Confidentiality and Competition in Concurrent Bargaining

Jangwoo Lee *

February 28, 2026

Abstract

I analyze a dynamic model of concurrent bargaining in which multiple prospective buyers compete to trade with an informed seller. When the seller maintains confidentiality over buyers' past offers, buyers may engage in competitive "price experimentation": buyers risk early losses to subsequently acquire informational advantages over competitors and expect to earn future information rents. Due to price experimentation, the seller may benefit from maintaining confidentiality over past offers and restricting buyer entry. The model has implications for the strategic choice between auctions and negotiations, and for the common use of "pre-qualification" in asset sales.

*Chinese University of Hong Kong. Email: matizoma@gmail.com. This is a substantially revised version of my doctoral thesis. I am deeply indebted to my supervisor, William Fuchs, for his guidance and support. I am also grateful for the advice I have received from Andres Almazan, Aydogan Alti, Sheridan Titman, and Thomas Wiseman. I have also benefited from helpful discussions with Piero Gottardi (discussant), Pablo Ruiz-Verdú (discussant), Qing Liu (discussant), Jian Sun (discussant), Heski Bar-Isaac, Kerry Back, Briana Chang, Sudipto Dasgupta, Felix Feng, Faruk Gul, Joonkyo Hong, Hyun Hwang, Yunzhi Hu, Ron Kaniel, John Kuong, Andrey Malenko, Moritz Meyer-ter-Vehn, Laura Veldkamp, Alex Priest, Tim Park, Harry Di Pei, Uday Rajan, Vasiliki Skreta, Maxwell Stinchcombe, Mahyar Sefidgaran, Taeuk Seo, Andrzej Skrzypacz, Yizhou Xiao, David Xiaoyu Xu, Liyan Yang, Yenan Wang, Mark Whitmeyer, and Jeffrey Zwiebel. I would also like to thank seminar participants at AEFIN Finance Forum, Barcelona BSE Summer Forum, China International Conference in Finance (CICF), Durham Economic Theory Conference, Econometric Society meetings, FTG summer school, GAMES, SITE (Dynamic Games, Contracts and Markets), Stony Brook International Conference on Game Theory, Korea University, Hong Kong Joint Finance Workshop, CUHK and UT Austin. A previous version of the paper was circulated under the title "Price Experimentation in Confidential Negotiations."

Concurrent negotiations are pervasive in corporate asset sales. For instance, in the market for corporate control, a target firm frequently enters into parallel merger talks with potential acquirers during the "private phase" of its takeover process [Boone and Mulherin, 2007, 2009, Eckbo, Malenko, and Thorburn, 2020]. Likewise, in bankruptcy fire sales, distressed asset sellers—e.g., debtors and secured creditors—frequently solicit interest from competing buyers to maximize the liquidation value of the underlying collateral.[1] Indeed, practitioners have long recognized the importance of "competition on the same side of the table" in real-world negotiations [Subramanian, 2011]. However, concurrent negotiations have received relatively scant attention in the literature on dynamic bargaining à la Coase [1972], despite its wide range of applications across finance and economics.[2]

Based on this motivation, I study a tractable bargaining model in which a seller concurrently negotiates the price of an indivisible asset with multiple buyers over two periods. At the beginning of the model, the seller has private information about the quality of the underlying asset. All buyers share a prior belief about asset quality and assign a common value to the asset. At the beginning of each period, buyers can submit price offers for the asset. After receiving the offers, the seller may accept an offer and close the deal, or reject them altogether. Neither the seller nor the buyers can commit to future terms of trade. All players are impatient and long-lived, so bargaining delays reduce gains from trade. Nevertheless, information asymmetry between buyers and the seller can generate a lemons problem [Akerlof, 1970] and may inefficiently protract trade. While this setup builds on the classic literature on dynamic bargaining, it also aligns with the fluid nature of the deal processes frequently used in recent corporate takeovers.[3]

Within this framework, I consider two alternative protocols: a *confidential negotiation process* (i.e., one in which buyers' offers are kept hidden from competing buyers) and a *public negotiation process* (i.e., one in which buyers' past offers are widely observable). Comparing bargaining dynamics across the two protocols allows me to revisit central issues in the theory of corporate takeovers and the broader literature on auctions versus negotiations [Bulow and Klemperer, 1996, 2009, Gentry and Stroup, 2019, Hoffmann and Vladimirov, 2025]. For instance, in

[1]The following practitioner articles describe typical procedures in expedited Section 363 sales and Article 9 foreclosure sales (accessed December 12, 2025): *"Purchasing Assets Out of Bankruptcy"* (Faegre Drinker Biddle & Reath LLP), available at `https://www.faegredrinker.com/webfiles/Buy%20it%20Cheap%20Buy%20it%20Right.pdf`; and *"UCC Article 9 Foreclosure Sales: An Alternative to Bankruptcy Code Section 363 to Buy and Sell Distressed Assets"* (ABA Commercial Law Newsletter), available at `https://www.sheppardmullin.com/assets/htmldocuments/Commercial%20Law%20Newsletter_November%202013.pdf`.

[2]Seminal works in the economics literature are Fudenberg and Tirole [1983], Gul, Sonnenschein, and Wilson [1986], Ausubel and Deneckere [1989], Deneckere and Liang [2006], Fuchs and Skrzypacz [2010], Daley and Green [2020], Gerardi, Maestri, and Monzón [2022]. Recently, a growing number of capital structure models have extended Coasean dynamics—and, more broadly, intertemporal competition with future selves—to study leverage dynamics without commitment (see, for instance, Admati, DeMarzo, Hellwig, and Pfleiderer [2018], DeMarzo and He [2021]).

[3]See [Liu, Officer, and Tu, 2023] for recent developments in corporate takeovers.

many corporate asset markets (e.g., M&A and bankruptcy sales), why do many sell-side advisors often restrict entry and maintain confidentiality over past offers? Why do offer confidentiality and restricted entry so often appear together in practice, and how do these two features interact with each other in concurrent negotiations [Subramanian, 2011, Roberts, 2009]?

The model produces two main results: (1) offer confidentiality encourages price experimentation and tends to benefit the seller; (2) the seller's expected revenue in the confidential negotiation process *decreases* with the number of buyers, as long as there are at least two buyers. By contrast, in the public negotiation process with multiple buyers, bargaining dynamics do not depend on the number of buyers.[4] Thus, these results highlight a novel interaction between offer confidentiality and buyer entry in concurrent bargaining.

The underlying intuition behind both results is that, in the confidential negotiation process, buyers screen the informed seller through "price experimentation." Namely, an experimenting buyer submits an aggressive offer and risks a loss in the earlier period so that the buyer can expect to reap an information rent in the future. On the one hand, if the seller accepts the earlier offer, the buyer overpays for the asset and realizes a loss. On the other hand, if the seller rejects the offer, the buyer learns that the seller's value is likely to be high and infers that the asset is also likely to be of high quality. Since the amount of the earlier offer is unknown to competitors in the confidential negotiation process,[5] the buyer subsequently acquires private information about the seller's asset quality and thus possesses an informational advantage over competitors. Thus, even in the face of competitive pressure, the buyer expects to earn an information rent in the later period, thereby offsetting the risk of a loss in the earlier period.

Importantly, an experimenting buyer pushes up the price level not only in the earlier period but also in the subsequent period. If a buyer makes an aggressive earlier offer and is rejected, the buyer becomes more optimistic about asset quality and thus more eager to win the asset in the later period. Subsequently, the buyer submits another offer to outbid non-experimenting buyers in the later period. As a result, price experimentation leads to a higher ensuing offer in the subsequent period and thus leads to higher revenue for the seller.

By contrast, no price experimentation occurs in the public negotiation process. Intuitively, when past offers are publicly observable, buyers are always symmetrically informed and thus compete away any information rent.

[4] In the public negotiation process, buyers are always symmetrically informed and assign a common value to the underlying asset. As a result, in each period, buyers compete for the asset in the classic Bertrand fashion.

[5] Whenever a buyer engages in price experimentation in the confidential negotiation process, the buyer randomizes its earlier offer amount to ensure that its information acquired through price experimentation is kept hidden from competing buyers.

Hence, even if a buyer attempts price experimentation, the buyer only risks a loss in the earlier period, without reaping any expected profit in the later period. As a result, buyers only submit conservative offers in every period, and thus the seller is worse off.

Notably, offer confidentiality does not benefit the seller at the expense of buyers. Since buyers are identical at the outset of the negotiation process, they expect to break even over their lifetime, regardless of offer confidentiality. Therefore, it must be the case that the seller enjoys efficiency gains from offer confidentiality. The intuition is that an experimenting buyer can expect an information rent in the later period, and thus does not bid up to its posterior expected value of the asset. Hence, from the seller's perspective, the buyer's ensuing offer tends to be less attractive relative to an experimenting buyer's earlier offer. As a result, the seller accepts the earlier offer more frequently. Since the players are all impatient, the higher likelihood of an earlier trade increases the expected total surplus. Such an efficiency gain ultimately accrues to the seller in the form of higher revenue.

A similar logic applies to the second finding: when a smaller number of buyers participate in the confidential negotiation process, trade efficiency increases, and the seller expects higher discounted revenue. With fewer competitors, an experimenting buyer faces less competitive pressure and need not make an offer so aggressive as to outbid other buyers in the later period. Subsequently, the buyer submits a more conservative offer to maximize its expected information rent in the later period. Hence, for a given level of the price in the earlier period, the price in the ensuing period tends to be more depressed, so the seller more frequently accepts the price in the earlier period. Since the higher likelihood of a bargaining agreement in the earlier period increases the efficiency of the negotiation process, the seller benefits from restricted entry.

While the relationship between expected prices and buyer entry in the confidential negotiation process is reminiscent of the winner's curse [Wilson, 1977, Milgrom, 1981, Bulow and Klemperer, 2002], the underlying mechanism is distinct.[6] In standard common-value auctions, when a buyer wins the underlying asset, the buyer infers that competitors had relatively pessimistic information about asset value, which leads to a downward revision in the buyer's valuation. Instead, in my model, the winning buyer is informed of a sufficient statistic for asset quality given all buyers' information *even prior to winning the asset*. As a result, winning does not lead to additional updating about expected asset value.

[6] An interesting work by Axelson and Makarov [2020] studies the role of an informational black hole in a static auction for project financing. Like my paper, they also find that an auctioneer can benefit from maintaining offer confidentiality and restricting the number of bidders. Their results are driven by the failure of information aggregation. By contrast, buyers are ex ante identical in the baseline model, so there is a limited role for information aggregation.

In addition to being of theoretical interest, my model can be extended to shed light on real-world asset sales in two ways. In the first extension, I incorporate heterogeneous information among buyers about their common value of the asset from the outset. This captures a salient feature of many corporate asset sales: in practice, buy-side financial advisors often form their own valuations of the underlying asset through due diligence and proprietary screening technologies.[7]

In this setting, I characterize the seller's revenue-maximizing number of prospective buyers. The analysis shows that when buyers hold dispersed private information, it is optimal for the seller to invite a larger number of buyers to the asset sale. Intuitively, when additional buyers participate in asset sales, the seller is more likely to face highly optimistic buyers. As a result, as buyers have increasingly dispersed private information, the seller's expected gain from the higher likelihood of facing more optimistic buyers may dominate the expected loss from diminished price experimentation. Hence, it is optimal for the seller to invite many prospective buyers to the asset sale.

This finding sheds new light on the classic tradeoff between auctions and negotiations. In particular, it helps understand differences in the typical number of prospective buyers per sale across corporate asset markets. The average bankruptcy sale attracts fewer than two serious buyers [LoPucki and Doherty, 2007], whereas the average auction of a large public company involves more than 12 potential acquirers, and even more when a financial buyer wins [Gorbenko and Malenko, 2014]. The paper suggests that this difference in practices may reflect heterogeneity in the relevant information frictions. Indeed, in many distressed asset sales, buyers face a tight timeline to form their own valuations and incur a high risk of adverse selection due to the seller's private information [Ayotte and Skeel Jr, 2013, Jacoby and Janger, 2013]. In contrast, in auctions of publicly traded companies, financial buyers are typically private equity firms with the expertise to produce their own assessments of the target, and are thus likely to be heterogeneously informed.[8] Hence, my paper illustrates how various informational frictions can have distinct implications for the strategic choice between auctions and negotiations.

In the second extension, I show how the seller can benefit from excluding less serious buyers — i.e., those who made less aggressive early offers — from participating in the later period, a process sometimes called "pre-

[7]For example, BCG states in its 2025 M&A report that its due diligence consulting team uses proprietary data analytics to reveal hidden value drivers and draws out answers based on its experience and expertise. See the following link for details (accessed December 2, 2025): `https://web-assets.bcg.com/c6/bd/d83b3b044a1eb67ef6822146d5f7/the-2025-m-a-report-oct-2025-web.pdf`.

[8]See, for instance, Cendrowski, Petro, Martin, and Wadecki [2012] for practitioners' accounts of due diligence conducted by private equity firms.

qualification" in the legal profession. The analysis shows that when the seller can credibly commit not to elicit later offers from excluded buyers, pre-qualification increases the seller's expected revenue in each period. Intuitively, pre-qualification protects information rents from price experimentation in the later period. In turn, this strengthens buyers' incentives to experiment with earlier offers and eventually increases the likelihood of early trade. Therefore, the seller can capture efficiency gains from pre-qualification in the form of higher expected revenue. This result is consistent with the common use of pre-qualification in distressed asset sales, where bankruptcy courts can serve as credible gatekeepers and frequently constrain non-qualified parties from entering the later stages of the sale.[9]

On a broader level, the paper puts forward a joint rationale for both offer confidentiality and restricted entry in many corporate asset sales. Since these two features frequently go hand in hand in practice, it is natural to conceive that they are connected. The paper illustrates a common economic force between the two practices, and thus suggests a new mechanism through which they may be interrelated.

Literature Review My work lies at the intersection of several literatures. First, my paper belongs to the rich literature on dynamic bargaining with interdependent values (Evans [1989], Vincent [1989], Deneckere and Liang [2006], Fuchs and Skrzypacz [2010]). My main contribution to this literature is to consider a model of concurrent bargaining between multiple long-lived buyers and a long-lived seller. The closest paper in the literature to mine is a breakthrough work by Daley and Green [2020]—henceforth, DG. The key innovation of DG's paper is to incorporate exogenous "news" into a model of bilateral bargaining, in which a monopsonistic buyer makes all the offers to a single informed seller. One of their key findings is that the presence of news can limit the degree to which the buyer can utilize bargaining delay to extract additional rents, and lead to more vigorous price experimentation by the buyer. As a result, the seller can potentially be better off by bargaining with a monopsonistic buyer and giving up all the bargaining power (i.e., letting the buyer make all offers), relative to facing a competitive pool of short-lived buyers in each period.

There are several ways in which my paper departs from DG's model of bilateral bargaining. Most importantly, my model of concurrent bargaining illustrates how multiple long-lived buyers can *competitively* engage in price experimentation in the absence of exogenous news. By contrast, DG's benchmark model for the competitive case

[9]For a general description of the Chapter 11 sale process supervised by bankruptcy courts, see Ayotte and Ellias [2022] and the following bankruptcy court's guidelines for the conduct of distressed asset sales: `https://www.nysb.uscourts.gov/sites/default/files/pdf/Guidelines_for_Asset_Sales.pdf`

involves a pool of *short-lived* buyers, who cannot engage in any form of price experimentation. Furthermore, my paper shows how offer confidentiality encourages price experimentation by multiple long-lived buyers. In so doing, the paper unearths a novel interaction between offer confidentiality and buyer entry. By contrast, since two players in bilateral bargaining can observe all previously rejected offers, offer confidentiality plays a limited role in DG's framework. Therefore, the two papers study separate (but closely related) issues in dynamic bargaining.

Second, the paper is part of the literature on auctions versus negotiations [Bulow and Klemperer, 1996, 2009, Gentry and Stroup, 2019, Hoffmann and Vladimirov, 2025]. I contribute to this literature in two ways. First, I show that offer confidentiality can play an integral role in this choice. Specifically, by protecting information rents generated through price experimentation, offer confidentiality can strengthen the seller's incentive to bargain with a small set of prospective buyers. In contrast, this literature tends to focus on English auctions where offer observability plays a limited role. Second, unlike most papers in this literature, I consider both information asymmetry among buyers and information asymmetry between buyers and the seller in my extension. The analysis shows that these two frictions have qualitatively different implications for the strategic choice between auctions and negotiations.

The paper also belongs to another strand of the literature on dynamic adverse selection that examines the effects of offer transparency (Swinkels [1999], Kremer and Skrzypacz [2007], Hörner and Vieille [2009], Kaya and Liu [2015], Fuchs, Öry, and Skrzypacz [2016]). This literature has considerably enriched our understanding of how offer confidentiality may increase trade efficiency in dynamic trading environments.[10] Despite recent progress, most of this literature focuses on confidential trading environments with *short-lived* uninformed players.

By incorporating multiple uninformed long-lived players, my paper differs from this literature in several ways. Most importantly, the price experimentation motive does not arise in the models considered in this literature because short-lived players cannot make use of price experimentation. Hence, I provide a distinct rationale for offer confidentiality from those identified in the literature. Relatedly, in a dynamic trading environment with multiple short-lived buyers in each period, the number of participating buyers does not affect trading dynamics. This occurs because short-lived buyers compete for the average-quality asset in the classic Bertrand fashion. This sharply contrasts with my model, where the seller's expected payoff monotonically decreases in the number of participating buyers in the confidential negotiation process.

[10] Asriyan, Fuchs, and Green [2017] study the effect of post-trade transparency in a dynamic trading environment with short-lived buyers. They show that post-trade transparency can lead to multiplicity of equilibria and can either increase or decrease trade efficiency depending on the equilibrium being played.

Chaves [2019] contributes to the literature in a similar direction. In particular, he endogenizes the entry of short-lived third parties in Fuchs and Skrzypacz [2010]'s framework and studies how offer confidentiality affects bargaining dynamics. My model of concurrent bargaining differs from his model in a number of ways. Most importantly, I study concurrent bargaining between multiple long-lived uninformed buyers and a long-lived seller, whereas he analyzes bilateral bargaining between two long-lived parties, which may be interrupted by short-lived third parties. Additionally, the key economic force behind my results is competitive price experimentation, whereas the price experimentation motive does not arise in his model. In this regard, the two papers concern distinct strategic aspects.

My paper is also related to the extensive literature on auctions with costly bidder entry [Samuelson, 1985, Hansen, 2001, French and McCormick, 1984, Quint and Hendricks, 2018]. The key insight of this literature is that a seller may find it optimal to restrict bidder entry in the presence of entry costs. Unlike these works, my model has no explicit entry costs. Hence, the mechanism of my paper is distinct from that proposed in this strand of the literature. Second, these models consider patient players, so these papers have limited implications for transaction speed. By contrast, in my model, offer confidentiality and restricted buyer entry in the confidential negotiation process increase expected revenue precisely because they accelerate transaction speed. Third, in contrast to much of this literature, my analysis allows for information asymmetry both among buyers, and between buyers and the seller.

Finally, a parallel strand of literature in market microstructure builds upon Glosten and Milgrom [1985] to study price experimentation [Bloomfield and O'Hara, 2000, Leach and Madhavan, 1993].[11] The main insight of this literature has to do with how market makers can engage in price experimentation to expedite price discovery. My model differs from these papers in two fundamental aspects. First, dynamic signaling is absent in these works. In these works, long-lived market makers face a sequence of short-lived counterparties who are either noise traders or informed traders. Therefore, informed traders are short-lived and do not delay their trades to signal their private information. Second, relatedly, offer confidentiality has ambiguous effects on welfare. When market makers engage in price experimentation in a confidential trading environment, early traders benefit at the expense of future traders. This differs from my setting where offer confidentiality can lead to a Pareto improvement.

[11]From a technical perspective, de Frutos and Manzano [2005] is the most closely related to my paper in this literature. In their paper, market makers in a confidential trading environment use mixed strategies to engage in price experimentation, whereas market makers in a public trading environment use pure strategies and do not engage in price experimentation.

I. Model

A. Setup

There are N (≥ 2) long-lived prospective buyers (henceforth called "buyers") and a single long-lived seller. I denote the collection of all buyers by $\mathcal{I}$, with a generic buyer i. At the beginning of the model, the seller has an indivisible asset and knows its asset quality $\theta \in \{L, H\}$, whereas the buyers share the prior belief that the seller's asset quality is low with probability $\pi_L \in (0, 1)$. The asset (seller) is referred to as the high-quality asset (seller) whenever $\theta = H$, whereas it is referred to as the low-quality asset (seller) whenever $\theta = L$.

Each buyer has two bargaining opportunities at $t \in \{1, 2\}$. At the beginning of each period, all buyers submit offers on the asset. After receiving the offers, the seller can either reject them altogether or accept the maximum offer, which is henceforth referred to as the "price." If the seller accepts the price, bargaining ends immediately. Consistent with standard terminology, offers that the seller would surely reject are referred to as "losing offers," and offers that the seller accepts with strictly positive probability are referred to as "serious offers."

At the end of each period, the asset generates a cash flow. More specifically, at the end of period 1, the asset generates an interim cash flow of $\$(1-\delta)c_\theta$ when the seller owns the asset, and $\$(1-\delta)v_\theta$ when a buyer owns the asset. At the end of period 2, the asset produces a final cash flow of $\$c_\theta$ when the seller owns the asset and $\$v_\theta$ when a buyer owns the asset. These cash flows are observable only to the owner.[12] Moreover, the asset generates higher cash flows when a buyer owns the asset (i.e., $v_\theta > c_\theta$). Also, when asset quality is high, the asset produces higher cash flows (i.e., $v_H > v_L$ and $c_H > c_L$). Therefore, each buyer derives a higher value from the asset than the seller does, so there is common knowledge of gains from trade. Nevertheless, when buyers are sufficiently pessimistic about asset quality, the trading outcome can be inefficient due to the adverse selection problem. All players have a common discount factor $\delta \in (0, 1)$. I also normalize $c_L = 0$ to simplify the exposition. All my analyses hold when $c_L \geq 0$.

If buyer i offers p_t^i and wins the asset of quality θ in period t, its profit is

$$\delta^{t-1}(v_\theta - p_t^i).$$

[12] I maintain this assumption solely for expositional simplicity. Alternatively, the players' payoffs can also be micro-founded as follows: (1) the asset yields a publicly observable cash flow at most once; (2) as long as the asset has not produced a cash flow, it generates a fixed cash flow with some probability that depends on the asset's underlying quality, and nothing otherwise, in each period; (3) the seller's marginal utility from consumption differs from that of all buyers.

If the θ-quality seller accepts the price $p_t := \left(\max_{i\in\mathcal{I}} p_t^i\right)$ in period t, its payoff is

$$(1-\delta^{t-1})c_\theta + \delta^{t-1}p_t.$$

In particular, if the seller rejects offers in both periods, the seller obtains its reservation value c_θ.

I consider two bargaining protocols: (1) the *public negotiation process* and (2) the *confidential negotiation process*. In the public negotiation process, all past offers are publicly observable. Formally, the public history h_1^P at the beginning of period 2 is given by the vector of previously rejected period-1 offers $(p_1^i)_{i\in\mathcal{I}}$. By convention, the public history h_0^P at the beginning of period 1 is set to $\emptyset$. In the confidential negotiation process, buyers can never observe their competitors' offer amounts. In particular, buyer i's private history h_1^i at the beginning of period 2 consists of its own period-1 offer, $h_1^i := (p_1^i)$. Also, a buyer's private history h_0^i at the beginning of period 1 is set to $\emptyset$. The seller can observe all the offers made by the buyers, so its information at its period-t decision node is given by the history of all the offers buyers have made up to period t: $(p_\tau^i)_{\substack{i\in\mathcal{I}\\ \tau\le t}}$.

For the purpose of this paper, it would be uninteresting to consider parametric configurations for which price experimentation can never occur.[13] To rule out such cases, I maintain the following assumptions:

$$\pi_L v_L + (1-\pi_L)v_H < c_H; \qquad \textbf{(SIC)}$$

$$\delta v_H > v_L; \qquad \textbf{(NFS)}$$

$$v_L > \delta c_H. \qquad \textbf{(EXP)}$$

Assumption (**SIC**) is also referred to as the "static incentive compatibility" condition in the literature (e.g., Deneckere and Liang [2006]). It states that adverse selection is severe enough to preclude the first-best pooling equilibrium, in which the seller always accepts the period-1 price and full efficiency is thus achieved. Furthermore, this assumption helps circumvent some technical issues associated with equilibrium refinements.[14] Assumption

[13] Assumptions (**NFS**) and (**EXP**) imply that the discount factor lies in some "intermediate" range $[\frac{v_L}{v_H}, \frac{v_L}{c_H}]$. In Section VI.C, I study a continuous-quality version of the model in which price experimentation occurs for high discount factors.

[14] An earlier work by Noldeke and Van Damme [1990] considers a dynamic version of the Spencian signaling model with publicly observable offers, which is isomorphic to the public negotiation process under the alternative assumption $\pi_L v_L + (1-\pi_L)v_H \geq c_H$. In their model, uninformed parties can implicitly collude by threatening to adopt incredible beliefs whenever one party "misbehaves" in period 1 and may make a strictly positive expected profit in equilibrium. This issue no longer arises under (**SIC**), because uninformed parties compete to trade with the low-quality informed party in period 1 in the classic Bertrand fashion under this assumption.

(**NFS**) eliminates fully separating equilibria in which the low-quality seller accepts the period-1 price with probability 1 and the high-quality seller trades in period 2 with probability 1. Assumption (**EXP**) ensures that buyers have sufficiently strong incentives to engage in price experimentation in the confidential negotiation process.[15] When combined, Assumptions (**SIC**), (**NFS**), and (**EXP**) ensure that the confidential negotiation process induces price experimentation on the equilibrium path.

REMARK 1: *Assumptions* (**NFS**) *and* (**EXP**) *together imply that the discount factor* δ *lies in an intermediate range. To present the mechanism behind price experimentation as transparently as possible, the paper focuses on the two-type specification with an intermediate discount factor* δ*. In Subsection VI.C, I extend the analysis to a continuum of types and show that price experimentation remains central to bargaining dynamics even as the discount factor* δ *approaches* 1.

B. Equilibrium Concept for Confidential Negotiation Process

The primary focus of my paper is the analysis of the confidential negotiation process, where characterization of equilibrium poses several challenges. Recall that in the confidential negotiation process, buyers never observe the amounts of their competitors' earlier offers. Thus, if the seller is more likely to accept an aggressive offer in the earlier period, a buyer who made an aggressive offer in the earlier period may acquire private information that the asset quality is likely to be high. Therefore, the continuation game in the later period may essentially become a first-price common-value auction among heterogeneously informed bidders, whose signals are endogenously determined by the seller's acceptance strategy in the earlier period. Hence, even when each buyer has at most two chances to bargain with the seller, the strategy and belief spaces of the confidential negotiation process can be intractably large. This challenge has been noted in the literature as early as Swinkels [1999] and Kremer and Skrzypacz [2007].

To facilitate the analysis of the confidential negotiation process, I impose additional structure on the strategy and belief spaces and develop a suitable equilibrium concept. I begin by formally defining the players' strategies. For any $i \in \mathcal{I}$, buyer i's pricing strategy is a sequence $\sigma^i := (\tilde{P}^i_t)_{t=1,2}$, in which the period-$t$ pricing strategy $\tilde{P}^i_t$ is a real-valued random variable that depends on buyer i's period-t history h^i_{t-1}. Since there exists no public randomization device, the collection of period-t pricing strategies $\left\{\tilde{P}^i_t(h^i_{t-1})\right\}_{i\in\mathcal{I}}$ is conditionally independent given

[15] While Assumption (**NFS**) differs from those previously used in the literature on dynamic signaling (e.g., [Asriyan et al., 2017]), the main messages of the paper continue to hold under the more general assumption that $\pi_L v_L + \delta(1-\pi_L)v_H > \delta c_H$.

the vector of their private histories $(h^i_{t-1})_{i\in\mathcal{I}}$. Since a buyer's private history at the beginning of period 2 consists of its own period-1 offer, I slightly abuse notation and let $\tilde{P}^i_2(p^i_1)$ denote buyer i's period-2 continuation pricing strategy conditional on its own period-1 offer p^i_1. The vector of all buyers' strategies is denoted by $\boldsymbol{\sigma}^B = (\sigma^i)_{i\in\mathcal{I}}$. Henceforth, buyer i's period-t offer p^i_t and the period-t price p_t should be understood as realizations of $\tilde{P}^i_t$ and $\max_{i\in\mathcal{I}} \tilde{P}^i_t$, respectively.

The seller's acceptance strategy is a sequence $\sigma^S := (x^\theta_t)_{\substack{t=1,2 \\ \theta\in\{L,H\}}}$, where for each $\theta \in \{L, H\}$ and each $t \in \{1, 2\}$, $x^\theta_t : \mathbb{R} \to [0, 1]$ specifies the θ-quality seller's period-t acceptance probability as a function of the period-t price $p_t := \max_{i\in\mathcal{I}} p^i_t$. Importantly, this specification requires that for any $\theta \in \{L, H\}$, the θ-quality seller's period-1 acceptance probability depend on the offer profile only through the price p_1. Hence, p_1 is a sufficient statistic for the seller's period-1 acceptance decision upon receiving the period-1 offer profile $(p^i_1)_{i\in\mathcal{I}}$. In period 2, since there is no further bargaining opportunity after rejecting the period-2 price, it is optimal for the seller to accept if and only if the price weakly exceeds its reservation value c_θ, so this specification entails no loss of generality.

This requirement facilitates the analysis by reducing the set of candidate equilibria. For instance, it rules out equilibrium multiplicity arising from the seller's use of losing offers as an implicit randomization device. Additionally, it precludes contrived equilibria in which buyers randomize their offer amounts in both periods (except possibly after offering the lowest possible equilibrium price in period 1), as shown in Subsection III.B.

DEFINITION 1: *A symmetric monotone equilibrium in the confidential negotiation process consists of the buyers' pricing strategies $\boldsymbol{\sigma}^B$, the seller's acceptance strategy σ^S, and the buyers' posterior belief process, which together satisfy the following conditions:*

*(i) **(Symmetry)***

(a) The collection of period-1 pricing strategies $\{\tilde{P}^i_1\}_{i\in\mathcal{I}}$ is independent and identically distributed.

(b) For each fixed period-1 offer $p_1 \in \mathbb{R}$, the collection of period-2 continuation pricing strategies $\{\tilde{P}^i_2(p_1)\}_{i\in\mathcal{I}}$ is independent and identically distributed.

*(ii) **(Monotonicity)***

(a) (Monotone Acceptance Strategy) For any period-1 price $p_1 \in \mathbb{R}$ and any quality $\theta \in \{L, H\}$, the θ-quality seller's period-1 acceptance probability $x^\theta_1(p_1)$ weakly increases in p_1.

(b) (Monotone Pricing Strategy) For any pair $(p_1^H, p_1^L) \in \mathbb{R}^2$ *with* $p_1^H > p_1^L$*, every period-2 offer in the support of* $\tilde{P}_2^i(p_1^H)$ *is weakly higher than every period-2 offer in the support of* $\tilde{P}_2^i(p_1^L)$*:*

$$\inf \operatorname{supp}(\tilde{P}_2^i(p_1^H)) \geq \sup \operatorname{supp}(\tilde{P}_2^i(p_1^L)).$$

(iii) ***(Sequential Rationality)*** *At every information set, each player's strategy is a best response to the others' strategies.*

(iv) ***(Belief Consistency)*** *Buyers use all available information to update their beliefs about the seller's asset quality by Bayes' rule whenever possible.*

(v) ***(Non-Singularity)*** *Offer distributions are either discrete, absolutely continuous, or mixtures of the two.*

These equilibrium conditions are primarily motivated by classic auction theory, the main focus of which lies on symmetric monotone pure-strategy equilibria (e.g., Milgrom and Weber [1982]). Symmetry conditions (i)-(a) and (i)-(b) state that after any history of rejected offers, all buyers use the same (potentially mixed) pricing strategies in both periods. The first monotonicity condition (ii)-(a) requires that the seller be more likely to accept a higher price in period 1. Combined with the "reverse skimming property" established later in Lemma 1, Condition (ii)-(a) implies that if a buyer's aggressive period-1 offer is rejected, the buyer subsequently receives a strong signal that the asset quality is likely to be high. Therefore, the rejected buyer becomes more optimistic about the asset quality and makes a more aggressive ensuing period-2 offer, in line with Condition (ii)-(b). Conditions (iii) and (iv) together imply that a symmetric monotone equilibrium is a Perfect Bayesian Equilibrium (PBE). Finally, Condition (v) is a mild technical assumption on the cumulative distribution function associated with buyers' pricing strategies. In particular, it rules out singular-continuous distributions (e.g., the Cantor distribution), which are rarely of economic interest in discrete-time models as mine.

C. *Preliminary Analysis*

I begin with a general property that holds in many bargaining models, including mine. Since the seller's reservation value c_θ increases in quality θ, the following lemma follows from the seller's sequential optimization given the distribution of price paths, regardless of offer confidentiality:

LEMMA 1 (Reverse Skimming Property): *After any history of the seller, if it is weakly optimal for the low-quality seller to reject price p_t in period t, it is strictly optimal for the high-quality seller to reject p_t in period t. Moreover, if it is weakly optimal for the high-quality seller to accept price p_t in period t, it is strictly optimal for the low-quality seller to accept p_t in period t.*

Lemma 1 is also referred to as the *reverse skimming property* in the dynamic signaling literature [Fuchs et al., 2016, Hörner and Vieille, 2009]. The proof is standard in the literature and thus omitted.[16] Intuitively, after rejecting the current price, the high-quality seller enjoys a higher cash flow at the end of the period than the low-quality seller. Therefore, the high-quality seller is more willing to wait for future price increases (if any) and is effectively more patient than the low-quality seller.

When coupled with the "reverse skimming property," the static incentive compatibility condition (**SIC**) implies that in equilibrium, only the low-quality asset seller potentially accepts the period-1 price:

LEMMA 2: *Regardless of offer confidentiality, in any equilibrium, the low-quality seller accepts the period-1 price with strictly positive probability, whereas the high-quality seller never accepts it.*

Intuitively, the low-quality seller is marginal in period 1 in the sense that it is indifferent between accepting the period-1 price and rejecting it, which is characteristic of the "cutoff-type" in prior works on adverse selection.

II. Public Negotiation Process

Proposition 1 fully characterizes the unique price path induced by Perfect Bayesian Equilibria in the public negotiation process:

PROPOSITION 1 (Unique Equilibrium Price Path in Public Negotiation Process): *All Perfect Bayesian Equilibria (PBEs) in the public negotiation process induce a unique price path. In equilibrium, the period-1 price is v_L, which only the low-quality seller accepts with strictly positive probability. The seller's period-1 acceptance probability is such that buyers' period-2 posterior expected asset value is $\frac{v_L}{\delta}$, which is also the period-2 price. In particular, the low-quality seller is indifferent between the two prices, whereas the high-quality seller strictly prefers*

[16] A similar *skimming property* holds in classic Coasian bargaining models, where a consumer with high private value trades first [Sobel and Takahashi, 1983, Fudenberg, Levine, and Tirole, 1985]. In particular, Fudenberg et al. [1985], Fudenberg and Tirole [1991] provide an elegant proof of the "skimming property" in this context. More relatedly to my paper, Fuchs et al. [2016] establish the reverse skimming property in a similar trading environment with short-lived buyers.

the period-2 price and accepts it with probability 1.

The public negotiation process provides a natural benchmark for the confidential negotiation process, which has richer bargaining dynamics. Since past offers are publicly observable in the public negotiation process, buyers are always symmetrically informed. As a result, equilibrium play in the public negotiation process essentially involves classic Bertrand competition between buyers in each period. This has the following two implications. First, in any period, the price is equal to buyers' posterior expected value of the asset traded in that period, and thus buyers expect to break even. Second, as long as there are $N \geq 2$ buyers, the number of buyers has no effect on pricing dynamics.

For future reference, I compute the low-quality seller's acceptance probability at $t = 1$ in the public negotiation process:

$$\overbrace{\pi_L(1 - x_1^L(p_1^i))\left(v_L - \frac{p_1^i}{\delta}\right) + (1 - \pi_L)\left(v_H - \frac{p_1^i}{\delta}\right) = 0}^{\text{Buyer } i \text{ expects to break even in the period-2 continuation game}} \iff x_1^L(p_1^i) = \frac{p_1^i - \delta(\pi_L v_L + (1 - \pi_L)v_H)}{\pi_L(p_1^i - \delta v_L)}, \tag{1}$$

for any $p_1^i \in [v_L, c_H)$.

REMARK 2: *The equilibrium path of prices given in Proposition 1 coincides with that in analogous dynamic trading environments with short-lived buyers, regardless of offer confidentiality.*[17] *Hence, the public negotiation process can also be viewed as the baseline case in which the inclusion of long-lived buyers has no material impact on pricing dynamics.*

III. Confidential Negotiation Process

I solve for a closed-form expression for the unique distribution of equilibrium price paths in the confidential negotiation process. In Subsection III.A, I present the basic equilibrium properties of the confidential negotiation process. In particular, I illustrate how buyers in the confidential negotiation process can profitably deviate from the equilibrium in the public negotiation process given in Proposition 1, and interpret the deviation as "price experimentation." In Subsection III.B, I heuristically construct the candidate equilibrium strategy profile in the confidential negotiation process. While the discussion in this subsection proceeds at an intuitive level, the details

[17] See Subsection 4.3 of Fuchs et al. [2016] for the full analysis of the environment with short-lived buyers.

of equilibrium construction may nevertheless be skipped at first reading. In Subsection III.C, I describe the unique distribution of equilibrium price paths in the confidential negotiation process, and illustrate how price experimentation plays a crucial role in shaping bargaining dynamics in the confidential negotiation process.

A few notations are introduced in order. For each period $t = 1, 2$, denote the equilibrium support of period-t pricing strategies $\tilde{P}_t^i$ by $\mathcal{P}_t$. The minimum equilibrium period-t offer is denoted by $\underline{p}_t := \min \mathcal{P}_t$, whereas the maximum equilibrium period-t offer is denoted by $\overline{p}_t := \max \mathcal{P}_t$.

A. Equilibrium Properties

Since buyers are ex ante homogeneous, intuition suggests that buyers should expect to break even over their lifetime. The next proposition verifies this conjecture:

PROPOSITION 2 (Zero Expected Lifetime Profit for Buyer): *In equilibrium, all buyers expect to earn zero lifetime profits. In particular, if a buyer submits the minimum equilibrium period-1 offer $\underline{p}_1$ in period 1, the buyer expects to earn zero profits in both periods.*

Importantly, Proposition 2 does not imply that buyers break even in each period. Specifically, it does not preclude dynamic pricing strategies under which a buyer risks a loss in period 1 to earn a positive expected period-2 continuation profit. Henceforth, I shall refer to such a dynamic strategy as *price experimentation*, and a buyer who engages in price experimentation as an *experimenting buyer*.

Since buyers have a common value for the asset, an experimenting buyer must possess an informational advantage over its competitors in period 2 in order to earn a positive expected rent. Otherwise, Bertrand competition drives their expected period-2 continuation profits to zero, which would make it difficult for an experimenting buyer to recoup its expected loss from period 1. Indeed, the next corollary formalizes this point. In equilibrium, buyers randomize period-1 offers, which ensures that the information acquired through price experimentation is kept hidden from competitors:

COROLLARY 1: *In equilibrium, buyers randomize over the amounts of their period-1 offers. Equivalently, an equilibrium period-1 pricing strategy $\tilde{P}_1^i$ is a nondegenerate random variable.*

Proof. Suppose, toward a contradiction, that there exists an equilibrium in which the $\tilde{P}_1^i$'s are degenerate. In this equilibrium, buyers are symmetrically informed in period 2. Hence, Bertrand competition between buyers in period 2 drives their expected equilibrium period-2 continuation profits down to zero. By Proposition 2, buyers must earn zero expected lifetime profits in equilibrium, so their equilibrium expected period-1 profits must also be zero. Additionally, since only the low-quality seller trades in period 1 by Lemma 2, the only possible period-1 price is v_L. Thus, an argument analogous to that in the proof of Proposition 1 shows that the equilibrium strategy profile in the confidential bargaining game must coincide with that in the public bargaining game.

It remains to show that there exists a profitable deviation from the strategy profile described in Proposition 1. Consider a deviating period-1 offer $v_L + \epsilon$ by buyer i. Observe that, regardless of buyer i's period-1 offer, other buyers always offer $\frac{v_L}{\delta}$ in period 2. Thus, even after offering $v_L + \epsilon$ in period 1, buyer i never finds it profitable to offer weakly more than $\frac{v_L+\epsilon}{\delta}$ in period 2. Since the expected period-2 price is strictly less than the deviating period-1 offer in present-value terms, the low-quality seller must accept the deviating period-1 offer. By contrast, the high-quality seller would never accept the offer because its amount falls short of the seller's reservation value c_H. Thus, if the deviating period-1 offer is rejected, buyer i will privately learn that the seller is of high type. Therefore, in the period-2 continuation game that follows the rejection of $v_L + \epsilon$, it is optimal for buyer i to offer $\frac{v_L}{\delta} + \epsilon' \left(< \frac{v_L+\epsilon}{\delta}\right)$, which is just enough to break the tie with competing buyers. Therefore, the deviating buyer expects to earn a lifetime profit of

$$\underbrace{\pi_L\big(v_L - (v_L + \epsilon)\big)}_{\text{Expected period-1 profit}} + \delta \underbrace{(1 - \pi_L)\left(v_H - \left(\frac{v_L}{\delta} + \epsilon'\right)\right)}_{\text{Expected period-2 continuation profit}},$$

which is positive by (**NFS**) as long as ϵ, ϵ' are both small. Since the deviation is profitable, the hypothesis that the $\tilde{P}_1^i$'s are degenerate is absurd, and this completes the proof. □

The deviation in the proof above can be viewed as price experimentation. In period 1, the deviating buyer makes an offer higher than the buyer's value of the low-quality asset, which can only be accepted by the low-quality seller. Thus, if the period-1 offer is accepted, the buyer will end up overpaying for the low-quality asset. However, if the offer is rejected, the buyer will *privately* learn that asset quality is likely to be high. In period 2, knowing that the asset is of high type, the buyer becomes more eager to win the asset and subsequently makes a more aggressive period-2 offer to outbid competing buyers. In essence, compared to the public negotiation process characterized in Proposition 1, the deviation can more effectively separate sellers of different quality levels, and the resulting intertemporal price discrimination produces an information rent.

B. *Heuristic Construction of Equilibrium Strategy Profile*

I provide an informal derivation of the unique candidate equilibrium strategy profile in the confidential negotiation game.[18] By backward induction, I present the players' strategies in reverse time order, from the terminal decision node back to the initial node. While the formal arguments are relegated to Appendix A.3, the details of the equilibrium construction may be skipped on a first reading.

B.1. Seller's Period-2 Acceptance Probability

Since there is no additional bargaining opportunity after the seller rejects the period-2 price, it is optimal for the seller to accept any price weakly greater than its reservation value. Thus, we have:

PROPERTY 1: *In equilibrium, a seller of quality $\theta \in \{L, H\}$ accepts the period-2 price if and only if it weakly exceeds the seller's reservation value c_θ.*

B.2. Buyers' Period-2 Continuation Pricing Strategy $\tilde{P}_2^i(p_1^i)$

I begin by showing that in equilibrium, buyers do not randomize their period-2 offer amounts—that is, for any equilibrium offer $p_1^i \in \mathcal{P}_1$, buyers use a pure period-2 continuation pricing strategy $\tilde{P}_2^i(p_1^i)$ in equilibrium.[19] In the remainder of III.B.2, I sketch the argument for the case with $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ for illustration. In the Online Appendix, I extend the analysis to the case with $p_1^i = \underline{p}_1$ as well.[20]

Suppose, to the contrary, that there exists an equilibrium period-1 offer $\hat{p}_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ such that buyers randomize period-2 offer amounts after submitting $\hat{p}_1^i$ in period 1. First, I consider the case in which a buyer's period-1 pricing strategy $\tilde{P}_1^i$ assigns positive probability mass at $\hat{p}_1^i$. In this case, the next two events occur with strictly positive probability in equilibrium: 1) only buyer A offers $\hat{p}_1^i$, and all its competitors $j \in \mathcal{I} \setminus \{A\}$ submit equilibrium period-1 offers strictly lower than $\hat{p}_1^i$; 2) exactly two buyers A, B submit $\hat{p}_1^i$ in period 1, and all their competitors $j \in \mathcal{I} \setminus \{A, B\}$ submit equilibrium period-1 offers strictly lower than $\hat{p}_1^i$. Moreover, since $\tilde{P}_2^i(p_1^i)$ is

[18] As is standard in the bargaining literature, the strategy profile derived in Subsection III.B is unique up to the specification of losing offers.

[19] If I instead impose the more general assumption $\pi_L v_L + \delta(1 - \pi_L) v_H > \delta c_H$ instead of (**EXP**), there may be strictly positive probability of buyers randomizing after having offered the minimum possible equilibrium period-1 offer $\underline{p}_1$.

[20] The heuristic argument is as follows. As I show later, $\underline{p}_1$ is a losing offer in period 1, and $\frac{\underline{p}_1}{\delta}$ is a losing offer in period 2. Hence, it entails no loss of generality to set $\tilde{P}_2^i(\underline{p}_1) = \frac{\underline{p}_1}{\delta}$.

random, monotonicity conditions (ii)-(b) imply that

$$\delta\mathbb{E}_{\boldsymbol{\sigma}}\Big(\max_{i\in\mathcal{I}}\tilde{P}_2^i(p_1^i)\Big|p_1^A=\hat{p}_1^i,\max_{j\in\mathcal{I}\setminus\{A\}}p_1^j<\hat{p}_1^i\Big)<\delta\mathbb{E}_{\boldsymbol{\sigma}}\Big(\max_{i\in\mathcal{I}}\tilde{P}_2^i(p_1^i)\Big|p_1^A=p_1^B=\hat{p}_1^i,\max_{j\in\mathcal{I}\setminus\{A,B\}}p_1^j<\hat{p}_1^i\Big),$$

where $\mathbb{E}_{\boldsymbol{\sigma}}$ is the expectation operator associated with the equilibrium strategy profile $\boldsymbol{\sigma} := (\boldsymbol{\sigma}^B, \sigma^S)$. This inequality shows that the equilibrium distribution of the period-2 price depends not only on the amount of the period-1 price, but also on the number of buyers submitting $\hat{p}_1^i$ in period 1. Nevertheless, this contradicts the requirement in Subsection I.B that the seller must make its period-1 acceptance decision solely based on the period-1 price p_1. This contradiction shows that $\tilde{P}_2^i(p_1^i)$ is non-random for any p_1^i at which $\tilde{P}_1^i$ assigns positive probability mass.

Second, I consider the alternative case in which a buyer's period-1 pricing strategy $\tilde{P}_1^i$ assigns zero probability mass at $\hat{p}_1^i$. Since $\tilde{P}_2^i(\hat{p}_1^i)$ is random, there exist distinct $p_2^i, \hat{p}_2^i$ in the support of $\tilde{P}_2^i(\hat{p}_1^i)$. Since there is zero probability mass at $\hat{p}_1^i$, the monotonicity condition (ii)-(b) implies that no competing buyers submit a period-2 offer between p_2^i and $\hat{p}_2^i$ with positive probability in equilibrium. Therefore, the two amounts $p_2^i, \hat{p}_2^i$ lead to the same winning probability, but yield different expected profits conditional upon winning in period 2. Hence, the two offer amounts lead to different expected period-2 continuation profits. Therefore, after offering $\hat{p}_1^i$ in period 1, a buyer cannot be indifferent between offering the two amounts p_2^i and $\hat{p}_2^i$, which is a contradiction. These arguments, together with those given in the Online Appendix, show that $\tilde{P}_2^i(p_1^i)$ is a degenerate random variable for any equilibrium period-1 offer p_1^i.

I am ready to pin down buyers' period-2 continuation pricing strategy $\tilde{P}_2^i(p_1^i)$ given an equilibrium period-1 offer $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$. By Lemma 2, the low-quality seller is of the so-called "cutoff type" in equilibrium, who is indifferent between accepting an equilibrium period-1 offer and rejecting it. Hence, buyer i's period-1 offer p_1^i must be equal to the present value of the expected period-2 offer under the equilibrium continuation pricing strategy $\tilde{P}_2^i(p_1^i)$:

$$\max_{i\in\mathcal{I}}\{p_1^i\}=\delta\mathbb{E}_{\boldsymbol{\sigma}}\Big(\max_{i\in\mathcal{I}}\tilde{P}_2^i(p_1^i)\Big|(p_1^i)_{i\in\mathcal{I}}\Big)\Longleftrightarrow\mathbb{E}_{\boldsymbol{\sigma}}\Big(\tilde{P}_2^i(p_1^i)\Big)=\frac{p_1^i}{\delta}.$$

Since $\tilde{P}_2^i(p_1^i)$ is nonrandom for each equilibrium period-1 offer $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$, we can drop the expectation operator in the equation above and pin down the period-2 continuation pricing strategy $\tilde{P}_2^i(p_1^i) = \frac{p_1^i}{\delta}$ for any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$. Together with the additional arguments in the Online Appendix, the discussion above can be summarized as follows:

PROPERTY 2 (Buyers' Period-2 Continuation Pricing Strategy): *In equilibrium, if a buyer offers a period-1 offer $p_1^i \in \mathcal{P}_1$ and is subsequently rejected at $t = 1$, the buyer will offer $\tilde{P}_2^i(p_1^i) = \frac{p_1^i}{\delta}$ at $t = 2$.*

B.3. Seller's Period-1 Acceptance Probability

To determine the players' period-1 strategies, I present an equilibrium property that characterizes the cumulative distribution function (CDF) of a buyer's period-1 pricing strategy $\tilde{P}_1^i$:

PROPERTY 3: *Buyer i's equilibrium period-1 pricing strategy $\tilde{P}_1^i$ is either:*

(1) a continuous random variable with support $[\underline{p}_1, \overline{p}_1]$ with $\underline{p}_1 \geq v_L$.

(2) a mixed random variable with support $[\underline{p}_1, \overline{p}_1]$, whose cumulative distribution function assigns a probability mass at the lower endpoint $\underline{p}_1 = v_L$ and is differentiable everywhere else in the support.

Since the proof of this property is quite involved and does not provide much intuition, it is deferred to the Online Appendix.

Let $G(\cdot)$ denote the CDF of the maximum offer $\max_{j \in \mathcal{I} \setminus \{i\}} \tilde{P}_1^j$ submitted by buyer i's competitors at $t = 1$. By Property 3 and Lemma 2, buyer i's expected lifetime payoff from submitting an equilibrium period-1 offer p_1^i can be expressed as follows:

$$G(p_1^i)\left(\overbrace{\pi_L x_1^L(p_1^i)(v_L - p_1^i)}^{\because \text{ Only } L \text{ accepts by Lemma 2}} + \delta \overbrace{\left(\pi_L \big(1 - x_1^L(p_1^i)\big) \left(v_L - \frac{p_1^i}{\delta} \right) + (1 - \pi_L) \left(v_H - \frac{p_1^i}{\delta} \right) \right)}^{\text{Since } \frac{p_1^i}{\delta} > c_H \text{ by } (\textbf{EXP}), \text{ the seller always accepts in period 2}} \right) = 0, \tag{2}$$

where the last equality follows from the fact that buyer i expects to break even over its lifetime by Proposition 2. After dividing both sides of (2) by $G(p_1^i)$,[21] we obtain a closed-form expression for $x_1^L(\cdot)$.

PROPERTY 4: *For any period-1 price $p_1 \in \mathcal{P}_1$, the low-quality seller's acceptance probability of p_1 can be expressed as follows:*

$$x_1^L(p_1) = \frac{p_1 - \delta(\pi_L v_L + (1 - \pi_L) v_H)}{(1 - \delta)\pi_L v_L}, \tag{3}$$

which strictly increases in p_1 on $\mathcal{P}_1$.

[21] More precisely, since it is possible to have $G(\underline{p}_1) = 0$, the argument for the minimum equilibrium period-1 offer $\underline{p}_1$ is separately provided in the Online Appendix.

B.4. Buyers' Period-1 Pricing Strategy

It remains to derive the CDF of buyers' equilibrium period-1 pricing strategy $\tilde{P}_1^i$. By Property 3, it suffices to pin down 1) the two endpoints $\underline{p}_1$ and $\overline{p}_1$ of support $\mathcal{P}_1$ and 2) the CDF of $\tilde{P}_1^i$ on $\mathcal{P}_1$.

First, I show that the maximum equilibrium period-1 offer is $\overline{p}_1 = \pi_L v_L + \delta(1-\pi_L)v_H$, which represents buyers' expected discounted asset value from trading exclusively with the low-quality seller at $t = 1$ and exclusively with the high-quality seller at $t = 2$. Suppose, to the contrary, that $x_1^L(\overline{p}_1) < 1$. Then, arguing as in Corollary 1, a buyer can profitably deviate by offering $\overline{p}_1 + \varepsilon$ at $t = 1$ and $\frac{\overline{p}_1}{\delta} + \varepsilon'$ at $t = 2$, which yields a strictly positive expected lifetime profit. This profitable deviation shows that $x_1^L(\overline{p}_1) = 1$, which is equivalent to $\overline{p}_1 = \pi_L v_L + \delta(1 - \pi_L)v_H$ by the seller's period-1 acceptance probability in (3).

Second, I obtain explicit expressions for the CDF of buyers' period-1 pricing strategy. Recall that $G(\cdot)$ denotes the CDF of the maximum competing offer $\max \tilde{\boldsymbol{P}}_1^{-i} := \max_{j \in \mathcal{I} \setminus \{i\}} \tilde{P}_1^j$ faced by buyer i at $t = 1$. After buyer i makes an equilibrium period-1 offer $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$, the buyer expects to earn the following period-2 continuation profit $F(p_2^i \mid p_1^i)$ by offering $p_2^i \geq c_H$ at $t = 2$:

$$
\begin{aligned}
F(p_2^i \mid p_1^i) := & \int_{\underline{p}_1 \leq \max \boldsymbol{p}_1^{-i} \leq \delta p_2^i} \Big(\pi_L\big(1 - x_1^L(\max(p_1^i, \boldsymbol{p}_1^{-i}))\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i)\Big)\, d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i}) \\
& \div \underbrace{\left\{\int_{\boldsymbol{p}_1^{-i} \in \mathcal{P}_1^{N-1}} \Big(\pi_L\big(1 - x_1^L(\max(p_1^i, \boldsymbol{p}_1^{-i}))\big) + 1 - \pi_L\Big)\, d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i})\right\}}_{=: R(p_1^i) \text{ (probability that the seller rejects the period-1 price } \max(p_1^i, \boldsymbol{p}_1^{-i}) \text{ at } t=1)} .
\end{aligned} \tag{4}
$$

where $\boldsymbol{p}_t^{-i} := (p_t^j)_{j \neq i}$ denotes buyer i's competing period-t offers. Henceforth, for any finite vector $\boldsymbol{x}$, I write $\max \boldsymbol{x}$ for its largest component. Accordingly, for each $t \in \{1, 2\}$, $\max(p_t^i, \boldsymbol{p}_t^{-i})$ expresses the period-t price p_t as the maximum component of the augmented vector $(p_t^i, \boldsymbol{p}_t^{-i})$, thereby explicitly expressing its dependence on buyer i's offer p_t^i.

Observe that the rejection probability $R(p_1^i)$ in the second line of (4) is strictly positive and unaffected by buyer i's period-2 offer p_2^i. Hence, I can replace the buyer's objective function $F(p_2^i \mid p_1^i)$ in (4) with its "ex-ante" expected continuation profit $\Pi(p_2^i \mid p_1^i) := F(p_2^i \mid p_1^i) \times R(p_1^i)$, which simplifies to the following expression for any period-2 offer $p_2^i \in (\frac{p_1^i-\epsilon}{\delta}, \frac{p_1^i}{\delta}]$:

$$\Pi(p_2^i \mid p_1^i) = \underbrace{G(\delta p_2^i)}_{\text{Winning prob at } p_2^i} \underbrace{\Big(\pi_L\big(1-x_1^L(p_1^i)\big)(v_L - p_2^i) + (1-\pi_L)(v_H - p_2^i)\Big)}_{\text{``Ex-ante'' expected period-2 payoff upon winning at } p_2^i \text{ after having offered } p_1^i}. \tag{5}$$

By Property 3, buyer i's ex-ante expected continuation profit in (5) is differentiable with respect to p_2^i on $(\frac{p_1^i-\epsilon}{\delta}, \frac{p_1^i}{\delta}]$. Since buyer i must find it sequentially optimal to offer $p_2^i = \frac{p_1^i}{\delta}$ at $t = 2$ after having offered p_1^i at $t = 1$, the first-order condition with respect to p_2^i yields:

$$\underbrace{\left(\frac{\pi_L(1-x_1^L(p_1^i))}{\pi_L(1-x_1^L(p_1^i))+1-\pi_L}\left(v_L - \frac{p_1^i}{\delta}\right) + \frac{1-\pi_L}{\pi_L(1-x_1^L(p_1^i))+1-\pi_L}\left(v_H - \frac{p_1^i}{\delta}\right)\right)}_{\text{Equilibrium posterior expected period-2 continuation payoff upon winning at price } \frac{p_1^i}{\delta}} = \frac{G(p_1^i)}{\delta G'(p_1^i)}. \tag{6}$$

Together with the fact that the maximum equilibrium period-1 offer is $\overline{p}_1 = \pi_L v_L + \delta(1-\pi_L)v_H$, the linear differential equation above yields the following unique solution:

$$G(p_1^i) = \left(\frac{p_1^i - v_L}{\pi_H(\delta v_H - v_L)}\right)^{\frac{\delta \pi_H (v_H - v_L)}{v_L - \delta \mathbb{E}_0(\tilde{v})}} \left(\frac{(1-\delta)\pi_L v_L}{p_1^i - \delta \mathbb{E}_0(\tilde{v})}\right)^{\frac{(1-\delta)v_L}{v_L - \delta \mathbb{E}_0(\tilde{v})}} \tag{7}$$

for any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$, where $\pi_H = 1 - \pi_L$ and $\mathbb{E}_0(\tilde{v}) := \pi_L v_L + (1-\pi_L)v_H$. Since buyers make use of identically distributed period-1 pricing strategies, the CDF of their period-1 offer amounts must be $G^{\frac{1}{N-1}}$ on $\mathcal{P}_1 \setminus \{\underline{p}_1\}$.

Finally, I show that the minimum equilibrium period-1 offer is $\underline{p}_1 = v_L$. By Property 3, we must have $\underline{p}_1 \geq v_L$, so it suffices to rule out the possibility that $\underline{p}_1 > v_L$. Suppose, to the contrary, that $\underline{p}_1 > v_L$. Direct computations show that if a buyer submits the minimum equilibrium period-1 offer $\underline{p}_1$ at $t = 1$, the buyer wins the asset at $t = 2$ with strictly positive probability (i.e., $G(\underline{p}_1) > 0$) by the CDF given in (7), and expects to earn a positive period-2 continuation profit upon winning (i.e., $\pi_L\big(1 - x_1^L(\underline{p}_1)\big)\left(v_L - \frac{\underline{p}_1}{\delta}\right) + (1-\pi_L)\left(v_H - \frac{\underline{p}_1}{\delta}\right) > 0$) by Property 2. Therefore, by (5), a buyer who offers $\underline{p}_1$ at $t = 1$ expects to make a positive continuation profit at $t = 2$. This contradicts Proposition 2, which implies that $\underline{p}_1 = v_L$.

The next property summarizes the discussion in this subsubsection:

PROPERTY 5: *In period 1, buyers randomize period-1 offers according to the CDF $G^{\frac{1}{N-1}}$ over support $[v_L, \pi_L v_L + \delta(1-\pi_L)v_H]$, where the explicit expression for G is given in* (7).

C. Unique Distribution of Equilibrium Price Paths

Based on the equilibrium strategy profile constructed so far, I now characterize the unique distribution of equilibrium price paths in the confidential negotiation process. In particular, the next theorem shows that in equilibrium, buyers always engage in price experimentation in the confidential negotiation process:

THEOREM 1 (Unique Distribution of Equilibrium Price Paths under Confidential Negotiation Process): *There exists a unique distribution of price paths induced by symmetric monotone equilibria, whose strategy profiles are characterized by Properties 1-5. In equilibrium, buyers make random offers $\tilde{P}_1^i > v_L$ in period 1, which only the low-quality seller accepts with positive probability. Hence, buyers incur losses when their initial offers are accepted. In period 2, buyers offer $\tilde{P}_2^i(p_1^i) = \frac{p_1^i}{\delta}$ and expect to earn a strictly positive information rent. Subsequently, the seller surely accepts the period-2 price, regardless of asset quality.*

The proof of Theorem 1 relies on the endogenous single-crossing property of buyers' expected period-2 continuation profit $\Pi(p_2^i \mid p_1^i)$ in (5) in (p_1^i, p_2^i). More formally, whenever $p_2^i \geq \frac{p_1^i}{\delta}$, partially differentiating $\Pi(p_2^i \mid p_1^i)$ with respect to p_1^i yields:

$$\frac{\partial}{\partial p_1^i}\Big[G(\delta p_2^i)\Big(\pi_L(1 - x_1^L(p_1^i))(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i)\Big)\Big] = \pi_L \underbrace{\frac{\partial x_1^L(p_1^i)}{\partial p_1^i}}_{>0 \text{ by Property 4}} G(\delta p_2^i)\,(p_2^i - v_L). \tag{8}$$

Since the right-hand side of Equation (8) strictly increases in p_2^i, a buyer's expected period-2 continuation profit $\Pi(p_2^i \mid p_1^i)$ in (5) has the single-crossing property in (p_1^i, p_2^i) when $p_2^i \geq \frac{p_1^i}{\delta}$. In the Online Appendix, I show that even when $p_2^i < \frac{p_1^i}{\delta}$, the expected continuation profit satisfies the single-crossing property in (p_1^i, p_2^i) as well.

The single-crossing property clarifies the mechanism behind price experimentation. Intuitively, if a buyer makes an aggressive period-1 offer and it is rejected afterwards, the buyer learns that the asset is likely to be of high quality, which can be seen from the strictly increasing acceptance probability in (3). As a result, the buyer faces less adverse selection risk in period 2 and thus becomes more eager to win the asset in period 2, as can be seen from Equation (8). In response, the buyer submits a more aggressive ensuing offer to outbid competitors.

IV. Main Results

A. Offer Confidentiality

I shall compare bargaining dynamics between two bargaining protocols. First, it is clear from Proposition 1 and Theorem 1 that in each period, the equilibrium price must be strictly higher in the confidential negotiation process.

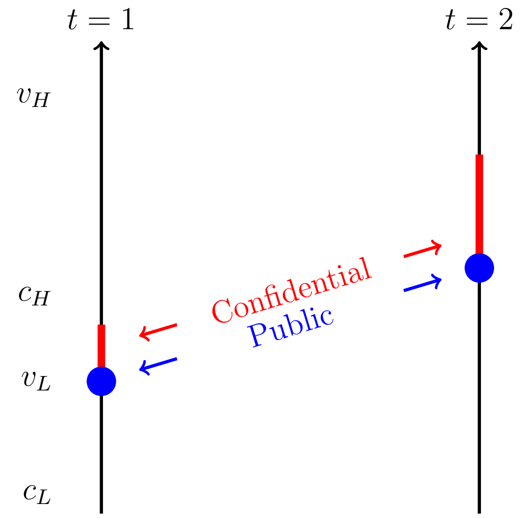


Figure 1. Price Comparison Between Two Negotiation Processes

Second, any equilibrium outcome in the confidential negotiation process Pareto dominates the outcome in the public negotiation process. To see why this property holds true, observe that the seller expects higher discounted revenue in the confidential negotiation process, whereas buyers make zero expected lifetime payoffs regardless of the information structure (see Proposition 2 and the discussion after Proposition 1). Therefore, the expected welfare gains from offer confidentiality eventually accrue to the seller.

Third, I shall compare transaction speed (i.e., the likelihood of trade in period 1) across the two informational structures. In this model, the present value of expected equilibrium welfare is an affine transformation of the present value of expected equilibrium gains from trade, which can be computed as

$$\left(\delta\pi_L + (1-\delta)\mathbb{P}_{\boldsymbol{\sigma}}\Big(\text{The low-quality seller accepts the price } \max_{i\in\mathcal{I}} \tilde{P}_1^i \text{ in period 1}\Big)\right) v_L + (1-\pi_L)\delta(v_H - c_H),$$

where $\mathbb{P}_{\boldsymbol{\sigma}}$ represents the probability measure induced by the equilibrium strategy profile $\boldsymbol{\sigma} := (\boldsymbol{\sigma}^B, \sigma^S)$. Since the present value of equilibrium expected discounted welfare is higher in the confidential negotiation process, the seller must accept $\max_{i \in \mathcal{I}} \tilde{P}_1^i$ with strictly higher probability in the confidential negotiation process. Therefore, offer confidentiality accelerates transaction speed. The discussion above can be summarized as follows:

IMPLICATION 1: Given any number of $N \geq 2$ buyers, compared to the public negotiation process, any equilibrium strategy profile in the confidential negotiation process induces (1) strictly higher prices in all periods, (2) strictly higher equilibrium transaction speed, and (3) a strictly higher present value of expected equilibrium total welfare.

Implication 1 is consistent with the widespread use of confidential sale processes across asset markets. For instance, in the M&A market, the financial advisor of a target firm often keeps potential acquirers in the dark about how much competing acquirers offered earlier [Subramanian, 2011, Roberts, 2009]. Likewise, in the commercial real estate sector, brokers frequently arrange "off-market" negotiations with interested buyers.[22]

There are two channels through which offer confidentiality accelerates trade. First, offer confidentiality raises the likelihood that the seller accepts a fixed level of an equilibrium period-1 price $p_1 (\geq v_L)$ in period 1, which can be seen from the seller's acceptance strategy (1) in the public negotiation process and Property 4 in the confidential negotiation process. Intuitively, since an experimenting buyer possesses private information about asset quality in period 2, the buyer makes a period-2 offer strictly less than its posterior expected value, as can be formally shown in (6). This tends to depress the seller's expected discounted continuation value after the rejection of the period-1 price, and thus the seller is more likely to accept the period-1 price.

Second, offer confidentiality increases the profitability of price experimentation, and thus leads buyers to submit more aggressive period-1 offers. In the public negotiation process, buyers are always symmetrically informed, so a buyer can never expect to reap an information rent in period 2, both on and off the equilibrium path. As a result, buyers' period-1 offers, and thus the period-1 price, remain at a relatively low level, v_L. By contrast, in the confidential negotiation process, buyers can earn positive expected period-2 continuation profits through price experimentation. Thus, this incentivizes buyers to engage in price experimentation with more aggressive period-1 offers, which the seller more frequently accepts by the monotonicity conditions (ii)-(a).

[22] In the language of the real estate sector, "off market" refers to activity that does not happen through a public marketing process or is not listed on the Multiple Listing Service (MLS). As an example usage, see the following press release of Colliers' acquisition of an office tower: `https://www.colliers.com/en/news/downtown-los-angeles-office-tower-largest-office-sale-since-2023`).

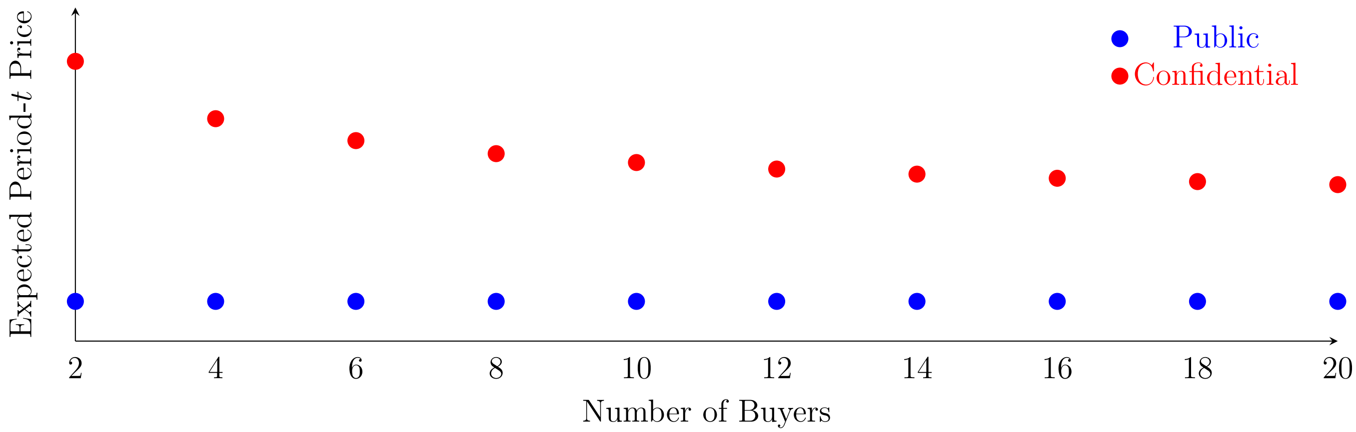


Figure 2. Number of Buyers and Expected Prices

B. Buyer Entry

I shall analyze how buyer entry (i.e., the number of buyers) affects bargaining dynamics in each information structure. Recall from the discussion subsequent to Proposition 1 that in the public negotiation process, buyer entry has no meaningful impact on the equilibrium path of expected prices, the present value of expected equilibrium welfare, and equilibrium transaction speed, as long as there are at least two buyers.

Let $\tilde{P}_t^{(N)}$ denote the period-t price in the confidential negotiation process with N buyers to emphasize its dependence on the number of buyers, N. Theorem 2 shows that expected prices are higher when *fewer* buyers participate in the confidential negotiation process:

THEOREM 2: For any $N \geq 2$ and any $t \in \{1, 2\}$, the equilibrium distribution of $\tilde{P}_t^{(N)}$ first-order stochastically dominates the equilibrium distribution of $\tilde{P}_t^{(N+1)}$.

Figure 2 depicts the relationship between the number of buyers N and the expected price in each period. Note that the expected price is maximized at $N = 2$ in the confidential negotiation process.

An immediate consequence of Theorem 2 is that restricted entry in the confidential negotiation process leads to a higher expected price in both periods, a Pareto improvement, and faster transaction speed. Since the arguments are very similar to those in Subsection IV.A, I only sketch them here. By Theorem 2, the expected price in each period is decreasing in the number of buyers. However, each buyer always expects to break even regardless of the number of buyers by Proposition 2. Hence, restricted entry increases the seller's expected lifetime payoff without making any buyer worse off. By arguing similarly to the previous Subsection IV.A, it is clear that trade is more efficient if and only if the seller accepts the period-1 price more frequently. Summarizing, we have:

IMPLICATION 2: Assume that there are $N \geq 2$ buyers.

(1) In the public negotiation process, the number of buyers has no effect on the equilibrium path of prices, the present value of expected equilibrium welfare, and equilibrium transaction speed;

(2) In the confidential negotiation process, the expected equilibrium price in each period, equilibrium transaction speed, and the present value of expected equilibrium welfare all strictly *decrease* in the number of buyers.

Implication 2 may help explain why a sell-side financial advisor in charge of a mid-market firm frequently invites a preselected group of potential acquirers to hold merger talks [Wasserstein, 2009]. Since mid-market firms often lack financial slack to realize their full business potential, they may derive large benefits from expediting the pace of their sale process. Therefore, these firms might opt to divest their assets via confidential negotiations with only a limited set of potential acquirers. Consistent with this logic, M&A financial advisors note that negotiations become exceedingly time-consuming once the target must engage with more than seven serious potential acquirers [Subramanian, 2011].

To build intuition behind Implication 2, I decompose the equilibrium distribution of the period-1 price with N buyers as follows:

$$\left(\underbrace{G^{\frac{1}{N-1}}}_{\text{CDF of } \tilde{P}_1^i \text{ with } N \text{ buyers}} \right)^{\overbrace{N}^{\text{Number of experimenting buyers}}}. \tag{9}$$

As can be seen from (9), changing N has two effects: (1) the first effect comes from the change in the number of experimenting buyers (i.e., N outside the parentheses), and (2) the second effect comes from the change in the equilibrium distribution $G^{\frac{1}{N-1}}$ of individual buyers' pricing strategies.

To illustrate the second effect, I heuristically show how an "unexpected" change from N to $N-1$ affects the players' optimal strategies. Absent any adjustment to other players' equilibrium strategies, it is profitable for buyer i to reduce the amount of the period-2 offer p_2^i after the buyer's period-1 offer p_1^i is rejected. Formally:

$$\frac{\partial}{\partial p_2^i} \left(\left(\underbrace{G^{\frac{1}{N-1}}}_{\text{CDF of } \tilde{P}_1^i \text{ with } N \text{ buyers}} (\delta p_2^i) \right)^{N-2} \left(\pi_L (1 - x_1^L(p_1^i))(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i) \right) \right) \Bigg|_{p_2^i = \frac{p_1^i}{\delta}} < 0, \tag{10}$$

which follows by logarithmic differentiation.[23] Now, suppose that a buyer who offered p_1^i in period 1 decreases the amount of the period-2 offer to $p_2^i = \frac{p_1^i}{\delta} - \varepsilon$. As a result, for a fixed amount of the period-1 price p_1, the expected period-2 price decreases. Therefore, the low-quality seller more frequently accepts the period-1 price (i.e., higher $x_1^L(p_1^i)$).

Given the adjustments to the strategies (i.e., lower p_2^i and higher $x_1^L(p_1^i)$), I can recalculate buyer i's expected lifetime payoff as of period 1 as

$$\Bigg(\underbrace{G^{\frac{1}{N-1}}}_{\text{CDF of } \tilde{P}_1^i \text{ with } N \text{ buyers}}(p_1^i)\Bigg)^{N-2} \Big(\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_1^i) + \delta\Big(\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i)\Big)\Big). \tag{11}$$

As can be seen from (11), while the changes in p_2^i and $x_1^L(p_1^i)$ improve the profitability of aggressive period-1 offers close to $\overline{p}_1$, they have limited impacts on that of conservative period-1 offers close to v_L.[24] This implies that it is more profitable to shift probability weight from lower period-1 offers to higher period-1 offers. Thus, buyers find it profitable to play stochastically dominant period-1 pricing strategies, and continue to adjust their strategies until their optimal continuation pricing strategy $\tilde{P}_2^i(p_1^i)$ re-equilibrates to $\frac{p_1^i}{\delta}$ as in Property 2.

I shall elaborate on why the second effect dominates the first effect in the CDF of the period-1 price given in (9). In both (10) and (11), let us substitute the underbraced CDFs with N buyers (i.e., $G^{\frac{1}{N-1}}(\cdot)$) for $G^{\frac{N}{(N-1)^2}}(\cdot)$. The idea here is to adjust individual buyers' pricing strategies so that the CDF of the period-1 price with $N-1$ buyers becomes $G^{\frac{N}{N-1}}$, which is the equilibrium CDF of the period-1 price with N buyers given in (9). Logarithmic differentiation shows that even after $G^{\frac{1}{N-1}}(\cdot)$ is replaced with $G^{\frac{N}{(N-1)^2}}(\cdot)$, the inequality in (10) still continues to hold, and the subsequent arguments do so as well. Hence, even after having adjusted period-1 pricing strategies so that the CDF of the period-1 price with $N-1$ buyers coincides with the equilibrium CDF of the period-1 price with N buyers, an experimenting buyer still finds it optimal to decrease the amount of its period-2 offer when there are fewer competitors.

[23]The log-derivative is $\left(\frac{N-2}{N-1}\right)\frac{\delta G'(p_1^i)}{G(p_1^i)} - \left(\frac{\pi_L(1-x_1^L(p_1^i))+1-\pi_L}{\pi_L(1-x_1^L(p_1^i))\left(v_L - \frac{p_1^i}{\delta}\right)+(1-\pi_L)\left(v_H - \frac{p_1^i}{\delta}\right)}\right)$. From (6), $\frac{\delta G'(p_1^i)}{G(p_1^i)}$ is equal to the reciprocal of the last term, so the log-derivative (and thus the original derivative) must be negative.

[24]For conservative period-1 offers close to v_L, regardless of the changes in p_2^i and $x_1^L(p_1^i)$, the probability that a buyer can outbid competitors by submitting such an offer is close to zero, so the expected lifetime payoff from making a conservative period-1 offer is close to zero as well.

To provide a verbal account of this formal argument, I apply the two channels introduced in the previous Subsection IV.A. Restricting entry accelerates trade in two ways. First, for a given level of the period-1 price, the seller tends to accept the price more frequently when a smaller number of buyers participate in the confidential negotiation process. As can be seen from the inequality in (10), with fewer competing buyers, an experimenting buyer faces less competitive pressure and need not make an aggressive period-2 offer to protect its expected information rent. Thus, an experimenting buyer's period-2 offer tends to become less aggressive. Hence, the low-quality seller finds it more attractive to accept the period-1 price, which results in faster trade.

Second, with fewer competitors, buyers engage in more aggressive price experimentation (i.e., use a stochastically dominant period-1 pricing strategy). As can be seen from (11), the changes in the optimal continuation strategies (i.e., lower p_2^i and higher $x_1^L(p_1^i)$) improve the profitability of aggressive period-1 offers. In response, buyers more aggressively engage in price experimentation with a stochastically dominant period-1 pricing strategy. Since price experimentation leads to a more aggressive expected period-1 price, a bargaining agreement is more likely to occur in period 1. Due to these two channels, bargaining is less likely to be inefficiently delayed, and the present value of expected equilibrium surplus increases. Since buyers always break even on a lifetime basis, these efficiency gains ultimately accrue to the seller in the form of higher expected discounted revenue.

REMARK 3: *Importantly, Implication 2 is distinct from the winner's curse in classic auction theory. If the result were driven by the winner's curse, then for a given distribution of an individual buyer's information variable (i.e.,* $G^{\frac{1}{N-1}}$*), restricting buyer entry in the confidential negotiation process would mitigate the winner's curse and thus lead to more aggressive continuation period-2 pricing strategies. Instead, the inequality in* (10) *shows that when* $G^{\frac{1}{N-1}}$ *is held fixed, continuation pricing strategies become less aggressive when there are fewer buyers.*

In my model, the winning buyer's expectation of asset quality is unaffected by competing buyers' information. More specifically, in period 1, the only information conveyed by the seller's acceptance decision is that the asset is of low quality (Lemma 2), so the winning buyer does not become more pessimistic due to competitors' information. Additionally, recall that the most optimistic buyer at the beginning of period 2, who will eventually win the asset in equilibrium, possesses a sufficient statistic for asset quality given all buyers' information variables before winning the asset. Therefore, as can also be seen from the optimality condition (6)*, winning the asset does not convey any negative news about asset type, so the winner's curse plays no role in the winning buyer's equilibrium posterior expected asset value.*

V. Extensions

I consider two extensions to explore additional implications of the paper. The first extension illustrates the role of various informational frictions in the seller's strategic choice between auctions and negotiations. The second extension considers a "pre-qualification" process that excludes buyers who made less aggressive early offers from participating in the later period.

A. *Asymmetric Information Among Buyers*

Up to this point, the paper has focused on a single friction: asymmetric information between the seller and the buyers. Nevertheless, in corporate asset sales, buy-side financial advisors frequently acquire private information through various means, such as independent due diligence and proprietary screening technology (see footnotes 7 and 8). Thus, it is natural to explore how information asymmetry among buyers shapes the seller's revenue-maximizing number of prospective buyers in confidential negotiations.

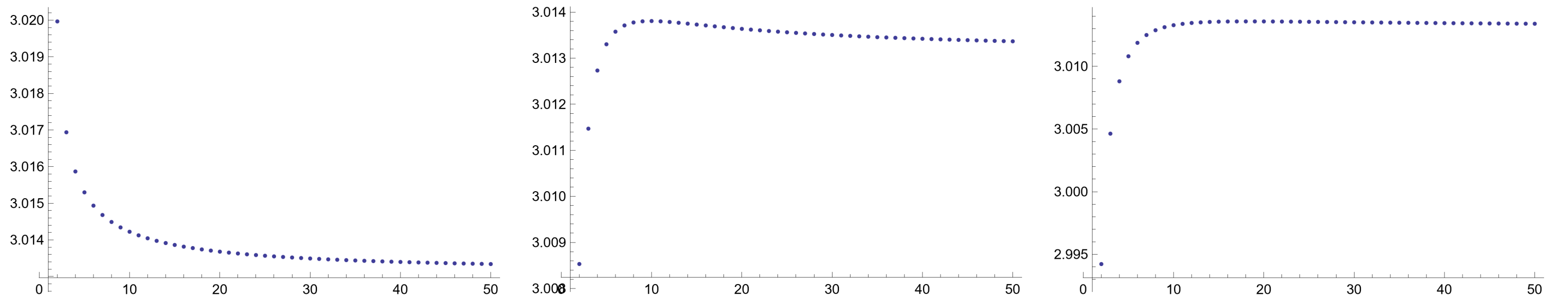
To this end, I modify the setup as follows. At the beginning of period 1, each buyer $i \in \mathcal{I}$ privately receives a signal ϵ_i, where the signals $(\epsilon_i)_{i=1}^N$ are independent draws from the uniform distribution on $[-\Delta, \Delta]$, where $\Delta > 0$ measures the dispersion of the buyers' private information about the common value of the underlying asset. If a buyer wins the asset of quality θ, the buyer's asset value is given by

$$v_\theta + \frac{1}{N}\sum_{i=1}^{N}\epsilon_i,$$

where the extra term $\frac{1}{N}\sum_{i=1}^{N}\epsilon_i$ is a variant of the common-value payoff in the standard "Wallet Game" [Klemperer, 1998, Bulow and Klemperer, 2002]. In particular, as dispersion $\Delta \to 0$, the buyer's asset value for a given quality $\theta \in \{L, H\}$ converges to that in the baseline environment.

In the Online Appendix IA.1, I "purify" the mixed equilibria in Theorem 1 à la Harsanyi [1973] to obtain the equilibria of the perturbed games with $\Delta > 0$. I rely on numerical methods to plot the relationship between the number of prospective buyers and the expected revenue for varying levels of $\Delta > 0$ in Figure 3. Hence, I obtain the following implication:

IMPLICATION 3: *The revenue-maximizing number of buyers increases in the dispersion of information asym-*



(a) $\Delta = 0.01$ **(b)** $\Delta = 0.05$ **(c)** $\Delta = 0.1$

Figure 3. Plots of the seller's expected revenue for $\Delta = 0.01$, $\Delta = 0.05$, and $\Delta = 0.1$. Other parameter values are $v_L = 3$, $v_H = 8$, $c_H = 4.5$, $\pi_L = 0.9$, and $\delta = 0.5$.

metry across the buyers, $\Delta > 0$.

Implication 3 sheds new light on the strategic choice between auctions and negotiations, and helps us better understand why the number of prospective buyers in a typical sale varies across different asset markets.

Empirically, the average sale of distressed corporate assets involves fewer than two serious buyers, and many "bankruptcy auctions" are effectively negotiated sales with only a few buyers [LoPucki and Doherty, 2007]. Likewise, legal analyses emphasize that typical sellers of distressed assets have private information about their value, whereas outsiders frequently have limited time to assess the underlying assets [Ayotte and Skeel Jr, 2013, Jacoby and Janger, 2013, LoPucki and Doherty, 2007]. In accordance with these accounts, Implication 3 asserts that if the buyers' private information has little dispersion (i.e., low $\Delta > 0$), the seller benefits from limiting the number of buyers participating in the confidential negotiation process. Heuristically, for sufficiently small $\Delta > 0$, the equilibrium is close to that in Theorem 1 in the baseline model. As a result, price experimentation remains crucial in determining the revenue-maximizing number of buyers, and restricting entry into the confidential negotiation process continues to be optimal.

By contrast, in the market for large publicly traded companies, the average takeover auction has approximately 12 potential acquirers, whereas the auctions won by financial buyers have 16 potential acquirers on average [Gorbenko and Malenko, 2014]. In practice, these financial buyers are often sophisticated private equity firms, who rely on proprietary screening and due diligence to form independent assessments of asset value. Correspondingly, Implication 3 shows that when buyers hold dispersed private information (i.e., large $\Delta > 0$), the seller's optimal sales strategy typically involves many buyers. Intuitively, if the seller increases the number of buyers, the seller faces a higher chance of at least one buyer receiving a very optimistic signal. As a result, the seller's expected gain from the higher likelihood of facing a more optimistic buyer dominates the expected loss from diminished price experimentation. Therefore, the seller's expected revenue tends to increase with the number of buyers.

REMARK 4: *The additional payoff component* $\frac{1}{N}\sum_{i=1}^{N}\epsilon_i$ *normalizes the common-value payoff in the standard Wallet Game by* $\frac{1}{N}$*. This normalization confers two advantages. First, in the standard Wallet Game, increasing the number of bidders tends to affect the expected value of the common-value payoff. As a result, even in the absence of information asymmetry, the seller's expected revenue depends on the number of buyers. The normalization by the number of buyers* N *circumvents this issue by isolating the strategic effects of added buyers from mechanical changes in the level of the expected value. Second, it ensures a positive social gain from trade. Specifically, the normalized payoff component* $\frac{1}{N}\sum_{i=1}^{N}\epsilon_i$ *has the same support independent of the number of buyers* N*. As a result, the buyers' total asset value,* $v_\theta + \frac{1}{N}\sum_{i=1}^{N}\epsilon_i$*, always exceeds the low-quality seller's asset value* c_θ*. Hence, there is common knowledge of gains from trade among the players, which is standard in the adverse selection literature.*

B. Pre-qualification

In many Chapter 11 sales, distressed sellers often winnow down the initial pool of prospective buyers to "pre-qualified" buyers and restrict subsequent communications about deal terms (e.g., requests for revised offers) to that group. Motivated by this practice, I extend my model to illustrate how the seller can benefit from such a pre-qualification process.

Formally, I consider the *confidential negotiation process with pre-qualification*, defined by the following modified procedures. At the beginning of period 1, $N \geq 3$ buyers simultaneously submit offers. If the seller rejects all offers at $t = 1$, the seller selects the M buyers who submitted the highest period-1 offers. These M buyers retain the right to negotiate with the seller in period 2, and the remaining $N - M$ buyers are excluded from the bargaining process thereafter. At the beginning of period 2, the remaining M buyers simultaneously submit new offers. The next proposition formally shows how the seller may benefit from pre-qualification:

PROPOSITION 3: *In the confidential negotiation process with pre-qualification, (1) expected prices in all periods, (2) equilibrium transaction speed, and (3) the present value of expected equilibrium total welfare all strictly decrease in* M*, for any pair of natural numbers* M, N *with* $2 \leq M < N$*.*

The intuition behind Proposition 3 parallels that of Theorem 2 in the baseline model. If the seller winnows down to a smaller set of buyers in period 2, an experimenting buyer faces less competitive pressure and therefore enjoys greater pricing power in period 2. Hence, holding fixed the period-1 offer, the buyer has a stronger incentive to reduce the amount of its period-2 offer. This makes it more attractive for the seller to accept the period-1 price in period 1, increasing the likelihood of trade in period 1. Therefore, the seller ultimately enjoys the resulting surplus from pre-qualification through higher expected revenue.

Importantly, in order to benefit from pre-qualification, the seller must commit not to entertain late offers from *excluded* buyers. If this commitment is not credible, the excluded buyers can infer that the seller has rejected competing buyers' aggressive period-1 offers, and accordingly revise their posterior asset values upward. Hence, the excluded buyers have an incentive to return with a surprise late offer that the seller is tempted to accept. Such a deviating offer would upset equilibrium play in Proposition 3 and unravel the gains from pre-qualification. Hence, on a fundamental level, pre-qualification can be viewed as the seller's commitment device: by preventing the seller from soliciting late offers from excluded buyers, it strengthens buyers' incentive to engage in price experimentation early on.

This requirement for credible commitment may help us better explain why pre-qualification is pervasive in distressed asset sales. In Chapter 11 sales, a bankruptcy court typically oversees the entire sale process and issues a judicial order that restricts non-qualified prospective buyers from "jumping in" at later stages [Ayotte and Ellias, 2022]. This court order can often provide the enforcement mechanism required to disregard later offers from excluded buyers.

VI. Discussion

A. Symmetry

Throughout the paper, I focus on symmetric equilibria. This requirement may appear restrictive because when there are $N \geq 3$ buyers, it is natural to conjecture the existence of asymmetric equilibria with "non-serious buyers," who submit losing offers in both periods on the equilibrium path and effectively opt out of negotiation. While this is a natural candidate, it cannot be sustained as an equilibrium:

PROPOSITION 4: For any pair of natural numbers M, N with $2 \leq M < N$, there exists no Perfect Bayesian Equilibrium in the confidential negotiation process in which M buyers randomize period-1 offers as in Property 5 and use a continuation pricing strategy given in Property 2, while the other $N - M$ buyers submit losing offers in both periods.

The gist of the proof is to show that a non-serious buyer can "free-ride" on other buyers' price experimentation by making a surprise serious period-2 offer and thus enjoy a profitable deviation.

Given that Proposition 4 rules out the most natural candidates for asymmetric equilibria, it is possible that no non-trivial asymmetric equilibria exist in my model. Unfortunately, a full-fledged analysis without symmetry is quite challenging. If the symmetry condition is relaxed, the equilibrium support of buyers' period-1 pricing strategies may take a very intricate form and make the analysis highly intractable. Therefore, I leave the full analysis of asymmetric equilibria to future research.

B. Additional Bargaining Opportunities

Thus far, I have assumed that each buyer has only two chances to bargain with the seller. In the Online Appendix IA.4, I relax this restriction and examine bargaining dynamics with a general number of periods $T \geq 3$. The bargaining dynamics of each negotiation process in this environment can be summarized as follows:

PROPOSITION 5:

(1) *In the public negotiation process, there exist Perfect Bayesian Equilibria satisfying the "independence of never weak best response" criterion (INWBR) à la Kohlberg and Mertens [1986], which proceed as follows:*

  (a) *Suppose that $\frac{v_L}{\delta^{T-1}} < \delta v_H + (1-\delta)c_H$. In period 1, at least two buyers offer v_L, which the low-quality seller accepts with positive probability and the high-quality seller surely rejects. Then buyers offer surely losing offers until period T. In period T, at least two buyers offer $\frac{v_L}{\delta^{T-1}}$ and expect to make zero profits upon winning.*

  (b) *Suppose that $\frac{v_L}{\delta} < \delta v_H + (1-\delta)c_H < \frac{v_L}{\delta^{T-1}}$. In period 1, at least two buyers offer v_L, which the low-quality seller accepts with positive probability and the high-quality seller surely rejects. Then buyers offer surely losing offers until some random time $\tilde{t} \leq T$ such that the low-quality seller is indifferent between accepting the period-1 price and rejecting it. In period $\tilde{t}$, at least two buyers offer $\delta v_H + (1-\delta)c_H$ and expect to make zero profits upon winning.*

  (c) *Suppose that $\frac{v_L}{\delta} > \delta v_H + (1-\delta)c_H$. Then, the equilibrium play coincides with the one given by Proposition 1.*

(2) In the confidential negotiation process, there exists a Public Bayesian Equilibrium whose play coincides with the one given by Theorem 1. If $\frac{v_L}{\delta} > \delta v_H + (1-\delta)c_H$*, the equilibrium also satisfies the INWBR criterion.*

Two takeaways emerge from Proposition 5. First, allowing for more bargaining opportunities does not eliminate price experimentation under confidentiality, because the confidential negotiation process still admits an equilibrium in which price experimentation plays a crucial role. Second, all the main results of the paper extend to this setting for an appropriate range of parametric configurations.

C. Continuum of Types With High Discount Factor

In Online Appendix IA.5, I consider a continuous-type version of my bargaining model with the discount factor close to 1. Proposition 6 shows that buyers compete à la Bertrand in each period of the public negotiation process:

PROPOSITION 6: In any equilibrium of the public negotiation process, all buyers expect to earn zero payoffs in both periods.

Consequently, by standard Bertrand logic, the number of buyers is irrelevant for the pricing dynamics of the public negotiation process with at least two buyers.

As in the two-type case, let $\underline{p}_1$ denote the minimum of the equilibrium support $\mathcal{P}_1$ of period-1 offers. The following analog of Theorem 1 shows that for sufficiently high discount factors, price experimentation plays a central role in shaping bargaining dynamics under confidentiality:

PROPOSITION 7: There exists $\delta^* \in (0,1)$ such that, for any $\delta > \delta^*$, there exists an equilibrium in the confidential negotiation process that proceeds as follows. In period 1, buyers randomize period-1 offers. If the seller accepts a period-1 offer, the winning buyer expects to earn a strictly negative payoff in period 1. In period 2, any buyer who submitted a period-1 offer strictly higher than the minimum equilibrium period-1 offer $\underline{p}_1$ expects to earn a strictly positive information rent in the continuation negotiation process.

I establish the following analogs of Properties 1 and 2 in Online Appendix IA.5.4:

PROPERTY' 1: For any fixed number of $N \geq 2$ buyers and δ strictly higher than δ^* given in Theorem 7, the equilibrium in the confidential negotiation process described in Theorem 7 induces strictly higher expected prices in all periods and a strictly higher present value of expected welfare compared to the public negotiation process.

PROPERTY' 2: Assume that there are $N \geq 2$ buyers.

(1) In the public negotiation process, the number of buyers has no effect on the path of expected prices and the present value of expected welfare in the most efficient equilibrium;

(2) In equilibria given in Proposition 7, the expected equilibrium price in each period, the expected equilibrium cutoff in each period, and the present value of expected equilibrium welfare all strictly *decrease* in the number of buyers as long as the discount factor $\delta > \delta^*$.

Thus, even with a continuum of asset quality levels and the discount factor close to 1, both offer confidentiality and restricted buyer entry in the confidential negotiation process can lead to more efficient outcomes. Hence, the qualitative conclusions of my paper do not depend on the two-type specification or on the discount factor lying in an intermediate range.

VII. Concluding Remarks

In this paper, I study a dynamic model of concurrent bargaining between multiple long-lived buyers and a long-lived seller. In the baseline analysis, I illustrate how offer confidentiality and restricted entry can lead to more efficient bargaining outcomes, and emphasize the role of price experimentation in shaping bargaining dynamics under confidentiality. My first extension shows how alternative informational frictions may have different implications for the strategic choice between auctions and negotiations, whereas my second extension sheds light on the "pre-qualification" process widely used in distressed asset sales.

These results are only one step toward a broader theory of concurrent bargaining in dynamic environments. Fruitful avenues for future research include asset divisibility [Fuchs, Gottardi, and Moreira, 2022, Gerardi et al., 2022] or information spillovers [Asriyan et al., 2017] in the context of concurrent bargaining. This investigation might help us better understand interactions between price experimentation and economic forces already unearthed in the prior literature. Additionally, my model can be interpreted as a theoretical framework for the "private phase" of takeover processes. Therefore, it would also be interesting to examine how the private phase considered in this paper interacts with the public phase, as emphasized by [Eckbo et al., 2020].

REFERENCES

Anat R Admati, Peter M DeMarzo, Martin F Hellwig, and Paul Pfleiderer. The leverage ratchet effect. *The Journal of Finance*, 73(1):145–198, 2018.

George A Akerlof. The market for 'lemons': Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500, 1970.

Vladimir Asriyan, William Fuchs, and Brett Green. Information spillovers in asset markets with correlated values. *American Economic Review*, 107(7):2007–40, 2017.

Lawrence M Ausubel and Raymond J Deneckere. Reputation in bargaining and durable goods monopoly. *Econometrica: Journal of the Econometric Society*, pages 511–531, 1989.

Ulf Axelson and Igor Makarov. Informational black holes in financial markets. *Journal of Finance, Forthcoming*, 2020.

Kenneth Ayotte and Jared A Ellias. Bankruptcy process for sale. *Yale J. on Reg.*, 39:1, 2022.

Kenneth Ayotte and David A Skeel Jr. Bankruptcy law as a liquidity provider. *The University of Chicago Law Review*, pages 1557–1624, 2013.

Robert Bloomfield and Maureen O'Hara. Can transparent markets survive? *Journal of Financial Economics*, 55 (3):425–459, 2000.

Audra L Boone and J Harold Mulherin. How are firms sold? *The Journal of Finance*, 62(2):847–875, 2007.

Audra L Boone and J Harold Mulherin. Is there one best way to sell a company? auctions versus negotiations and controlled sales. *Journal of Applied Corporate Finance*, 21(3):28–37, 2009.

Jeremy Bulow and Paul Klemperer. Auctions versus negotiations. *The American Economic Review*, 86(1):180–194, 1996.

Jeremy Bulow and Paul Klemperer. Prices and the winner's curse. *RAND journal of Economics*, pages 1–21, 2002.

Jeremy Bulow and Paul Klemperer. Why do sellers (usually) prefer auctions? *American Economic Review*, 99 (4):1544–75, 2009.

Harry Cendrowski, Louis W Petro, James P Martin, and Adam A Wadecki. *Private equity: History, governance, and operations*. John Wiley & Sons, 2012.

Isaıas N Chaves. Privacy in bargaining: The case of endogenous entry. *Available at SSRN*, 2019.

In-Koo Cho and David M Kreps. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221, 1987.

Ronald H Coase. Durability and monopoly. *The Journal of Law and Economics*, 15(1):143–149, 1972.

Brendan Daley and Brett Green. Bargaining and news. *American Economic Review*, 110(2):428–74, 2020.

M Ángeles de Frutos and Carolina Manzano. Trade disclosure and price dispersion. *Journal of Financial Markets*, 8(2):183–216, 2005.

Peter M DeMarzo and Zhiguo He. Leverage dynamics without commitment. *The Journal of Finance*, 76(3): 1195–1250, 2021.

Raymond Deneckere and Meng-Yu Liang. Bargaining with interdependent values. *Econometrica*, 74(5):1309–1364, 2006.

B Espen Eckbo, Andrey Malenko, and Karin S Thorburn. Strategic decisions in takeover auctions: Recent developments. *Annual Review of Financial Economics*, 12(1):237–276, 2020.

Robert Evans. Sequential bargaining with correlated values. *The Review of Economic Studies*, 56(4):499–510, 1989.

Kenneth R French and Robert E McCormick. Sealed bids, sunk costs, and the process of competition. *Journal of business*, pages 417–441, 1984.

William Fuchs and Andrzej Skrzypacz. Bargaining with arrival of new traders. *American Economic Review*, 100 (3):802–36, 2010.

William Fuchs, Aniko Öry, and Andrzej Skrzypacz. Transparency and distressed sales under asymmetric information. *Theoretical Economics*, 11(3):1103–1144, 2016.

William Fuchs, Piero Gottardi, and Humberto Moreira. *Time trumps quantity in the market for lemons*. Centre for Economic Policy Research, 2022.

Drew Fudenberg and Jean Tirole. Sequential bargaining with incomplete information. *The Review of Economic Studies*, 50(2):221–247, 1983.

Drew Fudenberg and Jean Tirole. Game theory, 1991. *Cambridge, Massachusetts*, 393(12):80, 1991.

Drew Fudenberg, David Levine, and Jean Tirole. Infinite-horizon models of bargaining with one-sided incomplete information. Technical report, David K. Levine, 1985.

Matthew Gentry and Caleb Stroup. Entry and competition in takeover auctions. *Journal of Financial Economics*, 132(2):298–324, 2019.

Dino Gerardi, Lucas Maestri, and Ignacio Monzón. Bargaining over a divisible good in the market for lemons. *American Economic Review*, 112(5):1591–1620, 2022.

Lawrence R Glosten and Paul R Milgrom. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):71–100, 1985.

Alexander S Gorbenko and Andrey Malenko. Strategic and financial bidders in takeover auctions. *The Journal of Finance*, 69(6):2513–2555, 2014.

Faruk Gul, Hugo Sonnenschein, and Robert Wilson. Foundations of dynamic monopoly and the coase conjecture. *Journal of economic Theory*, 39(1):155–190, 1986.

Robert G Hansen. Auctions of companies. *Economic Inquiry*, 39(1):30–43, 2001.

John C Harsanyi. Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory*, 2(1):1–23, 1973.

Florian Hoffmann and Vladimir Vladimirov. Auctions versus negotiations: The role of the payment structure. *The Journal of Finance*, 80(3):1769–1813, 2025.

Johannes Hörner and Nicolas Vieille. Public vs. private offers in the market for lemons. *Econometrica*, 77(1): 29–69, 2009.

Melissa B Jacoby and Edward J Janger. Ice cube bonds: allocating the price of process in chapter 11 bankruptcy. *Yale LJ*, 123:862, 2013.

Ayca Kaya and Qingmin Liu. Transparency and price formation. *Theoretical Economics*, 10(2):341–383, 2015.

Paul Klemperer. Auctions with almost common values: Thewallet game'and its applications. *European Economic Review*, 42(3-5):757–769, 1998.

Elon Kohlberg and Jean-Francois Mertens. On the strategic stability of equilibria. *Econometrica: Journal of the Econometric Society*, pages 1003–1037, 1986.

Ilan Kremer and Andrzej Skrzypacz. Dynamic signaling and market breakdown. *Journal of Economic Theory*, 133(1):58–82, 2007.

J Chris Leach and Ananth N Madhavan. Price experimentation and security market structure. *The Review of Financial Studies*, 6(2):375–404, 1993.

Tingting Liu, Micah S Officer, and Danni Tu. *Negotiation, auction, or negotiauction?! Evidence from the field*. SSRN, 2023.

Lynn M LoPucki and Joseph W Doherty. Bankruptcy fire sales. *Mich. L. Rev.*, 106:1, 2007.

Paul R Milgrom. Rational expectations, information acquisition, and competitive bidding. *Econometrica: Journal of the Econometric Society*, pages 921–943, 1981.

Paul R Milgrom and Robert J Weber. A theory of auctions and competitive bidding. *Econometrica: Journal of the Econometric Society*, pages 1089–1122, 1982.

Georg Noldeke and Eric Van Damme. Signalling in a dynamic labour market. *The Review of Economic Studies*, 57(1):1–23, 1990.

Daniel Quint and Kenneth Hendricks. A theory of indicative bidding. *American Economic Journal: Microeconomics*, 10(2):118–51, 2018.

Dennis Roberts. *Mergers & Acquisitions: An Insider's Guide to the Purchase and Sale of Middle Market Business Interests*. Wiley, 2009.

William F Samuelson. Competitive bidding with entry costs. *Economics Letters*, 17(1-2):53–57, 1985.

Joel Sobel and Ichiro Takahashi. A multistage model of bargaining. *The Review of Economic Studies*, 50(3): 411–426, 1983.

Guhan Subramanian. *Dealmaking: The New Strategy of Negotiauctions*. WW Norton & Company, 2011.

Jeroen M Swinkels. Education signalling with preemptive offers. *The Review of Economic Studies*, 66(4):949–970, 1999.

Gerald Teschl. *Ordinary Differential Equations and Dynamical Systems*. American Mathematical Society, 2012.

Donald M Topkis. *Supermodularity and complementarity*. Princeton university press, 1998.

Daniel R Vincent. Bargaining with common values. *Journal of Economic Theory*, 48(1):47–62, 1989.

Bruce Wasserstein. *Big deal: Mergers and acquisitions in the digital age*. Hachette UK, 2009.

Robert Wilson. A bidding model of perfect competition. *The Review of Economic Studies*, 44(3):511–518, 1977.

Appendix for Proofs

I introduce a few pieces of notation used throughout the appendix. For any offer path $(p_1^i, p_2^i) \in \mathbb{R}^2$, define

$$
\begin{aligned}
\Pi_2(p_2^i \mid p_1^i) :=& \int_{(\boldsymbol{p}_1^{-i}, \boldsymbol{p}_2^{-i}) \in \mathcal{P}_1^{N-1} \times \mathcal{P}_2^{N-1}} \left(\sum_{n=2}^{N} \frac{\mathbf{1}_{\{\text{ties with } n \text{ buyers at } p_2^i\}}}{n} + \mathbf{1}_{\{\max \boldsymbol{p}_2^{-i} < p_2^i\}} \right) \\
& \times \left(\sum_{\theta \in \{L,H\}} \pi_\theta \big(1 - x_1^\theta(\max(p_1^i, \boldsymbol{p}_1^{-i}))\big)\, x_2^\theta(\max(p_2^i, \boldsymbol{p}_2^{-i}))\, (v_\theta - p_2^i) \right) d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i}, \boldsymbol{p}_2^{-i}), \\
\Pi_2(p_1^i) :=& \max_{p_2^i \in \mathbb{R}} \Pi_2(p_2^i \mid p_1^i), \\
\pi_1(p_1^i) :=& \int_{\boldsymbol{p}_1^{-i} \in \mathcal{P}_1^{N-1}} \left(\sum_{n=2}^{N} \frac{\mathbf{1}_{\{\text{ties with } n \text{ buyers at } p_1^i\}}}{n} + \mathbf{1}_{\{\max \boldsymbol{p}_1^{-i} < p_1^i\}} \right) \left(\sum_{\theta \in \{L,H\}} \pi_\theta x_1^\theta(\max(p_1^i, \boldsymbol{p}_1^{-i}))(v_\theta - p_1^i) \right) d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i}).
\end{aligned}
\tag{12}
$$

where $\boldsymbol{p}_t^{-i} := (p_t^j)_{j \neq i}$ denotes the vector of buyer i's competing period-t offers for $t = 1, 2$. Recall that for any finite vector $\boldsymbol{z}$, I write $\max \boldsymbol{z}$ for its largest component. As a result, the period-t price may be written as the maximum component of the augmented vector $\max(p_t^i, \boldsymbol{p}_t^{-i})$, for both $t \in \{1, 2\}$. Verbally, $\Pi_2(p_2^i \mid p_1^i)$ is buyer i's expected period-2 continuation profit from offering p_2^i given p_1^i, $\Pi_2(p_1^i)$ is buyer i's period-2 value function at p_1^i, and $\pi_1(p_1^i)$ is buyer i's expected period-1 profit from offering $p_1^i \in \mathbb{R}$.

A.1 Preliminary Analysis

Lemma A.1.1 (Bertrand competition below c_H in period 2): *Fix any period-1 offer $p_1^i \in \mathcal{P}_1$. If buyer i makes a serious period-2 offer $p_2^i < c_H$ under an equilibrium pricing strategy σ^i, then $p_2^i = v_L$ and $\Pi_2(p_2^i \mid p_1^i) = 0$.*

Proof. Fix any such (p_1^i, p_2^i). By symmetry and independence, if buyer i makes a serious period-2 offer below c_H with positive probability, then at least two buyers make serious period-2 offers below c_H with strictly positive probability as well. Any offer below c_H can be accepted only by the low-quality seller, so buyers who make serious offers below c_H in period 2 effectively compete for the low-quality asset. Bertrand competition then implies that any serious offer below c_H must equal v_L, and the resulting continuation profit is $\Pi_2(p_2^i \mid p_1^i) = v_L - p_2^i = 0$. □

Proof of Lemma 2. I first show that the seller accepts the period-1 price with strictly positive probability in any equilibrium. Suppose, to the contrary, that there exists an equilibrium in which the seller never accepts the period-1 price. Then buyers' period-1 offers are always rejected, so their expected period-1 profits are all zero. Moreover, since the seller rejects with probability 1 in period 1, buyers' period-2 posterior beliefs about asset quality coincide with the prior beliefs.

Consider the period-2 continuation equilibrium. By Assumption (**SIC**), any serious period-2 offer $p_2 \geq c_H$ yields strictly negative expected profit ex ante, so no buyer makes such an offer in equilibrium. Hence any serious period-2 offer must satisfy $p_2 < c_H$. By Lemma A.1.1, it follows that $\tilde{P}_2^i = v_L$ for all i, and buyers earn zero period-2 profits. Therefore, buyers earn zero lifetime profits in this equilibrium. Since the low-quality seller rejects the period-1 price with probability 1 and expects to face the period-2 price v_L, sequential rationality of the low-quality seller implies that no buyer offers more than δv_L in period 1, i.e., $\tilde{P}_1^i \leq \delta v_L$ for all i.

Now fix any $\epsilon \in (0, (1-\delta)v_L)$ and consider a deviation by buyer i in period 1 to the offer $p_1^i = \delta v_L + \epsilon$. This offer is strictly higher than any equilibrium period-1 offer, so buyer i wins the period-1 price. Moreover, if the asset quality $\theta = L$, accepting $p_1^i = \delta v_L + \epsilon$ yields the seller a payoff $\delta v_L + \epsilon$, which strictly exceeds the present value of the continuation payoff δv_L from rejecting and trading in period 2. Hence the low-quality seller accepts this deviating offer with probability 1. Buyer i's expected profit from this deviation is therefore

$$\pi_L\big(v_L - (\delta v_L + \epsilon)\big) = \pi_L\big((1-\delta)v_L - \epsilon\big) > 0,$$

which contradicts equilibrium. This establishes that the period-1 price is accepted with strictly positive probability in any equilibrium.

It remains to show that the high-quality seller never accepts the period-1 price. Suppose, to the contrary, that the high-quality seller accepts the period-1 price with strictly positive probability in some equilibrium. Then there exists a period-1 price p_1 that occurs with strictly positive probability and is accepted by the high-quality seller with strictly positive probability. Since the high-quality seller's reservation value is c_H, sequential rationality implies $p_1 \geq c_H$. By the reverse skimming property (Lemma 1), the low-quality seller accepts p_1 with probability 1.

Consider a buyer who wins at price p_1. If asset quality $\theta = L$, the buyer's period-1 profit is at most $v_L - p_1 \leq v_L - c_H$. In state $\theta = H$, if trade occurs in period 1 its profit is at most $v_H - p_1 \leq v_H - c_H$, and if trade does not occur in period 1, its period-2 continuation profit is at most $\delta(v_H - c_H)$. Therefore, the buyer's expected lifetime profit from winning at p_1 is at most

$$\pi_L(v_L - c_H) + (1 - \pi_L)(v_H - c_H) < 0,$$

where the strict inequality follows from Assumption (**SIC**). Since any buyer can secure zero lifetime profit by submitting surely losing offers in both periods, no buyer can rationally make an offer that leads to winning at such a price. This is a contradiction. Hence the high-quality seller never accepts the period-1 price. □

A.2 Public Negotiation Process

Proof of Proposition 1. I proceed by backward induction. In the public negotiation process, buyers are always symmetrically informed, both on and off the equilibrium path. Hence, buyers engage in Bertrand competition and break even in period 2, regardless of the public history at the beginning of period 2.

Let π_2 denote buyers' period-2 posterior belief that the seller is of low quality, and let $\bar{\pi}_L$ be the threshold belief that makes buyers' posterior expected value equal to c_H, $\bar{\pi}_L v_L + (1 - \bar{\pi}_L)v_H = c_H$. In any period-2 continuation equilibrium, the period-2 price is

$$p_2 = \begin{cases} \pi_2 v_L + (1 - \pi_2)v_H & \text{if } \pi_2 < \bar{\pi}_L, \\ v_L \text{ or } c_H & \text{if } \pi_2 = \bar{\pi}_L, \\ v_L & \text{if } \pi_2 > \bar{\pi}_L. \end{cases} \tag{13}$$

I next show that in any continuation equilibrium following the rejection of a period-1 price $p_1 \in [v_L, \delta v_H)$, the (equilibrium) period-2 price satisfies $p_2 = p_1/\delta$. First, suppose $p_2 > p_1/\delta$. Then both types of the seller strictly prefer rejecting p_1 in period 1, so the seller rejects p_1 with probability 1. Hence $\pi_2 = \pi_L$. By Assumption (**SIC**), $\pi_L > \bar{\pi}_L$, and thus (13) implies $p_2 = v_L$, contradicting $p_2 > p_1/\delta$. Second, suppose $p_2 < p_1/\delta$. Then the low-quality seller strictly prefers accepting p_1 in period 1, so it accepts with probability 1. Therefore, conditional on reaching period 2 we must have $\pi_2 = 0$, and hence (13) implies $p_2 = v_H$. This contradicts $p_2 < p_1/\delta$ because $p_1 < \delta v_H$. Therefore, we must have $p_2 = p_1/\delta$.

By Lemma 2, only the low-quality seller potentially accepts in period 1. Hence, any period-1 trade implies

$\theta = L$, so buyers compete for the low-quality asset in Bertrand fashion in period 1. Therefore, the unique equilibrium period-1 price is $p_1 = v_L$, and the unique equilibrium period-2 price is $p_2 = v_L/\delta$. This completes the proof. □

A.3 *Confidential Negotiation Process*

The analysis of the confidential negotiation process in A.3 is organized as follows. In A.3.1, I show that a buyer expects to break even over its lifetime in any equilibrium of the confidential bargaining game (Proposition 2). In A.3.2, I begin by proving several auxiliary lemmas and subsequently establish Properties 1-5. These properties uniquely pin down a candidate equilibrium strategy profile in the confidential bargaining game. In A.3.3, I show that the candidate equilibrium admits no profitable deviation, which establishes Theorem 1.

A.3.1 Proof of Proposition 2: "Zero Expected Lifetime Profit for Buyers"

Lemma A.3.1: *Suppose buyer i's period-2 pricing strategy $\tilde{P}_2^i$ assigns strictly positive probability mass to some offer p_2^i. Then, in equilibrium, buyer i earns zero conditional expected period-2 profit given a tie at p_2^i.*

Proof. Suppose, to the contrary, that buyer i earns a nonzero conditional expected period-2 profit given a tie at p_2^i. Since there are at most countably many atoms, there exists a sufficiently small $\epsilon > 0$ so that no tie occurs at $p_2^i \pm \epsilon$. If it is negative, buyer i can profitably deviate to $p_2^i - \epsilon$ at which no tie occurs. If it is positive, buyer i can profitably deviate to $p_2^i + \epsilon$ at which no tie occurs. In either case, the deviation strictly increases buyer i's expected period-2 profit, contradicting the optimality of buyer i's equilibrium period-2 strategy. □

Lemma A.3.2: *In equilibrium, buyers expect to earn zero period-2 continuation profit after offering $\underline{p}_1$ in period 1:* $\Pi_2(\underline{p}_1) = 0$.

Proof. By the monotonicity condition (ii)-(b), if buyer i makes the minimum equilibrium period-1 offer $\underline{p}_1$ in period 1, the buyer must find it sequentially rational to offer the minimum equilibrium period-2 offer $\underline{p}_2$ in period 2 with strictly positive probability. By the definition of $\underline{p}_2$, other buyers never offer strictly less than $\underline{p}_2$ in period 2, so buyer i can win the asset in period 2 only by tying at $\underline{p}_2$. Hence, buyer i's conditional expected period-2 profit after offering $\underline{p}_1$ in period 1 must be zero by Lemma A.3.1, which completes the proof. □

Proof of Proposition 2: "Zero Expected Lifetime Profit for Buyers". By Lemma A.3.2, it suffices to show that buyer i expects to earn zero period-1 profit from offering $\underline{p}_1$, i.e., $\pi_1(\underline{p}_1) = 0$. Suppose, to the contrary, that $\pi_1(\underline{p}_1) > 0$. Then

$$\pi_1(\underline{p}_1) = \left(\sum_{n=1}^{N-1} \frac{\mathbb{P}_{\boldsymbol{\sigma}^{-i}}\big(\text{ties with } n \text{ other buyers at } \underline{p}_1\big)}{n+1} + \mathbb{P}_{\boldsymbol{\sigma}^{-i}}\Big(\max_{j \neq i} \tilde{P}_1^j < \underline{p}_1 \Big) \right) \pi_L x_1^L(\underline{p}_1)\,(v_L - \underline{p}_1) > 0.$$

Since $\underline{p}_1$ is the minimum equilibrium period-1 offer, $\tilde{P}_1^j \geq \underline{p}_1$ for all $j \neq i$, so $\mathbb{P}_{\boldsymbol{\sigma}^{-i}}(\max_{j \neq i} \tilde{P}_1^j < \underline{p}_1) = 0$. Hence, $\pi_1(\underline{p}_1) > 0$ implies that a tie at $\underline{p}_1$ occurs with strictly positive probability.

Pick $\epsilon > 0$ sufficiently small so that no tie occurs at $\underline{p}_1 + \epsilon$. Consider the deviation in which buyer i offers $\underline{p}_1 + \epsilon$ in period 1 and then submits a surely losing offer in period 2, so its period-2 continuation profit is zero. If all other buyers offer $\underline{p}_1$ in period 1, this deviation converts a tie at $\underline{p}_1$ (winning probability strictly less than 1) into a strict win at $\underline{p}_1 + \epsilon$ (winning probability 1), and the seller's period-1 acceptance probability weakly increases by Condition (ii)-(a). For ϵ sufficiently small, this deviation yields a strictly higher expected period-1 profit, and it weakly increases the buyer's expected lifetime profit. This contradicts sequential rationality, which completes the proof. □

A.3.2 Candidate Equilibrium Strategy Profile

I proceed by establishing Properties 2-5 in the main text.

Lemma A.3.3 (No Losing Offers in Period 1): *In equilibrium, buyers submit serious offers with probability* 1 *in period* 1*:* $x_1^L(\tilde{P}_1^i) > 0$ *almost surely.*

Proof. Assume, to the contrary, that buyers submit losing offers with strictly positive probability in equilibrium. I show that after a buyer makes a losing equilibrium period-1 offer $p_1^L \in \mathcal{P}_1$, its subsequent period-2 offer must be v_L, i.e., $\tilde{P}_2^i(p_1^L) = v_L$ almost surely in equilibrium. Assume, to the contrary, that $\tilde{P}_2^i(p_1^L) \neq v_L$ with strictly positive probability. Let p_2^L denote the supremum of the support of $\tilde{P}_2^i(p_1^L)$ on the event $\{\tilde{P}_2^i(p_1^L) \neq v_L\}$. Since $p_2^L \geq c_H$ by Lemma A.1.1, we have $\pi_L(v_L - p_2^L) + (1 - \pi_L)(v_H - p_2^L) < 0$ by Assumption (**SIC**). Therefore, there must exist an equilibrium serious period-1 offer $\hat{p}_1^i \in \mathcal{P}_1$ such that $\tilde{P}_2^i(\hat{p}_1^i) = p_2^L$ and

$$\pi_L\big(1 - x_1^L(\hat{p}_1^i)\big)(v_L - p_2^L) + (1 - \pi_L)(v_H - p_2^L) > 0,$$

so as to "subsidize" other buyers who offered p_1^L in period 1. However, this implies that a buyer who offers an equilibrium period-1 offer $\hat{p}_1^i$ must tie with buyers who made a losing offer in period 1 and make a strictly positive expected continuation profit by offering p_2^L in period 2, which contradicts Lemma A.3.1.

By the seller's sequential rationality, $\delta v_L \geq p_1^L$, which yields zero expected lifetime profit by Proposition 2. However, as in the first part of the proof of Corollary 2, a buyer can earn a strictly positive expected lifetime profit by offering $\delta v_L + \epsilon$ for sufficiently small $\epsilon > 0$. If the low-quality seller rejects this offer when all other buyers made losing offers, its discounted payoff as of $t = 1$ is δv_L. Since the seller can enjoy a strictly higher payoff by accepting $\delta v_L + \epsilon$, the low-quality seller accepts the deviating offer $\delta v_L + \epsilon$ with strictly positive probability when all competing buyers submit losing offers. Therefore, in this case, the deviating buyer earns a strictly positive profit in period 1, contradicting equilibrium. This completes the proof. □

Lemma A.3.4: *The minimum possible equilibrium period-1 offer $\underline{p}_1$ must be at least v_L.*

Proof. Suppose, to the contrary, that $\underline{p}_1 < v_L$. Fix $\epsilon > 0$ such that $\underline{p}_1 + \epsilon < v_L$. By Lemma A.3.3, buyers submit serious offers with probability 1 in period 1, so $x_1^L(\tilde{P}_1^i) > 0$ with probability 1. Since $\underline{p}_1$ is the minimum possible equilibrium period-1 offer, there exists a period-1 offer $\hat{p}_1 \in (\underline{p}_1, \underline{p}_1 + \epsilon)$ in the support of $\tilde{P}_1^i$ such that $x_1^L(\hat{p}_1) > 0$. Then, by the monotonicity requirement (a), we have $x_1^L(\underline{p}_1 + \epsilon) \geq x_1^L(\hat{p}_1) > 0$.

Consider a deviation in which buyer i offers $\underline{p}_1 + \epsilon$ in period 1 and then submits a surely losing offer in period 2, so its period-2 continuation profit is 0. With strictly positive probability, buyer i wins in period 1 (i.e., $\underline{p}_1 + \epsilon$ is the highest offer among all buyers), and conditional on $\theta = L$, the seller accepts with strictly positive probability. If this is the case, buyer i earns strictly positive period-1 profit $v_L - (\underline{p}_1 + \epsilon) > 0$. Therefore, the deviation yields a strictly positive expected lifetime profit, contradicting Proposition 2. □

Lemma A.3.5 (Decomposition of Buyer i's Period-2 Value Function): *For any offer path $(p_1^i, p_2^i) \in \mathcal{P}_1 \times \mathcal{P}_2$ induced by buyer i's equilibrium pricing strategy σ^i, buyer i's ex-ante expected period-2 continuation profit after offering $p_1^i \in \mathcal{P}_1$ can be written as*

$$\Pi_2(p_1^i) = \Pi_2(p_2^i \mid p_1^i) = \begin{cases} \underbrace{\mathbb{P}_{\boldsymbol{\sigma}}\left(\max_{j \neq i} \tilde{P}_2^j < p_2^i\right)}_{\text{Probability of winning}} \underbrace{\Big(\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i)\Big)}_{\text{Ex-ante expected period-2 profit upon winning at } p_2^i \text{ given } p_1^i} & \text{if } p_2^i \geq c_H, \\ 0 & \text{if } p_2^i < c_H. \end{cases} \tag{14}$$

Proof. Fix any offer path $(p_1^i, p_2^i) \in \mathcal{P}_1 \times \mathcal{P}_2$. If $p_2^i \geq c_H$, then

$$\begin{aligned}
\Pi_2(p_1^i) &= \Pi_2(p_2^i \mid p_1^i) \\
&= \int_{(\boldsymbol{p}_1^{-i}, \boldsymbol{p}_2^{-i}) \in \mathcal{P}_1^{N-1} \times \mathcal{P}_2^{N-1}} \mathbf{1}_{\{\max \boldsymbol{p}_2^{-i} < p_2^i\}} \left(\sum_{\theta \in \{L,H\}} \pi_\theta \Big(1 - x_1^\theta(\max(p_1^i, \boldsymbol{p}_1^{-i}))\Big) (v_\theta - p_2^i) \right) d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i}, \boldsymbol{p}_2^{-i}) \\
&= \mathbb{P}_{\boldsymbol{\sigma}} \left(\max_{j \neq i} \tilde{P}_2^j < p_2^i \right) \Big(\pi_L \big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i) \Big).
\end{aligned}$$

The first equality is the definition of Π_2 and buyer i must find it optimal to offer p_2^i in period 2 after offering p_1^i in period 1. The second equality follows the fact that ties yield zero conditional expected profit by Lemma A.3.1 and also the hypothesis that $p_2^i \geq c_H$, which is accepted by both seller types with probability 1 in period 2. For the last equality, note that under Condition (ii)-(b), the event $\{\max_{j \neq i} \tilde{P}_2^j < p_2^i\}$ can occur only if $\max \boldsymbol{p}_1^{-i} \leq p_1^i$, so $\max(p_1^i, \boldsymbol{p}_1^{-i}) = p_1^i$. Moreover, Corollary 2 implies $x_1^H(p_1^i) = 0$ for all $p_1^i \in \mathcal{P}_1$. If $p_2^i < c_H$, buyer i's period-2 continuation profit is 0 by Lemma A.1.1. □

Lemma A.3.6: *For each $i \in \mathcal{I}$, buyer i's value function $\Pi_2(p_1^i)$ at the beginning of period 2 and $x_1^L(p_1^i)$ both strictly increase in the period-1 offer p_1^i over $\mathcal{P}_1$.*

Proof. Since buyers expect to break even over their lifetime by Proposition 2, for any $p_1^i \in \mathcal{P}_1$, we have:

$$\Pi_2(p_1^i) = \underbrace{\pi_L x_1^L(p_1^i) \left(\frac{p_1^i - v_L}{\delta} \right) \Big(\sum_{n=2}^{N} \frac{\mathbb{P}_{\boldsymbol{\sigma}}(\text{Ties with } n \text{ other buyers at } p_1^i)}{n+1} + \prod_{j \neq i} \mathbb{P}_{\boldsymbol{\sigma}}(\tilde{P}_1^j < p_1^i) \Big)}_{= -\frac{\pi_1(p_1^i)}{\delta} \quad \text{(Expected Period-1 Profit)}}$$

The right-hand side in the last line above is strictly increasing in p_1^i by Lemma A.3.3 and the monotonicity requirement (ii)-(a). Hence, $\Pi_2(p_1^i)$ is a strictly increasing function of p_1^i in $\mathcal{P}_1$. It remains to establish the strict monotonicity of $x_1^L(\cdot)$ over $\mathcal{P}_1$. Suppose, to the contrary, that there exist two equilibrium period-1 offers $p_1^i, \hat{p}_1^i \in \mathcal{P}_1$ such that $p_1^i < \hat{p}_1^i$ and $x_1^L(p_1^i) = x_1^L(\hat{p}_1^i)$. Moreover, the decomposition in Lemma A.3.5 implies that $\Pi_2(p_1^i) = \Pi_2(\hat{p}_1^i)$, which contradicts the strict monotonicity of $\Pi_2(\cdot)$ established in the previous paragraph. This contradiction completes the proof. □

Lemma A.3.7: *Fix any equilibrium in which buyers use a mixed pricing strategy in period* 1 *(i.e., the support $\mathcal{P}_1$ contains at least two elements). For any equilibrium period-1 offer $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ at which the distribution of $\tilde{P}_1^i$*

has positive probability mass, the equilibrium period-2 continuation pricing strategy $\tilde{P}_2^i(p_1^i)$ is degenerate.

Proof. Suppose, to the contrary, that there exists $\hat{p}_1 \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ such that $\tilde{P}_1^i$ has an atom at $\hat{p}_1$ and buyers randomize in period 2 after offering $\hat{p}_1$ in period 1. Then the following two events occur with strictly positive probability:

(i) exactly one buyer offers $\hat{p}_1$ in period 1 and all other buyers offer strictly less than $\hat{p}_1$;

(ii) exactly two buyers offer $\hat{p}_1$ in period 1 and all other buyers offer strictly less than $\hat{p}_1$.

Because the continuation strategy after $\hat{p}_1$ is non-degenerate, the expected maximum of the relevant period-2 offers is strictly larger in event (ii) than in event (i). Hence, the seller's continuation value from rejecting the period-1 price $\hat{p}_1$ depends on the full period-1 offer profile, not only on the period-1 price. This contradicts the requirement that the seller's period-1 acceptance probability depends on the offer profile only through the period-1 price. Therefore, $\tilde{P}_2^i(\hat{p}_1)$ must be degenerate. □

Lemma A.3.8: *The equilibrium distribution of $\tilde{P}_1^i$ assigns zero probability mass to any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$.*

Proof. Suppose, to the contrary, that there exists $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ at which the equilibrium distribution of $\tilde{P}_1^i$ has positive probability mass. Observe that $\Pi_2(p_1^i) > \Pi_2(\underline{p}_1) = 0$, where the inequality follows from Lemma A.3.6 and the equality follows from Lemma A.3.2. Since there is positive probability mass at p_1^i, Lemma A.3.7 implies that there exists a unique $p_2^i \in \mathcal{P}_2$ such that $\tilde{P}_2^i(p_1^i) = p_2^i$ almost surely. Since $\Pi_2(p_1^i) > 0$, Lemma A.3.5 implies that buyer i earns strictly positive expected period-2 profit upon winning at p_2^i,

$$\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i) > 0.$$

Moreover, by symmetry conditions (a) and (b), other buyers offer p_1^i in period 1 with strictly positive probability and then offer p_2^i in period 2 with strictly positive probability. Hence, if buyer i offers p_2^i in period 2 after offering p_1^i in period 1, it ties with another buyer with strictly positive probability and earns strictly positive expected period-2 profit conditional on a tie, contradicting Lemma A.3.1. □

Lemma A.3.9 (Non-Randomness): *For any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$, $\tilde{P}_2^i(p_1^i)$ is a degenerate random variable.*

Proof. By Lemma A.3.7, it suffices to prove the statement for the case in which $\tilde{P}_1^i$ assigns zero probability mass to p_1^i. Suppose, to the contrary, that $\tilde{P}_2^i(p_1^i)$ is nondegenerate. Then there exist distinct $p_2^i, \hat{p}_2^i$ in the support of

$\tilde{P}_2^i(p_1^i)$. By Lemma A.3.6, $\Pi_2(p_1^i) > \Pi_2(\underline{p}_1) = 0$, where the equality follows from Lemma A.3.2. By Lemma A.1.1, this is only possible when $p_2^i, \hat{p}_2^i \geq c_H$.

Since $\tilde{P}_1^i$ assigns zero probability mass to p_1^i, and by the symmetry condition (b), each other buyer also assigns zero probability mass to p_1^i. Thus, conditional on buyer i having offered p_1^i in period 1, the event that any other buyer offered exactly p_1^i in period 1 has probability 0. By Condition (ii)-(b), this implies that the probability of a tie at either p_2^i or $\hat{p}_2^i$ in period 2 is 0.

Moreover, since the winning probability $\mathbb{P}_{\boldsymbol{\sigma}}(\max_{j\neq i} \tilde{P}_2^j < p_2)$ is the same for $p_2 = p_2^i$ and $p_2 = \hat{p}_2^i$ whenever there is zero probability mass of competitors at prices in between, the two offers p_2^i and $\hat{p}_2^i$ induce the same winning probability but different expected profit upon winning, because $p_2^i \neq \hat{p}_2^i$ and the term $\pi_L\big(1-x_1^L(p_1^i)\big)(v_L-p_2)+(1-\pi_L)(v_H-p_2)$ strictly decreases in p_2. Therefore, Lemma A.3.5 implies that the two offers yield different expected period-2 continuation profits, contradicting indifference required for both to lie in the support of $\tilde{P}_2^i(p_1^i)$. □

Lemma A.3.10: *For all equilibrium period-1 offer* $\hat{p}_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1, \overline{p}_1\}$*,* $\tilde{P}_2^i(\hat{p}_1^i) = \frac{\hat{p}_1^i}{\delta}$.

Proof. Fix any $\hat{p}_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1, \overline{p}_1\}$. By Lemma A.3.3, the low-quality seller accepts the offer $\hat{p}_1^i$ with strictly positive probability: $x_1^L(\hat{p}_1^i) > 0$. Additionally, the strict monotonicity of x_1^L (Lemma A.3.6) implies that $x_1^L(\hat{p}_1^i) < x_1^L(\overline{p}) \leq 1$. Thus, when the period-1 price $\max\limits_{i\in\mathcal{I}}\{p_1^i\} = \hat{p}_1^i$, the low-quality seller essentially mixes between accepting the price and rejecting it, and thus must be indifferent between the two actions. Hence, for any $(p_1^i)_{i\in\mathcal{I}}$ such that $\max\limits_{i\in\mathcal{I}}\{p_1^i\} = \hat{p}_1^i$, we have:

$$\hat{p}_1^i = \delta \mathbb{E}_{\boldsymbol{\sigma}}\Big(\max_{i\in\mathcal{I}}\{\tilde{P}_2^i(p_1^i)\} \Big|\ h_1^S = (p_1^i)_{i\in\mathcal{I}} \Big),$$

where $\mathbb{E}_{\boldsymbol{\sigma}}$ is the expectation operator associated with the strategy profile $\boldsymbol{\sigma} := (\boldsymbol{\sigma}^B, \sigma^S)$. By the monotonicity condition (ii)-(b), $\max\limits_{i\in\mathcal{I}}\{\tilde{P}_2^i(p_1^i)\} = \tilde{P}_2^i(\hat{p}_1^i)$, the right-hand side of which must be non-random by Lemma A.3.9. Hence, $\hat{p}_1^i = \delta\tilde{P}_2^i(\hat{p}_1^i)$, which completes the proof. □

Lemma A.3.11: $x_1^L(\overline{p}_1) = 1$.

Proof. Suppose, to the contrary, that $x_1^L(\overline{p}_1) \neq 1$. Then, $x_1^L(\overline{p}_1) \in (0,1)$ by Lemma A.3.3. Hence, the low-quality seller mixes between accepting $\overline{p}_1$ and rejecting it, and must be indifferent between the two options. Therefore, $\tilde{P}_2^i(\overline{p}_1) = \overline{p}_1/\delta$ by Lemmas A.3.8 and A.3.9.

Since $\overline{p}_1$ is the maximum equilibrium period-1 offer and the equilibrium distribution has no atom at $\overline{p}_1$ by Lemma A.3.8, a buyer who offers $\overline{p}_1$ wins with probability 1 in period 1. Thus, by offering $\overline{p}_1$, a buyer earns an expected lifetime profit of

$$(\delta\pi_L + (1-\delta)\pi_L x_1^L(\overline{p}_1))v_L + \delta(1-\pi_L)v_H - \overline{p}_1 = 0,$$

where the equality follows from Proposition 2. Hence,

$$\overline{p}_1 = (\delta\pi_L + (1-\delta)\pi_L x_1^L(\overline{p}_1))v_L + \delta(1-\pi_L)v_H < \pi_L v_L + \delta(1-\pi_L)v_H,$$

where the inequality follows from $x_1^L(\overline{p}_1) < 1$.

It remains to show that a buyer can profitably deviate by offering $\overline{p}_1 + \epsilon$ in period 1 given that $\overline{p}_1 < \pi_L v_L + \delta(1-\pi_L)v_H$. This can be shown by modifying the argument given in the proof of Corollary 1 in the manuscript (e.g., replacing v_L by $\overline{p}_1$ whenever necessary). □

Lemma A.3.12: *For any $p_1^i \in \mathcal{P}_1 \setminus \{\overline{p}_1, \underline{p}_1\}$, the seller's period-1 acceptance probability is*

$$x_1^L(p_1^i) = \frac{p_1^i - \delta(\pi_L v_L + (1-\pi_L)v_H)}{(1-\delta)\pi_L v_L}.$$

Proof. Fix any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1, \overline{p}_1\}$. By Lemma A.3.10, $\tilde{P}_2^i(p_1^i) = \frac{p_1^i}{\delta}$. Observe that $\frac{p_1^i}{\delta} \geq \frac{v_L}{\delta} > c_H$, where the first inequality follows from Lemma A.3.4, and the second from Assumption (**EXP**). Hence, the seller accepts $\frac{p_1^i}{\delta}$ with probability 1 in period 2. In addition, by Lemmas A.3.8 and A.3.10, conditional on buyer i offering p_1^i, there is zero probability of buyer i tying with a competitor in either period 1 or period 2. Therefore, denoting the equilibrium CDF of its competitors' period-1 offers by $G(\cdot)$, I can make use of the decomposition in Lemma A.3.5 and express buyer i's lifetime expected profit from offering p_1^i in period 1 as:

$$\pi_L x_1^L(p_1^i)G(p_1^i)(v_L - p_1^i) + \delta G(p_1^i)\Big(\pi_L\big(1 - x_1^L(p_1^i)\big)\Big(v_L - \frac{p_1^i}{\delta}\Big) + (1-\pi_L)\Big(v_H - \frac{p_1^i}{\delta}\Big)\Big) = 0,$$

where the equality follows from Proposition 2. Dividing the equation by $G(p_1^i) > 0$ and solving for $x_1^L(p_1^i)$ gives the desired expression. □

Lemma A.3.13: *If p_t^i is an isolated point of $\mathcal{P}_t$, then the distribution of the period-t amount $\tilde{P}_t^i$ induced by an equilibrium pricing strategy σ^i assigns positive probability mass to p_t^i for each $t \in \{1,2\}$.*

Proof. Suppose, to the contrary, that $\mathbb{P}_{\boldsymbol{\sigma}}(\tilde{P}_t^i = p_t^i) = 0$. Since p_t^i is an isolated point of $\mathcal{P}_t$, fix $\epsilon > 0$ such that

$(p_t^i-\epsilon,\, p_t^i+\epsilon)\cap(\mathcal{P}_t\setminus\{p_t^i\})=\varnothing$. Then $\tilde{P}_t^i\in(p_t^i-\epsilon,p_t^i+\epsilon)$ implies $\tilde{P}_t^i=p_t^i$, so $\mathbb{P}_{\boldsymbol{\sigma}}\left(\tilde{P}_t^i\in(p_t^i-\epsilon,p_t^i+\epsilon)\right)=\mathbb{P}_{\boldsymbol{\sigma}}(\tilde{P}_t^i=p_t^i)=0$, which contradicts the hypothesis that p_t^i is an element of support $\mathcal{P}_t$. □

Lemma A.3.14: *(i)* $\tilde{P}_2^i(\overline{p}_1)=\frac{\overline{p}_1}{\delta}$;

(ii) The maximum possible equilibrium period-1 offer satisfies $\overline{p}_1=\pi_L v_L+(1-\pi_L)\delta v_H$.

Proof. I first establish Lemma A.3.14-(i). By Lemma A.3.13, if $\overline{p}_1$ were an isolated point of $\mathcal{P}_1\setminus\{\underline{p}_1\}$, then the equilibrium distribution of $\tilde{P}_1^i$ would have positive probability mass at $\overline{p}_1$, contradicting Lemma A.3.8. Hence, $\overline{p}_1$ is a limit point of $\mathcal{P}_1\setminus\{\underline{p}_1\}$, so for any $\epsilon>0$ there exists $\epsilon'\in(0,\epsilon)$ such that $\overline{p}_1-\epsilon'\in\mathcal{P}_1\setminus\{\underline{p}_1\}$.

By Condition (ii)-(b) and Lemma A.3.10, we have

$$\inf\operatorname{supp}\left(\tilde{P}_2^i(\overline{p}_1)\right)\geq\sup\operatorname{supp}\left(\tilde{P}_2^i(\overline{p}_1-\epsilon')\right)=\frac{\overline{p}_1-\epsilon'}{\delta}>\frac{\overline{p}_1-\epsilon}{\delta}.$$

Since $\epsilon>0$ is arbitrary, this implies $\inf\operatorname{supp}(\tilde{P}_2^i(\overline{p}_1))\geq\frac{\overline{p}_1}{\delta}$, and hence $\tilde{P}_2^i(\overline{p}_1)\geq\frac{\overline{p}_1}{\delta}$.

By Lemma A.3.11, $x_1^L(\overline{p}_1)=1$. Therefore, upon receiving the period-1 price $\overline{p}_1$, the low-quality seller must find accepting $\overline{p}_1$ weakly optimal:

$$\overline{p}_1\geq\delta\mathbb{E}_{\boldsymbol{\sigma}}\left(\max_{j\in\mathcal{I}}\tilde{P}_2^j(p_1^j)\,\middle|\,(p_1^j)_{j\in\mathcal{I}}\right)=\delta\tilde{P}_2^i(\overline{p}_1),$$

where the last equality follows from Condition (ii)-(b) and Lemma A.3.9 as in Lemma A.3.10. Hence, $\tilde{P}_2^i(\overline{p}_1)\leq\frac{\overline{p}_1}{\delta}$, which together with the previous inequality yields $\tilde{P}_2^i(\overline{p}_1)=\frac{\overline{p}_1}{\delta}$. This establishes Lemma A.3.14-(i).

To establish Lemma A.3.14-(ii), note that $x_1^L(\overline{p}_1)=1$ and $x_1^H(\cdot)=0$ in equilibrium. Moreover, by Lemma A.3.14-(i), conditional on winning at $\overline{p}_1$ in period 1, a buyer's expected lifetime payoff equals

$$\pi_L(v_L-\overline{p}_1)+(1-\pi_L)\delta\left(v_H-\frac{\overline{p}_1}{\delta}\right)=\pi_L v_L+(1-\pi_L)\delta v_H-\overline{p}_1.$$

By Proposition 2, buyers earn zero expected lifetime profit in equilibrium, which implies $\pi_L v_L+(1-\pi_L)\delta v_H-\overline{p}_1=0$. Thus, $\overline{p}_1=\pi_L v_L+(1-\pi_L)\delta v_H$. □

Lemma A.3.15: *If buyer i makes an equilibrium period-1 offer $p_1^i\in\mathcal{P}_1\setminus\{\underline{p}_1\}$ in period 1, then* $\tilde{P}_2^i(p_1^i)=\frac{p_1^i}{\delta}$.

Proof of Lemma A.3.15. Immediate from Lemmas A.3.10 and A.3.14-(i). □

Lemma A.3.16: *For all* $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$*, the low-quality seller's period-1 acceptance rule is*

$$x_1^L(p_1^i) = \frac{p_1^i - \delta(\pi_L v_L + (1-\pi_L)v_H)}{(1-\delta)\pi_L v_L}.$$

Proof. Immediate from Lemmas A.3.11, A.3.12, and A.3.14-(ii). □

Lemma A.3.17: *The set* $\mathcal{P}_1 \setminus \{\underline{p}_1\}$ *is an interval.*

Proof. Suppose, to the contrary, that there is a gap in $\mathcal{P}_1 \setminus \{\underline{p}_1\}$: there exist $p_1^i, \hat{p}_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ with $p_1^i < \hat{p}_1^i$ such that

$$(p_1^i, \hat{p}_1^i) \cap \left(\mathcal{P}_1 \setminus \{\underline{p}_1\}\right) = \varnothing.$$

By Lemma A.3.15, competing buyers never make period-2 offers in $\left(\frac{p_1^i}{\delta}, \frac{\hat{p}_1^i}{\delta}\right)$ in equilibrium. Moreover, by Lemma A.3.8, there is zero probability of a tie at either endpoint. Hence, after offering $\hat{p}_1^i$ in period 1, if buyer i offers $\frac{p_1^i}{\delta}$ instead of $\frac{\hat{p}_1^i}{\delta}$ in period 2, the winning probability remains unchanged.

In addition, the buyer pays a smaller amount upon winning, while the expected asset value upon winning does not change. Therefore, buyer i attains a strictly higher (ex ante) expected period-2 profit upon winning. Hence, as can be seen from buyer i's expected continuation profit in Equation (14), buyer i would be strictly better off by offering $\frac{p_1^i}{\delta}$ instead of $\frac{\hat{p}_1^i}{\delta}$ in period 2 after having offered $\hat{p}_1^i$ in period 1, contradicting Lemma A.3.15. □

Lemma A.3.18: *For each* $i \in \mathcal{I}$*, the equilibrium cumulative distribution function (CDF)* G_i *of buyer* i*'s period-1 pricing strategy can be expressed on* $\mathcal{P}_1 \setminus \{\underline{p}_1\}$ *as follows:*

$$G_i(p_1^i) = \left\{ \left(\frac{p_1^i - v_L}{(1-\pi_L)(\delta v_H - v_L)}\right)^{\frac{\delta(1-\pi_L)(v_H - v_L)}{v_L - \delta(\pi_L v_L + (1-\pi_L)v_H)}} \left(\frac{(1-\delta)\pi_L v_L}{p_1^i - \delta(\pi_L v_L + (1-\pi_L)v_H)}\right)^{\frac{(1-\delta)v_L}{v_L - \delta(\pi_L v_L + (1-\pi_L)v_H)}} \right\}^{\frac{1}{N-1}}. \quad (15)$$

Proof. By Lemma A.3.1, for any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$ and any $p_2^i > \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta$, buyer i's ex ante expected period-2 continuation profit from offering p_2^i after having offered p_1^i in period 1 is

$$\int_{\underline{p}_1 \le \max \boldsymbol{p}_1^{-i} \le \delta p_2^i} \left(\pi_L\left(1 - x_1^L(\max(p_1^i, \boldsymbol{p}_1^{-i}))\right)(v_L - p_2^i) + (1-\pi_L)(v_H - p_2^i)\right) d\mathbb{P}_{\boldsymbol{\sigma}}(\boldsymbol{p}_1^{-i}), \quad (16)$$

where $\boldsymbol{p}_t^{-i} := (p_t^j)_{j \neq i}$ denotes the vector of buyer i's competing period-t offers for $t = 1, 2$.

In particular, whenever $p_2^i \in \big((p_1^i - \epsilon)/\delta,\, p_1^i/\delta\big]$, the expression (16) simplifies to

$$\underbrace{G(\delta p_2^i)}_{\text{Winning probability at } p_2^i} \underbrace{\Big(\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1-\pi_L)(v_H - p_2^i)\Big)}_{\text{Ex ante expected period-2 profit upon winning at } p_2^i \text{ given } p_1^i}, \tag{17}$$

where $G(\cdot)$ is the equilibrium CDF of $\max_{j\neq i}\{\tilde{P}_1^j\}$.

Lemma A.3.16 implies that for any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$, the expression in (17) must be maximized at $p_2^i = p_1^i/\delta$. By Lemma A.3.8, the equilibrium distribution of $\tilde{P}_1^i$ has no probability mass at any $p_1^i \in \mathcal{P}_1 \setminus \{\underline{p}_1\}$. Hence, the expected continuation profit in (17) is left differentiable at $p_2^i = p_1^i/\delta$, and the following first-order condition must hold:

$$\begin{aligned} &\frac{\partial}{\partial p_2^i}\Big(G(\delta p_2^i)\Big(\pi_L\big(1 - x_1^L(p_1^i)\big)(v_L - p_2^i) + (1-\pi_L)(v_H - p_2^i)\Big)\Big)\Bigg|_{p_2^i = p_1^i/\delta} = 0 \\ \Longleftrightarrow\;& \delta\Big(\frac{\pi_L(1 - x_1^L(p_1^i))}{\pi_L(1 - x_1^L(p_1^i)) + 1 - \pi_L}\Big(v_L - \frac{p_1^i}{\delta}\Big) + \frac{1-\pi_L}{\pi_L(1 - x_1^L(p_1^i)) + 1 - \pi_L}\Big(v_H - \frac{p_1^i}{\delta}\Big)\Big) = \frac{G(p_1^i)}{G'(p_1^i)}. \end{aligned} \tag{18}$$

By Lemma A.3.14-(ii), $\overline{p}_1 = \pi_L v_L + (1-\pi_L)\delta v_H$ is the maximum of the support of $G(\cdot)$, so $G(\overline{p}_1) = 1$. Subject to this boundary condition, the differential equation (18) admits a unique closed-form solution for G on $\mathcal{P}_1 \setminus \{\underline{p}_1\}$. Finally, by Condition (i)-(a) and the assumption that buyers randomize independently, the CDF of $\max_{j\neq i}\{\tilde{P}_1^j\}$ is G_i^{N-1}, so $G_i = G^{1/(N-1)}$, which yields (15). □

Lemma A.3.19: *The support satisfies* $\mathcal{P}_1 = \overline{\mathcal{P}_1 \setminus \{\underline{p}_1\}} = [v_L,\ \pi_L v_L + (1-\pi_L)\delta v_H]$.

Proof. By Lemma A.3.14-(ii) and Lemma A.3.17, $\mathcal{P}_1 \setminus \{\underline{p}_1\}$ is an interval with right endpoint $\pi_L v_L + (1-\pi_L)\delta v_H$. Hence, it remains to show that $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\}) = v_L$. Suppose, to the contrary, that $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\}) \neq v_L$. Since $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\}) \geq v_L$ by Lemma A.3.4, we have $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\}) > v_L$.

Since $\tilde{P}_1^i$ assigns zero probability mass to every point in $\mathcal{P}_1 \setminus \{\underline{p}_1\}$ and $G\big(\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})\big) > 0$ (which follows from (15)), it follows that $\tilde{P}_1^i$ must have positive probability mass at $\underline{p}_1$. By Proposition 2, a buyer makes zero expected period-1 profit by offering $\underline{p}_1$, i.e.,

$$\pi_1(\underline{p}_1) = \Big(\frac{1}{N}\prod_{j\neq i}\mathbb{P}_{\boldsymbol{\sigma}}(\tilde{P}_1^j = \underline{p}_1)\Big)\pi_L x_1^L(\underline{p}_1)(v_L - \underline{p}_1) = 0.$$

Since $x_1^L(\underline{p}_1) > 0$ by Lemma A.3.3 and $\mathbb{P}_{\boldsymbol{\sigma}}(\tilde{P}_1^j = \underline{p}_1) > 0$ for all $j \neq i$, we have $\underline{p}_1 = v_L$. Also, an argument similar to the one given in Proposition 1 shows that $\tilde{P}_2^i(v_L) = v_L/\delta$. Hence, a buyer never makes a period-2 offer in $(v_L/\delta,\ \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta)$.

The rest of the proof is similar to the proof of Lemma A.3.17. By Lemma A.3.5, for all $p_2^i \in (v_L/\delta,\ \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta]$, we have

$$\Pi_2(p_2^i \mid \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})) = \underbrace{G(\delta p_2^i)}_{\text{Winning probability at } p_2^i} \underbrace{\Big(\pi_L\big(1 - x_1^L(\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\}))\big)(v_L - p_2^i) + (1 - \pi_L)(v_H - p_2^i)\Big)}_{\text{Ex ante expected period-2 profit upon winning at } p_2^i},$$

where $G(\cdot)$ denotes the equilibrium CDF of $\max_{j \neq i} \tilde{P}_1^j$.

Recall that a competing buyer never makes a period-2 offer in $(v_L/\delta,\ \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta)$. Hence, if buyer i reduces the period-2 offer to $v_L/\delta + \epsilon < \inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta$ in period 2[25] after having offered $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})$ in period 1, the winning probability remains unchanged, whereas the ex ante expected period-2 profit upon winning increases. Hence, the buyer would be strictly better off by offering $v_L/\delta + \epsilon$ instead of $\inf(\mathcal{P}_1 \setminus \{\underline{p}_1\})/\delta$ in period 2, contradicting sequential rationality. □

Lemma A.3.20: *In equilibrium, all buyers continuously randomize period-1 offers according to the CDF G_i over the support $\mathcal{P}_1 = [v_L,\ \pi_L v_L + \delta(1 - \pi_L)v_H]$, with an explicit expression for G_i given in* (15).

Proof. Since $\mathcal{P}_1 \setminus \{\underline{p}_1\}$ is an interval by Lemma A.3.17 and $\overline{\mathcal{P}_1 \setminus \{\underline{p}_1\}} = \mathcal{P}_1$ is also an interval by Lemma A.3.19, the right-continuity of the CDF $G_i(\cdot)$ implies that $G_i(\cdot)$ is given by (15) on the entire support $\mathcal{P}_1$. □

Lemma A.3.21: *If a buyer submits the minimum equilibrium offer $\underline{p}_1 (= v_L)$ in period* 1*, it must submit a losing offer in period* 2 *on the equilibrium path. Moreover, it is weakly optimal for the buyer to submit $\frac{\underline{p}_1}{\delta} (= \frac{v_L}{\delta})$ in period* 2*, which is also a losing offer. Hence, it entails no loss of generality to set $\tilde{P}_2^i(v_L) = \frac{v_L}{\delta}$ for all $i \in \mathcal{I}$.*

Proof. Fix any $p_1' \in \mathcal{P}_1 \setminus \{v_L\}$. By Lemma A.3.15, $\tilde{P}_2^i(p_1') = \frac{p_1'}{\delta}$ (almost surely). By Condition (ii)-(b), every offer in the support of $\tilde{P}_2^i(v_L)$ is weakly below every offer in the support of $\tilde{P}_2^i(p_1')$. Hence, $\tilde{P}_2^i(v_L) \leq \frac{p_1'}{\delta}$ almost surely. Since v_L is the minimum of $\mathcal{P}_1$, letting $p_1' \downarrow v_L$ yields $\tilde{P}_2^i(v_L) \leq \frac{v_L}{\delta}$.

Moreover, on the equilibrium path, all buyers offer strictly greater than v_L in period 1 almost surely, and thus offer strictly greater than $\frac{v_L}{\delta}$ in period 2 almost surely by Lemmas A.3.15 and A.3.20. Therefore, if a buyer

[25] The increment $\epsilon > 0$ is necessary to break ties.

submits v_L in period 1, any period-2 offer in the support of $\tilde{P}_2^i(v_L)$ is a losing offer on the equilibrium path.

By Lemma A.3.2 and $\underline{p}_1 = v_L$, we have $\Pi_2(v_L) = 0$. By sequential rationality, the buyer cannot earn a strictly positive continuation profit by submitting any other period-2 offer. Since submitting $\frac{v_L}{\delta}$ is (also) a losing offer and thus yields zero continuation profit in period 2, it is weakly optimal for the buyer to offer $\frac{v_L}{\delta}$ in period 2 after offering v_L in period 1. Hence, it entails no loss of generality to set $\tilde{P}_2^i(v_L) = \frac{v_L}{\delta}$ for all $i \in \mathcal{I}$. □

Lemma A.3.22: *Suppose that $\tilde{P}_2^i(v_L) = \frac{v_L}{\delta}$ for all $i \in \mathcal{I}$ in equilibrium. Then, it is without loss of generality to set the low-quality seller's period-1 acceptance probability at $\underline{p}_1 = v_L$ to be*

$$x_1^L(v_L) = \frac{v_L - \delta(\pi_L v_L + (1-\pi_L)v_H)}{(1-\delta)\pi_L v_L}.$$

Proof. Suppose that a buyer wins the asset in period 2 by offering $\frac{v_L}{\delta}$ after offering v_L in period 1. Then, by Bayes' Rule, the buyer infers that it could have only won the asset because all the other buyers offered v_L in period 1, and $\frac{v_L}{\delta}$ in period 2. Thus, posterior expected value of the asset upon winning in period 2 equals Since the buyer earns zero period-2 continuation profit upon winning at price $\frac{v_L}{\delta}$ by Proposition 2, setting the buyer's posterior expected value upon winning in period 2, $\frac{\pi_L(1-x_1^L(v_L))}{1-\pi_L x_1^L(v_L)} v_L + \left(1 - \frac{\pi_L(1-x_1^L(v_L))}{1-\pi_L x_1^L(v_L)}\right) v_H$, equal to the period-2 price $\frac{v_L}{\delta}$ yields the desired acceptance probability. □

Property 1 directly follows from the argument in the main text, Property 2 from Lemmas A.3.15 and A.3.21, Property 4 from Lemmas A.3.15 and A.3.22, and Property 5 (and 3) from Lemma A.3.20.

A.3.3 Optimality of Equilibrium Strategy Profile

Proof of Theorem 1. Let us establish Theorem 1. Observe that Properties 1-5 uniquely determine the distribution of price paths under the candidate equilibrium. I shall check whether there is any profitable deviation from this candidate equilibrium. Clearly, the low-quality seller is indifferent between accepting $\max_{i\in\mathcal{I}} \tilde{P}_1^i$ in period 1 and rejecting it. Also, it is clear that the high-quality seller always finds it optimal to reject $\max_{i\in\mathcal{I}} \tilde{P}_1^i < c_H$ in period 1 and to accept $\frac{1}{\delta}\max_{i\in\mathcal{I}} \tilde{P}_1^i > c_H$ in period 2.

I show that buyer i finds it optimal to offer $\frac{p_1^i}{\delta}$ in period 2 after having offered p_1^i in period 1. As shown in (8),

$$\frac{\partial^2}{\partial \hat{p}_1^i \partial \hat{p}_2^i}\left(G(\delta\hat{p}_2^i)\Big(\pi_L\big(1 - x_1^L(\hat{p}_1^i)\big)(v_L - \hat{p}_2^i) + (1-\pi_L)(v_H - \hat{p}_2^i)\Big)\right)\Big|_{\hat{p}_1^i = p_1^i, \hat{p}_2^i = p_2^i} > 0$$

whenever $p_1^i \in (\underline{p}_1, \overline{p}_1]$ and $p_2^i \in \left[\frac{\underline{p}_1}{\delta}, \frac{p_1^i}{\delta}\right)$, implying:

$$\begin{aligned}
&\frac{\partial}{\partial \hat{p}_2^i}\Big(G(\delta\hat{p}_2^i)\Big(\pi_L\big(1-x_1^L(\hat{p}_1^i)\big)(v_L-\hat{p}_2^i)+(1-\pi_L)(v_H-\hat{p}_2^i)\Big)\Big)\Big|_{\hat{p}_1^i=p_1^i,\hat{p}_2^i=p_2^i} \\
>&\frac{\partial}{\partial \hat{p}_2^i}\Big(G(\delta\hat{p}_2^i)\Big(\pi_L\big(1-x_1^L(\hat{p}_1^i)\big)(v_L-\hat{p}_2^i)+(1-\pi_L)(v_H-\hat{p}_2^i)\Big)\Big)\Big|_{\hat{p}_1^i=\delta p_2^i,\hat{p}_2^i=p_2^i} \overset{(18)}{=} 0.
\end{aligned}$$

Clearly, a buyer earns a zero period-2 profit by offering less than $\frac{\underline{p}_1}{\delta}$. Thus, after having offered p_1^i in period 1, a buyer is strictly better off by offering $\frac{p_1^i}{\delta}$ in period 2, compared to offering a strictly lower period-2 offer.

Observe that for any $p_1^i \in [\underline{p}_1, \overline{p}_1)$ and any $\tilde{p}_1^i \in (p_1^i, \overline{p}_1]$, we have:

$$\begin{aligned}
&\frac{\partial}{\partial p_2^i}\left(\int\limits_{v_L\le \max(\mathbf{p}_1^{-i})\le \delta p_2^i} \pi_L\big(1-x_1^L(\max(p_1^i,\mathbf{p}_1^{-i}))\big)\, d\mathbb{P}_{\boldsymbol{\sigma}}(\mathbf{p}_1^{-i})\,(v_L-p_2^i)+(1-\pi_L)G(\delta p_2^i)\,(v_H-p_2^i)\right)\Bigg|_{p_2^i=\frac{\tilde{p}_1^i}{\delta}} \\
=&-\int\limits_{v_L\le \max(\mathbf{p}_1^{-i})\le \tilde{p}_1^i}\Big(\pi_L\big(1-\underbrace{x_1^L(\max(p_1^i,\mathbf{p}_1^{-i}))}_{<x_1^L(\tilde{p}_1^i)}\big)\Big)\, d\mathbb{P}_{\boldsymbol{\sigma}}(\mathbf{p}_1^{-i})-(1-\pi_L)G(\tilde{p}_1^i) \\
&+\delta\Big(\pi_L\big(1-x_1^L(\tilde{p}_1^i)\big)\Big(v_L-\frac{\tilde{p}_1^i}{\delta}\Big)+(1-\pi_L)\Big(v_H-\frac{\tilde{p}_1^i}{\delta}\Big)\Big)G'(\tilde{p}_1^i) \\
<&-G(\tilde{p}_1^i)\Big(1-\pi_L x_1^L(\tilde{p}_1^i)\Big)+\delta\Big(\pi_L\big(1-x_1^L(\tilde{p}_1^i)\big)\Big(v_L-\frac{\tilde{p}_1^i}{\delta}\Big)+(1-\pi_L)\Big(v_H-\frac{\tilde{p}_1^i}{\delta}\Big)\Big)G'(\tilde{p}_1^i) \overset{(18)}{=} 0.
\end{aligned} \tag{19}$$

In addition, since no buyer offers more than $\frac{\overline{p}_1}{\delta}$ in period 2, offering strictly more than $\frac{\overline{p}_1}{\delta}$ in period 2 is clearly suboptimal. Thus, after having offered p_1^i in period 1, buyer i is strictly better off by offering $\frac{p_1^i}{\delta}$, compared to offering a strictly higher period-2 offer.

It remains to check whether any deviating period-1 offer may potentially generate an expected lifetime profit strictly higher than zero, which is what buyers earn in the candidate equilibrium by Proposition 2. Obviously, a buyer earns a zero lifetime expected profit by offering $p_1^i < v_L$. Also, it is clear from the proof of Lemma 2 that a buyer expects a negative lifetime profit from any period-1 offer such that the high-quality seller accepts with positive probability. Additionally, if a buyer makes $p_1^i > \overline{p}_1$ such that the high-quality seller never accepts, the buyer pays more than $\overline{p}_1$ in period 1 but earns the same expected period-2 continuation profit as from making $\overline{p}_1$ in period 1. Hence, no period-1 offer amount leads to a strictly positive expected lifetime profit. This completes the proof.

□

A.4 Remaining Proofs

Proof of Theorem 2. By Theorem 1, $\tilde{P}_2^{(N)} = \frac{\tilde{P}_1^{(N)}}{\delta}$, so it suffices to establish the statement for period 1. Recall that the equilibrium CDF of $\tilde{P}_1^i$ with $N \geq 2$ buyers is $G^{\frac{1}{N-1}}$ by Property 5, and buyers randomize independently. Hence, the equilibrium CDF of the period-1 price $\tilde{P}_1^{(N)}$ is $G^{\frac{N}{N-1}}$. Since $\frac{N}{N-1}$ decreases in N and $G(p_1^i) \in (0,1)$ for any p_1^i in the interior of $\mathcal{P}_1$, $G^{\frac{N}{N-1}}(p_1^i)$ strictly increases in N for any p_1^i in the interior of $\mathcal{P}_1$. This is equivalent to the statement that the equilibrium distribution of $\tilde{P}_1^{(N)}$ first-order stochastically dominates the equilibrium distribution of $\tilde{P}_1^{(N+1)}$. This completes the proof. □

Proof of Proposition 3. Observe that the distribution of the maximal competing period-1 offers, G, is determined only by the number of buyers participating at $t = 2$. Hence, each buyer mixes its period-1 offer according to the CDF $G^{\frac{1}{M-1}}$ over $[v_L, \pi_L v_L + \delta(1-\pi_L)v_H]$. Therefore, the period-1 price has CDF $G^{\frac{N}{M-1}}$. Since $G(p) \in (0,1)$ on the interior of the support $[v_L, \pi_L v_L + \delta(1-\pi_L)v_H]$, an increase in M increases the CDF $G^{\frac{N}{M-1}}$ on the interior of the support. Since $\frac{N}{M-1}$ decreases in M and $G(p_1^i) \in (0,1)$ for any p_1^i in the interior of $\mathcal{P}_1$, $G^{\frac{N}{M-1}}(p_1^i)$ strictly increases in M for any p_1^i in the interior of $\mathcal{P}_1$. Hence, an increase in M reduces expected prices in all periods. Also, by arguing analogously as in Section IV, since expected prices in both periods strictly decrease in M, equilibrium transaction speed and the present value of expected equilibrium total welfare must also strictly decrease in M. □

Proof of Proposition 4. Suppose, to the contrary, that there exists a Perfect Bayesian Equilibrium in the confidential negotiation process that induces the strategies as prescribed in Proposition 4. I refer to buyers who are prescribed to make losing offers in both periods as "non-serious buyers." It remains to show that a non-serious buyer has a profitable deviation. In this equilibrium, non-serious buyers must earn zero expected period-t payoffs for both $t = 1, 2$. Since the equilibrium CDF of the period-1 price is $G^{\frac{M}{M-1}}$, the derivative of a non-serious buyer

i's ex ante expected period-2 continuation payoff with respect to its period-2 offer p_2^i can be written as:

$$\begin{aligned}
&\frac{\partial}{\partial p_2^i}\left(\int_{v_L}^{\delta p_2^i}\Big(\pi_L\big(1-x_1^L(p_1)\big)(v_L-p_2^i)+(1-\pi_L)(v_H-p_2^i)\Big)\,dG^{\frac{M}{M-1}}(p_1)\right)\\
=&\frac{M}{M-1}\underbrace{\delta\Big(\pi_L\big(1-x_1^L(\delta p_2^i)\big)(v_L-p_2^i)+(1-\pi_L)(v_H-p_2^i)\Big)G'(\delta p_2^i)}_{=G(\delta p_2^i)\left(1-\pi_L x_1^L(\delta p_2^i)\right)\text{ by Equation (6)}}G^{\frac{1}{M-1}}(\delta p_2^i)\\
&-\int_{v_L}^{\delta p_2^i}\Big(\pi_L\big(1-x_1^L(p_1)\big)+(1-\pi_L)\Big)\,dG^{\frac{M}{M-1}}(p_1)\\
=&G^{\frac{M}{M-1}}(\delta p_2^i)\underbrace{\left(\frac{M}{M-1}\Big(1-\pi_L x_1^L(\delta p_2^i)\Big)-\int_{v_L}^{\delta p_2^i}\Big(1-\pi_L x_1^L(p_1)\Big)\frac{dG^{\frac{M}{M-1}}(p_1)}{G^{\frac{M}{M-1}}(\delta p_2^i)}\right)}_{(*)}.
\end{aligned}$$

Note that as p_2^i tends to $\frac{v_L}{\delta}$, the integral inside $(*)$ converges to $1-\pi_L x_1^L(v_L)$, and thus the entire term $(*)$ converges to $\frac{1}{M-1}\Big(1-\pi_L x_1^L(v_L)\Big)>0$. Moreover, a non-serious buyer's ex ante expected period-2 continuation payoff from offering $\frac{v_L}{\delta}$ is zero, because any serious buyer's offer is strictly greater than $\frac{v_L}{\delta}$ with probability 1. This implies that a non-serious buyer can earn a strictly positive expected period-2 continuation payoff from offering $\frac{v_L}{\delta}+\varepsilon$ in period 2, which is a contradiction. □

Online Appendix

IA.1 Asymmetric Information Between Buyers

This subsection analyzes the confidential negotiation process in which the buyers receive independent private signals $(\epsilon_i)_{i\in\mathcal{I}}$ uniformly distributed on $[-\Delta,\Delta]$, where $\Delta>0$ satisfies the following conditions:

$$\pi_L v_L + (1-\pi_L)v_H + \Delta < c_H; \quad \textbf{(SIC')}$$

$$\delta(v_H - \Delta) > (v_L - \Delta); \quad \textbf{(NFS')}$$

$$v_L - \Delta > \delta c_H. \quad \textbf{(EXP')}$$

$$\Delta < (1-\pi_L)(v_H - v_L). \quad \textbf{(NOL)}$$

The first three conditions are the counterparts of (**SIC'**),(**NFS'**), and (**EXP'**) in the manuscript. The last condition (**NOL**) rules out an overly large dispersion of signals Δ, which preserves various mathematical properties of equilibrium objects (e.g., supermodularity of buyers' continuation profit functions). Under these parametric assumptions, I construct a candidate equilibrium analogous to the one in Theorem 1 and verify that the candidate equilibrium is indeed an equilibrium.

IA.2 Construction of Candidate Equilibrium

I conjecture that the candidate equilibrium is characterized by a pair of strictly increasing, differentiable functions (p,x) with the same domain $[-\Delta,\Delta]$. More specifically, $p(\epsilon_i)\in\mathbb{R}^1$ denotes the equilibrium period-1 offer made by a buyer whose signal realization is $\epsilon_i\in[-\Delta,\Delta]$, whereas $x(\max_{i\in\mathcal{I}}\epsilon_i)\in[0,1]$ denotes the low–quality seller's equilibrium probability of accepting the period-1 price submitted by the buyer whose signal realization is $\max_{i\in\mathcal{I}}\epsilon_i\in[-\Delta,\Delta]$. Moreover, the candidate equilibrium play proceeds as follows:

Candidate Equilibrium Characterized by Function Pair (p,x)***:*** *For each $i\in\mathcal{I}$, buyer i receives a private signal ϵ_i. At the beginning of period 1, buyer i submits $p(\epsilon_i)$ for all $i\in\mathcal{I}$. Then, the low-type seller accepts $p(\max_{i\in\mathcal{I}}\epsilon_i)$ with probability $x(\max_{i\in\mathcal{I}}\epsilon_i)$, or rejects all the offers with residual probability. At the beginning of period 2, buyer i submits $\frac{p(\epsilon_i)}{\delta}$ for all $i\in\mathcal{I}$. Then, for each $\theta\in\{L,H\}$, the θ-quality seller accepts the period-2 price if* *and only if* $\frac{p(\max_{i\in\mathcal{I}} \epsilon_i)}{\delta} \geq c_\theta$.

The remainder of Subsection IA.2 is organized as follows. I compute the conditional expectation of the additional common value component upon winning in each period and derive a set of necessary conditions under which the players find it optimal to adhere to the strategies prescribed by the candidate equilibrium. Then, I rely on these conditions to pin down the function pair (p, x), which fully describes the candidate equilibrium strategies above.

IA.2.1 Expectation of Additional Common Value Components Conditional Upon Winning

A few notations are introduced in turn. Let $\epsilon_{-i} := \max_{j\neq i} \epsilon_j$ denote the highest signal received by buyer i's competitors, whose CDF is denoted by F and the associated PDF is denoted by f. If a buyer initially receives a signal $\epsilon_i = \epsilon_0$, the conditional expectation of the buyer's additional payoff component upon winning at the price $p(\hat{\epsilon}_1)$ in period 1 can be calculated as:

$$\mathbb{E}\left[\frac{1}{N}\sum_{j\in\mathcal{I}} \epsilon_j \,\middle|\, \epsilon_i = \epsilon_0,\ \text{won at the price of } p(\hat{\epsilon}_1) \text{ at } t = 1\right]$$
$$= \frac{1}{N}\left(\epsilon_0 + \mathbb{E}\left[\sum_{j\in\mathcal{I}\setminus\{i\}} \epsilon_j \,\middle|\, \max_{j\neq i} \epsilon_j \leq \hat{\epsilon}_1\right]\right) = \frac{1}{N}\left(\epsilon_0 + (N-1)\,\frac{\hat{\epsilon}_1 - \Delta}{2}\right).$$

Likewise, if a buyer initially receives a signal $\epsilon_i = \epsilon_0$ and submits $p(\hat{\epsilon}_1)$ in period 1, the conditional expectation of the buyer's additional payoff component upon winning at the price $\frac{p(\hat{\epsilon}_2)}{\delta}$ in period 2 can be calculated as:

$$\mathbb{E}\left[\frac{1}{N}\sum_{j\in\mathcal{I}} \epsilon_j \,\middle|\, \epsilon_i = \epsilon_0,\ \text{rejected } p(\hat{\epsilon}_1) \text{ at } t = 1,\ \text{won at the price of } \frac{p(\hat{\epsilon}_2)}{\delta} \text{ at } t = 2\right]$$
$$= \frac{1}{N}\left(\epsilon_0 + \mathbb{E}\left[\sum_{j\in\mathcal{I}\setminus\{i\}} \epsilon_j \,\middle|\, \text{rejected } p(\hat{\epsilon}_1) \text{ at } t = 1,\ \text{won at the price of } \frac{p(\hat{\epsilon}_2)}{\delta} \text{ at } t = 2\right]\right)$$
$$= \frac{1}{N}\left(\epsilon_0 + \frac{\int_{[-\Delta,\hat{\epsilon}_2]^{N-1}} \left(\sum_{j\in\mathcal{I}\setminus\{i\}} \epsilon_j\right)\left(\pi_L\big(1 - x(\max\{\epsilon_{-i}, \hat{\epsilon}_1\})\big) + (1-\pi_L)\right) d\mathbb{P}(\epsilon_1, \ldots, \epsilon_{-i})}{\int_{[-\Delta,\hat{\epsilon}_2]^{N-1}} \left(\pi_L\big(1 - x(\max\{\epsilon_{-i}, \hat{\epsilon}_1\})\big) + (1-\pi_L)\right) d\mathbb{P}(\epsilon_1, \ldots, \epsilon_{-i})}\right).$$

In particular, when $\hat{\epsilon}_2 \leq \hat{\epsilon}_1$, the expression above simplifies to:

$$\frac{1}{N}\left(\epsilon_0 + (N-1)\,\frac{\hat{\epsilon}_2 - \Delta}{2}\right).$$

IA.2.2 Pricing Function p

I rely on buyers' optimality conditions in period 2 to pin down buyers' period-1 pricing function p. For any $(\epsilon_0, \hat{\epsilon}_1, \hat{\epsilon}_2)$, define

$$\begin{aligned}&\Pi_2(\hat{\epsilon}_2|\hat{\epsilon}_1,\epsilon_0)\\ :=&\pi_L\mathbb{E}\left[\mathbf{1}\{\epsilon_{-i} < \hat{\epsilon}_2\}\Big(1 - x(\max\{\hat{\epsilon}_1, \epsilon_{-i}\})\Big)\left(v_L + \frac{1}{N}\Big(\epsilon_0 + \sum_{j\neq i}\epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right)\right] + (1-\pi_L)\mathbb{E}\left[v_H + \frac{1}{N}\Big(\epsilon_0 + \sum_{j\neq i}\epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right]\end{aligned} \tag{20}$$

to be a buyer's "ex-ante" continuation profit from submitting $p(\hat{\epsilon}_2)$ in period 2, after receiving the initial signal ϵ_0 and submitting $p(\hat{\epsilon}_1)$ in period 1. Here, $\mathbf{1}\{\epsilon_{-i} \leq \hat{\epsilon}_2\}$ is the indicator function for the event that buyer i wins at the price of $\frac{p(\hat{\epsilon}_2)}{\delta}$ at $t = 2$.

In particular, when $\hat{\epsilon}_2 \leq \hat{\epsilon}_1$, $\Pi_2(\hat{\epsilon}_2|\hat{\epsilon}_1, \epsilon_0)$ can be simplified as follows:

$$\pi_L\big(1 - x(\hat{\epsilon}_1)\big)F(\hat{\epsilon}_2)\left(v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\,\frac{\hat{\epsilon}_2 - \Delta}{2}\right) - \frac{p(\hat{\epsilon}_2)}{\delta}\right) + (1-\pi_L)F(\hat{\epsilon}_2)\left(v_H + \frac{1}{N}\left(\epsilon_0 + (N-1)\,\frac{\hat{\epsilon}_2 - \Delta}{2}\right) - \frac{p(\hat{\epsilon}_2)}{\delta}\right).$$

After a buyer receives the initial signal ϵ_0 and then offers $p(\epsilon_0)$ in period 1, then the buyer must find it sequentially optimal to offer $\frac{p(\epsilon_0)}{\delta}$. Hence, the first-order condition $\frac{\partial \Pi_2(\hat{\epsilon}_2|\hat{\epsilon}_1,\epsilon_0)}{\partial \hat{\epsilon}_2}\Big|_{\hat{\epsilon}_1=\hat{\epsilon}_2=\epsilon_0} = 0$ must hold, which is equivalent to:

$$p'(\epsilon_0) = \delta\,\frac{N-1}{2N} + \delta\,\frac{f(\epsilon_0)}{F(\epsilon_0)}\left[\frac{\pi_L(1-x(\epsilon_0))v_L + (1-\pi_L)v_H}{1 - \pi_L x(\epsilon_0)} + \frac{1}{N}\Big(\epsilon_0 + (N-1)\frac{\epsilon_0 - \Delta}{2}\Big) - \frac{p(\epsilon_0)}{\delta}\right]. \tag{21}$$

In the next lemma, I compute the pricing function $p(\cdot)$ and its lower boundary value $p(-\Delta)$, which is useful for numerical exercises.

Lemma IA.2.1: *If the candidate equilibrium characterized by the function pair (p, x) is indeed an equilibrium, the* *pricing function can be expressed as follows:*

$$p(\epsilon_0) = \delta\left(\frac{(N-1)(N+2)\epsilon_0 - (N^2-N+2)\Delta}{2N^2}\right) + \delta\mathbb{E}\left[\frac{\pi_L(1-x(\epsilon_{-i}))v_L + (1-\pi_L)v_H}{1-\pi_L x(\epsilon_{-i})}\middle|\epsilon_{-i} \leq \epsilon_0\right]. \tag{22}$$

In particular,

$$p(-\Delta) = \delta\left(\frac{\pi_L(1-x(-\Delta))v_L + (1-\pi_L)v_H}{1-\pi_L x(-\Delta)} - \Delta\right). \tag{23}$$

Proof. I take $x(\cdot)$ as given and express $p(\cdot)$ in the following integral form:

$$\begin{aligned} p(\epsilon_0) =& \frac{p(-\Delta)F(-\Delta) + \delta\,\frac{N-1}{2N}\int_{-\Delta}^{\epsilon_0} F(\epsilon)\,d\epsilon + \delta\int_{-\Delta}^{\epsilon_0} f(\epsilon)\left[\frac{\pi_L(1-x(\epsilon))v_L+(1-\pi_L)v_H}{1-\pi_L x(\epsilon)} + \frac{1}{N}\left(\epsilon + (N-1)\frac{\epsilon-\Delta}{2}\right)\right]d\epsilon}{F(\epsilon_0)} \\ =& \frac{\delta\,\frac{N-1}{2N}\int_{-\Delta}^{\epsilon_0} F(\epsilon)\,d\epsilon + \delta\int_{-\Delta}^{\epsilon_0} f(\epsilon)\left[\frac{\pi_L(1-x(\epsilon))v_L+(1-\pi_L)v_H}{1-\pi_L x(\epsilon)} + \frac{1}{N}\left(\epsilon + (N-1)\frac{\epsilon-\Delta}{2}\right)\right]d\epsilon}{F(\epsilon_0)}, \end{aligned} \tag{24}$$

where the second equality follows from the facts that 1) $F(-\Delta) = 0$ and 2) $p(-\Delta)$ is finite. By applying l'Hopital's Rule, I can take the limit of (24) as $\epsilon_0 \downarrow -\Delta$ and obtain (23), as desired.

Moreover, since $\epsilon_{-i} = \max_{j\neq i}\epsilon_j$ is the maximum of $N-1$ independent signals uniformly distributed on $[-\Delta, \Delta]$, its CDF is $F(\epsilon_0) = \left(\frac{\epsilon_0+\Delta}{2\Delta}\right)^{N-1}$ with the associated PDF f. Hence, the equation above can be rewritten as

$$\begin{aligned} \frac{p(\epsilon_0)}{\delta} =& \left(\frac{N-1}{2N^2}\right)(\epsilon_0+\Delta) + \mathbb{E}\left[\frac{\pi_L(1-x(\epsilon_{-i}))v_L + (1-\pi_L)v_H}{1-\pi_L x(\epsilon_{-i})}\middle|\epsilon_{-i} \leq \epsilon_0\right] \\ &+ \underbrace{\frac{1}{N}\mathbb{E}\left[\left(\epsilon_{-i} + (N-1)\frac{\epsilon_{-i}-\Delta}{2}\right)\middle|\epsilon_{-i} \leq \epsilon_0\right]}_{=\frac{(N^2-1)\epsilon_0-(N^2+1)\Delta}{2N^2}}. \\ =& \frac{(N-1)(N+2)\epsilon_0 - (N^2-N+2)\Delta}{2N^2} + \mathbb{E}\left[\frac{\pi_L(1-x(\epsilon_{-i}))v_L + (1-\pi_L)v_H}{1-\pi_L x(\epsilon_{-i})}\middle|\epsilon_{-i} \leq \epsilon_0\right], \end{aligned} \tag{25}$$

for any $\epsilon_0 \in [-\Delta, \Delta]$, as desired. □

IA.2.3 Acceptance Function x

I make use of buyers' optimality conditions in period 1 to pin down the low-quality seller's period-1 acceptance function x. Moreover, if a buyer initially receives signal ϵ_0, the buyer's expected lifetime payoff from offering $p(\hat{\epsilon}_1)$

in period 1 can be expressed as follows:

$$\Pi_1(\hat{\epsilon}_1 \mid \epsilon_0) := \pi_L x(\hat{\epsilon}_1) F(\hat{\epsilon}_1) \left(v_L + \frac{1}{N} \left(\epsilon_0 + (N-1) \frac{\hat{\epsilon}_1 - \Delta}{2} \right) - p(\hat{\epsilon}_1) \right) + \delta \max_{\hat{\epsilon}_2} \Pi_2(\hat{\epsilon}_2 \mid \hat{\epsilon}_1, \epsilon_0). \tag{26}$$

Since a buyer who received an initial signal ϵ_0 must find it optimal to offer $p(\epsilon_0)$ in period 1, the following first-order condition $\frac{\partial \Pi_1(\hat{\epsilon}_1 | \epsilon_0)}{\partial \hat{\epsilon}_1}\Big|_{\hat{\epsilon}_1 = \epsilon_0} = 0$ must be satisfied:

$$\begin{aligned} &\left[\left(\frac{x'(\epsilon_0)}{x(\epsilon_0)} + \frac{f(\epsilon_0)}{F(\epsilon_0)} \right) \left(v_L + \frac{1}{N}\Big(\epsilon_0 + (N-1) \frac{\epsilon_0 - \Delta}{2} \Big) - p(\epsilon_0) \right) + \left(\frac{N-1}{2N} - p'(\epsilon_0) \right) \right] \\ &- \delta \left(\frac{x'(\epsilon_0)}{x(\epsilon_0)} \right) \left(v_L + \frac{1}{N}\Big(\epsilon_0 + (N-1) \frac{\epsilon_0 - \Delta}{2} \Big) - \frac{p(\epsilon_0)}{\delta} \right) = 0, \end{aligned} \tag{27}$$

for any $\epsilon_0 \in (-\Delta, \Delta]$.

The next lemma provides a boundary condition for the differential equation governing the evolution of x. In particular, it shows that the low-quality seller must accept $p(\Delta)$ (i.e., the equilibrium offer made by the buyer who received the highest signal Δ) with probability 1.

Lemma IA.2.2: *If the candidate equilibrium characterized by the function pair (p, x) is indeed an equilibrium,*

$$x(\Delta) = 1. \tag{28}$$

Proof. The argument is analogous to the proof of Corollary 1 in the main text. Suppose, to the contrary, that there exists an equilibrium with $x(\Delta) < 1$ that proceeds as the candidate equilibrium.

It suffices to show that if buyer i receives the initial signal Δ, then the buyer can profitably deviate by offering $p(\Delta) + \epsilon$ in period 1 for a sufficiently small $\epsilon > 0$. Fix any $\epsilon > 0$. Since $\epsilon_{-i} \leq \Delta$ almost surely, buyer i wins in period 1 regardless of whether it offers $p(\Delta)$ or $p(\Delta) + \epsilon$.

If buyer i offers $p(\Delta)$ in period 1 and then offers $\frac{p(\Delta)}{\delta}$ in period 2, its expected lifetime payoff equals

$$\pi_L \left[x(\Delta)\Big(v_L + \frac{\Delta}{N} - p(\Delta) \Big) + \delta\big(1 - x(\Delta)\big) \left(v_L + \frac{\Delta}{N} - \frac{p(\Delta)}{\delta} \right) \right] + \delta(1 - \pi_L)\Big(v_H + \frac{\Delta}{N} - \frac{p(\Delta)}{\delta} \Big).$$

Now consider the deviation in which buyer i offers $p(\Delta) + \epsilon$ in period 1 and, if rejected, offers $\frac{p(\Delta)}{\delta}$ in period 2.

If the low-quality seller rejects $p(\Delta)+\epsilon$ in period 1, then its period-2 price is almost surely no higher than $\frac{p(\Delta)}{\delta}$, so its discounted continuation value is no higher than $\delta \cdot \frac{p(\Delta)}{\delta} = p(\Delta)$, which is strictly less than $p(\Delta)+\epsilon$. Hence, the low-quality seller accepts $p(\Delta)+\epsilon$ with probability 1.

Therefore, the deviating buyer's expected lifetime payoff is at least

$$\pi_L\Big(v_L + \frac{\Delta}{N} - p(\Delta) - \epsilon\Big) + \delta(1-\pi_L)\Big(v_H + \frac{\Delta}{N} - \frac{p(\Delta)}{\delta}\Big).$$

Since $v_L + \frac{\Delta}{N} > 0$, choosing $\epsilon > 0$ sufficiently small implies that this payoff is strictly higher than the equilibrium payoff under $x(\Delta) < 1$, which is a contradiction. Therefore, $x(\Delta) = 1$. □

By combining (21) and (27), I obtain the following initial value problem for x that does not depend on p:

$$\frac{x'(\epsilon_0)}{x(\epsilon_0)} = \underbrace{\frac{\delta\,\frac{f(\epsilon_0)}{F(\epsilon_0)}\left(\frac{\pi_L(1-x(\epsilon_0))v_L + (1-\pi_L)v_H}{1-\pi_L x(\epsilon_0)}\right) - \frac{f(\epsilon_0)}{F(\epsilon_0)}v_L - (1-\delta)\frac{f(\epsilon_0)}{F(\epsilon_0)}\frac{1}{N}\Big(\epsilon_0 + (N-1)\frac{\epsilon_0-\Delta}{2}\Big) - (1-\delta)\frac{N-1}{2N}}{(1-\delta)\Big(v_L + \frac{1}{N}\big(\epsilon_0 + (N-1)\frac{\epsilon_0-\Delta}{2}\big)\Big)}}_{=:G(\epsilon_0, x(\epsilon_0))}, \tag{29}$$

for all $\epsilon_0 \in [-\Delta, \Delta]$, with the initial condition (28).

Lemma IA.2.3: *Suppose that the differential equation* (29) *with the initial condition* (28) *admits a strictly increasing solution x on $[-\Delta, \Delta]$. Then, the function pair (x, p) must be unique and strictly positive on $[-\Delta, \Delta]$.*

Proof. Since $x(\Delta) = 1$ and x is strictly increasing on $(-\Delta, \Delta]$, I have $x(\epsilon_0) \leq 1$ for all $\epsilon_0 \in (-\Delta, \Delta]$, and hence $1 - \pi_L x(\epsilon_0) \geq 1 - \pi_L > 0$ for all $\epsilon_0 \in (-\Delta, \Delta]$. Fix any $\eta \in (0, \Delta)$. On $[-\Delta+\eta, \Delta]$ I also have $F(\epsilon_0) \geq F(-\Delta+\eta) > 0$. Therefore, for each fixed $\eta \in (0,\Delta)$, the function $G(\epsilon_0, x)$ is Lipschitz continuous in x on $[-\Delta+\eta, \Delta] \times (0, 1]$, uniformly in ϵ_0. Hence, standard uniqueness results for ODEs imply that the initial value problem (29) with the initial condition (28) admits at most one solution on $[-\Delta+\eta, \Delta]$.[26]

Since $\eta \in (0, \Delta)$ is arbitrary, uniqueness holds on $(-\Delta, \Delta]$ as well. Logarithmic integration of (29) with respect to ϵ_0 over $[\epsilon, \Delta]$ yields

$$x(\epsilon) = \underbrace{x(\Delta)}_{=1 \text{ by } (28)} \exp\Big(-\int_{\epsilon}^{\Delta} G(\epsilon_0, x(\epsilon_0))\, d\epsilon_0\Big) > 0.$$

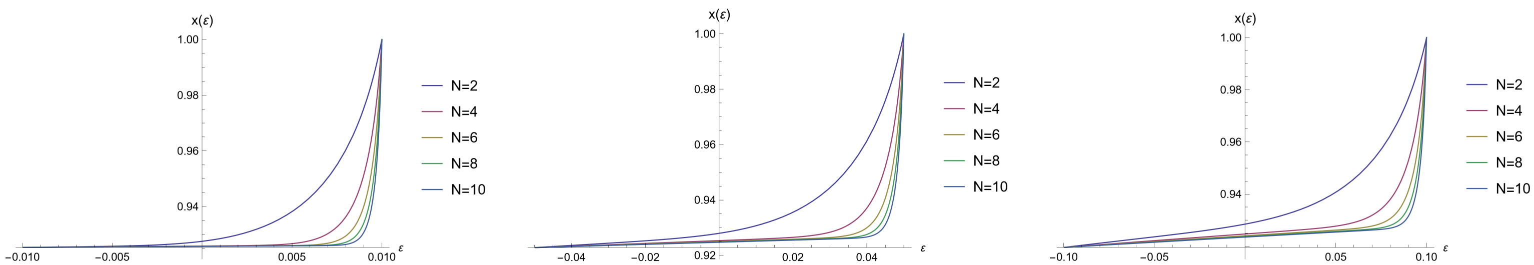
[26]This argument is standard in the theory of differential equations. More precisely, combining Theorem 2.5 and Corollary 2.6 in Teschl [2012] implies the following: *if a function $f(t,x)$ is Lipschitz continuous in x (uniformly in t) on $[t_0, t_1] \times \mathbb{R}$, then the initial value problem $x'(t) = f(t, x(t))$ with $x(t_0) = x_0$ admits a unique solution on $[t_0, t_1]$.*

Since x is weakly increasing and strictly positive on $(-\Delta, \Delta]$, the one-sided limit $\lim_{\epsilon \downarrow -\Delta} x(\epsilon)$ exists and is weakly positive. Hence, x admits a unique continuous extension to $[-\Delta, \Delta]$. Finally, by the representation (25), this unique solution for x uniquely determines p, which establishes the desired result. □

IA.3 Verification

IA.3.1 Numerical Verification of Existence and Strict Monotonicity of x

I rely on numerical methods to verify the existence of a strictly increasing function x satisfying the differential equation on $[-\Delta, \Delta]$ subject to the initial condition (28) for a set of parametric configurations satisfying (**SIC'**), (**NFS'**), (**EXP'**), and (**NOL**).



(a) $\Delta = 0.01$ **(b)** $\Delta = 0.05$ **(c)** $\Delta = 0.1$

Figure 4. Plots of the low-quality seller's acceptance function x for $\Delta = 0.01$, $\Delta = 0.05$, and $\Delta = 0.1$. Other parameter values are $v_L = 3$, $v_H = 8$, $c_H = 4.5$, $\pi_L = 0.9$, and $\delta = 0.5$.

IA.3.2 Supermodularity

I establish the supermodularity of the buyers' continuation profit functions in both periods, which ensures the optimality of the candidate equilibrium play. The next lemma provides a lower bound on p' and p respectively, and shows that the pricing function p must be strictly increasing on $[-\Delta, \Delta]$.

Lemma IA.3.1: *Suppose that the differential equation* (29) *with the initial condition* (28) *admits a strictly increasing solution x on* $[-\Delta, \Delta]$. *Then, for every* $\hat{\epsilon}_2 \in [-\Delta, \Delta]$,

$$\frac{p'(\hat{\epsilon}_2)}{\delta} > \frac{N-1}{2N}, \qquad \frac{p(\hat{\epsilon}_2)}{\delta} \geq v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\right) \tag{30}$$

for any $\epsilon_0 \in [\hat{\epsilon}_2, \Delta]$.

Proof. Since x is strictly increasing on $[-\Delta, \Delta]$, the map $\epsilon \mapsto \frac{\pi_L(1-x(\epsilon))v_L+(1-\pi_L)v_H}{1-\pi_L x(\epsilon)}$ is increasing as well. Hence, the conditional expectation term in (25) is increasing in ϵ_0. Differentiating (25) with respect to ϵ_0 yields

$$\frac{p'(\epsilon_0)}{\delta} = \frac{(N-1)(N+2)}{2N^2} + \underbrace{\frac{\partial}{\partial \epsilon_0}\left(\mathbb{E}\left[\frac{\pi_L(1-x(\epsilon_{-i}))v_L+(1-\pi_L)v_H}{1-\pi_L x(\epsilon_{-i})}\middle|\epsilon_{-i} \le \epsilon_0\right]\right)}_{\ge 0} > \frac{N-1}{2N}, \tag{31}$$

for any $\epsilon_0 \in [-\Delta, \Delta]$.

Moreover, for any pair $(\epsilon_0, \hat{\epsilon}_2)$ with $\hat{\epsilon}_2 \in [-\Delta, \Delta]$ and $\epsilon_0 \in [\hat{\epsilon}_2, \Delta]$, Equation (25) implies that

$$\begin{aligned} &v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\right) - \frac{p(\hat{\epsilon}_2)}{\delta} \\ &= v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\right) - \left(\frac{(N-1)(N+2)\hat{\epsilon}_2 - (N^2-N+2)\Delta}{2N^2}\right) \\ &\quad - \underbrace{\mathbb{E}\left[\frac{\pi_L(1-x(\epsilon_{-i}))v_L+(1-\pi_L)v_H}{1-\pi_L x(\epsilon_{-i})}\middle|\epsilon_{-i} \le \hat{\epsilon}_2\right]}_{\ge \pi_L v_L + (1-\pi_L)v_H \ (\because\ x\in[0,1])}. \end{aligned} \tag{32}$$

Using $\epsilon_0 \le \Delta$ and $\hat{\epsilon}_2 \ge -\Delta$, I obtain

$$v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\right) - \frac{p(\hat{\epsilon}_2)}{\delta} \le -(1-\pi_L)(v_H - v_L) + \frac{2\Delta}{N}.$$

Since $N \ge 2$, Assumption (**NOL**) yields $-(1-\pi_L)(v_H - v_L) + \frac{2\Delta}{N} < 0$. Therefore,

$$v_L + \frac{1}{N}\left(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\right) \le \frac{p(\hat{\epsilon}_2)}{\delta},$$

as desired. □

Define $\Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0) := \Pi_2(\hat{\epsilon}_2 \mid \epsilon_0, \epsilon_0)$ to be a buyer's period-2 continuation payoff from offering $\frac{p(\hat{\epsilon}_2)}{\delta}$ in period 2, after receiving the initial signal ϵ_0 and submitting $p(\epsilon_0)$ in period 1. The next two lemmata establish the supermodularity of Π_2^{Path} in $(\epsilon_0, \hat{\epsilon}_2)$ and of Π_1^{Path} in $(\epsilon_0, \hat{\epsilon}_1)$.

Lemma IA.3.2: *Suppose that the differential equation* (29) *with the initial condition* (28) *admits a strictly in-*

creasing solution x on $[-\Delta, \Delta]$. For any $(\epsilon_0, \hat{\epsilon}_2) \in [-\Delta, \Delta] \times [-\Delta, \Delta]$,

$$\frac{\partial^2 \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \hat{\epsilon}_2 \, \partial \epsilon_0} > 0.$$

Proof. By the definition of $\Pi_2(\hat{\epsilon}_2 \mid \hat{\epsilon}_1, \epsilon_0)$ in (20), $\Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)$ can be written as

$$\begin{aligned}\Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0) =& \pi_L \mathbb{E}\left[\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\left\{\Big(1 - x(\max\{\epsilon_0, \epsilon_{-i}\})\Big)\left(v_L + \frac{1}{N}\Big(\epsilon_0 + \sum_{j \neq i} \epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right)\right\}\right] \\ &+ (1 - \pi_L)\mathbb{E}\left[\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\left(v_H + \frac{1}{N}\Big(\epsilon_0 + \sum_{j \neq i} \epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right)\right].\end{aligned}$$

By Leibniz's rule, differentiation with respect to ϵ_0 yields

$$\begin{aligned}\frac{\partial \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \epsilon_0} =& \frac{1}{N}\, \mathbb{E}\Big[\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\big((1 - \pi_L) + \pi_L(1 - x(\max\{\epsilon_0, \epsilon_{-i}\}))\big)\Big] \\ &- \pi_L x'(\epsilon_0)\, \mathbb{E}\left[\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\mathbf{1}\{\epsilon_{-i} < \epsilon_0\}\left(v_L + \frac{1}{N}\Big(\epsilon_0 + \sum_{j \neq i} \epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right)\right].\end{aligned}$$

Case 1) $\hat{\epsilon}_2 > \epsilon_0$. In this case, $\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\mathbf{1}\{\epsilon_{-i} < \epsilon_0\} = \mathbf{1}\{\epsilon_{-i} < \epsilon_0\}$. Hence,

$$\begin{aligned}\frac{\partial \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \epsilon_0} =& \frac{1}{N} \int_{-\Delta}^{\hat{\epsilon}_2} \big((1 - \pi_L) + \pi_L(1 - x(\max\{\epsilon_0, \epsilon_{-i}\}))\big)\, dF(\epsilon_{-i}) \\ &- \pi_L x'(\epsilon_0)\, \mathbb{E}\left[\mathbf{1}\{\epsilon_{-i} < \epsilon_0\}\left(v_L + \frac{1}{N}\Big(\epsilon_0 + \sum_{j \neq i} \epsilon_j\Big) - \frac{p(\hat{\epsilon}_2)}{\delta}\right)\right].\end{aligned}$$

Differentiating with respect to $\hat{\epsilon}_2$ gives

$$\begin{aligned}\frac{\partial^2 \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \hat{\epsilon}_2 \, \partial \epsilon_0} =& \frac{f(\hat{\epsilon}_2)}{N}\Big((1 - \pi_L) + \pi_L(1 - x(\hat{\epsilon}_2))\Big) + \pi_L x'(\epsilon_0)\mathbb{E}\Big[\mathbf{1}\{\epsilon_{-i} < \epsilon_0\}\Big]\frac{p'(\hat{\epsilon}_2)}{\delta} \\ =& \frac{f(\hat{\epsilon}_2)}{N}\Big(1 - \pi_L x(\hat{\epsilon}_2)\Big) + \pi_L x'(\epsilon_0) F(\epsilon_0)\frac{p'(\hat{\epsilon}_2)}{\delta},\end{aligned}$$

which is strictly positive since $f(\hat{\epsilon}_2) > 0$ and $1 - \pi_L x(\hat{\epsilon}_2) \ge 1 - \pi_L > 0$.

Case 2) $\hat{\epsilon}_2 \le \epsilon_0$. In this case, $\mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}\mathbf{1}\{\epsilon_{-i} < \epsilon_0\} = \mathbf{1}\{\epsilon_{-i} \le \hat{\epsilon}_2\}$, and moreover $x(\max\{\epsilon_0, \epsilon_{-i}\}) = x(\epsilon_0)$

on $\{\epsilon_{-i} \le \hat{\epsilon}_2\}$. Hence,

$$\frac{\partial \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \epsilon_0} = \frac{1}{N} \int_{-\Delta}^{\hat{\epsilon}_2} \big((1-\pi_L) + \pi_L(1 - x(\epsilon_0))\big)\, dF(\epsilon_{-i})$$
$$-\pi_L x'(\epsilon_0)\, F(\hat{\epsilon}_2) \left[v_L + \frac{1}{N}\Big(\epsilon_0 + (N-1)\left(\frac{\hat{\epsilon}_2 - \Delta}{2}\right)\Big) - \frac{p(\hat{\epsilon}_2)}{\delta} \right].$$

Differentiating with respect to $\hat{\epsilon}_2$ yields

$$\frac{\partial^2 \Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)}{\partial \hat{\epsilon}_2\, \partial \epsilon_0} = f(\hat{\epsilon}_2) \left[\frac{1 - \pi_L x(\epsilon_0)}{N} + \pi_L x'(\epsilon_0) \left(\frac{p(\hat{\epsilon}_2)}{\delta} - v_L - \frac{1}{N}\Big(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_2 - \Delta}{2}\Big) \right) \right]$$
$$+ \pi_L x'(\epsilon_0) F(\hat{\epsilon}_2) \left(\frac{p'(\hat{\epsilon}_2)}{\delta} - \frac{N-1}{2N} \right).$$

The first bracket is strictly positive because $f(\hat{\epsilon}_2) > 0$ and $1 - \pi_L x(\epsilon_0) \ge 1 - \pi_L > 0$, while the remaining terms are weakly positive by (30). Therefore, the cross-partial derivative is strictly positive. This establishes the desired result. □

Lemma IA.3.3 (Supermodularity of Π_1): *Suppose that the differential equation* (29) *with the initial condition* (28) *admits a strictly increasing solution x on* $[-\Delta, \Delta]$. *Then $\Pi_1(\hat{\epsilon}_1 \mid \epsilon_0)$ has increasing differences in* $(\hat{\epsilon}_1, \epsilon_0)$ *on* $[-\Delta, \Delta]^2$. *Moreover, for each fixed $\hat{\epsilon}_2 \in [-\Delta, \Delta]$, $\Phi(\hat{\epsilon}_1, \hat{\epsilon}_2, \epsilon_0)$ has strictly increasing differences in* $(\hat{\epsilon}_1, \epsilon_0)$ *on* $(-\Delta, \Delta]^2$.

Proof. Define

$$\Phi(\hat{\epsilon}_1, \hat{\epsilon}_2, \epsilon_0) := \pi_L x(\hat{\epsilon}_1) F(\hat{\epsilon}_1) \left(v_L + \frac{1}{N}\Big(\epsilon_0 + (N-1)\frac{\hat{\epsilon}_1 - \Delta}{2}\Big) - p(\hat{\epsilon}_1) \right) + \delta\, \Pi_2(\hat{\epsilon}_2 \mid \hat{\epsilon}_1, \epsilon_0), \tag{33}$$

so that

$$\Pi_1(\hat{\epsilon}_1 \mid \epsilon_0) = \max_{\hat{\epsilon}_2 \in [-\Delta, \Delta]} \Phi(\hat{\epsilon}_1, \hat{\epsilon}_2, \epsilon_0).$$

In our model, ϵ_0 enters $\Pi_2(\hat{\epsilon}_2 \mid \hat{\epsilon}_1, \epsilon_0)$ only through the term $\frac{1}{N}\big(\epsilon_0 + \sum_{j \ne i} \epsilon_j\big)$, whose coefficient equals $1 - \pi_L x(\max\{\hat{\epsilon}_1, \epsilon_{-i}\})$. Hence,

$$\frac{\partial^2 \Pi_2(\hat{\epsilon}_2 \mid \hat{\epsilon}_1, \epsilon_0)}{\partial \hat{\epsilon}_1\, \partial \epsilon_0} = -\frac{\pi_L}{N} x'(\hat{\epsilon}_1)\, F\big(\min\{\hat{\epsilon}_1, \hat{\epsilon}_2\}\big).$$

Therefore, for any $(\hat{\epsilon}_2, \hat{\epsilon}_1, \epsilon_0) \in [-\Delta, \Delta]^3$,

$$\frac{\partial^2 \Phi(\hat{\epsilon}_1, \hat{\epsilon}_2, \epsilon_0)}{\partial \hat{\epsilon}_1 \, \partial \epsilon_0} = \frac{\pi_L}{N} \Big[x'(\hat{\epsilon}_1) F(\hat{\epsilon}_1) + x(\hat{\epsilon}_1) f(\hat{\epsilon}_1) - \delta \, x'(\hat{\epsilon}_1) F\big(\min\{\hat{\epsilon}_1, \hat{\epsilon}_2\}\big) \Big].$$

Since $F(\min\{\hat{\epsilon}_1, \hat{\epsilon}_2\}) \leq F(\hat{\epsilon}_1)$ and $\delta \in (0, 1)$,

$$x'(\hat{\epsilon}_1) F(\hat{\epsilon}_1) - \delta x'(\hat{\epsilon}_1) F(\min\{\hat{\epsilon}_1, \hat{\epsilon}_2\}) \geq 0,$$

so

$$\frac{\partial^2 \Phi(\hat{\epsilon}_1, \hat{\epsilon}_2, \epsilon_0)}{\partial \hat{\epsilon}_1 \, \partial \epsilon_0} \geq \frac{\pi_L}{N} x(\hat{\epsilon}_1) f(\hat{\epsilon}_1) \geq 0.$$

Moreover, on $(-\Delta, \Delta]$ I have $f(\hat{\epsilon}_1) > 0$ and (by monotonicity with $x(\Delta) = 1$) $x(\hat{\epsilon}_1) > 0$, so the last term is strictly positive, which implies strict increasing differences on $(-\Delta, \Delta]^2$ for each fixed $\hat{\epsilon}_2$.

Pointwise maximization over $\hat{\epsilon}_2$ preserves strict increasing differences in $(\hat{\epsilon}_1, \epsilon_0)$ (Theorem 2.7.6 in Topkis [1998]), hence $\Pi_1(\hat{\epsilon}_1 \mid \epsilon_0)$ has strict increasing differences on $(-\Delta, \Delta]^2$. The strict increasing differences on $(-\Delta, \Delta]^2$ follow from the strict increasing differences for Φ, which is preserved under pointwise maximization over $\hat{\epsilon}_2$ (Theorem 2.7.7 in Topkis [1998]). □

IA.3.3 Verifying Optimality of Candidate Equilibrium

I am ready to show that the candidate equilibrium play is indeed optimal:

Proposition IA.1: *Suppose that the differential equation* (29) *with the initial condition* (28) *admits a strictly increasing solution x on* $[-\Delta, \Delta]$. *Then, the candidate equilibrium play characterized by the function pair* (p, x) *is indeed an equilibrium.*

Proof. I verify the optimality of the conjectured equilibrium strategy profile for each player, who takes other players' strategies as given. The proof is organized as follows. In **Step 1**, I show that any deviation from the candidate equilibrium play in period 1 is suboptimal. In **Step 2**, I show that conditional on a buyer's private history in which the buyer made an equilibrium period-1 offer and was subsequently rejected, any deviation from the candidate equilibrium play in period 2 is suboptimal. Throughout the proof, I assume that buyer i has received an initial signal $\epsilon_0 \in [-\Delta, \Delta]$ at $t = 0$.

Step 1) Deviations at $t = 1$

Case 1: $\hat{p}_1 \in (p(\epsilon_0), p(\Delta)]$

Suppose first that the buyer who received the initial signal of ϵ_0 offers $\hat{p}_1 > p(\epsilon_0)$ in period 1. By the monotonicity and continuity of p, there exists a unique $\hat{\epsilon}_1^+ \in (\epsilon_0, \Delta]$ such that $\hat{p}_1 = p(\hat{\epsilon}_1^+)$. By the fundamental theorem of calculus,

$$\Pi_1(\hat{\epsilon}_1^+ \mid \epsilon_0) - \Pi_1(\epsilon_0 \mid \epsilon_0) = \int_{\epsilon_0}^{\hat{\epsilon}_1^+} \frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \epsilon_0)\, d\tilde{\epsilon}.$$

By Lemma IA.3.3, $\Pi_1(\hat{\epsilon}_1 \mid \epsilon_0)$ has strictly increasing differences in $(\hat{\epsilon}_1, \epsilon_0)$ on $(-\Delta, \Delta]^2$. Hence, for every $\tilde{\epsilon} \in (\epsilon_0, \hat{\epsilon}_1^+]$,

$$\frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \epsilon_0) < \frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \tilde{\epsilon}).$$

By the first-order condition, $\frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \tilde{\epsilon}) = 0$ for all $\tilde{\epsilon} \in [-\Delta, \Delta]$. Therefore the integrand is strictly negative on $(\epsilon_0, \hat{\epsilon}_1^+]$, so the integral is strictly negative and

$$\Pi_1(\hat{\epsilon}_1^+ \mid \epsilon_0) < \Pi_1(\epsilon_0 \mid \epsilon_0).$$

Case 2: $\hat{p}_1 \in [p(-\Delta), p(\epsilon_0))$

By the monotonicity and continuity of p, there exists a unique $\hat{\epsilon}_1^- \in [-\Delta, \epsilon_0)$ such that $\hat{p}_1 = p(\hat{\epsilon}_1^-)$. Similarly as in Case 1,

$$\Pi_1(\epsilon_0 \mid \epsilon_0) - \Pi_1(\hat{\epsilon}_1^- \mid \epsilon_0) = \int_{\hat{\epsilon}_1^-}^{\epsilon_0} \frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \epsilon_0)\, d\tilde{\epsilon}.$$

For $\tilde{\epsilon} \in [\hat{\epsilon}_1^-, \epsilon_0)$, Lemma (IA.3.3) implies

$$\frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \epsilon_0) > \frac{\partial}{\partial \hat{\epsilon}_1} \Pi_1(\tilde{\epsilon} \mid \tilde{\epsilon}) = 0,$$

so the right-hand side is strictly positive. Hence, the deviation is unprofitable.

Case 3: $\hat{p}_1 < p(-\Delta)$

In this case, if a buyer makes an offer $\hat{p}_1$ below $p(-\Delta)$, the buyer's offer is almost surely strictly lower than competitors' offers in equilibrium, and hence almost surely a losing offer. Thus, this deviation is payoff equivalent to the buyer making the offer $p(-\Delta)$, which is also almost surely a losing offer in equilibrium. By **Case 2**, if a buyer receives an initial signal ϵ_0, the buyer is weakly better off offering $p(\epsilon_0)$ than offering $p(-\Delta)$, which establishes the suboptimality of the deviating offer $\hat{p}_1 < p(-\Delta)$.

Case 4: $\hat{p}_1 > p(\Delta)$ I first show that $p(\Delta) < c_H$. Suppose, to the contrary, that $p(\Delta) \geq c_H$. Under this hypothesis, a buyer's expected continuation payoff from offering $p(\Delta)$ is bounded above by

$$\pi_L v_L + (1 - \pi_L)v_H + \Delta - p(\Delta) \underbrace{<}_{\textbf{(SIC')}} c_H - p(\Delta) \leq 0.$$

A losing offer yields payoff 0 (the buyer loses in both periods almost surely). Hence, when $\epsilon_0 = \Delta$, offering $p(\Delta)$ would be weakly worse than making an almost surely losing offer $p(-\Delta)$. This contradicts **Case 2**, which implies that when $\epsilon_0 = \Delta$ the buyer is strictly better off offering $p(\Delta)$ than offering $p(-\Delta)$. Therefore $p(\Delta) < c_H$.

Next, consider any deviating offer $\hat{p}_1 \in (p(\Delta), c_H)$. Since $\hat{p}_1 > p(\Delta)$, the deviating buyer wins at $t = 1$ almost surely. Moreover, because $x(\Delta) = 1$, the low-quality seller accepts $p(\Delta)$ with probability 1, and hence also accepts $\hat{p}_1$ with probability 1. Since $\hat{p}_1 < c_H$, the high-quality seller rejects in period 1 under both offers. Therefore, relative to offering $p(\Delta)$, the deviation $\hat{p}_1 \in (p(\Delta), c_H)$ only increases the payment in the event of period-1 trade and does not affect the continuation payoff. Thus the deviation is strictly worse than offering $p(\Delta)$, and hence is suboptimal.

Finally, consider any deviation in period 1 with $\hat{p}_1 \geq c_H$. If the high-quality seller accepts $\hat{p}_1$ in period 1 with positive probability, then by the reverse skimming property the low-quality seller's acceptance probability is weakly higher. Hence, conditional on period-1 acceptance, the deviating buyer's expected lifetime asset value is bounded above by $\pi_L v_L + (1 - \pi_L)v_H + \Delta$, and so the buyer's payoff conditional on acceptance is at most

$$\pi_L v_L + (1 - \pi_L)v_H + \Delta - \hat{p}_1 \leq \pi_L v_L + (1 - \pi_L)v_H + \Delta - c_H \underbrace{<}_{\textbf{(SIC')}} 0.$$

Thus, if $\hat{p}_1 \geq c_H$ is accepted with positive probability, the deviation yields strictly negative expected payoff on that event. On the other hand, the buyer can always guarantee a weakly positive lifetime payoff by submitting an almost surely losing offer in period 1 (e.g., any $\hat{p}_1 < p(-\Delta)$) and then behaving optimally thereafter. Therefore, no deviation with $\hat{p}_1 \geq c_H$ can be profitable.

Alternatively, suppose that the high-quality seller accepts in period 1 with zero probability. Just as in the case of a deviating offer $\hat{p}_1 \in (p(\Delta), c_H)$, the deviating buyer ends up paying the higher price to the low-quality seller compared to offering $p(\Delta)$, which is suboptimal.

Step 2) Deviations at $t = 2$

In Step 1, I have established that the buyer finds it optimal to adhere to the equilibrium offer $p(\epsilon_0)$ at $t = 1$. Hence, I only check whether it is optimal for the buyer to submit a deviating offer $\hat{p}_2 \neq \frac{p(\epsilon_0)}{\delta}$ at $t = 2$, given that the buyer already made an equilibrium period-1 offer at $t = 1$ and was rejected.

Case 1: $\hat{p}_2 \in \left(\frac{p(\epsilon_0)}{\delta}, \frac{p(\Delta)}{\delta}\right]$

By the monotonicity and continuity of p, there exists a unique $\hat{\epsilon}_2^+ \in (\epsilon_0, \Delta]$ such that $\hat{p}_2 = \frac{p(\hat{\epsilon}_2^+)}{\delta}$. By the fundamental theorem of calculus,

$$\Pi_2^{\text{Path}}(\hat{\epsilon}_2^+ \mid \epsilon_0) - \Pi_2^{\text{Path}}(\epsilon_0 \mid \epsilon_0) = \int_{\epsilon_0}^{\hat{\epsilon}_2^+} \frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \epsilon_0) \, d\tilde{\epsilon}.$$

By Lemma IA.3.2, $\Pi_2^{\text{Path}}(\hat{\epsilon}_2 \mid \epsilon_0)$ has strictly increasing differences in $(\hat{\epsilon}_2, \epsilon_0)$ on $[-\Delta, \Delta]^2$. Hence, for every $\tilde{\epsilon} \in (\epsilon_0, \hat{\epsilon}_2^+]$,

$$\frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \epsilon_0) < \frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \tilde{\epsilon}).$$

By the first-order condition (evaluated on path), $\frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \tilde{\epsilon}) = 0$ for all $\tilde{\epsilon} \in [-\Delta, \Delta]$. Therefore the integrand is strictly negative on $(\epsilon_0, \hat{\epsilon}_2^+]$, which implies the deviation is unprofitable.

Case 2: $\hat{p}_2 \in \left[\frac{p(-\Delta)}{\delta}, \frac{p(\epsilon_0)}{\delta}\right)$

By the monotonicity and continuity of p, there exists a unique $\hat{\epsilon}_2^- \in [-\Delta, \epsilon_0)$ such that $\hat{p}_2 = \frac{p(\hat{\epsilon}_2^-)}{\delta}$. Similarly,

$$\Pi_2^{\text{Path}}(\epsilon_0 \mid \epsilon_0) - \Pi_2^{\text{Path}}(\hat{\epsilon}_2^- \mid \epsilon_0) = \int_{\hat{\epsilon}_2^-}^{\epsilon_0} \frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \epsilon_0) \, d\tilde{\epsilon}.$$

For $\tilde{\epsilon} \in [\hat{\epsilon}_2^-, \epsilon_0)$, strict increasing differences of Π_2^{Path} implies

$$\frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \epsilon_0) > \frac{\partial}{\partial \hat{\epsilon}_2} \Pi_2^{\text{Path}}(\tilde{\epsilon} \mid \tilde{\epsilon}) = 0,$$

so the right-hand side is strictly positive and the deviation is unprofitable.

Case 3: $\hat{p}_2 < \frac{p(-\Delta)}{\delta}$

In this case, if a buyer makes an offer $\hat{p}_2$ below $\frac{p(-\Delta)}{\delta}$, the buyer's period-2 offer is almost surely strictly lower than competitors' offers in equilibrium and hence almost surely a losing offer. Thus, this deviation is payoff equivalent to the buyer making the offer $\frac{p(-\Delta)}{\delta}$, which is also almost surely a losing offer in equilibrium. By **Case 2** above, the buyer is weakly better off offering $\frac{p(\epsilon_0)}{\delta}$ than offering $\frac{p(-\Delta)}{\delta}$, which establishes the (weak) suboptimality of the deviating offer $\hat{p}_2 < \frac{p(-\Delta)}{\delta}$.

Case 4: $\hat{p}_2 > \frac{p(\Delta)}{\delta}$

Suppose that a buyer makes a deviating offer $\hat{p}_2 > \frac{p(\Delta)}{\delta}$ in period 2. Competing buyers' period-2 offers are strictly less than $\frac{p(\Delta)}{\delta}$ almost surely, so raising the offer above $\frac{p(\Delta)}{\delta}$ does not increase the probability of winning. Moreover, for each $\theta \in \{L, H\}$ the seller's acceptance decision at $t = 2$ is a threshold rule in the period-2 price, so offering strictly more than $\frac{p(\Delta)}{\delta}$ cannot increase the seller's acceptance probability beyond one. Hence, such a deviation is weakly dominated by offering $\frac{p(\Delta)}{\delta}$ and is suboptimal. This completes the proof. □

IA.4 General Number of Bargaining Opportunities

Proof of Proposition 5. I begin by proving (1). Fix any equilibrium outcome in which the seller accepts a period-t price $p_t \leq (1-\delta)c_H + \delta v_H$ after a public history h^P_{t-1}. By the reverse skimming property (Lemma 1), it is possible to construct continuation equilibria after which the low-quality seller finds it strictly suboptimal to reject p_t, and the high-quality seller finds it strictly optimal to reject p_t, conditional on the history h^P_{t-1}.[27]

Case 1: $\frac{v_L}{\delta^{T-1}} < (1-\delta)c_H + \delta v_H$**.**

In the last period, buyers engage in Bertrand competition over the average-quality asset, so they have no incentive to deviate in period T. In particular, on the equilibrium path, buyers' average asset value in period T is $\frac{v_L}{\delta^{T-1}}$, which yields the seller's reservation value regardless of asset quality. Hence, the last period proceeds as specified in the proposition.

Fix any period $t < T$. Since the seller can expect to earn the price $\frac{v_L}{\delta^{T-1}}$ in period T on the equilibrium path, any offer strictly less than $\frac{v_L}{\delta^{t-1}}$ is rejected by both types and is therefore a losing offer in period t. Hence, it suffices to consider a deviation $p^i_t > \frac{v_L}{\delta^{t-1}}$.

(i) If $p^i_t \geq (1-\delta)c_H + \delta v_H$, then both seller types accept p^i_t. Since the expected asset value of the remaining seller

[27]The INWBR criterion à la Kohlberg and Mertens [1986] requires that after any public history of rejected offers, buyers assign zero probability weight to any seller type for whom rejecting all previously offered prices is never a weak best response. See Cho and Kreps [1987], Noldeke and Van Damme [1990] for analyses of pruning rules based on INWBR in signaling games.

types $\frac{v_L}{\delta^{T-1}} < (1-\delta)c_H + \delta v_H$ by hypothesis, such a deviation necessarily offers strictly more than the period-T competitive level $\frac{v_L}{\delta^{T-1}}$ at which buyers earn zero expected profits upon winning. Hence, buyer i expects to incur a loss, so this deviation is suboptimal.

(ii) If $p_t^i \in \left[\max\{\delta v_H, \frac{v_L}{\delta^{t-1}}\}, (1-\delta)c_H + \delta v_H\right)$, consider a continuation equilibrium in which the high-quality seller rejects p_t^i surely and the low-quality seller accepts p_t^i with strictly positive probability and rejects it otherwise. Choose the low-quality seller's mixing probability so that buyers' posterior asset value v_{t+1}^i in the next period satisfies

$$p_t^i = \delta v_{t+1}^i + (1-\delta)c_H,$$

and specify that in period $t+1$ at least two buyers engage in Bertrand competition, yielding the period-$(t+1)$ price $p_{t+1} = v_{t+1}^i$ and zero expected profits upon winning. In this case, conditional on acceptance, buyer i wins only when the seller is low-quality. Since $p_t^i \geq \frac{v_L}{\delta^{t-1}} \geq v_L$, buyer i's payoff upon acceptance is $v_L - p_t^i \leq 0$, and upon rejection buyer i's payoff is 0. Hence, the deviation is suboptimal.

(iii) If $\frac{v_L}{\delta^{t-1}} < \delta v_H$ *and* $p_t^i \in \left(\frac{v_L}{\delta^{t-1}}, \delta v_H\right)$, I proceed by induction.

Inductive hypothesis. Fix any period $t < T$ and any deviation $p_t^i \in \left(\frac{v_L}{\delta^{t-1}}, \delta v_H\right)$. There exists an off-path continuation equilibrium satisfying INWBR after the public history (h_{t-1}^P, p_t^i) such that: the high-quality seller rejects p_t^i surely; the low-quality seller accepts p_t^i with strictly positive probability and rejects it otherwise; and, conditional on rejection of p_t^i at time t, continuation play from period $t+1$ onward unfolds exactly as in a period-$(t+1)$ continuation equilibrium after the rejection of the period-$(t+1)$ deviating offer $\frac{p_t^i}{\delta}$. Moreover, under this continuation equilibrium the deviation p_t^i is suboptimal.

Base step ($t = T-1$). Fix any deviation $p_{T-1}^i \in \left(\frac{v_L}{\delta^{T-2}}, \delta v_H\right)$ in period $T-1$. Consider a continuation equilibrium such that the high-quality seller rejects p_{T-1}^i surely and the low-quality seller accepts p_{T-1}^i with strictly positive probability and rejects it otherwise. Furthermore, conditional on rejection at time $T-1$, at least two buyers engage in Bertrand competition and offer $\frac{p_{T-1}^i}{\delta}$ in period T, so buyers expect to make zero profits upon winning. Then the low-quality seller's continuation value at time $T-1$ from rejecting p_{T-1}^i equals

$$\delta \cdot \frac{p_{T-1}^i}{\delta} = p_{T-1}^i,$$

so the low-quality seller is indifferent between accepting p_{T-1}^i and rejecting it. Finally, buyer i's payoff upon acceptance is $v_L - p^i_{T-1} < 0$ since $p^i_{T-1} > \frac{v_L}{\delta^{T-2}} \geq v_L$, whereas buyer i's payoff upon rejection is 0. Hence, the deviation is suboptimal.

Induction step. Suppose the inductive hypothesis holds for periods $t+1, t+2, \ldots, T-1$. Fix any deviation $p^i_t \in \left(\frac{v_L}{\delta^{t-1}}, \delta v_H\right)$ in period t. Consider a continuation equilibrium in which the high-quality seller rejects p^i_t surely and the low-quality seller accepts p^i_t with strictly positive probability and rejects it otherwise. Conditional on rejection of p^i_t at time t, let continuation play from period $t+1$ onward unfold as follows: (1) buyers submit losing offers in period $t+1$; (2) the continuation equilibrium unfolds exactly as in the period-$(t+1)$ continuation equilibrium after the rejection of the period-$(t+1)$ deviating offer $\frac{p^i_t}{\delta}$, whose existence is guaranteed by the inductive hypothesis. By construction, in that continuation equilibrium the low-quality seller's continuation value at time $t+1$ equals $\frac{p^i_t}{\delta}$. Therefore, at time t the low-quality seller's continuation value from rejecting p^i_t equals $\delta \cdot \frac{p^i_t}{\delta} = p^i_t$, so the low-quality seller is indifferent between accepting p^i_t and rejecting it. Moreover, buyer i's payoff upon acceptance is $v_L - p^i_t < 0$ since $p^i_t > \frac{v_L}{\delta^{t-1}} \geq v_L$, whereas buyer i's payoff upon rejection is 0 because the ensuing Bertrand competition yields zero expected profits. Hence, the deviation is suboptimal.

This establishes the inductive hypothesis and thus rules out profitable deviations in any period $t < T$, completing the proof of **Case 1**.

Case 2: $\frac{v_L}{\delta} < (1-\delta)c_H + \delta v_H < \frac{v_L}{\delta^{T-1}}$.

The proof is analogous to **Case 1**. Let us choose the distribution of the random time $\tilde{t}$ so that (1) $\mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-1}((1-\delta)c_H + \delta v_H)) = v_L$, so that the low-quality seller is indifferent between mixing and accepting in period 1; and (2) $\mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-t}((1-\delta)c_H + \delta v_H) \mid \tilde{t} > t) \geq v_L$ for all $t = 1, 2, \ldots, T-1$.

Let us verify the sequential rationality of the candidate equilibrium strategy profile. In period $\tilde{t}$, buyers engage in Bertrand competition over the average-quality asset, so they have no incentive to deviate in period $\tilde{t}$. On the equilibrium path, buyers' average asset value in period $\tilde{t}$ is $(1-\delta)c_H + \delta v_H$, which yields the seller's reservation value regardless of asset quality.

Now fix any period $t < \tilde{t} \leq T$ before the terminal competitive price $(1-\delta)c_H + \delta v_H$ is offered. Along the candidate equilibrium path, if the seller rejects all offers in period t, then (for either seller type) the continuation value equals the expected discounted value of the terminal competitive price at time $\tilde{t}$, conditional on $\tilde{t} > t$, namely

$$\mathbb{E}_{\boldsymbol{\sigma}}\left(\delta^{\tilde{t}-t}\big((1-\delta)c_H+\delta v_H\big)\;\Big|\;\tilde{t}>t\right).$$

Therefore, any offer strictly below this amount is rejected by both types and is a losing offer in period t. Hence, it suffices to consider a deviation

$$p_t^i>\mathbb{E}_{\boldsymbol{\sigma}}\left(\delta^{\tilde{t}-t}\big((1-\delta)c_H+\delta v_H\big)\;\Big|\;\tilde{t}>t\right).$$

(i) If $p_t^i \geq (1-\delta)c_H+\delta v_H$, then both seller types accept p_t^i. Since buyer i can guarantee a payoff of 0 by not deviating and competing in period $\tilde{t}$ (where Bertrand competition yields zero expected profits at price $(1-\delta)c_H+\delta v_H$), it suffices to show that acceptance at this deviation node yields buyer i a weakly nonpositive expected payoff. Since $(1-\delta)c_H+\delta v_H>v_L$, the deviating buyer risks overpaying for the low-quality asset, so the deviation is suboptimal.

(ii) If $p_t^i \in \big[\max\{\delta v_H, \mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-t}((1-\delta)c_H+\delta v_H)\mid \tilde{t}>t)\}, (1-\delta)c_H+\delta v_H\big)$, consider a continuation equilibrium after (h_{t-1}^P, p_t^i) in which the high-quality seller rejects p_t^i surely and the low-quality seller accepts p_t^i with strictly positive probability. Choose the low-quality seller's mixing probability so that buyers' posterior asset value v_{t+1}^i satisfies $p_t^i=\delta v_{t+1}^i+(1-\delta)c_H$, and specify Bertrand competition in period $t+1$. Since $p_t^i\geq \delta v_H>v_L$ by **NFS**, the deviating buyer risks overpaying for the low-quality asset, so the deviation is suboptimal.

(iii) Suppose that $\mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-t}((1-\delta)c_H+\delta v_H)\mid\tilde{t}>t)<\delta v_H$ and $p_t^i\in\big(\mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-t}((1-\delta)c_H+\delta v_H)\mid\tilde{t}>t), \delta v_H\big)$. Proceed by induction analogously to **Case 1(iii)**. By the hypothesis that $\mathbb{E}_{\boldsymbol{\sigma}}(\delta^{\tilde{t}-t}((1-\delta)c_H+\delta v_H)\mid\tilde{t}>t)\geq v_L$ for all $t=1,2,\ldots,T-1$, the deviating buyer risks overpaying for the low-quality asset under the resulting continuation equilibrium, so the deviation is suboptimal.

Case 3: $\frac{v_L}{\delta}>(1-\delta)c_H+\delta v_H$.

In this case, the INWBR criterion does not impose meaningful restrictions on the belief specification of the equilibrium characterized in Proposition 1, because the high-quality seller rejects almost surely in period 1, and the period-2 price $\frac{v_L}{\delta}$ exceeds $(1-\delta)c_H+\delta v_H$ in this equilibrium. Hence, the equilibrium in Proposition 1 satisfies the INWBR criterion.

Finally, I proceed to establish (2). Specify off-path beliefs and continuation play so that after any rejection in period 2 and onwards buyers do not update their belief about asset quality and continue to submit their period-2 offer amounts afterward. Given this off-path belief and continuation play, it is optimal for the seller to adhere to the equilibrium play characterized in Theorem 1. Moreover, if $\frac{v_L}{\delta} > (1-\delta)c_H + \delta v_H$, then rejecting the equilibrium period-2 price is never a weak best response for any seller type along the relevant histories, so the INWBR criterion does not assign zero probability weight to any seller type. Hence, this equilibrium satisfies the INWBR criterion. □

IA.5 Continuous-Type Version

The Online Appendix IA.5 is organized as follows. In IA.5.1, I begin by specifying the type space and establishing a technical property that holds regardless of the information structure. In IA.5.2, I analyze the equilibria of the public negotiation process. In IA.5.3, I construct a candidate equilibrium in the confidential negotiation process and show that there is no profitable deviation from it. In IA.5.4, I provide formal arguments for Properties' 1 and 2 to investigate how offer confidentiality and buyer entry affect bargaining dynamics.

IA.5.1 Setup

The buyers share the prior belief that q is drawn from a distribution function F on $[0,1]$. If the asset quality is q, the asset (seller) is sometimes referred to as the q-quality asset (seller). For any $q \in [0,1]$, all buyers assign the same value $v(q)$ to the asset, whereas the seller's value of the asset is q. The valuation function $v(q)$ is assumed to be strictly increasing in q, which captures the idea that an asset of higher quality is preferable.

Moreover, I assume that both $v(q)$ and $F(q)$ are strictly increasing and twice continuously differentiable on $[0,1]$, with $f(q) := F'(q) > 0$. For any $q < 1$, $v(q) - q$ is assumed to be strictly positive, so there is common knowledge of gains from trade. Furthermore, in line with the prior literature (e.g., Fuchs et al. [2016], henceforth "FOS"), it is convenient to consider primitives under which full efficiency is never achieved, so I assume that $v(1) = 1$.

A few additional notations are introduced in order. Define $W(p \mid k) := \int_k^p (v(q) - p)\, dF(q)$ to be a buyer's expected payoff upon winning the asset at price p in a static negotiation process in which its prior belief about seller types is given by a left truncation of F with support $[k,1]$. Henceforth, I denote its partial derivatives by $\partial_1 W(p \mid k) := \frac{\partial W(p \mid k)}{\partial p}$ and $\partial_2 W(p \mid k) := \frac{\partial W(p \mid k)}{\partial k}$.

I impose additional regularity Assumptions 1 and 2, both of which are satisfied by the affine version of the model with $F(q) = q$ and $v(q) = \alpha q + (1-\alpha)$ with $\alpha \in (0,1)$ and thus are not overly restrictive. Assumption 1 states that for any $k < 1$, $W(p \mid k)$ is a strictly concave function of p on $[k, 1]$:

ASSUMPTION 1: The function $f(q)(v(q) - q) - F(q)$ strictly decreases in q.

Straightforward calculations show that for any given $k \in [0,1]$, there exists (1) a unique $p^M(k)$ in $[k,1]$ that maximizes $W(p \mid k)$ with respect to p (Assumption 1), and (2) a unique $p^C(k)$ in $(k,1]$ such that $W(p^C(k) \mid k) = 0$.[28]

To gain some intuition behind Assumption 1, consider two static negotiation processes, one with a monopsonistic buyer and another with a competitive pool of buyers. Assumption 1 guarantees that if a buyer's prior belief is given by a left truncation of F with support $[k,1]$ in either static negotiation process, its equilibrium price must be uniquely determined. In particular, given any left truncation of F with support $[k,1]$ with $k \in [0,1]$, $p^M(k)$ corresponds to the unique price charged by a monopsonistic buyer in a static bargaining game, whereas $p^C(k)$ corresponds to the unique price charged by a competitive pool of buyers in a static negotiation process.

With the notations $p^M(\cdot)$ and $p^C(\cdot)$, I further assume:

ASSUMPTION 2: (1) $p^M(p^M(0)) < p^C(0)$; (2) $v(p^M(0)) \leq p^C(0)$.

Similar to (**EXP**) in the two-quality case, Assumption 2 ensures sufficient incentives for buyers to engage in price experimentation in the confidential negotiation process. Intuitively, the first part of Assumption 2 guarantees that an experimenting buyer has no incentive to make an overly aggressive period-2 offer in the continuation game, which can reduce the profitability of price experimentation. The second part ensures that a non-experimenting buyer has no incentive to make a period-2 offer as aggressive as an experimenting buyer, which protects its expected information rent from price experimentation.

Lemma IA.5.1: Both $p^C(k)$ and $p^M(k)$ strictly increase in k.

[28]Observe that $\frac{\partial}{\partial p} W(p \mid k)\Big|_{p=k} = F(k)(v(k) - k) > 0$ and $\frac{\partial}{\partial p} W(p \mid k)\Big|_{p=1} = F(k) - F(1) < 0$. Coupled with the concavity of $W(p \mid k)$ in p, this implies that there exists a unique $p^M(k) \in (k,1)$ such that $W(p \mid k)$ strictly increases in p on $[k, p^M(k)]$ and strictly decreases in p on $[p^M(k), 1]$. Also, since $W(1 \mid k) < 0$, the intermediate value theorem implies that there exists a unique zero $p^C(k)$ of $W(p \mid k)$ in the set $p \in [p^M(k), 1]$.

Proof. By the construction of $p^C(k)$, $W(p^C(k) \mid k) = 0$. Implicitly differentiating this equation gives:

$$\frac{dp^C(k)}{dk} = -\frac{(p^C(k) - v(k))f(k)}{\Big(v(p^C(k)) - p^C(k)\Big)f(p^C(k)) - F(p^C(k)) + F(k)}. \tag{34}$$

Since $p^C(k) \in (p^M(k), 1]$, $\partial_1 W(p^C(k) \mid k) < 0$. Hence, the denominator $\Big(v(p^C(k)) - p^C(k)\Big)f(p^C(k)) - F(p^C(k)) + F(k)$ is negative. Observe that $p^C(k) - v(k)$ must be positive, because assuming that $p^C(k) - v(k) \leq 0$ leads to an immediate contradiction.[29] Hence, $\frac{dp^C(k)}{dk} > 0$.

Additionally, the first-order condition $\partial_1 W(p^M(k) \mid k) = 0$ can be written as

$$\Big(v(p^M(k)) - p^M(k)\Big)f(p^M(k)) - F(p^M(k)) + F(k) = 0,$$

which implies

$$\frac{dp^M(k)}{dk} = -\frac{f(k)}{\partial_p\Big((v(p) - p)f(p) - F(p)\Big)\big|_{p=p^M(k)}}.$$

By Assumption 1, $\partial_p\Big((v(p) - p)f(p) - F(p)\Big)\big|_{p=p^M(k)} < 0$, which implies that $p^M(k)$ strictly increases in k. □

IA.5.2 Public Negotiation Process

Proof of Proposition 6. I first analyze the period-2 continuation game. Note that buyers are symmetrically informed and attach a common value to the asset in period 2. By standard Bertrand competition logic, all buyers must earn zero expected period-2 payoffs regardless of the history.

Suppose, en route to a contradiction, that there exists an equilibrium in which buyers expect strictly positive period-1 payoffs. Since buyers are indifferent between all equilibrium period-1 offers, buyers' expected period-1 payoff from the minimum possible equilibrium period-1 offer $\underline{p}_1 := \min \mathcal{P}_1$ is strictly positive as well. Observe that if a buyer i offers $\underline{p}_1$, the buyer can win the asset in period 1 only by tying with competitors. Hence, if buyer i offers $\underline{p}_1 + \varepsilon$, the buyer's winning probability discontinuously increases in ε, whereas the payoff upon winning (i.e., $\int_0^{\kappa_1(\underline{p}_1+\varepsilon)} (v(q) - \underline{p}_1 - \varepsilon)\, dF(q)$) changes continuously in ε by the right-continuity of $\kappa_1(\cdot)$. Hence, compared to when the buyer offers $\underline{p}_1$, the buyer expects a strictly larger payoff by offering $\underline{p}_1 + \varepsilon$, which is a contradiction. □

[29] Assume, to the contrary, that $p^C(k) - v(k) \leq 0$. Then, for any $q > k$, $p^C(k) < v(q)$ because $v(\cdot)$ is strictly increasing. Hence, $\int_k^{p^C(k)} (v(q) - p^C(k))\, dF(q) > 0$, which is a contradiction.

Lemma IA.5.2: There exists $\delta^* > 0$ such that for all $\delta \in (\delta^*, 1)$, no trade occurs in period 1 in any equilibrium of the public negotiation process.

Proof. This follows from FOS's Proposition 1 ("Quiet Period"), as their proof does not hinge on buyers being short-lived in the public negotiation process. □

IA.5.3 Confidential Negotiation Process

I begin by constructing the candidate equilibrium in the confidential negotiation process. Additional notation is introduced in order. Consider the equation

$$(1-\delta)W(k_1^i \mid 0) + \delta W(p_2^i \mid 0) = 0.$$

Observe that for any $k_1^i \in [0, p^M(0)]$ and any $p_2^i \in [p^C(0), 1]$, $\partial_1 W(p_2^i \mid 0) < 0$ and $\partial_1 W(k_1^i \mid 0) \geq 0$. Thus, the implicit function theorem implies that there exists a unique differentiable, strictly increasing function $P_2 : [0, p^M(0)] \to [p^C(0), 1]$ such that for any $k_1^i \in [0, p^M(0)]$,

$$(1-\delta)W(k_1^i \mid 0) + \delta W(P_2(k_1^i) \mid 0) = 0. \tag{35}$$

Since $W(p \mid 0)$ uniquely vanishes at $p = p^C(0)$ on $(0, 1]$ (see the remark after Assumption 1), $P_2(0) = p^C(0)$.

With the notation $P_2(\cdot)$, define functions $P_1(\cdot)$ and $H(\cdot)$ as follows:

$$P_1(k_1^i) = (1-\delta)k_1^i + \delta P_2(k_1^i), \quad H(k_1^i) = \exp\left(\int_{k_1^i}^{p^M(0)} \left(\frac{\partial_1 W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)\, P_2'(\hat{k}_1^i)}{W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)}\right) d\hat{k}_1^i\right). \tag{36}$$

A straightforward argument shows that $H(k_1^i)$ strictly increases in k_1^i on $[0, p^M(0)]$.[30] Also, it is clear from (36) that $H(p^M(0)) = 1$. Hence, $H(k_1^i)$ is a well-defined CDF that has no probability mass at every point of its support $[0, p^M(0)]$, potentially except at $k_1^i = 0$. In addition, $\underline{p}_1 = P_1(0)$ in the confidential negotiation process.

Based on these functions, suppose that the candidate equilibrium proceeds as follows:

[30] Observe that $P_2(\hat{k}_1) \geq p^C(0) > p^M(p^M(0)) > p^M(k_1)$, where the first inequality follows from (35), the second from Assumption 2, and the last from Lemma IA.5.1. Hence, $P_2(\hat{k}_1) > p^M(k_1)$, which implies that the integrand on the right-hand side of (36) is strictly negative. Thus, $H(k_1^i)$ is a positive function whose log-derivative is positive, so $H(k_1^i)$ strictly increases in k_1^i.

Candidate Equilibrium.) For all $i \in \mathcal{I}$, random variables $\tilde{K}_1^i$ are independently drawn from the distribution function $H^{\frac{1}{N-1}}$, where H is given by (36). Denote the realization of $\tilde{K}_1^i$ by k_1^i, which is henceforth referred to as buyer i's "cutoff" in period 1. At the beginning of period 1, for all $i \in \mathcal{I}$, buyer i offers $P_1(k_1^i)$. The seller accepts the period-1 price $\max_{i\in\mathcal{I}} P_1(k_1^i)$ if the seller's asset quality is below $\max_{i\in\mathcal{I}} k_1^i$, and rejects otherwise. In period 2, for all $i \in \mathcal{I}$, buyer i offers $P_2(k_1^i)$. The seller accepts the period-2 price $\max_{i\in\mathcal{I}} P_2(k_1^i)$ if the seller's asset quality is below the period-2 price amount, and rejects otherwise.

Proof of Proposition 7. I shall verify whether there is a profitable deviation from the candidate equilibrium strategy profile above. I first examine the seller's decision. Clearly, the seller's period-2 acceptance rule is optimal. Assume that the equilibrium period-1 price is equal to $P_1(k_1^i)$ for some $k_1^i \in [0, p^M(0)]$. For any seller of quality q with $q \le k_1^i$, the definition of $P_1(\cdot)$ in (36) implies that $P_1(k_1^i) \ge (1-\delta)q + \delta P_2(k_1^i)$. Since its optimal decision at the end of period 2 is to accept $P_2(k_1^i) \ge k_1^i \ge q$, the q-quality seller's expected continuation payoff from rejection in period 1 is equal to $(1-\delta)q + \delta P_2(k_1^i)$. Hence, it is optimal for any seller of quality q with $q \le k_1^i$ to accept $P_1(k_1^i)$ in period 1.

For any seller with asset quality $q > k_1^i$, $P_1(k_1^i) < (1-\delta)q + \delta P_2(k_1^i)$. The right-hand side of the inequality represents the q-quality seller's expected period-1 payoff under any acceptance strategy such that the seller always accepts the equilibrium period-2 price. Since the q-quality seller can choose whether to accept an offer or not at the end of period 2, the seller's expected continuation payoff from rejection in period 1 is weakly higher than $(1-\delta)q + \delta P_2(k_1^i)$. Hence, it is optimal for any seller of quality q with $q > k_1^i$ to reject $P_1(k_1^i)$ in period 1.

I also check whether a buyer has a profitable deviation. By construction, the equilibrium distribution of $\tilde{K}_1^i$ has no probability mass at any point of its support $[0, p^M(0)]$ (potentially except at $k_1^i = 0$). Hence, for any $k_1^i \in [0, p^M(0)]$ and any $\hat{k}_1^i \in (0, p^M(0)]$, if a buyer's cutoff is equal to k_1^i, its expected continuation payoff $\Pi_2(P_2(\hat{k}_1^i) \mid k_1^i)$ from making $P_2(\hat{k}_1^i)$ in period 2 is expressible as:

$$\Pi_2(P_2(\hat{k}_1^i) \mid k_1^i) = \int_{0 \le \max \boldsymbol{k}_1^{-i} \le \hat{k}_1^i} \underbrace{\left(\int_{\max\{k_1^i, \boldsymbol{k}_1^{-i}\}}^{P_2(\hat{k}_1^i)} \left(v(q) - P_2(\hat{k}_1^i) \right) dF(q) \right)}_{= W(P_2(\hat{k}_1^i) \mid \max\{k_1^i, \boldsymbol{k}_1^{-i}\})} d\mathbb{P}_{\boldsymbol{\sigma}^{-i}}(\boldsymbol{k}_1^{-i}). \tag{37}$$

If a buyer makes $P_2(0)$ in period 2, it can win the asset only upon a tie with all its competitors. Thus, if a buyer's cutoff is given by $k_1^i \in [0, p^M(0)]$, its expected payoff from making $P_2(0)$ in period 2 is

$$\frac{1}{N} \lim_{\hat{k}_1^i \to 0} \Pi_2(P_2(\hat{k}_1^i) \mid k_1^i) = \frac{1}{N} H(0)\, W(P_2(0) \mid k_1^i) \geq 0,$$

where the inequality follows from the fact that $H(0) \geq 0$ (by the definition (36)), and $W(P_2(0) \mid k_1^i) \geq 0$ for any $k_1^i \in [0, p^M(0)]$.[31]

Since $P_2(\cdot)$ is a strictly increasing function on $[0, p^M(0)]$, the buyer's choice over the period-2 offer amount p_2^i can be viewed as the choice over its cutoff $\hat{k}_1^i$ in (37). Based on this observation, I show that for any $k_1^i \in [0, p^M(0))$ and $\hat{k}_1^i \in (k_1^i, p^M(0)]$, $\frac{\partial}{\partial \hat{k}_1} \Pi_2(P_2(\hat{k}_1) \mid k_1)\big|_{\substack{\hat{k}_1 = \hat{k}_1^i \\ k_1 = k_1^i}}$, which is expressible as

$$\int\limits_{0 \leq \max \boldsymbol{k}_1^{-i} \leq \hat{k}_1^i} \partial_1 W(P_2(\hat{k}_1^i) \mid \max\{k_1^i, \boldsymbol{k}_1^{-i}\})\, P_2'(\hat{k}_1^i)\, dH(\max \boldsymbol{k}_1^{-i}) \;+\; W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)\, H'(\hat{k}_1^i),$$

is strictly negative. Since $\partial_1 W(p \mid k) := f(p)(v(p) - p) - F(p) + F(k)$, it strictly increases in k. Hence, the expression above is strictly less than

$$\frac{\partial}{\partial \hat{k}_1} \Pi_2(P_2(\hat{k}_1) \mid k_1)\big|_{\substack{\hat{k}_1 = \hat{k}_1^i \\ k_1 = \hat{k}_1^i}} = \partial_1 W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)\, P_2'(\hat{k}_1^i)\, H(\hat{k}_1^i) + W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)\, H'(\hat{k}_1^i) = 0, \tag{38}$$

where the equality in the last line follows from the definition of $H(\cdot)$ in (36). Additionally, for any period-2 offer $p_2^i \geq p^C(0)$,

$$\frac{\partial}{\partial p} \Pi_2(p \mid k_1^i)\Big|_{p = p_2^i} = \int\limits_{0 \leq \max \boldsymbol{k}_1^{-i} \leq p^M(0)} \partial_1 W(p \mid \max\{k_1^i, \boldsymbol{k}_1^{-i}\})\, dH(\max \boldsymbol{k}_1^{-i}),$$

whose integrand is negative because $p \geq p^C(0) > p^M(p^M(0)) \geq p^M\big(\max\{k_1^i, \boldsymbol{k}_1^{-i}\}\big)$ by Lemma IA.5.1 and Assumption 2, and $W(\cdot \mid k)$ is strictly concave in p by Assumption 1. Therefore, a buyer whose cutoff is given by $k_1^i \in [0, p^M(0)]$ has no incentive to make a period-2 offer strictly higher than $P_2(k_1^i)$.

[31]Since $W(P_2(0) \mid 0) = W(p^C(0) \mid 0) = 0$ and $\partial_2 W(p^C(0) \mid k_1^i) = f(k_1^i)\big(p^C(0) - v(k_1^i)\big) > 0$, we have $W(P_2(0) \mid k_1^i) \geq 0$.

By the expression (37), for any $k_1^i \in (0, p^M(0)]$ and $\hat{k}_1^i \in (0, k_1^i)$, we have:

$$\begin{aligned}
\frac{\partial^2 \Pi_2(P_2(\hat{k}_1) \mid k_1)}{\partial \hat{k}_1 \partial k_1}\Big|_{\substack{\hat{k}_1=\hat{k}_1^i \\ k_1=k_1^i}} &= \frac{\partial^2}{\partial \hat{k}_1 \partial k_1}\Big(H(\hat{k}_1) W(P_2(\hat{k}_1) \mid k_1)\Big)\Big|_{\substack{\hat{k}_1=\hat{k}_1^i \\ k_1=k_1^i}} \\
&= \frac{\partial}{\partial \hat{k}_1}\Big(H(\hat{k}_1) \underbrace{\frac{\partial}{\partial k_1} W(P_2(\hat{k}_1) \mid k_1)}_{=f(k_1)(P_2(\hat{k}_1)-v(k_1))}\Big)\Big|_{\substack{\hat{k}_1=\hat{k}_1^i \\ k_1=k_1^i}} \\
&= H'(\hat{k}_1^i)\, f(k_1^i)\big(P_2(\hat{k}_1^i) - v(k_1^i)\big) + H(\hat{k}_1^i)\, f(k_1^i)\, P_2'(\hat{k}_1^i).
\end{aligned} \tag{39}$$

Recall that by the construction of $P_2(\cdot)$ preceding (35), $P_2(\hat{k}_1^i) \in [p^C(0), 1]$ for any $\hat{k}_1^i \in [0, p^M(0)]$. Hence, $v(k_1^i) \leq v(p^M(0)) \leq p^C(0) \leq P_2(\hat{k}_1^i)$ by Lemma IA.5.1 and Assumption 2, so $v(k_1^i) \leq P_2(\hat{k}_1^i)$. Additionally, $H(\cdot)$, $H'(\cdot)$, $P_2'(\cdot)$, and $f(\cdot)$ are all strictly positive, so the cross-derivative in (39) is positive for any $\hat{k}_1^i \in (0, k_1^i)$, implying:

$$\frac{\partial}{\partial \hat{k}_1} \Pi_2(P_2(\hat{k}_1) \mid k_1)\Big|_{\substack{\hat{k}_1=\hat{k}_1^i \\ k_1=k_1^i}} > \frac{\partial}{\partial \hat{k}_1} \Pi_2(P_2(\hat{k}_1) \mid k_1)\Big|_{\substack{\hat{k}_1=\hat{k}_1^i \\ k_1=\hat{k}_1^i}} \underbrace{=}_{(38)} 0. \tag{40}$$

Therefore, a buyer whose cutoff is given by $k_1^i \in (0, p^M(0)]$ is strictly better off by making $P_2(k_1^i)$ in period 2, compared to making a period-2 offer in $(P_2(0), P_2(k_1^i))$.

Additionally, recall that if a buyer's cutoff is given by $k_1^i \in (0, p^M(0)]$, it expects to earn $\frac{1}{N} \lim_{\hat{k}_1^i \to 0} \Pi_2(P_2(\hat{k}_1^i) \mid k_1^i) \geq 0$ by making $P_2(0)$ in period 2. Hence, its expected payoff from $P_2(0)$ cannot exceed the expected payoff from $P_2(0) + \varepsilon$, which in turn cannot exceed the expected payoff from $P_2(k_1^i)$ by (40).

For any $\hat{k}_1^i \in [0, p^M(0)]$, a buyer's expected lifetime payoff from making $P_1(\hat{k}_1^i)$ can be computed as:

$$\begin{aligned}
& H(\hat{k}_1^i)\left(\int_0^{\hat{k}_1^i} (v(q) - P_1(\hat{k}_1^i))\, dF(q) + \delta W(P_2(\hat{k}_1^i) \mid \hat{k}_1^i)\right) \\
=& H(\hat{k}_1^i)\left(\int_0^{\hat{k}_1^i} v(q)\, dF(q) - \underbrace{\big((1-\delta)\hat{k}_1^i + \delta P_2(\hat{k}_1^i)\big)}_{=P_1(\hat{k}_1^i) \text{ by } (36)} F(\hat{k}_1^i) + \delta \int_{\hat{k}_1^i}^{P_2(\hat{k}_1^i)} v(q)\, dF(q) - \delta P_2(\hat{k}_1^i)\big(F(P_2(\hat{k}_1^i)) - F(\hat{k}_1^i)\big)\right) \\
=& H(\hat{k}_1^i)\Big((1-\delta) W(\hat{k}_1^i \mid 0) + \delta W(P_2(\hat{k}_1^i) \mid 0)\Big) \underbrace{=}_{(35)} 0.
\end{aligned} \tag{41}$$

Hence, for any $\hat{k}_1^i \in [0, p^M(0)]$, a buyer makes a zero expected lifetime payoff from making $P_1(\hat{k}_1^i)$ in period 1. Since $P_1(\cdot)$ is strictly increasing on $[0, p^M(0)]$, any period-1 offer $p_1^i \in [P_1(0), P_1(p^M(0))]$ uniquely corresponds to a cutoff $\hat{k}_1^i \in [0, p^M(0)]$ via $p_1^i = P_1(\hat{k}_1^i)$. Under the candidate equilibrium continuation strategies, buyer i's period-2 continuation payoff after such an offer is given by (37) with cutoff $\hat{k}_1^i$. Consequently, buyers are indifferent among all period-1 offers in $[P_1(0), P_1(p^M(0))]$.

If $p_1^i < P_1(0)$, then buyer i loses for sure in period 1, so its expected lifetime payoff equals its optimal period-2 continuation payoff when the cutoff is 0, which is 0 by the period-2 analysis above. Additionally, a straightforward argument shows that buyer i expects to make a strictly negative lifetime payoff by making an offer $p_1^i > P_1(p^M(0))$.[32] Hence, no player can profitably deviate from the candidate equilibrium.

Finally, recall that for any $k_1^i \in [0, p^M(0)]$, $\lim_{\hat{k}_1^i \to 0} \Pi_2(P_2(\hat{k}_1^i) \mid k_1^i) = H(0)W(P_2(0) \mid k_1^i) \geq 0$. Additionally, recall from (40) that $\frac{\partial}{\partial \hat{k}_1} \Pi_2(P_2(\hat{k}_1) \mid k_1)\big|_{\substack{\hat{k}_1 = \hat{k}_1^i \\ k_1 = k_1^i}} > 0$ for any $k_1^i \in [0, p^M(0)]$ and any $\hat{k}_1^i \in (0, k_1^i)$. Hence, a buyer whose cutoff is given by $k_1^i \in (0, p^M(0)]$ expects to earn a positive equilibrium period-2 continuation payoff $\Pi_2(P_2(k_1^i) \mid k_1^i) > 0$. Since a buyer makes a zero expected equilibrium lifetime payoff by (41), any serious equilibrium period-1 offer (i.e., any $P_1(k_1^i)$ with $0 < k_1^i \leq p^M(0)$) must generate a loss to the winning buyer upon the seller's acceptance, which completes the proof. □

IA.5.4 Formal Arguments for Properties' 1 and 2 in Continuous-Type Version

I first examine Property 1. By Lemma IA.5.2, for any $\delta > \delta^*$, no trade occurs in period 1 of the public negotiation process, whereas trade occurs with strictly positive probability in the confidential negotiation process (Proposition 7). Hence, the expected period-1 cutoff must be strictly higher in the confidential negotiation process. In addition, the period-2 price in the confidential negotiation process is at least $P_2(0) = p^C(0)$, which is the period-2 price in the public negotiation process for any $\delta > \delta^*$. Thus, the expected period-2 price, as well as the expected period-2 cutoff, are both strictly higher in the confidential negotiation process. Since the expected period-2 price and the expected period-1 cutoff are both strictly higher in the confidential negotiation process, the definition of $P_1(\cdot)$ given in (36) implies that the period-1 price in the confidential negotiation process must also be strictly higher.

As for welfare, recall that the expected equilibrium price is always higher in every period of the confidential negotiation process. Hence, the seller's expected discounted payoff is higher in the confidential negotiation process.

[32]If a buyer makes a period-1 offer $p_1^i > P_1(p^M(0))$, the buyer's period-1 cutoff is strictly higher than $p^M(0)$ and its period-2 offer is weakly higher than $P_2(p^M(0))$, which yields a negative expected lifetime payoff.

The buyers make zero expected lifetime payoffs regardless of the information structure. Therefore, the equilibrium in the confidential negotiation process Pareto dominates the equilibrium in the public negotiation process.

To establish Property 2, I define the maximum equilibrium period-1 cutoff in the confidential negotiation process with $N \geq 2$ buyers as the random variable $\tilde{K}_1^{(N)} := \max_{i \in \mathcal{I}} \tilde{K}_1^i$, in order to emphasize that the equilibrium distribution of the period-1 maximum cutoff depends on the number of buyers. Since $P_2(\cdot)$ is strictly increasing, the maximum period-2 offer is $\max_{i \in \mathcal{I}} P_2(\tilde{K}_1^i) = P_2(\tilde{K}_1^{(N)})$. I denote the corresponding period-2 cutoff by $\tilde{K}_2^{(N)} := P_2(\tilde{K}_1^{(N)})$. I start with an analog of Theorem 2 in the continuous-type version of the model:

Lemma IA.5.3: For any $N \geq 2$ and $t \in \{1, 2\}$, the equilibrium distribution of $\tilde{K}_t^{(N)}$ first-order stochastically dominates the equilibrium distribution of $\tilde{K}_t^{(N+1)}$.

Proof. The proof is analogous to that of Theorem 2. In particular, replace the equilibrium CDF G in Theorem 2 with H, replace the equilibrium support $\mathcal{P}_1$ with $[0, p^M(0)]$, and replace the equilibrium random variable for the maximum period-1 object with $\tilde{K}_1^{(N)}$. Since $\tilde{K}_2^{(N)} = P_2(\tilde{K}_1^{(N)})$ and $P_2(\cdot)$ is strictly increasing, first-order stochastic dominance for $\tilde{K}_1^{(N)}$ implies the stochastic dominance for $\tilde{K}_2^{(N)}$. □

Since the period-t price is $\tilde{P}_t^{(N)} = P_t(\tilde{K}_1^{(N)})$ for both $t = 1, 2$, the equilibrium distribution of $\tilde{P}_t^{(N)}$ first-order stochastically dominates the equilibrium distribution of $\tilde{P}_t^{(N+1)}$ for all $N \geq 2$. Also, since the seller's expected discounted revenue decreases in N, the seller's expected lifetime payoff decreases in N. However, all buyers make zero expected lifetime payoff, so the present value of expected equilibrium welfare decreases in N.