The Structure of Credit Markets with New Screening Technologies

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Abstract

We develop a model of credit market competition with endogenous screening technologies and interest rates, and use it to study two implications of the technological transformation reshaping credit markets. First, lenders deploying similar AI systems and overlapping data may produce increasingly correlated screening errors. We show that correlated mistakes lead ex-ante identical lenders to endogenously specialize across distinct market segments, resulting in a *hockey-stick interest rate schedule* that echoes the coexistence of traditional banking, fintech lending and private credit, and high-rate indiscriminate lending. Within each segment, lenders charge lower rates and face fewer non-performing loans than absent specialization; yet because credit supply reallocates toward higher-rate segments, the average borrower may end up paying more. Second, technological and regulatory changes affect screening costs unevenly. Big data innovations expand financial inclusion, while broader Open Banking adoption can harm the financially excluded and increase inequality in financial access.

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The competitive landscape of credit markets is rapidly changing. A house buyer might obtain a mortgage from a traditional bank or a fintech lender. A small business owner might apply for a bank loan, fintech credit, or a high-interest credit card. A firm might approach a loan syndicate or a private credit fund. These diverse lenders increasingly rely on similar data sources and algorithmic screening methods, yet charge different interest rates and serve different borrower segments. A central question is how AI and big data are reshaping this landscape.

Our analysis is motivated by two possible implications of this technological change. The first is that lenders deploying similar AI and machine learning systems, sharing training data, foundation models, or algorithmic architectures, and using overlapping data sets can produce *increasingly correlated screening errors*. As we discuss below, early evidence on error structures in AI systems points in this direction. We embed correlated screening mistakes in a model of credit markets and show that, *relative to the standard assumption of independent screening errors*, they radically change the equilibrium structure. Ex-ante identical lenders endogenously specialize into distinct market segments that echo traditional banking, fintech lending and private credit, and high-rate indiscriminate lending. Within each segment, lenders face fewer non-performing loans and charge lower rates than they would under independent information—yet because credit supply reallocates toward the higher-rate segments, the average borrower may end up paying *more*.

The second implication is that technological and regulatory changes introduce *directed technological change* to credit market screening, reducing the cost of screening unevenly across the precision spectrum. We study two leading applications of this idea. In the first, we interpret big data, machine learning, and AI as primarily reducing the cost of screening the hardest-to-assess borrowers. In the second, we interpret data-sharing policies such as Open Banking as primarily reducing the cost of screening relatively transparent borrowers. Because the credit market is segmented, a directed cost reduction in one segment spills over to others through the composition of the residual borrower pool—with consequences that, as we show, depend on which part of the spectrum is affected.

We build a parsimonious model of credit market competition with the above two ingredients. Ex-ante identical lenders choose both the precision of their screening technology and the interest rate they charge. Borrowers' objective is to obtain credit at the lowest rate at which they are served. In the centre of our analysis is a *pool externality*: borrowers rejected by lenders offering a low rate reapply at others, so each lender's screening choices affect other lenders' applicant pools.

Borrowers in our model have a two-dimensional type. The first dimension is creditworthiness: good borrowers repay, bad borrowers default. The second dimension is *opacity*—a non-payoff-relevant characteristic that determines how difficult it is for lenders to correctly assess a borrower's creditworthiness after conditioning on their observable characteristics.

Each lender chooses the precision of their screening technology at a cost. A higher-precision technology is more costly, but can correctly identify borrowers as good or bad up to a higher opacity threshold. Whether a lender correctly identifies a borrower depends on the lender's precision relative to the borrower's opacity, so lenders with similar precision make similar mistakes. The resulting information structure is *nested*: a high-precision lender makes a strict subset of the errors made by a low-precision lender. As such, opacity is the key modeling device that generates correlated screening errors.

Empirical evidence on AI error structures supports our focus on nested information and the role of unobservable opacity. Research on *algorithmic monoculture* (Kleinberg and Raghavan, 2021; Bommasani et al., 2022) finds that decision-makers deploying similar systems produce correlated outcomes: Bommasani et al. (2022) show that data-sharing reliably increases *outcome homogenization*, and that this error correlation persists after conditioning on observables, consistent with our formulation. In the LLM domain, Kim et al. (2025) find that models agree on wrong answers far more than chance predicts, with more accurate models exhibiting more correlated errors. Crucially, Ritchie et al. (2026) documents that weaker models fail on both fundamental and advanced tasks, while stronger models primarily fail only on advanced tasks, consistent with approximate nesting. Our nested information structure captures these patterns in stylized form.

Moreover, introducing opacity allows us to model both correlated and independent information structures as special cases of a unified framework and characterize the equilibrium under both while holding all other assumptions constant. This comparison illustrates that the market structure is remarkably sensitive to which one prevails.

We begin our analysis with the independent information benchmark and show that the equilibrium features a smooth, fully separating market structure: interest rates increase continuously with lender skill. The borrower pool deteriorates through standard cream-skimming, and lenders with higher skill serve more adversely selected pools, charging higher interest rates to compensate both for the worse pool quality and the cost of their technology.

Under nested information, the equilibrium is strikingly different. A segmented market structure emerges with three qualitatively distinct regions, which we call the *hockey stick interest rate schedule*.[1] The first segment resembles traditional banking: lenders offer a uniform low interest rate but reject borrowers they cannot reliably assess. The second segment resembles high-tech lending: lenders invest heavily in screening technology to serve harder-to-assess borrowers, charging rates that increase with borrower opacity. The third segment features indiscriminate lending at the very high rate, resembling high-rate credit cards or low-documentation mortgages, where lenders forgo screening entirely and compensate through high rates. Ex-ante homogeneous lenders endogenously differentiate into heterogeneous screening technologies, and the matching between borrowers and lenders is non-assortative: the lowest-skilled lenders serve the most opaque borrowers at the highest rates. Crucially, all borrowers are eventually served, and the pool available to higher-skilled lenders is *cleansed* rather than diluted: traditional banks inadvertently absorb hard-to-recognize bad borrowers, improving the pool for other segments. This cross-segment dependence, whereby conditions in one market segment affect others, is what makes directed technological shocks to spill over across segments, the object we formalize in Section 4.

Our unified framework allows us to directly compare equilibrium outcomes across infor-

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[1] This pattern is consistent with empirical evidence from credit markets. In SME lending, Berger and Udell (2006) document that firms with higher financial transparency benefit from clearer risk assessments and lower costs of credit, while Strahan (1999) finds that borrowers harder for outside investors to value pay more for their loans.Berger and Udell (2002) finds that small banks lend at higher interest rate to more opaque borrowers than large banks. By defining the segments differently, Gopal et al. (2024) find that Fintech lenders lend at higher rates to more opaque borrowers compared to traditional banks.

mation structures, holding all other primitives constant. The contrasting pool externalities described above have direct implications for interest rates and default. Under nested information, pool-cleansing gradually raises the repayment probability γ as we move up the skill spectrum, approaching one. The interest-rate consequences differ across two segments. In the traditional-banking segment, all lenders charge the same low rate; as skill rises, the higher cost of screening is exactly offset by the improving repayment probability, keeping profits constant without any rate increase. In the high-tech segment, default has become rare, and interest rates rise only as much as the higher cost of screening requires. Under independent information, by contrast, cream-skimming progressively worsens the pool, forcing lenders to charge higher rates to compensate for greater default risk. As a result, at any given skill level, lenders under nested information charge strictly lower interest rates and enjoy strictly higher repayment probabilities than under independent information.

These forces also reshape the allocation of credit supply across market segments. In equilibrium, the way credit is distributed across segments must balance the cost of screening, the quality of the applicant pool, and the interest rate borrowers pay to compensate lenders for both. How this balance is achieved depends on the impact of low-skill lenders on the pool. How many lenders gather at a given rate is determined by how much each one changes the pool for others: equilibrium fixes the total amount of pool change required between adjacent rates, so the smaller each lender's effect, the more of them must enter. With independent information, low-skill lenders provide barely any useful screening: their decisions are nearly random. With each one doing so little, many must enter to serve the lowest-rate market, and credit ends up concentrated there. With nested information, however, even a low-skill lender does meaningful work, picking off the easy-to-identify good borrowers along with the hard-to-identify bad ones. Consequently, fewer lenders are needed at the bottom of the interest rate spectrum and credit supply reallocates toward the higher-rate segments. As a consequence, under independent information most borrowers are served near the lowest interest rate, while under nested information they spread across a wider range of rates. Perhaps surprisingly, despite the pointwise rate advantage of nested information, the average interest rate borrowers actually pay can be higher: precisely because independent information concentrates most lending where rates are lowest. Which information structure ultimately benefits borrowers depends on the balance between these two forces.

We exploit this framework to study how directed technological change spills over across market segments. To focus on the short-run impact, we consider new entrants with improved screening technology into a market populated by incumbents who cannot change their technology immediately, but are able to change their terms of lending or exit the market if no longer profitable. The hockey-stick structure of interest rates under nested information means that conditions in one market segment affect others through the composition of the borrower pool. When technological progress reduces screening costs in a directed fashion, affecting some parts of the screening technology spectrum more than others, the resulting entry spills over to segments where no new lender enters. Crucially, the *direction* of the spillover depends on which borrowers the new entrants absorb from the pool. When entrants screen at the high end of the precision spectrum, they compete directly with indiscriminate lenders and draw opaque borrowers away from the highest-rate segment. When entrants screen at the low end, they affect the composition of the borrower pool that remains for higher-rate segments, and the sign of spillover can go in either direction. We show that the hockey-stick market structure is robust to entry under quite general conditions, and illustrate the implications of our model for technological and policy spillover with two leading applications.

As a first application, we interpret big data innovation as a cost reduction for screening the most opaque borrowers, where richer signals and the ability to process non-traditional data have the highest marginal value. High-tech entrants invest in sophisticated screening and serve borrowers who would otherwise rely on indiscriminate lenders at the highest rates. This can lead to the opening of new markets,[2] and also generates positive spillovers for the financially excluded: high-tech entrants draw business from indiscriminate lenders, who respond by lowering rates to deploy their capital. Big data adoption thus increases financial inclusion.

Second, we consider a disproportionate reduction in screening cost of borrowers who are relatively transparent. This shift can represent the adoption of Open Banking policies in lending markets, as Open Banking enables the data of borrowers that already exist in the market to be more widely available to more lenders. As expected, this shift benefits these borrowers by intensifying lender competition in this segment. As such, the terms of borrowing for borrowers who were already being served at favorable rates improves even further. However, the direction of the spillover to other segments depends critically on the scope of adoption. Under limited adoption—when cost reductions are confined to only very transparent borrowers—new entrants in the pooling market disproportionately absorb hard-to-recognize bad borrowers, improving the quality of the pool available to indiscriminate lenders and lowering the highest interest rate. However, under broader adoption—when cost reductions extend to intermediate transparency levels—new entrants instead cream-skim good transparent borrowers from the pooling segment, *worsening* the residual pool for indiscriminate lenders. As such, counterintuitively, mandatory data sharing can thus *harm* the financially excluded and increase inequality in financial access.

Related literature. At the heart of our model is a *pool externality*: each lender's screening and pricing decision changes the pool of borrowers that subsequent lenders face. This mechanism is the unifying force behind a large theoretical literature on competition between imperfectly informed traders who can select among potential trading partners of heterogeneous quality. This force has been explored in abstract trading models (Lauermann and Wolinsky, 2016; Kurlat, 2016; Kaya and Kim, 2018), in the context of credit markets (Broecker, 1990; Riordan, 1993; Shaffer, 1998; Direr, 2008; Dell'Ariccia and Marquez, 2006; Hauswald and Marquez, 2003, 2006; He et al., 2023; Li and Shimer, 2019; Farboodi and Kondor, 2022, 2023) and in the context of labor markets (Lockwood, 1991; Board et al., 2017; Kurlat and Scheuer, 2020).

The pool externality can take different forms depending on the equilibrium concept. In one-shot auction-like models like Broecker (1990) it may take the form of a winner's curse: the presence of competitors affects the distribution of counterparty quality conditional on

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[2] Similar to how the internet facilitated creation of new markets characterized by rapid innovation and the collection and use of detailed consumer and market data during the tech boom, as discussed in Levin (2011).

actually trading. In dynamic models like Lockwood (1991), the composition of the pool evolves over time as trades take place. Our equilibrium concept is closest to that in Board et al. (2017) and Kurlat and Scheuer (2020): borrowers apply for credit at the best price available and if they are rejected they continue to apply at progressively less favorable prices, so the pool evolves over the course of trading even though no actual time elapses.

Relative to these studies, our paper adds two new ingredients. First, we let ex-ante identical lenders choose the precision of their information, generating an endogenous distribution of lender skill rather than treating lenders as ex-ante asymmetric. Second, we analyse two polar information structures, independent and nested screening errors, in a unified framework. We show that the standard "cream-skimming" form of the pool externality (screening by one trader worsens the pool for others) and a less-studied "cleansing" form (less-skilled traders *improve* the pool for others) are two possibilities within this framework, and which of them prevails depends on the correlation stucture of traders' information.

The closest precendents for the cleansing form of the externality are in our own earlier work. Kurlat (2016) analyzes a knife-edge case where, because signals are nested and false-positive-only, trading by less-informed traders is neutral for subsequent more-informed traders. In Farboodi and Kondor (2022), cleansing emerges in a credit market where lenders make only one either false-positive or false-negative errors; Farboodi and Kondor (2023) further develop the cleansing mechanism in a dynamic credit-cycle setting. The present paper generalizes these in two dimensions. First, those only papers consider pure false-poistive or false-negative signals; we consider the case where lenders make both types of mistakes while still allowing us to contrast nested and independent information. Second, those papers take the wealth distribution of lenders as exogenous; we show that when ex-ante identical lenders choose their precision, a unique heterogeneous distribution emerges endogenously, which makes it possible to analyse entry and directed technological change. Outside our own work, the closest theoretical antecedent is Kleinberg and Raghavan (2021), who study an exogenous monoculture algorithm in a selection market and show informally that correlated errors can "lock out" rejected applicants from subsequent decision-makers; we capture this lock-out in a fully-specified equilibrium with endogenous prices, endogenous screening choices, and endogenous market structure.

Two further features of our equilibrium are distinctive. First, our hockey-stick equilibrium features the endogenous coexistence of pooling and separating segments within a single equilibrium, a combination that does not arise the methodologically similar literature, as far as we know. The closest precedent is Inderst and Mueller (2006), who obtain pooling-and-separating coexistence from a security-design mechanism rather than from the pool externality. Second, ex-ante identical lenders endogenously specialize into a continuous distribution of screening precisions and serve different market segments at different rates, a form of endogenous market structure for which we are not aware of a direct precedent.

We also contribute to the recent theoretical literature on Open Banking and the use of borrowers' data in credit markets (Goldstein et al., 2022; He et al., 2023; Parlour et al., 2022; Babina et al., 2025). Relative to that literature, which typically models data sharing as a binary informed/uninformed shift, our framework allows a directed cost reduction along a continuous precision spectrum; this is what generates the sign flip between limited and broad Open Banking adoption that we document in Section 4.

Outline. Section 1 presents our model. Section 2 presents the equilibrium concept, constructs equilibria under both information structures, and compares the implied market structures. Section 3 introduces new entrants and characterizes their short-run impact and provides illustrating examples. Section 4 presents the implications of the model about the direct and indirect impact of directed technological change in the credit market, through the lens of two applications. Section 5 concludes.

1 The Economy

There are two dates, $t = 1, 2$ and two types of agents: lenders and borrowers. Borrowers borrow at $t = 1$, promising to pay back in period $t = 2$.

Each borrower has a two-dimensional type, (τ, ω). The first dimension, $\tau \in \{G, B\}$ controls borrower repayment. A good borrower ($\tau = G$) pays back fully, while a bad borrower ($\tau = B$) defaults. The second dimension ω is only relevant for describing the information that lenders observe about borrowers. In the case of a nested information structure, ω can be interpreted as a measure of the borrower's "opacity", for reasons that will be made clear below. The measures of good and bad borrowers are denoted by $\mathscr{G}(\omega)$ (with density $\mathscr{g}(\omega)$) and $\mathscr{B}(\omega)$ (with density $\mathscr{b}(\omega)$) respectively.

Any borrower who is offered a loan at rate r will borrow $D(r)$, regardless of their type, and then pay back or not depending on their type. The most straightforward interpretation of why the loan demand of all types is the same is that borrowers do not know their own type at the time of borrowing. Appendix D presents an alternative micro-foundation where borrowers know their type but a collateral constraint determines the same borrowing limit for each type.

Lenders are ex-ante identical. They have an endowment of 1, which they can consume or lend out, are risk neutral, and do not discount the future. The total mass of lenders is $\bar{W}$. Lenders can choose to screen borrowers. For each borrower, they observe a noisy binary signal $s \in \{g, b\}$. These signals are governed by a function $s : \{G, B\} \times [0, 1] \to [0, 1]$. The value $s(\tau, \omega)$ is the probability with which a lender will observe $s = g$ when evaluating a lender of type (τ, ω). Conditional on (τ, ω), realizations of signals are independent across all lender-borrower pairs. Lenders may choose to only lend to borrowers for whom they observe $s = g$. We refer to this as being "selective".

We will focus on examples where a lender's screening ability is indexed by a scalar $\alpha \in [0, 1]$, which governs the precision of their information. We denote the function that governs the signals observed by a lender of ability α as $s(\tau, \omega; \alpha)$. Precision is chosen individually by each lender at a cost $C(\alpha)$.[3] The cost is specified directly in utility terms, so it does not subtract from the endowment they can lend. $C(\alpha)$ is strictly increasing and continuous, with $C(0) = 0$. We will refer to lenders choosing higher α as more skilled. Two special cases are of particular interest: a nested information structure and an independent information structure.

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[3] For comparison,in Appendix C we discuss the special case where the distribution of lenders' screening technology is exogenously given.

Definition 1 **(Nested Information structure).** *The function* $s(\tau, \omega; \alpha)$ *is:*

$$s(\tau, \omega; \alpha) = \begin{cases} 1 & \text{if } \tau = G, \omega < \omega_g(\alpha) \\ 0 & \text{if } \tau = G, \omega \geq \omega_g(\alpha) \\ 0 & \text{if } \tau = B, \omega < \omega_b(\alpha) \\ 1 & \text{if } \tau = B, \omega \geq \omega_b(\alpha) \end{cases}$$

where

$$\omega_g(\alpha) \equiv \beta + \alpha(1 - \beta)$$
$$\omega_b(\alpha) \equiv (1 - \beta) + \alpha\beta$$

for some $\beta \in [0, 1]$.

Figure 1 illustrates the information structure. A lender with skill α observes the correct g signal with probability 1 for G borrowers with $\omega < \omega_g(\alpha)$ and the correct b signal with probability 1 for B borrowers with $\omega < \omega_b(\alpha)$, and the incorrect signal otherwise. Both thresholds move to the right with higher α, so higher skill reduces both Type I and Type II errors.

If there is a distribution of skill α among lenders, higher-ω borrowers will be misclassified by a wider range of lenders, which is why we refer to ω as a measure of opacity. Intuitively, an opaque bad borrower might be able to provide rich documentation which for a low-skilled lender looks immaculate, leading to a false positive mistake. At the same time, an opaque good borrower might have irregular documentation and this is why low-skilled lenders mistakenly take them as bad.

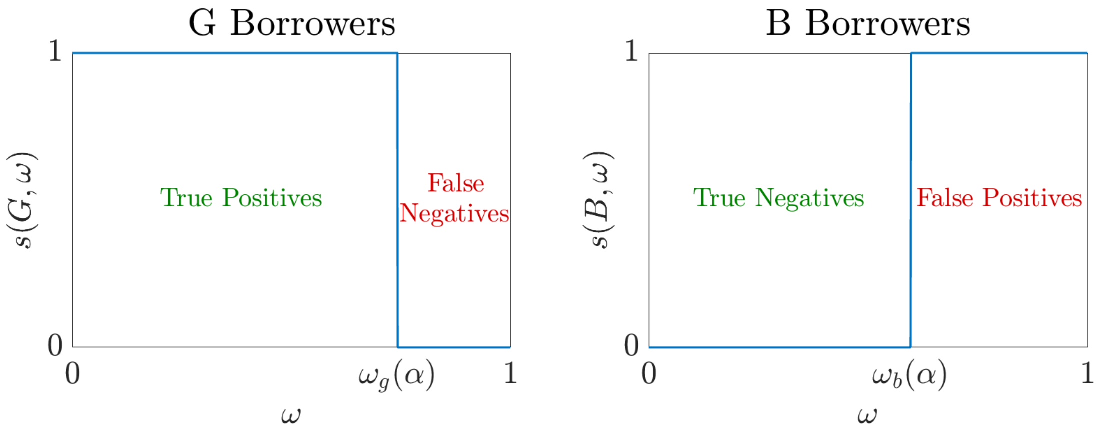

Figure 1: Nested information structure

The parameter β governs the relative frequency of Type I (false positive) and Type II (false negative) errors. Higher β moves the ω_g threshold to the right and the ω_b threshold to the left, resulting in fewer false negatives but more false positives. For $\beta = 0$, we have $\omega_g = \alpha$ and $\omega_b = 1$, so the lender makes only false negative mistakes. Conversely, for $\beta = 1$, we have $\omega_g = 1$ and $\omega_b = \alpha$, so the lender makes only false positive mistakes. Farboodi and

Kondor (2023) associate (the extreme values of) parameter β with aggregate business cycle conditions, with false positives in good times and false negatives in bad times. Here instead we keep β fixed.

Throughout, we adopt the natural normalization that zero skill conveys no information about borrower type:

$$\frac{\mathcal{G}(\beta)}{\mathcal{G}(1)}+\frac{\mathcal{B}(1-\beta)}{\mathcal{B}(1)} \;=\; 1.$$

This is automatic under uniform priors, where the left-hand side reduces to $\beta+(1-\beta)=1$.

Importantly, under Definition 1, lenders make correlated mistakes. Two lenders choosing the same skill observe identical signals on the same borrower. For two with different skill, the less-skilled lender makes all the Type I and Type II to errors of the more-skilled lender plus some more. We call this property *nestedness* and it plays a crucial role in our analysis.

The second special case we focus on has the opposite property: lenders make uncorrelated mistakes.

Definition 2 (Independent Information structure). *Using the borrower measure functions $\mathcal{G},\mathcal{B}$ (with strictly positive densities $g \equiv \mathcal{G}'$ and $\mathscr{b} \equiv \mathcal{B}'$), define the conditional probabilities of a correct signal:*

$$\Omega_g(\alpha) \equiv \frac{\mathcal{G}(\omega_g(\alpha))}{\mathcal{G}(1)}, \qquad\qquad \Omega_b(\alpha) \equiv \frac{\mathcal{B}(\omega_b(\alpha))}{\mathcal{B}(1)}. \tag{1}$$

The function $s(\tau,\omega;\alpha)$ is:

$$s\left(\tau,\omega;\alpha\right)=\begin{cases} \Omega_g(\alpha) & \text{if } \tau=G \\ 1-\Omega_b(\alpha) & \text{if } \tau=B \end{cases}$$

Figure 2 illustrates the independent information structure. By construction, the unconditional probabilities of Type I and Type II errors are exactly the same as in the nested case. A randomly selected good borrower is correctly classified with probability $\mathcal{G}(\omega_g(\alpha))/\mathcal{G}(1)=\Omega_g(\alpha)$, and a randomly selected bad borrower is correctly classified with probability $\mathcal{B}(\omega_b(\alpha))/\mathcal{B}(1)=\Omega_b(\alpha)$. (For the special case of a uniform distribution, we have $\Omega_g=\omega_g$ and $\Omega_b=\omega_b$). As in the nested case, higher α means higher Ω_g and Ω_b, reducing both types of mistakes; higher β means higher Ω_g but lower Ω_b, resulting in fewer false negatives but more false positives.

The difference with the nested case is that now mistakes are not correlated across lenders or across borrowers: all type τ borrowers have the same chances of misleading lenders, and the second dimension of borrower types plays no role. Any difference in outcomes between the two information structures is purely a result of this difference rather than a difference in the quality of information.

Other information structures (i.e. other $s(\tau,\omega)$ functions) are possible, and we define equilibrium for a general case, but our analysis will focus on these two polar cases.

2 Equilibrium

Equilibrium works as follows. First, lenders simultaneously choose their precision α. Then the lending markets open. Each possible interest rate r defines a different market. Borrowers

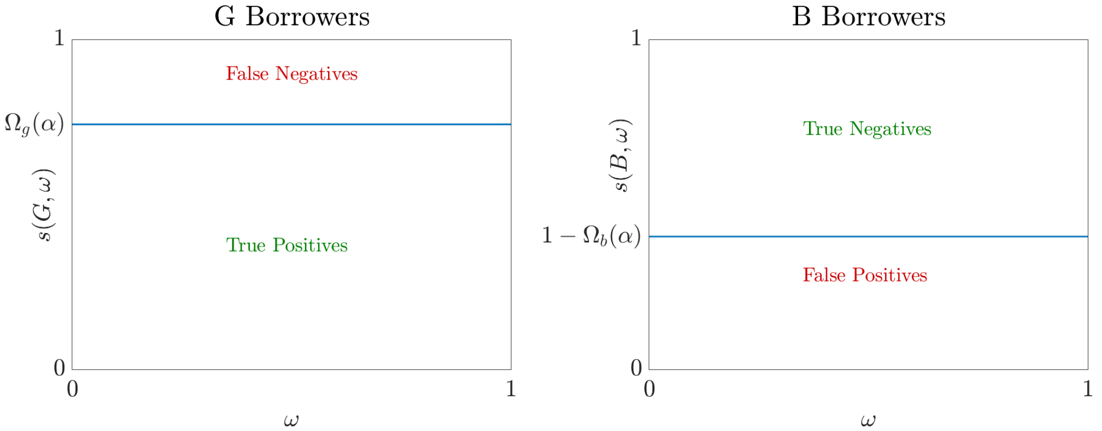

Figure 2: Independent information structure

submit applications to various markets sequentially, starting from the lowest interest rate. If their application is accepted in market r, they borrow $D(r)$ and exit; if it is rejected, they continue to apply to higher interest rates.

Lenders who choose to lend in market r have to decide whether to be selective, in which case they only lend to applicants for whom they observe $s = g$, or non-selective, in which case they lend to anyone. Each lender has one unit of capital and makes loans of size $D(r)$, so it finances $1/D(r)$ borrowers, drawn at random from the applicants it finds acceptable.

If many lenders lend in the same market, the pool of applicants each lender faces depends on the order in which they lend, since borrowers who have already been served exit the pool. We will assume that lenders are queued in order of increasing α, so those with lower precision go first (and non-selective lenders before everyone else). We later show that, perhaps surprisingly, in the nested information case all lenders prefer this ordering, so that if we generalized our definition of equilibrium to encompass an endogenous ordering, as in Kurlat (2016), this is the ordering that would emerge. In the independent information case, instead, we show each lender visits a different market (except perhaps those with perfect information) so in equilibrium the issue of how to order them does not arise.

Because each lender has limited capital, the two sides of a market need not balance, and we close the protocol with the standard short-side rule. If acceptable applicants are the short side, they are all served, and any capital remaining once the acceptable pool is exhausted stays idle and earns zero (so its owner earns $-C(\alpha)$ net of cost).[4] If capacity is the short side, lenders deploy in full and the unserved applicants apply at higher rates. A borrower thus moves up the rate ladder for either of two reasons: no active lender found him acceptable, or he was acceptable but capacity was exhausted before his turn.

We use the following notation. The functions $r(\alpha)$ and $z(\alpha)$ denote, respectively, the choice of market and selectivity by a lender with precision α, with $z(\alpha) = 1$ representing the decision to be selective and $z(\alpha) = 0$ the choice to be nonselective. The function $\gamma\left(r, z, \alpha\right)$ denotes the probability that a borrower accepted by a lender with precision α and selectivity

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[4] As we will see, this eventuality never happens in equilibrium. A lender expecting rationing would offer a lower rate instead and lend out all its capital.

z in market r is a good borrower. The measures $G(\cdot; r, z, \alpha)$ and $B(\cdot; r, z, \alpha)$ (defined over the space of opacity $\omega \in [0,1]$) denote how many good and bad borrowers respectively of each opacity are in the pool of applicants in market r by the time it's the turn of lender α with selectivity z. The measure $W(\cdot)$ (defined over the space of precision $\alpha \in [0,1]$) denotes how many lenders choose each precision.

The problem of a lender can be divided into two parts. Conditional on a given precision α, the lender must choose a market r and selectivity z to solve:

$$\tilde{\Pi}(\alpha) = \max_{r,z} \gamma(r, z; \alpha)(1+r) - 1 \tag{2}$$

The lender lends out 1 and, with probability $\gamma(r, z; \alpha)$, gets $1+r$ in return, so the expected gross profit is $\tilde{\Pi}(\alpha)$. The choice-of-precision problem is to maximize net profit, that is, gross profit minus the cost of precision:

$$\Pi = \max_{\alpha} \tilde{\Pi}(\alpha) - C(\alpha). \tag{3}$$

Any precision α chosen in equilibrium by a positive measure of lenders must imply the same profit Π. We later show that Π is decreasing in the total mass of lenders, $\bar{W}$. In this sense, $\bar{W}$ indexes the intensity of lender competition in our economy.

The quality $\gamma(r, z; \alpha)$ faced by the lender can be computed as follows. Define

$$S(\tau, \omega; \alpha, z) = \begin{cases} s(\tau, \omega; \alpha) & \text{if } z = 1 \\ 1 & \text{if } z = 0 \end{cases}$$

$S(\tau, \omega; \alpha, z)$ is the probability that a lender of ability α and selectivity z will find a borrower of type (τ, ω) acceptable. If the lender, is not selective, this probability is equal to 1; if the lender is selective, this equals the probability of observing a signal g.

Now let X^G and X^B be any subsets of good and bad borrowers. If lender α is chooses z in market r, the probability of getting a borrower who belongs in one of these subsets is, respectively:

$$\Pr{}^G\left(X^G; r, z, \alpha\right) = \frac{\int_{\omega \in X^G} S(G, \omega, z)\, dG(\omega; r, z, \alpha)}{\int S(G, \omega, z)\, dG(\omega; r, z, \alpha) + \int S(B, \omega, z)\, dB(\omega; r, z, \alpha)} \tag{4}$$

$$\Pr{}^B\left(X^B; r, z, \alpha\right) = \frac{\int_{\omega \in X^B} S(B, \omega, z)\, dB(\omega; r, z, \alpha)}{\int S(G, \omega, z)\, dG(\omega; r, z, \alpha) + \int S(B, \omega, z)\, dB(\omega; r, z, \alpha)} \tag{5}$$

when the denominators are positive, or zero otherwise. The denominators in (4) and (5) are the measure of all borrowers that are acceptable to lender α, and the numerators are the measures in subsets X^G and X^B respectively. Using (4) and (5), the probability that a lender α with selectivity z in market r gets a good borrower is:

$$\gamma(r, z; \alpha) = \Pr{}^G([0,1]; r, z, \alpha) \tag{6}$$

It remains to compute the measures $G(\cdot; r, z, \alpha)$ and $B(\cdot; r, z, \alpha)$. For this we need to subtract from the original pool of borrowers those who have been served in lower-r markets or in market r by lower-α or non-selective lenders. Let

$$A(r, z, \alpha) = \{\tilde{\alpha} : r(\tilde{\alpha}) < r\} \cup \{\tilde{\alpha} : r(\tilde{\alpha}) = r, z(\tilde{\alpha})\tilde{\alpha} < z(\alpha)\alpha\} \tag{7}$$

be the set of lenders that choose a lower-interest-rate market than r or choose r but pick before α. Recall that each of these lenders lends to $1/D(r(\alpha))$ borrowers in the market they visit, distributed across opacity levels according to (4) and (5). Hence, the distributions faced by lender α with selectivity z in market r are:

$$G\left(X^G; r, z, \alpha\right) = \mathcal{G}\left(X^G\right) - \int_{A(r,z,\alpha)} \Pr{}^G\left(X^G; r\left(\alpha\right), z\left(\alpha\right), \alpha\right) \frac{1}{D\left(r\left(\alpha\right)\right)} dW\left(\alpha\right) \tag{8}$$

and

$$B\left(X^B; r, z, \alpha\right) = \mathcal{B}\left(X^B\right) - \int_{A(r,z,\alpha)} \Pr{}^B\left(X^B; r\left(\alpha\right), z\left(\alpha\right), \alpha\right) \frac{1}{D\left(r\left(\alpha\right)\right)} dW\left(\alpha\right) \tag{9}$$

Although the market clearing protocol sounds dynamic—borrowers applying market by market and lenders drawing in queue order as the pool is progressively depleted—it is encoded entirely in static objects, and no time actually elapses. The set $A(r, z, \alpha)$ in (7) records the order, collecting the lenders who act before lender α: those at lower rates, and those at the same rate but earlier in the queue. Given any candidate profile $(W, r(\cdot), z(\cdot))$, equations (8) and (9) then map that profile into the pool of applicants $G(\cdot; r, z, \alpha)$ and $B(\cdot; r, z, \alpha)$ that each lender faces, by subtracting from the primitive pool everything already served by the lenders in $A(r, z, \alpha)$. The information regime enters this mapping in exactly one place: through the acceptance function S—the general counterpart of s—in (4) and (5), which fixes who is acceptable to whom and hence the composition of what each lender draws. Everything else, including the order bookkeeping in $A(r, z, \alpha)$ and the capital constraint $1/D(r)$, is common to both information structures. This is what makes the comparison in the rest of the section clean: independent and nested information are the same construction with different S.

We can now formally define an equilibrium:

Definition 3 (**Equilibrium**). *The equilibrium consists of*

1. *A measure W over lender skill precision such that $W([0, 1]) = \bar{W}$,*
2. *A choice-of-market function $r(\alpha)$ and a choice-of selectiveness function $z(\alpha)$ for each lender α in the support of W,*
3. *Measures of good and bad borrowers available to lender α with selectivity z in market r: $G(\cdot; r, z, \alpha)$ and $B(\cdot; r, z, \alpha)$*

such that

1. *Given α, $r(\alpha)$ and $z(\alpha)$ solve the lender's problem* (2)*, with γ defined by* (4), (5) *and* (6)*,*
2. *Every α in the support of W solves* (3)*,*
3. *The measures $G(\cdot; r, z, \alpha)$ and $B(\cdot; r, z, \alpha)$ satisfy* (8) *and* (9) *respectively.*

2.1 Equilibrium Construction I: The Independent Information Case

In this section, we construct the equilibrium in our economy when the information structure satisfies Definition 2. The next proposition states the main result. Then we present its contsruction and spell out the economic intuition behind it.

Proposition 1 (Equilibrium: Independent Information). *There is a unique equilibrium with the following properties.*

1. *All lenders are selective, $z(\alpha) = 1$.*
2. *$W(\cdot)$ has continuous support in the range $[\alpha_0, \bar{\alpha})$ and an atom at $\alpha = 1$ with some thresholds on lender screening precision: α_0, $\bar{\alpha}$ satisfying $0 \leq \alpha_0 \leq \bar{\alpha} \leq 1$. When $\bar{\alpha} \neq 1$, no lender chooses a skill level in the interval $[\bar{\alpha}, 1)$.*
3. *The equilibrium interest rate schedule in the range $[\alpha_0, \bar{\alpha})$ is a strictly increasing function $r(\alpha)$ and $r(\bar{\alpha}) = r(1)$.*
4. *At each interest rate $r(\alpha)$, $\alpha < 1$ both bad and good borrowers are served. At $r(1)$ only good borrowers are served.*
5. *The repayment probability $\gamma\left(r(\alpha), 1; \alpha\right)$ is strictly decreasing in α at α_0.*

If in addition, we specify a regularity condition[5] *in Appendix A which is sufficient for $\gamma\left(r(\alpha), 1; \alpha\right)$ being quasi-convex in α on $[\alpha_0, \bar{\alpha}]$: it can changes monotonicity at most once, and any critical point is a minimum. When $\bar{\alpha} = 1$, γ is U-shaped with $\gamma\left(r(1), 1, 1\right) = 1$.*

We construct the equilibrium by finding the measure $W(\alpha)$ and the function $r(\alpha)$ that describe lenders' choices of skill and interest rate respectively. We refer to the market where the interest rate is $r(\alpha)$ simply as market α (we show below that $r(\alpha)$ is strictly increasing, so this is well defined).

As a preliminary result, we show there cannot be a positive measure of lenders who choose the same interest rate, unless they are perfect screeners. Suppose to the contrary that a lender $\alpha \in [0, 1)$ lends in a market where a positive measure of selective lenders with $\alpha \in A \subseteq (0, 1]$ is ahead of him in the queue, and lender α is given the option of moving ahead of them. Let γ be the average quality lender α gets with his original position and γ' be the average quality after moving ahead.

Lemma 1 (Cream-Skimming under Independent Information). *For any $A \subseteq (0, 1]$, $\gamma' > \gamma$*

The reason for Lemma 1 is the standard logic of cream-skimming. Any lenders that pick before lender α will leave behind an adversely-selected remainder, so lender α would strictly prefer to pick before them. This implies that it cannot be part of an equilibrium for many lenders to coexist in the same market r since any of them that is not first in line would gain

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[5] Under uniform priors $\mathcal{G}(\omega) = \mathcal{G}(1)\,\omega$ and $\mathcal{B}(\omega) = \mathcal{B}(1)\,\omega$, the regularity condition is $3\big(C''(\alpha)\big)^2 \geq 2\,C'(\alpha)\,C'''(\alpha)$ for all $\alpha \in [\alpha_0, \bar{\alpha}]$. The condition is satisfied by any cost with non-increasing C''/C' including the linear, quadratic, cubic and exponential cost functions.

by deviating to $r-\epsilon$ to guarantee himself a less adversely selected pool. This unraveling forces lenders to separate across rates, with higher rates compensating for a worse pool. Note that this does not depend on α being chosen endogenously, and would be true even if lenders were exogenously endowed with identical skill. As we show below (Lemma 3), this logic does not extend to the case with nested information.

Now let

$$G(\alpha) \equiv G([0,1], r(\alpha), 1, \alpha) \tag{10}$$

$$B(\alpha) \equiv B([0,1], r(\alpha), 1, \alpha) \tag{11}$$

denote the (endogenous) remaining masses of good and bad borrowers in the pool available to lenders choosing market α, and define the pool quality:

$$q(\alpha) \equiv \frac{G(\alpha)}{G(\alpha)+B(\alpha)} \tag{12}$$

The repayment probability for a lender of skill α who chooses to lend in market $\tilde{\alpha}$ is:

$$\gamma(\alpha,\tilde{\alpha}) = \frac{q(\tilde{\alpha})\Omega_g(\alpha)}{q(\tilde{\alpha})\Omega_g(\alpha)+(1-q(\tilde{\alpha}))(1-\Omega_b(\alpha))} \tag{13}$$

Conjecture a profit level Π that all lenders make in equilibrium and define:

$$K(\alpha) \equiv \Pi + C(\alpha) \tag{14}$$

$K(\alpha)$ is the gross profit that a lender must make in equilibrium to be willing to choose skill α, so it can be interpreted as a modified cost function.

The first step is to find the skill α_0 of the lender offering the lowest interest rate. Since this rate attracts all borrowers, the lender faces the entire initial pool, so no matter what the value of α_0 is, the average quality of the pool is

$$q_0 = \frac{\mathcal{G}(1)}{\mathcal{G}(1)+\mathcal{B}(1)}, \tag{15}$$

and the repayment probability is

$$\gamma_0(\alpha) = \frac{q_0\Omega_g(\alpha)}{q_0\Omega_g(\alpha)+(1-q_0)(1-\Omega_b(\alpha))} \tag{16}$$

From (2) and (3), the interest rate required for the first lender to earn profit Π is:

$$r_0(\alpha) = \frac{K(\alpha)+1}{\gamma_0(\alpha)} - 1 \tag{17}$$

The first lender is whoever can charge the lowest possible interest rate and still make profits Π:

$$\alpha_0 = \arg\min_{\alpha} r_0(\alpha) \tag{18}$$

and

$$r(\alpha_0) = r_0(\alpha_0). \tag{19}$$

The next Lemma and the subsequent discussion help to understand the intuition of how the skill level α_0 of the lender who offers the lowest interest rate $r_0(\alpha_0)$ is determined.

Lemma 2 (Comparative statics of the lowest-rate lender)**.**

(i) At any interior optimum $\alpha_0 \in (0,1)$, a higher cost scale λ (replacing C by λC) raises $r_0(\alpha_0)$ and lowers α_0, and a higher prior pool quality q_0 lowers both $r_0(\alpha_0)$ and α_0.

(ii) There exist thresholds $\bar{\lambda} < \infty$, $\bar{q} < 1$, and $\underline{\beta} > 0$ such that $\alpha_0 = 0$ (and $r_0(\alpha_0) = r_0(0)$) whenever $\lambda \geq \bar{\lambda}$, $q_0 \geq \bar{q}$, or $\beta \leq \underline{\beta}$.

At any interior α_0, the lender's FOC for choosing α, together with equation (17), imply that the proportional marginal cost of skill is equal to that proportional marginal quality gain:

$$\underbrace{\frac{K'(\alpha_0)}{1+K(\alpha_0)}}_{\text{marginal cost of skill}} \quad = \quad \underbrace{\frac{\gamma_0'(\alpha_0)}{\gamma_0(\alpha_0)}}_{\text{marginal quality gain from skill}} \qquad .$$

Each comparative static in Part (i) is a tilt in this balance.

A higher cost scale λ raises the left-hand side at every α: each extra unit of skill is proportionally more costly. The lender shifts down to a lower α_0 where γ_0 is still steep enough to clear the FOC. The lowest break-even rate $r_0(\alpha_0)$ rises because that lender now operates against a higher total cost.

A higher prior pool quality q_0 shrinks the right-hand side in two reinforcing ways: γ_0' falls (fewer bad apples to catch) and γ_0 rises toward 1 (the pool is already good even at $\alpha = 0$). Both push α_0 down, so $r_0(\alpha_0)$ falls.

Part (ii) states that the left hand side of the FOC might be larger for any α than the right hand side pushing the equilbrium into the corner. When λ is large enough, or q_0 close enough to 1, or β close enough to 0, the proportional marginal cost of even the first unit of skill exceeds the proportional marginal gain at $\alpha = 0$. No skill is bought; the marginal lender operates at $\alpha = 0$ and charges the zero-skill rate $r_0(0)$.

We now characterize how lenders choose their skill level and sort across markets with different interest rates, starting at α_0. The profit a lender obtains if they choose skill α and market $\tilde{\alpha}$ (or, equivalently, interest rate $r(\tilde{\alpha})$) is:

$$\Pi(\alpha, \tilde{\alpha}) = \gamma(\alpha, \tilde{\alpha})(1 + r(\tilde{\alpha})) - 1 - C(\alpha) \tag{20}$$

The first order condition with respect to α is:

$$\frac{\partial \gamma}{\partial \alpha}(\alpha, \tilde{\alpha})(1 + r(\alpha)) = C'(\alpha) \tag{21}$$

This says that the marginal benefit of better selection due to higher skill must equal the marginal cost of skill.

The first-order condition with respect to $\tilde{\alpha}$ is:

$$\frac{\partial \gamma}{\partial \tilde{\alpha}}(\alpha, \tilde{\alpha})(1 + r(\alpha)) + \gamma(\alpha, \tilde{\alpha})r'(\alpha) = 0 \tag{22}$$

This says that the worse selection the lender obtains by going to a higher market must be exactly offest by a higher interest rate. In order to evaluate this expression, we need to compute $\frac{\partial \gamma}{\partial \tilde{\alpha}}$, i.e. how much selection worsens at higher markets. Using (13):

$$\frac{\partial \gamma\left(\alpha, \tilde{\alpha}\right)}{\partial \tilde{\alpha}} = q'\left(\tilde{\alpha}\right) \frac{\Omega_g\left(\alpha\right)\left(1 - \Omega_b\left(\alpha\right)\right)}{\left[q\left(\tilde{\alpha}\right)\Omega_g\left(\alpha\right) + \left(1 - q\left(\tilde{\alpha}\right)\right)\left(1 - \Omega_b\left(\alpha\right)\right)\right]^2} \tag{23}$$

Substituting (23) into the market-choice FOC (22) relates the slope of the interest-rate schedule $r'(\alpha)$ to the rate at which pool quality deteriorates, $q'(\alpha)$. This single equation, however, does not pin down the equilibrium: it links $r'(\alpha)$ and $q'(\alpha)$ but determines neither on its own.

In order to solve for both $r'(\alpha)$ and $q'(\alpha)$, we need to ensure that (21), the FOC for α, holds. Inspecting (13), the FOC imposes a relationship between the $r(\alpha)$ and $q(\alpha)$: the marginal benefit of additional skill depends on both the quality of the pool and the interest rate. At α_0, (21) holds by construction. In order to ensure that it is satisfied for every α, we compute the total derivative with respect to α and set it to zero. Informally, this ensures that if the FOC holds when evaluated at α, it also holds when evaluated at $\alpha + d\alpha$. In Appendix A, we show that this results in a second differential equation which, when paired with the market-choice FOC (22), can be reduced to a single ODE in terms of $q(\alpha)$.

The ODE[6] is:

$$q'(\alpha) = \left(\begin{array}{c} \left(\frac{C''(\alpha)}{C'(\alpha)} - \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}\right)\left[q(\alpha)\Omega_g(\alpha) + \left(1 - q(\alpha)\right)\left(1 - \Omega_b(\alpha)\right)\right] \\ +2\left(q(\alpha)\Omega_g'(\alpha) - \left(1 - q(\alpha)\right)\Omega_b'(\alpha)\right) \end{array} \right) \frac{q(\alpha) - 1}{\Omega_g(\alpha)} \tag{24}$$

where

$$\Gamma(\alpha) \equiv \Omega_g'(\alpha)\bigl(1 - \Omega_b(\alpha)\bigr) + \Omega_b'(\alpha)\,\Omega_g(\alpha), \tag{25}$$

with initial condition $q\left(\alpha_0\right) = q_0$. This ODE pins down a path $q(\alpha)$ which is consistent with the first-order conditions. Then, using (13) and the equiprofit condition $\Pi = \gamma(\alpha, \alpha)(1 + r(\alpha) - 1 - C(\alpha))$, we can calculate the corresponding interest rate path $r(\alpha)$. When the resulting $q(\alpha)$ is monotonically decreasing in the range $\alpha \in [\alpha_0, 1]$, then the implied $r(\alpha)$ is monotonically increasing.

It remains to recover the density of entering lenders $w(\alpha)$ that supports this path. Suppose the density at α is $w(\alpha)$. Then approximately $w(\alpha)d\alpha$ lenders choose skills in the interval $[\alpha, \alpha + d\alpha]$, and they lend to $\frac{w(\alpha)d\alpha}{D(r(\alpha))}$ borrowers, of whom a fraction $\frac{w(\alpha)d\alpha}{D(r(\alpha))}$ good. Therefore:

$$G'(\alpha) = -\frac{w(\alpha)}{D(r(\alpha))}\gamma(\alpha, \alpha) \tag{26}$$

$$B'(\alpha) = -\frac{w(\alpha)}{D(r(\alpha))}[1 - \gamma(\alpha, \alpha)] \tag{27}$$

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[6] Under uniform priors $\mathscr{G}(\omega) = \mathscr{G}(1)\,\omega$ and $\mathscr{B}(\omega) = \mathscr{B}(1)\,\omega$, the probabilities are linear, $\Omega_g(\alpha) = \beta + \alpha(1 - \beta)$ and $\Omega_b(\alpha) = (1 - \beta) + \alpha\beta$, and $\Gamma(\alpha) = \beta$ is constant. The ODE (24) then reduces to

$$q'(\alpha) = \left[\frac{C''(\alpha)}{C'(\alpha)}\bigl(q\alpha + \beta(1 - \alpha)\bigr) + 2\bigl(q - \beta\bigr)\right] \frac{q - 1}{\beta + \alpha(1 - \beta)}.$$

Using (26), (27) and (12), we solve for the evolution of pool quality:

$$q'(\alpha) = -\frac{w(\alpha)}{D(r(\alpha))}\frac{\gamma(\alpha,\alpha) - q(\alpha)}{G(\alpha) + B(\alpha)} \tag{28}$$

Note that $\gamma(\alpha, \alpha) > q(\alpha)$ trivially holds, i.e. a lender with positive skill gets a better quality portfolio than the unselected pool. Therefore pool quality must worsen, as long as positive density of lenders enters, $w(\alpha) > 0$. Furthermore, the deterioration of selection is proportional to the density of lenders who enter at skill α: the more lenders enter and draw from the pool, the faster the quality worsens.

Replacing (23) and (28) into (22), we obtain:

$$r'(\alpha) = \frac{w(\alpha)}{D\left(r(\alpha)\right)} \cdot \frac{1 + r(\alpha)}{\gamma\left(\alpha, \alpha\right)} \cdot \frac{\gamma\left(\alpha, \alpha\right) - q(\alpha)}{G(\alpha) + B(\alpha)} \cdot \frac{\Omega_g(\alpha)\left(1 - \Omega_b(\alpha)\right)}{\left(q(\alpha)\Omega_g(\alpha) + \left(1 - q(\alpha)\right)\left(1 - \Omega_b(\alpha)\right)\right)^2} \tag{29}$$

Equation (29) gives a relationship between $r'(\alpha)$, the slope of the relationship between skill and interest rates, and $w(\alpha)$, the density of lenders who enter with skill α. If more lenders enter at skill α, the quality of the pool worsens faster, so a steeper increase in interest rates is needed to make up for it.

Together with the implied $w(\alpha) > 0$ by (26)-(28), we obtain an equilibrium illustrated on Figure 3.

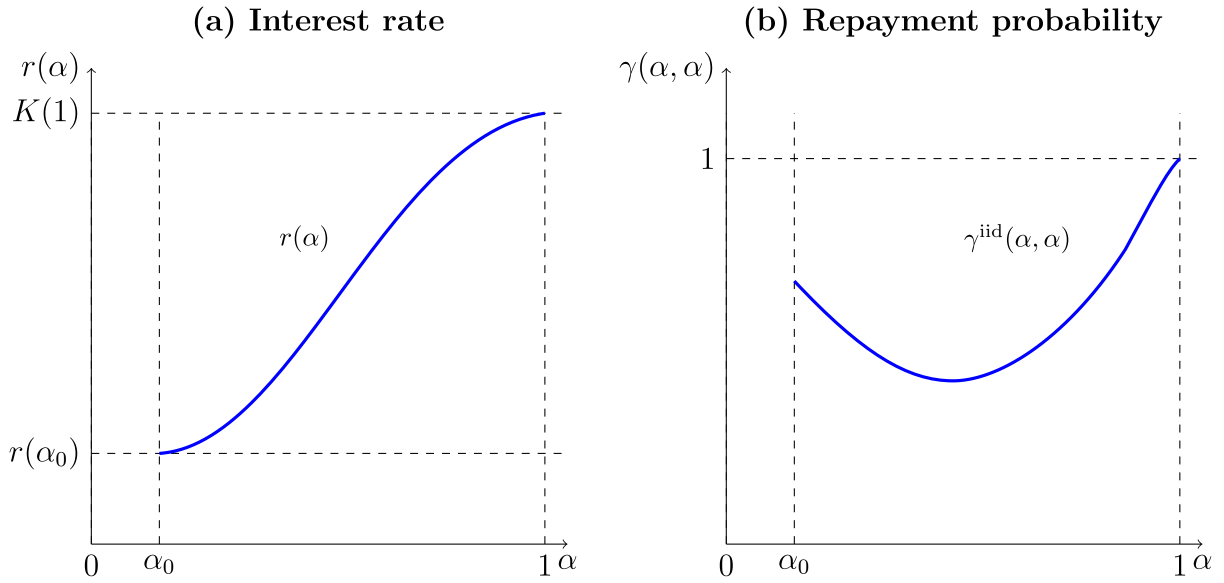

Figure 3: Equilibrium under independent information. Panel (a): The interest rate $r(\alpha)$ is strictly increasing. Panel (b): The repayment probability $\gamma(\alpha, q(\alpha))$ is U-shaped, initially decreasing due to adverse selection, then increasing as higher-skilled lenders achieve better screening.

The left panel depicts the interest rate path offered by lenders with skill α. This path hits $r(1) = K(1)$ at $\alpha = 1$. This is intuitive as this is the level of interest rate at which perfects screeners, $\alpha = 1$ are making Π profit. These lenders will obtain a clean portfolio of good borrowers, ($\gamma(1,1) = 1$) regardless of the pool quality $q(1)$.[7]

This is a relatively standard credit market equilibrium with (endogenously) heterogenous skill.[8] As we move to higher-α markets, pool quality $q(\alpha)$ deteriorates because lower-α lenders cream-skim good borrowers. This adverse selection, along with the higher cost for better skill necessary to compensate for it, forces lenders to charge higher interest rates to obtain the same net profit Π. Approaching the market $r(1)$, some good borrowers are still in the available pool as lenders with $\alpha < 1$ all make mistakes. The mass of the entering perfect screeners must be such that they can absorb all the remaining good borrowers $G(1)$.

The right panel depicts the corresponding repayment probability $\gamma(\alpha, \alpha)$. Initially, the worsening pool quality dominates and lenders with higher skill obtain a worse quality portfolio. However, given that this function has to converge to 1 for perfect screeners, there must be a point when the higher skill dominates, and higher skill agents can obtain a better quality portfolio. The regularity condition guarantees that this function changes monotonicity only once giving the U-shape pattern. When this condition does not hold, $\gamma(\alpha, \alpha)$ might change monotonicity an odd number of times.

2.2 Nested Information Structure

In this section, we analyze what happens when information is nested as in Definition 1. In Section 2.2.1, we provide the formal equilibrium construction and in Section 2.2.2 we discuss the economic logic that makes the equilibrium behave the way it does.

2.2.1 Equilibrium Construction

The market structure that emerges is segmented in a way that we label a "hockey stick interest rate schedule"[9].

Proposition 2 (Equilibrium: The hockey stick interest rate schedule). *There is a unique equilibrium defined by thresholds on lender screening precision:* α_0*,* α_1*,* α_2 *satisfying* $0 \leq \alpha_0 \leq \alpha_1 \leq \alpha_2 \leq 1$ *and a measure of lenders* $W(\cdot)$*, which may have an atom at* $\alpha = 0$ *and otherwise has a continuous support in the range* $[\alpha_0, \alpha_2]$*.*

---

[7] Proposition 1 allows for a variant of this equilibrium with the difference that no lenders choose skill in the interval $\alpha \in [\bar{\alpha}, 1)$. This arises when the implied $q(\alpha)$ by (24) has an increasing section. By (28), this would imply $w(\alpha) < 0$ which is not economically meaningful. We show, that in this case, there is always an $\bar{\alpha} < 1$ for which the implied interest rate is $r(\bar{\alpha}) = K(1)$ the same what perfect screeners would offer. Furthermore, in the interval $\alpha \in [\alpha_0, \bar{\alpha}]$, $q(\alpha)$ is monotonically decreasing and $r(\alpha)$ is monotonically increasing. Then, any lender with skill $\alpha \in [\bar{\alpha}, 1)$ would need to charge $r > K(1)$ to break even. But in this case borrowers would instead choose to be perfect screeners. The mass of entering perfect screeners must be still such that they can absorb all the remaining good borrowers $G(\bar{\alpha})$, so that no perfect screener holds idle capital, while the bad borrowers remaining in the pool go unfinanced because none is acceptable to a perfect screener.

[8] The model generating the most similar equilibrium is Board et al. (2017). The main difference, apart from their exogenous information structure, is that they focus on an economy where highest skill is associated with the best quality pool. We have the opposite.

[9] Under a loose interpretation of what a hockey stick looks like.

The equilibrium interest rate schedule consists of (at most) three segments, ordered by increasing interest rates:

1. **Region I:** *A low interest rate r_p where both good and bad borrowers borrow.*

   Every transparent good borrower with $\omega \leq \omega_g(\alpha_1)$, every opaque bad borrower with $\omega > \omega_b(\alpha_1)$ and some moderately-opaque bad borrowers with $\omega \in [\omega_b(\alpha_0), \omega_b(\alpha_1)]$ borrow at r_p.

   Lenders with intermediate degrees of skill, $\alpha \in [\alpha_0, \alpha_1]$ lend at r_p. All lenders in this market are selective.

2. **Region II:** *An increasing interest rate schedule for good borrowers only.*

   Every moderately-opaque good borrower, with $\omega \in [\omega_g(\alpha_1), \omega_g(\alpha_2)]$ borrows at a single interest rate $r(\omega)$ within this range, where $r(\omega)$ is increasing. No bad borrower borrows in this range.

   Lenders with high skill, $\alpha \in [\alpha_1, \alpha_2]$, lend in this segment. All lenders in these markets are selective.

3. **Region III:** *A high interest rate r_{NS} where both good and bad borrowers borrow.*

   All opque good borrowers with $\omega > \omega_g(\alpha_2)$, all transparent bad borrowers with $\omega < \omega_b(\alpha_0)$ and some moderately-opaque bad borrowers with $\omega \in [\omega_b(\alpha_0), \omega_b(\alpha_1)]$ borrow at r_{NS}.

   Lenders with the lowest technology level, $\alpha = 0$ who are non-selective lend at r_{NS}.

The interest rate schedule is continuous in ω for good borrowers, that is $r(\omega_g(\alpha_1)) = r_p$ and $r(\omega_g(\alpha_2)) = r_{NS}$. If Region III exists, all borrowers, good and bad, are served by some lenders.

Figure 4 shows and example of a hockey-stick schedule. The left panel shows the interest rate at which good borrowers obtain credit, as a function of their opacity ω, which has the hockey-stick shape. The right panel shows the interest rate chosen by lenders, as a function of their skill α.

In order to construct the equilibrium, we first verify the property that is built into equation (7) in our definition of equilibrium: if lenders with different skill who lend in the same market could choose in what order to lend, as in the definition of equilibrium from Kurlat (2016), they would all choose to be ordered by increasing α. Suppose a given lender α lends in a market where other lenders are also active. Starting from any ordering, lender α is given the option of moving further back in the queue, letting other lenders with types in some set $A \subseteq [0, 1]$ in front of him . Let γ be the average quality lender α gets with his original position and γ' the average quality he gets if he moves back.

Lemma 3 (Endogenous Ordering). *If $\tilde{\alpha} < \alpha$ for all $\tilde{\alpha} \in A$, then $\gamma' > \gamma$. Conversely, if $\tilde{\alpha} > \alpha$ for all $\tilde{\alpha} \in A$, then $\gamma' < \gamma$.*

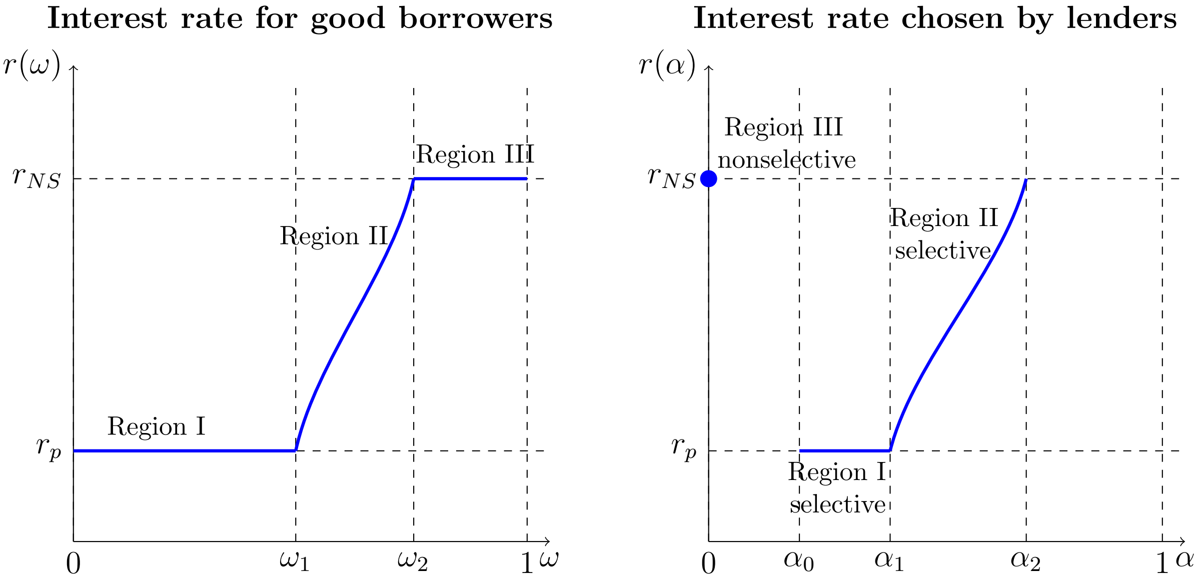

Figure 4: A hockey-stick interest rate schedule

Lemma 3 says that any lender α benefits from lending after lower-skilled lenders but is hurt by lending after higher-skilled lenders. The fact that they prefer to come before more-skilled lenders is standard: more skilled lenders pick out good borrowers, leaving behind an more adversely selected pool. What is perhaps more surprising is the lenders are happy to come after less-skilled colleagues. Would they not also leave a somewhat adversely selected sample? The reason this is not undesirable has to do with the way the information is nested. A less-skilled lender lends to a subset of the good borrowers and a superset of the bad borrowers that are acceptable to a more-skilled lender.

The top row of Figure 5 illustrates this by showing how the pool of good and bad borrowers that lender $\alpha_0 + \Delta\alpha$ faces changes if more capital with a lower precision, α_0, enters. Before any entry, the pool is the unit rectangle in each panel. A lender with a given precision α will accept bad borrowers with $\omega \geq \omega_b(\alpha)$ and good borrowers with $\omega \leq \omega_g(\alpha)$. These thresholds are denoted by the vertical red and green dashed lines for lenders with precision α_0 and $\alpha_0 + \Delta\alpha$, respectively, in each panel. The lender with precision α_0 accepts borrowers above the solid red line. The arrows show that the size of these rectangles depends on the mass of borrowers $w(\alpha_0)$ entering at that precision. More entry pushes the solid red line downwards. If $w(\alpha_0) = 0$, the second group of lenders with higher precision would face the original pool and could accept all the borrowers in the grey rectangles (including the hatched areas) on each panel plus the green rectangle on the right panel. That is, $\hat{B}$ mass of bad and $\hat{G} + O$ mass of good ones. Instead, as lenders with precision α_0 take the borrowers in the hatched rectangles pro rata in each pool, the higher precision lenders end up with the remaining, non-hatched grey $(1 - \phi)\hat{B}$ mass of bad and non-hatched grey and green $(1 - \phi)\hat{G} + O$ good borrowers for some ϕ. The larger is $w(\alpha_0)$, the larger is ϕ. The critical observation is that whenever a group with higher α comes in, they always have an *own slice* O of good borrowers — in this case, the goods in $[\omega_g(\alpha_0), \omega_g(\alpha_0 + \Delta\alpha)]$ that only this group recognizes as good. This is the green rectangle in our example. Taking stock, the accepted masses and resulting selection for the lenders with precision $\alpha_0 + \Delta\alpha$ are:

$$G(\alpha_0{+}\Delta\alpha) = (1{-}\phi)\hat{G}{+}O, \quad B(\alpha_0{+}\Delta\alpha) = (1{-}\phi)\hat{B}, \qquad \gamma = \frac{G}{G+B} = \frac{(1-\phi)\hat{G}+O}{(1-\phi)(\hat{G}+\hat{B})+O}.$$

With little α_0 entry ($w(\alpha_0)$ hence ϕ small) this is close to the prior quality $(\hat{G}+O)/(\hat{G}+\hat{B}+O)$; as more α_0 lenders enter ϕ rises, the red line descends, and the shared masses $(1-\phi)\hat{G}, (1-\phi)\hat{B}$ shrink in equal proportion while the all-good own slice O stays fixed—so γ rises, reaching 1 once α_0 entry is large enough to clear the shared pool ($\phi \to 1$). That is, lower α borrowers leave behind a positively selected pool for higher α borrowers. This is the cleansing of Lemma 3.

We now use this property to construct the equilibrium, starting from Region I.

Region I Conjecture a profit level Π and consider the corresponding modified cost function $K(\alpha)$ as defined in (14).

We start by finding the interest rate r_p and the endpoints α_0 and α_1 of Region I. Since r_p is the lowest interest rate available, it must attract all the borrowers. Therefore the lowest-α selective lender who is active in market r_p obtains an average quality of:

$$\gamma_0(\alpha) = \frac{\mathcal{G}(\omega_g(\alpha))}{\mathcal{G}(\omega_g(\alpha)) + \mathcal{B}(1) - \mathcal{B}(\omega_b(\alpha))} \tag{30}$$

Since $\mathcal{G}(\omega_g) = \mathcal{G}(1)\,\Omega_g$ and $\mathcal{B}(1) - \mathcal{B}(\omega_b) = \mathcal{B}(1)(1-\Omega_b)$, dividing numerator and denominator by $\mathcal{G}(1)+\mathcal{B}(1)$ gives $\gamma_0(\alpha) = q_0\,\Omega_g(\alpha)/[q_0\,\Omega_g(\alpha) + (1-q_0)(1-\Omega_b(\alpha))]$, identical to the independent-signals formula (16). Consequently, the choice of skill at the lowest offered interest rate follows the same logic as in the independent information case. The interest rate that lender α needs to charge in order to make profit Π, is $r_0(\alpha)$ as defined in (17) and the optimal choice α_0 is determined by (18). Therefore,

$$r_p = r_0(\alpha_0). \tag{31}$$

However, Lemma 3 implies that lenders who choose $\alpha > \alpha_0$ and lend in market r_p will get a better pool than if they were first in line. Accordingly, we construct the density $w(\alpha)$ in Region I by finding how many entrants must enter at each α to ensure that all entrants make profits Π.

The other boundary of Region I is given by α_1, which solves:[10]

$$K(\alpha_1) = r_p \tag{32}$$

Skill level α_1 is defined as the skill level such that, if the lender only lends to good borrowers at rate r_p, he will make net profits Π (gross profits $\Pi + C(\alpha_1)$). The equilibrium construction ensures that this will indeed be the case. No lender would be willing to choose $\alpha > \alpha_1$ and

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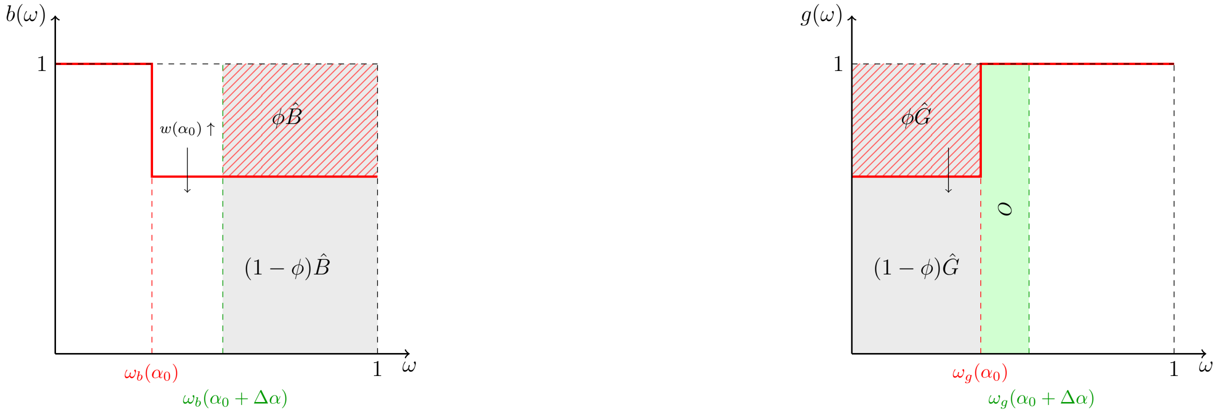
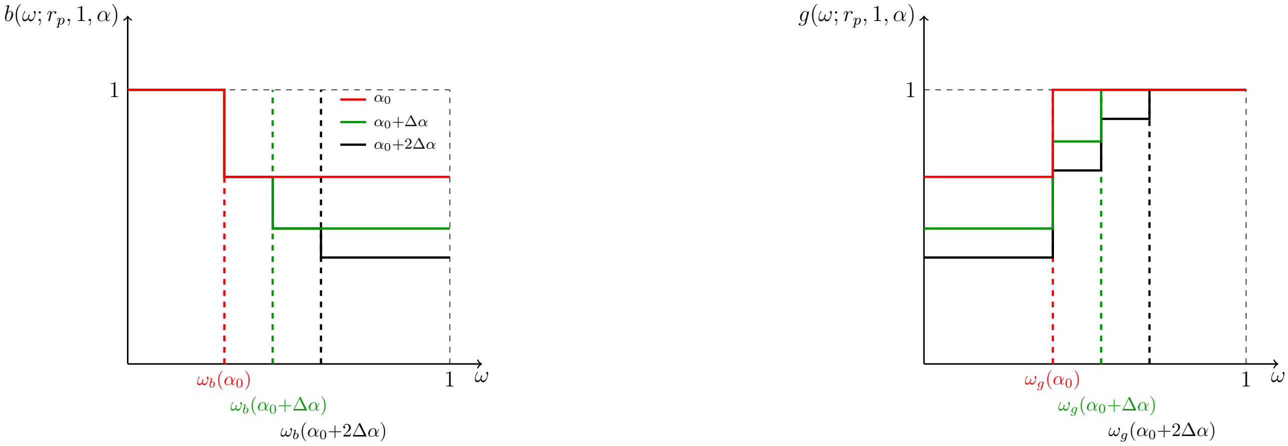
[10] If $r_p > K(\alpha)$ for all α, then $\alpha_1 = 1$ and regions II and III do not exist.



(a) Bad pool $\alpha_0+\Delta\alpha$ faces with less or more entry of α_0 lenders

(b) Good pool $\alpha_0+\Delta\alpha$ faces with less or more entry of α_0 lenders



(c) Bad pool: three discrete entrants

(d) Good pool: three discrete entrants

(e) Continuous limit ($\Delta\alpha\to0$): bad pool

(f) Continuous limit ($\Delta\alpha\to0$): good pool

Figure 5: Evolution of the borrower pool along the precision queue: lender $\alpha_0+\Delta\alpha$'s pool with and without prior entry of red α_0 (a,b); three discrete entrants α_0,α_0+$\Delta\alpha,\alpha_0$+2$\Delta\alpha$ in turn (c,d); the continuous limit $\Delta\alpha\to0$ (e,f). In every row the left panel is bad borrowers and the right panel is good borrowers.

lend in market r_p because even perfect selection would not be sufficient to offset the cost of information.

Before solving for $w(\alpha)$ we describe how the pool of borrowers evolves. Define:

$$\begin{aligned} G\left(\alpha\right) &\equiv \left(\left[0, \omega_g(\alpha)\right], r_p, 1, \alpha\right) \\ B\left(\alpha\right) &\equiv \left(\left[1-\omega_b(\alpha)\right], r_p, 1, \alpha\right) \end{aligned}$$

The functions $G\left(\alpha\right)$ and $B\left(\alpha\right)$ denote, respectively, the mass of good and bad borrowers who are available to lender α and acceptable to him. The total mass of acceptable available borrowers is:

$$T(\alpha) \equiv G\left(\alpha\right) + B\left(\alpha\right)$$

These accepted masses are the continuum counterpart of the single-cohort decomposition in Figure 5(a,b): there one cohort left green $G = (1-\phi)\hat{G} + O$ and $B = (1-\phi)\hat{B}$. Letting the whole queue of lenders act turns the served fraction ϕ into the depletion rate θ defined in (33) below and makes $G(\alpha), B(\alpha)$ evolve, while the rising good threshold keeps contributing fresh own slices. The next paragraphs derive this evolution.

Consider an interval of lenders $[\alpha, \alpha + d\alpha]$. When they lend, they serve $\frac{w(\alpha)d\alpha}{D(r_p)}$ borrowers, pro-rated among the $T(\alpha)$ acceptable ones, so for every borrower type that α finds acceptable, the remaining pool shrinks by a fraction $\theta d\alpha$, where:

$$\theta(\alpha) = \frac{w(\alpha)}{D(r_p)T(\alpha)} \tag{33}$$

Using (8)-(9), this means that density at ω of the applicant pools must satisfy the following differential equations:

$$\begin{aligned} \frac{\partial g\left(\omega; r_p, 1, \alpha\right)}{\partial \alpha} &= -\theta(\alpha)\mathbb{I}\left(\omega \le \omega_g(\alpha)\right) g\left(\omega; r_p, 1, \alpha\right) && (34) \\ \frac{\partial b\left(\omega; r_p, 1, \alpha\right)}{\partial \alpha} &= -\theta(\alpha)\mathbb{I}\left(\omega \ge \omega_b(\alpha)\right) b\left(\omega; r_p, 1, \alpha\right) && (35) \end{aligned}$$

Recall that for each α, $g\left(\omega; r_p, 1, \alpha\right)$ and $b\left(\omega; r_p, 1, \alpha\right)$ is the density of borrowers with opacity ω. Equations (34) and (35) and initial condition $g\left(\omega; r_p, 1, \alpha_0\right) = g(\omega)$ and $b\left(\omega; r_p, 1, \alpha_0\right) = \mathit{b}\left(\omega\right)$ fully define these functions for a given density of lenders $w(\cdot)$.

Importantly, the nested information structure and market clearing put tight restrictions on how the applicant pools can evolve. Figure 5 illustrates this. The top row is the single-cohort decomposition discussed above; the lower rows iterate it over the whole queue of lenders. The middle row conveys the mechanism in stylized form, taking W to be a step function so that three entrants of distinct skill α_0, $\alpha_0 + \Delta\alpha$ and $\alpha_0 + 2\Delta\alpha$ act one after another: each entrant takes a pro-rata slice from its acceptance region through $A(r, z, \alpha)$ in (7), building a staircase that descends on the bad side and ascends, with fresh own-slices, on the good side. The bottom row shows the resulting continuous limit for uniform priors as $\Delta\alpha \to 0$: each curve shows the density of remaining borrowers after all lenders up to skill α have lent.

Bad borrowers (left column of Figure 5). At α_0 the density is at its prior level for all ω. Lender α accepts borrowers with $\omega \geq \omega_b(\alpha)$ and takes a pro-rata slice of them, so in the accepted region the density falls uniformly at rate θ, given by (33). Once it's the turn of lenders for whom $\omega_b(\alpha)$ rises past a given borrower's opacity ω, that borrower is no longer acceptable to any subsequent lender, and the density freezes at whatever level it had reached, hence the staircase pattern in the figure. For borrowers with $\omega > \omega_b(\alpha_1)$ the density never freezes and falls all the way to zero. The total mass $B(\alpha)$ of acceptable bad borrowers therefore falls with α for two reasons: depletion within the acceptance region, and the threshold $\omega_b(\alpha)$ rising by $\beta\, d\alpha$, which drops the marginal borrowers, who have density $b(\alpha) \equiv b(\omega_b(\alpha);\, r_p, 1, \alpha)$. This gives:

$$B\left(\alpha + d\alpha\right) \approx \left(1 - \theta\, d\alpha\right) B(\alpha) - \beta\, b(\alpha)\, d\alpha. \tag{36}$$

Good borrowers (right column of Figure 5). Depletion at rate θ shrinks the existing pool, just as on the bad side. But the threshold ω_g also rises by $(1-\beta)\, d\alpha$, *adding* a fresh own slice $[\omega_g(\alpha),\, \omega_g(\alpha + d\alpha)]$—the continuum version of the own slice O of Figure 5(b), goods no previous lender could recognize. Their density enters at the undepleted prior level $g(\alpha) = g\left(\omega_g(\alpha)\right)$, visible as the step-up at $\omega_g(\alpha)$. Thus:

$$G\left(\alpha + d\alpha\right) \approx \left(1 - \theta\, d\alpha\right) G(\alpha) + (1-\beta)\, g(\alpha)\, d\alpha. \tag{37}$$

The key asymmetry is that on the bad side both effects reduce the pool, while on the good side the own-slice partially replenishes it. As α approaches α_1, both pools deplete.

Expressions (37) and (36) pin down how the average quality evolves. A Taylor approximation gives the quality that lender $\alpha + d\alpha$ obtains:

$$\begin{aligned}\gamma(\alpha + d\alpha) &\approx \frac{G(\alpha)}{T(\alpha)} + \frac{1}{T(\alpha)}\Big[(1-\beta)\, g(\alpha) - \theta\, G(\alpha)\Big]\, d\alpha - \frac{G(\alpha)}{T(\alpha)^2}\Big[(1-\beta)\, g(\alpha) - \beta\, b(\alpha) - \theta\, T(\alpha)\Big]\, d\alpha \\ &= \frac{G(\alpha)}{T(\alpha)} + \frac{(1-\beta)\, g(\alpha)\, (T(\alpha) - G(\alpha)) + \beta\, b(\alpha)\, G(\alpha)}{T(\alpha)^2}\, d\alpha \\ &= \frac{G(\alpha)}{T(\alpha)} + \frac{(1-\beta)\, g(\alpha)\, B(\alpha) + \beta\, b(\alpha)\, G(\alpha)}{T(\alpha)^2}\, d\alpha, \end{aligned} \tag{38}$$

where we used the definition that $\gamma(\alpha) = \frac{G(\alpha)}{T(\alpha)}$. Note that in the second line the θ-terms cancel ($-\theta G/T + \theta G/T = 0$).

Now we turn to how to determine lenders' information choice and entry in equilibrium, that is, the equilibrium density $w\left(\alpha\right)$ that ensures that each active lender offering interest rate r_p makes profits Π. This requires that:

$$\gamma\left(r_p, 1, \alpha\right) = \frac{1 + K\left(\alpha\right)}{1 + r_p} \qquad \text{for all } \alpha \in [\alpha_0, \alpha_1] \tag{39}$$

Taking a first-order approximation of (39), we also have that:

$$\gamma\left(r_p, 1, \alpha + d\alpha\right) \approx \gamma\left(r_p, 1, \alpha\right) + \frac{K'\left(\alpha\right)}{1 + r_p} d\alpha = \frac{G(\alpha)}{T(\alpha)} + \frac{K'\left(\alpha\right)}{1 + r_p} d\alpha \tag{40}$$

must hold in equilibrium.

As we show in the Appendix, equating the first order terms in (40) and (38), after some algebraic manipulation, leads to a quasi-closed form expression of the total mass of acceptable borrowers

$$T(\alpha) = \frac{(1-\beta)\,\mathscr{g}(\omega_g(\alpha))\,(r_p - K(\alpha))\;(1-\mathcal{B}(\omega_b(\alpha)))\;(1+r_p)}{K'(\alpha)\;(1-\mathcal{B}(\omega_b(\alpha)))\;(1+r_p) - \beta\,\mathscr{b}(\omega_b(\alpha))\,(1+K(\alpha))(r_p - K(\alpha))}. \tag{41}$$

This is the critical step of the equilibrium construction. (41) gives the measure of the acceptable pool for any lender with skill α such that the implied profit from lending compensates the given lender for the cost of obtaining that skill. Note that for this expression there is no need to solve ODEs.

Also, the rest of the equilibrium objects in this region follows easily. The bad and good types from (39) are

$$B(\alpha) = \frac{r_p - K(\alpha)}{1+r_p}\,T(\alpha), \qquad\qquad G(\alpha) = \frac{1+K(\alpha)}{1+r_p}\,T(\alpha),. \tag{42}$$

Then, we can work out what rate[11] do lenders need to drain the pool of borrowers to obtain exactly $B(\alpha)$

$$\theta(\alpha) = -\frac{d}{d\alpha}\ln B(\alpha) + \frac{\beta\,\mathscr{b}(\omega_b(\alpha))}{(1-\mathcal{B}(\omega_b(\alpha)))}. \tag{43}$$

which in turn gives the equilibrium lender density:

$$w(\alpha) = \theta(\alpha)\,D(r_p)\,T(\alpha). \tag{44}$$

Overall, Region I is constructed as follows. For any profit level, Π, the modified cost function $K(\alpha)$, and the initial distributions $\mathcal{B}, \mathcal{G}$ pins down the lowest skill chosen α_0 and the interest rate r_p for this region. Then, (41) gives the measure of the acceptable pool which ensures the same profit level for each lender. Obtaining exactly that partciular pool requires entry at a specific rate which determines the equilibrium mass of lenders, $w(\alpha)$ at each skill level. Note that by construction, we have $\gamma(r_p, 1, \alpha_1) = 1$, which can only be the case if lender α_1 only lends to his own slice, as illustrated in Figure 5.

Region II In Region II we find $r(\alpha)$ by the indifference condition

$$r(\alpha) = K(\alpha). \tag{45}$$

Since in Region II lenders only lend to good borrowers, the interest rate has to be exactly enough to compensate for information costs. Then we find the density $w(\alpha)$ by condition

$$w(\alpha) = D(r(\alpha))\,\mathscr{g}(\omega_g(\alpha))\,(1-\beta) \tag{46}$$

which equates supply and demand for each interest rate in this region.

---

[11] Expression (43) is not necessarily positive. For the case where it becomes negative for some α it is necessary to use an ironing algorithm to solve for $w(\alpha)$. This results in some intervals where the density is zero. See Appendix REF for details

Region III Finally, we find Region III, if it exists. This happens when for some α the interest rate $r(\alpha)$ implied by (45) is relatively high compared to the quality of remaining borrowers. Then, it might be the case that a lender can obtain profit Π by not investing in precision at all, instead lending non-selectively to all remaining borrowers at the given interest rate $r(\alpha)$.

To be more precise, the average quality that a non-selective lender would get in market $r(\alpha)$ is:

$$\gamma^{NS}(\alpha; \alpha_0, \alpha_1) = \frac{G([\omega_g(\alpha), 1])}{G([\omega_g(\alpha), 1]) + B([0,1]; r_p, 1, \alpha_1)} \tag{47}$$

Here the quantity $G([\omega_g(\alpha), 1])$ is the total mass of good borrowers who have not been served yet because they are too opaque for the lenders with lower skill than α, while $B([0,1]; r_p, 1, \alpha_1)$ is the total mass of bad borrowers who were not served by lenders $\alpha \in [\alpha_0, \alpha_1]$ in Region I at interest rate r_p. We will refer to $B([0,1]; r_p, 1, \alpha_1)$ as the leftover bad borrowers.

For each $r \in [r_p, r(1)]$, compute:

$$\Pi^{NS}(r) = \gamma^{NS}\left(K^{-1}(r); \alpha_0, \alpha_1\right)(1+r) - 1 \tag{48}$$

The quantity $\Pi^{NS}(r)$ represents the gross profit a nonselective lender would make if they lent at interest rate r Let r_{NS} be defined by the minimum value of r within the interval $[0, r(1)]$ such that $\Pi^{NS}(r) \geq \Pi$, if such a value exists.[12]. Lenders who enter with $\alpha = 0$ choose $r(0) = r_{NS}$ and $z(0) = 0$, that is they lend non-selectively in market r_{NS}. Let $\alpha_2 \equiv K^{-1}(r_{NS})$ define the the boundary between Region II and Region III. If r_{NS} exists, then W has a mass point at $\alpha = 0$, with mass

$$w^{NS} \equiv \frac{1}{D(r_{NS})}\left[G([0,1]; r_{NS}, 0) + B([0,1]; r_{NS}, 0)\right].$$

That is, the non-selective entry w^{NS} is sufficient to clear all the remaining demand of both bad and good borrowers. This also implies no entry from lenders with precision $\alpha > \alpha_2$ who would require a $r > r_{NS}$ in order to earn Π.

Equilibrium Construction The construction above, for any conjectured Π, fully defines the measure W of lenders who enter at each precision α. In the Appendix, we prove that the resulting total measure of lenders $W([0,1])$ is monotonically decreasing in Π in a range $\Pi \in [0, \Pi^{max}]$. Hence we can simply find the equilibrium by finding the unique level of Π such that $\bar{W} = W([0,1])$.

The equilibrium is therefore:

1. The measure W defined by the construction above
2. Choice of markets and selectiveness

$$r(\alpha) = \begin{cases} r_{NS} & \text{if } \alpha = 0 \\ r_p & \text{if } \alpha \in [\alpha_0, \alpha_1] \\ K(\alpha) & \text{if } \alpha > \alpha_1 \end{cases}$$

---

[12] This includes as a special case $r_{NS} \leq r_p$, in which case the only market is non-selective

$$z\left(\alpha\right)=\left\{\begin{array}{ll}0 & \text{if } \alpha=0\\ 1 & \text{otherwise}\end{array}\right.$$

3. Measures G and B defined by the construction above

The resulting portfolio quality for selective lenders is:

$$\gamma\left(\alpha\right)=\begin{cases}\gamma_0\left(\alpha\right) & if \quad \alpha\in\left[0,\alpha_0\right]\\ \gamma\left(r_p,1,\alpha\right) & if \quad \alpha\in\left[\alpha_0,\alpha_1\right]\\ 1 & if \quad \alpha\in\left[\alpha_1,1\right]\end{cases}. \tag{49}$$

and $\gamma^{NS}(\alpha_2,\alpha_0,\alpha_1)$ for non-selective lenders in Region III.

2.2.2 Equilibrium Properties in the Nested Case

As Figure 4 shows, a group of low opacity good borrowers, that is, the ones which can be recognized as good at low cost, can borrow at a uniformly low rate, r_p. This is Region I. Because these good borrowers are served by low-precision lenders with $\alpha\in[\alpha_0,\alpha_1]$, the most opaque bad borrowers will be served at this segment too. These are the borrowers whom low precision lenders cannot distinguish from the good ones. Hence, this market is characterized by a moderate but positive default rate.

Intuitively, this market resembles traditional lending. There is an advertised low rate at which any borrowers can apply. Lenders invest some in due diligence, but prefer to reject those borrowers who are too costly to identify as good. Lenders also do not try to avoid default at all cost.

Lenders with skill $\alpha\in[\alpha_1,\alpha_2]$ lend to good borrowers only with opacity $\omega\in[\omega_1,\omega_2]$. This is Region II. Each type of lender charges a different interest rate $r(\alpha)$. in this region. $r(\alpha)$ has to be increasing in order to compensate for the cost of the higher skill. In fact,

$$\frac{\partial r(\alpha)}{\partial\alpha}=\frac{\partial K(\alpha)}{\partial\alpha}.$$

Then, the mass of lenders $w(\alpha)$ offering that interest rate is determined by the market clearing condition (46). That is, the mass of lenders with skill α entering has to be such that their capital is sufficient to serve the lenders whom they can just recognize as good.

Intuitively, this market resembles specialized or high-tech lending. The lenders present in this market invest a lot in their screening technology to serve the good borrowers who are hard to assess as good. This is expensive, therefore lenders ask for a high interest rate as compensation.

Note that the existence of Region II critically builds on the existence of Region I: the fact that less-skilled lenders cleanse the pool from the hardest-to-recognize bad borrowers in Region I (that is, the fact that $\gamma(r_p,1,\alpha_1)\equiv 1$), makes it possible for high-skilled lenders in Region II to lend to good borrowers only.

As the interest rate is increasing in Region II, there might be a high enough $r(\alpha)$ to tempt some lenders to lend indiscriminately to all the remaining borrowers. The advantage is that such non-selective lenders can save the cost of precision $C(\alpha)$. The disadvantage is that they obtain a loan portfolio contaminated by the leftover bad borrowers, $B([0,1]; r_p, 1, \alpha_1)$. If such α_2 exists, then there is a Region III where non-selective lenders offer the interest rate r_{NS} to any borrower who takes it. The mass of lenders who enter non-selectively with no precision $\alpha = 0$ is just enough to clear all demand from hard-to-recognize bad and good borrowers who were not served at any lower rate.

Intuitively, depending on the context, this region resembles a market with loan-sharks, high-rate credit cards, or low-documentation mortgages. Lenders are not skilled in due diligence. Instead they ask for a high interest rate to compensate for adverse selection. This market exists to profit form the hard-to-recognize good borrowers who cannot obtain loan anywhere else. The interest rate is high, but no applicants are rejected.

Note that similarly to Region II, market conditions in Region III crucially depend on what is happening in Region I. The interest rate in Region III is higher whenever there are more leftover bad borrowers, $B([0,1]; r_p, 1, \alpha_1)$. This quantity is endogenously determined by the mass of lenders entering with different skills in Region I.

Note also that in the presence of Region III, our economy features non-assortative matching between lenders and borrowers. Throughout most of the market structure, more precise lenders lend to harder-to-recognize borrowers at higher interest rates. However, this pattern breaks down in Region III: there, lenders with the least precise screening technology lend to the hardest-to-recognize borrowers at the highest interest rate. As we will see, the dependence across regions through the pool externality, between lenders with highest and lowest precision, has important consequences for the impact of new information technologies on financial inclusion.

Finally, recall that despite of the heterogeneity between the structure of the credit market across regions, the net return on lending is the same everywhere, Π. The heterogeneity across regions comes from the different margins along which lenders can obtain the same profit. In Region I, the increasing cost of skill is compensated by increasing loan quality. In Region II, it is compensated by increasing interest rates. While in Region III, lenders do not invest in more skill, but the adverse selection implied by the remaining leftover bad borrowers have to be compensated by the high interest rates.

We finish our analysis with the effect of more intense overall competition, that is, higher $\bar{W}$, to our equilibrium.

Proposition 3. *A higher mass of lenders, $\bar{W}$, implies*

1. *lower level of profit, Π, for each lender,*
2. *a strictly lower interest rate $r(\alpha)$ for each precision in each region*
3. *strictly lower thresholds α_0, α_1*
4. *a smaller mass of bad left-over borrowers.*
5. *strictly better selection, $\gamma(\alpha)$ in Region I for each precision*

The negative relationship between the intensity of competition and profits is intuitive. A larger mass of lenders implies less profit for each. As we mentioned, we exploit this strictly monotonic relationship in our equilibrium construction. This also implies that there is an equivalent way to set up and interpret our model. Instead of taking $\bar{W}$ as primitive and derive Π, we could entertain a large mass of potential entrants, think of Π as their exogenous cost of capital, and treat $K(\alpha)$ as the total cost of entry. Then, a free entry condition would give $\bar{W}$ as the equilibrium mass of active lenders. With this interpretation, each statement in Proposition 3 can be interpreted as the effect of lower cost of capital Π.

The second statement shows that more competition benefits each good borrower in the form of lower rates and more credit, whichever region they borrow at.

The third and fourth statements imply that bad borrowers are also strictly better off. More intense competition implies both the least and most skilled type lending at the lowest interest rate r_p have lower precision. Intuitively, the curves on Figure 5 are pushed to the left. Therefore, by the end of Region 1, at $\alpha = \alpha_1$ there are less bad borrowers are left-over to be cleared at higher interest rates. This makes it possible for interest rate in Region 3 be smaller. To sum up, bad borrowers who has been served at the lowest rate, r_p in Region I, are still served in Region I, but a lower interest rate. Some bad borrowers, who were served in Region III, now are served in Region I. Bad borrowers who remained to be served in Region III, are served at a lower rate, r_{NS}.

The last statement characterizes the change in the relative mass of good to bad borrowers, who are served by lenders with a given $\alpha \in [\alpha_0, \alpha_1]$. For any given precision, this ratio, that is, lenders' portfolio quality improves. This is in line with the positive selection we emphasized by Lemma 3. When competition is more intense, each lender type in the interior of Region 1 enjoys a higher degree of positive selection coming from less skilled entrants. Intuitively, less skilled entrants, by accepting some bad very opaque bad borrowers, cleanse the pool for them.

2.3 Market Structure: Nested vs Independent Information

To highlight the role of the information structure, we compare equilibria under nested and independent information holding all other primitives constant. Specifically, we assume the same initial borrower pool with the same measures $\mathcal{G}$ and $\mathcal{B}$, the same cost function $C(\alpha)$, and the same equilibrium profit level Π for lenders. Figure 6 displays the resulting equilibrium outcomes in a special case when the same Π also implies the same total investor wealth $\bar{W}$ for easier interpretation.

The Figure echoes our observations from before. The two information structures generate strikingly different market outcomes. The nested structure produces a fragmented credit market with three qualitatively distinct regions: pooling at a low rate, separation with increasing rates, and non-selective lending at a high rate. In contrast, the independent information structure yields a relatively homogeneous market where interest rates increase smoothly without discrete jumps or qualitative transitions. Second, the mechanisms governing pool quality differ fundamentally. Under independent information, the market exhibits standard cream-skimming: as interest rates rise, the borrower pool deteriorates because good borrowers are progressively served, leaving a worse pool for higher-rate lenders. Under nested information, lower-skilled lenders instead cleanse the pool for higher-skilled lenders by removing the hard-to-recognize bad borrowers, improving pool quality in subsequent markets.

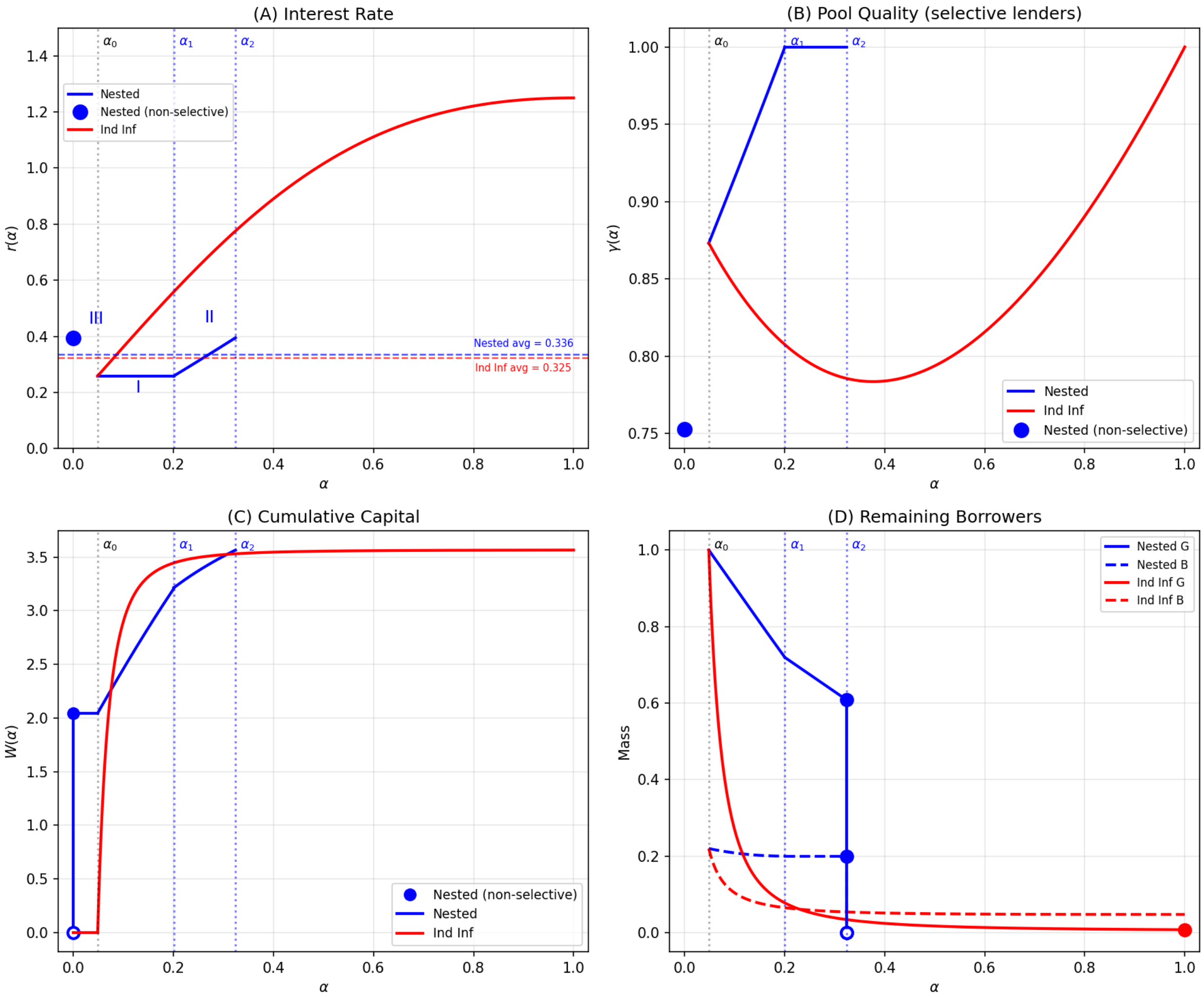


Figure 6: Equilibrium comparison: Nested vs Independent Information. Panel (A): Interest rate schedule; horizontal dashed lines show the average interest rate paid by good borrowers. Panel (B): Repayment probability $\gamma(\alpha, \alpha)$. Panel (C): Cumulative wealth distribution of lenders. Panel (D): Remaining masses of good (G) and bad (B) borrowers. Under nested information, the good and bad borrowers remaining at α_2 are served by non-selective lenders at rate r_{NS}. Under independent information, a mass of bad borrowers remains unserved at $\alpha = 1$ (dashed red line).

Third, the two structures differ in borrower coverage. Under nested information, eventually all borrowers—including all bad borrowers—are served, either by selective lenders or by non-selective lenders in Region III. Under independent information, some bad borrowers are always excluded from the market.

Interestingly, as our general structure includes the two extreme informational assumptions as special cases, we can directly compare the equilibrium objects. The next proposition states the first set of our analytical results.

Proposition 4. *Under the regularity condition* (A.1), *comparing the equilibrium with the same implied net profit* Π *under each information structure,*

1. *The interest rate a lender with skill* α *offers is higher under the independent information structure. More precisely,* $r^{indep}(\alpha_0) = r^{nested}(\alpha_0)$ *and* $r^{indep}(\alpha) > r^{nested}(\alpha)$ *for any other* α

2. *The default probability a lender with skill* α *suffers is higher under independent information. More precisely,* $\gamma^{indep}(\alpha_0) = \gamma^{nested}(\alpha_0)$ *and* $\gamma^{indep}(\alpha) < \gamma^{nested}(\alpha)$.

Panels (A) and (B) compare the interest rate schedules and the corresponding repayment probabilities $\gamma(\alpha, \alpha)$. Both equilibria share the same α_0, the skill level of the marginal lender who can profitably serve the initial pool at the lowest rate. The nested interest rate lies strictly below the independent rate for all $\alpha > \alpha_0$, and the nested schedule terminates at r_{NS}, strictly below $K(1)$, the rate charged by perfect screeners under independent information. The repayment probabilities differ markedly as well: under independent information, γ is U-shaped—high at low and high skill levels, with a minimum in between—eventually reaching $\gamma = 1$ for perfect screeners; under nested information, γ rises immediately to 1 and remains there throughout the selective regions, with the blue dot indicating the lower γ for non-selective lenders at $\alpha = 0$. Proposition 4 formalizes the pointwise comparison. The two panels are linked by the equilibrium first-order conditions: the slope of the interest rate schedule depends on how $\gamma(\alpha, \alpha)$ changes with α. Under independent information, decreasing γ requires steeper interest rate increases to maintain lender indifference; under nested information, $\gamma = 1$ pins the interest rate to track the cost $C(\alpha)$. Panel (A) also displays the average interest rate at which borrowers are actually served under each structure (horizontal lines). Perhaps surprisingly, the average borrowing rate is higher under nested information in this example, despite the pointwise ordering $r^{\text{iid}}(\alpha) > r^{\text{nested}}(\alpha)$. We return to this observation below.

Panels (C) and (D) illustrate the distribution of lenders and borrowers across skill levels. Panel (C) shows the cumulative wealth of lenders; Panel (D) tracks the remaining masses of good and bad borrowers across the skill spectrum. Under independent information, lender wealth concentrates heavily at low skill levels—most entry occurs near α_0—and consequently both G and B decline rapidly at low α: a large fraction of borrowers, good and bad, is served by low-skill lenders. Under nested information, by contrast, lender entry is spread out across the skill distribution, the decline in G and B is more gradual, and a substantial mass of bad borrowers persists until absorbed by non-selective lenders in Region III.

The contrasting γ shapes (Panel B) and lender distributions (Panel C) both arise from two distinct properties of the pool externality. The first is its *direction*: under independent information, lower-skill lenders cream-skim the pool for subsequent lenders (Lemma 1); under nested information, they cleanse it (Lemma 3). The direction property accounts for the contrast in γ shapes: a negative externality drives γ below 1 under independent information (yielding the U-shape, since better screening eventually dominates), while a positive externality pins γ at 1 throughout the selective regions under nested information. The second property is the *strength* of the externality, particularly at low skill levels. As we explain below, this property accounts for both the rapid initial decline of γ near α_0 under independent information and the concentration of $w(\alpha)$ near α_0.

To see how the strength matters, consider a lender with vanishingly low skill, α close to zero. Under independent information, such a lender has little effect on the pool: their signal is essentially pure noise, so they accept a representative cross-section of every type and leave the pool's composition unchanged. The per-lender cream-skimming contribution vanishes as $\alpha \to 0$. Under nested information, by contrast, a lender with arbitrarily low skill still absorbs a positive measure of hard-to-recognize bad borrowers, cleansing the pool for every higher-skill lender. The per-lender cleansing effect is bounded away from zero no matter how small α is. This property is not an artifact of our particular formalization. If screening errors are correlated, the bad borrowers that low-skilled lenders mistakenly serve naturally make screening easier for other lenders: those hard-to-assess borrowers no longer apply for loans elsewhere.

It is this strength asymmetry that produces both the rapid initial decline of γ near α_0 under independent information and the difference in $w(\alpha_0)$ across the two regimes. Under independent information, generating the equilibrium pool deterioration at α_0 requires a large mass of low-skill lenders, hence the concentration of $w(\alpha)$ near α_0.[13] Under nested information, a bounded mass of low-skill lenders suffices to generate the cleansing required by the equilibrium.

This entry concentration also explains the average borrowing rates noted in Panel (A). Although Proposition 4 establishes that, at each skill level, the interest rate is lower under nested information, the average rate at which borrowers are actually served can be higher. Under independent information, the concentration of entry near α_0 means that the majority of borrowers are served at rates close to $r(\alpha_0)$, the lowest rate in the market. Under nested information, lending is spread across skill levels, exposing more borrowers to the higher rates in Regions I and II. Which information structure is better for borrowers therefore depends on whether the pointwise rate advantage of nested information or the concentration of lending at low rates under independent information dominates.

We conclude this section with a proposition that formalizes this entry concentration effect: the insight that correlated screening errors lead to less low-skilled entry. The second part of the result, via Lemma 2, identifies sufficient parametric conditions under which it is most pronounced.

Proposition 5. *Comparing equilibria with the same implied net profit* Π*:*

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[13] For the formal counterpart of this statement, observe that the ODE (24) derived from (21)–(22) determines the rate $|q'(\alpha_0)|$ at which the pool must deteriorate at α_0. The pool-accounting equation (28) produces this deterioration through entry. As $\gamma_0(\alpha_0) - q_0$ vanishes linearly with α_0, the mass $w^{\text{iid}}(\alpha_0)$ must compensate.

1. *There exists* $\underline{\alpha} > 0$ *such that* $w^{iid}(\alpha_0) > w^{nested}(\alpha_0)$ *whenever* $\alpha_0 < \underline{\alpha}$.
2. *By Lemma 2,* $\alpha_0 < \underline{\alpha}$ *whenever any one of the following holds:*
   - *(a) the cost scale* λ *is sufficiently high;*
   - *(b) the prior pool quality* q_0 *is sufficiently high;*
   - *(c) the false-positive rate* β *is sufficiently low .*

3 New Entrants: The Short Run Impact

We next investigate the impact of new lenders entering the market. In this Section, we are interested in the short-run impact. In particular, assume an incumbent equilibrium has formed at the beginning of period $t = 1$ and let $w(\alpha)$ denote the incumbent wealth distribution. We consider unexpected entry of a positive measure of new lenders at the end of period $t = 1$, before borrowing and lending takes place. These new entrants are endowed with the same basic technology β as incumbents, but they have a potentially different cost of entry

$$K^E(\alpha) = \Pi^E + C^E(\alpha)$$

for a chosen precision α. Let $\bar{W}^E$ denote the aggregate wealth of active entrants, and $w^E(\alpha)$ their endogenous wealth distribution. In order to study the short run impact of entry, we assume the incumbent lenders cannot change their precision, α, in response to the unexpected entry of the new lenders. However, they can change the interest rate they advertise. For simplicity, we assume that all incumbent cost is sunk, hence they stay active even if their lending activity provides less profit than Π given the new entrants.

Our new Entry Equilibrium follows closely the definition of the Incumbent Equilibrium under Case 2. The critical difference is that new entrants understand that there is a mass of $w(\alpha)$ incumbents who are present at the economy and whom new entrants has to compete against. Still, new entrants solve the analogous problem to incumbents given by

$$\tilde{\Pi}^E(\alpha) = \max_{r,z} \gamma^E(r, z; \alpha)(1 + r) - 1 \tag{50}$$

and

$$0 = \max_{\alpha} \tilde{\Pi}^E(\alpha) - K^E(\alpha). \tag{51}$$

However, the probability of a new entrant with precision α serves a good borrower, $\gamma^E(r, z; \alpha)$ is determined by the evolution of the measures of good and bad borrowers

$$G^E\left(X^G; r, z, \alpha\right) = G^E\left(X^G\right) - \int_{A(r,z,\alpha)} \frac{G^E\left(X^G \cap I^G(\alpha, z); r, z, \alpha\right)}{G^E(I^G(\alpha, z); r, z, \alpha) + B^E(I^B(\alpha, z); r, z, \alpha)} \frac{1}{D(r(\alpha))} d\left(W(\alpha) + W^E(\alpha)\right) \tag{52}$$

$$B^E\left(X^B; r, z, \alpha\right) = B^E\left(X^B\right) - \int_{A(r,z,\alpha)} \frac{B^E\left(X^B \cap I^B(\alpha, z); r, z, \alpha\right)}{G^E(I^G(\alpha, z); r, z, \alpha) + B^E(I^B(\alpha, z); r, z, \alpha)} \frac{1}{D(r(\alpha))} d\left(W(\alpha) + W^E(\alpha)\right) \tag{53}$$

which take into account the present of incumbents. Then, the definition of the equilibrium is as follows.

Definition 4 (Entry Equilibrium). *For any given W measure of incumbents with various screening precision, the Entry Equilibrium consists of*

1. *A measure W^E over new entrants screening precision such that $W^E([0,1]) = \bar{W}^E$,*
2. *choice-of-market function $r^E(\alpha)$ and a (binary) choice-of selectiveness function $z^E(\alpha)$ for each lender α in the support of W^E,*
3. *measures of good and bad borrowers available to lender α with selectivity z^E in market r^E: $G^E(\cdot; r, z, \alpha)$ and $B^E(\cdot; r, z, \alpha)$*

such that

1. *Given α, $r^E(\alpha)$ and $z^E(\alpha)$ solve the new entrants' problem* (50)*, with γ^E defined by the appropriately modified versions of* (4), (5) *and* (6),
2. *Every α in the support of W^E solves the entrants problem* (51),
3. *The measures $G^E(\cdot; r, z, \alpha)$ and $B^E(\cdot; r, z, \alpha)$ satisfy* (52) *and* (53) *respectively.*

3.1 Equilibrium Construction and Properties

As we show in this section, the structure of the equilibrium remains similar. The Entry Equilibrium still features the hockey stick schedule, with one new possibility: the schedule can be "broken", as described in the proposition below.

The following proposition states the main result of this section.

Proposition 6 (Entry Equilibrium: The broken hockey stick interest rate schedule). *Consider an incumbent equilibrium where lenders make profit Π. A mass $\bar{W}^E$ of ex-ante identical new lenders enter with cost of capital Π^E and cost of precision $C(\alpha)$.*

The unique entry equilibrium is heterogeneous in lender precision and every incumbent lender makes $0 \leq \Pi' \leq \Pi$ profits.

The equilibrium features a hockey-stick interest rate schedule described in Proposition 2 with the following modifications.

1. *The interest rate schedule can discretely jump at points α_1 and α_2: $r(\omega_g(\alpha_1)) \geq r_p$ and $r(\omega_g(\alpha_2)) \leq r_{NS}$.*
2. *Region II is divided into*
   - *Region IIa: where good borrowers with $\omega \in [\omega_1, \omega_2']$ borrows at a single interest rate served by lenders with a moderately high degree of precision $\alpha \in [\alpha_1, \alpha_2']$, as in Region II in Proposition 2 and*

- *Region IIb: where incumbent lenders with the highest degree of precision $\alpha \in [\alpha_2', \alpha_2]$ compete with a zero measure of non-selective lenders to serve good borrowers in each market characterized by an increasing interest rate schedule $\tilde{r}(\omega)$ for $\omega \in [\omega_2', \omega_2]]$.*

When any of these modifications apply, we refer to the interest rate schedule as having the form of a broken hockey stick.

The construction follows the steps of the construction of an Incumbent Equilibrium in Case 2. We describe these steps in the Appendix in detail. Here we give only a draft and highlight the main forces and properties. Then, in the next section we illustrate our main insights with a few economically relevant examples.

The main difference compared to the construction of an Incumbent Equilibrium is that in every step we have to check whether new entrants wish to enter in a given region. This decision is mostly driven by their comparative advantage for the given level of skill α. In particular, when the adjusted cost of entry at a given precision α, $K^E(\alpha)$ is small relative to $K(\alpha)$ for incumbents, new entrants tend to choose the given α and enter in the corresponding region. When entrants decide to do so, they affect the equilibrium along two channels. First, they enter because they can offer a lower interest rate creating losses for incumbents and gains for borrowers served in the given market. Second, new entrants change the pool of borrowers for all lenders with higher skill or offering a higher interest rate. This potentially creates spillovers over the economy: a main focus of our analysis.

Region I (Pooling). Conjecture a profit level Π^E for entrants. The marginal entrant in the pooling region has skill

$$\alpha_0^E = \arg\min_{\alpha} \frac{K^E(\alpha) + 1}{\gamma_0(\alpha)} - 1,$$

lending at rate

$$r_p^E = \frac{K^E(\alpha_0^E) + 1}{\gamma_0(\alpha_0^E)} - 1. \tag{54}$$

If $r_p^E \geq r_p$ there is no entry in the pooling region.

Otherwise, we construct the combined (incumbents + entrants) equilibrium in the pooling region using the same approach as for the Incumbent Equilibrium. In the combined system, both incumbents and entrants lend at rate r_p^E and are selective. The total lending density at skill α is $w(\alpha) + w^E(\alpha)$, and the zero-profit condition for entrants requires

$$\gamma^E(r_p^E, 1, \alpha) = \frac{1 + K^E(\alpha)}{1 + r_p^E} \qquad \text{for all } \alpha \in [\alpha_0^E, \alpha_1^E]. \tag{55}$$

Just as in the incumbent case, the own-slice property applies: since no lower-skill lender can recognize borrowers at opacity $\omega_g(\alpha)$ under nested information, the good-borrower density at this threshold remains at the prior level: $g(\omega_g(\alpha); r_p^E, 1, \alpha) = g(\omega_g(\alpha))$. Similarly, bad borrowers in the acceptance region are uniformly depleted: $b(\omega; r_p^E, 1, \alpha) = \beta(\omega)\, E^E(\alpha)$ for $\omega \geq \omega_b(\alpha)$.

Equating the first-order terms from the pool evolution and the zero-profit condition gives the total mass of acceptable borrowers in the combined system:

$$T^E(\alpha) = \frac{(1-\beta)\, \mathcal{g}(\omega_g(\alpha))\, (r_p^E - K^E(\alpha))\ (1-\mathcal{B}(\omega_b(\alpha)))\ (1+r_p^E)}{(K^E)'(\alpha)\ (1-\mathcal{B}(\omega_b(\alpha)))\ (1+r_p^E) - \beta\, \mathcal{b}(\omega_b(\alpha))\, (1+K^E(\alpha))(r_p^E - K^E(\alpha))}. \tag{56}$$

This is the same formula as in the incumbent case (41), with K^E and r_p^E replacing K and r_p.

The combined depletion rate and entry density follow:

$$B^E(\alpha) = \frac{r_p^E - K^E(\alpha)}{1+r_p^E}\, T^E(\alpha), \qquad\qquad G^E(\alpha) = \frac{1+K^E(\alpha)}{1+r_p^E}\, T^E(\alpha), \tag{57}$$

$$\theta^E(\alpha) = -\frac{d}{d\alpha} \ln B^E(\alpha) - \frac{\beta\, \mathcal{b}(\omega_b(\alpha))}{1-\mathcal{B}(\omega_b(\alpha))}. \tag{58}$$

The total lending density at skill α is $\theta^E(\alpha)\, D(r_p^E)\, T^E(\alpha)$, and the **entrant density** is

$$w^E(\alpha) = \theta^E(\alpha)\, D(r_p^E)\, T^E(\alpha) - w(\alpha). \tag{59}$$

Entry occurs at skill α if and only if $w^E(\alpha) > 0$, that is, when the combined equilibrium requires more lending than incumbents alone provide.

If $w^E(\alpha) < 0$ for some interval, ironing is required: the entrant density is set to zero on that interval, and the boundaries of the active entry region are adjusted accordingly.

The upper boundary of Region I is $\alpha_1^E = \max(\alpha_1'^E, \alpha_1''^E)$, where:

- $\alpha_1'^E$ is the smallest α such that incumbent capital alone suffices for all $\alpha' \in [\alpha, \alpha_1]$, i.e. $w(\alpha') \geq D(\hat{r}(\alpha'))\, \mathcal{g}(\omega_g(\alpha'))\, (1-\beta)$ and $\hat{r}(\alpha') \geq r_p^E$.
- $\alpha_1''^E$ solves the indifference condition $r_p^E - C^E(\alpha_1''^E) = \Pi^E$.

Region II As stated in Proposition 6 this region might feature a new segment.

Region IIa is similar to Region II of the incumbent equilibrium. That is, for the endogenous thresholds $\alpha \in [\alpha_1^E, \alpha_2^E]$ the interest rate follows

$$r^E(\alpha) = \min\left(K^E(\alpha), K(\alpha)\right). \tag{60}$$

The expression illustrates that if for any skill-level new entrants have a comparative advantage, they enter and push down interest rates accordingly.

Region IIb arises in an Entry Equilibrium when non-selective lenders find it profitable to compete with incumbent high-tech selective lenders with precision $\alpha \in [\alpha_2^E, \alpha_2]$. While only zero measure of them enter at a given market, their threat of entry is sufficient to push the interest rate down to a level $\tilde{r}(\alpha) < \min\left(K^E(\alpha), K(\alpha)\right)$. At that interest rate non-selective lenders, serving a mixture of hard-to-recognize bad and good borrowers make the same profit as in Region III below. However, the incumbent lenders suffer even largest losses than in region IIa.

In the next section, we provide some economically intuitive examples when this case arises.

Region III The outcome in Region III depends on which group has the comparative advantage to lend to good borrowers just above opacity $\omega = \beta + \alpha_2(1-\beta)$. These are the least opaque borrowers who were served in Region III in the Incumbent Equilibrium. Namely, we have to compare three interest rates.

$$\begin{aligned}
&D^{-1}\left(\frac{w^{NS}}{(L^E(\alpha_0^E,\alpha_1^E)+[G(1)-G(\omega_g(\alpha_2))])}\right)=r' \\
&\frac{K^E(0)}{\gamma^{NS,E}(\alpha_2)}-1=r'' \\
&K^E(\alpha_2)=r'''
\end{aligned}$$

where $L^E(\alpha_0^E,\alpha_1^E)$ and $\gamma^{NS,E}(\alpha_2)$ are the left-over bad borrowers and the probability a non-selective lender entering at interest rate $r^E(\alpha_2)$ serves a good borrower. Both these objects are defined analogously to their counterpart in the Incumbent Equilibrium.

If $r' = \min(r', r'', r'')$ then non-selective incumbents have the comparative advantage over new entrants, and there will not be new entrants in this region. Whether the interest rate goes up or down critically depends on the whether there are more or less left-over bad borrowers after entry in Region I.

If $r'' = \min(r', r'', r'')$, than non-selective entrants have the comparative advantage, leading to a smaller interest rate in this region.

Perhaps it is useful to note that Region IIb we described in the previous part arises if in any of these cases $\min(r', r'', r'') < r_{NS}$. That is, when the interest rate what non-selective incumbents or entrants can offer is smaller than the non-selective interest rate in the Incumbent Equilibrium. This is the case, when non-selectives can compete with high-skilled incumbents.

Finally, if $r''' = \min(r', r'', r'')$ then the cost advantage of new entrants is sufficiently large that high-skilled entrants serve some of the good borrowers who previously were served by non-selective incumbents. It implies that Region II extends to the right.

3.2 A Benchmark: Entrants with identical technology

Before turning to our applications, it is useful to introduce the simplest case of an entry equilibrium as a benchmark. In this example, entrants are endowed with the same cost function as incumbents, $C^E(\alpha) = C(\alpha)$.

Corollary 1 (Entry Equilibrium with Homogeneous Technology). *Consider an incumbent equilibrium with mass of lenders'* $\bar{W}$*, implied level of profit* Π*, and cost function* $C(\alpha)$*. Consider new entrants with total wealth* $\bar{W}^E$*, and identical information technology* $C^E(\alpha) = C(\alpha)$*.*

The unique Entry Equilibrium is equivalent to an Incumbent Equilibrium with a single group of lenders with total mass $\bar{W} + \bar{W}^E$ *and information technology* $C(\alpha)$*.*

The equilibrium has the same, hockey stick structure as stated in Proposition 2.

This result is intuitive. It simply says that when the new entrants have no technological advantage relative to incumbents, they spread out across the full spectrum of incumbent lender precision distribution. At each precision α all lenders, incumbents and new entrants, have paid the same cost, lend to the same portfolio of borrowers at the same interest rate and face the same default rate. As such, they will all be as well of as each other.

The proof proceeds by showing that and increase in incumbent aggregate wealth leads to a pointwise increase at wealth at every precision α which is chosen in the original incumbent equilibrium $\alpha \in \{0 \cup [\alpha_0, \alpha_2]\}$, and no increase in wealth outside this range, $\alpha \notin \{0 \cup [\alpha_0, \alpha_2]\}$. This benchmark result illustrates that without any technological improvement, an increase in supply of lender capital benefits all borrowers. Every incumbent lender makes less profit as the supply of capital has increased.

4 Directed Technological Change in Credit Markets

Technological progress and policy implementation in credit markets do not reduce screening costs uniformly. Different innovations affect different parts of the precision spectrum. In this section, we use our framework to explore the impact of two concurrent changes to credit markets. In both applications, we give special attention to cross-segment spillover from changes that only impact limited segments directly.

In what follows we study how the market structure changes with the entry of new lenders, whose screening cost differs from that of incumbents, only in limited market segments. This analysis allows us to identify the borrowers who benefit or are harmed by entry, and in particular whether there is any *spillover* in equilibrium. To be precise, Definition 5 defines the equilibrium spillover in our framework.

Definition 5 (Equilibrium Spillover). *There is equilibrium spillover if there is a change in market conditions in markets where no new lender with improved screening technology enters.*

Our two applications corresponds to cost reductions at different parts of the precision spectrum. In order to abstract away from the direct impact of an increase in the supply of capital in the credit market, we restrict attention to when Π^E is high in all of these applications. As we have noted earlier in the paper, this corresponds to limited capital of new entrants.

These two applications correspond to a directed screening cost reduction at the top and bottom of the screening cost function. Big data, machine learning, and AI bring richer signals and more sophisticated pattern recognition helps lenders correctly assess borrowers whose available information is ambiguous or misleading. Traditional screening methods can resolve borrower type when the available data is straightforward to interpret, but fail for the most opaque borrowers. The additional signals provided by new technologies have the highest marginal value precisely for these borrowers: for those who are already easy to assess, extra data dimensions add little, but for opaque borrowers the additional signal can be decisive. These innovative technologies therefore corresponds to a reduction in $C(\alpha)$ concentrated at high α. In contrast, data-sharing policies such as Open Banking make existing financial data of already-served borrowers more broadly available, reducing the cost of screening borrowers who are already relatively transparent—a reduction in $C(\alpha)$ at low α.

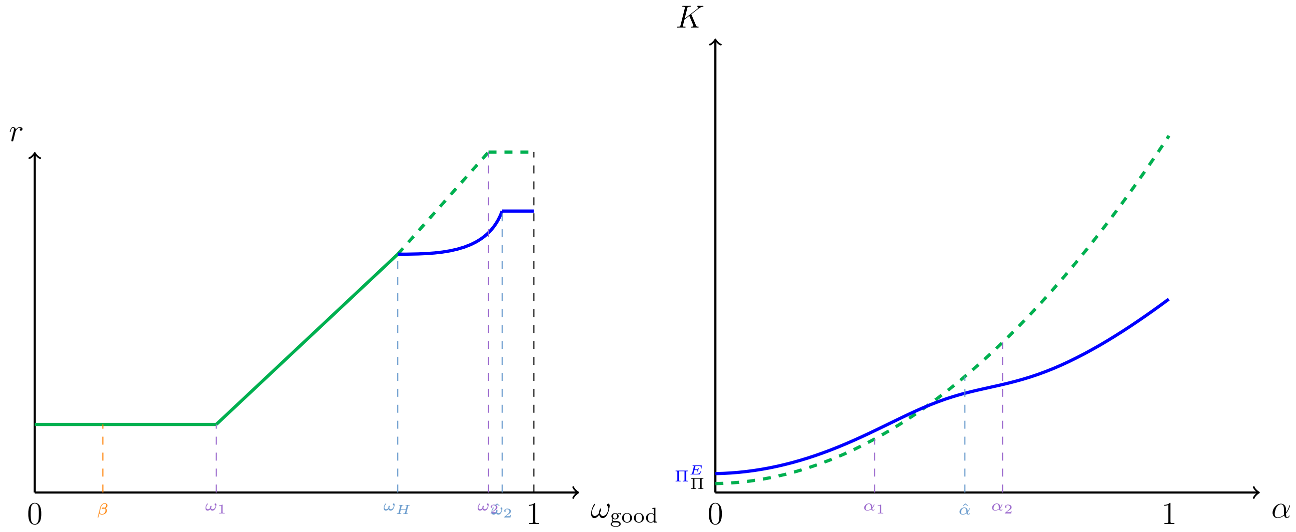

Figure 7: Impact of technological innovations in the credit market, which reduces the cost of screening opaque borrowers. Left panel: equilibrium rate schedule $r(\omega)$. Right panel: capital cost schedule $K(\alpha) = \Pi + C(\alpha)$ for the incumbent (green dashed) and $K^E(\alpha) = \Pi^E + C^E(\alpha)$ for the entrant (blue solid).

4.1 Screening Cost Reduction at the Top: Frontier Technology Adoption

As discussed above, Big data, machine learning, and AI technologies add signal dimensions that are most valuable for the most opaque borrowers, where traditional screening methods fail to resolve creditworthiness. We therefore model these new innovations as a directed cost reduction for high levels of screening technology. In particular, assume there is $\hat{\alpha}$ such that $C^E(\alpha) < C(\alpha)$ for $\alpha > \bar{\alpha}$. Figure 7 depicts the impact of this innovation on the credit market structure.[14] This innovation can lead to the opening of new markets:[15] new entrants will enter at the high end of the market, serving the most opaque borrowers. Importantly, there is no spillover in markets within the upward sloping range of interest rates. However, there is an interesting spillover to the financially excluded, as the lowest technology lenders who serve the most opaque borrowers now face fiercer competition from high-tech lenders and are forced to offer lower interest rates. Here, there is *positive equilibrium spillover*.[16]

There is wide empirical evidence that technological innovation in financial markets affects market structure and access. This evidence illustrates that improvements in screening

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[14] The figures in this section are stylized schematics drawn to illustrate the qualitative shape of the entry equilibrium. Appendix E provides numerical realizations of all three examples (Figures 7 and 8) together with the exact parameter values and the location of the script generating each set of panels.

[15] Similar to how the internet facilitated creation of new markets characterized by rapid innovation and the collection and use of detailed consumer and market data during the tech boom, as discussed in Levin (2011).

[16] In the Region III analysis of Proposition 6, this case is the branch where the high-skill entrants' rate $r''' = K^E(\alpha_2)$ is the smallest of the three Region III candidates: Region II is then extended to the right ($\alpha_2^E > \alpha_2$), with high-skilled entrants taking over good borrowers previously served at the non-selective rate.

technology—particularly through the use of alternative data sources—lead to new markets opening up or substantial expansion of existing markets.

In the context of credit scoring using alternative data for thin-file borrowers, Berg et al. (2020) provide experimental evidence that digital footprint data (e.g., e-commerce behavior) improves credit scoring for borrowers with thin credit files. The technology reduces default rates by 50% for marginal borrowers, enables lending to previously unscoreable individuals, and creates market access for over one billion underbanked globally. This demonstrates how new information technology opens markets for previously excluded populations.

Improved screening technology also expands access in mortgage and consumer lending markets. Fuster et al. (2019) provide evidence that better screening technology reduces mortgage default rates by 10–15% for a given approval rate, and increases approvals by 5–10% for a given default rate, enabling lending to previously marginal borrowers. Finally, Jagtiani and Lemieux (2018) show that fintech lenders using alternative data sources serve borrowers rejected by traditional banks: 20–30% of borrowers have subprime credit scores. They document that this technology enables lending in credit deserts (areas without bank branches) and creates a large market in previously unfunded segments.

4.2 Screening Cost Reduction at the Bottom: Open Banking

Next, consider a directed change in screening cost which is concentrated at lower levels of screening expertise. Recall that these lenders are those who lend to the most transparent borrowers in the credit market, i.e. those who are easiest to screen. Such a change resembles the impact of policies that enforce mandatory data sharing among financial institutions, if requested by their clients—*Open Banking*. Open Banking makes the data of already-served borrowers more broadly available, thus reducing the cost of screening them for creditors, in a directed fashion.

Note that in our model, the best served borrowers are served by low α lenders at low rates—many of them at the (lowest) pooling interest rate. Thus, we model Open Banking as a directed change in reducing cost of lower α. To be more specific, we assume there is $\hat{\alpha}$ such that $C^E(\alpha) < C(\alpha)$ for $\alpha < \hat{\alpha}$.

Interestingly, we find that adopting Open Banking has expected and unexpected implications for credit market conditions, which depends on the detail of implementation and which lenders primarily enter the market. Figure 8 illustrates two different possible outcomes that can happen if Open Banking is adopted in a credit market. The left panel corresponds to a directed cost reduction for $C(\alpha)$ for very low screening technology levels. We interpret this as a limited adoption of Open Banking. Alternatively, the right panel corresponds to a directed cost reduction for intermediate screening technology levels, which we interpret as a broader adoption of Open Banking.

Let us start from the more expected impact of Open Banking. As cost of low and/or intermediate screening technology decreases, borrowers who are served in the markets which are (partially) served by directly impacted lenders will benefit, irrespective of whether they are served by the new entrants or existing incumbents. In particular, assume $\hat{\alpha} < \alpha(\omega_1)$. There will be new lenders who choose screening technology $\alpha < \hat{\alpha}$ and enter the pooling market. Supply of capital will increase in this market and the pooling interest rate r_p falls.

The left hand side of both panels in Figure 8 depicts the decrease in the prevailing interest rate in the pooling market, from baseline green to blue, as a result of adoption of Open Banking.

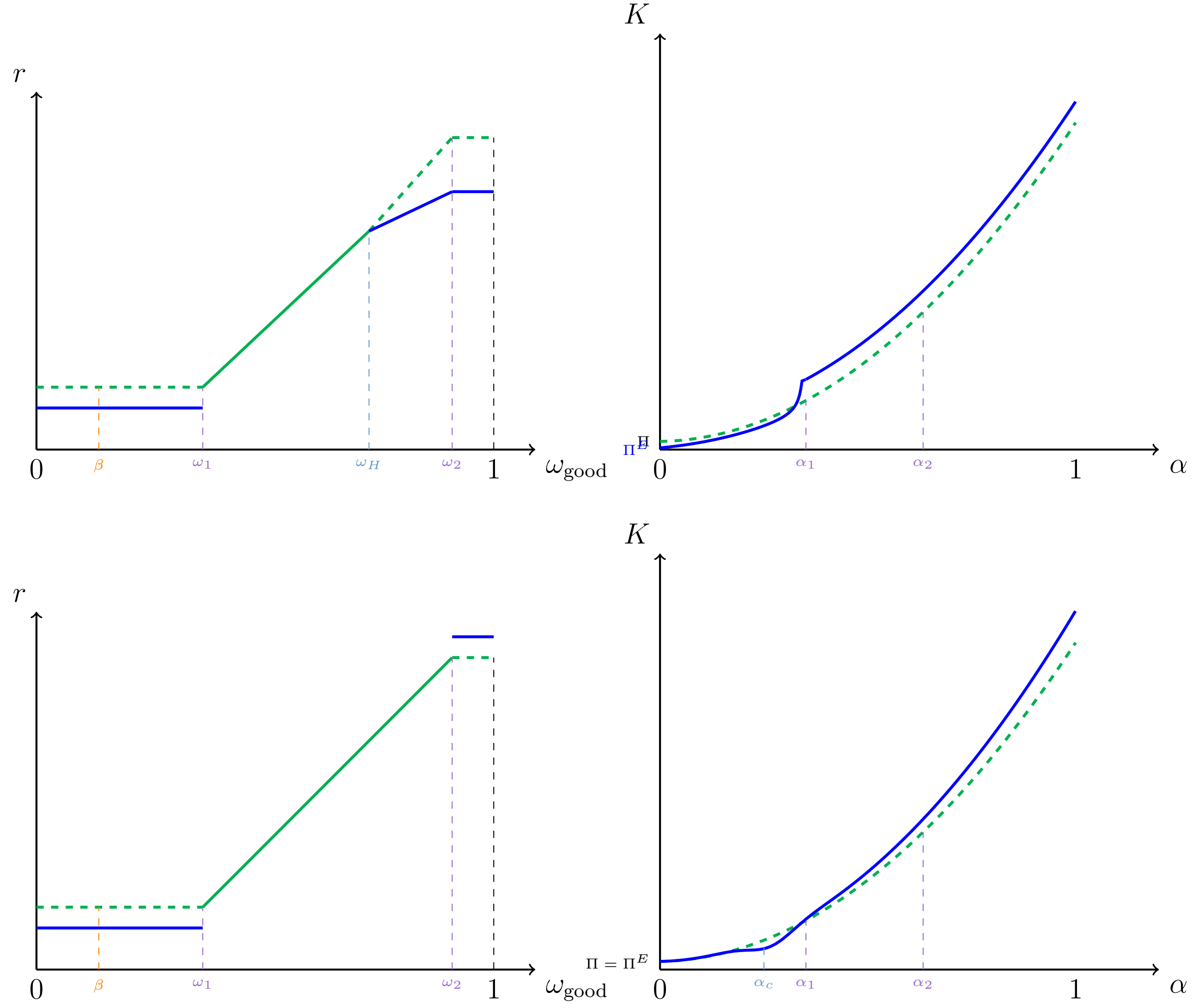

Figure 8: Adoption of Open Banking policy. Left column: equilibrium rate schedule $r(\omega)$. Right column: capital cost schedule $K(\alpha)$ for incumbent (green dashed) and entrant (blue solid). Top row (limited Open Banking) corresponds to a directed cost reduction for very low screening technology levels; bottom row (broad Open Banking) corresponds to a directed cost reduction for intermediate screening technology levels.

The more interesting impact of adoption of Open Banking in the credit market is through a spillover to market segments that are served at higher interest rates. Counter-intuitively, the interest rates in these segments can increase or decrease, depending on the exact implementation and scale of adoption of Open Banking. Lets call the borrowers who are served on the non-selective segment of the credit market, at the highest interest rates, the *financially* *excluded*, and consider why Open Banking can impact them positively or negatively.

The left panel of Figure 8 depicts the case where the financially excluded benefit from adoption of Open Banking, as displayed in the right end of the panel. This happens when $\hat{\alpha}$ is quite low, which we interpret as a limited adoption of Open Banking, when only the data of most served borrowers is shared even more broadly. In other words, $C^E(\alpha)$ reduces only for very low levels of α. This implies that all the new entrants have a relatively low level of screening expertise, thus, they disproportionately absorb the demand by bad opaque borrowers in the pooling segment. This in turn implies that the quality of the pool of remaining borrowers to be served at the highest interest rate improves. Recall that the lender-borrower matching is non-assortative in that region and only lenders with $\alpha = 0$ serve that market—incumbents or new entrants. Thus, as quality of the pool improves the interest rate falls, and there can be some extra spillover to the left, to the borrowers who were served at high interest rates in the right end of the cash-in-the-market region. This segment is referred to as Region IIb in Proposition 6. Thus, there is *positive equilibrium spillover*.[17]

Alternatively, the right panel of Figure 8 depicts the case where the financially excluded are harmed by adoption of Open Banking and now face an even higher interest rate, as displayed in the right end of the panel. This happens when $\hat{\alpha}$ is in an intermediate range. We interpret this as a widespread adoption of Open Banking, as the data of a wider range of borrowers is shared among the institutions. This implies that the new entrants in the pooling market have an intermediate level of screening expertise, thus, they disproportionately absorb the demand by good relatively transparent borrowers in the pooling segment. This in turn implies that the quality of the pool of remaining borrowers to be served at the highest interest rate worsens. Thus, the lenders with the most basic level of expertise who lend in this market are only willing to provide credit at a higher interest rate and the credit conditions for these borrowers worsen. Thus, there is *negative equilibrium spillover*.[18]

5 Conclusion

We develop an equilibrium model of credit markets with adverse selection and two-sided heterogeneity in lender and borrower types. Borrower type is two-dimensional, they are heterogeneous in both creditworthiness and opacity. Lenders have nested information structures and choose the precision of their screening technology to reduce the type I and II error rates that they make about borrowers' creditworthiness. Lenders also set interest rates to be compensated for different types of error that they will make. Borrowers choose interest rates and their quantity demanded to maximize their payoff.

In equilibrium, ex-ante homogeneous lenders choose heterogeneous levels of screening

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[17] In the Region III analysis of Proposition 6, this case is the branch where the non-selective entrants' rate r'' is the smallest of the three Region III candidates: non-selective entrants have the comparative advantage, the rate r_{NS} falls, and there is new entry at $\alpha = 0$.

[18] In the Region III analysis of Proposition 6, this case is the branch where the post-entry non-selective incumbents' rate r' is the smallest of the three Region III candidates and exceeds the baseline r_{NS}. Non-selective incumbents retain the comparative advantage—no new entry in Region III—and $\alpha_2^E = \alpha_2$; the rate rises because Region I entry by intermediate-skill entrants leaves a worse non-selective pool.

precision. The market structure is segmented with variable degrees of fragmentation across different level of borrower opacity and a hockey stick interest rate schedule. We then show that this market structure is robust to entry of new lenders and use our framework to investigate the impact of changes in big data technologies and policies on the financial sector. We find that adoption of AI technology benefits borrower who face high rates and improves financial inclusion. However, a mandatory data sharing policy not only does not have any spillover to the under-served population, but also exacerbates the inequality in financial access.

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Appendix

A Proofs

A.1 Equilibrium construction: Independent information, Proof of Proposition 1

We construct the equilibrium in several steps: deriving the first-order conditions, identifying the extra condition needed to pin down the equilibrium, combining these into an ODE for pool quality, and characterizing the solution.

The regularity condition is

$$\widetilde{R}(\alpha)^2 - 2\widetilde{R}'(\alpha) \;\geq\; 4\max\left(\frac{\Omega_g''(\alpha)}{\Omega_g(\alpha)},\ \frac{-\Omega_b''(\alpha)}{1-\Omega_b(\alpha)}\right) \quad \text{for all } \alpha \in [\alpha_0, \bar{\alpha}] \tag{A.1}$$

holds, where

$$\widetilde{R}(\alpha) \equiv \frac{C''(\alpha)}{C'(\alpha)} - \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}. \tag{A.2}$$

Throughout, we use the notation from the main text: $\Omega_g(\alpha) = \mathcal{G}(\omega_g(\alpha))/\mathcal{G}(1)$, $\Omega_b(\alpha) = \mathcal{B}(\omega_b(\alpha))/\mathcal{B}(1)$.

Step 1: Preliminaries. Recall that $q(\alpha)$ denotes the quality of the borrower pool (fraction of good borrowers) available to lenders in market α. A selective lender with skill α choosing market $\tilde{\alpha}$ faces repayment probability $\gamma(\alpha, q(\tilde{\alpha}))$ as defined in (13). The partial derivatives are:

$$\frac{\partial \gamma}{\partial \alpha} = \frac{q(1-q)\,\Gamma(\alpha)}{\left[q\,\Omega_g(\alpha) + (1-q)(1-\Omega_b(\alpha))\right]^2} \tag{A.3}$$

$$\frac{\partial \gamma}{\partial q} = \frac{\Omega_g(\alpha)\,(1-\Omega_b(\alpha))}{\left[q\,\Omega_g(\alpha) + (1-q)(1-\Omega_b(\alpha))\right]^2} \tag{A.4}$$

For (A.3), the chain rule gives $\frac{\partial}{\partial\alpha}[q\,\Omega_g] = q\,\Omega_g'$ and $\frac{\partial}{\partial\alpha}[(1-q)(1-\Omega_b)] = (1-q)\Omega_b'$, so the cross terms yield $\Omega_g'(1-\Omega_b) + \Omega_b'\Omega_g = \Gamma(\alpha)$ in the numerator.

Step 2: First-Order Conditions. Consider a lender with skill α choosing market $\tilde{\alpha}$. From the lender's problem (2)–(3), the first-order condition with respect to skill choice is:

$$\frac{\partial \gamma}{\partial \alpha}(\alpha, q(\tilde{\alpha}))(1 + r(\tilde{\alpha})) = C'(\alpha) \tag{A.5}$$

The first-order condition with respect to market choice is:

$$\frac{\partial \gamma}{\partial q}(\alpha, q(\tilde{\alpha}))q'(\tilde{\alpha})(1 + r(\tilde{\alpha})) + \gamma(\alpha, q(\tilde{\alpha}))r'(\tilde{\alpha}) = 0 \tag{A.6}$$

In equilibrium, lenders choose $\tilde{\alpha} = \alpha$, so both conditions must hold at (α, α).

At α_0, the skill-choice FOC (A.5) holds by construction: α_0 is chosen to minimize the break-even interest rate $r_0(\alpha) = \frac{K(\alpha)+1}{\gamma_0(\alpha)} - 1$ where $K(\alpha) = \Pi + C(\alpha)$.

Evaluating the market FOC at $\tilde{\alpha} = \alpha$ and using $\frac{\partial \gamma}{\partial \tilde{\alpha}} = \frac{\partial \gamma}{\partial q}\, q'$:

$$r' = -\frac{(1-\Omega_b)\, q'\,(1+r)}{q\left[q\,\Omega_g + (1-q)(1-\Omega_b)\right]}. \tag{A.7}$$

Step 3: The Indeterminacy Problem and the Extra Condition. The market-choice FOC (A.6) is a single equation, but involves two unknowns: $r'(\alpha)$ and $q'(\alpha)$. Equivalently, $q'(\alpha)$ is determined by the lender entry density $w(\alpha)$ through:

$$q'(\alpha) = -\frac{w(\alpha)}{D(r(\alpha))} \frac{\gamma(\alpha, q(\alpha)) - q(\alpha)}{G(\alpha) + B(\alpha)} \tag{A.8}$$

Thus, the market-choice FOC alone does not pin down the equilibrium—there is an extra degree of freedom.

The second equation comes from requiring the skill FOC (A.5) at every α in the support. Define $F(\alpha) \equiv \frac{\partial \gamma}{\partial \alpha}\Big|_{(\alpha, q(\alpha))} = \frac{q(1-q)\Gamma}{[q\,\Omega_g + (1-q)(1-\Omega_b)]^2}$. The skill FOC is $F(1+r) = C'$. Differentiating:

$$F'(1+r) + F\, r' = C''. \tag{A.9}$$

Using $F = C'/(1+r)$ and substituting r' from (A.7):

$$\frac{F'}{F} = R + \frac{(1-\Omega_b)\, q'}{q\left[q\,\Omega_g + (1-q)(1-\Omega_b)\right]}. \tag{A.10}$$

Step 4: Derivation of the ODE for $q(\alpha)$. We compute F'/F independently from $\ln F = \ln q + \ln(1-q) + \ln \Gamma - 2\ln[q\,\Omega_g + (1-q)(1-\Omega_b)]$:

$$\frac{F'}{F} = \frac{(1-2q)\, q'}{q(1-q)} + \frac{\Gamma'}{\Gamma} - \frac{2\left[q'(\Omega_g - (1-\Omega_b)) + q\Omega_g' - (1-q)\Omega_b'\right]}{q\,\Omega_g + (1-q)(1-\Omega_b)}. \tag{A.11}$$

Equating (A.10) and (A.11) and collecting terms, the coefficient of q' on the left-hand side simplifies to $\frac{-\Omega_g}{(1-q)\left[q\,\Omega_g + (1-q)(1-\Omega_b)\right]}$ (see Lemma below). Defining

$$\Phi \equiv \widetilde{R}\left[q\,\Omega_g + (1-q)(1-\Omega_b)\right] + 2\left(q\,\Omega_g' - (1-q)\,\Omega_b'\right), \tag{A.12}$$

we obtain the ODE $q' = (q-1)\Phi/\Omega_g$, which is (24).

Lemma A.1. *The coefficient of q' obtained by subtracting the right-hand side of* (A.10) *from that of* (A.11) *equals* $\frac{-\Omega_g}{(1-q)\left[q\,\Omega_g + (1-q)(1-\Omega_b)\right]}$.

Proof. Over the common denominator $q(1-q)[q\,\Omega_g + (1-q)(1-\Omega_b)]$, the numerator is:

$$
\begin{aligned}
&(1-2q)\big[q\,\Omega_g + (1-q)(1-\Omega_b)\big] - 2q(1-q)(\Omega_g - (1-\Omega_b)) - (1-q)(1-\Omega_b) \\
&= q\Omega_g(1-2q) - 2q(1-q)\Omega_g + (1-q)(1-\Omega_b)\big[(1-2q)+2q-1\big] = -q\Omega_g.
\end{aligned}
$$

□

Step 5: Initial Conditions. The initial lender skill α_0 minimizes the break-even rate when facing the full borrower pool:

$$\alpha_0 = \operatorname*{argmin}_{\alpha} \frac{K(\alpha)+1}{\gamma_0(\alpha)} - 1$$

where $\gamma_0(\alpha) = \frac{q_0\Omega_g(\alpha)}{q_0\Omega_g(\alpha)+(1-q_0)(1-\Omega_b(\alpha))}$ and $q_0 = \frac{\mathcal{G}(1)}{\mathcal{G}(1)+\mathcal{B}(1)}$ is the initial pool quality. The initial condition for the ODE is $q(\alpha_0) = q_0$.

Step 6: Sign of $q'(\alpha_0)$.

Lemma A.2. *If $\alpha_0 \in (0,1)$ is an interior solution, then $q'(\alpha_0) < 0$.*

Proof. The first-order condition for the minimization defining α_0 is $C'(\alpha_0)\gamma_0(\alpha_0) = \frac{\partial\gamma_0}{\partial\alpha}(\alpha_0)(K(\alpha_0)+1)$, and the second-order condition requires $(d^2/d\alpha^2)[(K+1)/\gamma_0]|_{\alpha_0} > 0$.

Define $\widehat{F}(\alpha) = \Omega_g(\alpha)^2\,(d/d\alpha)[(K+1)/\gamma_0]$. Then $\widehat{F}(\alpha_0) = 0$ and $\widehat{F}'(\alpha_0) > 0$. Computing $\widehat{F}'(\alpha_0)$ using $\gamma_0' = q_0(1-q_0)\Gamma/[q_0\Omega_g + (1-q_0)(1-\Omega_b)]^2$, the SOC reduces to $\Phi(\alpha_0, q_0) > 0$. From (24), $q'(\alpha_0) = (q_0-1)\Phi/\Omega_g < 0$. □

Step 7: $q(\alpha)$ is decreasing on $[\alpha_0, \bar{\alpha}]$.

Lemma A.3. *The pool quality $q(\alpha)$ is strictly decreasing on $[\alpha_0, \bar{\alpha}]$. No regularity condition is needed.*

Proof. By Lemma A.2, $\Phi(\alpha_0) > 0$, so $q'(\alpha_0) = (q_0-1)\Phi(\alpha_0)/\Omega_g(\alpha_0) < 0$. By continuity, $\Phi > 0$ on some interval $[\alpha_0, \alpha_0+\epsilon)$. Define $\bar{\alpha}$ as the earlier of: (i) the first $\alpha > \alpha_0$ where $\Phi(\alpha) = 0$, or (ii) the α where $r(\alpha) = K(1)$, or (iii) $\alpha = 1$ if neither occurs. On $[\alpha_0, \bar{\alpha}]$, $\Phi \geq 0$ by construction (since $\bar{\alpha}$ is at most the first zero of Φ). Therefore $q' = (q-1)\Phi/\Omega_g \leq 0$, with strict inequality on $[\alpha_0, \bar{\alpha})$.

Note: at the first zero of Φ (if one exists), $d\Phi/d\alpha \leq 0$ holds automatically—Φ arrives from above. The regularity condition would only be needed to prevent Φ from crossing back to positive after going negative, but the equilibrium is constructed on $[\alpha_0, \bar{\alpha}]$, where this does not arise. □

Step 7b: Characterization of Equilibrium. Starting from (α_0, q_0) with $q'(\alpha_0) < 0$, the ODE (24) determines a path $q(\alpha)$. Two cases arise:

Case 1: q is decreasing on $[\alpha_0, 1]$. If $\Phi(\alpha) > 0$ on the entire interval (so q is monotonically decreasing), the equilibrium is fully separating with $\bar{\alpha} = 1$. There is continuous entry on $[\alpha_0, 1)$ and an atom at $\alpha = 1$.

Case 2: Φ reaches zero. If $\Phi(\alpha^*) = 0$ at some $\alpha^* \in (\alpha_0, 1)$, the ODE would imply $q'(\alpha) \geq 0$ for $\alpha \geq \alpha^*$, requiring $w(\alpha) \leq 0$ by (A.8)—not economically meaningful. From the market-choice FOC, $\text{sign}(r') = -\text{sign}(q')$, so r switches from increasing to decreasing at the same point. Since $r(\alpha_0) < K(1)$ and r starts increasing, there exists $\bar{\alpha} \in (\alpha_0, \alpha^*]$ with $r(\bar{\alpha}) = K(1)$. For $\alpha \in [\bar{\alpha}, 1)$, lenders would need $r > K(1)$, but borrowers prefer perfect screeners. An atom at $\alpha = 1$ absorbs the remaining good borrowers.

Step 8: Quasi-convexity of $\gamma(\alpha, q(\alpha))$.

Lemma A.4 (Quasi-convexity of γ)**.** *Under the regularity condition* (A.1)*, the function $\gamma(\alpha, q(\alpha))$ changes monotonicity at most once on $[\alpha_0, \bar{\alpha}]$, and any critical point is a minimum.*

Step 8a: Critical point condition. Using (A.3), (A.4), and the ODE $q' = (q-1)\Phi/\Omega_g$:

$$\frac{d\gamma}{d\alpha} = \frac{1-q}{\big[q\,\Omega_g + (1-q)(1-\Omega_b)\big]^2}\big[q\Gamma - (1-\Omega_b)\Phi\big]. \tag{A.13}$$

Since $(1-q)/[q\,\Omega_g + (1-q)(1-\Omega_b)]^2 > 0$:

$$\frac{d\gamma}{d\alpha} = 0 \quad\iff\quad \Phi(\alpha^*) = \frac{q(\alpha^*)\,\Gamma(\alpha^*)}{1-\Omega_b(\alpha^*)}. \tag{A.14}$$

Note that at a γ-critical point, $\Phi > 0$ (since $q, \Gamma > 0$), so $q' < 0$: critical points occur where pool quality is still declining.

Step 8b: Structure of γ''. Define $h(\alpha) \equiv q\Gamma - (1-\Omega_b)\Phi$. At a critical point $h(\alpha^*) = 0$:

$$\left.\frac{d^2\gamma}{d\alpha^2}\right|_{\alpha^*} = \frac{1-q}{\big[q\,\Omega_g + (1-q)(1-\Omega_b)\big]^2}\,h'(\alpha^*), \tag{A.15}$$

since the derivative of $(1-q)/[q\,\Omega_g + (1-q)(1-\Omega_b)]^2$ is multiplied by $h = 0$. We need $h'(\alpha^*) > 0$.

Since $h = \Gamma\big[q - \frac{1-\Omega_b}{\Gamma}\Phi\big]$ and $\Gamma > 0$, the sign of h' at a zero of h equals the sign of

$$\frac{d}{d\alpha}\left[q - \frac{1-\Omega_b}{\Gamma}\,\Phi\right]\Bigg|_{\alpha^*}. \tag{A.16}$$

Step 8c: Decomposition and sign analysis. At the critical point, $\Phi = q\Gamma/(1-\Omega_b)$ and $q' = (q-1)q\Gamma/[(1-\Omega_b)\Omega_g]$. Expanding (A.16):

$$\frac{d}{d\alpha}\left[q - \frac{1-\Omega_b}{\Gamma}\,\Phi\right] = q' + \left(\frac{\Omega_b'}{\Gamma} - \frac{(1-\Omega_b)\Gamma'}{\Gamma^2}\right)\Phi - \frac{1-\Omega_b}{\Gamma}\,\Phi'.$$

Substituting the critical-point values and the expression for Φ', the result can be organized into three groups:

1. *Terms depending only on q, Ω_g, Ω_b, and their first derivatives* (not on $\widetilde{R}$ or second derivatives of Ω). Using the critical-point identity $\widetilde{R}[q\,\Omega_g + (1-q)(1-\Omega_b)] = q\Gamma/(1-\Omega_b) - 2(q\Omega_g' - (1-q)\Omega_b')$ to eliminate $\widetilde{R}$, these terms combine into a rational function of q, Ω_g, $1-\Omega_b$, Ω_g', Ω_b', Γ with a manifestly positive numerator. This group is **unconditionally positive**.

2. *Terms involving $\widetilde{R}'$.* After eliminating $\widetilde{R}$ via the critical-point identity, these produce $\frac{(1-\Omega_b)}{2\Gamma}\big[q\,\Omega_g + (1-q)(1-\Omega_b)\big](\widetilde{R}^2 - 2\widetilde{R}')$.

3. *Prior-curvature terms.* The second derivatives Ω_g'', Ω_b'' contribute $-\frac{2(1-\Omega_b)}{\Gamma}\big[q\Omega_g'' - (1-q)\Omega_b''\big]$.

Groups (2) and (3) sum to $\frac{1-\Omega_b}{2\Gamma}\big\{[q\,\Omega_g + (1-q)(1-\Omega_b)](\widetilde{R}^2 - 2\widetilde{R}') - 4(q\Omega_g'' - (1-q)\Omega_b'')\big\}$. The expression $\frac{4[q\Omega_g'' - (1-q)\Omega_b'']}{q\,\Omega_g + (1-q)(1-\Omega_b)}$, viewed as a function of q for fixed α, is a Möbius transformation and therefore monotone. Its supremum over $q \in (0,1)$ is attained at one of the endpoints $\frac{4\Omega_g''}{\Omega_g}$ (as $q \to 1$) or $\frac{-4\Omega_b''}{1-\Omega_b}$ (as $q \to 0$). Under (A.1), the bracket is non-negative for every q, so groups (2)+(3) are non-negative.

Proof of Lemma A.4. A γ-critical point α^* lies in $[\alpha_0, \bar{\alpha}]$, so (A.1) applies at α^*. By the decomposition above, group (1) is strictly positive and groups (2)+(3) are non-negative. Therefore (A.16) is strictly positive, $h'(\alpha^*) > 0$, and $\gamma''(\alpha^*) > 0$ by (A.15). Every critical point is a strict local minimum, so γ' changes sign at most once. □

Remark 1 (Role of the regularity condition)**.** *The regularity condition* (A.1) *is only needed for the quasi-convexity of γ (Lemma A.4), not for the existence of the equilibrium (items 1–5 of Proposition 1). The unconditionally positive group (1) means that even mild violations of* (A.1) *would not overturn the quasi-convexity conclusion. For uniform priors ($\mathcal{G}(x) = \mathcal{B}(x) = x$), $\Omega_g = \omega_g$, $\Omega_b = \omega_b$, $\Gamma = \beta$ (constant), $\widetilde{R} = R$, and the regularity condition reduces to $R^2 \geq 2R'$. For log-concave priors, $\Omega_g'' \leq 0$, $\Omega_b'' \leq 0$, and groups (2)+(3) are automatically non-negative.*

Step 9: Verification. *All lenders are selective.* A non-selective lender sets $\alpha = 0$, $C(0) = 0$, and obtains $\gamma(0, q) = q$. The profit from entering market $\tilde{\alpha}$ is $\pi^{NS}(\tilde{\alpha}) = q(\tilde{\alpha})(1 + r(\tilde{\alpha})) - 1$.

(i) Best market is $\tilde{\alpha} = \alpha_0$. Using the equilibrium FOCs to substitute r', the profit derivative involves $f(\alpha) \equiv \frac{1-\Omega_b(\alpha)}{q\,\Omega_g(\alpha) + (1-q)(1-\Omega_b(\alpha))}$. Since $f'(\alpha) = -q\,\Gamma/[q\,\Omega_g + (1-q)(1-\Omega_b)]^2 <$

0, a lender with $\alpha < \tilde{\alpha}$ has $f(\alpha) > f(\tilde{\alpha})$, so the profit derivative with respect to $\tilde{\alpha}$ is negative (given $q' < 0$). For $\alpha = 0$, profit is strictly decreasing in $\tilde{\alpha}$, so the best market is $\tilde{\alpha} = \alpha_0$.

(ii) At $r(\alpha_0)$, a non-selective lender earns at most Π. By the optimality of α_0: $\gamma_0(\alpha)(1 + r(\alpha_0)) \leq 1 + K(\alpha)$ for all α. Setting $\alpha = 0$: $q_0(1 + r(\alpha_0)) \leq 1 + \Pi$, with strict inequality when $\alpha_0 > 0$.

Interest rate monotonicity. On $[\alpha_0, \bar{\alpha})$, $q' < 0$ and (A.7) gives $r' = -(1 - \Omega_b)q'(1 + r)/\{q[q\,\Omega_g + (1 - q)(1 - \Omega_b)]\} > 0$.

Perfect screeners. At $\alpha = 1$: $\Omega_g(1) = \mathcal{G}(1)/\mathcal{G}(1) = 1$ and $\Omega_b(1) = \mathcal{B}(1)/\mathcal{B}(1) = 1$, so $1 - \Omega_b(1) = 0$. Lenders with $\alpha = 1$ perfectly identify all borrowers, achieve $\gamma = 1$ regardless of q, charge $r(1) = K(1)$, and lend exclusively to good borrowers.

This completes the construction and verifies all properties stated in Proposition 1.

A.2 Equilibrium construction: Nested Information, The Proofs of Proposition 2 and 3

Here we give the remaining details of the equilibrium construction for our baseline case. The comparative statics results in Proposition 3 are part of the proof. Following the description in the main text, we construct the three equilibrium regions in sequence for a given profit level Π.

Region I: boundaries and interest rate. The marginal lender α_0 minimizes the interest rate needed to break even:

$$\alpha_0 = \arg\min_{\alpha} \frac{\Pi + C\left(\alpha\right) + 1}{\gamma_0\left(\alpha\right)} - 1,$$

where γ_0 is defined in (30). The pooling interest rate is

$$r_p = \frac{\Pi + C\left(\alpha_0\right) + 1}{\gamma_0\left(\alpha_0\right)} - 1.$$

The upper boundary α_1 is the highest-skill lender in the r_p market. The indifference condition $r_p - C(\alpha_1) = \Pi$ gives $\alpha_1 = C^{-1}(r_p - \Pi)$, unless $r_p - C(\alpha) > \Pi$ for all α, in which case $\alpha_1 = 1$.

Region I: lender density. To determine the density $w(\alpha)$ for $\alpha \in [\alpha_0, \alpha_1]$, we first establish two structural properties of the borrower pools.

Lemma A.5 (Own-slice good density is undepleted)**.** *For any priors, the good-borrower density at the acceptance threshold is the original prior density:*

$$g\big(\omega_g(\alpha); r_p, 1, \alpha\big) = g(\omega_g(\alpha)) \qquad \text{for all } \alpha \in [\alpha_0, \alpha_1].$$

Proof. A good borrower at opacity $\omega_g(\alpha)$ satisfies $\omega > \omega_g(\alpha')$ for all $\alpha' < \alpha$ (since ω_g is increasing). No lender $\alpha' < \alpha$ recognized this borrower as good, so this borrower was never in any previous lender's acceptance region. Hence the density is undepleted. □

Lemma A.6 (Bad borrowers in the acceptance region are uniformly depleted)**.** *For any priors, there exists a common depletion factor $E(\alpha) \in (0,1]$ such that*

$$b\big(\omega;\, r_p, 1, \alpha\big) = \mathscr{b}(\omega)\, E(\alpha) \qquad \text{for all } \omega \geq \omega_b(\alpha).$$

Proof. A bad borrower at opacity $\omega \geq \omega_b(\alpha)$ satisfies $\omega \geq \omega_b(\alpha')$ for all $\alpha' \leq \alpha$ (since ω_b is increasing). Every lender from α_0 through α accepted this borrower, each depleting the density at the common rate $\theta(\alpha')$. Therefore $b(\omega; \alpha) = \mathscr{b}(\omega)\, \exp\!\big(-\int_{\alpha_0}^{\alpha} \theta(s)\, ds\big) \equiv \mathscr{b}(\omega)\, E(\alpha)$, which is independent of ω. $\square$

These two lemmas, together with the equal-profit condition (39), lead to the following equilibrium condition for the pool evolution.

Lemma A.7 (Equilibrium condition for pool evolution)**.** *The equal-profit condition* (39) *implies*

$$\frac{K'(\alpha)}{1 + r_p} = \frac{(1-\beta)\, g(\alpha)\, B(\alpha) + \beta\, b(\alpha)\, G(\alpha)}{T(\alpha)^2}, \tag{A.17}$$

where $g(\alpha)$ and $b(\alpha)$ denote the good- and bad-borrower densities at the acceptance thresholds $\omega_g(\alpha)$ and $\omega_b(\alpha)$, and $T(\alpha) = G(\alpha) + B(\alpha)$.

Proof. Step 1: Pool evolution. If the density of lenders at α is $w(\alpha)$, each pool is depleted at rate $\theta(\alpha) = w(\alpha)/(D(r_p)\, T(\alpha))$. As α increases by $d\alpha$, the good pool gains the own-slice $[\omega_g(\alpha), \omega_g(\alpha + d\alpha)]$ of width $(1-\beta)\, d\alpha$ at density $g(\alpha)$, and the bad pool loses $[\omega_b(\alpha), \omega_b(\alpha + d\alpha)]$ of width $\beta\, d\alpha$ at density $b(\alpha)$. Together with the common depletion, this gives the pool evolution equations (37)–(36).

Step 2: Quality from the evolved pool. Dividing (37) by $G(\alpha + d\alpha) + B(\alpha + d\alpha)$ and keeping first-order terms in $d\alpha$:

$$\gamma(\alpha + d\alpha) = \frac{G + \big[(1-\beta)\, g - \theta\, G\big]\, d\alpha}{T + \big[(1-\beta)\, g - \beta\, b - \theta\, T\big]\, d\alpha}.$$

Writing $\gamma = G/T$ and expanding $1/(T + \epsilon) \approx (1 - \epsilon/T)/T$:

$$\begin{aligned}
\gamma(\alpha + d\alpha) &\approx \frac{G}{T} + \frac{1}{T}\Big[(1-\beta)\, g - \theta\, G\Big]\, d\alpha - \frac{G}{T^2}\Big[(1-\beta)\, g - \beta\, b - \theta\, T\Big]\, d\alpha \\
&= \frac{G}{T} + \frac{(1-\beta)\, g\, (T - G) + \beta\, b\, G}{T^2}\, d\alpha \\
&= \frac{G}{T} + \frac{(1-\beta)\, g\, B + \beta\, b\, G}{T^2}\, d\alpha,
\end{aligned}$$

where in the second line the θ-terms cancel ($-\theta G/T + \theta G/T = 0$) and $T - G = B$.

Step 3: Quality from the equal-profit condition. Taylor-expanding $\gamma(\alpha) = (1 + K(\alpha))/(1 + r_p)$ gives:

$$\gamma(\alpha + d\alpha) \approx \frac{G(\alpha)}{T(\alpha)} + \frac{K'(\alpha)}{1 + r_p}\, d\alpha.$$

Step 4: Equating. Matching the first-order terms in Steps 2 and 3 yields (A.17). $\square$

Region I: algebraic solution. From Lemma A.6, the total bad mass in the acceptance region is

$$B(\alpha) = E(\alpha)\ (1 - \mathscr{B}(\omega_b(\alpha)))\,. \tag{A.18}$$

By (39), $G = (1+K)\,T/(1+r_p)$ and $B = (r_p - K)\,T/(1+r_p)$. Substituting into (A.17) (with $g(\alpha) = \mathscr{g}(\omega_g(\alpha))$ by Lemma A.5 and $b(\alpha) = \mathscr{b}(\omega_b(\alpha))\,E(\alpha)$ by Lemma A.6) and eliminating E via $E = B/(1 - \mathscr{B}(\omega_b)) = (r_p - K)\,T/\big((1 + r_p)(1 - \mathscr{B}(\omega_b))\big)$, we solve for T, obtaining (41). The good and bad pools follow from (42), and the depletion factor is

$$E(\alpha) = \frac{B(\alpha)}{1 - \mathscr{B}(\omega_b(\alpha))}. \tag{A.19}$$

Since E is now a known function, the depletion rate (43) and the lender density $w(\alpha) = \theta(\alpha)\,D(r_p)\,T(\alpha)$ follow.

The leftover bad borrower mass after Region I is:

$$B^{\text{left}}(\alpha_1) = \mathscr{B}\big([0, \omega_b(\alpha_0)]\big) + \beta \int_{\alpha_0}^{\alpha_1} \mathscr{b}(\omega_b(\alpha))\,E(\alpha)\,d\alpha + \underbrace{B(\alpha_1)}_{\to 0}. \tag{A.20}$$

The first term is the mass of bad borrowers who were never in any lender's acceptance region; the second is the mass who left at some $\alpha' \in [\alpha_0, \alpha_1]$ with partial depletion $E(\alpha')$. By construction, $\gamma(r_p, 1, \alpha_1) = 1$: lender α_1 lends exclusively to his own slice, as illustrated in Figure 5.

Region II. For $\alpha > \alpha_1$, lenders lend only to their own-slice good borrowers. The interest rate is pinned down by the indifference condition

$$r\,(\alpha) - C\,(\alpha) = \Pi,$$

and the lender density by the requirement that each lender absorbs exactly his own slice:

$$w\,(\alpha) = D\,(r\,(\alpha))\ \mathscr{g}(\beta + \alpha\,(1 - \beta))\,(1 - \beta)\,.$$

Non-selective region. The non-selective region exists if unskilled lenders ($\alpha = 0$) can earn profit Π by lending to the entire remaining pool. For each $r \in [r_p,\ r\,(1)]$, compute

$$\Pi^{NS}\,(r) = \gamma^{NS}\,(r, 0)\,(1 + r) - 1.$$

Let r_{NS} be the minimum r in $[0,\ r(1)]$ such that $\Pi^{NS}(r) \geq \Pi$, if such a value exists. (The special case $r_{NS} \leq r_p$ means the only market is non-selective.)

If r_{NS} exists, then W has a mass point at $\alpha = 0$ with mass

$$\frac{1}{D\,(r_{NS})}\left[G\,([0,1]\,; r_{NS}, 0) + B\,([0,1]\,; r_{NS}, 0)\right],$$

i.e. enough to serve all remaining borrowers. Unskilled lenders choose market r_{NS} and are non-selective ($z(0) = 0$). For any $A \subset [C^{-1}(r_{NS} - \Pi),\ 1]$, $W(A) = 0$: there is no entry at skill levels that would require $r > r_{NS}$ to earn Π.

Closing the equilibrium. The construction above determines the total mass of entrants $W(\Pi)$ as a function of the profit level. The last step is to invert this function to find the Π consistent with the exogenous total lender wealth W. The resulting equilibrium is:

$$r(\alpha) = \begin{cases} r_{NS} & \text{if } \alpha = 0, \\ r_p & \text{if } \alpha \in [\alpha_0, \alpha_1], \\ \Pi + C(\alpha) & \text{if } \alpha > \alpha_1, \end{cases} \qquad z(\alpha) = \begin{cases} 0 & \text{if } \alpha = 0, \\ 1 & \text{otherwise,} \end{cases}$$

with the measure W and the borrower pools G, B constructed as above.

Verifying optimality The above construction is an equilibrium as

1. Given their choice of α, all lenders are optimizing over r and s
   - Optimization over s is immediate because of the way we define the non-selective point.
   - Optimization over r:
     - For $\alpha > \alpha_1$; at higher r they would have to be non-selective and at lower r they get the same γ but a worse price
     - For $\alpha \in [\alpha_0, \alpha_1]$: at higher r they would have to be non-selective and at lower r they get weakly worse γ(same γ only for α_0) but a worse price
     - For $\alpha = 0$: by construction it's the only market where they can make at least Π, so they are optimizing
2. Lenders are optimizing over α
   - By construction, they are indifferent
3. The accounting conditions hold by construction

Showing Uniqueness and Proving Comparative Statics The following steps prove uniqueness and Proposition 3.

Proposition A.1. *Taking Π as given, the only equilibrium is the one that follows the construction above.*

Proof. We start by showing that the lowest-α active lender α_0 and lowest-active interest rate $r(\alpha_0)$ must be as in our equilibrium construction.

Lemma A.8. *Let α_0 be the lowest α in the support of W. Then*

$$\alpha_0 = \arg\min_{\alpha} \frac{\Pi + C(\alpha) + 1}{\gamma_0(\alpha)} - 1 \tag{A.21}$$

and

$$r(\alpha_0) = \frac{\Pi + C(\alpha) + 1}{\gamma_0(\alpha)} - 1 \tag{A.22}$$

Proof. Define, as in our construction

$$\gamma_0(\alpha) = \frac{G([0, \beta + \alpha(1-\beta)])}{G([0, \beta + \alpha(1-\beta)]) + B([1-\beta+\alpha\beta, 1])}$$

In any equilibrium, it must be that

1. α_0 makes profits Π
$$\Pi = (1 + r(\alpha_0))\gamma_0(\alpha_0) - 1 - C(\alpha_0)$$
2. No lender can make profits higher than Π at interest rate $r(\alpha_0)$ when faced with the original pool
$$(1 + r(\alpha_0))\gamma_0(\alpha)(r(\alpha_0), \alpha) - 1 - C(\alpha) \leq \Pi \qquad \text{for all } \alpha$$

This immediately implies (A.21) and (A.22) □

Now Let $\tilde{A}(r) = \{\tilde{\alpha} : r(\tilde{\alpha}) \leq r\}$ be the set of lenders that in equilibrium choose interest rate r or less. The remainder of good borrowers in market r, defined for an arbitrary subset $X^G \subseteq [0,1]$, is:

$$R\left(X^G; r\right) = G\left(X^G\right) - \int_{\tilde{A}(r)} \left[G^S\left(X^G; r(\alpha), \alpha\right) s(\alpha) + G^{NS}\left(X^G; r(\alpha), \alpha\right)[1 - s(\alpha)]\right] dW(\alpha)$$

Lemma A.9. *There can be no interest rate r such that a lender α in the support of W chooses $r(\alpha) = r$, $s(\alpha) = 1$ and has a positive-measure remainder of acceptable good assets, i.e. $R([0, \beta + \alpha(1-\beta)], r) > 0$*

Proof. Assume the contrary for market r. Let α be the highest type lender that chooses market r. Lender α's profits are $(1+r)\gamma^S(r, \alpha) - 1 - C(\alpha)$. Since $R([0, \beta + \alpha(1-\beta)], r) > 0$ then for small enough ϵwe have $G([0, \beta + \alpha(1-\beta)], r + \epsilon, \alpha) > 0$ (i.e. the adjacent markets have acceptable good assets available). If lender α chooses market $r + \epsilon$, profits are:

$$(1 + r + \epsilon)\gamma^S(r + \epsilon, \alpha) - 1 - C(\alpha) \tag{A.23}$$

where

$$\gamma^S(r + \epsilon, \alpha) = \frac{G^S([0,1]; r + \epsilon, \alpha)}{G^S([0,1]; r + \epsilon, \alpha) + B^S([0,1]; r + \epsilon, \alpha)}$$

The fact that $G([0, \beta + \alpha(1-\beta)], r + \epsilon, \alpha) > 0$ implies that $\gamma^S(r + \epsilon, \alpha) > 0$. The derivative with respect to ϵ is:

$$\begin{aligned}
\frac{\partial\gamma^S(r + \epsilon, \alpha)}{\partial\epsilon} &= \frac{\frac{\partial G^S}{\partial\epsilon}\left[G^S + B^S\right] - \left(\frac{\partial G^S}{\partial\epsilon} + \frac{\partial B^S}{\partial\epsilon}\right)G^S}{\left[G^S + B^S\right]^2} \\
&= \frac{\frac{\partial G^S}{\partial\epsilon}B^S - \frac{\partial B^S}{\partial\epsilon}G^S}{\left[G^S + B^S\right]^2} \\
&= \frac{\gamma^S(r + \epsilon, \alpha)B^S - \left(1 - \gamma^S(r + \epsilon, \alpha)\right)G^S}{\left[G^S + B^S\right]^2} \frac{w\left(r^{-1}(r + \epsilon)\right)}{d(r + \epsilon)}
\end{aligned}$$

Evaluating it at $\epsilon = 0$, this is

$$\begin{aligned}
\left.\frac{\partial \gamma^S (r+\epsilon, \alpha)}{\partial \epsilon}\right|_{\epsilon=0} &= \frac{\gamma^S (r, \alpha) B^S - \left(1 - \gamma^S (r, \alpha)\right) G^S}{\left[G^S + B^S\right]^2} \frac{w(\alpha)}{d(r)} \\
&= \frac{\frac{G^S}{G^S+B^S} B^S - \frac{B^S}{G^S+B^S} G^S}{\left[G^S + B^S\right]^2} \frac{w(\alpha)}{d(r)} \\
&= 0
\end{aligned}$$

Therefore, expression (A.23) is strictly increasing at $\epsilon = 0$, so choosing r cannot have been optimal for lender α. $\square$

We now use Lemmas (A.8) and (A.9) to show that the equilibrium must be as in our equilibrium construction

Lemma A.10. *In any equilibrium with profit level* Π*, the measure of lenders who choose* $\alpha \in [0, \alpha_1]$, $r(\alpha) = r(\alpha_0)$ *and* $s(\alpha) = 1$ *must coincide with the one defined by the construction above.*

Proof. Assume the contrary. By Lemma A.8, no one chooses $\alpha < \alpha_0$ and $s = 1$, so the measures coincide on $[0, \alpha_0)$.

Now suppose that for some $\tilde{\alpha}$, the measure over $[\alpha_0, \tilde{\alpha})$ is strictly lower than the one defined by the construction above. Since, by Lemma REF, profits for $\alpha' > \tilde{\alpha}$ are increasing in the mass of lenders who choose $\alpha < \alpha'$, this implies that lenders cannot make profits Π by choosing $\alpha' > \tilde{\alpha}$ and $r(\alpha_0)$, so no lenders do so. But this implies that $R([0, \beta + \tilde{\alpha}(1-\beta)], r) > 0$, which contradicts Lemma A.9.

Conversely, suppose that for some $\tilde{\alpha}$,the measure over $[\alpha_0, \tilde{\alpha})$ is strictly greater than the one defined by the construction above. By Lemma REF, lenders can make profits higher than Π by choosing $\alpha > \tilde{\alpha}$ and $r(\alpha_0)$, which contradicts that there is an equilibrium with profits Π. $\square$

Lemma A.11. *In any equilibrium with profit level* Π*, the measure of lenders who choose* $\alpha \in \left[\alpha_1, \alpha^{NS}\right]$ *and their choice of* r *and* s *must coincide with the one defined by the construction above.*

Proof. Conditional on α, the choice of r as in the construction above is the only one consistent with profits Π, so it remains to show that the measure coincides. By the same reasoning as in Lemma A.10, a lower measure would mean a positive remainder, contradicting Lemma A.9. Conversely, a greater measure would mean a negative remainder, which contradicts the definition of equilibrium. $\square$

Lemma A.12. *The measure of entrants who choose* $\alpha = 0$*,* $s = 0$ *and* r_{NS} *must coincide with the one defined by the construction above.*

Proof. Given that the choices of selective lenders must coincide with construction ADD REFERENCE, lenders can make profits exactly Π only in the market r_{NS}. If the mass of non-selective entrants at r_{NS} was lower, then there would be a positive remainder of good assets, so lenders could make higher profits at market $r_{NS} + \epsilon$. If the mass of non-selective entrants at r_{NS} was higher, it would lead to a negative measure of remaining assets, which contradicts the definition of equilibrium. ☐

Together, Lemmas A.10, A.11 and A.12 imply the result. ☐

Proposition A.2. *Let $W(\Pi)$ be the total wealth of entering lenders as a function of their equilibrium profit level. $W(\Pi)$ is strictly decreasing*

Proof. We start by showing how the values of α_0, α_1 and $\gamma(\alpha)$ depend on Π.

Lemma A.13. *α_0 is increasing in Π*

Proof. Let

$$R(\alpha, \Pi) \equiv \frac{\Pi + C(\alpha) + 1}{\gamma_0(\alpha)} - 1$$

be the interest rate that will make a lender who chooses α make profits Π, conditional on facing the entire pool of borrowers.

By definition, α_0 solves

$$\alpha_0(\Pi) = \arg\min_{\alpha} R(\alpha, \Pi)$$

with FOC and SOC:

$$R_1(\alpha_0(\Pi), \Pi) = 0 \tag{A.24}$$

$$R_{11}(\alpha_0(\Pi), \Pi) > 0 \tag{A.25}$$

By the implicit function theorem:

$$R_{11}(\cdot)\,\alpha_0'(\Pi) + R_{12}(\cdot) = 0$$

$$\alpha_0'(\Pi) = -\frac{R_{12}(\cdot)}{R_{11}(\cdot)}$$

Compute $R_{12}(\cdot)$:

$$R_{12}(\cdot) = -\gamma_0(\alpha)^{-1}\gamma_0'(\alpha) < 0$$

which, together with the SOC (A.25), implies

$$\alpha_0'(\Pi) > 0$$

☐

Lemma A.14. *α_1 is increasing in Π*

Proof. Since lender α_1 lends at $r_p = R(\alpha_0(\Pi), \Pi)$ and faces $\gamma(\alpha_1) = 1$, α_1 satisfies:

$$R(\alpha_0(\Pi), \Pi) - C(\alpha_1(\Pi)) = \Pi$$

Take derivatives on both sides:

$$R_1(\alpha_0(\Pi), \Pi)\,\alpha_0'(\Pi) + R_2(\cdot) - C'(\cdot)\,\alpha_1'(\Pi) = 1$$

Use the FOC (A.24) and rearrange:

$$\begin{aligned}
\alpha_1'(\Pi) &= \frac{R_2(\cdot)-1}{C'(\cdot)} \\
&= \frac{\frac{1}{\gamma_0(\alpha)}-1}{C'(\cdot)} > 0
\end{aligned}$$

□

Lemma A.15. *For any $\alpha \in [\alpha_0, \alpha_1]$, $\gamma(\alpha;\Pi)$ is decreasing in Π*

Proof. In an equilibrium with profits Π $\gamma(\alpha;\Pi)$ must satisfy:

$$\gamma(\alpha;\Pi)(1+R(\alpha_0(\Pi),\Pi)) - C(\alpha) - 1 = \Pi$$

Rearrange:

$$\gamma(\alpha;\Pi) = \frac{1+\Pi+C(\alpha)}{(1+R(\alpha_0(\Pi),\Pi))}$$

Compute the derivative w.r.t. Π, use the FOC (A.24) and rearrange:

$$\begin{aligned}
\frac{\partial\gamma(\alpha;\Pi)}{\partial\Pi} &= \frac{(1+R(\alpha_0(\Pi),\Pi)) - [R_1(\cdot)\alpha_0'(\Pi)+R_2(\cdot)][1+\Pi+C(\alpha)]}{(1+R(\alpha_0(\Pi),\Pi))^2} \\
&= \frac{(1+R(\alpha_0(\Pi),\Pi)) - \frac{1}{\gamma_0(\alpha)}[1+\Pi+C(\alpha)]}{(1+R(\alpha_0(\Pi),\Pi))^2} \\
&= \frac{(1+R(\alpha_0(\Pi),\Pi)) - \frac{1}{\gamma_0(\alpha)}[\gamma(\alpha;\Pi)(1+R(\alpha_0(\Pi),\Pi))]}{(1+R(\alpha_0(\Pi),\Pi))^2} \\
&= \frac{1-\frac{\gamma(\alpha;\Pi)}{\gamma_0(\alpha)}}{(1+R(\alpha_0(\Pi),\Pi))} < 0
\end{aligned}$$

□

Lemma A.16. *Let $B^R(X^B) = B(X^B) - \int_{\tilde{A}(r(\alpha_0))} B^S(X^B; r(\alpha_0), \alpha)\, dW(\alpha)$ be the measure (with density b^R) of bad borrowers who remain after market $r(\alpha_0)$. $b^R(\omega)$ is increasing in Π for all ω.*

Proof. Consider two equilibria, with profits Π and $\hat{\Pi} > \Pi$ respectively.

Define $\omega_0 \equiv \omega_b(\alpha_0) = 1-\beta+\alpha_0\beta$ and $\hat{\omega}_0 \equiv \omega_b(\hat{\alpha}_0) = 1-\beta+\hat{\alpha}_0\beta$. By Lemma A.13, $\hat{\omega}_0 > \omega_0$. We know that borrowers with $\omega < \omega_0$ cannot borrow in either equilibrium, so $b^R(\omega) = \hat{b}^R(\omega) = b(\omega)$. Borrowers $\omega \in [\omega_0, \hat{\omega}_0]$ borrow with positive probability in the Π equilibrium but cannot borrow in the $\hat{\Pi}$ equilibrium, so $b^R(\omega) < \hat{b}^R(\omega)$ in this range.

Similarly, define $\omega_1 \equiv \omega_b(\alpha_1) = 1-\beta+\alpha_1\beta$ and $\hat{\omega}_1 \equiv \omega_b(\hat{\alpha}_1) = 1-\beta+\hat{\alpha}_1\beta$. By Lemma A.13, $\hat{\omega}_1 > \omega_1$. We know that borrowers with $\omega > \hat{\omega}_1$ borrow with probability 1 in both equilibria, so $b^R(\omega) = \hat{b}^R(\omega) = 0$. Borrowers $\omega \in [\omega_1, \hat{\omega}_1]$ borrow with probability 1 in the Π equilibrium but probability less than 1 in the $\hat{\Pi}$ equilibrium, so so $b^R(\omega) < \hat{b}^R(\omega)$ in this range.

Now assume the statement is not true. Since $b^R(\omega) \leq \hat{b}^R(\omega)$ for all $\omega \leq \hat{\omega}_0$ and $\omega \geq \omega_1$, there must be values $\omega \in (\hat{\omega}_0, \omega_1)$ such that $b^R(\omega) > \hat{b}^R(\omega)$. By continuity, this implies that there are at least two values $\omega \in (\hat{\omega}_0, \omega_1)$ such that $b^R(\omega) = \hat{b}^R(\omega)$. Let ω^* be the highest of these values, and define $\alpha^* \equiv \frac{\omega^* - (1-\beta)}{\beta}$. Let $B(\cdot; r_p, \alpha^*)$ and $\hat{B}(\cdot; \hat{r}_p, \alpha^*)$ be the measures of borrowers faced by lender α^* in the two equilibria. Since any lender that accepts bad borrower ω^* also accepts all bad borrowers $\omega > \omega^*$, the measures B and $\hat{B}$ coincide on the set $[\omega^*, 1]$. By Lemma A.15, $\gamma(\alpha^*) > \hat{\gamma}(\alpha^*)$, and since $\gamma(\alpha^*) = \frac{G([0,\beta+\alpha^*(1-\beta)]; r_p \alpha^*)}{G([0,\beta+\alpha^*(1-\beta)]; r_p \alpha^*) + B([\omega^*,1]; r_p, \alpha^*)}$, this implies that

$$G([0, \beta + \alpha^*(1-\beta)]; r_p, \alpha^*) > \hat{G}([0, \beta + \alpha^*(1-\beta)]; \hat{r}_p, \alpha^*)$$

Now consider a discrete approximation where lenders choose between α^* and $\alpha^* + \Delta$ for a fixed Δ. The mass of entrants at α^* must be such that lenders are willing to also enter at $\alpha^* + \Delta$. In the Π equilibrium, this means that:

$$\begin{aligned}
\Pi &= \gamma(\alpha^* + \Delta)(1 + r_p) - 1 - C(\alpha^* + \Delta) \\
\Rightarrow \gamma(\alpha^* + \Delta) &= \frac{\Pi + 1 + C(\alpha^* + \Delta)}{1 + r_p} \\
&\approx \gamma(\alpha^*) + \frac{C'(\alpha)}{1 + r_p}\Delta \\
&= \frac{G}{G + B} + \frac{C'(\alpha)}{1 + r_p}\Delta \qquad \text{(A.26)}
\end{aligned}$$

where

$$\begin{aligned}
G &= G([0, \beta + \alpha^*(1-\beta)]; r_p, \alpha^*) \\
B &= B([\omega^*, 1]; r_p, \alpha^*)
\end{aligned}$$

Suppose that the entrants into market α^* are enough to lend to a fraction θ of acceptable borrowers in that market. Given the construction above, this implies that:

$$\gamma(\alpha^* + \Delta) \approx \frac{(1-\theta)G + (1-\beta)g\Delta}{(1-\theta)G + (1-\beta)g\Delta + (1-\theta)B - \beta b\Delta} \qquad \text{(A.27)}$$

where:

$$\begin{aligned}
g &= g(\beta + \alpha^*(1-\beta)) \\
b &= b^R(\omega^*)
\end{aligned}$$

Equate A.26 and A.27 and solve for $1 - \theta$ to obtain:

$$\begin{aligned}
\frac{G}{G+B} + \frac{C'(\alpha)}{1+r_p}\Delta &= \frac{(1-\theta)G + (1-\beta)g\Delta}{(1-\theta)G + (1-\beta)g\Delta + (1-\theta)B - \beta b\Delta} \\
\Rightarrow 1 - \theta &= (1 + r_p)\frac{(1-\beta)gB + \beta bG}{(G+B)^2 C'(\alpha)} + \frac{[\beta b - (1-\beta)g]}{G+B}\Delta \qquad \text{(A.28)}
\end{aligned}$$

Now take the derivative with respect to Δ in A.26 and A.27, evaluate them at and equate them:

$$\frac{C'(\alpha)}{1+r_p} = \frac{(1-\beta)\,gB + \beta bG}{(G+B)^2}$$

$$\Rightarrow (1+r_p)\,\frac{(1-\beta)\,gB + \beta bG}{(G+B)^2\,C'(\alpha)} = 1$$

Replace this in (A.28):

$$\theta = \frac{(1-\beta)\,g - \beta b}{G+B}\Delta$$

Since g is the original density, it is the same in both the Π and the Π' equilibrium; by assumption b is the same in both equilibria, and we proved above that this implies B is the same in both equilibria, and that $G > \hat{G}$. This implies that:

$$\hat{\theta} > \theta$$

This implies that at any point ω^* where the densities $b^R(\omega^*)$ and $\hat{b}^R(\omega^*)$ are equal, the density $\hat{b}^R(\omega^*)$ must have a lower (more negative) slope than $b^R(\omega^*)$. But since $\hat{b}^R(\omega) > b^R(\omega)$ for $\omega > \omega_1$, this cannot be true of the last time the densities intersect, which represents a contradiction. ☐

Lemma A.17. *For every borrower, the interest rate at which they borrow is increasing in* Π

Proof. Start with good borrowers. For $\omega \leq \beta + \alpha_1(1-\beta)$, they borrow at r_p. By (A.22) and the FOC (A.24), this is increasing in Π. For $\omega \geq \beta + \alpha_1(1-\beta)$, indifference for lenders implies that they borrow at $r = \Pi + C\left(\frac{\omega-\beta}{1-\beta}\right) + 1 - 1$, which is also increasing in Π . For bad borrowers, consider first the case where there is no non-selective region. In that case, bad borrowers borrow only in the r_p market, where the interest rate is increasing in Π. Instead, in the case where there is a nonselective region, Lemma A.16 implies that $\gamma^{NS}(r,\alpha)$ and therefore $\Pi^{NS}(r)$ is decreasing in Π. This in turn implies that r_{NS} is increasing in Π, so the interest rate increases both for bad borrowers who borrow at r_p and those who borrow at r_{NS} ☐

In the case where the equilibrium has a non-selective region, all borrowers end up borrowing but, by Lemma A.17, at higher interest rates when Π is higher. Therefore, since $d(r)$ is a decreasing function, the total wealth needed to lend to them is lower when Π is higher. In the case where there is no non-selective region, all good borrowers end up borrowing and, by Lemma A.16 fewer borrowers end up borrowing when Π is higher, so the same conclusion applies. ☐

A.3 Proof of Proposition 4

For the first part of the Proposition, we will build on the following Lemma.

Lemma A.18 (Single crossing of r' and C')**.** *In the independent information case, under the regularity condition* (A.1)*, $r'(\alpha)$ crosses $C'(\alpha)$ at most once on $[\alpha_0, \bar{\alpha}]$.*

Proof. From the zero-profit condition $\gamma(\alpha, q(\alpha))(1 + r(\alpha)) = K(\alpha) + 1$, we have $1 + r = (K + 1)/\gamma$, so:

$$r' = \frac{C'}{\gamma} - \frac{(K+1)\gamma'}{\gamma^2}. \tag{A.29}$$

Using $C' = (1 + r)\frac{\partial\gamma}{\partial\alpha}$ from the skill-choice FOC and $K + 1 = \gamma(1 + r)$:

$$r' - C' = (1+r)\left[\frac{\partial\gamma/\partial\alpha}{(}1-\gamma) - \gamma'\right].$$

Substituting $\gamma' = \frac{\partial\gamma}{\partial\alpha} + \frac{\partial\gamma}{\partial q}q'$ and simplifying:

$$r' - C' = -(1+r)\left[\gamma\frac{\partial\gamma}{\partial\alpha} + \frac{\partial\gamma}{\partial q}q'\right].$$

Using $\frac{\partial\gamma}{\partial\alpha} = \frac{\Gamma\, q(1-q)}{\Delta^2}$, $\frac{\partial\gamma}{\partial q} = \frac{\Omega_g(1-\Omega_b)}{\Delta^2}$, and $q' = \Phi(q-1)/\Omega_g$:

$$\gamma\frac{\partial\gamma}{\partial\alpha} + \frac{\partial\gamma}{\partial q}q' = \frac{1-q}{\Delta^2}\big[\gamma\Gamma q - (1-\Omega_b)\Phi\big],$$

where the computation parallels that leading to (A.14), but with γq replacing q in the first term. Since $(1 + r)(1 - q)/\Delta^2 > 0$:

$$r' > C' \quad\iff\quad (1-\Omega_b)\Phi > \gamma\Gamma q. \tag{A.30}$$

We now show that $(1 - \Omega_b)\Phi$ crosses $\gamma\Gamma q$ at most once. Define $h_1 \equiv \Gamma q - (1 - \Omega_b)\Phi$ and $h_2 \equiv \gamma\Gamma q - (1 - \Omega_b)\Phi = h_1 - (1 - \gamma)\Gamma q$. By Lemma A.4, h_1 has a unique zero at α^* (the γ-minimum), with $h_1 < 0$ for $\alpha < \alpha^*$ and $h_1 > 0$ for $\alpha > \alpha^*$. Since $(1 - \gamma)\Gamma q > 0$:

$$h_2 < h_1 \quad \text{for all } \alpha.$$

In particular, $h_2(\alpha^*) = -(1 - \gamma)\Gamma q < 0$, so any zero of h_2 must occur strictly after α^*.

It remains to show that at any $\hat{\alpha} > \alpha^*$ where $h_2 = 0$, the crossing is transversal from below, i.e.:

$$h_2'(\hat{\alpha}) > 0. \tag{A.31}$$

Since $\Gamma > 0$, the sign of h_2 equals the sign of $\gamma q - \frac{(1-\Omega_b)}{\Gamma}\Phi$. Differentiating:

$$\frac{d}{d\alpha}\left[\gamma q - \frac{1-\Omega_b}{\Gamma}\,\Phi\right] = \gamma' q + (\gamma - 1)q' + \frac{d}{d\alpha}\left[q - \frac{1-\Omega_b}{\Gamma}\,\Phi\right]. \tag{A.32}$$

At $\hat{\alpha}$, each of the three terms in (A.32) is positive:

Term 1: $\gamma' q > 0$. Since $\hat{\alpha} > \alpha^*$ and γ is increasing after its minimum (Lemma A.4), $\gamma' > 0$. And $q > 0$.

Term 2: $(\gamma - 1)q' > 0$. On $[\alpha_0, \bar{\alpha}]$, $\gamma < 1$ and $q' < 0$.

Term 3: $\frac{d}{d\alpha}[q-\frac{1-\Omega_b}{\Gamma}\Phi] > 0$. This is exactly the expression (A.16) analyzed in Step 8c, but evaluated at $\hat{\alpha}$ where $\frac{(1-\Omega_b)}{\Gamma}\Phi = \gamma q$ instead of the γ-critical condition $\frac{(1-\Omega_b)}{\Gamma}\Phi = q$. We apply the same three-group decomposition of Step 8c. The only change is the critical-point identity: $\widetilde{R}\,\Delta = \gamma q\Gamma/(1-\Omega_b) - 2(q\Omega_g' - (1-q)\Omega_b')$ instead of $\widetilde{R}\,\Delta = q\Gamma/(1-\Omega_b) - 2(q\Omega_g' - (1-q)\Omega_b')$. Since $\gamma < 1$, the γq terms yield smaller $\widetilde{R}\,\Delta$, and when $\widetilde{R}$ is eliminated via this identity, group (1) acquires additional positive corrections proportional to $(1-\gamma)$. Groups (2) and (3) involve $\widetilde{R}^2 - 2\widetilde{R}'$ and the prior curvatures exactly as before, and are non-negative under (A.1). Therefore Term 3 is strictly positive.

Since all three terms are positive, (A.31) holds, proving that any zero of h_2 is a transversal crossing from below. This implies at most one crossing. □

Using this Lemma, the proof of the first statement proceeds in three steps. First, note that α_0 and $r(\alpha_0)$ coincide under the two information structures: since $\mathcal{G}(\omega_g) = \mathcal{G}(1)\Omega_g$ and $\mathcal{B}(1) - \mathcal{B}(\omega_b) = \mathcal{B}(1)(1-\Omega_b)$, the nested γ_0 equals $q_0\Omega_g/[q_0\Omega_g + (1-q_0)(1-\Omega_b)]$, which is identical to the independent case (16). Therefore the break-even rate $r_0(\alpha) = (K(\alpha)+1)/\gamma_0(\alpha) - 1$ is the same objective, and $\alpha_0 = \arg\min r_0(\alpha)$ coincides. Next, we show that the interest rate under independent information is above $K(\alpha)$ everywhere. Then we conclude by showing that the interest rate under nested information is bounded above by $K(\alpha)$.

$r^{\text{iid}}(\alpha) \geq K(\alpha)$ on $[\alpha_0, \bar{\alpha}]$. Define $h(\alpha) = r^{\text{iid}}(\alpha) - K(\alpha)$. We claim $h(\alpha) \geq 0$ on $[\alpha_0, \bar{\alpha}]$, with equality only at $\bar{\alpha}$ (or $\alpha = 1$ when $\bar{\alpha} = 1$).

Boundary values. At α_0, the zero-profit condition gives $\gamma_0(\alpha_0)(1+r_p) = K(\alpha_0) + 1$, i.e. $1 + r_p = (K(\alpha_0)+1)/\gamma_0$. Since $\gamma_0(\alpha_0) < 1$:

$$h(\alpha_0) = r_p - K(\alpha_0) = \frac{K(\alpha_0)+1}{\gamma_0(\alpha_0)} - 1 - K(\alpha_0) = \frac{(1-\gamma_0(\alpha_0))(1+K(\alpha_0))}{\gamma_0(\alpha_0)} > 0.$$

At $\bar{\alpha}$: if $\bar{\alpha} = 1$, then $r^{\text{iid}}(1) = K(1)$ (perfect screeners achieve $\gamma = 1$), so $h(1) = 0$. If $\bar{\alpha} < 1$, then $r^{\text{iid}}(\bar{\alpha}) = K(1) > K(\bar{\alpha})$, so $h(\bar{\alpha}) > 0$.

By Lemma A.18, $h'(\alpha) = r'^{\text{iid}}(\alpha) - K'(\alpha)$ changes sign at most once on $[\alpha_0, \bar{\alpha}]$. Since $h(\alpha_0) > 0$ and $h(\bar{\alpha}) \geq 0$, and h' has at most one sign change, h cannot become negative: if it did, h would have to return to $h(\bar{\alpha}) \geq 0$, requiring h' to change sign at least twice. Therefore:

$$r^{\text{iid}}(\alpha) \geq K(\alpha) \quad \text{for all } \alpha \in [\alpha_0, \bar{\alpha}]. \tag{A.33}$$

Showing that $r^{\text{nested}}(\alpha) \leq K(\alpha)$ with $\bar{\alpha} < 1$ implying strict inequality. Recall the structure of the nested equilibrium:

- *Region I* ($\alpha \in [\alpha_0, \alpha_1]$): $r^{\text{nested}}(\alpha) = r_p$, which is constant. Since r^{iid} is strictly increasing from r_p at α_0, we have $r^{\text{iid}}(\alpha) > r_p = r^{\text{nested}}(\alpha)$ for all $\alpha \in (\alpha_0, \alpha_1]$.

- *Region II* ($\alpha \in [\alpha_1, \alpha_2]$): $r^{\text{nested}}(\alpha) = K(\alpha)$, since lenders serve only good borrowers ($\gamma = 1$) and the interest rate exactly compensates for cost. By (A.33), $r^{\text{iid}}(\alpha) \geq$

$K(\alpha) = r^{\text{nested}}(\alpha)$. To obtain strict inequality, note that at α_1:

$$r^{\text{iid}}(\alpha_1) > r_p = K(\alpha_1) = r^{\text{nested}}(\alpha_1),$$

so $h(\alpha_1) > 0$. If $h(\alpha^{**}) = 0$ for some $\alpha^{**} \in (\alpha_1, \bar{\alpha}]$, then $h > 0$ on $[\alpha_1, \alpha^{**})$ by the single crossing property. At α^{**}, $r^{\text{iid}} = K(\alpha^{**})$. But this can only occur at $\bar{\alpha}$ (when $\bar{\alpha} = 1$), since h can cross zero at most once. For $\alpha < \alpha^{**}$, $h(\alpha) > 0$ strictly. Hence $r^{\text{iid}}(\alpha) > K(\alpha) = r^{\text{nested}}(\alpha)$ on (α_1, α_2).

- *Region III* ($\alpha_2 < 1$, if it exists): $r^{\text{nested}} = r_{NS} = K(\alpha_2)$. For any $\alpha > \alpha_2$ in the support of the independent equilibrium, $r^{\text{iid}}(\alpha) \geq K(\alpha) > K(\alpha_2) = r^{\text{nested}}$.

Combining all three regions: $r^{\text{iid}}(\alpha) > r^{\text{nested}}(\alpha)$ for all $\alpha \neq \alpha_0$. □

The first part of the Proposition implies the second part as follows.

In both equilibria, every active lender earns the same profit Π, so the zero-profit condition

$$\gamma(\alpha)(1 + r(\alpha)) = K(\alpha) + 1$$

holds throughout. Solving for γ:

$$\gamma(\alpha) = \frac{K(\alpha) + 1}{1 + r(\alpha)}. \tag{A.34}$$

Since $K(\alpha)$ is the same under both information structures, γ is strictly decreasing in r for each α. By the first part of the Proposition, $r^{\text{iid}}(\alpha) > r^{\text{nested}}(\alpha)$ for all $\alpha \neq \alpha_0$. Substituting into (A.34):

$$\gamma^{\text{iid}}(\alpha) = \frac{K(\alpha) + 1}{1 + r^{\text{iid}}(\alpha)} < \frac{K(\alpha) + 1}{1 + r^{\text{nested}}(\alpha)} = \gamma^{\text{nested}}(\alpha).$$

At α_0, the interest rates coincide, so $\gamma^{\text{iid}}(\alpha_0) = \gamma^{\text{nested}}(\alpha_0) = \gamma_0(\alpha_0)$. □

A.4 Proof of Lemma 2

Write $\Delta_0(\alpha) \equiv q_0\,\Omega_g(\alpha) + (1-q_0)(1-\Omega_b(\alpha))$ and recall $\Gamma(\alpha) \equiv \Omega_g'(\alpha)(1-\Omega_b(\alpha)) + \Omega_b'(\alpha)\,\Omega_g(\alpha)$ from (25). Then $\gamma_0(\alpha) = q_0\Omega_g(\alpha)/\Delta_0(\alpha)$ and, by direct computation, $\gamma_0'(\alpha) = q_0(1 - q_0)\Gamma(\alpha)/\Delta_0(\alpha)^2$. With $r_0(\alpha) = (1 + K(\alpha))/\gamma_0(\alpha) - 1$ and $K(\alpha) = \Pi + \lambda C(\alpha)$, the FOC at any interior α_0 rewrites as

$$\frac{K'(\alpha_0)}{1 + K(\alpha_0)} = \frac{\gamma_0'(\alpha_0)}{\gamma_0(\alpha_0)} = \frac{(1 - q_0)\,\Gamma(\alpha_0)}{\Omega_g(\alpha_0)\,\Delta_0(\alpha_0)}, \tag{A.35}$$

and the second-order condition gives $r_0''(\alpha_0) > 0$.

Part (i): Monotone comparative statics. By the implicit function theorem, $d\alpha_0/d\theta = -[\partial^2 r_0/(\partial\alpha\,\partial\theta)|_{\alpha_0}]\,/\,r_0''(\alpha_0)$, and by the envelope theorem, $dr_0(\alpha_0)/d\theta = \partial r_0/\partial\theta|_{\alpha_0}$. Since $r_0''(\alpha_0) > 0$, the sign of $d\alpha_0/d\theta$ is opposite to that of the cross-partial.

Cost scale λ. $\partial r_0/\partial\lambda = C(\alpha)/\gamma_0(\alpha) > 0$, so $dr_0(\alpha_0)/d\lambda > 0$. Differentiating and substituting $\gamma_0'(\alpha_0) = \gamma_0(\alpha_0)\,\lambda C'(\alpha_0)/(1+K(\alpha_0))$ from (A.35):

$$\left.\frac{\partial^2 r_0}{\partial\alpha\,\partial\lambda}\right|_{\alpha_0} \;=\; \frac{C'(\alpha_0)\gamma_0(\alpha_0) - C(\alpha_0)\gamma_0'(\alpha_0)}{\gamma_0(\alpha_0)^2} \;=\; \frac{C'(\alpha_0)\,(1+\Pi)}{\gamma_0(\alpha_0)\,(1+K(\alpha_0))} \;>\; 0,$$

so $d\alpha_0/d\lambda < 0$.

Prior pool quality q_0. A direct computation gives $\partial\gamma_0/\partial q_0 = \Omega_g(1-\Omega_b)/\Delta_0^2 > 0$, hence

$$\frac{\partial r_0}{\partial q_0} \;=\; -\,\frac{(1+K)(1-\Omega_b)}{q_0^2\,\Omega_g} \;<\; 0,$$

giving $dr_0(\alpha_0)/dq_0 < 0$. For the cross-partial, use $(d/d\alpha)[(1-\Omega_b)/\Omega_g] = -\Gamma/\Omega_g^2$ to obtain

$$\frac{\partial^2 r_0}{\partial\alpha\,\partial q_0} \;=\; -\,\frac{1}{q_0^2\Omega_g^2}\Big[K'(1-\Omega_b)\Omega_g - (1+K)\Gamma\Big].$$

Substituting $K'\Omega_g = (1+K)(1-q_0)\Gamma/\Delta_0$ from (A.35), the bracket reduces to $-(1+K)\Gamma\,q_0\Omega_g/\Delta_0$, so

$$\left.\frac{\partial^2 r_0}{\partial\alpha\,\partial q_0}\right|_{\alpha_0} \;=\; \frac{(1+K(\alpha_0))\,\Gamma(\alpha_0)}{q_0\,\Omega_g(\alpha_0)\,\Delta_0(\alpha_0)} \;>\; 0,$$

so $d\alpha_0/dq_0 < 0$.

Part (ii): Corner thresholds. The corner $\alpha_0 = 0$ obtains whenever $r_0'(0) \geq 0$, which by the same FOC algebra as (A.35) is equivalent to

$$\frac{\lambda C'(0)}{1+\Pi+\lambda C(0)} \;\geq\; \frac{(1-q_0)\,\Gamma(0)}{\Omega_g(0)\,\Delta_0(0)}. \tag{A.36}$$

With $C(0) = 0$ and $C'(0) > 0$, the left-hand side is strictly increasing in λ and diverges as $\lambda \to \infty$, while the right-hand side is independent of λ; this yields a finite $\bar{\lambda}$. The right-hand side vanishes as $q_0 \to 1$ (factor $(1-q_0)$), giving $\bar{q} < 1$. Finally, as $\beta \to 0$, $\omega_b(\alpha) \to 1$ uniformly, so $\Omega_b'(\alpha) = \beta\,\mathscr{b}(\omega_b)/\mathcal{B}(1) \to 0$ and $\Gamma(0) \to 0$, so the right-hand side again vanishes; this yields $\underline{\beta} > 0$. $\square$

A.5 Proof of Lemma 1

Suppose the fraction of good boorowers faced by lender α is $q = \frac{G}{G+B}$. If lender α instead skips ahead of some lender $\tilde{\alpha} \in A$ who was lending to a total of x borrowers, the pool he faces includes all the borrowers that lender $\tilde{\alpha}$ would have chosen, so the fraction of good borrowers he would face would be:

$$q' = \frac{G + \gamma\left(\tilde{\alpha}\right)x}{G + \gamma\left(\tilde{\alpha}\right)x + B + \left(1 - \gamma\left(\tilde{\alpha}\right)\right)} > \frac{G + \frac{G}{G+B}x}{G + \frac{G}{G+B}x + B + \frac{B}{G+B}x} = \frac{G}{G+B} = q$$

where the inequality holds because $\gamma\left(\tilde{\alpha}\right) > q$, that is because lender α selects better than random. $\square$

A.6 Proof of Proposition 5

Part 1. We derive the entry density $w(\alpha_0)$ under each information structure and show $w^{\mathrm{iid}}(\alpha_0)/w^{\mathrm{nested}}(\alpha_0) \to \infty$ as $\alpha_0 \to 0^+$.

Independent Information density. At α_0 the pool is fresh, so $G(\alpha_0)+B(\alpha_0) = \mathcal{G}(1)+\mathcal{B}(1)$ and $q(\alpha_0) = q_0$. From the pool-quality equation (A.8):

$$q'(\alpha_0) = -\frac{w^{\mathrm{iid}}(\alpha_0)}{D(r_p)}\,\frac{\gamma_0(\alpha_0) - q_0}{\mathcal{G}(1) + \mathcal{B}(1)}. \tag{A.37}$$

From (16), writing $\Delta_0 \equiv q_0\,\Omega_g(\alpha_0) + (1 - q_0)(1 - \Omega_b(\alpha_0))$:

$$\gamma_0(\alpha_0) - q_0 = \frac{q_0(1 - q_0)\big(\Omega_g(\alpha_0) + \Omega_b(\alpha_0) - 1\big)}{\Delta_0}.$$

Moreover, $q_0\,\Omega_g = \mathcal{G}(\omega_g)/(\mathcal{G}(1)+\mathcal{B}(1))$ and $(1-q_0)(1-\Omega_b) = (\mathcal{B}(1)-\mathcal{B}(\omega_b))/(\mathcal{G}(1)+\mathcal{B}(1))$, so

$$\Delta_0 = \frac{\mathcal{G}(\omega_g(\alpha_0)) + \mathcal{B}(1) - \mathcal{B}(\omega_b(\alpha_0))}{\mathcal{G}(1) + \mathcal{B}(1)} = \frac{T(\alpha_0)}{\mathcal{G}(1) + \mathcal{B}(1)}, \tag{A.38}$$

where $T(\alpha_0)$ is the total mass of acceptable borrowers at α_0 as defined in (41). The skill-choice ODE (24) gives $q'(\alpha_0) = (q_0 - 1)\Phi(\alpha_0, q_0)/\Omega_g(\alpha_0)$, where $\Phi(\alpha_0, q_0) > 0$ by the second-order condition at α_0 (Lemma A.2). Equating with (A.37):

$$w^{\mathrm{iid}}(\alpha_0) = \frac{\Phi(\alpha_0, q_0)\,T(\alpha_0)\,D(r_p)}{q_0\,\Omega_g(\alpha_0)\,\big(\Omega_g(\alpha_0) + \Omega_b(\alpha_0) - 1\big)}. \tag{A.39}$$

Nested density. From the nested equilibrium construction, $w^{\mathrm{nested}}(\alpha_0) = \theta(\alpha_0)\,D(r_p)\,T(\alpha_0)$, where $\theta(\alpha_0)$ is given by (43).

Ratio and limit. Dividing:

$$\frac{w^{\mathrm{iid}}(\alpha_0)}{w^{\mathrm{nested}}(\alpha_0)} = \frac{\Phi(\alpha_0, q_0)}{q_0\,\Omega_g(\alpha_0)\,\big(\Omega_g(\alpha_0) + \Omega_b(\alpha_0) - 1\big)\,\theta(\alpha_0)}. \tag{A.40}$$

By the standing assumption $\mathcal{G}(\beta)/\mathcal{G}(1) + \mathcal{B}(1-\beta)/\mathcal{B}(1) = 1$ we have $\Omega_g(0) + \Omega_b(0) = 1$, so as $\alpha_0 \to 0^+$,

$$\Omega_g(\alpha_0) + \Omega_b(\alpha_0) - 1 = \big[\Omega_g'(0) + \Omega_b'(0)\big]\,\alpha_0 + O(\alpha_0^2),$$

where $\Omega_g'(0) = (1 - \beta)\,\mathscr{g}(\beta)/\mathcal{G}(1) > 0$ and $\Omega_b'(0) = \beta\,\mathscr{b}(1 - \beta)/\mathcal{B}(1) > 0$. Hence the denominator of (A.39) vanishes like α_0, so $w^{\mathrm{iid}}(\alpha_0) \geq C/\alpha_0$ for some constant $C > 0$.

Meanwhile, $w^{\mathrm{nested}}(\alpha_0) = \theta(\alpha_0)\,D(r_p)\,T(\alpha_0)$ remains bounded as $\alpha_0 \to 0^+$: $T(\alpha_0)$ converges to a positive limit (the T formula (41) is a ratio of smooth functions with positive denominator); and $\theta(\alpha_0)$ from (43) has two terms, $-(d/d\alpha)\ln B|_{\alpha_0}$ (bounded) and $\beta\,\mathscr{b}(\omega_b)/(\mathcal{B}(1) - \mathcal{B}(\omega_b))$, which converges to $\beta\,\mathscr{b}(1 - \beta)/(\mathcal{B}(1) - \mathcal{B}(1 - \beta))$, finite since $1 - \Omega_b(0) = \Omega_g(0) > 0$ by assumption.

Therefore $w^{\mathrm{iid}}/w^{\mathrm{nested}} \to \infty$, proving Part 1.

Part 2. Each of the three sufficient conditions follows from a different combination of results in Lemma 2.

(a) λ sufficiently high. By Part (i) of Lemma 2, α_0 is strictly decreasing in λ at any interior optimum, with smooth dependence by the implicit function theorem. By Part (ii), there exists a finite corner threshold $\bar{\lambda}$ with $\alpha_0(\bar{\lambda}) = 0$. Hence $\alpha_0(\lambda)$ is continuous and strictly decreasing on the interior region and hits 0 at $\bar{\lambda}$. By the intermediate value theorem, for any $\underline{\alpha} > 0$ there exists $\lambda^* \leq \bar{\lambda}$ such that $\alpha_0(\lambda) < \underline{\alpha}$ for all $\lambda \geq \lambda^*$.

(b) q_0 sufficiently close to 1. Analogous to (a): Part (i) gives strict monotonicity of α_0 in q_0 (decreasing) and Part (ii) gives a finite corner threshold $\bar{q} < 1$. By the same intermediate-value argument, for any $\underline{\alpha} > 0$ there exists $q^* \leq \bar{q}$ such that $\alpha_0(q_0) < \underline{\alpha}$ for all $q_0 \geq q^*$.

(c) β sufficiently close to 0. Lemma 2 provides no monotonicity result for β under general priors. We use Part (ii) directly: the corner threshold $\underline{\beta} > 0$ exists, so $\alpha_0(\beta) = 0 < \underline{\alpha}$ whenever $\beta \leq \underline{\beta}$. ☐

B Proof of Proposition 6

Proof. Suppose we have constructed an incumbent equilibrium with measure W, thresholds $\alpha_0, \alpha_1, \alpha_2$, interest rate schedule $r(\alpha)$, where incumbents have cost function $C(\alpha)$ and cost of capital Π. We construct the entry equilibrium in steps.

Step 1: Entrant quality function. Let $\gamma^E(\alpha; r, \tilde{W}^E)$ be the probability that an atomistic selective lender with precision α gets a good borrower in market r, given incumbents enter with measure W and entrants with measure $\tilde{W}^E$. The pool-evolution equations are

$$G\big(X^G; r, 1, \alpha\big) = \mathcal{G}(X^G) - \int_{A(r,z,\alpha)} \Pr_G\big(X^G; r(\alpha), z(\alpha), \alpha\big)\frac{1}{D(r(\alpha))}\, d\big(W(\alpha) + \tilde{W}^E(\alpha)\big), \tag{B.41}$$

$$B\big(X^B; r, 1, \alpha\big) = \mathcal{B}(X^B) - \int_{A(r,z,\alpha)} \Pr_B\big(X^B; r(\alpha), z(\alpha), \alpha\big)\frac{1}{D(r(\alpha))}\, d\big(W(\alpha) + \tilde{W}^E(\alpha)\big). \tag{B.42}$$

Step 2: Marginal entrant and pooling rate. Define

$$\alpha_0^E(r) \equiv \arg\min_{\alpha \leq \alpha_1} \frac{\Pi^E + C^E(\alpha) + 1}{\gamma^E(\alpha; r, 0)} - 1$$

and solve for the pooling rate r_p^E:

$$r_p^E = \frac{\Pi^E + C^E(\alpha_0^E(r_p^E)) + 1}{\gamma^E(\alpha_0^E(r_p^E); r_p^E, 0)} - 1. \tag{B.43}$$

If $r_p^E \geq r_p$, there is no entry in the pooling region: $W^E([0, \alpha_1]) = 0$, $\alpha_1^E = \alpha_1$, and the construction proceeds to Step 6.

Step 3: CIM rate and upper boundary of pooling. Find the implied cash-in-the-market interest rate $\hat{r}(\alpha)$ by

$$w(\alpha) = D(\hat{r}(\alpha))\, g(\beta + \alpha(1-\beta))\,(1-\beta)$$

for all α. Let $\alpha_1'^E$ be the smallest element of the set $\{\alpha : \hat{r}(\alpha') \geq r_p^E \text{ for all } \alpha' \in [\alpha, \alpha_1]\}$.

Step 4: Indifference boundary. Find $\alpha_1''^E$ by the indifference condition:

$$r_p^E - C^E(\alpha_1''^E) = \Pi^E. \tag{B.44}$$

Let $\alpha_1^E = \max(\alpha_1'^E, \alpha_1''^E)$.

Step 5: Pooling-region entry measure via $T(\alpha)$. We construct W^E on $[\alpha_0^E, \alpha_1^E]$ using the algebraic approach from the incumbent equilibrium. In the combined system, the zero-profit condition for entrants is

$$\gamma^E(r_p^E, 1, \alpha) = \frac{1 + K^E(\alpha)}{1 + r_p^E} \qquad \text{for all } \alpha \in [\alpha_0^E, \alpha_1^E]. \tag{B.45}$$

We verify that the structural lemmas underlying the $T(\alpha)$ formula carry over to the combined system.

Lemma B.19 (Own-slice good density is undepleted in the combined system)**.** *In the combined incumbent-plus-entrant system, for all $\alpha \in [\alpha_0^E, \alpha_1^E]$:*

$$g\big(\omega_g(\alpha);\, r_p^E, 1, \alpha\big) = g(\omega_g(\alpha)).$$

Proof. A good borrower at opacity $\omega_g(\alpha)$ satisfies $\omega > \omega_g(\alpha')$ for all $\alpha' < \alpha$. No lender—incumbent or entrant—with skill $\alpha' < \alpha$ could identify this borrower as good. All lenders in the pooling region are selective, so this borrower was never in any previous lender's acceptance region. The density is undepleted. □

Lemma B.20 (Bad borrowers are uniformly depleted in the combined system)**.** *In the combined system, there exists a common depletion factor $E^E(\alpha) \in (0, 1]$ such that*

$$b\big(\omega;\, r_p^E, 1, \alpha\big) = \mathscr{b}(\omega)\, E^E(\alpha) \qquad \textit{for all } \omega \geq \omega_b(\alpha).$$

Proof. A bad borrower at opacity $\omega \geq \omega_b(\alpha)$ satisfies $\omega \geq \omega_b(\alpha')$ for all $\alpha' \leq \alpha$. Every lender (incumbent or entrant) from α_0^E through α accepted this borrower, each depleting the density at the common rate $\theta^E(\alpha')$. Therefore

$$b(\omega; \alpha) = \mathscr{b}(\omega)\, \exp\Big(-\int_{\alpha_0^E}^{\alpha} \theta^E(s)\, ds\Big) \equiv \mathscr{b}(\omega)\, E^E(\alpha),$$

which is independent of ω. □

With these lemmas in hand, the pool evolution in the combined system is governed by the same equations as in the incumbent case:

$$G^E(\alpha + d\alpha) \approx (1 - \theta^E\, d\alpha)\, G^E(\alpha) + (1 - \beta)\, \mathscr{g}(\omega_g(\alpha))\, d\alpha, \tag{B.46}$$

$$B^E(\alpha + d\alpha) \approx (1 - \theta^E\, d\alpha)\, B^E(\alpha) - \beta\, \mathscr{b}(\omega_b(\alpha))\, E^E(\alpha)\, d\alpha, \tag{B.47}$$

where $\theta^E(\alpha) = (w(\alpha) + w^E(\alpha))/(D(r_p^E)\, T^E(\alpha))$ and $T^E = G^E + B^E$.

From Lemma B.19, $g(\alpha) = \mathscr{g}(\omega_g(\alpha))$. From the zero-profit condition (B.45),

$$G^E(\alpha) = \frac{1 + K^E(\alpha)}{1 + r_p^E}\, T^E(\alpha), \qquad B^E(\alpha) = \frac{r_p^E - K^E(\alpha)}{1 + r_p^E}\, T^E(\alpha). \tag{B.48}$$

Equating the first-order quality increment from the pool evolution (as in eq. (38) of the incumbent case) with the increment from the zero-profit condition, and using Lemmas B.19–B.20 to substitute $g(\alpha) = \mathscr{g}(\omega_g(\alpha))$ and $b(\alpha) = \mathscr{b}(\omega_b(\alpha))\, E^E(\alpha)$, and eliminating E^E via

$$E^E(\alpha) = \frac{B^E(\alpha)}{1 - \mathscr{B}(\omega_b(\alpha))} = \frac{(r_p^E - K^E(\alpha))\, T^E(\alpha)}{(1 + r_p^E)(1 - \mathscr{B}(\omega_b(\alpha)))},$$

we solve for $T^E(\alpha)$:

$$\boxed{T^E(\alpha) = \frac{(1 - \beta)\, \mathscr{g}(\omega_g(\alpha))\, (r_p^E - K^E(\alpha))\, \big(1 - \mathscr{B}(\omega_b(\alpha))\big)\, (1 + r_p^E)}{(K^E)'(\alpha)\, \big(1 - \mathscr{B}(\omega_b(\alpha))\big)\, (1 + r_p^E) - \beta\, \mathscr{b}(\omega_b(\alpha))\, (1 + K^E(\alpha))(r_p^E - K^E(\alpha))}}. \tag{B.49}$$

This is the same formula as the incumbent (41) with (K, r_p) replaced by (K^E, r_p^E).

The depletion rate is

$$\theta^E(\alpha) = -\frac{d}{d\alpha} \ln B^E(\alpha) - \frac{\beta\, \mathscr{b}(\omega_b(\alpha))}{1 - \mathscr{B}(\omega_b(\alpha))}, \tag{B.50}$$

where $B^E(\alpha) = (r_p^E - K^E(\alpha))\, T^E(\alpha)/(1 + r_p^E)$.

The total lending density at skill α in the combined system is

$$w(\alpha) + w^E(\alpha) = \theta^E(\alpha)\, D(r_p^E)\, T^E(\alpha),$$

yielding the entrant density:

$$w^E(\alpha) = \theta^E(\alpha)\, D(r_p^E)\, T^E(\alpha) - w(\alpha). \tag{B.51}$$

Entry occurs at α if and only if $w^E(\alpha) > 0$.

Step 6: Ironing. If $\alpha_1^E < \alpha_1$ and $w^E(\alpha) < 0$ on some subinterval of $[\alpha_0^E, \alpha_1^E]$, ironing is required: on intervals where $w^E < 0$, no entry occurs, the pool evolves according to incumbent lending alone, and the boundaries of the active entry region are adjusted. The ironed entry measure is

$$w^E(\alpha) = \max\big(0,\ \theta^E(\alpha)\, D(r_p^E)\, T^E(\alpha) - w(\alpha)\big),$$

with $T^E(\alpha)$ recomputed on the ironed intervals (where the zero-profit condition may not hold with equality, because incumbent capital alone may generate excess quality).

Step 7: Point of non-selective entry. Let $B^{NS,E} = B([0,1]; r_p^E, 1, \alpha_1^E)$ be all bad borrowers who did not borrow at the pooling market. Let

$$\gamma^{NS,E}(\alpha) = \frac{\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))}{\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta)) + B^{NS,E}}$$

be the fraction of good applicants a non-selective entrant gets at market $r_{\text{CIM}}(\alpha)$.

Determine the group of lenders who will serve good borrowers with opacity just above $\beta + \alpha_2(1-\beta)$ by comparing three interest rates:

$$\begin{aligned} r' &= D^{-1}\left(\frac{w^{NS}}{B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha_2(1-\beta))}\right), \\ r'' &= \frac{1 + \Pi^E}{\gamma^{NS,E}(\alpha_2)} - 1, \\ r''' &= C^E(\alpha_2) + \Pi^E. \end{aligned}$$

(a) If $r' = \min(r', r'', r''')$: incumbent non-selectives serve these borrowers. There are no non-selective entrants.

   (i) If $r' > r_{NS}$: $r^{NS,E} = r'$, $\alpha_2^E = \alpha_2$ (with a possible jump).

   (ii) If $r' < r_{NS}$: Region IIb arises with $\alpha_2^E < \alpha_2$. The incumbent non-selectives are reallocated in the new equilibrium (see Step 8, Case A).

(b) If $r'' = \min(r', r'', r''')$: entrant non-selectives serve these borrowers.

   (i) If $r'' > r_{NS}$: $r^{NS,E} = r''$, $\alpha_2^E = \alpha_2$.

   (ii) If $r'' < r_{NS}$: Region IIb arises with $\alpha_2^E < \alpha_2$, and entrant non-selectives enter along the path $\tilde{r}(\alpha)$ (see Step 8, Case B).

(c) If $r''' = \min(r', r'', r''')$: skilled entrants extend the CIM region to the right, $\alpha_2^E > \alpha_2$. Find α_2^E as the intercept where the entrant CIM rate meets the non-selective entry rate.

Step 8: Region IIb — determining $\tilde{r}(\alpha)$ and α_2^E. When Region IIb arises, we distinguish two sub-cases depending on whether the non-selective lenders are entrants or incumbents.

Case B (Step 7b-ii): non-selective entrants. In this case, α_2^E is the solution of

$$\gamma^{NS,E}(\alpha_2^E)\left(1 + \min(C^E(\alpha_2^E) + \Pi^E,\ C(\alpha_2^E) + \Pi)\right) = 1 + \Pi^E,$$

and $\tilde{r}(\alpha)$ for $\alpha \in [\alpha_2^E, \alpha_2]$ is given by the constant-profit condition

$$\gamma^{NS,E}(\alpha)\,(1 + \tilde{r}(\alpha)) = 1 + \Pi^E, \tag{B.52}$$

or, equivalently,

$$\gamma^{NS,E}(\alpha)\,(1 + \tilde{r}(\alpha)) = \gamma^{NS,E}(\alpha_2^E)\,(1 + r_{\text{CIM}}^E(\alpha_2^E)). \tag{B.53}$$

The mass of non-selective entrants entering along with incumbents in markets $r \in [r^E_{\mathrm{CIM}}(\alpha_2^E),\, r^{NS,E}]$ is

$$w^{NS,E}(\alpha) = -\phi'(\alpha)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right) D(\tilde{r}(\alpha)), \tag{B.54}$$

and

$$\phi(\alpha) = \frac{w(\alpha)}{D(\tilde{r}(\alpha))\, g(\beta + \alpha(1-\beta))\,(1-\beta)}, \tag{B.55}$$

with boundary condition $\phi(\alpha_2^E) = 1$.

In this case, the total required capital strictly exceeds W^{NS}:

$$\int_{\alpha_2^E}^{\alpha_2} -\phi'(\alpha)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right) D(\tilde{r}(\alpha))\, d\alpha \\ + D(\tilde{r}(\alpha_2))\,\phi(\alpha_2)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha_2(1-\beta))\right) > W^{NS}.$$

The excess is absorbed by an atom of non-selective entrant capital at $\tilde{r}(\alpha_2)$, whose mass equals the difference of the two sides of this inequality.

Case A (Step 7a-ii): incumbent non-selectives. In this case there is no non-selective entry. Instead, the incumbent non-selectives are reallocated in the new equilibrium. We conjecture α_2^E, which by (B.53) determines $\tilde{r}(\alpha)$ and by (B.55) determines $\phi(\alpha)$. The value of α_2^E is pinned down by the capital-clearing condition

$$\int_{\alpha_2^E}^{\alpha_2} -\phi'(\alpha)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right) D(\tilde{r}(\alpha))\, d\alpha \\ + D(\tilde{r}(\alpha_2))\,\phi(\alpha_2)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha_2(1-\beta))\right) = W^{NS}.$$

Verification of non-selective profit. Under either case, all non-selective lenders earn the same profit when entering at any $r \in [r^E_{\mathrm{CIM}}(\alpha_2^E),\, \tilde{r}(\alpha_2)]$, and all good borrowers with $\alpha > \alpha_2^E$ are cleared at a weakly increasing interest rate in α.

Let $B(\alpha)$ denote the total number of bad borrowers applying to market α, and define

$$\phi(\alpha) = \frac{B(\alpha)}{B^{NS,E}}, \tag{B.56}$$

the fraction of bad borrowers remaining relative to $B^{NS,E}$. The total measure of borrowers that non-selectives face in market α is then

$$\underbrace{B(\alpha)}_{\text{bad borrowers}} \quad + \quad \underbrace{\phi(\alpha)\left[\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right]}_{\text{good borrowers not yet served by non-selectives before } \alpha} \\ = \phi(\alpha)\left(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right),$$

which explains why (B.54) is a market-clearing condition.

The profit of non-selectives in market α is

$$\frac{\phi(\alpha)\left[\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right]}{\phi(\alpha)\left[\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))\right] + \phi(\alpha)\, B^{NS,E}}\,(1 + \tilde{r}(\alpha)) - 1 - C^E(0) \\ = \frac{\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta))}{\mathcal{G}(1) - \mathcal{G}(\beta + \alpha(1-\beta)) + B^{NS,E}}\,(1 + \tilde{r}(\alpha)) - 1 - C^E(0) \\ = \gamma^{NS,E}(\alpha)\,(1 + \tilde{r}(\alpha)) - 1 - C^E(0).$$

The $\phi(\alpha)$ factors cancel, confirming that all non-selectives earn the same profit regardless of their market. The definition of $\tilde{r}(\alpha)$ in (B.52) then implies that this common profit equals $\Pi^E - C^E(0)$.

The function ϕ satisfies (B.55) because the incumbent with precision α and one unit of capital serves exactly the good types at $\omega = \beta + \alpha(1-\beta)$:

$$D(\tilde{r}(\alpha))\,\phi(\alpha)\,g(\beta+\alpha(1-\beta))\,(1-\beta) = w(\alpha).$$

Intuitively, the left-hand side is the demand for capital from good applicants with $\omega = \beta+\alpha(1-\beta)$, while the right-hand side is the supply of incumbents with exactly that precision.

Derivation of the ϕ recursion. By definition, $\phi(\alpha_2^E) = 1$. For $\epsilon > 0$:

$$\begin{aligned}
\phi(\alpha+\epsilon) &= \phi(\alpha) - \frac{w^{NS,E}(\alpha)\,\epsilon}{\big(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta+\alpha(1-\beta))\big)\,D(\tilde{r}(\alpha))} \\
&= \phi(\alpha) - \frac{\phi(\alpha)\,D(\tilde{r}(\alpha)) - \dfrac{w(\alpha)}{g(\beta+\alpha(1-\beta))\,(1-\beta)}}{D(\tilde{r}(\alpha))} \\
&= \frac{w(\alpha)}{D(\tilde{r}(\alpha))\,g(\beta+\alpha(1-\beta))\,(1-\beta)},
\end{aligned}$$

which also implies

$$\frac{\phi(\alpha+\epsilon)-\phi(\alpha)}{\epsilon} = -\frac{w^{NS,E}(\alpha)}{\big(B^{NS,E} + \mathcal{G}(1) - \mathcal{G}(\beta+\alpha(1-\beta))\big)\,D(\tilde{r}(\alpha))},$$

or, in the limit $\epsilon \to 0$, equation (B.54).

Step 9: CIM region. The CIM region is $\alpha \in [\alpha_1^E, \alpha_2^E]$ with

$$r^E_{\text{CIM}}(\alpha) = \begin{cases} \min(\hat{r}(\alpha),\, C^E(\alpha)+\Pi^E) & \text{if } \alpha < \alpha_2, \\ C^E(\alpha)+\Pi^E & \text{otherwise.} \end{cases}$$

Entry in the CIM region with precision α is

$$w^E(\alpha) = \max\!\big(0,\, D(r^E_{\text{CIM}}(\alpha))\,g(\beta+\alpha(1-\beta))\,(1-\beta) - w(\alpha)\big).$$

□

C Exogenous Distribution of Screening Technology

This appendix analyzes the equilibrium when the distribution of lender skill $w(\alpha)$ is exogenous rather than determined by endogenous entry both for the independent information and the nested information case.

C.1 Independent Information

The equilibrium construction follows the same logic as in Section 2.1, so we focus on the key differences. With exogenous $w(\alpha)$, the skill choice first-order condition (21) no longer applies, since lenders do not choose α. However, the market choice first-order condition (22) still holds: each lender must be indifferent across markets in equilibrium.

The definitions of pool quality $q(\alpha)$ in (12), portfolio quality $\gamma(\alpha, q(\tilde{\alpha}))$ in (13), and the pool evolution equations carry over unchanged. In particular, the laws of motion for the good and bad borrower pools are the same as (26)–(27):

$$\begin{aligned}\frac{dG(\alpha)}{d\alpha} &= -\frac{w(\alpha)}{D(r(\alpha))}\gamma(\alpha, q(\alpha)) \\ \frac{dB(\alpha)}{d\alpha} &= -\frac{w(\alpha)}{D(r(\alpha))}[1-\gamma(\alpha, q(\alpha))]\end{aligned}$$

and the implied pool quality evolution is (28). The only difference is that $w(\alpha)$ is now a predetermined function rather than an equilibrium object.

In the endogenous case, the two first-order conditions (21) and (22) combine into a single ODE for $q(\alpha)$ given by (24), which is independent of $w(\alpha)$. The density $w(\alpha)$ is then backed out from the pool evolution (28). With exogenous $w(\alpha)$, the skill choice condition is absent, and that single ODE no longer applies. Instead, the equilibrium is characterized by a coupled system of two ODEs. The first is the pool evolution (28) with the given $w(\alpha)$. The second is the interest rate ODE obtained by rearranging the market choice condition (22):

$$r'(\alpha) = -\frac{\frac{\partial\gamma}{\partial\tilde{\alpha}}(\alpha, \alpha)}{\gamma(\alpha, q(\alpha))}(1 + r(\alpha)) \tag{C.57}$$

where $\frac{\partial\gamma}{\partial\tilde{\alpha}}$ depends on $q'(\alpha)$ through the chain rule. Substituting (28) into (C.57) yields a system in $(q(\alpha), r(\alpha))$ that can be integrated forward given initial conditions and the exogenous $w(\alpha)$.

Boundary conditions and the role of α_0. Let Π denote the minimum required profit for entry. Define α_0 as the lowest active lender, i.e., the lender with the lowest skill who finds it profitable to participate. The initial pool quality is the unconditional average:

$$q(\alpha_0) = \frac{\mathcal{G}(1)}{\mathcal{G}(1) + \mathcal{B}(1)} \tag{C.58}$$

as in (15). The interest rate $r(\alpha_0)$ is pinned down by the break-even condition for the marginal lender:

$$\gamma(\alpha_0, q(\alpha_0))(1 + r(\alpha_0)) = 1 + \Pi \tag{C.59}$$

That is, the least-skilled active lender earns exactly Π.

The terminal condition requires that all good borrowers are served: $G(1) = 0$. In the endogenous case (when $\bar{\alpha} = 1$), this condition is straightforward to satisfy: the mass of perfect screeners at $\alpha = 1$ is an equilibrium object, so one simply adjusts the size of this atom to absorb all remaining good borrowers $G(1)$. With exogenous $w(\alpha)$, this is no longer possible—the mass of perfect screeners $w(1)$ is given. Instead, α_0 serves as the free parameter: it must be chosen so that, after integrating the ODE system forward from the initial conditions $(q(\alpha_0), r(\alpha_0))$ using the given $w(\alpha)$, the terminal condition $G(1) = 0$ is satisfied. In other words, α_0 is determined by a shooting condition: the right starting point ensures the borrower pool is exactly exhausted at $\alpha = 1$.

A further difference from the endogenous case is that lenders no longer earn equal profits. Higher-α lenders generally earn profits above Π because their superior screening allows them to extract rents from better pool selection, net of the interest rate adjustment. With endogenous entry, competition through the skill choice margin eliminates these rents. With exogenous $w(\alpha)$, the only equilibrium force is the market choice condition, which equalizes the marginal benefit of moving across markets but not the level of profits.

C.2 Nested Information

The equilibrium construction follows the same logic as in Section 2.2, so we focus on the key differences. The hockey-stick interest rate schedule is robust to this change, but some borrowers may be excluded from credit.

Proposition C.3 (Equilibrium with Exogenous Skill Distribution). *Assume the wealth distribution of lenders $w(\alpha)$ is exogenous. An equilibrium exists, is unique, and features a hockey-stick interest rate schedule with thresholds α_0, α_1, α_2 satisfying* $0 \leq \alpha_0 \leq \alpha_1 \leq \alpha_2 \leq 1$. *The three regions differ from the endogenous case (Proposition 2) as follows:*

1. **Region I:** *In addition to every transparent good borrower with $\omega \leq \omega_g(\alpha_1)$ and every opaque bad borrower with $\omega > \omega_b(\alpha_1)$, some transparent bad borrowers with $\omega < \omega_b(\alpha_1)$ also borrow at r_p.*

2. **Region II:** *No change from the endogenous case.*

3. **Region III:** *Lenders with low skill $\alpha < \alpha_0$ (not just $\alpha = 0$) lend non-selectively at r_{NS}, along with high-skill lenders $\alpha > \alpha_2$ who are selective. Some transparent bad borrowers with $\omega < \omega_b(\alpha_0)$ are not served.*

C.3 Region I: The Smooth Pasting Condition

With endogenous entry, the density $w(\alpha)$ in Region I is pinned down by equation (44), which ensures that each lender makes profit Π. With exogenous $w(\alpha)$, this equation no longer applies. Instead, we need a condition to determine the boundary α_0.

The evolution of the borrower pool follows the same differential equations (34)-(35) as in the main text, with $\theta(\alpha)$ given by (33). However, since $w(\alpha)$ is now exogenous, $\theta(\alpha)$ is determined by the given distribution rather than by the equal-profit condition.

The key new condition is *smooth pasting* at the boundary α_1. Define the total mass of acceptable borrowers as $T(\alpha) = G(\alpha) + B(\alpha)$, where $G(\alpha)$ and $B(\alpha)$ are the masses of acceptable good and bad borrowers at lender α's turn. Taking the derivative:

$$T'(\alpha) = -\frac{w(\alpha)}{D(r_p)} + (1-\beta)g(\omega_g(\alpha)) - \beta b(\omega_b(\alpha);\alpha) \tag{C.60}$$

At α_1, the pool of acceptable borrowers is exhausted: $T(\alpha_1) = 0$. For the interest rate schedule to be continuous at the transition to Region II, we require:

$$T'(\alpha_1) = 0 \tag{C.61}$$

This smooth pasting condition (C.61), together with $T(\alpha_1) = 0$, pins down both α_0 and α_1 given the exogenous $w(\alpha)$.

C.4 Region II: Market Clearing with Exogenous Supply

In Region II, selective lenders serve only good borrowers. The interest rate schedule $r(\alpha) = K(\alpha)$ from (45) still applies: lenders must be compensated for their information costs.

The difference is in the market clearing condition. With endogenous entry, equation (46) determines $w(\alpha)$. With exogenous $w(\alpha)$, market clearing instead determines which lenders are active:

$$w(\alpha) = D(r(\alpha))g(\omega_g(\alpha))(1-\beta) \tag{C.62}$$

If $w(\alpha)$ exceeds the right-hand side of (C.62), lender α cannot be fully active in Region II. This excess capacity shifts to Region III.

C.5 Region III: Partial Exclusion of Bad Borrowers

The most significant difference from the endogenous case arises in Region III. With endogenous entry, uninformed lenders ($\alpha = 0$) enter in sufficient mass to serve all remaining borrowers, ensuring no exclusion. With exogenous $w(\alpha)$, this need not hold.

Lenders with $\alpha < \alpha_0$ cannot profitably lend selectively at r_p (they lack sufficient skill). Instead, they lend non-selectively at r_{NS}. Their total mass is $\int_0^{\alpha_0} w(\alpha)d\alpha$, which is exogenously given. If this mass is insufficient to serve all remaining borrowers, some are excluded.

Specifically, the remaining good borrowers (with $\omega > \omega_g(\alpha_2)$) are served by high-skill lenders $\alpha > \alpha_2$ who find it profitable to lend selectively at r_{NS}. The leftover bad borrowers $B([0,1]; r_p, 1, \alpha_1)$ compete for the remaining non-selective lending capacity. Bad borrowers with the lowest opacity $\omega < \omega_b(\alpha_0)$ may be excluded from credit entirely—unlike the endogenous case where all borrowers are eventually served.

C.6 Equilibrium Characterization

The equilibrium is determined by the following system. Given exogenous $w(\alpha)$ and profit level Π for the marginal lender:

1. Find α_0 and α_1 from the smooth pasting conditions: $T(\alpha_1; \alpha_0) = 0$ and $T'(\alpha_1; \alpha_0) = 0$
2. Find r_p from the break-even condition: $\gamma_0(\alpha_0)(1 + r_p) = 1 + \Pi$

3. Find α_2 from the non-selective indifference condition: $\gamma^{NS}(\alpha_2)(1 + r(\alpha_2)) = 1 + \Pi$

4. Verify market clearing at all interest rates

Proposition C.4 (Duality). *For any exogenous lender distribution $w(\alpha)$ with full support, there exists an increasing cost function $C(\alpha)$ such that $w(\alpha)$ arises as the equilibrium distribution when lenders choose α endogenously. Moreover, $C(\alpha)$ is unique on the support of $w(\alpha)$.*

This duality result implies that qualitative features of the equilibrium—the hockey-stick structure, the three regions, the relationship between skill and interest rates—are robust across the exogenous and endogenous formulations. The key difference is quantitative: the exogenous case may feature exclusion when the given $w(\alpha)$ does not provide sufficient non-selective lending capacity.

D Microfoundation for Borrowers Demand

Consider a borrower with type (τ, ω) endowed with a unit of capital and a project. She wants to obtain a loan $\ell(\tau, \omega)$ to invest $i(\tau, \omega)$ in period 1 to consume the proceeds in period 2. Each unit of investment in the morning produces ρ return. The cost of investment has to be covered by the borrower's initial endowment or credit, implying the following budget constraint

$$i(\tau, \omega) = 1 + \ell(\tau, \omega). \tag{D.63}$$

Furthermore, each borrower has to pledge her investment as collateral to obtain credit. Seizing the collateral is the only threat to enforce repayment from the borrowers, thus $(1 + r_t(\tau, \omega))\ell_t(\tau, \omega) \leq i_t(\tau, \omega)$. Using (D.63) this simplifies to

$$\ell_t(\tau, \omega) \leq \frac{1}{r_t(\tau, \omega)}. \tag{D.64}$$

Given the linear technology, all borrowers would like to borrow the maximum

$$D(r) = \frac{1}{r}$$

at the minimal interest rate they can obtain loans as described in Assumption 1.

E Online Appendix: Numerical Examples

This appendix records the parametrizations under which Figures 7 and 8 were generated as numerical realizations of the stylized schematics in the main text. All three examples share a common incumbent baseline; they differ only in the entrant capital cost Π^E and the entrant screening cost $C^E(\alpha)$.

E.1 Common incumbent baseline

$$\Pi = 0.235, \qquad \beta = 0.5, \qquad B/G = 1, \qquad C(\alpha) = 9\alpha^2 + 0.2\,\alpha.$$

The Incumbent Equilibrium then yields $\alpha_0 \approx 0.056$, $\alpha_1 \approx 0.351$, $\alpha_2 \approx 0.633$, pooling rate $r_p \approx 1.43$ and non-selective rate $r_{NS} \approx 3.97$.

All three sets of panels below are produced by Python scripts that share a common solver module `mainE_python.py` (`https://github.com/Kopeti/pabloproject/blob/main/mainE_python.py`) and a common plotting module `panel_plots.py` (`https://github.com/Kopeti/pabloproject/blob/main/panel_plots.py`); the per-figure scripts `make_figure8.py`, `make_figure9a.py` and `make_figure9b.py` are thin drivers that call the solver with the parameter set listed in each subsection below and pass the result to the plotting helpers.

The three panels. For each of the three examples below we plot the same three diagnostics:

(a) $r(\alpha)$: the equilibrium interest rate as a function of the lender's screening precision α. The incumbent's hockey-stick rate schedule (green dashed) has a flat pooling segment r_p on $[\alpha_0, \alpha_1]$, a rising CIM segment on $[\alpha_1, \alpha_2]$, and a flat NS plateau r_{NS} on $[\alpha_2, 1]$, plus an atom at $\alpha = 0$ corresponding to non-selective lenders. The entrant's post-entry schedule is overlaid (blue solid).

(b) $r(\omega)$: the same equilibrium rates re-indexed by borrower opacity $\omega = \omega_g(\alpha) = \beta + \alpha(1-\beta)$, with vertical dashed lines marking the region boundaries ω_1, ω_2 and the example-specific thresholds (ω_H, $\hat{\omega}_2$, ω_2').

(c) $K(\alpha)$: the effective capital cost $K(\alpha) = \Pi + C(\alpha)$ for the incumbent (green dashed) and $K^E(\alpha) = \Pi^E + C^E(\alpha)$ for the entrant (blue solid). This panel shows the underlying cost-function comparison that drives the equilibrium shapes in panels (a) and (b).

E.2 Big-Data Innovation (Figure 7)

Entrant parameters: $\Pi^E = 0.5$, and

$$C^E(\alpha) \;=\; w(\alpha)\,C(\alpha), \qquad w(\alpha) \;=\; 1 - (1-\lambda)\,\sigma\!\left(\tfrac{\alpha - \hat{\alpha}}{s}\right),$$

with $\hat{\alpha} = 0.55$, $\lambda = 0.5$, $s = 0.08$, and σ the logistic. This places the cost reduction in the high-α range, consistent with the main-text discussion: entrants are uniformly more expensive (or equal) at low and mid α and cheaper only for $\alpha > \hat{\alpha}$.

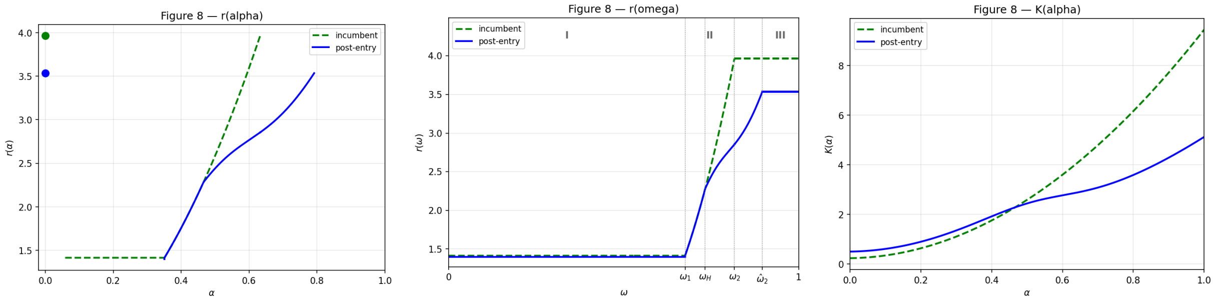

Figure E.1: Numerical realization of Figure 7.

This calibration is on the r'''-min branch of Proposition 6 (the branch identified in the footnote to Figure 7): $\alpha_2^E \approx 0.79 > \alpha_2$, so Region II is extended to the right on $[\alpha_2, \alpha_2^E]$ and no Region IIb opens. The non-selective rate falls from $r_{NS} \approx 3.97$ to $r_{NS}^E \approx 3.54$, i.e. a positive spillover to the financially excluded of about 0.43.

The three panels are produced by running `make_figure8.py` (`https://github.com/Kopeti/pabloproject/blob/main/make_figure8.py`).

E.3 Limited Open Banking (Figure 8, left panel)

Entrant parameters: $\Pi^E = 0.168$, and for $\alpha \le \alpha_1$

$$C^E(\alpha) \;=\; C(\alpha) \;+\; c_{\text{lift}}\,\alpha(\bar{\alpha} - \alpha) \;+\; 0.1 \cdot \min\!\Big(0.1\,\big(\tfrac{1}{\alpha_1 - \alpha} - \tfrac{1}{\alpha_1}\big),\, \text{cap}\Big),$$

where $\bar{\alpha} = (\alpha_0 + \alpha_1)/2$, $c_{\text{lift}} = 4$, cap $= 10$. For $\alpha > \alpha_1$, C^E extends parallel to C with a constant offset $0.1\,\text{cap} + c_{\text{lift}}\,\alpha_1(\bar{\alpha} - \alpha_1)$ chosen for continuity at $\alpha = \alpha_1$. The construction ensures $K^E(0) = \Pi^E$ since $C(0) = 0$, the lift term vanishes at $\alpha = 0$, and the add-on vanishes at $\alpha = 0$.

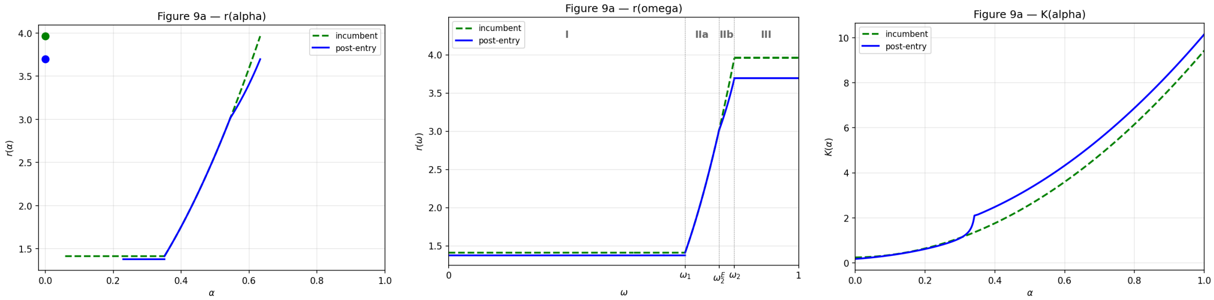

Figure E.2: Numerical realization of Figure 8 (left panel).

Region IIb is non-empty with width $\alpha_2 - \alpha_2^E \approx 0.088$ on the α axis; the non-selective rate drops from $r_{NS} \approx 3.97$ to $r_{NS}^E \approx 3.70$, a positive spillover of about 0.27.

The three panels are produced by running `make_figure9a.py` (`https://github.com/Kopeti/pabloproject/blob/main/make_figure9a.py`).

E.4 Broad Open Banking (Figure 8, right panel)

Entrant parameters: $\Pi^E = 0.235$, and

$$C^E(\alpha) \;=\; C(\alpha)\,\Big[1 + \delta_b - \delta_d\,\exp\!\Big(-\tfrac{(\alpha-\alpha_c)^2}{2\sigma^2}\Big)\Big],$$

with $\alpha_c = 0.25$, $\sigma = 0.05$, $\delta_d = 0.50$, $\delta_b = 0.10$. The multiplicative form ensures $K^E(0) = \Pi^E$ since $C(0) = 0$.

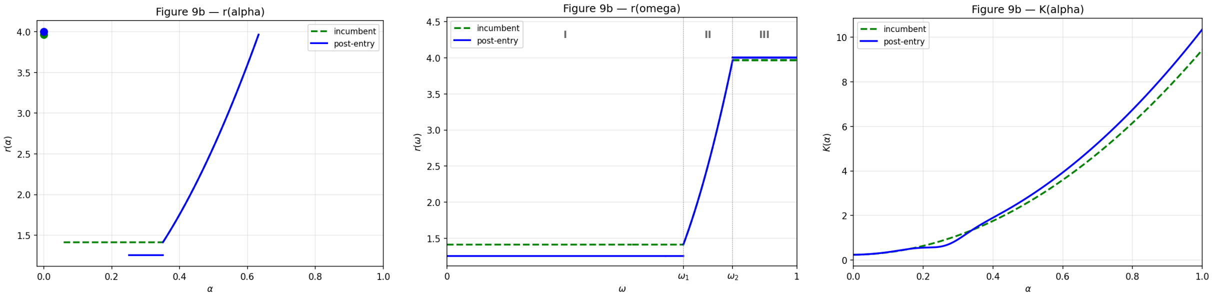

Figure E.3: Numerical realization of Figure 8 (right panel).

There is no Region IIb ($\alpha_2^E = \alpha_2$); the non-selective rate rises slightly from $r_{NS} \approx 3.97$ to $r_{NS}^E \approx 4.00$, a small negative spillover.

The three panels are produced by running `make_figure9b.py` (`https://github.com/Kopeti/pabloproject/blob/main/make_figure9b.py`).