Simplifying Auction Designs via Market Feedback[*]

Dan Bernhardt
University of Illinois, University of Warwick, danber@illinois.edu

Tingjun Liu
University of Hong Kong, tjliu@hku.hk

Xiaorong Ma
University of Macau, rachelma@um.edu.mo

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Abstract: Implementation of optimal or near-optimal mechanisms with heterogeneous bidders is informationally demanding for auctioneers. Such mechanisms invariably employ discriminatory winning and payment rules—creating legal and moral hazard concerns. We show how sellers can exploit information feedback from capital markets, linking auction outcomes to post-auction market prices to obtain high revenues even when arbitrarily heterogeneous bidders pay with different securities. Steeper securities always generate greater revenues, and we identify conditions where near-optimal revenues obtain. Crucially, the market collects information and responds to details ex post when pricing the winner, so the selling mechanism can be nondiscriminatory and *detail-free* ex ante.

Keywords: market feedback; security bidding/security design; mechanism robustness; simple contracts

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1 Introduction

A common, fundamental critique of theoretically-derived optimal or near-optimal mechanism designs with heterogeneous agents is that they are invariably complicated and informationally demanding for auctioneers.[1] When bidders in an auction are ex-ante heterogeneous—as is the case in practice—two complications arise. First, optimal designs are sensitive to the details of the auction environment, requiring the seller to know the specific characteristics of all agents to determine the winner and payments. Second, and crucially, even if the seller knows everything, optimal auction rules are discriminatory, requiring different rules for different bidders and favoring some bidders at the expense of others. Such discriminatory designs give rise to legal and moral hazard concerns.

These demands for information and discrimination are magnified in security-bid auctions, in which the winner pays with securities (claims to future cash flows) rather than cash, as commonly is the case in takeover and merger settings. Unlike cash bids, the monetary values of security bids are sensitive to the fine details of the underlying cash flow distributions, and because bidder cash flows are private information, a security bid's monetary value is not transparent. Without intricate and discriminatory adjustments for differences in bidder characteristics (e.g., standalone values or distributions of cash flows) and the possibly different securities bidders use, the "wrong" (lower valuation) winning bid is likely to be selected. Moreover, the probabilities and costs of such selections rise when bidders pay with steeper securities whose values are more sensitive to cash flows. This can drive revenue in nondiscriminatory security auctions below that in cash auctions, with greater revenue decreases for steeper securities.

Our paper develops a simple and nondiscriminatory mechanism for security-bid auctions, which is detail free and the seller does not need to know anything about bidder characteristics. Regardless of how bidders differ, our mechanism treats bidders identically—yet it always generates higher revenues than cash auctions and revenues are even higher for steeper securities. The key to our mechanism is that it uses information

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[1] See e.g., Hurwicz 1972, Wilson 1985, Dasgupta and Maskin 2000, Bergemann and Morris 2005.

aggregation by financial markets, linking auction outcomes to security prices set by the market post-auction to alleviate informational burdens and avoid discrimination. Because the post-auction price of a bidder's security incorporates the market's information about the bidder's specific characteristics, linking auction outcomes to market prices lets a seller *achieve de facto discrimination without explicitly discriminating.*

Our "dollar-denominated" security auctions consist of two stages. In the first stage, bidders, who are privately informed about their synergies (the value added by their control) with the auctioned asset, submit cash bids in a second-price auction and the winning price is publicly announced.[2] In the second stage, in lieu of cash, the winner pays with securities priced by a competitive financial market that incorporates all public information, so that the post-auction market value of the security paid equals the winning price of the auction (the second-highest cash bid).

More specifically, the security paid is from a set of securities ordered by a parameter, e.g., the set of equities ordered by share, or the set of debt contracts ordered by face value. We adopt the standard assumption in the literature (DeMarzo, Kremer, and Skrzypacz 2005) that this set is either pre-specified by a seller or exogenously determined; unlike the literature we also allow the sets to differ across bidders. The security parameter paid is such that the security's expected payoff, given the winning price and the market's knowledge of the publicly-traded winner's characteristics, equals the winning price.[3]

Thus, bids in our security auctions are denominated in dollars, but are ultimately paid in securities. This distinguishes our auction design from the "face-value-denominated" security-bid auctions typically studied, where bids are both denominated and paid in

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[2] Our focus on second-price auctions where the winning price announced is consistent with the standard way of modeling takeover contests via ascending (English) auctions, in which bids are raised continuously until only one bidder remains.

[3] For example, consider a bidder i with a standalone value of \$400 who enters a dollar-denominated equity auction to acquire a target with a standalone value of zero and wins at a cash price of \$200. Rather than pay cash, i pays with equity claims to the merged firm. For example, if the market's post-auction estimate of i's merger synergy v_i given the winning price of \$200 is \$300, the market values the merged firm at \$300+\$400=\$700, so i pays the target an equity share of \$200/\$700=2/7 in lieu of cash (as \$700*2/7=\$200). See Example 1 in the main text. There we show that if, pre-takeover, the market believes $v_i \sim unif[0, \$440]$, then its updated belief given the winning price of \$200 is that $v_i \sim unif[\$160, \$440]$, yielding an expected synergy of \$300.

securities—i.e., where the security bids submitted in the auction determine the security parameter paid. Our design is a hybrid of face-value-denominated auctions and standard cash auctions, and we show it only inherits their good features.

Importantly, dollar-denominated security auctions correspond to the actual practices in M&A and takeovers, where the prices are often reported in dollars even though the payment might occur in securities and different bidders may pay different securities to the seller. We establish an equivalence in terms of both bidding strategies and seller revenues between our design and cash bidding that is financed after the auction with public issues of equity or debt by the winner; the equivalence reflects that a bidder does not care who gets the security—the seller or the market. Our design also corresponds to the "collar clauses" that appear in many equity payment agreements, where the bidder's equity payment adjusts according to the post-takeover market price of the bidder's stock, so that the equity payment delivers a guaranteed cash value ex post. From a regulatory perspective, our design has another advantage: it eases legal and moral hazard concerns for discriminatory auctions rules that arise in face-value-denominated security auctions. For example, in takeovers, fiduciary duties imposed by regulators may require a target to accept the "highest" bid. Our design complies with this requirement.

We start our analysis with the benchmark setting of existing studies—ex-ante identical bidders who use the same ordered set of securities. In such settings, DeMarzo, Kremer, and Skrzypacz (2005) show that in nondiscriminatory face-value-denominated security auctions, expected seller revenue is higher when bidders use steeper securities (e.g., equity is steeper than debt, which is steeper than cash). We derive a revenue equivalence result for homogeneous bidders, establishing that dollar-denominated and face-value-denominated auction designs deliver the same expected revenue.

Revenue equivalence breaks down with heterogeneous bidders. Even when bidders use the same security, when bidders' standalone values differ sufficiently, revenues in nondiscriminatory face-value-denominated auctions fall even below those in a cash auction. In contrast, in nondiscriminatory dollar-denominated security auctions, revenues always exceed those in cash auctions. That is, by modifying the auction format to have a dollar-denominated design with market feedback, we show how to extend the insights of DeMarzo, Kremer, Skrzypacz (2005) on the advantages of steeper securities to settings with heterogeneous bidders and heterogeneous securities.

To establish the robustness of dollar-denominated security auctions, we consider a general framework with *arbitrary* bidder heterogeneities in (1) standalone values, (2) distributions of expected synergies, (3) conditional distributions of future payoffs given expected synergies, and (4) the types of ordered securities used by different bidders. We prove that, no matter how bidders differ, our simple and nondiscriminatory auction design has the following properties: (i) it always generates higher revenues than cash auctions, and (ii) revenues are always higher with steeper securities than with flatter securities, and (iii) revenues rise when the standalone value of any *single* bidder is reduced. These results hold even if different bidders use different types of securities, for example, if some bidders use equity and others use debt. Thus, result (ii) means that revenues rise if one bidder switches to a steeper security type, and the other bidders use the same security types that they had used before.

To understand these results, note that in a dollar-denominated security auction, a bidder breaks even when it barely wins, i.e., when the first and second-highest bids are equal. When this happens, the market only knows that the winning bid is at least as high as the publicly-revealed second-highest bid, inferring that the winner's synergies are at least as high as those associated with the bid, but likely higher. As a result, at the break-even point, the market overestimates the winner's synergies. This provides bidders incentives to submit more aggressive cash bids than in cash auctions, yielding property (i). Property (ii) follows because the monetary value of a steeper security is more sensitive to a bidder's type. This increases a bidder's incentives to signal a high type, as the value of being grouped with higher types is larger, leading to more aggressive cash bids. Property (iii), that revenues in dollar-denominated security auctions rise when the standalone value of any *single* bidder is reduced, reflects that a bidder with a lower standalone value cares more about the market inferences about its synergies, as it pays a larger fraction of cash flows upon winning. This incentivizes the bidder to make more aggressive bids.

These results stand in sharp contrast to nondiscriminatory face-value-denominated auctions, where *each* of these properties is violated. In nondiscriminatory face-value-denominated auctions, (i) security auctions may generate lower revenues than cash auctions, and (ii) revenues with steeper securities may be less than with flatter securities, and (iii) revenues rise if *all* bidders switch to the same steeper security, or if their common standalone values fall by the same amount—but if only one bidder's standalone value is reduced, revenues eventually fall below those in a cash auction.

The key to our design's robust ability to deliver high revenues is that bidders submit cash bids in dollar-denominated security auctions, but they submit security bids (e.g., equity share) in face-value-denominated auctions. When bidders have different characteristics, some bid more aggressively than others. More aggressive bidding by *any subset of bidders* always benefits a seller in dollar-denominated security auctions—no matter how bidders differ, *more cash is always good.* In contrast, more aggressive bidding by a subset of bidders in face-value-denominated security auctions may hurt a seller because the security bid with the highest face value may not have the highest monetary value, as it could be from an incentivized bidder with a low valuation. Put differently, bids in face-value-denominated auctions are in non-transparent units, as their monetary values depend on a firm's features and information—whereas dollar-denominated designs convert all bids into a common unit, cash, making it easy to correctly rank bids even when bidders have different characteristics or use different types of securities.

We conclude our analysis by identifying conditions under which revenues from a dollar-denominated auction approach the theoretical maximum that no any mechanism can surpass. We prove that if bidder standalone values are negligible relative to the sum of the seller's standalone value and expected synergies then nondiscriminatory dollar-denominated security auctions extract full rents even if: (1) the rates at which standalone values go to zero *vary* across bidders, (2) different bidders use different types of securities, and (3) the common distribution for expected synergies is arbitrary. By contrast, nondiscriminatory face-value-denominated security auctions do not extract full rents unless all bidders' standalone values go to zero at the same rate.

The ability of our nondiscriminatory mechanism to accommodate heterogeneous bidders differentiates it from existing designs. Its virtues include:

1. It is simple, detail-free and invariant to the environment: no matter how bidders or their securities differ, the same mechanism is used and the high cash bid wins.
2. It removes legal and moral hazard concerns about discriminatory auctions. In takeovers, it lets a seller satisfy its fiduciary duties to accept the "highest" bid even when bidders have very different characteristics.
3. It imposes no information demands on a seller, and the market only needs to learn about the winner—and it can do so post-auction, after a winner is selected.
4. It generates higher revenues than cash auctions no matter how bidders differ.

Relation to Literature. Dollar-denominated auctions[4] were first analyzed in Liu (2012). Liu studied an ascending-price format with equity payments, showing how signaling incentives arise and interact with the multiple-bid nature of ascending-price auctions. By contrast, we analyze the revenue properties of dollar-denominated auctions with general securities, allowing for arbitrary heterogeneity in bidder characteristics and types of securities used.[5]

Researchers have stressed the desirability of robust mechanisms that do not depend on agents' common knowledge and work well in a range of settings (Hurwicz 1972, Wilson 1985, Dasgupta and Maskin 2000, Bergemann and Morris 2005). With heterogeneous bidders, a seller in a face-value-denominated auction must discriminate according to all details of *all* bidders, else the auction can generate lower revenues than cash auctions.

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[4] Our previous drafts have used alternative terms such as auctions with floating-parameter offers, floating-rate offers (as in Liu 2012), or cash equivalent of security payments.

[5] More generally, our paper relates to a vast literature on security auction design, including Hansen (1985), Crémer (1987), Samuelson (1987), Nachman and Noe (1994), Dasgupta and Tsui (2004), Povel and Singh (2010), Gorbenko and Malenko (2011), Kogan and Morgan (2010), Abhishek, Hajek, and William (2015), Sogo (2017), Cong (2020), and Fioriti and Hernandez-Chanto (2022). Dasgupta and Tsui (2003), Gorbenko and Malenko (2014) and Boulatov and Severinov (2021) analyze auctions with asymmetric bidders, and Burkart (1995), Singh (1998), Biais et al. (2007), Burkart and Lee (2015), Ekmekci, Kos and Vohra (2016), Lee and Rajan (2018), Eckbo, Malenko and Thorburn (2020) analyze auction/security design in financing and takeover settings.

In contrast, dollar-denominated security auctions are nondiscriminatory, detail-free and invariant to the environment: no matter how bidders or their securities differ, the same mechanism is used. The design also shifts information burdens to the market and bidders have weakly dominant strategies so they need not know anything about rival bidders.

Gorbenko and Malenko (2018) develop a model of takeovers in cash auctions followed by the winner's choice of security mix subject to the "no default" constraint that the mix's value as assessed by the seller equals the winning price. They endogenize the auction timing and the winner's choice of the mix between cash and equity when cash is costly to use due to financing constraints. In our setting, the market rather than the seller evaluates the value of securities. This lets a seller circumvent discrimination, and it obviates a seller's need to be informed. Unlike existing mechanisms studied, our auction design has the property that a steeper set of securities always generate higher revenues than any flatter set, regardless of any heterogeneities in bidder characteristics or security types. In this way, our design also relates to work on robust auctions that obtain maximum revenue guarantees in worst case scenarios about unknown distributions of bidder valuations or bidder beliefs and equilibrium selection (Bergemann, Brooks, and Morris (2016, 2017, 2019), Brooks and Du (2021), Du (2018)). Our design's nondiscriminatory nature also makes it robust to moral hazard concerns, as it treats the cash bids of all bidders identically. In contrast, with discriminatory auctions, a biased designer, e.g., a firm's board, can favor its desired winner in possibly undetectable ways.

Our work contributes to research on the role of financial markets, especially that on how market feedback affects real outcomes and stock prices (Bagnoli and Lipman 1996; Bond, Edmans and Goldstein 2012; Bond and Goldstein 2015; Goldstein and Yang 2019; Banerjee, Breon-Drish and Smith 2025).[6] To our knowledge, we are the first to show how financial markets can be used to facilitate auction design. We exploit the fact that market

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[6] Our auction design with market feedback also relates to the two-stage mechanism structure of Mezzetti (2004) in which outcomes (allocations) are determined first, and then payments are determined, depending on the information revealed in the first stage. The simplicity of our design relates our paper to work on the performance of "simple contracts" relative to the optimal complex menu (e.g., McAfee, McMillan, and Reny (1989); Rogerson (2003) and Chu and Sappington (2007); Chu, Leslie and Sorensen (2011), Bose, Pal and Sappington (2011), Chassang (2013), and Carroll (2015)).

price of the winning bidder's securities *incorporates* the market's knowledge of the winner's specific characteristics, so the seller herself doesn't need to discriminate. Also, as the market responds to details ex post, the selling mechanism can be detail-free ex ante.[7]

2 The Model

An asset being auctioned has a standalone value of V_T if retained by the seller. There are n risk-neutral, possibly heterogeneous, bidders. Bidder i has a standalone value of $X_i > 0$. If bidder i acquires the asset, it yields a contractible stochastic payoff/cash flow of Z_i that equals the sum of the combined standalone values of the seller and bidder *plus* the synergies (value added) created by bidder i's control. Bidder i receives a private signal θ_i that is informative of Z_i: conditional on θ_i, the expected value of Z_i is

$$E(Z_i|\theta_i) = X_i + V_T + \theta_i. \tag{1}$$

Thus, the expected synergy that bidder i generates is $E(Z_i|\theta_i) - X_i - V_T = \theta_i$. We also refer to θ_i as bidder i's type. Conditional on θ_i, Z_i is distributed according to a pdf $h_i(\cdot|\theta_i)$ with full support on $[0, \infty)$, where the family $\{h_i(\cdot|\theta_i)\}$ has the *strict monotone likelihood ratio property (sMLRP)*: $h_i(z|\theta_i)/h_i(z|\theta_i')$ is increasing in z for $\theta_i > \theta_i'$, i.e., higher signals represent better news. Bidder i's type θ_i is distributed according to cdf $F_i(\cdot)$ with full support over $\left[\underline{\theta}_i, \overline{\theta}_i\right]$, $\overline{\theta}_i > \underline{\theta}_i \geq 0$, and types are independently distributed across bidders.

The winner pays the seller with a security from an *ordered set* whose elements are indexed by a parameter s. Let $S(s, z)$ denote the value of security s when the cash flow is z. For example, with equities, equity share s has value $S(s, z) = sz$; for debt, its face value s has value $S(s, z) = min\{s, z\}$. More generally, an ordered set of securities is a set $\mathcal{S} = \{S(s, \cdot) : s \in [\underline{s}, \overline{s}]\}$ such that (i) for all s, both $S(s, z)$ and $z - S(s, z)$ are nonnegative

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[7] More generally, our work suggests how settings in which outsiders/market participants observe a game can ease informational demands and reduce dependence on the specific details of the economic environment in implementing mechanisms. In such settings a mechanism designer chooses which information about agent actions to disclose and the forms of payoff functions. For example, in our model a seller only discloses the winning price. Our approach can also be used to examine the effects of other information disclosures as the advantages of dollar-denominated designs extend.

and weakly increasing in z; (ii) for any given bidder i, $ES^i(s,\theta) \equiv E(S(s,Z_i)|\theta_i = \theta)$, the *expected* value of security $S(s,\cdot)$ derived from cash flows generated by bidder i of type θ, is differentiable and strictly increasing in both s and θ; and (iii) there is sufficient range in the security index that $ES^i(\underline{s},\underline{\theta}_i) \leq \underline{\theta}_i + V_T$ and $ES^i(\overline{s},\overline{\theta}_i) > V_T + \overline{\theta}_i$ for all i.

We depart from existing theories on security auctions in two ways. First, we consider heterogeneous bidders, so $ES^i(s,\theta)$ will depend on bidder i's characteristics. For instance, with equities, equation (1) yields $ES^i(s,\theta) = s\,(X_i + V_T + \theta)$, which depends on bidder i's standalone value X_i. For non-linear securities such as debt, $ES^i(s,\theta)$ will also depend on the specific details of the distributions F_i and h_i. Second, we consider the possibility that bidders *pay with different types of securities.* For example, some bidders may use equity, while others use debt.

We use the widely-used notion of steepness introduced in DeMarzo, Kremer and Skrzypacz (2005) and used, e.g., by Malenko and Gorbenko 2011 and Sogo, Bernhardt and Liu 2016: an ordered set of securities $\mathcal{S}_A$ is *steeper* than $\mathcal{S}_B$ if for all $s_A \in [\underline{s}_A, \overline{s}_A]$, $s_B \in [\underline{s}_B, \overline{s}_B]$ and bidders i, $ES^i_A\,(s_A,\theta^*) = ES^i_B\,(s_B,\theta^*)$ implies that $\partial ES^i_A(s_A,\theta^*)/\partial\theta > \partial ES^i_B(s_B,\theta^*)/\partial\theta$. Thus, if a bidder i with type θ^* expects to pay the same amount with the two securities, then i expects to pay strictly more with the steeper security than with the flatter security when i has a higher type $\theta > \theta^*$—the payment of the steeper security is tied more tightly to a bidder's valuation. Thus, call options are steeper than equities, which are steeper than debt, which is steeper than cash.

Let $\mathcal{S}^i$ denote the ordered set of securities used by bidder i and let $\underline{s}_i$ and $\overline{s}_i$ denote the lower and upper bounds on feasible security parameters. We assume that $\mathcal{S}^i$ is determined exogenously prior to the auction. Alternatively, the seller could specify $\mathcal{S}^i$ prior to the auction.[8] Our auction also design also captures scenarios in which the winner finances its cash bid by issuing securities to the market after the auction—reflecting that

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[8] A seller would select the steepest set of feasible ordered securities for a given bidder. DeMarzo, Kremer and Skrzypacz (2005) provide reasons for why certain sets of securities may be infeasible, possibly for bidder-specific reasons (e.g., moral hazard). One can also show that a bidder i would prefer the flattest feasible type of security. In practice, bidders often have exogenous reasons for using different security types (e.g., institutional rigidities such as tax, bankruptcy risk, or leverage concerns can lead to the use of debt rather than equity or vice versa, Faccio and Masulis (2005)).

a bidder does not care whether the seller or market participants receive the securities.

Bidders maximize long-run (i.e., after all information is revealed) expected profits. If the asset is not sold, all agents receive zero profit. If the asset is sold, a losing bidder's profit is zero. When winner i pays with security s, expected seller revenue is $ES^i(s, \theta_i)$, i.e., expected seller profit is $ES^i(s, \theta_i) - V_T$, and i's expected profit is

$$\pi_i = V_T + \theta_i - ES^i(s, \theta_i), \tag{2}$$

where $V_T + \theta_i$ is the expected value of the asset under i's control.

2.1 Face-value-denominated security auctions

The literature on security auctions has studied what we term "*face-value-denominated security auctions*," in which each bidder i submits a bid $s_i \in [\underline{s}_i, \bar{s}_i]$, where s_i is a parameter from i's ordered set of securities, $\mathcal{S}^i$. The seller commits to an evaluation rule for selecting the winner and for determining the security parameter that the winner pays.

When bidders have the same characteristics and use the same type of security, standard first- or second-price formats are sensible ways to conduct face-value-denominated auctions. For example, in a second-price face-value-denominated equity auction, each bidder i submits a bid $s_i \in [0, 1]$, the highest bid wins, and the winner pays an equity share equal to the second-highest bid. Thus, if bidder i wins and the second-highest bid is s_j, the seller's revenue is $s_j\,(X_i + V_T + \theta_i)$.

With heterogeneous bidders, evaluation rules in face-value-denominated security auctions need discriminatory adjustments to account for the heterogeneities (see Hansen 1985; DeMarzo, Kremer and Skrzypacz 2005; Liu 2016), else the revenues could be less than those from cash auctions. There are multiple challenges with such adjustments. First, a seller must know the attributes of each bidder i (i.e., X_i, $F_i(\cdot)$, and $h_i(\cdot|\cdot)$). Second, even if the seller knows those details, the optimal adjustments are not even known when bidders with different characteristics (e.g., standalone values) pay with securities from different sets.[9] Indeed, when bidders pay with different securities, existing theories have yet to study how to conduct face-value-denominated security auctions, even with otherwise identical bidders. Third, the adjustments, whatever they are, are necessarily discriminatory, creating legal and moral hazard concerns. By contrast, the dollar-denominated auction design that we describe below circumvents all of these challenges.

2.2 Dollar-denominated security auctions

Information Structure. In our *dollar-denominated second-price security auctions*, the seller does not need to know about the characteristics of any bidder.

- To emphasize this point, we assume that the seller is totally uninformed about (X_i, F_i, and $h_i(\cdot|\cdot)$) for all i (results are unchanged if the seller is informed).
- The market only has to be informed about the winning bidder i's attributes (X_i, F_i, and $h_i(\cdot|\cdot)$) after the auction and it does not need to know about losing bidders.
- A bidder i knows his own characteristics (X_i, F_i, and $h_i(\cdot|\cdot)$), but does not need to know about other bidders (we show bidders employ weakly-dominant strategies).

Auction Protocol. Bidders submit *cash* bids, the highest cash bid wins and the price p paid by the winner equals the second-highest cash bid. The seller publicly announces p. However, rather than pay p with cash, the winner pays with a security whose post-auction monetary value, as determined by a competitive (efficient) capital market, equals p.

Specifically, after the auction, the market forms beliefs about winner i's type θ_i based on the winning cash price p, and bidder i's characteristics (X_i, F_i, and $h_i(\cdot|\cdot)$). Winner i pays with a security from $\mathcal{S}^i$, and the security parameter s paid (e.g., the share of equity) is such that the market's assessment of the security's value, based on the market's post-auction beliefs about the winner's type, equals the winning price p:

$$E\left[ES^i(s,\theta_i)|i \text{ wins at price } p;\, X_i, F_i, h_i\right] = p.^{10} \tag{3}$$

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[9] The optimal design is only known if all bidders pay with equities.

[10] For completeness of description, we assume that if the maximal security index $\bar{s}_i$ is too low, i.e.,

In both face-value-denominated and dollar-denominated security auction designs, bidders ultimately pay with securities from a pre-specified set. The difference is that face-value-denominated auctions are denominated in the face value of the security, but dollar-denominated security auctions are denominated in cash. This lets our dollar-denominated security design circumvent the issues detailed above for face-value-denominated designs. First, as the seller just selects the high cash bid, the design only relies on market investors collectively knowing the (typically actively-traded) *winning* bidder's characteristics. Second, our auction rules are simple, non-discriminatory and invariant to the environment. That is, the highest cash bid wins and the monetary value of the winner's security payment, as determined by the market, equals the second-highest cash bid, regardless of how bidder characteristics differ or the possibly different security types used.

2.3 Practical Relevance of Dollar-denominated Auctions

Our design is particularly germane for takeover auctions where bidders' equity or debt are actively traded. For example, with equity,[11] suppose that after observing bidder i win at a cash price of p, the market assesses the expected value of the merged firm at V_{merge},[12] then bidder i pays the seller an equity claim of $s = \frac{p}{V_{merge}}$ to the merged firm.

This dollar-denominated equity design corresponds to "collars" in equity payments that are often used in takeovers. With collars, bidders bid with cash but effectively pay with equities that have a guaranteed post-takeover market value—the equity payment adjusts according to the post-takeover market price of the bidder's stock—the shares paid increase if the bidder's post-takeover stock price falls, and vice versa.[13]

The literature has recognized that collars reduce the volatility in seller revenues, in-

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if the left-hand side of (3) is less than p even when $s = \bar{s}_i$, then the solution to (3) is $s = \bar{s}_i$ (i.e., the winner pays $\bar{s}_i$). We prove that this will not happen; Proposition 1 shows that $s \in [\underline{s}_i, \bar{s}^i)$ in equilibrium.

[11] Equity is commonly used in takeover settings: Andrade, Mitchell, and Stafford (2001) report that 58% of mergers and acquisitions are paid entirely in equity, and 70% involve at least some equity.

[12] V_{merge} is the sum of bidder i and the target's standalone values, plus the market's estimation of bidder i's synergy based on the winning price.

[13] Officer (2004) finds that two-thirds of collars guarantee a dollar value if a bidder's stock price stays within specified bounds around the effective merger date, and the other third details a constant exchange ratio over a range of bidder stock prices, with adjustment outside the bounds. Dollar-denominated equity offers correspond to an infinite range for the first type of collar and a zero range for the second.

sulating a seller from fluctuations in the winning bidder's stock price. We show that *even risk-neutral sellers* benefit from collars: (1) collars increase the *level* of seller revenue by inducing bidders to make more aggressive cash bids, (2) collars make revenues *less sensitive* to bidder heterogeneities, and (3) collars *relieve* sellers of the need to discriminate.

Dollar-denominated security auctions also correspond to auctions with external financing, where publicly-traded bidders bid and pay the winning price with cash, but finance the cash payment by issuing securities in the financial market after the auction. To see this, note that in dollar-denominated security auctions, the winner pays the seller a security, while in auctions with external financing, the winner sells a security to the market for the cash that the winner uses to pay the seller. Bidding strategies and seller revenues in the two settings are the same, as a bidder does not care who gets the security—the seller or the market. We formally establish this equivalence in online Appendix B.

Thus, our analysis captures the common practice of externally financing cash acquisitions with debt or equity issuance: the bidding strategies and the implications for seller revenues in dollar-denominated debt or equity auctions apply directly to cash auctions with debt or equity financing.[14] Our model also applies to *hybrid* settings, e.g., where some bidders pay with collared equities, others make cash bids that are financed with a debt issue,[15] and the remaining (cash rich) bidders make cash bids and pay with internal cash. The broad applicability of our model reflects that dollar-denominated security designs convert security bids to cash bids, allowing cash and security bids of any type to be evaluated *on the same footing.* Such settings with hybrid payment methods and heterogeneous bidders are widespread in corporate takeovers, but there have been almost no theoretical studies of them. We provide a tractable way to analyze such settings.

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[14] Martynova and Renneboog (2009) find that one-third of cash acquisitions are financed with security issues; 30% obtain equity finance and 70% use debt. See Vladimirov (2015) and Gorbenko and Malenko (2018) for theoretical analyses of takeovers when bidders need financing.

[15] In practice, bidders often have exogenous reasons for using different security types, e.g., institutional rigidities such as tax, bankruptcy risk, or leverage concerns can lead to the use of debt rather than equity or vice versa, Faccio and Masulis (2005). DeMarzo, Kremer and Skrzypacz (2005) provide reasons for why certain sets of securities may be infeasible, possibly for bidder-specific reasons (e.g., moral hazard).

3 Analysis of bidding with general securities

Denote bidder i's bidding strategy in the dollar-denominated security auction by $\beta_i(\theta_i)$. We focus on Perfect Bayesian Equilibria that satisfy two mild refinement conditions, under which $\beta_i(\theta_i)$ is uniquely determined.

Assumption 1: Bidders do not play weakly dominated strategies.

Given Assumption 1, any equilibrium bidding strategy must be weakly increasing:

Lemma 1 *Under Assumption 1, if $\theta_i^a > \theta_i^b$, then $\beta_i(\theta_i^a) \geq \beta_i(\theta_i^b)$.*

After the auction, the market forms beliefs about the winner's type based on the winning price and the winner's characteristics. For generality, we denote the market's belief about θ_i by a cdf $M(\theta_i; p, i)$ that has support over $\left[\underline{\theta}_i, \bar{\theta}_i\right]$. That is, given the winning price p, the market believes θ_i is distributed according to $M(\cdot; p, i)$.

In our setting, any $p \leq \beta_i(\bar{\theta}_i)$ is on the equilibrium path, while any higher p is off the equilibrium path (bidder i cannot win in equilibrium if the price exceeds its maximum possible bid of $\beta_i(\bar{\theta}_i)$). For any on-path price p, we require that market beliefs be consistent with i's strategy so that Bayes' rule holds: for any $p \leq \beta_i(\bar{\theta}_i)$, the market believes winner i's type θ_i is in the set $\{\theta_i : \beta_i(\theta_i) \geq p\}$. Because bidding strategies are weakly increasing (Lemma 1), market beliefs reduce to $\theta_i \geq \hat{\theta}_i(p)$, where $\hat{\theta}_i(p)$ is the smallest value of θ_i that satisfies $\beta_i(\theta_i) \geq p$. Thus, $M(\cdot; p, i)$ is given by the original distribution $F_i(\cdot)$ truncated at $\hat{\theta}_i(p)$. It follows that for any $p_1 > p_2$ on the equilibrium path, $M(\cdot; p_1, i)$ weakly first-order stochastically dominates $M(\cdot; p_2, i)$ (because $\hat{\theta}_i(p)$ weakly increases in p). We now impose this property off-the-equilibrium-path:

Assumption 2: Market beliefs about winner i's type weakly increase in the winning price: for $p_1 > p_2$, $M(\cdot; p_1, i)$ weakly first-order stochastically dominates $M(\cdot; p_2, i)$.

Under Assumptions 1 and 2, there is a unique equilibrium and bidding strategies are strictly increasing:

Proposition 1 *Under Assumptions 1 and 2, bidding strategies are uniquely determined. Bidder i's bidding strategy is strictly increasing, given by*

$$\beta_i^{dd}(\theta_i) = E_{\theta;i}[ES^i(s(\theta_i),\theta)|\theta \geq \theta_{i}], \tag{4}$$

where $E_{\theta;i}$ denotes the expectation over θ (where $\theta \sim F_i(\cdot)$), and $s(\theta_i)$ solves

$$ES^i(s,\theta_{i}) = \theta_i + V_T, \tag{5}$$

which has a unique solution for $s \in [\underline{s}_i, \bar{s}^i)$. Using $(\beta_i^{dd})^{-1}(\cdot)$ to denote the inverse function, market beliefs about winner i's type given the winning price p are given by

$$\begin{cases} \theta_i = \bar{\theta}_i & \text{if} \quad p > \beta_i^{dd}(\bar{\theta}_i) \\ \theta_i \geq (\beta_i^{dd})^{-1}(p) & \text{if} \quad p \in \left[\beta_i^{dd}(\underline{\theta}_i), \beta_i^{dd}(\bar{\theta}_i)\right] \\ \theta_i \geq \underline{\theta}_i & \text{if} \quad p < \beta_i^{dd}(\underline{\theta}_i) \end{cases} . \tag{6}$$

Proof: See the appendix. □

To convey the intuition, we provide a heuristic derivation of the bidding strategy β_i, where we assume that β_i is strictly increasing; the appendix provides a complete proof without this assumption. By standard reasoning, the bidding strategy in second-price auctions leaves a bidder indifferent between winning and losing at its bid. That is, if bidder i barely wins, i.e., if i's bid equals the highest losing bid (which is the winning price p), i's expected profit is zero. Let s_{bare} be the index of the security that bidder i of type θ_i pays when i barely wins (so that $p = \beta_i^{dd}(\theta_i)$). Then s_{bare} solves (3), with p replacing $\beta_i^{dd}(\theta_i)$ on both sides of (3). Then equation (3) becomes

$$E_{\theta;i}[ES^i(s_{bare},\theta)|\beta_i^{dd}(\theta) \geq \beta_i^{dd}(\theta_i)] = \beta_i^{dd}(\theta_i). \tag{7}$$

Because a strictly-increasing bidding strategy is invertible, the condition $\beta_i^{dd}(\theta) \geq \beta_i^{dd}(\theta_i)$ reduces to $\theta \geq \theta_i$, hence (7) simplifies to

$$E_{\theta;i}[ES^i(s_{bare},\theta)|\theta \geq \theta_i] = \beta_i^{dd}(\theta_i). \tag{8}$$

The break-even condition pins down s_{bare}: setting winner i's expected profit (the right-hand side of (2)) to zero yields

$$V_T + \theta_i - ES^i(s_{bare}, \theta_i) = 0. \tag{9}$$

Equation (9) implies that s_{bare} is a function of θ_i. Relabeling s_{bare} as s, equations (8) and (9) become (4) and (5) in Proposition 1.

Illustration of equity bidding formulation. Suppose bidder i's expected synergy θ_i is uniformly distributed on $\left[0, \bar{\theta}\right]$. In a dollar-denominated auction, the bidding strategy is determined by (4) and (5). With equity payments,

$$ES^i(s, \theta_i) = s(X_i + V_T + \theta_i). \tag{10}$$

Combining (10) with (5) yields

$$s(X_i + V_T + \theta_i) = V_T + \theta_i,$$

which we solve for $s(\theta_i) = \frac{V_T+\theta_i}{X_i+V_T+\theta_i}$. Substituting this back into (4) yields

$$ES^i(s\left(\theta_i\right), \theta) = \frac{V_T + \theta_i}{X_i + V_T + \theta_i}(X_i + V_T + \theta). \tag{11}$$

As $\mathrm{E}[\theta|\theta \geq \theta_i] = 0.5\left(\bar{\theta} + \theta_i\right)$, (4) yields the bidding strategy

$$\beta_i^{dd}(\theta_i) = \frac{V_T + \theta_i}{X_i + V_T + \theta_i}(X_i + V_T + 0.5\left(\bar{\theta} + \theta_i\right)) = (V_T + \theta_i)\left(1 + \frac{0.5(\bar{\theta} - \theta_i)}{X_i + V_T + \theta_i}\right). \tag{12}$$

Example 1. Suppose bidder i has a standalone value of 400 with an expected synergy θ_i that is uniformly distributed over $[0, 440]$. It participates in a dollar-denominated *equity* auction and wins at a cash price of \$200. Based on the winning price, what is the market's post-takeover estimate of the bidder's expected synergy? By (12), i's bidding strategy is:

$$\beta_i^{dd}(\theta_i) = \theta_i\left(1 + \frac{0.5(440 - \theta_i)}{400 + \theta_i}\right).$$

The winning price reveals that $\beta_i^{dd}(\theta_i) \geq 200$, or $\theta_i\left(1 + \frac{0.5(440-\theta_i)}{400+\theta_i}\right) \geq 200$, yielding

$\theta_i \geq 160$. Thus, the market's post-auction belief is that θ_i is uniformly distributed over $[160, 440]$, yielding an expected synergy of 300. Thus, the merged firm has a market value of \$300+\$400=\$700, so bidder i pays the target a share of \$200/\$700=2/7 of the merged firm's equity in lieu of cash (as \$700*2/7=\$200).

Bidder i can implement this equity payment by issuing new shares. For instance, if bidder i has one share outstanding and issues N shares after the auction to pay the target, then N equals the winning price of \$200 divided by i's post-takeover stock price p_{post}. To calculate p_{post}, note that acquiring the target yields bidder i an expected NPV gain of \$100 (\$300 expected synergy minus the \$200 cash price). After winning, prior to issuing new shares, the market value of i's one share was $p_{post} = \$500$ (standalone value plus expected NPV). After issuing the N shares, bidder i's stock price remains at \$500 (reflecting market efficiency), so i must give the target $N = 200/500 = 0.4$ new shares.[16] Thus, the target receives a $\frac{0.4}{1+0.4} = \frac{2}{7}$ share of the merged firm, which has value \$700. $\square$

Even though a winner pays with a security in the dollar-denominated security design, a seller's expected revenue is exactly the same as if the bidder paid the winning cash price. This reflects the law of iterated expectations and the fact that the expected value of the security payment from the market's perspective equals the cash price. Thus, the expected revenue in the dollar-denominated security design is the same as in a *hypothetical* cash auction in which bidders bid according to Proposition 1. However, dollar-denominated security designs induce more aggressive bidding than second-price cash auctions—the cash bids exceed bidders' true values, as $E_{\theta;i}[ES^i(s,\theta)|\theta \geq \theta_i] \geq ES^i(s,\theta_i)$. Thus, $\beta_i^{dd}(\theta_i) \geq \theta_i + V_T$, and this inequality is strict for all $\theta_i < \bar{\theta}_i$. By way of comparison, bidders in cash auctions bid their actual values:

$$\beta_i^{cash}(\theta_i) = \theta_i + V_T.$$

Thus,

$$\beta_i^{dd}(\theta_i) > \beta_i^{cash}(\theta_i), \text{ for all } \theta_i < \bar{\theta}_i. \tag{13}$$

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[16] Equivalently, one can solve the system of equations, $N = \frac{200}{p_{post}}$ and $p_{post} = \frac{400+300}{1+N}$, for N and p_{post}.

The more aggressive bidding in our auction design reflects that bidder i's strategy in a second-price auction is determined by outcomes at the point where i barely wins, i.e., where its bid equals the highest losing bid. Crucially, at this point the market does not know that i is about to lose, inferring only that i has some type $\theta \geq \theta_i$. This means that when i barely wins, the market groups it with higher types—so i pays a lower security—incentivizing i to bid above its true valuation. The proof of Proposition 1 also establishes that bidders employ weakly-dominant strategies; indeed, as (4) and (5) make clear, i's bidding strategy β_i^{dd} only depends on i's characteristics, and not those of other bidders. Thus, a bidder does not need to know anything about rival bidders.

Dollar-denominated security auctions are nondiscriminatory by construction. We next examine their revenues and the sensitivities of their revenues to bidder heterogeneities, and contrast them with those of nondiscriminatory face-value-denominated security auctions (henceforth "face-value-denominated security auctions"). To begin, we relate the bidding strategies in the two designs.

In a second-price face-value-denominated auction where bidders pay with the same type of securities, bidding strategies are given by (5): a bidder breaks even when she pays her bid. Let $s_i^{fvd}(\theta_i)$ be the bid of bidder i with type θ_i. Then,

$$ES^i(s_i^{fvd}(\theta_i), \theta_i) = \theta_i + V_T. \tag{14}$$

For example, with equities, (10) yields

$$ES^i(s_i^{fvd}(\theta_i), \theta_i) = s_i^{fvd}(\theta_i) s(X_i + V_T + \theta_i),$$

which, upon plugging into the left-hand side of (14), yields

$$s_i^{fvd}(\theta_i) = \frac{V_T + \theta_i}{X_i + \theta_i + V_T}. \tag{15}$$

This is the well-known second-price equity auction bidding strategy (Hansen (1985)).

Inspection of equations (14), (4) and (5) reveals the relation between bidding strategies in dollar-denominated and face-value-denominated security auction designs:

$$\beta_i^{dd}(\theta_i) = E_{\theta;i}[ES^i(s_i^{fvd}(\theta_i), \theta)|\theta \geq \theta_i]. \qquad (16)$$

Before comparing the robustness of the two auction designs to bidder heterogeneities, we consider a benchmark setting of ex-ante identical bidders who use the same set of securities.

Definition 1 *Bidders are ex-ante identical if and only if* $X_i = X_j$*,* $F_i = F_j$*,* $h_i(\cdot|\theta) = h_j(\cdot|\theta)$*, and* $\mathcal{S}^i = \mathcal{S}^j$ *for all* i, j*, and* θ*.*

Proposition 2 *With ex-ante identical bidders and a common ordered set of securities, dollar-denominated and face-value-denominated second-price security auctions yield the same expected revenue.*

Proof: Let θ_1 and θ_2 denote the highest and second highest signals, respectively. With ex-ante identical bidders, we suppress the i index in ES^i and $E_{\theta;i}$ and simply write ES and E_θ. As discusssed above, in face-value-denominated auctions, bidding strategies are given by (5): a bidder in face-value-denominated auctions breaks even when she pays that bid. Let $s^{fvd}(\theta_2)$ be the bid by type θ_2: $s^{fvd}(\theta_2)$ is the value of s that solves (5) (or (14)) with θ_i replaced by θ_2. Hence the expected value of the winner's payment is $ES(s^{fvd}(\theta_2), \theta_1)$. Thus, conditional on the second-highest signal being θ_2 (and $\theta_1 \geq \theta_2$), expected revenue the face-value denominated auction is $E_{\theta_1}[ES(s^{fvd}(\theta_2), \theta_1)|\theta_1 \geq \theta_2]$. In the dollar-denominated auction, expected revenue equals bidder 2's cash bid. By equation (16), this cash bid is $E_\theta[ES^i(s^{fvd}(\theta_2), \theta)|\theta \geq \theta_2]$, which equals $E_{\theta_1}[ES(s^{fvd}(\theta_2), \theta_1)|\theta_1 \geq \theta_2]$ (expected revenue in the face-value denominated auction conditional on the second-highest signal being θ_2). By the law of iterated expectations, the proposition follows. □

Example 2: Revenue equivalence with ex-ante identical bidders and equity. Suppose $V_T = 0$ and there are two bidders with expected synergies distributed i.i.d., uniformly on $[0, 1]$. In a face-value-denominated equity auction, substituting $V_T = 0$ into

(15) yields that a type θ_i bidder i, i=1, 2, has a weakly dominant strategy to bid the share

$$s_i^{fvd}(\theta_i) = \frac{\theta_i}{X+\theta_i}$$

that breaks even when i pays its bid. As $X \to 0$, both bidders bid close to 100%, but because X is not quite zero, the higher-type bids slightly more, winning the auction. Expected seller revenue is $\mathrm{E}[\max\{\theta_1, \theta_2\} \times 100\%] = \frac{2}{3}$, where $\max\{\theta_1, \theta_2\}$ is the value of the merged firm (as the higher type bidder wins), and 100% is the losing bid.

In a dollar-denominated equity auction, substituting $V_T = 0$ into (12) yields i's bid:

$$\beta_i^{dd}(\theta_i) = \theta_i\left(1 + \frac{0.5(1-\theta_i)}{X+\theta_i}\right).$$

As X goes to zero, $\beta_i^{dd}(\theta_i)$ reduces to $\frac{1}{2}(1+\theta_i)$ for all $\theta_i \in (0,1]$. As $\frac{1}{2}(1+\theta_i)$ is strictly increasing in θ_i, the higher-type bidder again wins and pays the lower-type's bid of $\frac{1}{2}(1+\min\{\theta_1,\theta_2\})$, yielding expected seller revenue of $\frac{1}{2}(1+E[\min\{\theta_1,\theta_2\}]) = \frac{1+1/3}{2} = \frac{2}{3}$, as in the face-value-denominated auction. $\square$

With heterogeneous bidders, i.e., with bidders who are not ex-ante identical, revenue equivalence breaks down. Proposition 2's proof suggests why: it uses the fact that with ex-ante identical bidders, (i) the winner is the same (highest-signal bidder) in both auction designs, and (ii) the expectations of the synergies $E_{\theta;i}$ and security values ES^i do not vary with a bidder's identity i. With heterogeneous bidders, these features do not hold.

Example 2 (continued): Equivalence breakdown with heterogeneous bidders. Suppose both X_1 and X_2 go to zero, but they do so at different rates, with X_1 declining quadratically and X_2 declining linearly, so that X_1/X_2 also goes to zero. Then, in a face-value-denominated equity auction, while both bidders' equity offers approach 100%, for $\theta_1 > 0$, bidder 1's offer is always slightly larger even when $\theta_1 < \theta_2$.[17] Thus, bidder 1

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[17] To see this, note that

$$\frac{s_2^{fvd}(\theta_2)}{s_1^{fvd}(\theta_1)} = \frac{\frac{\theta_2}{X_2+1}}{\frac{\theta_1}{X_1+\theta_1}} \le \frac{\frac{1}{X_2+1}}{\frac{\theta_1}{X_1+\theta_1}} = \frac{X_1+\theta_1}{\theta_1(X_2+1)} = \frac{\frac{X_1}{\theta_1}+1}{X_2+1},$$

where the first inequality follows from $\theta_2 \le 1$. The right-hand side, $\frac{\frac{X_1}{\theta_1}+1}{X_2+1}$, is strictly less than 1 when wins with probability one, and pays 100% (bidder 2's bid) of the combined firm. Hence, expected seller revenue is only $\mathrm{E}[\theta_1 \times 100\%] = \frac{1}{2}$.

By contrast, bidding strategies in the dollar-denominated equity auction remain separating: substituting $V_T = 0$ into (12) yields $\lim_{X_i \to 0} \beta_i^{dd}(\theta_i) = \frac{1+\theta_i}{2}$, which strictly increases in θ_i. Thus, the higher-type bidder still wins, regardless of differences in the rates at which standalone values approach zero, so expected seller revenue remains $\frac{2}{3}$, corresponding to full rent extraction, just as when bidders are ex ante identical. Moreover, when there are more than two bidders, the extent of the breakdown of revenue equivalence widens: if the standalone value of one bidder goes to zero much faster than those of other bidders, expected revenue in a face-value-denominated auction remains $\frac{1}{2}$ but revenue in a dollar-denominated security auction rises past $\frac{2}{3}$. $\square$

As the example illustrates, this breakdown of revenue equivalence turns out to be desirable. With ex-ante heterogeneous bidders, face-value-denominated auction designs need discriminatory adjustments, else they could generate even lower revenues than cash auctions. In contrast, our dollar-denominated security design remains nondiscriminatory, yet, as we will show, it extends the insights of DeMarzo, Kremer and Skrzypacz (2005) regarding the advantages of securities over cash (and steeper securities over flatter ones) to settings with heterogeneous bidders.

To show this, we consider a setting with general securities and standalone values that need not be small. For general securities, a security's value depends on the distribution of cash flows conditional on a bidder type, $h_i(\cdot|\theta_i)$. To ease analysis, we assume that the cash flow distribution scales with the total valuation:

Assumption 3: For a bidder i, the final cash flow is given by $\tau_i(\theta_i + X_i + V_T)$, where τ_i is an independently-distributed random variable with support $(0, \infty)$ and a mean of 1.[18]

We next provide sufficient conditions on bidder heterogeneities for nondiscriminatory face-value-denominated security auctions to generate lower revenues than cash auctions,

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$\frac{X_1}{X_2} < \theta_1$. Thus, as $\frac{X_1}{X_2}$ approaches zero, $s_1^{fvd}(\theta_1) > s_2^{fvd}(\theta_2)$ even if $\theta_1 < \theta_2$.

[18] Our results would not change if τ_i were correlated across bidders. This is because at the time of bidding, bidder i only observes θ_i; τ_i is only realized after bidder i wins, and only one bidder can win.

with steeper securities generating *less* than flatter securities:

Lemma 2 *Consider two bidders 1 and 2 with identical synergy distributions on support* $\left[\underline{\theta}, \overline{\theta}\right]$. *Suppose the difference in bidder standalone values is large enough that*

$$X_2 - X_1 > \max\left\{\frac{\bar{\theta} - \underline{\theta}}{\underline{\theta} + V_T} X_1, \bar{\theta} - \underline{\theta}, \frac{E\left[\theta\right] - \underline{\theta}}{\underline{\theta} + V_T}\left(X_1 + E\left[\theta\right] + V_T\right)\right\}.$$

Then, under Assumption 3, for nondiscriminatory face-value-denominated auctions,

(i) equity generates less revenues than cash.

(ii) steeper-than-equity securities generate less revenue than equities, and the revenue strictly decreases as securities grow even steeper.

Proof: See the appendix. □

Unlike face-value-denominated security auctions, dollar-denominated security auctions are robust to bidder heterogeneities even if bidders differ arbitrarily in their characteristics and use different types of securities. We next formalize this robustness result. As dollar-denominated auctions accommodate settings where different bidders pay with different types of securities, we extend the notion of steepness to a *profile* of security sets:

Definition 2 *Let* $\{\mathcal{S}^i\}_{i=1}^n$ *be the profile of ordered sets of securities used by the bidders. Profile A is steeper than B if* $\mathcal{S}_A^j$ *is weakly steeper than* $\mathcal{S}_B^j$ *for all* $j \in \{1, ..., n\}$, *with strict inequality for at least one* j.

For example, a profile of security sets is steeper if at least one bidder switches to using a steeper set of securities, while all other bidders use the same sets of securities as before.

Theorem 1 *Let profile A of ordered sets of securities be steeper than profile B. Then, regardless of how bidder characteristics differ, expected revenue in a non-discriminatory dollar-denominated security auction is higher with profile A than profile B.*

Proof: See the appendix. □

The key to the proof is a lemma (Lemma 4 in the appendix) establishing that if the ordered set of securities $\mathcal{S}_A^i$ is steeper than $\mathcal{S}_B^i$, then bidder i bids more aggressively in a dollar-denominated security auction if he pays with $\mathcal{S}_A^i$ than with $\mathcal{S}_B^i$. This result reflects that dollar-denominated security payments induce signaling incentives, with steeper securities inducing greater signaling incentives. The economic intuition is that the monetary value of steeper securities is more sensitive to a bidder's type. Consequently, with steeper securities, the value of being grouped with higher types is greater, inducing bidders to make more aggressive cash bids.

In appearance, this outcome may seem similar to the classical result for face-value-denominated security auctions that, with steeper securities, bids are more aggressive. However, there is a key difference. In face–value-denominated designs, the more aggressive bids are in the face value of a security, and, as Lemma 2 illustrated, a higher face value bid may have a lower monetary value as the higher bid may come from an incentivized (small) bidder with a lower valuation. In contrast, in dollar-denominated security auctions, the more aggressive bids are in *cash*, and more aggressive cash bids from *any bidder* always benefit a seller—no matter how bidders differ, more cash is always good.

As one can view cash as the flattest type of security, an immediate corollary of Theorem 1 is that seller revenues in dollar-denominated security auctions exceed those in cash auctions. More precisely, from the bidding strategy in (4) and (13), it follows that

$$\beta_i^{dd}(\theta_i) > V_T + \theta_i, \quad \text{for all } \theta_i < \bar{\theta}_i. \tag{17}$$

That is, bidders bid *more than* their true valuations for the asset, and hence seller revenue always exceeds that in a standard cash auction in which bidders bid their true valuations.

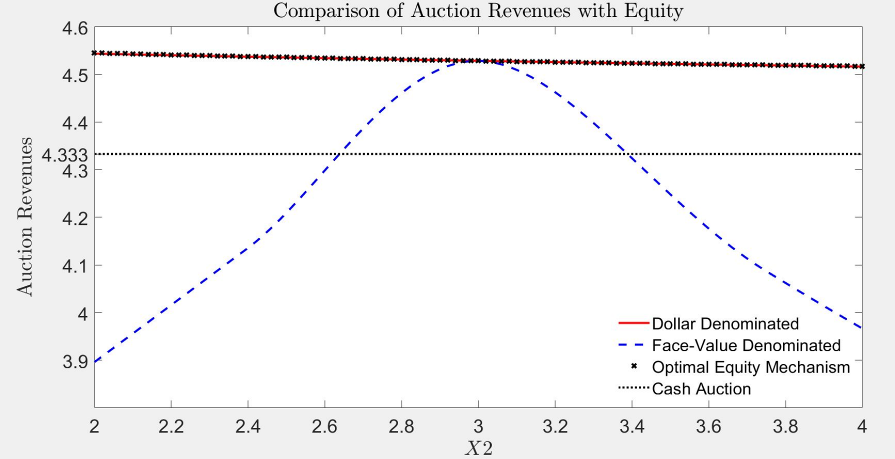
More generally, Theorem 1 establishes that dollar-denominated security auctions have the property that a *steeper set of securities always generate higher revenues than any flatter set of securities*, regardless of how bidders differ ex ante. In practice, bidder heterogeneity is widespread, and the literature has made little progress on the design of security auctions with heterogeneous bidders. In this regard, Theorem 1 *represents an* *advance*: it shows how security auctions can be designed in a non-discriminatory fashion to generate higher revenues than standard cash auctions, with steeper sets of securities generating still more no matter how bidders differ in their characteristics or types of securities they use. To the best of our knowledge, no existing mechanism has this property.

3.1 Numerical Illustration and Additional Properties of Dollar-Denominated Security Auctions

We next numerically illustrate Theorem 1. We then identify further advantages of dollar-denominated security designs.

Example 3: The target's standalone value $V_T = 3$. There are two bidders 1 and 2. Bidder $i = 1, 2$'s expected synergy θ_i is distributed i.i.d., uniformly on $[1, 2]$. Bidder 1's standalone value is $X_1 = 3$. We vary X_2 and contrast revenues in dollar-denominated equity auctions, nondiscriminatory face-value-denominated equity auctions, cash auctions, and optimally-designed equity auctions (that require complicated and discriminatory adjustments for bidder heterogeneities; Liu 2016).

Figure 1: Revenues for different auction designs



Notes: The figure plots expected revenues in dollar-denominated equity auctions, nondiscriminatory face-value-denominated equity auctions, optimal equity auctions and cash auctions as a function of bidder 2's standalone value. Bidder 1 and the target both have standalone values of 3. Each bidder's expected synergy is drawn from a uniform distribution on $[1, 2]$.

Figure 1 illustrates that dollar-denominated and face-value-denominated designs generate the same expected revenue *only* when $X_1 = X_2 = 3$ so that the two bidders are ex ante identical, in which case our revenue equivalence result, Proposition 2, holds and expected revenues exceed those from cash auctions. Once X_2 differs from X_1 by more than small amounts, revenues in nondiscriminatory face-value-denominated design decrease sharply, falling below those from cash auctions, consistent with Lemma 2. By contrast, revenues in dollar-denominated auctions are insensitive to differences between X_2 and X_1: revenues vary monotonically and smoothly with X_2, and revenues always exceed those from cash auctions. Indeed, Figure 1 shows that revenues from the dollar-denominated equity auction are remarkably close to those from the *optimal* equity mechanism: almost no differences can be discerned visually even when X_1 and X_2 differ substantially. This closeness obtains even though the optimal equity mechanism employs complicated and discriminatory adjustments for bidder heterogeneities, whereas our dollar-denominated equity design remains nondiscriminatory. We return to this closeness in Proposition 4. $\square$

As Lemma 2 establishes and Figure 1 illustrates, in a nondiscriminatory face-value-denominated auction, reducing only one bidder's standalone value sufficiently causes revenues to fall below those in cash auctions. We now prove that, in sharp contrast, reducing *any* bidder's standalone value in a nondiscriminatory dollar-denominated security auction *always increases* seller revenue:

Proposition 3 *Suppose a bidder i uses equity, call options or debt securities. Then, under Assumption 3, in a dollar-denominated security auction, reductions in i's standalone value (fixing those of other bidders) lead to strictly greater expected seller revenue.*

Proof: See the appendix.

The intuition for Proposition 3 is that in dollar-denominated security auctions, a bidder with a smaller standalone value pays a larger fraction of its cash flows upon winning. In turn, this raises the value of being perceived by the market as a higher type, increasing the bidder's signaling incentives, and hence its cash bid.

Proposition 3 underscores the advantages of dollar-denominated designs. We next illustrate quantitatively how the standalone values and the steepness of securities affect revenues when bidders can have different standalone values and use different types of securities.

Example 3 (continued): To extend Example 3 to securities other than equity, one must fully specify the cash flow process. Accordingly, we assume that the final cash flow is distributed according to $\tau_i(\theta_i + X_i + V_T)$, where τ_i is a lognormal distribution with a mean of 1 and standard deviation of 0.5.

Table 1 reveals how four types of securities (cash, debt, equity, and call) affect revenues in dollar-denominated security auctions for different combinations of X_1 and X_2. First, consistent with Theorem 1, it illustrates that steeper securities always generate more revenues, even with heterogeneous bidders. Thus, call options generate the highest revenues, and cash generates the lowest. Second, consistent with Proposition 3, revenues decrease monotonically in the standalone value X_i of any single bidder i.

Table 1: Revenues in dollar-denominated auctions when bidders use the same security

Market Capitalization	Cash	Debt	Equity	Call
$X_1 = 3, X_2 = 3$	4.33	4.36	4.53	4.66
$X_1 = 3, X_2 = 4$	4.33	4.36	4.52	4.66
$X_1 = 3, X_2 = 6$	4.33	4.35	4.50	4.65
$X_1 = 6, X_2 = 6$	4.33	4.34	4.47	4.63

Table 2 shows how the steepness of securities affects revenues in dollar-denominated auctions when heterogeneous bidders ($X_1 = 3,\ X_2 = 6$) use *different* types of securities. In line with Theorem 1 and Proposition 3, seller revenues rise if any subset of bidders switches to using a steeper set of securities or the standalone value of any bidder falls.

More subtly, Table 2 shows that the size of the increase depends on which bidder switches to a steeper security and the steepness of the two securities. When switching from cash to debt, revenues increase by more if the *smaller* bidder switches to debt

Table 2: Revenues in dollar-denominated auctions when bidders use different securities

Securities Used	Expected Revenues	Securities Used	Expected Revenues
($\mathcal{S}_1$=Cash, $\mathcal{S}_2$=Cash)	4.333		
($\mathcal{S}_1$=Cash, $\mathcal{S}_2$=Debt)	4.336	($\mathcal{S}_1$=Debt, $\mathcal{S}_2$=Cash)	4.348
($\mathcal{S}_1$=Debt, $\mathcal{S}_2$=Debt)	4.351		
($\mathcal{S}_1$=Equity, $\mathcal{S}_2$=Debt)	4.42	($\mathcal{S}_1$=Debt, $\mathcal{S}_2$=Equity)	4.41
($\mathcal{S}_1$=Equity, $\mathcal{S}_2$=Equity)	4.50		
($\mathcal{S}_1$=Equity, $\mathcal{S}_2$=Call)	4.58	($\mathcal{S}_1$=Call, $\mathcal{S}_2$=Equity)	4.55
($\mathcal{S}_1$=Call, $\mathcal{S}_2$=Call)	4.65		

Notes: Market capitalizations of the two bidders are $(X_1 = 3, X_2 = 6)$.

than if the larger bidder does. Conversely, when switching from equity to call options, revenues increase by more if the *larger* bidder switches. The intuition for this difference is that with a flat security, a bidder's information advantage is insensitive to her standalone value. In particular, with cash, standalone values have no effect on a bidder's information advantage. Payments with securities reduce a bidder's information advantage by more when her standalone values are smaller. Thus, with a switch from cash to a steeper security, the reduction in a bidder's information advantage is greatest when her standalone value is smaller. When, instead the security used is very steep, a bidder's informational advantage will already be close to zero unless her standalone value is large. As a result, with a switch to an even steeper security, the increase in seller revenues will be small unless the bidder's standalone value is large. $\square$

Collectively, the results in Theorem 1, Proposition 3 and Example 3 highlight the robustness of dollar-denominated security designs to bidder heterogeneity: revenues exhibit a monotonicity property *with respect to any single bidder, both in terms of the bidder's standalone value and the type of security used.* In particular, revenues when bidders have different standalone values and use different types of securities *always ex-*

ceed those in a homogeneous setting in which bidders have the same largest standalone value and use the same flattest set of securities.

To our knowledge, the results in Tables 1 and 2 comprise the first to be reported for security auctions where *heterogeneous bidders pay with different securities*. They highlight how dollar-denominated designs can be used to model and analyze heterogeneous/hybrid settings, which arise frequently in M&A and takeovers where the prices are often reported in dollars but the payments are in securities, and different bidders may pay with different securities.

The results in Figure 1 hint that even with heterogeneous bidders, revenues in nondiscriminatory dollar-denominated equity auctions are close to those in an optimal equity mechanism. We conclude analysis by identifying conditions under which revenues from a dollar-denominated auction approach the theoretical maximum that no any mechanism can surpass. We prove that if bidder standalone values are negligible relative to the sum of a seller's standalone value and expected synergies then nondiscriminatory dollar-denominated security auctions extract full rents, achieving the first-best outcome that no mechanism can beat. This result holds even if (1) the standalone values of different bidders go to zero at different rates, (2) the common distribution for expected synergies is arbitrary, and (3) different bidders use different types of securities (not just equity).

Proposition 4 *Suppose expected synergies of all bidders are drawn from a common distribution $F(\cdot)$, conditional cash flow distributions follow Assumption 3, and different bidders use possibly different types of ordered securities. Then, when X_i goes to zero for all i, possibly at different rates, nondiscriminatory dollar-denominated security auctions extract full rents. In contrast, nondiscriminatory face-value-denominated security auctions do not extract full rents unless all X_i go to zero at the same rate.*

Proof: We first show that for any security set $\mathcal{S}^i$ that bidder i uses, the solution s to (5) approaches the "full security" (a security that pays $S(z) = z$ for all z, i.e., paying out all cash flows) as X_i goes to 0. This follows since, under Assumption 3, for any given s, the left-hand-side of (5) decreases as X_i decreases. To maintain equality, the solution

s to (5) must increase as X_i decreases, and hence must reach a limit as X_i goes to 0. If this limit did not correspond to a full security, then as X_i gets arbitrarily close to 0, the left-hand side would be strictly less than the right-hand side, a contradiction of the finding in the proof of Proposition 1 that (5) has a solution.

Next, observe that when X_i goes to zero and s is a full security, the term $ES^i(s, \theta)$ on the right-hand side of (4) becomes $V_T + \theta$, for any security set $\mathcal{S}^i$. Hence (4) yields

$$\lim_{X_i \to 0} \beta_i^{dd}(\theta_i) = E_\theta[V_T + \theta | \theta \geq \theta_i] = V_T + E_\theta[\theta | \theta \geq \theta_i], \tag{18}$$

where E_θ is the expectation over $\theta \sim F(\cdot)$. Because the right-hand side of (18) strictly increases in θ_i, the highest-type bidder bids the most and wins. Expected seller profit is the expected value of the second-highest (cash) bid, yielding

$$\lim_{X_i \to 0 \text{ for all } i} \pi_s^{dd} = E\left[E_\theta[\theta | \theta \geq Y_2]\right], \tag{19}$$

where Y_2 denotes the second-highest value among the n random draws $\{\theta_1, \theta_2, ..., \theta_n\}$, and E is the expectation over the realizations of these n random draws. Further,

$$E_\theta[\theta | \theta \geq Y_2] = E[Y_1 | Y_2],$$

where Y_1 is the highest of the n random draws $\{\theta_1, \theta_2, ..., \theta_n\}$, E is the expectation over the realizations of $\{\theta_1, \theta_2, ..., \theta_n\}$ (which differs from E_θ), and $E[Y_1 | Y_2]$ is the expected value of Y_1 conditional on a given value of Y_2. Plugging this into (19) yields

$$\lim_{X_i \to 0 \text{ for all } i} \pi_s^{dd} = E\left[E[Y_1 | Y_2]\right] = E\left[Y_1\right].$$

The final equality follows from the law of iterated expectations. As Y_1 is the highest NPV from all bidders, the dollar-denominated security auction extracts the full NPV. □

When the rates at which X_i's go to zero vary with i, face-value-denominated auctions must use discriminatory adjustments to select the right winner while nondiscriminatory dollar-denominated security auctions always extract full rents. This difference reflects that when X_i goes to 0, the bidding strategy in nondiscriminatory face-value-denominated auctions approaches pooling on the full security, and, because bids are insensitive to bidder types, differences in standalone values determine winner selection. In contrast, bidding strategies in the analogous dollar-denominated auction remain fully separating, strictly increasing in bidder type (see equation (18)) so the highest bidder wins. So, too, face-value-denominated auctions need adjustments if bidders pay with different types of securities even when bidders have the same standalone value,[19] whereas dollar-denominated security auctions do not.

4 Conclusions

A serious drawback of optimal mechanisms with heterogeneous bidders is that they invariably require complex discriminatory winner selection and payment rules, and they impose strong information demands on the auctioneer, as the optimal design is sensitive to each bidder's features. These concerns especially manifest themselves in security auctions, where the monetary values of bids depend on the fine details of the underlying cash flow distributions, and nondiscriminatory designs risk selecting the wrong winning bidder and generating lower revenues than cash auctions.

We identify a simple mechanism that imposes no information burdens on the seller and is invariant to the environment, yet generates high revenues no matter how bidders differ in their attributes or securities used. In our second-price dollar-denominated security auction, bidders submit cash bids, the high bid wins, and the winner pays with securities priced in a competitive capital market that sees the winning price. That is, bidders offer securities with guaranteed cash equivalent values, where the security parameter paid equates the security's expected value to the winning cash price. The design:

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[19] For instance, consider three bidders with the same standalone value, $X_1 = X_2 = X_3 \equiv X$, but bidder 1 offers debt, bidder 2 offers equity and bidder 3 offers call options. As X goes to zero, then regardless of their types, bidder 1's offer of the face value of debt approaches infinity, bidder 2's offer of equity fraction approaches one, and the strike price in bidder 3's offer approaches zero. This means that winner selection requires delicate adjustments, which will depend sensitively on the precise value of X and the details of the conditional cash flow distributions $h(\cdot|\cdot)$.

1. Is detail-free and nondiscriminatory, removing legal and moral hazard concerns about discriminatory auctions. In takeovers, our mechanism lets a seller satisfy fiduciary duties requiring it to accept the "highest" bid. The seller uses the same mechanism no matter how bidders differ or how their securities differ.

2. Alleviates information demands. A seller can be uninformed. The market only needs to learn about the winner, and it can acquire the information post-auction.

3. Is *practically relevant*. It applies to common takeover settings where bidders use security bids with collars or cash bids that are financed post auction with securities.

4. Generates higher revenues than cash, and is robust to bidder heterogeneity, with revenues rising if any bidder uses a steeper security, or has a lower standalone value.

More broadly, our work highlights the potential of using financial markets to aid auction designs.

5 Appendix A: Proofs

Proof of Lemma 1: Denoting by $s_i(p)$ the solution to (3) when bidder i wins at price p, we rewrite (2), the expected profit of winner i with θ_i, as

$$\pi_i(\theta_i, p) = V_T + \theta_i - ES^i(s_i(p), \theta_i). \tag{20}$$

To economize on language, we say that a security is a "full security" if $S(z) = z$ for all z; that is, if the security pays out 100% of the cash flows.

Claim 1: $\pi_i(\theta_i^a, p) \geq \pi_i(\theta_i^b, p)$ for all p, where the inequality is strict unless the security corresponding to $s_i(p)$ is a full security.

Proof: We use Lemma 1 of DeMarzo, Kremer and Skrzypacz (2005). In our setting their lemma implies that the derivative of $ES^i(s_i(p), \theta_i)$ with respect to θ_i is strictly less than 1 if the security corresponding to $s_i(p)$ is not a full security. With a full security, the derivative equals 1. Given our premise that $\theta_i^a > \theta_i^b$, Claim 1 follows.

Claim 2: If for $p > 0$ the security corresponding to $s_i(p)$ is a full security, then $\pi_i(\theta_i, p) < 0$.

Proof: With a full security, (20) yields $\pi_i(\theta_i, p) = V_T + \theta_i - (V_T + \theta_i + X_i) = -X_i < 0$.

Claim 3: For any $\hat{b} < \beta_i^{dd}(\theta_i^b)$, there is a positive measure of $p \in \left[\hat{b}, \beta_i^{dd}(\theta_i^b)\right]$ such that the security corresponding to $s_i(p)$ is not a full security.

Proof: Let $\hat{\pi}_i(\theta_i, b)$ be the expected profit of bidder i of type θ_i when it bids some b, and let $K(\cdot)$ be the cdf for the highest bid among all bidders other than i. Then

$$\hat{\pi}_i(\theta_i, b) = \int_0^b \pi_i(\theta_i, t) dK(t).$$

If bidder i with θ_i^b bids its equilibrium bid $\beta_i^{dd}(\theta_i^b)$, then i's expected profit is

$$\hat{\pi}_i(\theta_i^b, \beta_i^{dd}(\theta_i^b)) = \int_0^{\beta_i^{dd}(\theta_i^b)} \pi_i(\theta_i, t) dK(t).$$

If bidder i instead deviates and bids $\hat{b}$, the gain from the deviation is

$$\hat{\pi}_i(\theta_i^b, \hat{b}) - \hat{\pi}_i(\theta_i^b, \beta_i^{dd}(\theta_i^b)) = -\int_{\hat{b}}^{\beta_i^{dd}(\theta_i^b)} \pi_i(\theta_i^b, t)dK(t).$$

Suppose contrary to Claim 3 that for all (full measure) $p \in [\hat{b}, \beta_i^{dd}(\theta_i^b)]$, the security corresponding to $s_i(p)$ is a full security. Then Claim 2 would yield that the deviation gain $\hat{\pi}_i(\theta_i^b, \hat{b}) - \hat{\pi}_i(\theta_i^b, \beta_i^{dd}(\theta_i^b)) \geq 0$ for all functions $K(t)$, with strict inequality for some $K(\cdot)$. Thus, bidding the posited equilibrium bid $\beta_i^{dd}(\theta_i^b)$ would be weakly dominated by bidding $\hat{b}$, violating Assumption 1. This proves Claim 3, and thus the Lemma. $\square$

Proof of Proposition 1: We first prove that bidding strategies are strictly increasing under Assumptions 1 and 2.

Claim 1: If bidder i with type θ_i wins at price $p = \beta_i^{dd}(\theta_i)$, then (i) i's expected profit $\pi_i(\theta_i, p)$ (in (20)) is nonnegative; and (ii) the security corresponding to $s_i(p)$ is not a full security.

Proof: Were $\pi_i(\theta_i, p)$ strictly negative, then following standard arguments for second-price auctions, bidding the posited equilibrium bid $\beta_i^{dd}(\theta_i)$ would be weakly dominated by bidding slightly less than $\beta_i^{dd}(\theta_i)$, a violation of Assumption 1. This proves part (i). To prove part (ii), note that if it were a full security, then by Claim 2 in the proof of Lemma 1, i's expected profit would be strictly negative if it wins at $\beta_i^{dd}(\theta_i)$.

Claim 2: Any equilibrium bidding strategy is strictly increasing: if $\theta_i^a > \theta_i^b$ then $\beta_i^{dd}(\theta_i^a) > \beta_i^{dd}(\theta_i^b)$. Here in Claim 2 and its proof, we slightly abuse notation by using β_i^{dd} to denote the bidding strategy in any equilibrium, where β_i^{dd} need not be given by (4).

Proof: Suppose that Claim 2 does not hold. Then by Lemma 1, the equilibrium bid of type θ_i^a must equal that of type θ_i^b; i.e., $\beta_i^{dd}(\theta_i^a)=\beta_i^{dd}(\theta_i^b)$. Next we show that a contradiction would arise, thereby establishing Claim 2.

By Claim 1 above and Claim 1 in the proof of Lemma 1, $\pi(\theta_i^a, \beta_i^{dd}(\theta_i^b)) > \pi(\theta_i^b, \beta_i^{dd}(\theta_i^b)) \geq 0$; i.e., type θ_i^a's $\beta_i^{dd}(\theta_i^b)$ is strictly positive. We now show that bidding slightly above $\beta_i^{dd}(\theta_i^a)$ is a profitable deviation for type θ_i^a. First, suppose that when winning at a higher (than $\beta_i^{dd}(\theta_i^a)$) price, the market's beliefs about i's synergies are the same as if i

wins at $\beta_i^{dd}(\theta_i^a)$. By the continuity of the profit function, if i wins at a price that exceeds $\beta_i^{dd}(\theta_i^a)$ by an arbitrarily small amount, i's profit must still be strictly positive. Second, by Assumption 2, the market's beliefs are weakly more optimistic when a bidder wins at a higher price. Thus, relaxing the premise that beliefs are the same from winning at a slightly higher price as those from winning at $\beta_i^{dd}(\theta_i^a)$ can only further increase i's expected profit. Hence, following standard arguments for second-price auctions, bidding i's posited equilibrium bid $\beta_i^{dd}(\theta_i^a)$ is weakly dominated by bidding slightly above $\beta_i^{dd}(\theta_i^a)$, violating Assumption 1. This contradiction establishes Claim 2.

We next prove that the bidding strategy in (4) is well-defined and strictly increasing.

Claim 3: For any θ_i, (5) has a unique solution, $s \in [\underline{s}_i, \bar{s}^i)$ and s weakly increases in θ_i.

Proof: We use Lemma 1 of DeMarzo, Kremer and Skrzypacz (2005), which yields that for any s, the derivative of $ES^i(s, \theta_i)$ with respect to θ_i is no greater than 1. The premise of sufficient range in the index (i.e., $ES^i(\underline{s}, \underline{\theta}_i) \leq \underline{\theta}_i + V_T$ and $ES^i(\overline{s}, \overline{\theta}_i) > V_T + \overline{\theta}_i$) implies that $ES^i(\underline{s}, \theta_i) \leq \theta_i + V_T$ and $ES^i(\overline{s}, \theta_i) > V_T + \theta_i$. Then Claim 3 follows by the intermediate value theorem and the properties that (i) $ES^i(s, \theta_i)$ is strictly increasing in s, and (ii) the derivative of the left-hand side of (5), $ES^i(s, \theta_i)$, with respect to θ_i is not greater than 1, whereas the derivative of the right-hand side of (5) with respect to θ_i equals 1.

Claim 4: $\beta_i^{dd}(\theta_i)$ as defined in (4) is strictly increasing in θ_i.

Proof: This follows because (i) for any $\theta_1 > \theta_2$, the distribution of θ conditional on $\theta \geq \theta_1$ strictly first-order stochastically dominates the distribution of θ conditional on $\theta \geq \theta_2$ (so the right-hand side of (4) strictly increases in θ_i for a fixed index s); and (ii) the right-hand side of (4) increases in s, and by Claim 3, the index s is weakly higher for higher θ_i.

We next prove that the bidding strategy in (4) and the market's belief in (6) comprise an equilibrium.

Claim 5: If the bidding strategy is (4), then the market's belief is given by (6).

Proof: The second and third lines in (6) follow from the consistency requirements on the equilibrium path—market beliefs must satisfy Bayes rule. The first line in (6) follows from Assumption 2 that market beliefs are non-decreasing in the winning price.

Claim 6: When market beliefs are given by (6), bidding according to (4) is weakly dominant for bidders.

Proof: We prove the claim in two steps. Consider a generic bidder i with type θ_i.

Step 1: We show that if i bids above $\beta_i^{dd}(\theta_i)$ and wins at a price $p > \beta_i^{dd}(\theta_i)$, then i's expected profit is negative (i.e., bidding above $\beta_i^{dd}(\theta_i)$ is sub-optimal).

Case 1: $p \leq \beta_i^{dd}(\bar{\theta}_i)$. Then, there exists some $\theta^* > \theta_i$ such that $p = \beta_i^{dd}(\theta^*)$. Let s^* denote the resulting security index of its payment as determined by the market. Because the expected value of payment equals the winning price $\beta_i^{dd}(\theta^*)$, s^* satisfies

$$E_{\theta;i}[ES^i(s^*, \theta)|\theta \geq \theta^*] = \beta_i^{dd}(\theta^*).$$

Note that if bidder i were of type θ^* and won at price $p = \beta_i^{dd}(\theta^*)$, then the index of the security paid would be s^*, and its expected profit would be zero (as a bidder i of type θ^* is indifferent between winning at $\beta_i^{dd}(\theta^*)$ and losing). Thus, (2) yields

$$V_T + \theta^* - ES^i(s^*, \theta^*) = 0. \tag{21}$$

Next we show that

$$ES^i(s^*, \theta^*) - ES^i(s^*, \theta_i) \leq \theta^* - \theta_i. \tag{22}$$

Let Z_i and Z_i^* be the random cash flows when bidder i's types are θ_i, and θ^*, respectively. Then $ES^i(s^*, \theta^*) - ES^i(s^*, \theta_i) = E[S(s^*, Z_i^*)] - E[S(s^*, Z_i)]$. Because the distribution of Z_i^* first-order stochastically dominates that of Z_i (due to the sMLRP), it follows that Z_i^* can be expressed as the sum of the random variable Z_i plus another random variable $\widetilde{\epsilon}$, where $\widetilde{\epsilon}$ is nonnegative and $\mathrm{E}[\widetilde{\epsilon}] = \theta^* - \theta_i$. Thus,

$$E[S(s^*, Z_i^*)] = E[S(s^*, Z_i + \widetilde{\epsilon})] \leq E[S(s^*, Z_i) + \widetilde{\epsilon}] = E[S(s^*, Z_i)] + \theta^* - \theta_i.$$

This establishes (22), which, by (21), (2), and the inequality $\theta^* > \theta_i$ yields that its expected profit is negative, $V_T + \theta_i - ES^i(s^*, \theta_i) < 0$.

Case 2: $p > \beta_i^{dd}(\bar{\theta}_i)$. As the market's belief about i's type cannot exceed $\bar{\theta}_i$, i's expected profit does not exceed that if it won at $p = \beta_i^{dd}(\bar{\theta}_i)$, which Case 1 showed was negative.

Step 2: We show bidder i's expected profit is positive if it wins at a price $p < \beta_i^{dd}(\theta_i)$ (i.e., bidding below $\beta_i^{dd}(\theta_i)$ is sub-optimal).

Case 1: $p \geq \beta_i^{dd}(\underline{\theta}_i)$. Then, there exists $\theta^* < \theta_i$ so that $p = \beta_i^{dd}(\theta^*)$. Let s^* denote the security index of i's payment as determined by the market. Then (21) follows by a similar argument as in the above step. By (21) and (2), its expected profit is positive.

Case 2: $p < \beta_i^{dd}(\underline{\theta}_i)$. Because market beliefs about the winner's type are the same as if i wins at $p = \beta_i^{dd}(\underline{\theta}_i)$ (see third equation in (6); this reflects that winning at a price below $\beta_i^{dd}(\underline{\theta}_i)$ is on the equilibrium path), i's expected profit is no less than what would be if it wins at $p = \beta_i^{dd}(\underline{\theta}_i)$, which is positive from Case 1.

Combining Cases 1 and 2 establishes Claim 6.

By Claims 5 and 6, bidding strategy (4) and market beliefs (6) constitute an equilibrium. Equilibrium uniqueness follows from Claim 2 (strategies are strictly increasing) and the arguments in the text that show strictly increasing strategies must be given by (4)). □

Proof of Lemma 2: As a preliminary step, we first prove Lemma 3:

Lemma 3 *Consider nondiscriminatory face-value-denominated auctions using ordered securities $\mathcal{S}_A$ and $\mathcal{S}_B$, where $\mathcal{S}_A$ is steeper than $\mathcal{S}_B$. If bidder 1 with expected synergy θ_1 wins against bidder 2 with expected synergy θ_2 under $\mathcal{S}_B$, then (i) bidder 1 also wins against bidder 2 under $\mathcal{S}_A$, and (ii) seller revenue under $\mathcal{S}_A$ is strictly less than that under $\mathcal{S}_B$ given θ_1 and θ_2.*

Proof: By Assumption 3, the cash flow distribution depends only on the expected total valuation of the bidder (bidder identity is irrelevant). Suppressing the superscript i in ES, we use $ES(s,\theta)_i + X_i + V_T)$ to denote the expected value of the security with index s. Let s_A^j, s_B^j, be the bids of bidder $j = 1, 2$ under $\mathcal{S}_A$ and $\mathcal{S}_B$, respectively. Then

$$ES_B(s_B^1, \theta_1 + X_1 + V_T) = \theta_1 + V_T, \tag{23}$$

$$ES_B(s_B^2, \theta_2 + X_2 + V_T) = \theta_2 + V_T, \tag{24}$$

$$ES_A(s_A^1, \theta_1 + X_1 + V_T) = \theta_1 + V_T, \tag{25}$$

and

$$ES_A(s_A^2, \theta_2 + X_2 + V_T) = \theta_2 + V_T. \tag{26}$$

Because bidder 1 wins against bidder 2 under $\mathcal{S}_B$, $s_B^2 < s_B^1$. By (23) we have

$$ES_B(s_B^2, \theta_1 + X_1 + V_T) < \theta_1 + V_T. \tag{27}$$

By (24) and (26), we have

$$ES_B(s_B^2, \theta_2 + X_2 + V_T) = ES_A(s_A^2, \theta_2 + X_2 + V_T).$$

Because $\mathcal{S}_A$ is steeper than $\mathcal{S}_B$, and $\theta_1 + X_1 + V_T < \theta_2 + X_2 + V_T$ by the premise of Lemma 2, we have

$$ES_B(s_B^2, \theta_1 + X_1 + V_T) > ES_A(s_A^2, \theta_1 + X_1 + V_T), \tag{28}$$

which, together with (27), yields

$$\theta_1 + V_T > ES_A(s_A^2, \theta_1 + X_1 + V_T),$$

which, by (25), yields $s_A^1 > s_A^2$: bidder 1 also wins against bidder 2 under $\mathcal{S}_A$. Thus, seller revenues under $\mathcal{S}_A$ and $\mathcal{S}_B$ are $ES_A(s_A^2, \theta_1 + X_1 + V_T)$ and $ES_B(s_B^2, \theta_1 + X_1 + V_T)$. By (28), revenue under $\mathcal{S}_A$ is strictly less than that under $\mathcal{S}_B$. This proves Lemma 3.

Next, consider equity. Bidder 1 bids $\frac{\theta_1+V_T}{\theta_1+V_T+X_1} \geq \frac{1}{1+\frac{X_1}{\underline{\theta}+V_T}}$, and bidder 2 bids $\frac{\theta_2+V_T}{\theta_2+V_T+X_2} \leq \frac{1}{1+\frac{X_2}{\bar{\theta}+V_T}}$. As $X_2 > \frac{\bar{\theta}+V_T}{\underline{\theta}+V_T} X_1$ by the premise of Lemma 2, bidder 1 wins against bidder 2.

Expected seller revenue is

$$\begin{aligned}
E\left[\theta_1+V_T+X_1\right]E\Big[\frac{\theta_2+V_T}{\theta_2+V_T+X_2}\Big] &= \left(E\left[\theta\right]+V_T+X_1\right)E\Big[1-\frac{X_2}{\theta_2+V_T+X_2}\Big] \\
&\leq \left(E\left[\theta\right]+V_T+X_1\right)\Big(1-\frac{X_2}{E\left[\theta\right]+V_T+X_2}\Big) \\
&= \frac{E\left[\theta\right]+V_T+X_1}{E\left[\theta\right]+V_T+X_2}\left(E\left[\theta\right]+V_T\right),
\end{aligned}$$

where the inequality is by Jensen's inequality. By the premise of Lemma 2, $X_2-X_1>\frac{E[\theta]-\underline{\theta}}{\underline{\theta}+V_T}\left(X_1+E\left[\theta\right]+V_T\right)$. Thus,

$$\begin{aligned}
\frac{E[\theta]+V_T+X_1}{E[\theta]+V_T+X_2}\left(E[\theta]+V_T\right) &< \frac{E[\theta]+V_T+X_1}{E[\theta]+V_T+X_1+\frac{E[\theta]-\underline{\theta}}{V_T+\underline{\theta}}\left(E[\theta]+V_T+X_1\right)}(E[\theta]+V_T) \\
&= \frac{1}{1+\frac{E[\theta]-\underline{\theta}}{V_T+\underline{\theta}}}(E[\theta]+V_T) = V_T+\underline{\theta}.
\end{aligned}$$

Thus, expected seller revenue in an equity auction is less than $V_T+\underline{\theta}$, while revenue in a cash auction exceeds $V_T+\underline{\theta}$. This, combined with Lemma 3, establishes Lemma 2. $\square$

Proof of Theorem 1. We begin with a preliminary lemma.

Lemma 4 *Suppose ordered set of securities* $\mathcal{S}_A^i$ *is steeper than* $\mathcal{S}_B^i$. *Then bidder* i *bids more aggressively in a dollar-denominated security auction with* $\mathcal{S}_A^i$ *than with* $\mathcal{S}_B^i$:

$$\beta_{(A)i}^{dd}(\theta_i) \geq \beta_{(B)i}^{dd}(\theta_i),$$

for all θ_i, *with strict inequality for all* $\theta_i < \bar{\theta}_i$.

Proof. From Proposition 1, $\beta_{(A)i}^{dd}(\theta_i)=E_{\theta;i}[ES^i(s_A,\theta)|\theta\geq\theta_i]$, where s_A solves

$$ES^i(s_A,\theta_i)=\theta_i+V_T, \tag{29}$$

and $\beta_{(B)i}^{dd}(\theta_i)=E_{\theta;i}[ES^i(s_B,\theta)|\theta\geq\theta_i]$, where s_B solves

$$ES^i(s_B,\theta_i)=\theta_i+V_T. \tag{30}$$

Here s_A and s_B denote the corresponding security in sets $\mathcal{S}_A^i$ and $\mathcal{S}_B^i$. By (29) and (30),

$$ES_A(s_A, \theta_i) = ES_B(s_B, \theta_i). \tag{31}$$

By the property of steeper securities, a bidder who expects to pay the same amount with a steeper security as with a flatter security for a given private valuation expects to pay strictly more with the steeper security than the flatter security if its private valuation is higher. Thus, (31) yields $ES_A(s_A, \theta) > ES_B(s_B, \theta)$ for all $\theta > \theta_i$, and hence

$$E_{\theta;i}\left[ES_A(s_A, \theta)|\theta \geq \theta_i\right] \geq E_{\theta;i}\left[ES_B(s_B, \theta)|\theta \geq \theta_i\right],$$

where the inequality is strict for $\theta_i < \bar{\theta}_i$. □

Lemma 4 establishes that when the profile of security sets is steeper, some bidders place more aggressive cash bids and the cash bids of all other bidders are weakly higher. Because the expected value of the security payment equals the winning cash price, the theorem follows by the law of iterated expectations. □

Proof of Proposition 3: Consider any bidder i. Recall that $ES^i(s, \theta) \equiv E(S(s, Z_i)|\theta_i = \theta)$ is the expected value of security with index s, derived from cash flows generated by a type θ bidder. To ease notation we replace "ES^i" with "g", where g is a function of s, the standalone value X, and the expected synergy θ of bidder i. We derive a sufficient condition for the bidding strategy to decrease in X:

Lemma 5 *If the ratio $\frac{\partial g}{\partial X}/\frac{\partial g}{\partial s}$ strictly decreases in the expected synergy θ at any given s and X, then the bidder's bidding strategy in dollar-denominated security design weakly decreases in X, where the decrease is strict for all $\theta < \bar{\theta}_i$.*

Proof: Use the notation of this proof to rewrite (5) as:

$$g(s, X, \theta = \theta_i) = \theta_i + V_T, \tag{32}$$

where θ_i is a constant. In (32), s is an implicit function of X, which we write as $s(X)$; that is, $s = s(X)$ and X satisfies (32).

Next examine the derivative of $g(s = s(X), X, \theta)$ with respect to X:

$$\frac{dg(s(X), X, \theta)}{dX} = \frac{\partial g}{\partial X}|_{s=s(X)} + \frac{\partial g}{\partial s}|_{s=s(X)} \frac{ds(X)}{dX}, \tag{33}$$

where using the implicit function theorem on (32) yields

$$\frac{ds(X)}{dX} = -\frac{\frac{\partial g(s,X,\theta_i)}{\partial X}}{\frac{\partial g(s,X,\theta_i)}{\partial s}}|_{s=s(X)}$$

Plugging the solution for $ds(X)/dX$ into (33) yields

$$\begin{aligned} \frac{dg(s(X), X, \theta)}{dX} &= \frac{\partial g(s, X, \theta)}{\partial X}|_{s=s(X)} - \frac{\partial g(s, X, \theta)}{\partial s}|_{s=s(X)} \frac{\frac{\partial g(s,X,\theta_i)}{\partial X}}{\frac{\partial g(s,X,\theta_i)}{\partial s}}|_{s=s(X)} \\ &= \frac{\partial g(s, X, \theta)}{\partial s}|_{s=s(X)} \left\{ \frac{\frac{\partial g(s,X,\theta)}{\partial X}}{\frac{\partial g(s,X,\theta)}{\partial s}} - \frac{\frac{\partial g(s,X,\theta_i)}{\partial X}}{\frac{\partial g(s,X,\theta_i)}{\partial s}} \right\} |_{s=s(X)}. \end{aligned} \tag{34}$$

Use the notation in this proof to rewrite i's bidding strategy (4), as

$$\beta_i^{dd}(\theta_i) = E[g(s(X), X, \theta)|\theta \geq \theta_i]. \tag{35}$$

For any $\theta > \theta_i$, by the premise of the lemma that the ratio $\frac{\partial g}{\partial X}/\frac{\partial g}{\partial s}$ strictly decreases in θ, the term inside the curly brackets on the right-hand side of (34) is strictly negative. Hence, by $\frac{\partial g(s,X,\theta)}{\partial s}|_{s=s(X)} > 0$, we have $\frac{dg(s(X),X,\theta)}{dX} < 0$. Thus, by (35), we have $\frac{d}{dX}\beta_i(\theta_i) \leq 0$, where strict inequality holds for $\theta_i < \bar{\theta}_i$. $\square$

Next we show that for debt and call, the ratio $\frac{\partial g}{\partial X}/\frac{\partial g}{\partial s}$ strictly decreases in θ. First note that both $\frac{\partial g(s,X,\theta)}{\partial s}$ and $\frac{\partial g(s,X,\theta)}{\partial X}$ are strictly positive.

Consider debt with face value s. Using the short-hand notation τ for τ_i, we have

$$g = \theta + X + V_T - E\left[\max\Big(\left((\theta + X + V_T)\,\tau - s\right), 0\Big)\right],$$

which yields

$$\frac{\partial g}{\partial s} = \text{prob}\Big(\tau \geq \frac{s}{\theta + X + V_T}\Big) \text{ and } \frac{\partial g}{\partial X} = 1 - E\Big[\tau|\tau \geq \frac{s}{\theta + X + V_T}\Big]\text{prob}\Big(\tau \geq \frac{s}{\theta + X + V_T}\Big),$$

where "prob" denotes the probability. Both prob$(\tau \geq \frac{s}{\theta+X+V_T})$ and $E[\tau|\tau \geq \frac{s}{\theta+X+V_T}]$prob$(\tau \geq \frac{s}{\theta+X+V_T})$ increase in θ (to see the latter result note that it equals $\int_{\frac{s}{\theta+X+V_T}}^{\infty} \tau dR(\tau)$, where $R(\tau)$ is the cdf of τ, and then differentiate with respect to θ). Hence, $\frac{\partial g}{\partial s}$ increases in θ and $\frac{\partial g}{\partial X}$ decreases in θ. Thus, the ratio $\frac{\partial g}{\partial X}/\frac{\partial g}{\partial s}$ strictly decreases in θ.

Next, consider call options. Let K be the strike price. Given our convention that a larger s means a larger security, but a larger K means a smaller call, we set $s = -K$ to preserve the order. We have $g = E[\max(((\theta + X + V_T)\tau - K), 0)]$, which yields

$$\frac{\partial g}{\partial s} = \text{prob}\left(\tau \geq \frac{K}{\theta + X + V_T}\right) \text{ and } \frac{\partial g}{\partial X} = E\left[\tau|\tau \geq \frac{K}{\theta + X + V_T}\right] \text{prob}\left(\tau \geq \frac{K}{\theta + X + V_T}\right).$$

Hence

$$\frac{\partial g}{\partial X}/\frac{\partial g}{\partial s} = E\left[\tau|\tau \geq \frac{K}{\theta + X + V_T}\right],$$

which strictly decreases in θ.

Thus, debt and call satisfy the premise of Lemma 5. Thus, the bidding strategy in the dollar-denominated security design strictly increases as X decreases for $\theta_i < \bar{\theta}_i$. Further, from (11), this also holds for equity. Because more aggressive cash bidding by any bidder leads to a higher expected profit, the proposition follows. □

Bibliography

Andrade G., M. Mitchell, and E. Stafford, 2001, “New Evidence and Perspectives on Mergers”, *Journal of Economic Perspectives*, 103–120.

Abhishek, V., B. Hajek and S. William, 2015, “On Bidding with Securities: Risk Aversion and Positive Dependence”, *Games and Economic Behavior*, 66-80.

Bagnoli, M.B. and B. Lipman, 1996, “Stock Price Manipulation Through Takeover Bids,” *RAND Journal of Economics*, 124–147.

Banerjee, S, B. Breon-Drish, and K. Smith, 2025, “Feedback Effects and Systematic Risk Exposures,” *Journal of Finance*,981-1028.

Bergemann, D. and S. Morris, 2005, “Robust Mechanism Design”, *Econometrica*, 1771–1813.

Bergemann, D., B. Brooks, and S. Morris, 2016, “Informationally robust optimal auction design,” working paper.

Bergemann, D., B. Brooks, and S. Morris, 2017, “First-Price Auctions With General Information Structures: Implications for Bidding and Revenue,” *Econometrica*, 107–143.

Bergemann, D., B. Brooks, and S. Morris, 2019, “Revenue Guarantee Equivalence,” *American Economic Review*, 1911–1929.

Bergemann, D., B. Brooks, and S. Morris, 2019, “Revenue Guarantee Equivalence,” *American Economic Review*, 1911–1929.

Betton, S., E. Eckbo, and K. Thorburn, 2008, “Corporate takeovers,” Handbook of Empirical Corporate Finance (Elsevier).

Biais, B., T. Mariotti, G. Plantin, and J.C. Rochet, 2007, “Dynamic security design: Convergence to continuous time and asset pricing implications,” *Review of Economic Studies*, 345–390.

Bond, P., A. Edmans, and I. Goldstein, 2012, “The real effects of financial markets,” *Annual Review of Financial Economics*, 339–360.

Bond, P. and I. Goldstein, 2015, “Government intervention and information aggregation by prices,” *Journal of Finance*, 2777–2812.

Bose, A., D. Pal, D. and D. Sappington, 2011, “On the performance of linear contracts,” *Journal of Economics and Management Strategy*, 159–193.

Boulatov, A. and S. Severinov, 2021, “Optimal and efficient mechanisms with asymmetrically budget constrained buyers,” *Games and Economic Behavior*, 155–178.

Brooks, B. and S. Du., 2021,“Optimal auction design with common values: An informationally-robust approach.” *Econometrica*, 1313-1360.

Burkart, Mike, 1995. ”Initial Shareholdings and Overbidding in Takeover Contests,” *Journal of Finance,* 50:1491–515.

Burkart, Mike and Samuel Lee, 2015. “Signalling to Dispersed Shareholders and Corporate Control,” *Review of Economic Studies*, 82(3), 922-962.

Carroll, G., “Robustness and linear contracts,” 2015, *American Economic Review*, 536-563.

Chassang, S., 2013, “Calibrated incentive contracts,” *Econometrica*, 1935-1971.

Chu, C., P. Leslie and A. Sorensen, 2011, “Bundle-size pricing as an approximation to mixed bundling,” *American Economic Review*, 263-303.

Chu, L. and D. Sappington, 2007, “Simple cost-sharing contracts,” *American Economic Review*, 419-428.

Cong, Lin W., 2020, ”Timing of Auctions of Real Options,” *Management Science*, 3956-3976.

Dasgupta, P. and E. Maskin, 2000, “Efficient Auctions,” *Quarterly Journal of Economics*, 341-388.

Dasgupta, S. and K. Tsui, 2003, “A ‘Matching Auction’ for Targets with Heterogeneous Bidders”, *Journal of Financial Intermediation*, 331-364.

Dasgupta, S. and K. Tsui, 2004, “Auctions with Cross-Shareholding”, *Economic Theory*, 163-194.

DeMarzo, P., I. Kremer, and A. Skrzypacz, 2005, “Bidding with Securities: Auctions and Security Design,” *American Economic Review*, 936-59.

Du, S., 2018, “Robust mechanisms under common valuation.” *Econometrica*, 1569-1588.

Eckbo, E., A. Malenko, and K. Thorburn, 2020, “Strategic Decisions in Takeover Auctions: Recent Developments.” *Annual Review of Financial Economics*, 237-276.

Ekmekci, M., N. Kos, and R. Vohra, 2016, “Just enough or all: Selling a firm”, *American Economic Journal: Microeconomics*, 223-56.

Faccio, M. and R. Masulis, 2005, “The choice of payment method in European mergers and acquisitions,” *Journal of Finance*, 1345–1388.

Fioriti, Andres and Allan Hernandez-Chanto, 2022, “Leveling the Playing Field for Risk-Averse Agents in Security-Bid Auctions,” *Management Science*.

Goldstein, I. and L. Yang, (2019). “Good Disclosure, Bad Disclosure,” *Journal of Financial Economics*, 118-138.

Gorbenko, A. and A. Malenko, 2011, “Competition among Sellers in Securities Auctions”, *American Economic Review*, 1-38.

Gorbenko, A. and A. Malenko, 2014, “Strategic and Financial Bidders in Takeover Auctions”, *Journal of Finance*, 2513-2555.

Gorbenko, A. and A. Malenko, 2018, “The timing and method of payment in mergers when acquirers are financially constrained,” *Review of Financial Studies*, 3937-3978.

Hansen, Robert G. 1985. “Auctions with Contingent Payments.” *American Economic Review*, 862–65.

Hurwicz, L., 1972, “On Informationally Decentralized Systems,” in *Decision and Organization*, ed. by R. Radner and C. McGuire. Amsterdam: North-Holland, 297–336.

Kogan, S. and J. Morgan, 2010, ”Securities Auctions under Moral Hazard: Theory and Experiments”, Review of Finance 14, 477-520.

Liu, T., 2012, “Takeover Bidding with Signaling Incentives”, *Review of Financial Studies*, 522-556.

Liu, T., 2016, “Optimal Equity Auctions with Heterogeneous Bidders”, *Journal of Economic Theory*, 94-123.

Lee, S. and U. Rajan, 2018, “Robust Security Design,” U. Michigan, working paper.

Martynova, M., and L. Renneboog, 2009, “What Determines the Financing Decision in

Corporate Takeovers: Cost of Capital, Agency Problems, or the Means of Payment?" *Journal of Corporate Finance*, 290-315.

McAfee, R. P., J. McMillan, and P. Reny. 1989. "Extracting the surplus in the common-value auction," *Econometrica*, 1451-1459.

Mezzetti, C. 2004, "Mechanism design with interdependent valuations: Efficiency", *Econometrica*, 1617-1626.

Nachman, D. C. and T. H. Noe. 1994. "Optimal design of securities under asymmetric information," *Review of Financial Studies*, 1-44.

Officer, M., 2004, "Collars and Renegotiation in Mergers and Acquisitions," *Journal of Finance*, 2719-43.

Povel, P. and R. Singh, 2010, "Stapled Finance", *Journal of Finance*, 927-953.

Rogerson, W., 2003, "Simple menus of contracts in cost-based procurement and regulation", *American Economic Review*, 919-926.

Samuelson, William. 1987. "Auctions with Contingent Payments: Comment." *American Economic Review*, 740-745.

Singh, Raj. 1998. "Takeover Bidding with Toeholds: The Case of the Owner's Curse." *Review of Financial Studies*, 11:679–704.

Sogo, T., D. Bernhardt and T. Liu, 2016, "Endogenous Entry to Security-Bid Auctions", *American Economic Review*, 3577-3589.

Sogo, T., 2017, "Effects of Seller's Information Disclosure in Equity Auctions Requiring Post-Auction Investment", *International Journal of Industrial Organization*, 166-181.

Wilson, R., 1985, "Incentive Efficiency in Double Auctions," *Econometrica*, 1101-1115.

Vladimirov, V., 2015. "Financing bidders in takeover contests", *Journal of Financial Economics*, 534-557.

6 Online Appendix B: Financing Formulation

In a *non-discriminatory dollar-denominated second-price security auction*, bidders submit *monetary* bids, the highest bid wins, the cash price p equals the publicly-announced second-highest bid, and the bidder pays the seller the security equivalent of p as determined by the financial market. In the *financing formulation*, rather than pay the seller with securities, the winner instead raises the cash to pay the seller p by issuing security claims to the merged firm in the financial market. The means of financing is exogenously specified for a given firm, but can differ across firms. The financing terms—the equity share of the merged firm or the face value of its debt—are determined after the auction by a competitive capital market given its information set, which includes: cash price p, winner i's standalone value X_i, the distribution $F_i(\cdot)$ over i's expected synergies, and the distribution $h_i(\cdot|\theta_i)$ of i's future cash flows conditional on θ_i.

In the financing formulation, a firm must raise funds to pay p, while in our base (dollar-denominated) model the capital market prices the winner's securities to determine the fraction of equity or face value of debt that the winner pays the seller. Phrased differently, in the base model the winner pays a security to the seller, while in the financing version, the winner sells a security to the market to raise cash p and then pays p to the seller. As we observed earlier, these two models are equivalent because a bidder does not care whether he sells a security to the capital market or to the seller.

Below we formally establish this equivalence for general securities. To do this, we show the bidding strategy and market beliefs in equations (4) – (6) of Proposition 1 also comprise an equilibrium in the second-price auction with external financing model.

Proposition 1$'$: In the financing formulation for general securities, the bidding strategy and market beliefs in equations (4) – (6) of Proposition 1 comprise an equilibrium.

Proof: We prove Proposition 1$'$ by borrowing elements from the proof of Proposition 1. We proceed in three steps. In the proof of Proposition 1:

1. Claim 4 established that the bidding strategy in (4) is strictly increasing.

2. Claim 5 established that when the bidding strategy is (4), the market's beliefs are given by (6).

3. Claim 6 established that when market beliefs are given by (6), bidding according to (4) is weakly dominant for bidders (specifically the claim showed that it would be suboptimal for a bidder to bid either above, or below the strategy stipulated in (4)). While Claim 6 was established in the dollar-denominated auctions model, the proof applies to the financing model here because a bidder does not care whether he sells a security to the capital market or to the seller—hence the bidder's expected profit takes the same form in the two models.

The above steps establish Proposition $1'$. $\square$

Above we proved the equivalence between the financing and dollar-denominated models (for general securities) by adopting the proof of Proposition 1. To underscore the equivalence, we now specialize to equities and use an alternative approach to derive the equivalence from scratch. We consider an arbitrary bidder i who would finance the winning cash price p by issuing equity to the market.

Claim: Bidder i's equilibrium bidding strategy is strictly increasing and given by:

$$\beta_i(\theta_i) = (V_T + \theta_i)\,\frac{V_T + X_i + E_{\theta;i}[\theta|\theta \geq \theta_i]}{V_T + X_i + \theta_i}, \tag{36}$$

where $E_{\theta;i}$ denotes the expectation given $\theta \sim F_i$. The market's belief about θ_i, conditional on i winning the auction at price p, is

$$\begin{cases} \theta_i = \bar{\theta}_i & \text{if} \quad p > \beta_i(\bar{\theta}_i) \\ \theta_i \geq \beta_i^{-1}(p) & \text{if} \quad p \in \left[\beta_i(\underline{\theta}_i), \beta_i\left(\bar{\theta}_i\right)\right] \\ \theta_i \geq \underline{\theta}_i & \text{if} \quad p < \beta_i(\underline{\theta}_i) \end{cases}, \tag{37}$$

where β_i^{-1} is the inverse function of β_i (equation (36)).

Proof: To connect this to our base model, note that with equity, (36) follows from substituting s from (5) into (4). Equation (4) says $\beta_i(\theta_i) = E_{\theta;i}[ES^i(s,\theta)|\theta \geq \theta_i]$. With equity,

$$ES^i(s,\theta_i) \equiv s\left(V_T + X_i + \theta_i\right). \tag{38}$$

Substituting (38) for $ES^i(s,\theta_i)$ into equation (5) yields

$$\beta_i(\theta_i) = E_{\theta;i}[s\left(V_T + X_i + \theta\right))|\theta \geq \theta_i]. \tag{39}$$

Equation (5) says $ES^i(s,\theta_i) = \theta_i + V_T$, and thus $s(V_T + X_i + \theta_i) = \theta_i + V_T$, yielding

$$s = \frac{\theta_i + V_T}{V_T + X_i + \theta_i}. \tag{40}$$

Substituting (40) for s into (39) yields i's bidding strategy given in (36):

$$\begin{aligned}
\beta_i(\theta_i) &= E_{\theta;i}\left[\frac{\theta_i + V_T}{V_T + X_i + \theta_i}\left(V_T + X_i + \theta\right))|\theta \geq \theta_i\right] \\
&= \left(\frac{\theta_i + V_T}{V_T + X_i + \theta_i}\right)\left(V_T + X_i + E_{\theta;i}\left[\theta|\theta \geq \theta_i\right]\right).
\end{aligned}$$

Inspection yields that the bidding strategy is strictly increasing in θ_i. Further, market beliefs in (37) are the same as those in (6). Next we prove Proposition 1' in two steps.

Step 1: It is immediate that market beliefs (37) about θ_i are consistent with i's bidding strategy (36): market beliefs satisfy Bayes rule on the equilibrium path. For instance, when i bids according to (36), and wins at a price $p \in \left[\beta_i(\underline{\theta}_i), \beta_i\left(\bar{\theta}_i\right)\right]$, the market updates to infer that $\beta_i(\theta_i) \geq p$, which, upon inverting, yields $\theta_i \geq \beta_i^{-1}(p)$.

Step 2: We show it is optimal for i to bid according to (36) when market beliefs are given by (37). Let $\pi(\theta_i, p)$ be i's expected profit when it is type θ_i and wins at price p (i.e., the highest losing bidder bids p and i bids above p and wins) and let $\theta_i^*(p)$ be the market's belief about i's expected type when i wins at price p. By (37),

$$\theta_i^*(p) = \begin{cases} \bar{\theta}_i & \text{if} \quad p > \beta_i(\bar{\theta}_i) \\ E_{\theta;i}[\theta|\theta \geq \beta_i^{-1}(p)] & \text{if} \quad p \in \left[\beta_i(\underline{\theta}_i), \beta_i\left(\bar{\theta}_i\right)\right] \\ E_{\theta;i}[\theta] & \text{if} \quad p < \beta_i(\underline{\theta}_i) \end{cases}. \tag{41}$$

After the auction, the capital market values the merged firm at $V_T + X_i + \theta_i^*(p)$. In return for providing cash amount p, the market demands a share $s = \frac{p}{V_T + X_i + \theta_i^*(p)}$ of the merged firm. Thus, bidder i retains share $1 - s$ and its expected profit is

$$\begin{aligned}\pi(\theta_i, p) &= (1-s)(V_T + X_i + \theta_i) - X_i && (42)\\ &= V_T + \theta_i - p\frac{V_T + X_i + \theta_i}{V_T + X_i + \theta_i^*(p)}, && (43)\end{aligned}$$

which is the value of i's retained share of the merged firm less i's standalone value. p does not appear in (42) because i raises p from the market but then pays p to the seller. Equation (43) follows from (42) upon substituting $s = \frac{p}{V_T + X_i + \theta_i^*(p)}$. We next prove that

$$\begin{cases} \pi(\theta_i, p) = 0 & \text{if} \quad p = \beta_i(\theta_i) \\ \pi(\theta_i, p) > 0 & \text{if} \quad p < \beta_i(\theta_i) \\ \pi(\theta_i, p) < 0 & \text{if} \quad p > \beta_i(\theta_i) \end{cases}, \qquad (44)$$

where $\beta_i(\theta_i)$ is given by (36). Note that we are not taking a position on whether $\beta_i(\theta_i)$ is the equilibrium bidding strategy; we simply interpret $\beta_i(\theta_i)$ as being defined by (36). We establish that (44) holds for each of the three pricing scenarios in (41).

Scenario 1: $p \in \left[\beta_i(\underline{\theta}_i), \beta_i\left(\bar{\theta}_i\right)\right]$. Then there exists a $\theta_i^a \in [\underline{\theta}_i, \bar{\theta}_i]$ such that $p = \beta_i(\theta_i^a)$. Plugging $p = \beta_i(\theta_i^a)$ into (41) yields $\theta_i^*(p) = E_{\theta;i}[\theta | \theta \geq \theta_i^a]$. Plugging this into (43) yields

$$\begin{aligned}\pi(\theta_i, p) &= V_T + \theta_i - p\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta|\theta \geq \theta_i^a]}\\ &= V_T + \theta_i - \beta_i(\theta_i^a)\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta|\theta \geq \theta_i^a]}\\ &= V_T + \theta_i - \left[(V_T + \theta_i^a)\,\frac{V_T + X_i + E_{\theta;i}[\theta|\theta \geq \theta_i^a]}{V_T + X_i + \theta_i^a}\right]\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta|\theta \geq \theta_i^a]}\\ &= V_T + \theta_i - \left[(V_T + X_i + \theta_i)\,\frac{V_T + \theta_i^a}{V_T + X_i + \theta_i^a}\right]. && (45)\end{aligned}$$

Line 3 follows by substituting (36) for $\beta_i(\theta_i^a)$ on the RHS of line 2 (using $\theta_i = \theta_i^a$).

To prove that $\pi(\theta_i, p) = 0$ when $p = \beta_i(\theta_i)$, note that $p = \beta_i(\theta_i)$ implies $\theta_i^a = \theta_i$, which, upon plugging into (45), yields

$$\pi(\theta_i, p) = V_T + \theta_i - \left[(V_T + X_i + \theta_i)\,\frac{V_T + \theta_i}{V_T + X_i + \theta_i}\right] = 0.$$

To prove that $\pi(\theta_i, p) > 0$ when $p < \beta_i(\theta_i)$, note that $p < \beta_i(\theta_i)$ is equivalent to $\theta_i^a < \theta_i$. Substituting $1 - \frac{X_i}{V_T + X_i + \theta_i^a}$ for $\frac{V_T + \theta_i^a}{V_T + X_i + \theta_i^a}$ into equation (45) then yields

$$\begin{aligned}\pi(\theta_i, p) &= V_T + \theta_i - \left[(V_T + X_i + \theta_i)\left(1 - \frac{X_i}{V_T + X_i + \theta_i^a}\right)\right]\\ &< V_T + \theta_i - \left[(V_T + X_i + \theta_i)\left(1 - \frac{X_i}{V_T + X_i + \theta_i}\right)\right] = 0, \qquad (46)\end{aligned}$$

where the inequality in (46) follows from $\theta_i^a < \theta_i$. The proof that $\pi(\theta_i, p) < 0$ when $p > \beta_i(\theta_i)$ follows that for $\pi(\theta_i, p) > 0$ when $p < \beta_i(\theta_i)$ after noting that $p > \beta_i(\theta_i)$ is equivalent to $\theta_i^a > \theta_i$, which reverses the inequality in (46).

Scenario 2: $p < \beta_i(\underline{\theta}_i)$. Plugging $\theta_i^*(p) = E_{\theta;i}[\theta]$ from equation (41) into (43) yields

$$\begin{aligned}\pi(\theta_i, p) &= V_T + \theta_i - p\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta]}\\ &> V_T + \theta_i - \beta_i(\underline{\theta}_i)\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta]}\\ &= V_T + \theta_i - \left[(V_T + \underline{\theta}_i)\,\frac{V_T + X_i + E_{\theta;i}[\theta]}{V_T + X_i + \underline{\theta}_i}\right]\frac{V_T + X_i + \theta_i}{V_T + X_i + E_{\theta;i}[\theta]}\\ &= V_T + \theta_i - \left[(V_T + \underline{\theta}_i)\,\frac{V_T + X_i + \theta_i}{V_T + X_i + \underline{\theta}_i}\right]\\ &= V_T + \theta_i - \left[(V_T + \underline{\theta}_i)\left(1 + \frac{\theta_i - \underline{\theta}_i}{V_T + X_i + \underline{\theta}_i}\right)\right]\\ &\geq V_T + \theta_i - \left[(V_T + \underline{\theta}_i)\left(1 + \frac{\theta_i - \underline{\theta}_i}{V_T + \underline{\theta}_i}\right)\right] = 0,\end{aligned}$$

where the first inequality follows from $p < \beta_i(\underline{\theta}_i)$, and second follows from $\theta_i \geq \underline{\theta}_i$. This establishes line 2 of (44): $p < \beta_i(\underline{\theta}_i)$ and $\theta_i \geq \underline{\theta}_i$ yields $p < \beta_i(\theta_i)$.

Scenario 3: $p > \beta_i(\bar{\theta}_i)$. Plugging $\theta_i^*(p) = \bar{\theta}_i$ from equation (41) into (43) yields

$$\begin{aligned}
\pi(\theta_i, p) &= V_T + \theta_i - p\frac{V_T + X_i + \theta_i}{V_T + X_i + \bar{\theta}_i} \\
&< V_T + \theta_i - \beta_i(\bar{\theta}_i)\frac{V_T + X_i + \theta_i}{V_T + X_i + \bar{\theta}_i} \\
&= V_T + \theta_i - \left[\left(V_T + \bar{\theta}_i\right)\frac{V_T + X_i + \bar{\theta}_i}{V_T + X_i + \bar{\theta}_i}\right]\frac{V_T + X_i + \theta_i}{V_T + X_i + \bar{\theta}_i} \\
&= V_T + \theta_i - \left[(V_T + X_i + \theta_i)\left(1 - \frac{X_i}{V_T + X_i + \bar{\theta}_i}\right)\right] \\
&\leq V_T + \theta_i - \left[(V_T + X_i + \theta_i)\left(1 - \frac{X_i}{V_T + X_i + \theta_i}\right)\right] = 0,
\end{aligned}$$

where the first inequality follows from $p < \beta_i(\underline{\theta}_i)$, and the second follows from $\theta_i \leq \bar{\theta}_i$. This establishes line 3 of (44): $p > \beta_i(\bar{\theta}_i)$ and $\theta_i \leq \bar{\theta}_i$ yields $p > \beta_i(\theta_i)$.

This completes the proof of (44). By (44), standard arguments for second-price auctions yield that bidding according to (36) is a weakly-dominant strategy for bidder i. $\square$