} \begin{bmatrix} 1 \\ D_{t+1} + \phi_{t+1} \end{bmatrix} \begin{bmatrix} 1 \\ D_{t+1} + p_{t+1} \end{bmatrix}' + H_{it}, \tag{A.21}$$

and we recognize that strategic households do not internalize that they can affect the equity lending fee ϕ_t because it is set by the large lending intermediary. The second matrix modifies the diagonal entry $\partial_{s_i} p_t$ because a marginal purchase of firm equity impacts the technological choice of firms. This is captured by the additional diagonal term

$$\begin{aligned} h_{it,22} &= (z-1)\, \partial_{D_{t+1}} \left(q_t \left(D_{t+1} + p_{t+1}\right)\right) \\ &= (z-1)\, q_t \left(1 - \left(D_{t+1} + \phi_{t+1}\right) \frac{\partial_{D_{t+1}} W_{ft+1}}{W_{ft+1}}\right), \end{aligned}$$

where

$$\begin{aligned} \partial_{D_{t+1}} W_{ft+1} =& s_{ft} + \left(p_{t+1} - \phi_{t+1}\right) s_{ft} \frac{\partial_{D_{t+1}} W_{ft+1}}{W_{ft+1}} \\ =& \frac{W_{ft+1} s_{ft}}{b_{ft} + \left(D_{t+1} + \phi_{t+1}\right) s_{ft}}, \end{aligned} \tag{A.22}$$

and which together imply

$$h_{it,22} = (z-1)\, q_t \frac{b_{ft}}{b_{ft} + \left(D_{t+1} + \phi_{t+1}\right) s_{ft}} \geq 0, \tag{A.23}$$

because the denominator must be positive in equilibrium.

We can then further recover

$$h_{it,12} = -\,(z-1)\, q_t \frac{s_{ft}}{b_{ft} + \left(D_{t+1} + \phi_{t+1}\right) s_{ft}} G'\left(\chi_t\right) < 0.$$

Because $\partial_{s_{St}} \chi_t < 0$ and $\partial_{s_{Ft}} \chi_t > 0$ when voting has value, price impact from a change in the marginal vote must be positive for the F household and negative for the S household. The effect is the same for the price of the risk-free bond ($q_t = \frac{1}{r_t}$).

By the Saddle-Point Theorem, if $\{c_f, b_f, s_f\}$ solves the saddle-point problem A.2, then it also solves the original primal problem 4. Less obvious is that if $\{c_f, b_f, s_f\}$ solves the convex dual problem A.4, then it also solves the saddle-point problem A.2. See, for instance, Marcet and Marimon [2019].

Step 4: Optimal Voting

Because a competitive household prices firm equity according to equation (A.17), she aims to maximize the excess return on her portfolio. Consequently, she always votes for the dirty technology when she is long equity, i.e.,

$$x_{ft} = s_{ft} - L_{ft}. \tag{A.24}$$

Because competitive households view themselves as atomistic, they will never assign value to their vote, and will be the first to lend their shares.

Step 5: Linear Homogeneity of Investment in Wealth

Further, we now recognize now from equation (3) thatW_{ft+1} is linear in the fringe's asset positions from its perspective because it behaves competitively. Consequently, let us conjecture that its optimal asset positions a_{ft}, b_{ft}, s_{ft} are linear in the fringe's current wealth, W_{ft}, at dates 0 and 1, i.e., $a_{ft} = \hat{a}_{ft} W_{ft}$ and $s_{ft} = \hat{s}_{ft} W_{ft}$. We then have from equations (A.15), (A.16), and (A.17) that

$$\frac{1}{r_t} = \beta \frac{h(t)}{h(t+1)} \frac{1}{\hat{b}_{ft} + (D_{t+1} + p_{t+1})\, \hat{s}_{ft}},$$

and

$$p_t = \beta \frac{h(t)}{h(t+1)} \frac{D_{t+1} + p_{t+1}}{\hat{b}_{ft} + (D_{t+1} + p_{t+1})\, \hat{s}_{ft}} + \phi_t,$$

which confirms the conjecture. Consequently, we can instead solve for $\hat{b}_{ft}, \hat{s}_{ft}$ when finding prices and the fringe's optimal policies.

Step 6: Moving to Infinite-Horizon

Notice our characterizations of the optimal policies of competitive households in Proposition 1 remain valid as $T \to \infty$, provided we take the limits as $T \to \infty$ for the finite-horizon policies and adjust the boundary condition for $\beta^T \Lambda_{fT} W_{fT}$ to a transversality condition. Taking the limit of (A.9) as $T \to \infty$ in the infinite-horizon case, we arrive at the asymptotic transversality condition

$$\lim_{T \to \infty} \beta^T \Lambda_{fT} W_{fT} = 0. \tag{A.25}$$

With this, the solution to the competitive household's problem remains valid for the infinite-horizon problem.

Proof of Proposition 2:

We follow a similar approach to how we solved the competitive fringe's problem in Proposition 1. We first start with a finite T-horizon version of the problem, and then take the limit to infinite horizon. This nests both the $T = 2$ economy and the infinite-horizon economies in one approach.

Step 1: The Saddle-Point Problem and Hamiltonian

We first combine equations (11) and (12) to write the law of motion of strategic household i's wealth as

$$W_{it+1} \quad \leq \quad r_t \left(\phi_t L_{it} + W_{it} - c_{it}\right) + \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{it}<0\}} - p_t\right)\right) s_{it}, \tag{A.26}$$

which will be the law of motion we force strategic household i's optimization to respect.

Let $\beta^t \Lambda_{it}$ be the Lagrange multiplier on the law of motion of household i's wealth A.26, $\beta^t \zeta_{it}$ household i's multiplier on the law of motion of the social resource constraint 10, and and $\beta^t \vartheta_{it}$ the multiplier on the lending constraint $L_{it} \leq s_{it}$ when $s_{it} > 0$ (i.e., the household cannot lend shares it does not own). Let d_{it} be the number of dirty technology votes and e_{it} the number of clean technology votes, such that $x_{it} = d_{it}$. Finally, $\beta^t \xi_{it}$ is the multiplier on the voting constraint that the number of votes cast not exceed $s_{it} - L_{it}$ for firms. It follows that we can express the problem of household i using the Lagrangian as the saddle-point problem for a finite-horizon T :

$$\begin{aligned}
\mathcal{L}_{i0} \;=\; & \sup_{\{c_i, b_i, s_i, d_i, e_i, L_i, W_i, \epsilon\}} \inf_{\Lambda_i, \zeta_i, \xi_i, \vartheta_i} \sum_{t=0}^{T} \beta^t \left(\log (c_{it}) + v_{i,t} (\epsilon_t) - \beta \Lambda_{it+1} r_t (c_{it} - \phi_t L_{it}) \right) \\
& + \sum_{t=0}^{T} \beta^{t+1} \Lambda_{it+1} \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{it}<0\}} - p_t \right) \right) s_{it} \\
& - \sum_{t=0}^{T} \beta^{t+1} \zeta_{it+1} \left(\epsilon_{t+1} - \epsilon_t + \eta G (\chi_t) \right) - \beta^t \vartheta_{it} \left(L_{it} - s_{it} \mathbf{1}_{\{s_{jit} \geq 0\}} \right) \\
& - \sum_{t=0}^{T} \beta^t \Lambda_{it+1} \left(W_{it+1} - r_t W_{it} \right) - \sum_{t=0}^{T-1} \beta^t \xi_{it} \left(d_{it} + e_{it} - (s_{it} - L_{it}) \right),
\end{aligned} \tag{A.27}$$

where the law of motion for the social resource constraint is given by equation (10). We can recover b_{it} from the budget constraint (11).

By summation-by-parts, with $\Lambda_{i0} = 1$,

$$\begin{aligned}
\sum_{t=0}^{T} \beta^{t+1} \Lambda_{it+1} \left(W_{it+1} - r_t W_{it} \right) \;=\; & \beta^{T+1} \Lambda_{iT+1} W_{iT+1} - \Lambda_{i0} W_{i0} \\
& - \sum_{t=0}^{T} \beta^t W_{it} \left(\beta \Lambda_{it+1} r_t - \Lambda_{it} \right),
\end{aligned} \tag{A.28}$$

and

$$\sum_{t=0}^{T} \beta^{t+1} \zeta_{it+1} \left(\epsilon_{t+1} - \epsilon_t \right) \;=\; \beta^{T+1} \zeta_{iT+1} \epsilon_{T+1} - \zeta_{i0} \epsilon_0 - \sum_{t=0}^{T} \beta^t \epsilon_t \left(\beta \zeta_{it+1} - \zeta_{it} \right), \tag{A.29}$$

and (A.27) becomes the Hamiltonian

$$
\begin{aligned}
\mathcal{L}_{i0} \quad = \quad & \sup_{\left\{c_i, b_i, s_{ij}, d_{ij}, e_{ij}, L_{ij}, W_i, \epsilon\right\}} \inf_{\Lambda_i, \zeta_{ij}, \xi_{ij}, \vartheta_{ij}} \sum_{t=0}^{T} \beta^t \left(\log\left(c_{it}\right) + v_{i,t}\left(\epsilon_t\right) - \beta \Lambda_{it+1} r_t \left(c_{it} - \phi_t L_{it}\right)\right) \\
& + \sum_{t=0}^{T} \beta^{t+1} \Lambda_{it+1} \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{it}<0\}} - p_t\right)\right) s_{it} \\
& + \sum_{t=0}^{T} \beta^t \left(\epsilon_t \left(\beta \zeta_{it+1} - \zeta_{it}\right) + \eta \beta \zeta_{it+1} G\left(\chi_t\right)\right) - \sum_{t=0}^{T} \beta^t \vartheta_{ijt} \left(L_{it} - s_{it} \mathbf{1}_{\{s_{it} \geq 0\}}\right) \\
& + \sum_{t=0}^{T} \beta^t W_{it} \left(\beta \Lambda_{it+1} r_t - \Lambda_{it}\right) - \sum_{t=0}^{T-1} \beta^t \xi_{it} \left(d_{it} + e_{it} - \left(s_{it} - L_{it}\right)\right) \\
& + \Lambda_{i0} W_0 + \zeta_{i0} \epsilon_0 - \beta^{T+1} \Lambda_{iT+1} W_{iT+1} - \beta^{T+1} \zeta_{iT+1} \epsilon_{T+1}.
\end{aligned} \tag{A.30}
$$

Complementary slackness in the Hamiltonian A.30 for the state variable W_{iT+1} in the finite-horizon case where T imposes the boundary condition

$$\beta^T \Lambda_{iT+1} W_{iT+1} = 0, \tag{A.31}$$

and for ϵ_3

$$\beta^T \zeta_{iT+1} \epsilon_{T+1} = 0. \tag{A.32}$$

We impose these by specifying

$$\Lambda_{iT+1} = \zeta_{iT+1} = 0, \tag{A.33}$$

so that time $T+1$ objects are not valued after the game ends.

Maximizing over the state variable W_{it} in the Hamiltonian A.30, recognizing that W_{it} impacts asset prices by altering b_{it} through the budget constraint (11),

$$\beta \Lambda_{it+1} \partial_{W_i} b_{it} \quad = \quad \Lambda_{it}. \tag{A.34}$$

Recognizing W_{it} by the Envelope Conditions impacts prices only through b_{it}, we have from the strategic household's budget constraint (11)

$$\partial_{W_i} b_{it} = r_t - \frac{1}{nm_f} r_t \partial_{W_i} \left(\frac{1}{r_t}\right) b_{it} - \frac{1}{nm_f} r_t \partial_{W_i} p_t \left(s_{it} - s_{ijt-1}\right), \tag{A.35}$$

where the s_{it-1} term comes from differentiating $W_{it} = b_{it-1} + (D_t + p_t)\, p_{jt-1}$, and because $\partial_{W_i}\left(\frac{1}{r_t}\right) = \partial_{W_i} b_{it} \partial_{b_i}\left(\frac{1}{r_t}\right)$ and $\partial_{W_i}\left(\frac{1}{r_t}\right) = \partial_{W_i} b_{it} \partial_{b_i} p_t$, we have

$$\partial_{W_i} b_{it} = \frac{1}{\frac{1}{r_t} + \partial_{b_i}\left(\frac{1}{r_t}\right) b_{it} + \frac{1}{nm_f} \partial_{b_i} p_t \left(s_{it} - s_{ijt-1}\right)}. \tag{A.36}$$

Substituting equation A.36 into the first-order condition A.34, we arrive at

$$\frac{\beta \Lambda_{it+1}}{\Lambda_{it}} = \frac{1}{r_t} + \partial_{b_i}\left(\frac{1}{r_t}\right) b_{it} + \frac{1}{nm_f} \partial_{b_i} p_t \left(s_{it} - s_{ijt-1}\right), \tag{A.37}$$

which, substituting with price impact matrix J_t from Proposition (1), reduces to

$$\frac{\beta \Lambda_{it+1}}{\Lambda_{it}} = \frac{1}{r_t} + \frac{1}{nm_f} \boldsymbol{e}_1' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}. \tag{A.38}$$

Maximizing over the state variable ϵ_t in the Hamiltonian A.30, and dropping the β^t exponents, further implies

$$\zeta_{it} = v_{i,t}'(\epsilon_t) + \beta \zeta_{it+1} = \sum_{s=t}^{T} \beta^{s-t} v_{i,t}'(\epsilon_s), \tag{A.39}$$

which is household i's shadow cost of depleting the social resource. The shadow value of the social resource constraint consequently reflects her marginal disutility from its depletion. Since there is no disutility from the resource at time $T+1$, this reinforces that $\zeta_{iT} = 0$, and that the transversality condition is trivially satisfied.

For notational convenience, we define in the sequel

$$\Xi_{it} = \zeta_{it+1}. \tag{A.40}$$

Step 2: Optimal Voting and the Value of Control

We solve for strategic households i' time-consistent policies in the finite-horizon game. At time t, strategic household i faces the continuation problem

$$\{d_{it}, e_{it}\} = \arg\sup_{d'_{it}, e'_{it}} \beta \Lambda_{it+1} W_{it+1} - \beta \zeta_{it+1} \eta G(\chi_t) - \xi_{it}\left(d'_{it} + e'_{it}\right). \tag{A.41}$$

Because from equation 1, a marginal increase in x_{it} increases firms' collective output by $z-1$, the optimal choice of d_{it} satisfies

$$\frac{\beta}{n}\left(\Lambda_{it+1}(z-1)\,\partial_{D_{t+1}} W_{it+1} - \eta \Xi_{it}\right) G'(\chi_t)\,\mathbf{1}_{\{x_{it}<1\}} - \xi_{it} = 0, \tag{A.42}$$

while the optimal choice of e_{it} satisfies

$$\frac{\beta}{n}\left(-\Lambda_{it+1}(z-1)\,\partial_{D_{t+1}} W_{it+1} + \eta \Xi_{it}\right) G'(\chi_t) - \xi_{it} = 0, \tag{A.43}$$

where, substituting with equation (A.22)

$$\begin{aligned} \partial_{D_{t+1}} W_{it+1} &= s_{it} + \frac{\partial_{D_{t+1}} W_{ft+1}}{W_{ft+1}}\left(p_{t+1} - \phi_{t+1}\right) s_{it} \\ &= s_{it} + \frac{\left(p_{t+1} - \phi_{t+1}\right) s_{it}}{b_{ft} + \left(D_{t+1} + \phi_{jt+1}\right) s_{ft}} s_{ft}, \end{aligned} \tag{A.44}$$

because strategic agents take future trading and voting decisions by other strategic agents as given. Consequently, the derivative captures the wealth effect of the fringe having a slightly higher dividend from firm production tomorrow. In the above, the $\frac{1}{n}$ terms arises because each strategic household has a mass of $\frac{1}{n}$. Furthermore, because a strategic households votes at the end of a period but trades at the beginning on date t, she does not account for the impact of her vote on prices at date t.

Define

$$\Theta_{it} = \Lambda_{it+1} (z-1) \partial_{D_{t+1}} W_{it+1}, \tag{A.45}$$

which, substituting with equation (A.44) and market clearing, becomes

$$\Theta_{it} = \Lambda_{it+1} (z-1) \left(s_{it} + \frac{(p_{t+1} - \phi_{t+1}) s_{it}}{b_{ft} + (D_{t+1} + \phi_{t+1}) s_{ft}} s_{ft} \right). \tag{A.46}$$

It then follows from equations A.42, A.43, and equation A.46 that

$$\xi_{it} = \frac{\beta}{n} |\Theta_{it} - \eta \Xi_{it}| G' (\chi_t) \left(1 - \mathbf{1}_{\{\Theta_{it} > \eta \zeta_{it+1} \wedge \chi_t = 1\}}\right) \tag{A.47}$$

is the marginal value of her vote, and that her optimal voting policy when she votes satisfies

$$x_{it} = \mathbf{1}_{\{\Theta_{it} > \eta \zeta_{it+1}\}}. \tag{A.48}$$

Then,

$$v_{it} = \frac{\beta}{n} \frac{|\Theta_{it} - \eta \Xi_{it}|}{\Lambda_{it}} G' (\chi_t) \left(1 - \mathbf{1}_{\{\Theta_{it} > \eta \Xi_{it} \wedge \chi_t = 1\}}\right) \tag{A.49}$$

is the marginal value of her vote. Her vote has no value when voting is at a corner solution for the dirty technology because firm productivity cannot increase any further, and when the resource is already fully depleted ($\varepsilon_t = 0$).

A strategic household votes for the dirty technology if the marginal benefit to her wealth outweighs her private cost of depleting the social resource. While a vote for the clean technology always has value because the financially-motivated F strategic household will always vote for the dirty technology when $\chi_t = 0$, voting has no value if there is a consensus for the dirty technology.

Step 3: Optimal Consumption and Share Lending

Assuming that the Minimax Theorem is valid, we can switch the order of the sup and inf operators to take first-order necessary conditions of the Hamiltonian (A.30) with respect to the household's consumption and asset positions. For optimal consumption c_{it}, the first-order necessary condition substituting with (11) is

$$0 \geq \frac{1}{c_{it}} + \beta \Lambda_{it+1} \partial_{c_i} b_{it}, \tag{A.50}$$

with equality if $c_{it} > 0$.

Notice further by the strategic household's budget constraint (11), that

$$\partial_{c_i} b_{it} = -\partial_{W_i} b_{it}. \tag{A.51}$$

Intuitively, an additional dollar of consumption is akin to reducing the household's holdings in the one-period bond b_{it}, and the derivative takes into account the total impact of also altering prices on the change in the household's holdings.

Substituting with equation A.34 into equation A.51 at equality, we find that

$$\beta \Lambda_{it+1} \partial_{c_i} b_{it} \quad = \quad -\Lambda_{it} \tag{A.52}$$

As such, it follows from equation A.52 that the first-order condition for optimal consumption c_{it} A.50 reduces to

$$\frac{1}{c_{it}} \leq \Lambda_{it} \ \left(= \ if \ c_{it} > 0\right). \tag{A.53}$$

Because a strategic household has log utility, then $c_{it} > 0$ and the first-order condition A.50 holds with equality.

Consequently, optimal consumption satisfies the equivalent concave problem

$$H\left(\Lambda_i\right) = \sup_c \left[\log\left(c\right) - \Lambda_i c\right], \tag{A.54}$$

where $H(\Lambda_i)$ is the Fenchel-Legendre transform, which has a unique maximizer.

Because the game ends at $t = T$, it must be the case that optimally $c_{iT} = W_{iT}$ and because $c_{iT} > 0$, $\Lambda_{iT} = \frac{1}{W_{iT}}$.

Complementary slackness for share-lending L_{it} reveals

$$\vartheta_{ijt} = \max\{\beta\Lambda_{it+1}\partial_{L_i}b_{it} - \xi_{it}, 0\}. \tag{A.55}$$

By similar arguments to those for consumption

$$\beta\Lambda_{i,t+1}\partial_{L_i}b_{it} \quad = \quad \phi_t\Lambda_{it}, \tag{A.56}$$

from which follows, substituting also with equation A.47, that

$$\vartheta_{it} = \max\{\phi_t\Lambda_{it} - \nu_{it}, 0\}. \tag{A.57}$$

Step 4: Optimal Investment in Firm Equity and Share-lending

Assuming that the Minimax Theorem is valid, maximizing over the equity position s_{it}, substituting with the budget constraint 11, and dropping the β^t exponents imposes

$$\beta\Lambda_{it+1}\left(D_{t+1} + p_{t+1} + \partial_{s_i}b_{it}\right) + \xi_{it} + \vartheta_{it}\mathbf{1}_{\{s_{it}\geq 0\}} = 0, \tag{A.58}$$

where $\partial_{s_i}D_{t+1} = 0$ because we are only considering the direct effect (i.e., partial derivative) of having more shares (and not altering voting).

Notice further by the strategic household's budget constraint (11), that

$$\partial_{s_i}b_{it} = -\Bigg(p_t - \phi_t\mathbf{1}_{\{s_{it}<0\}} + \frac{1}{nm_f}\partial_{s_i}\left(\frac{1}{r_t}\right)b_{it} + \frac{1}{nm_f}\partial_{s_i}p_t\left(s_{it} - s_{it-1}\right)\Bigg)\partial_{W_i}b_{it}. \tag{A.59}$$

Intuitively, an additional dollar invested in j-firms' equity is akin to reducing the household's holdings in the one-period bonds b_{it}, and the derivative takes into account the total impact of also altering prices on the change in the household's existing holdings and the per-unit cost of the equity $p_t - \phi_t \mathbf{1}_{\{s_{it}<0\}}$. At the initial date, household i inherits $s_{ij-1} = \frac{1}{2+m_f}$ shares of both firms, and internalizes that her trading impacts the value of her initial wealth in each period.

Substituting with equation (A.34) into (A.59), we consequently find that

$$
\begin{aligned}
\beta \Lambda_{it+1} \partial_{s_{ij}} b_{it} &= -\Big(p_t - \phi_t \mathbf{1}_{\{s_{it}<0\}} + \tfrac{1}{nm_f} \partial_{s_i} \left(\tfrac{1}{r_t}\right) b_{it} \\
&\quad + \tfrac{1}{nm_f} \partial_{s_i} p_t \left(s_{it} - s_{it-1}\right) \Big) \Lambda_{it}.
\end{aligned} \tag{A.60}
$$

As such, substituting with equation (A.60), the first-order condition for optimal equity holdings, s_{it}, equation (A.58) reduces to

$$
\begin{aligned}
p_t &= \frac{\beta \Lambda_{it+1}}{\Lambda_{it}} \left(D_{t+1} + p_{t+1}\right) + \phi_t \mathbf{1}_{\{s_{it}<0\}} + \frac{1}{\Lambda_{it}} \xi_{it} + \frac{1}{\Lambda_{it}} \vartheta_{it} \mathbf{1}_{\{s_{it}\geq 0\}} \\
&\quad - \frac{1}{nm_f} \partial_{s_i} \left(\frac{1}{r_t}\right) b_{it} - \frac{1}{nm_f} \partial_{s_i} p_t \left(s_{it} - s_{it-1}\right).
\end{aligned} \tag{A.61}
$$

Finally, substituting for ξ_{ijt} with equation A.47, for ϑ_{ijt} with equation A.57, $\Lambda_{it} = \frac{1}{c_{it}}$ for time t into A.61, and the price impact matrix J_t from Proposition (1), we arrive at

$$
\begin{aligned}
p_t &= \frac{\beta c_{it}}{c_{it+1}} \left(D_{t+1} + p_{t+1}\right) + \phi_t \mathbf{1}_{\{s_{it}<0\}} + \max\{\phi_t, \nu_{it}\} \mathbf{1}_{\{s_{it}\geq 0\}} \\
&\quad - \frac{1}{nm_f} \boldsymbol{e}_2' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}.
\end{aligned} \tag{A.62}
$$

Because all her shares are purchased at the same market-clearing price, and a strategic household receives the dividend and capital gain regardless if she votes or lends out her shares, she lends her shares if the marginal value of control is less than the present-value of the share-lending fee, or

$$\phi_t \geq \nu_{it}, \tag{A.63}$$

We can consequently express equation (A.62) when household i is short firm equity as

$$p_t = \frac{\beta c_{it}}{c_{it+1}} \left(D_{t+1} + p_{t+1} \right) + \phi_t - \frac{1}{nm_f} \boldsymbol{e}_2' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}. \tag{A.64}$$

and when is long firm equity as

$$p_t = \frac{\beta c_{it}}{c_{it+1}} \left(D_{t+1} + p_{t+1} \right) + \max\{\phi_t, \nu_{it}\} - \frac{1}{nm_f} \boldsymbol{e}_2' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix}. \tag{A.65}$$

Step 5: Present-value Budget Constraint

Consider now the law of motion of the household's wealth (A.26). Multiplying equation (A.26) by $\frac{\beta \Lambda_{it+1}}{\Lambda_{it}}$ and substituting with equations (11), and (A.62), we find that

$$c_{it} + \left(\max\{\phi_t, \nu_{it}\} s_{it} - \phi_t L_{it} \right) \mathbf{1}_{\{s_{it} \geq 0\}} - PI_{it} + \frac{\beta \Lambda_{it+1}}{\Lambda_{it}} W_{it+1} = W_{it}, \tag{A.66}$$

where

$$PI_{it} = \frac{1}{nm_f} \boldsymbol{e}_1' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix} b_{it} + \frac{1}{nm_f} \boldsymbol{e}_2' J_{it} \begin{bmatrix} b_{it} \\ s_{it} - s_{it-1} \end{bmatrix} s_{it}, \tag{A.67}$$

is the price impact term.

Iterating forward on the law of motion of wealth from equation A.66 and imposing $\Lambda_{it} = \frac{1}{c_{it}}$ and A.31

$$\begin{aligned} W_{i0} &= \sum_{t=0}^{T} \beta^t \frac{c_{i0}}{c_{it}} \left(c_{it} + \nu_{it} \left(s_{it} - L_{it} \right) \mathbf{1}_{\{s_{it} > 0\}} - PI_{it} \right) \\ &= \frac{1 - \beta^{T+1}}{1 - \beta} c_{i0} + \sum_{t=0}^{T-1} \beta^t \frac{c_{i0}}{c_{it}} \left(\nu_{it} \left(s_{it} - L_{it} \right) \mathbf{1}_{\{s_{it} > 0\}} - PI_{it} \right), \end{aligned} \tag{A.68}$$

from which we can recover Λ_{i0} given $W_{i0} = \frac{D_0+p_0}{2+m_f}$. That the second sum counts to $T-1$ reflects that there is no trading on the final date T. Strategic households consequently earn strategic rents from putting a wedge between their state prices and asset prices. For the consumption bundle of household i to be marketed, her consumption must consequently satisfy the present-value budget constraint (A.68).

Step 6: Moving to Infinite-horizon

In the infinite-horizon case, as $T \to \infty$ from (A.27) instead becomes the standard transversality condition

$$\lim_{T\to\infty} \beta^{T+1} \Lambda_{iT+1} W_{iT+1} = 0, \tag{A.69}$$

otherwise the household could achieve arbitrarily high value by making W_{iT+1} arbitrarily positive or negative, and for ϵ_{T+1}

$$\lim_{T\to\infty} \beta^{T+1} \zeta_{iT+1} \epsilon_{T+1} = 0. \tag{A.70}$$

Because ϵ_T is bounded, we need only be concerned that ζ_{iT} grows too fast as the resource depletes, and our solution remains valid for the infinite-horizon problem.

Step 7: Time-Consistency of Optimal Policies

Time-consistency of the optimal policies, consistent with a Markov Perfect Equilibrium, follows from two observations. The first is that the objective of a strategic household in Problem 13 is dynamically consistent, i.e., preferences are additively-time-separable with exponential discounting. The second is that all optimal policies are based on forward-looking shadow costs, Λ_{it} and ξ_{it}, rather than backward-looking commitment multipliers on forward-looking constraints (e.g., Marcet and Marimon [2019]).

Proof of Corollary 3:

Because competitive households are the first to sell their shares (since they have the lowest valuation for firm equity), they will not be eligible to lend to potential short-sellers. However, once they have zero shares, then they have incentive to short-sell firm equity if its price rises above its present-discounted value of future cash flows. In this case, the price of equity must rise to incorporate the share-lending fee, ϕ_t, because the two strategic households are now the marginal buyers. For the marginal strategic household that sells or lends its share to be indifferent, it must be the case that

$$\phi_t = \nu_{*t}. \tag{A.71}$$

for the marginal strategic household $*$ that has the lower value of control, ν_{*t}.

Proof of Proposition 5:

We begin with the finite-horizon case with T periods. Notice that the total resources in the economy are bounded at each date by the total production of firms, and consequently so is total financial wealth (and asset positions). The constraint set for each household's optimization problem is therefore closed and bounded (and compact by the Heine-Borel Theorem). Because households' have continuous objectives and face a compact constraint set, by Berges' Theory of the Maximum, there exists a solution to each household's problem and their optimal policies (i.e., consumption, asset positions, voting) are upper hemicontinuous correspondences from prices and states to prices and (endogenous) states. This is a sequence of optimal policies $\mathcal{G}_i = \{c_{it}, a_{it}, b_{it}, s_{it}, L_{it}, x_{it}\}_{t=0}^{T}$ for strategic households and $\mathcal{G}_f = \{c_{ft}, a_{ft}, b_{ft}, s_{ft}, L_{ft}\}_{t=0}^{T}$ for the competitive fringe given initial conditions $\{\{W_{S,0}, W_{F,0}, W_{f0}\}$.

By invoking Roxin's condition, we recognize that the budget constraint of a strategic household is both compact and convex because it is linear in consumption, and consequently the optimal policy correspondence is convex. Since the aggregate production of firms is a ray in $G(\chi_t)$, the aggregate production set from the perspective of shareholders is also convex. We can then stack the market clearing conditions for consumption, asset, and share-lending markets to construct a correspondence from optimal policies $\{\{\mathcal{G}_i'\}_{i=1}^N, \mathcal{G}_f'\} \in \Omega$ into optimal policies $\{\{\mathcal{G}_S'', \mathcal{G}_F'', \mathcal{G}_f''\} \in \Omega$ using the optimal policy correspondences, where Ω is a compact and convex space. An equilibrium is then a fixed point of this (vector-valued) upper hemi-continuous correspondence given the initial conditions $\{\{W_{S,0}, W_{F,0}, W_{f0}\}$.

Because we have an upper hemi-continuous correspondence from a compact, convex space Ω into itself, we can apply Kakutani's Fixed-Point Theorem to conclude that a fixed point exists for any horizon T. However, the choice of T was arbitrary, and we can extend this for arbitrarily large T by substituting boundary for transversality conditions. Because the long-run of the model is stationary (i.e., either the resource is fully depleted or all firms use the clean technology at each date), the economy remains bounded and well-defined in the limit as $T \to \infty$. As such, an equilibrium in our model exists.

Proof of Proposition 12:

Step 1: Constrained Optimal Production Rule

We derive a modified version of the Dréze rule in the case of production externalities by examining how a Planner would assign production rights when maximizing utilitarian social welfare with transfers (e.g., what are referred to as extended Dréze equilibria). The allowance of transfers potentially expands the space of allocations beyond what stock market equilibria can achieve (Dierker and Dierker [2010]).

To derive the rule, we modify the approach of Geanakoplos et al. [1990], and instead of voting consider a Planner who can influence production decisions and engage in household-specific transfers at dates 0 and 1, τ_{it}, such that

$$\sum_{i \in F,S} \tau_{it} + m_f \tau_{ft} = 0, \; \forall \; t.$$

Suppose that the Planner maximizes welfare subject to households' sequential budget constraints and optimal policies when choosing the fraction of dirty firms χ_t

$$\begin{aligned}\mathcal{U}_0 \quad = \sup_{\chi,\tau_i} \quad & \sum_{i\in f,F,S} m_i \sup_{\{c_i,b_i,s_i,L_i,W_i,\epsilon\}} \inf_{\Lambda_i,\zeta_i} \sum_{t=0}^{2} \beta^t \left(\log\left(c_{it}\right) + v_{i,t}\left(\epsilon_t\right)\right) \\ & - \sum_{t=0}^{2} \beta^{t+1} \Lambda_{it+1} r_t \left(c_{it} - \tau_{it} - \phi_t L_{it}\right) \\ & + \sum_{t=0}^{2} \beta^{t+1} \Lambda_{it+1} \left(D_{t+1} + p_{t+1} + r_t \left(\phi_t \mathbf{1}_{\{s_{it}<0\}} - p_t\right)\right) s_{it} \\ & - \sum_{t=0}^{2} \beta^{t+1} \zeta_{it+1} \left(\epsilon_{t+1} - \epsilon_t + \eta \chi_{jt}\right) - \sum_{t=0}^{T} \beta^t \Lambda_{it+1} \left(W_{it+1} - r_t W_{it}\right),\end{aligned} \tag{A.72}$$

with the convention that $m_i = 1$ for $i \in F, S$ and m_f is the size of the competitive fringe. In the optimization program (A.72), we suppress the constraint on share lending, L_{it}, from the proof of Proposition 2 for brevity, and we follow the convention $v_f(\epsilon_t) \equiv 0$. Similar to households, the Planner choose production at the beginning of each period.

We proceed by a calculus of variations argument. Suppose the Planner considers variations of the fraction of firms that use the dirty technology, $\hat{\chi}_t = \chi_t + \Delta\chi_t$, and the transfers at each date to households $\hat{\tau}_{it} = \tau_{it} + \Delta\tau_{it}$. By the Envelope Condition for households' optimal policies, the impact on household $i's$ continuation problem in optimization problem (A.72) from this variation is

$$\Delta\mathcal{L}_i = m_i \sum_{t=1}^{2} \left(\Theta_{it} - \eta \Xi_{it}\right) \Delta\chi_t + \beta \Lambda_{it} \partial_{\tau_{it-1}} b_{it-1} m_i \Delta\tau_{it-1}, \tag{A.73}$$

where Θ_{it} and Ξ_{it} are defined in the proof of Proposition 2, and measures the shadow cost of depleting the social resource. Recognizing from the proof of Proposition 2 that $\Lambda_{it} = \frac{1}{c_{it}}$, $\partial_{\tau_{it}} b_{it} = -\partial_{W_{it}} b_{it}$ because they enter symmetrically into the budget constraint, and $\beta\Lambda_{it+1}\partial_{W_{it}} b_{it} = \Lambda_{it}$ from equation A.34 , we have from equation A.52 that equation (A.79) reduces to

$$\Delta\mathcal{L}_i = m_i \sum_{t=1}^{2} \left(\Theta_{it} - \eta \Xi_{it}\right) \Delta\chi_t - \frac{1}{c_{it}} m_i \Delta\tau_{it-1}. \tag{A.74}$$

Consider the contribution of the variation to household $i's$ continuation problem at date t. If we divide at each date by household i's state price deflator, $1/c_{it}$, and sum across households, we have that social welfare changes according to

$$\sum_{i \in f,F,S} \frac{\Delta \mathcal{L}_i}{1/c_i} = \sum_{t=1}^{2} \sum_{i \in f,F,S} m_i \frac{\Theta_{it} - \eta \Xi_{it}}{1/c_{it}} \Delta \chi_t - \sum_{i \in f,F,S} m_i \Delta \tau_{it-1},$$
$$= \sum_{t=1}^{2} \sum_{i \in f,F,S} m_i \frac{\Theta_{it} - \eta \Xi_{it}}{1/c_{it}} \Delta \chi_t, \tag{A.75}$$

because $\Delta \tau_{Ft-1} + \Delta \tau_{St-1} + m_f \Delta \tau_{ft-1} = 0$ by construction. Because the choice of variation $\Delta \chi_t$ is arbitrary, at an optimum, we have from equation (A.75) that at a stationary point that

$$\frac{\Theta_{Fjt}}{1/c_{Ft}} + m_f \frac{\Theta_{fjt}}{1/c_{ft}} + \frac{\Theta_{Sjt} - \eta \Xi_{St}}{1/c_{St}} = 0, \tag{A.76}$$

which substituting with equation (A.45), we can rewrite as

$$\left(\sum_{i \in F,S} \frac{c_{it}}{c_{it+1}} \partial_{D_{t+1}} W_{it+1} + m_f \frac{c_{ft}}{c_{ft+1}} \partial_{D_{t+1}} W_{ft+1} \right) (z-1) = \frac{\eta \Xi_{St}}{1/c_{St}}.$$

We consequently conclude that a valid stochastic discount factor for the firm is

$$\Lambda^*_{t+1} = \sum_{i \in F,S} \frac{c_{it}}{c_{it+1}} s_{it} + m_f \frac{c_{ft}}{c_{ft+1}} s_{ft}. \tag{A.77}$$

Define from equation (A.45)

$$\partial_{D_{t+1}} p_{t+1} = \frac{s_{ft}}{b_{ft} + (D_{t+1} + \phi_{t+1}) s_{ft}} (p_{t+1} - \phi_{t+1}). \tag{A.78}$$

This allows us to rewrite equation (A.79) as the condition at an interior solution

$$\Lambda^*_{t+1} \left(1 + \partial_{D_{t+1}} p_{t+1}\right) (z-1) = \frac{\eta \Xi_{St}}{1/c_{St}}. \tag{A.79}$$

Notice that we can repeat this analysis for either a finite horizon T, with an infinite-horizon, or for any set of Pareto weights for the Planner and arrive at the same first-order conditions. Consequently, any constrained efficient equilibrium implies the first-order conditions of a Dréze equilibrium.[19]

Step 2: Decentralizing the Constrained Optimal Rule through Voting

Suppose that the planner can choose an optimal voting rule $G^*(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ that maps the number of votes in favor of the dirty technology at firms, χ_t, into the fraction of firms that actually employ dirty technology, $G^*(\chi_t)$. We impose that $G^*(\chi_t)$ is $\mathcal{C}^1(\mathbb{R})$-differentiable, (weakly) increasing, and that it respects unanimity, i.e., $G^*(0) = 0$ and $G^*(1) = 1$.

In addition, we allow the planner to engage in household-specific transfers at dates 0 and 1, τ_{it}, such that

$$\sum_{i \in F,S} \tau_{it} + m_f \tau_{ft} = 0, \; \forall \, t.$$

Suppose the Planner considers variations of the fraction of firms that use the dirty technology, $\hat{\chi}_t = \chi_t + \Delta\chi_j$, and the transfers at each date to households $\hat{\tau}_{it} = \tau_{it} + \Delta\tau_{it}$. By the Envelope Condition for households' optimal policies, the change in utility of an agent at date t for an arbitrary variation in policy is

$$\Delta U_i = (\Theta_{it} - \Xi_{it}) \, G^{*\prime}(\chi_t) \, \Delta\chi_t + \frac{1}{c_{it}} \Delta\tau_i, \tag{A.80}$$

Multiplying equation (A.80) by c_{it} and aggregating across agents at a constrained optimal solution implies

$$\left(\Lambda^*_{t+1} \left(1 + \partial_{D_{t+1}} p_{t+1}\right)(z-1) - c_{St}\eta\Xi_{St}\right) G^{*\prime}(\chi_t) = 0, \tag{A.81}$$

[19] In incomplete markets with one good at each date and convex production technologies, all Dréze equilibria are constrained efficient (Geanakoplos et al. [1990]). Inefficiency arises here if the firm ignores the impact of dirty production on the S household's utility.

because $\Delta\tau_{Ft-1} + \Delta\tau_{St-1} + m_f \Delta\tau_{ft-1} = 0$ by construction. The argument in parentheses is the net dollar value of a marginal increase in χ_{jt}.

Notice if $G^*(\chi_t)$ satisfies (A.76), then the argument inside the parentheses of (A.81) is zero at an interior solution, while it is positive if $G^*(\chi_t) = 1$ and negative if $G^*(\chi_t) = 0$, at which points $G^{*\prime}(\chi_t) = 0$ (i.e., the rule is flat over the regions in which $G^*(\chi_t)$ at a corner solution at 0 or 1). Consequently, $G^{*\prime}(\chi_t) = 0$ only at corner solutions for the optimal fraction of dirty firms.

Therefore the constrained optimal voting rule achieves the constrained efficient level of production from Step 1.

Step 3: Properties of $G^*(\chi_t)$

Notice we can express the optimal voting rule in equation (A.81) as

$$\left(\frac{\Theta_{Ft}}{1/c_{Ft}} + m_f \frac{\Theta_{ft}}{1/c_{ft}} + \frac{\Theta_{St} - \eta \Xi_{St}}{1/c_{St}} \right) G^{*\prime}(\chi_t) = 0.$$

At an interior equilibrium for $G^*(\chi_t)$, the argument in parentheses equals zero. As a benchmark for our analysis, let us conjecture $G^*(\chi_t) = \chi_t$, i.e., a linear aggregation rule. For this special case, the argument in parentheses is precisely the first-order condition for the optimal χ_t.

Consider the case in which the S household's shadow cost of resource depletion, Ξ_{St}, is such that $\eta\Xi_{St} = \Theta_{St}$. At this point, all households have $\frac{1}{2+m_f}$ shares of firm equity, and

$$\frac{\Theta_{Ft}}{1/c_{Ft}} + m_f \frac{\Theta_{ft}}{1/c_{ft}} + \frac{\Theta_{St} - \eta \Xi_{St}}{1/c_{St}} = \frac{\Theta_{Ft}}{1/c_{Ft}} + m_f \frac{\Theta_{ft}}{1/c_{ft}} > 0.$$

Consequently, the planner would choose for all firms continue to use the dirty technology. Notice this is true until Ξ_{St} rises sufficiently that

$$\frac{\eta \Xi_{St}}{1/c_{St}} \geq \frac{\Theta_{Ft}}{1/c_{Ft}} + \frac{\Theta_{St}}{1/c_{St}} + m_f \frac{\Theta_{ft}}{1/c_{ft}},$$

in which case the planner would choose for some firms to implement the clean technology, although less than s_{St} because the socially responsible household votes based on its private value for resource conservation. The optimal voting protocol therefore has a region in which $G^*(\chi_t) > \chi_t$ when χ_t is sufficiently large.

In contrast, when the competitive fringe has sold all its shares and the S household now buys shares from the F household, equation (A.81) reduces to

$$\left(\frac{\Theta_{Ft}}{1/c_{Ft}} - \frac{\eta\Xi_{St} - \Theta_{St}}{1/c_{St}}\right) G^{*\prime}(\chi_t) = 0.$$

By construction, in this case $\frac{\eta\Xi_{St} - \Theta_{St}}{1/c_{St}} > \frac{\Theta_{Ft}}{1/c_{Ft}}$, and therefore the planner would want more firms to use the clean technology than s_{St}. The optimal voting protocol therefore has a region in which $G^*(\chi_t) < \chi_t$ when χ_t is sufficiently small.

We conclude that $G^*(\chi_t) < \chi_t$ when χ_t is sufficiently small and $G^*(\chi_t) > \chi_t$ when it is sufficiently large. Because $G^*(\chi_t)$ is continuous, by the Intermediate Value Theorem there must be at least one point such that $G^*(\chi_t^*) = \chi_t^*$, above which $G^*(\chi_t) > \chi_t$ for $\chi_t > \chi_t^*$, and a χ_t^{**}such that $G^*(\chi_t) < \chi_t$ for $\chi_t < \chi_t^{**}$. Given $G^*(\chi_t)$ is continuous and $G^*(\chi_t) < \chi_t$ for χ_t sufficiently small, $G^*(\chi_t)$ must cross the 45-degree line an odd number of times. However, if it crossed more than once, then $G^*(\chi_t)$ would oscillate from being above χ_t to being below it, which contradicts the monotonicity of a socially responsible household's holdings in her shadow cost of depleting the social resource, Ξ_{St}. Consequently, there is only one intersection with the 45-degree line.

Online Appendix for Building Voice in Socially Responsible Investing

Richard Lowery and Michael Sockin

OA.A. Additional Proofs

Proof of Proposition 7:

We prove this result for the special case that $T = 2$, but the results generalize to arbitrary T, including $T \to \infty$. Suppose all households behaved competitively, which we recover by taking the limit as $n \to \infty$. Then large households do not value their vote; as such, they all agree on the firm equity price and there are no gains from trade. However, the competitive S household will continue to vote according to the voting protocol in Proposition (2).

Given that $c_{ft} = \frac{1-\beta}{1-\beta^{3-t}} W_{ft}$ and $q_t = \beta \frac{c_{ft}}{c_{ft+1}}$ from Proposition 1 , we recognize if each household owns $\frac{1}{2+m_f}$ shares of stock that

$$
\begin{aligned}
c_{f2} &= \frac{1 + G(\chi_1)(z-1)}{2+m_f}, \\
c_{f1} &= \frac{1-\beta}{1-\beta^2}\left(\frac{1+G(\chi_0)(z-1)}{2+m_f} + q_1 c_{f2}\right) = \frac{1}{1+\beta}\left(\frac{1+G(\chi_0)(z-1)}{2+m_f} + \beta c_{f1}\right) \\
&= \frac{1+G(\chi_0)(z-1)}{2+m_f}, \\
c_{f0} &= \frac{1-\beta}{1-\beta^3}\left(\frac{D_0}{2+m_f} + q_0\left(c_{f1} + q_1 c_{f2}\right)\right) \\
&= \frac{1-\beta}{1-\beta^3}\left(\frac{D_0}{2+m_f} + \beta(1+\beta) c_{f0}\right) = \frac{D_0}{2+m_f}.
\end{aligned}
$$

By market-clearing, large households consume c_{it} given by

$$c_{it} = \frac{1}{2}\left(y_t - m_f c_{ft}\right), \tag{OA.1}$$

from which follows that

$$\begin{aligned}
c_{i2} &= \frac{1}{2}\left(1 + G\left(\chi_1\right)(z-1) - \frac{m_f}{2+m_f}\left(1 + G\left(\chi_1\right)(z-1)\right)\right) = \frac{1 + G\left(\chi_1\right)(z-1)}{2+m_f}, \\
c_{i1} &= \frac{1 + G\left(\chi_0\right)(z-1)}{2+m_f}, \\
c_{i0} &= \frac{D_0}{2+m_f}.
\end{aligned} \tag{OA.2}$$

It is then immediate that the system OA.2 reduces to

$$c_{it} = c_{ft} = \frac{y_{jt}}{2+m_f} = \frac{1 + G\left(\chi_{t-1}\right)(z-1)}{2+m_f}, \tag{OA.3}$$

Because all households have the same consumption and log utility, it follows that they all agree that the price of an Arrow asset is

$$q_t = \beta \frac{y_t}{y_{t+1}}. \tag{OA.4}$$

Generalizing this to a T-period horizon and taking the limit as $T \to \infty$, we have that

$$c_{it} = \frac{1 + G\left(\chi_{t-1}\right)(z-1)}{2+m_f}. \tag{OA.5}$$

Consequently the economy is an autarky that features no trade in which $b_{it} = 0$ and $b_{ft} = 0$. This confirms the equilibrium.

Proof of Proposition 8:

Step 1: Autarky Region with Dirty Technology

We first recognize that if the S household wants to employ the dirty technology at both dates, then the equilibrium is autarky in which $b_{S0} = b_{S1} = 0$ and $s_{S0} = s_{S1} = \frac{1}{2}$. At date 1, this occurs from Proposition 2 when

$$\frac{z-1}{1+\chi_1(z-1)} \geq \eta B. \tag{OA.6}$$

Similarly, at date 0, this occurs when

$$\frac{z-1}{1+\chi_0(z-1)}\left(1+\frac{p_1}{D_1}\right) \geq \beta\eta B, \tag{OA.7}$$

Because the incentive to use the dirty technology is stronger at date 0, at date 1 if

$$B \leq \underline{B} = \frac{1}{\eta}\frac{z-1}{z}, \tag{OA.8}$$

then the solution is autarky and all firms use the dirty technology at both dates.

Step 2: Autarky Region in which Some Firms Use Clean Technology

Suppose $B > \underline{B}$, then the S household will first alter its production decisions at date 1 because the disutility from resource depletion is highest at date 1.

The S household consequently starts voting for the clean technology with (some of) its shares. Because voting for the clean technology with its own shares is the same as buying from the fringe and voting clean with those shares, the S household will first use its shares to vote clean. This is because the value of a clean vote is zero (i.e., $\nu_{S1} = 0$) in this interval (which pins down the equilibrium χ_1), and consequently there are no gains from trade with the fringe. The S household's first-order conditions for her optimal asset holdings from equations (16) and (17) are satisfied at no trade.

This gives rise to a range of B during which the portfolio allocation is autarky, but the S household votes for the clean technology with some fraction of its shares at date 1.

From equation (OA.6), we can solve for this fraction, χ_1, as a function of B

$$\chi_1 = \frac{1}{\eta B} - \frac{1}{z-1}, \tag{OA.9}$$

until $\chi_1 = \frac{1}{2}$ when

$$B = B^* = \frac{2}{\eta} \frac{z-1}{1+\frac{1}{2}(z-1)}. \tag{OA.10}$$

Consequently, for $B \in [\underline{B}, B^*]$, the S household does not change its portfolio and starts voting its date 1 shares clean until it votes completely clean.

Step 2: All Clean Technology Limit

Suppose that the disutility from resource depletion is so severe that the S household buys all firm equity and votes for the clean technology, i.e., $s_{S0} = s_{S1} = 1$. To buy all equity in both firm types, the S household must issue debt to the competitive fringe at return r_0 and roll b_{S1} over at return r_1 at date 1,

Given the competitive fringe's optimal consumption rule from Proposition 1, and $W_{f0} = \frac{1}{2}(D_0 + p_0)$, $W_{f1} = b_{f0}$, and $W_{f2} = b_{f1}$, firm equity prices are given by

$$p_0 = \frac{1}{r_0}(1+p_1) = \frac{\beta W_{f0}}{1+\beta+\beta^2}\left(\frac{1+\beta}{W_{f1}} + \frac{\beta}{W_{f2}}\right) = \frac{\frac{1}{2}\beta D_0 \left(\frac{1+\beta}{b_{f0}} + \frac{\beta}{b_{f1}}\right)}{1+\beta+\beta^2 - \frac{\beta}{2}\left(\frac{1+\beta}{b_{f0}} + \frac{\beta}{b_{f1}}\right)}, \tag{OA.11}$$

$$p_1 = \frac{1}{r_1} = \frac{\beta}{1+\beta}\frac{W_{f1}}{W_{f2}} = \frac{\beta}{1+\beta}\frac{b_{f0}}{b_{f1}}, \tag{OA.12}$$

and all dividends at dates 1 and 2 are 1, where

$$\frac{1}{r_0} = \beta\frac{1+\beta}{1+\beta+\beta^2}\frac{W_{f0}}{W_{f1}} = \beta\frac{1+\beta}{1+\beta+\beta^2}\frac{1}{2}\frac{D_0+p_0}{b_{f0}}. \tag{OA.13}$$

This matches the cash flows the S household owes to the fringe with the cash flows paid by firms at dates 1 and 2.

Notice that price impact in both assets is

$$J_{St} = \frac{1}{W_{ft+1}} \begin{bmatrix} \frac{1}{r_t} & p_t \\ \frac{1}{r_t}\left(1+p_{t+1}\right) & p_t\left(1+p_{t+1}\right) \end{bmatrix} + \frac{1}{r_t} \begin{bmatrix} 0 & 0 \\ 0 & z-1 \end{bmatrix}. \tag{OA.14}$$

We focus on the first-order necessary conditions for debt at both dates. From the condition for debt for the S household at date 1, we have because $s_{S1} = s_{S0}$ and market clearing in the consumption market that

$$\frac{\beta}{1+\beta}\frac{b_{f0}}{b_{f1}} = \beta\frac{1-\frac{b_{f0}}{1+\beta}}{1-b_{f1}} + \frac{1}{n}\frac{1}{r_1}\frac{1}{W_{f2}}b_{f1} = \beta\frac{1-\frac{b_{f0}}{1+\beta}}{1-b_{f1}} + \frac{1}{n}\frac{\beta}{1+\beta}\frac{b_{f0}}{b_{f1}}, \tag{OA.15}$$

from which follows that

$$b_{f1} = \frac{1}{\frac{n}{n-1}\frac{1+\beta}{b_{f0}} - \frac{1}{n}}. \tag{OA.16}$$

At date 0, the first-order condition for debt for the S household is

$$\beta\frac{1+\beta}{1+\beta+\beta^2}\frac{1}{2}\frac{D_0+p_0}{b_{f0}} = \beta\frac{D_0 - \frac{1}{2}\frac{D_0+p_0}{1+\beta+\beta^2}}{1-\frac{b_{f0}}{1+\beta}} - \frac{1}{n}\frac{1}{b_{f0}}\left(\frac{1}{r_0}b_{S0}+p_0\right), \tag{OA.17}$$

which we can rewrite, substituting for p_0, as

$$\frac{n+1}{n}\left(\frac{1+\beta}{b_{f0}}-1\right) = 1+2\beta+2\beta^2 - \left(\beta + \frac{1}{n}\frac{1}{1+\beta}\left(\frac{1+\beta}{b_{f0}}-1\right)\right)\left(\frac{1+\beta}{b_{f0}}+\frac{\beta}{b_{f1}}\right). \tag{OA.18}$$

Substituting with b_{f1}, we arrive at the quadratic equation for $x = \frac{1+\beta}{b_{f0}}$

$$\begin{aligned} 0 \;=\; & \frac{1}{n}\frac{1}{1+\beta}\left(1+\beta\frac{n}{n-1}\right)x^2 + \left(1+\left(\beta-\frac{1}{n}\frac{1}{1+\beta}\right)\left(1+\beta\frac{n}{n-1}\right)+\frac{1}{n}\left(1-\frac{1}{n}\frac{\beta}{1+\beta}\right)\right)x \\ & -\left(2+2\beta+2\beta^2+\frac{\beta^2}{n}+\frac{1}{n}\left(1-\frac{1}{n}\frac{\beta}{1+\beta}\right)\right), \end{aligned} \tag{OA.19}$$

from which we can recover $b_{f0} = b_{f0}\left(n,\beta\right)$, $b_{f1} = b_{f1}\left(n,\beta\right)$ from equation (OA.16), $c_{S2} =$

$1 - b_{f1}$, and

$$c_{S0} = D_0 - \frac{1}{2}\frac{D_0 + p_0}{1+\beta+\beta^2}, \tag{OA.20}$$

$$c_{S1} = 1 - \frac{b_{f0}}{1+\beta}. \tag{OA.21}$$

With these expressions, we can then evaluate the first-order necessary condition for the S household's optimal equity position at date 1 when $s_{f1} = 0$ to find

$$p_1 \leq \beta \frac{c_{S1}}{c_{S2}} + \frac{\beta}{n} c_{S1} \left(B_2 \eta - \frac{1}{c_{S2}} (z-1) \right) + \frac{1}{n}\frac{1}{r_1} - \frac{1}{n}\frac{1}{r_1}(z-1), \tag{OA.22}$$

from which follows because $p_1 = \frac{1}{r_1}$

$$B \geq \bar{B}_1 = \frac{1}{\eta} \left(\frac{n - 1 + (z-1)}{1+\beta} \frac{b_{f0}}{b_{f1}} \frac{1}{1 - \frac{b_{f0}}{1+\beta}} - \frac{n - (z-1)}{1 - b_{f1}} \right), \tag{OA.23}$$

We can then evaluate the first-order necessary condition for the S household's optimal equity position at date 0 when $s_{f0} = 0$ to find

$$p_0 \leq \beta \frac{c_{S0}}{c_{S1}} (1 + p_1) + \frac{\beta}{n} c_{S0} \left(B_2 \eta - \frac{1}{c_{S1}} (z-1) \right) + \frac{1}{n}\frac{1}{r_0} b_{f0} - \frac{1}{n}\frac{1}{b_{f0}} p_0 + \frac{1}{n}\frac{1}{r_0}(z-1), \tag{OA.24}$$

from which it follows that

$$\begin{aligned} B \geq \bar{B}_0 = \frac{1}{\eta} \frac{1}{1 - \frac{b_{f0}}{1+\beta}} \Bigg(\frac{1}{\beta} \left(n + \frac{1}{b_{f0}} \right) p_0 + \left(z - 1 - n - n \frac{\beta}{1+\beta} \frac{b_{f1}}{b_{f2}} \right) \\ - \frac{1+\beta}{1+\beta+\beta^2} \frac{1}{2} \frac{D_0 + p_0}{b_{f1}} \left(b_{f0} + z - 1 \right) \Bigg). \end{aligned} \tag{OA.25}$$

Consequently, this equilibrium is sustainable if

$$B \geq \bar{B} = \max\{\bar{B}_0, \bar{B}_1\}. \tag{OA.26}$$

For disutilities higher than $\bar{B}$, the S household is willing to pay at least p_0 to own all equity in the economy and would buy more if shares were not in unit supply.

Step 4: Region with Trade

For $B \in \{B^*, \bar{B}\}$, the S household cannot change any more of its votes at date 1 to clean and does not yet own all firm equity in the economy.

For these values of B, the marginal value of her vote at date 2, ν_{S1}, must be positive when she now buys more shares at date 1. This is because her optimal asset position, s_{S1}, cannot simultaneously satisfy both her first-order condition for her optimal equity holdings and equation (OA.6) with equality when there is trade. Consequently, voting must have value to the S household.

Proof of Proposition 9:

We consider the case of one S and one F household.

Step 1: Autarky Region with Dirty Technology

The autarky region $B \leq \underline{B}$ is the same as characterized in the case with the competitive fringe in Proposition (8) because it is based only on the incentives of the S household.

Step 2: Region of Strategic Delay by F Household

Consider $B = \underline{B} + \delta$ for δ arbitrarily small. In this case, the S household has a marginally positive value of control for the clean technology at date 1 is

$$\nu_{S1} = \kappa\beta \left(\frac{z}{2}\eta B - (z-1) \right). \tag{OA.27}$$

However, the F household has a more positive value of control

$$\nu_{F1} = \kappa\beta\,(z-1) > \nu_{S1}. \tag{OA.28}$$

In both expressions, we recognize under autarky, each household consumes z at date 1.

Consequently, the F household will buy the shares at date 1 from the S household because she marginally values using the dirty technology more than the S household marginally values using the clean technology.

This is true for $B \geq \underline{B}$ until $\nu_{S1} > \nu_{F1}$, at which point the equilibrium must return to autarky or

$$B < B^s = \frac{4}{\eta}\frac{z-1}{z}, \tag{OA.29}$$

where $c_{S1}\left(B^s\right) \geq z$ because the F household pays the S household's inflated marginal valuation beyond the present value of future cash flows to acquire the shares. Consequently, for $B \in [\underline{B}, B^s]$, the F household buys equity to delay the transition to the clean technology.

Step 3: All Clean Technology Limit

Suppose that B is sufficiently large that the S household buys all firm equity at date 0, i.e., $s_{S0} = s_{S1} = 1$. In this case, the wealth of the F household is given by $W_{F1} = b_{F0}$ and $W_{F2} = b_{F1}$, and consumption by the budget constraint satisfies $c_{F0} = \frac{1}{2}\left(D_0 + p_0\right) - \frac{1}{r_0}b_{F0}$ $c_{F1} = b_{F0} - \frac{1}{r_1}b_{F1}$, and $c_{F2} = b_{F1}$.

Notice if we sum the first-order conditions for the optimal choice of debt at dates 0 and 1, we have by market clearing

$$\frac{1}{r_0} = \frac{\beta}{2}\left(\frac{\frac{1}{2}\left(D_0 + p_0\right) - \frac{1}{r_0}b_{F0}}{b_{F0} - \frac{1}{r_1}b_{F1}} + \frac{\frac{1}{2}\left(D_0 - p_0\right) + \frac{1}{r_0}b_{F0}}{1 - b_{F0} + \frac{1}{r_1}b_{F1}}\right), \tag{OA.30}$$

and

$$\frac{1}{r_1} = \frac{\beta}{2}\left(\frac{b_{F0} - \frac{1}{r_1}b_{F1}}{b_{F1}} + \frac{1 - b_{F0} - \frac{1}{r_1}b_{F1}}{1 - b_{F1}}\right). \tag{OA.31}$$

Similarly, for the equity price because $s_{S0} - s_0 = s_0 - s_{F0}$

$$
\begin{aligned}
p_0 \;\; = \;\; & \frac{\beta\kappa}{2}\left(\left(\frac{1}{2}\left(D_0 - p_0\right) + \frac{1}{r_0} b_{F0}\right)\beta\eta B_2 + \left(\frac{\frac{1}{2}\left(D_0 + p_0\right) - \frac{1}{r_0} b_{F0}}{b_{F0} - \frac{1}{r_1} b_{F1}} - \frac{\frac{1}{2}\left(D_0 - p_0\right) + \frac{1}{r_0} b_{F0}}{1 - b_{F0} + \frac{1}{r_1} b_{F1}}\right)(z-1)\right) \\
& + \frac{1}{r_0}\left(1 + p_1\right) + \frac{1}{n}\frac{1}{r_0}\left(z - 1\right). \qquad \text{(OA.32)}
\end{aligned}
$$

and because there is no trade in equity at date 1

$$
p_1 = \frac{1}{r_1} + \frac{\beta\kappa}{2}\left(\left(1 - b_{F0} + \frac{1}{r_1} b_{F1}\right)\eta B_2 + \left(\frac{b_{F0} - \frac{1}{r_1} b_{F1}}{b_{F1}} - \frac{1 - b_{F0} + \frac{1}{r_1} b_{F1}}{1 - b_{F1}}\right)(z-1)\right). \qquad \text{(OA.33)}
$$

Given prices and market clearing, we can use the first-order conditions for the optimal asset positions of the F household to find the F household's two debt positions, b_{F0} and b_{F1}. However, we need to compute the price impact matrices using the fringe's pricing functions, which we can do following the algorithm we outline in Online Appendix OA.B.2.

The first-order conditions for household F's optimal debt at dates 0 and 1 are given, respectively, by

$$
\frac{1}{r_0} = \beta \frac{\frac{1}{2}\left(D_0 + p_0\right) - \frac{1}{r_0} b_{F0}}{b_{F0} - \frac{1}{r_1} b_{F1}} - \kappa \frac{1}{W_{f1}}\left(\frac{1}{r_0} b_{F0} - p_0\right), \qquad \text{(OA.34)}
$$

and

$$
\frac{1}{r_1} = \beta \frac{b_{F0} - \frac{1}{r_1} b_{F1}}{b_{F1}} - \kappa \frac{1}{W_{f2}} \frac{1}{r_1} b_{F1}, \qquad \text{(OA.35)}
$$

where W_{f1} and W_{f2} are the implied wealth of the competitive fringe at dates 1 and 2, respectively. Substituting for prices, we can recover b_{F0} and b_{F1}, which solves for the optimal allocations.

For all clean technology to be an equilibrium, we must find the lower bound on prices such that the F household does not want to buy any firm equity from the S household.

At date 1, this requires

$$p_1 \geq \beta \frac{b_{F0} - \frac{1}{r_1} b_{F1}}{b_{F1}} + \beta\kappa \frac{b_{F0} - \frac{1}{r_1} b_{F1}}{b_{F1}} (z-1) - \kappa \frac{1}{W_{f2}} \frac{1}{r_1} b_{F1}, \tag{OA.36}$$

which substituting with equations (OA.33) and (OA.35) reduces to

$$B \geq \bar{B}_1 = \left(\frac{1}{b_{F1}} \frac{b_{F0} - \frac{1}{r_1} b_{F1}}{1 - b_{F0} + \frac{1}{r_1} b_{F1}} + \frac{1}{1 - b_{F1}} \right) (z-1) . \tag{OA.37}$$

At date 0, this requires

$$\begin{aligned} p_0 \quad \geq \quad & \beta \frac{\frac{1}{2}(D_0 + p_0) - \frac{1}{r_0} b_{F0}}{b_{F0} - \frac{1}{r_1} b_{F1}} (1 + p_1) + \beta\kappa \frac{\frac{1}{2}(D_0 + p_0) - \frac{1}{r_0} b_{F0}}{b_{F0} - \frac{1}{r_1} b_{F1}} (z-1) \\ & - \kappa \frac{1}{W_{f1}} (1 + p_1) \left(\frac{1}{r_0} b_{F0} - p_0 \right) + \kappa \frac{1}{r_0} (z-1) , \end{aligned} \tag{OA.38}$$

which substituting with equations (OA.32) and (OA.34) reduces to

$$B \geq \bar{B}_0 = \frac{1}{\beta\eta} \left(\frac{1}{b_{F0} - \frac{1}{r_1} b_{F1}} \frac{\frac{1}{2}(D_0 + p_0) - \frac{1}{r_0} b_{F0}}{\frac{1}{2}(D_0 - p_0) + \frac{1}{r_0} b_{F0}} + \frac{1}{1 - b_{F0} + \frac{1}{r_1} b_{F1}} \right) (z-1) , \tag{OA.39}$$

Consequently, this equilibrium is sustainable if

$$B \geq \bar{B}_s = \max\{\bar{B}_0, \bar{B}_1\}. \tag{OA.40}$$

For disutilities higher than $\bar{B}_s$, the S household is willing to pay at least p_0 to own all equity in the economy, and would buy more if shares were not in unit supply.

Notice if the size of the F household were to tend to zero, and it behaved competitively, the equilibrium would reduce to that with the competitive fringe characterized in Proposition 8. It is then immediate because the F household prices the firm equity with a premium based on the marginal value of her vote that the S household consumes less and delays the full transition to the clean technology (i.e., higher $\bar{B}_s$) compared to the case with a competitive household.

Step 4: Region in which S Household Buys Some Equity

Once $B > B$, the S household has a higher value of control, $\nu_{S1} > \nu_{F1}$, and begins voting for the clean technology with her shares and buying additional shares from the F household.

Proof of Proposition 11:

Step 1: First-Best Allocation

Fix the horizon T. Let X_t be the fraction of firms that use the dirty technology at date t. In the social planner's economy, the planner aims to maximize the welfare of all strategic agents and the competitive fringe subject to the social resource constraint (23), i.e.,

$$\begin{aligned} U_{i0} &= \sup_{\left\{\{c_i\}_{i\in\{f,F,S\}},X_t\right\}} \mathbb{E}\left[\sum_{t=0}^{T}\beta^t\left(\sum_{i\in\{F.S\}}\omega_i\left(\log\left(c_{it}\right)+v_{i,t}\left(\epsilon_t\right)\right)+\omega_f m_f \log\left(c_{ft}\right)\right)\right] \\ s.t. &: (23), \end{aligned} \tag{OA.41}$$

where ω_i are the Pareto weights the planner assigns at $t = 0$ to each agent type, and we choose the normalization

$$\omega_F + \omega_S + m_f\omega_f = 1. \tag{OA.42}$$

Let λ_{yt} be the Lagrange multiplier on the consumption resource constraint (23) at date t. Given these Pareto weights and the sequence of Lagrange multipliers, the first-order conditions for optimal consumption are

$$\frac{\omega_i}{c_{it}} = \lambda_{yt}, \tag{OA.43}$$

$$\frac{\omega_f}{c_{ft}} = \lambda_{yt}, \tag{OA.44}$$

and for the depletion of the social resource

$$\lambda_{yt+1}\left(z-1\right)=\eta\sum_{i\in F,S}\omega_{i}\Xi_{it}\ \ \left(<\ if\ X_{t}=0,>\ if\ X_{t}=1\right), \tag{OA.45}$$

where

$$\Xi_{it}=\sum_{s=t+1}^{T}\beta^{s-t}v_{i,t}'\left(\epsilon_{s}\right), \tag{OA.46}$$

is the marginal life-time cost for household i from depleting the social resource. For financially-motivated households, $\Xi_{Ft}\equiv 0$.

Non-satiation of preferences implies that the social resource constraint (23) will bind.

From the resource constraint (23), we can recover the Lagrange multiplier according to

$$\frac{\sum_{i\in\{F,S\}}\omega_{i}+m_{f}\omega_{f}}{\lambda_{yt+1}}=1+\left(z-1\right)X_{t}, \tag{OA.47}$$

from which follows, along with equation (OA.42) and X_t, that

$$\lambda_{yt+1}=\frac{\sum_{i\in\{F.S\}}\omega_{i}+m_{f}\omega_{f}}{1+\left(z-1\right)X_{t}}=\frac{1}{1+\left(z-1\right)X_{t}}. \tag{OA.48}$$

Substituting equation (OA.48) into (OA.45) , we have that

$$\frac{z-1}{1+\left(z-1\right)\chi_{t}}=\eta\omega_{S}\Xi_{St}\ \ \left(<\ if\ X_{t}=0,>\ if\ X_{t}=1\right). \tag{OA.49}$$

Notice that the left-hand side of equation (OA.49), which we call LHS, is strictly decreasing in X_t because

$$\frac{dLHS}{dX_{t}}=-\frac{\left(z-1\right)^{2}}{\left(1+\left(z-1\right)X_{t}\right)^{2}}<0,$$

while the right-hand side is strictly increasing in X_t because it is a disutility from more pollution. Because X_t is bounded between 0 and 1, it follows that there exists a unique solution to X_t. Given X_t, we can solve for all other equilibrium objects that, conditional on X_t, are also unique. Consequently, there exists a unique First-best equilibrium.

In addition, because the marginal utility from resource depletion (weakly) increases over time, it follows that X_t is decreasing over time since the left-hand side is decreasing in X_t.

Substituting with equation (OA.48) into equation (OA.43), we have that

$$c_{it} = \omega_i \left(1 + (z-1)\, X_t\right) = \omega_i y_t. \tag{OA.50}$$

Step 2: Pareto Weights

The equations in Step 1 characterize an infinite sequence of Pareto optimal equilibria, each parameterized by a different set of the Pareto weights. We choose a sensible criterion for selecting among this multitude by appealing to a planner equilibrium without wealth transfers. That is, the Planner can use any tax and transfer schemes to allocate resources and make production decisions subject that the consumption of all agents must be marketed by their initial wealth.

To determine the cost of a consumption allocation for agent i, we must first determine asset prices, and specifically the interest rate. To do this, we assume the Planner can, in principle, employ storage S_t at a price q_t, and restrict the supply of storage to be 0 in equilibrium. This is the interest rate in the planner's economy and the shadow cost of marginally reallocating consumption between dates t and $t+1$.

It is immediate by solving for the first-order condition for the optimal S_t that

$$q_t = \beta \frac{\lambda_{yt+1}}{\lambda_{yt}}. \tag{OA.51}$$

The shadow cost of a consumption allocation $\{c_{it}\}_{t=0}^{T}$, C_{i0}, is then the inter-temporal pseudo-budget constraint

$$C_{i0} = \sum_{t=0}^{T} \prod_{s=0}^{t} q_{s-1} c_{it}. \tag{OA.52}$$

Our equilibrium selection criterion for the First-Best equilibrium is then to impose that

$C_{i0} = W_{i0}$.

Substituting equations (OA.44) and (OA.51) into equation (OA.52) for the competitive fringe when $C_{f0} = W_{f0}$, we have

$$W_{f0} = \sum_{t=0}^{T} \beta^t \frac{\lambda_{yt}}{\lambda_{y0}} c_{ft} = \frac{1-\beta^{T+1}}{1-\beta} \frac{\omega_f}{\lambda_{y0}}, \tag{OA.53}$$

from which follows that

$$\omega_f = \frac{1-\beta}{1-\beta^{T+1}} \lambda_{y0} W_{f0}. \tag{OA.54}$$

For the strategic agents, recall that the consumption utility of strategic agents is also log. Then, from equations (OA.52), (OA.43), and (OA.51), we have when $C_{i0} = W_{i0}$ that

$$\omega_i = \frac{1-\beta}{1-\beta^{T+1}} \lambda_{y0} W_{i0}. \tag{OA.55}$$

It is then immediate that conditional on the sequence of λ_{yt} that the Pareto weights based on the initial wealth distribution exist and are unique. Further, these weights are continuous in this sequence. From equations (OA.53), (OA.52), and (OA.42), we further have that

$$\lambda_{y0} = \frac{1-\beta^{T+1}}{1-\beta} \frac{1}{2W_{i0} + m_f W_{f0}}, \tag{OA.56}$$

from which follows from equation (OA.55) that

$$\omega_i = \frac{W_{i0}}{2W_{i0} + m_f W_{f0}} = \frac{1}{2+m_f}, \tag{OA.57}$$

because $W_{i0} = W_{f0}$. Consequently, we can decentralize the First-Best equilibrium, and it is unique.

Step 3: Moving to Infinite

Taking the limit as $T \to \infty$, our arguments remain valid, and consequently we have characterized the first-best equilibrium in the economy.

Step 4: Connection to Grossman and Hart [1979]

Notice that we can rewrite equation (OA.49), substituting with (OA.50) and (OA.57), at an interior solution as

$$\frac{1}{2+m_f}\left(\sum_{i\in F,S}\frac{\beta c_{it}}{c_{it+1}}+m_f\frac{\beta c_{ft}}{c_{ft+1}}\right)(z-1)=\frac{\beta\eta\Xi_{St}}{1/c_{St}}. \tag{OA.58}$$

The left-hand side term in equation (OA.58) corresponds to the voting criterion in Grossman and Hart [1979] in which each shareholder votes based on initial equity positions at date 0. This rule is modified by the pecuniary value of the marginal disutility of the socially responsible household S, $\frac{\beta\eta\Xi_{St}}{1/c_{St}}$. With voting decisions fixed by initial equity, there is no value to trading equity in financial markets to acquire or sell votes. Consequently, there are no gains from trade and the equilibrium allocation is again autarky.

OA.B. Numerical Algorithm

OA.B.1 One SRI Household and Competitive Fringe:

With only one strategic household, we can assume that the S household follows identical strategies in both assets. Consequently, we can drop the subscript from stock prices and positions.

We can specify the state space for the economy as the vector $a_t=\begin{bmatrix}\epsilon_t & b_{St-1} & s_{St-1} & \chi_{t-1}\end{bmatrix}$ that lies in a bounded set $\mathcal{A}$, and construct functions $p(a_t)$ and $\Xi_i(a_t)$. We can then recover household consumption according to

$$c_{ft}=(1-\beta)W_{ft},$$
$$c_{it}=D_t-m_fc_{ft}=(1+\chi_{t-1}(z-1))-(1-\beta)m_fW_{ft},$$

and the wealth of the strategic S household by market clearing in asset markets as

$$W_{ft} = b_{ft-1} + (1 + \chi_t (z-1) + p_t) s_{ft-1}, \tag{OA.59}$$

$$W_{it} = (1 + \chi_t (z-1) + p_t) - m_f W_{ft}. \tag{OA.60}$$

Using Proposition 1, we solve for asset prices according to

$$p_t = \beta \frac{W_{ft}}{W_{ft+1}} \left((1 + \chi_t (z-1)) + p_{t+1} (a_{t+1}) \right), \tag{OA.61}$$

which we can rewrite equation (OA.59) as

$$p_t = \frac{\left(b_{ft-1} + (1 + \chi_t (z-1)) s_{ft-1} \right) \beta \frac{1+\chi_t(z-1)+p(a_{t+1})}{W_{ft+1}}}{1 - s_{ft-1} \beta \frac{1+\chi_t(z-1)+p(a_{t+1})}{W_{ft+1}}}, \tag{OA.62}$$

We can use equation (OA.62) to find p_0 and p_1 by recognizing that $p_2 = 0$.

For a given conjecture of $\{s_{S0}, s_{S1}\}$ and $\{b_{S0}, b_{S1}\}$ for stock and bond positions at dates 0 and 1, there is an unique path for whether the S household chooses the dirty or clean technology at each date. We solve for the wealth and consumption of the S household and the competitive fringe f at date 2 for the conjectured path. We then solve backwards for date 1 wealth, consumption, and prices, and the S household's technology choices based on its optimization program. We repeat this procedure to solve for date 0 wealth, consumption, and prices and the S household's technology choices.

Given prices and consumption, we then calculate the residuals from the S household's Euler equations. We then search for a fixed point that constitutes an equilibrium.

To bound the controls, we recognize that stock holdings when there is no short-selling is bounded between 0 and 1. Further, we can bound bond positions by the total maximum wealth in the economy required to buy all an agent's stock at each date; for date 2, this is z, at date 1, it is $(1+\beta) z$, and date 0, it is $\left(1 + \beta + \beta^2\right) z$,

We then search for an equilibrium as a fixed point over the S household's asset positions at date 0 and dates 1 using his four first-order conditions for b_{St} and s_{St}.

OA.B.2 One SRI Household and One Financially-motivated Household:

We consider the special case in which the competitive fringe becomes arbitrarily small, i.e., $m_f \to 0$. In this case, we can recover the asset positions of the F household from those of the S household by market clearing.

We can specify the state space for the economy as the vector $a_t = \begin{bmatrix} \epsilon_t & b_{St-1} & s_{St-1} & \chi_{t-1} \end{bmatrix}$ that lies in a bounded set $\mathcal{A}$, and construct functions $p(a_t)$ and $\Xi_i(a_t)$. We can then recover household consumption according to

$$c_{St} = W_{St} - \frac{1}{r_t} b_{St} - p_t s_{St}, \tag{OA.63}$$

$$c_{Ft} = D_t - c_{St}, \tag{OA.64}$$

and the wealth of the strategic S household can be recovered as

$$W_{St} = b_{St-1} + (D_t + p_t) s_{St-1}, \tag{OA.65}$$

and substituting with equation (OA.63)

$$c_{St} = b_{St-1} - \frac{1}{r_t} b_{St} + D_t s_{St-1} - p_t (s_{St} - s_{St-1}). \tag{OA.66}$$

We can then summarize prices by summing strategic agents' first-order conditions as

$$\begin{aligned} p_t &= \beta \left(\frac{D_{t+1} + p_{t+1}}{c_{St+1}} - \frac{D_{t+1} + p_{t+1}}{c_{Ft+1}} \right) \frac{c_{St}}{2} + \frac{D_t}{2c_{Ft+1}} (D_{t+1} + p_{t+1}) \\ &\quad + \frac{1}{2} (\nu_{St} + \nu_{Ft}) + \frac{1}{2} h_{St,22} (s_{St} - s_{St-1}) - \frac{1}{2} h_{St,22} (s_{Ft} - s_{Fjt-1}), \end{aligned} \tag{OA.67}$$

where $s_{Ft} - s_{Ft-1} = -(s_{St} - s_{St-1})$ by market clearing, and

$$\frac{1}{r_t} = \beta\left(\frac{1}{c_{St+1}} - \frac{1}{c_{Ft+1}}\right)\frac{c_{St}}{2} + \beta\frac{D_t}{2c_{Ft+1}} + h_{St,12}\left(s_{St} - s_{St-1}\right), \tag{OA.68}$$

because the direct price impact terms cancel. From equation (OA.68), we recognize that the fringe's state prices can be recovered as

$$q_t = \beta\frac{1}{2}\left(\frac{c_{St}}{c_{St+1}} + \frac{c_{Ft}}{c_{Ft+1}}\right) + \frac{1}{2}h_{St,12}\left(s_{St} - s_{St-1}\right) - \frac{1}{2}h_{St,12}\left(s_{Ft} - s_{Fjt-1}\right). \tag{OA.69}$$

Define

$$a_r = \beta\left(\frac{1}{c_{St+1}} - \frac{1}{c_{Ft+1}}\right), \tag{OA.70}$$

and

$$a_j = \beta\left(\frac{1}{c_{St+1}} - \frac{1}{c_{Ft+1}}\right)\left(D_{t+1} + p_{t+1}\right). \tag{OA.71}$$

Substituting with equation (OA.66) into equations (OA.67) and (OA.68), we arrive at the matrix equation

$$A\begin{bmatrix} p_t & \frac{1}{r_t}\end{bmatrix}' = \boldsymbol{d}. \tag{OA.72}$$

and

$$A = \begin{bmatrix} 2 + a\left(s_{St} - s_{St-1}\right) & a_A b_{St} \\ a_r\left(s_{St} - s_{St-1}\right) & 2 + a_r b_{St}\end{bmatrix} \tag{OA.73}$$

and

$$\boldsymbol{d} = \beta\begin{bmatrix}\left(\frac{D_{t+1}+p_{t+1}}{c_{St+1}} - \frac{D_{t+1}+p_{t+1}}{c_{Ft+1}}\right)\left(b_{St-1} + D_t s_{St-1}\right) \\ \left(\frac{1}{c_{St+1}} - \frac{1}{c_{Ft+1}}\right)\left(b_{St-1} + D_t s_{St-1}\right)\end{bmatrix} + \begin{bmatrix}\frac{D_t}{c_{Ft+1}}\left(D_{t+1} + p_{t+1}\right) + \frac{1}{2}\left(\nu_{St} + \nu_{Ft}\right) + h_{St,22}\left(s_{St} - s_{St-1}\right) \\ \frac{D_t}{c_{Ft+1}} + h_{St,12}\left(s_{St} - s_{St-1}\right)\end{bmatrix}. \tag{OA.74}$$

We can then recover asset prices as

$$\begin{bmatrix} p_t \\ \frac{1}{r_t} \end{bmatrix} = A^{-1}\boldsymbol{d}. \tag{OA.75}$$

What remains is to recover price impact. Although the level asset prices are pinned down by the average marginal utility of strategic households, we must select a price impact function by parameterizing the fringe's wealth process. To do this, we assume that the fringe is also endowed with s_0 units of the stock of each firm and zero debt; because they are infinitesimal relative to strategic households, this initial allocation does not impact equilibrium allocations.

The state prices and sequential budget constraints of the fringe then provide a system of equations to identify their pseudo-holdings in stocks and bonds at dates 0 and 1.

For instance, at date 0, the system of equations is

$$\begin{bmatrix} \frac{1}{r_0} & p_0 \\ 1 & p_1 \end{bmatrix} \begin{bmatrix} b_{f0} \\ s_{S0} \end{bmatrix} = (1+\beta)\,\beta c_{f0} \begin{bmatrix} 1 \\ 1/q_0 \end{bmatrix}, \tag{OA.76}$$

where $c_{f0} = \frac{W_{f0}}{1+\beta+\beta^2}$ because the fringe has log utility, and $W_{f0} = p_0 s_0$ by our assumption of the fringe's initial endowment.

Given our pseudo process for fringe asset holdings and wealth at each date, we can calculate price impact using our expressions from Proposition 1.

We then search for an equilibrium as a fixed point over the S household's asset positions at date 0 and dates 1 in the high and low state using his two first-order conditions for b_{St} and s_{St}.