Buy High, Cry Later: Beliefs-driven Cycles of Stock Market Participation

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Abstract

We show that stock market participants are optimists, while exit coincides with a deterioration in beliefs. Accordingly, belief-cycles drive participation-cycles in our model: Inexperienced cohorts enter with high leverage during episodes of market exuberance and low risk premia and exit during market downturns. In equilibrium, experienced cohorts typically remain participants throughout as the market clearing risk premium tends to gravitate towards their beliefs. These interactions generate procyclical participation, feedback between participation and experience-based learning, and welfare losses. Feeding the model with realized shocks reproduces fluctuations in participation, entry, and exit rates from multiple countries.

1   Introduction

One of the fundamental insights of standard portfolio theory[1] is that everyone should participate in the stock market. However, even in the United States with its investment culture, total stock market participation is only slightly above fifty percent.[2] Our analysis starts with highlighting the relevance of beliefs for fluctuations in participation. Specifically, using the Michigan Surveys of Consumers, we document that participants are optimists and nonparticipants are pessimists about future stock market returns, and entry and exit are pervasive and associated with sizable revisions in beliefs towards a "cutoff." Motivated by these facts, we then build a dynamic consumption-based general equilibrium economy to study how cycles in exuberance and pessimism drive stock market entry, exit, and reentry and how these decisions, in turn, affect asset prices and welfare.

The model has three key components: overlapping generations, experience-based learning, and a friction capturing in a reduced-form discouragement that leads pessimists to exit the stock market. The experience-based learning and the assumption that participants pay more attention to the economy and hence follow an additional signal in their learning problem imply a large and time-varying cross section of beliefs about growth. This feature of the model allows us to reproduce entry and exit rates that are realistic and associated with revisions in beliefs. In equilibrium, the fluctuations of perceived expected returns drive the trading strategies of agents, which endogenously determine fluctuations in the real short rate of interest, market price of risk, stock market volatility, and participation.

To gain insight into how the economy works, it is important to look at who is in the market and how that depends on the extent of optimism versus pessimism. When the inexperienced are the optimists, they take levered long positions in the stock market. Instead, when the inexperienced are the pessimists, they usually exit. This is what one would expect. However, for the experienced, we see a different pattern. When the experienced are the pessimists,

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[1]See, for example, the seminal works by Markowitz (1952), Merton (1969) and Merton (1971).

[2]A compilation of stock market participation rates around the world over 2023-2024 can be found here: https://hellosafe.com/investing/broker/percentage-of-population-investing-in-stock-market-by-country.

Figure 1: **The Stock Market and Participation.** The plots show the time-series of price-dividend ratios, which are calculated using $\frac{P_t}{D_t} = \frac{R_t^{price}}{R_t^S - R_t^{price}}$, where R_t^{price} is the price return, and R_t^S is the total return including dividends. Juxtaposed with the price-dividend ratios are detrended stock market participation rates with preserved original means. The data on price-dividend ratios are from Amit Goyal's website for the United States, and from Ken French's website for the other countries. Participation data are from Statistics Finland, Deutsches Aktieninstitut, Statistics Norway, and the Internal Revenue Services.
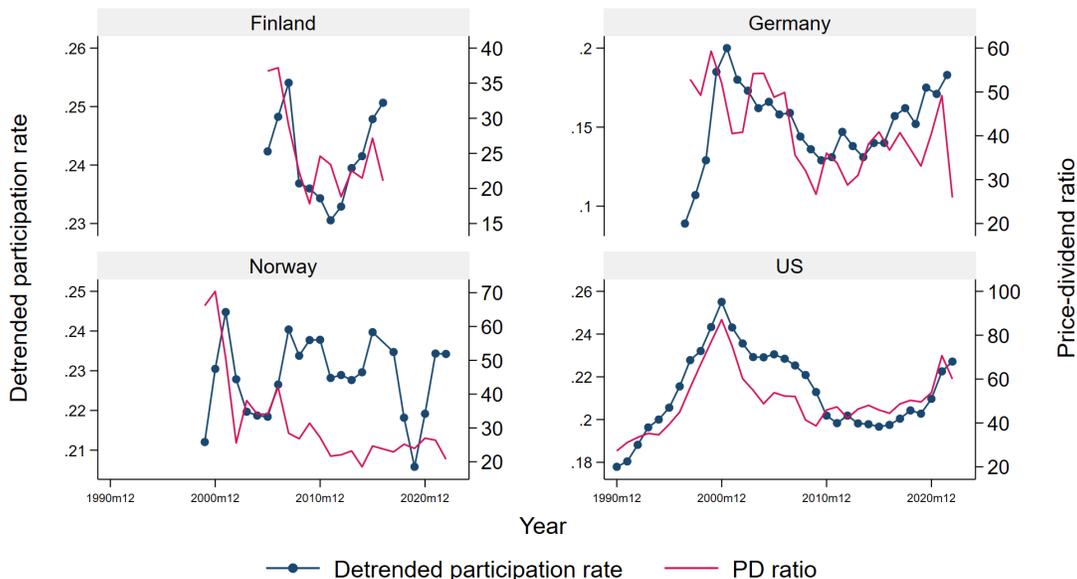
they typically remain in the market throughout. This is because the endogenous market clearing cutoff between participants and nonparticipants is not just determined by beliefs, but also significantly driven by the consumption distribution, which often gravitates towards the experienced. In other words, the experienced and rich pull the cutoff down with their beliefs. This asymmetric response to waves of pessimism leads to a time-varying wedge in consumption growth between experienced and inexperienced. In particular, exuberant inexperienced tend to enter the stock market with high leverage when the true market price of risk is low. These are times when participants have lower consumption growth than nonparticipants. Subsequently, agents may exit the stock market after a period of disappointing returns.

In this regard, Figure 1 suggests that market participation in Finland, Germany, Norway,

and the United States tends to increase during times of elevated valuation ratios. As alluded to above, such behavior may prove costly, as high past returns or elevated valuation ratios often signal lower, rather than higher future risk premia. This suggests that many investors inadvertently adopt a procyclical entry-exit strategy that undermines long-term performance.

What is the cost of cyclical entry-exit strategies? To answer this question, we equate average expected lifetime utility in our model with lifetime utility in a frictionless benchmark economy with complete information, where all cohorts participate. This is achieved by adjusting growth in the benchmark economy à la Lucas (1987). What this exercise shows is a reduction in consumption growth from the actual 2% down to at most 1.3%.[3] The obvious policy response to such large costs from poorly timed sequences of entry, exit, and reentry must be a concerted effort to educate investors on the merits of buy-and-hold strategies. Here, we point to the fact that nowadays every professional investment advisor advertises the benefits of diversification. Building on that, investment advisors should also be required to emphasize the benefits of passive strategies rather than trying to time the market.[4]

Reproducing the time-series of stock market participation, entry, and exit reflects the essence of our analysis. To tease out realistic beliefs-driven effects, we feed —in the spirit of Lettau, Ludvigson, and Wachter (2007)— our model with shocks extracted from the stock market and from various macroeconomic data standing in for the signal. This exercise isolates our proposed mechanism and generates implied time-series of participation rates for Finland, Germany, Norway, and the United States. Broadly speaking, the correlations between the actual time-series of stock market participation and the model counterparts are statistically significant. For Finland and Norway, we can increase the requirement on the model and compare the actual time-series of entry and exit to the counterparts of the model. In particular, while the participation rate is tightly linked to entry and exit in the model,

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[3]See Bhamra and Uppal (2019) for an equilibrium model that studies the welfare loss from underdiversification.

[4]Consistent with our concern that investors might inadvertently adopt a procyclical entry-exit strategy, Duraj, Grunow, Haliassos, Laudenbach, and Siegel (2025) report that nonparticipants and even active participants believe that "participation requires selecting safe stocks, avoiding bad ones, and timing the market through monitoring and frequent trading."

this does not directly imply that reproducing the overall dynamics of the participation rate in a country also reproduces its entry and exit dynamics. Hence, it is reassuring that this exercise also produces statistically significant entry and exit correlations between the model and the data counterparts.

Our main contribution is theoretical and relates closely to the literature that explains stock market nonparticipation through one-time entry costs or per period monitoring costs in an equilibrium overlapping generations setting such as the classical work of Gomes and Michaelides (2008),[5] with focus on the one-time decision to either enter or not to enter.[6] Technically, our continuous-time general equilibrium model with overlapping generations of agents builds on Gârleanu and Panageas (2015), that focuses on asset pricing, and Ehling, Graniero, and Heyerdahl-Larsen (2018b), that focuses on how lifetime experiences affect expectations in an otherwise frictionless market.[7] Our contribution vis-à-vis the literature on endogenous nonparticipation is a dynamic return-driven mechanism explaining entry, exit, and reentry, which then simultaneously determines movements in asset prices. For instance, the time-varying participation rate predicts stock market returns in the model and the data. A recent complementary work by Pan, Su, Wang, and Yu (2025) also studies participation dynamics. They empirically show how new participation growth drives overreaction and amplifies momentum, and present a model with extrapolative trading.

Close to our work in its focus on entry and exit is Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2025). In their life-cycle portfolio choice model with constant expectations, a rare disaster or an increased labor income uncertainty, along with various participation costs produce realistic frequencies for stock market entry and exit, which is a mechanism that is complementary to ours. Another recent paper also using a life-cycle model with constant ex-

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[5]See also the seminal paper on nonparticipation by Basak and Cuoco (1998) and Guvenen (2009), which matches asset prices with a participant and a nonparticipant.

[6]See Khorrami (2022) for a recent model that requires nonparticipants to incur a one-time cost to enter the stock market.

[7]For empirical evidence on lifetime experiences affecting expectations, preferences, and choices see Malmendier and Nagel (2011), Malmendier and Nagel (2016), Knüpfer, Rantapuska, and Sarvimäki (2017) and more recently Cocco, Gomes, and Lopes (2025). See also the quantitative experience-driven equilibrium model by Collin-Dufresne, Johannes, and Lochstoer (2017).

pectations is Gomes and Smirnova (2023); through empirically motivated participation costs, decreasing relative risk aversion, and human capital, the model generates participation that is a hump-shaped function of age. Further, Galaasen and Raja (2024) present a portfolio choice problem with experience effects to explain participation dynamics without requiring high per-period participation costs. They document exit and reentry margins showing among other things that the longer Norwegian households participate, the less likely they are to exit. Our approach emphasizes endogenous fluctuations in entry and exit, which through asset pricing implications lead to time-varying welfare consequences of nonparticipation.

Our paper builds on a vast literature of heterogeneous beliefs models.[8, 9] Further, we build on models in this extended literature that incorporate an additional signal as in Detemple and Murthy (1997) or the sentiment index in Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Xiong and Yan (2010), where the signal or index may improve or inhibit learning about growth. We contribute to the agree-to-disagree literature, in which updating occurs independently of equilibrium prices, by considering an economy which features endogenous feedback between stock market participation, experience-based learning, and asset prices.[10]

2   Motivating Evidence

We use data from the Surveys of Consumers by the University of Michigan to motivate the assumptions of our overlapping generations model and provide empirical context.

The surveys are statistically designed to be representative of households in the United

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[8]From this body of literature see, for example, Harris and Raviv (1993), Detemple and Murthy (1994), Jouini and Napp (2007), Cvitanić, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018a).

[9]We mention Gârleanu, Panageas, and Zheng (2025), a recent study that features heterogeneous beliefs in a continuous-time overlapping generations environment, where their focus is on the performance of a short seller facing a shorting fee.

[10]On a technical note, we solve the incomplete market with participants and nonparticipants as a complete market with fictitious state prices as in He and Pearson (1991) and Karatzas, Lehoczky, Shreve, and Xu (1991). Works that also use fictitious state prices include Cuoco (1997), Basak and Croitoru (2000), Gallmeyer and Hollifield (2008) and Dieckmann (2011).

States. Each month, roughly 400 interviews are conducted and approximately 200 respondents are interviewed a second time after 6 months. Utilizing the panel structure, we define stock market participants[11] as respondents who report investment in an interview, nonparticipants who report no investment in an interviews, and entering (exiting) households as those respondents who are nonparticipants (participants) in the first interview but have become participants (nonparticipants) in the second interview. A pattern in the data that is noteworthy at the onset is how pervasive entry and exit is: On average, 6.02% of the respondents in the panel sample report entry into and 5.74% report exit from the stock market within the 6-month interval.

From June 2002 onward, the survey contains the following question: "[PSTK] What do you think the chances are that $1,000 invested in a diversified stock mutual fund will increase in value in the year ahead, so that it is worth more than $1,000 a year from now?", where respondents can answer by selecting a number between 0 and 100. Specifically, the revealed probabilities allow us to uniquely rank households based on their beliefs. Hence, we use PSTK/100 to measure beliefs throughout the analysis in this section.[12]

2.1 Participants are Optimists while Nonparticipants are Pessimists

Figure 2 plots the average beliefs reported in the first interview. The data show that households that participate in both interviews (Remain participant) are optimistic relative to households that do not participate in both interviews (Remain nonparticipant). This difference between the beliefs of the two groups in the first interview is economically large, 0.54 versus 0.37, and highly statistically significant. Furthermore, respondents who subsequently enter or exit the stock market are more likely to have beliefs that are close to a "cutoff," rather than beliefs that are very optimistic or pessimistic.

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[11]The survey question is: "[INVEST] Do you (and your family living there) have any investments in the stock market, including any publicly traded stock that is directly owned, stocks in mutual funds, or stocks in any of your retirement accounts, including 401(K)s, IRAs, or Keogh accounts? Considering all of your (family's) investments in the stock market, overall about how much would your investments be worth today?"

[12]Summary statistics of all variables and additional results are delegated to A.1 in the Online Appendix.

Figure 2: **Beliefs of Participants and Nonparticipants.** The plot shows the average belief from the first interview of households in the Michigan survey split into: respondents who (1) do not participate in the first round but participate in the second round (Entry), (2) participate in the first round but do not participate in the second round (Exit), (3) do not participate in the stock market in both rounds of interviews (Remain nonparticipant), and (4) participate in the stock market in both rounds (Remain participant). T-statistics for the difference in the means of beliefs with the following null hypotheses Belief(Remain participant)=Belief(Remain nonparticipant), Belief(Remain participant)=Belief(Exit), and Belief(Entry)=Belief(Remain nonparticipant) are 24.68, 13.50, and 6.02, respectively.
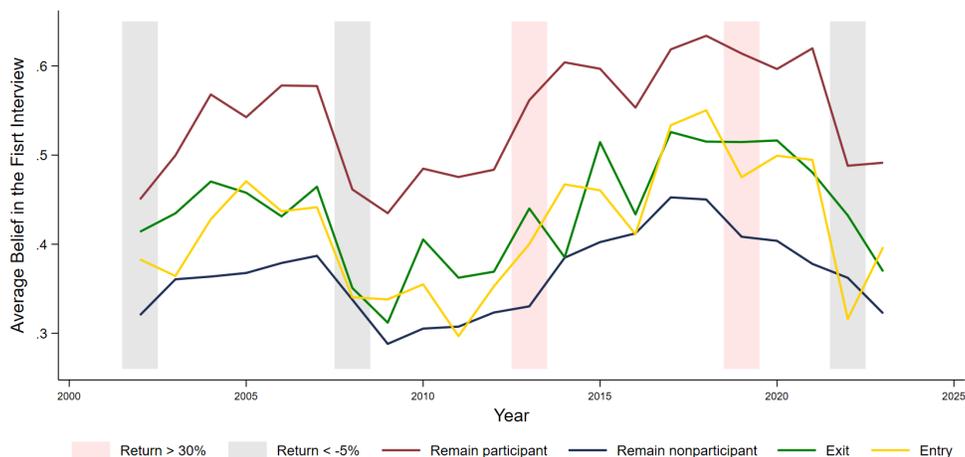
To substantiate that differences in beliefs among these four groups could drive participation and investment, we regress participation (the extensive margin), as well as the dollar amount invested in the stock market conditional on participation (the intensive margin) on beliefs, a host of household characteristics, and interview wave fixed effects. These regressions are summarized in columns (1)-(2) in Table 1; they show that a higher level of optimism about the stock market is statistically significantly associated with a larger likelihood of participating and an increase in the dollar amount invested, conditional on investment.

2.2 Revisions in Beliefs Drive Entry and Exit

How do changes in beliefs affect the actions of households? In this regard, we regress change in participation, change in dollar investment, entry and exit on change in beliefs between the two rounds of interviews, the prior belief, controls and year fixed effects. We learn

	(1) Participation$_t$	(2) Investment$_t$	(3) ΔParticipation$_{t,t+6m}$	(4) ΔInvestment$_{t,t+6m}$	(5) Entry$_{t,t+6m}$	(6) Exit$_{t,t+6m}$
Belief$_t$	0.191***	52435.041***	0.009	13872.360*	0.463***	-0.579***
	(0.005)	(11973.868)	(0.007)	(8260.758)	(0.035)	(0.064)
ΔBelief$_{t,t+6m}$			0.031***	13476.850**	0.101***	-0.446***
			(0.006)	(6306.125)	(0.038)	(0.041)
Controls	Y	Y	Y	Y	Y	Y
Fixed effects	Year-month	Year-month	Year-month	Year-month	Year	Year
R-squared	.367	.251	.022	.011	-	-
N	74539	41436	44959	38334	74539	74539
Regression	OLS	OLS	OLS	OLS	Heckman probit	Heckman probit

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 1: **Participation and Beliefs.** Belief is computed as PSTK/100. Participation is a dummy that equals 1 if participating and 0 otherwise. Investment is the dollar amount invested in the stock market, conditional on investment. Change of belief (ΔBelief) is belief in the second round of interview minus that in the first round. ΔParticipation equals 1 if entry, 0 if no change between two interviews, and -1 if exit. For Heckman probit regressions, the first step is selection into the respective entry sample (not participating in the first round) and exit sample (participating in the first round). Controls include dummy variables for wealth quartiles, income quartiles, age groups (<40, 40-60, >60), level of education (from lowest to highest grade completed), the housing situation of the household (own or rent), the region of residence of the household (North, South, Northeast, Central North), gender of respondent, and political affiliation (democrat, republican, non affiliated). Include year-month (interview wave) fixed effects. For Heckman probit regressions, we include belief, controls, and year fixed effects in the first stage. Returns on the stock market are from Amit Goyal's website.

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from columns (3)-(4) in Table 1 that a positive change in belief is associated with a higher propensity of participating and a higher dollar investment in stocks, funds or other stock-like investments. For entry and exit, we use a probit model with Heckman sample selection and find that the regressions in columns (5)-(6) support the view that a positive change in belief associates with a higher propensity of entry into the stock market and lower propensity of exit.

2.3   Experience-based Learning

How do the experiences of households shape their beliefs? To answer this question, we regress the changes in beliefs between the two rounds of interviews on the stock returns during the same interval. According to Table 2, households on average become more optimistic after experiencing positive returns (column 1), and the sensitivity of the belief updating on returns relates to whether the household has been investing previously in the stock market (column (2)). Furthermore, we test whether households who experience different lifetime stock market returns form different beliefs and whether they trade differently. To capture experience heterogeneity, we label households below the age of 40 as young and those above 60 as old, and use the average stock return of the past 20 years as a proxy for the experience of the young and that of the past 50 years as a proxy for the experience of the old. Based on this definition, we calculate the time-series of the difference (gap) between old and young in average belief, participation rate, and investment amount. The regression results, columns (3) to (5) in Table 2, indicate that the experience gap could affect the participation rate gap and the investment gap, by affecting the belief gap (first stage IV).

Overall, the empirical evidence suggests that belief-cycles drive participation-cycles. Participants tend to be optimists while nonparticipants are pessimists; furthermore, entry and exit are frequent and are related to belief updates formed through experiences with stock market fluctuations. Consequently, our model features experience-based learning and that pessimists leave the stock market.

	(1) $\Delta\text{Belief}_{t,t+6m}$	(2) $\Delta\text{Belief}_{t,t+6m}$	(3) First stage	(4) Participation gap	(5) Investment gap
$\text{Return}_{t,t+6m}$	0.245***	0.138***			
	(0.017)	(0.025)			
Belief_t	-0.639***	-0.650***			
	(0.005)	(0.005)			
Participation_t		0.049***			
		(0.004)			
$\text{Participation}_t \times \text{R}_{t,t+6m}$		0.146***			
		(0.026)			
Experience gap			0.393***		
			(0.062)		
Linear prediction				0.382***	0.363***
				(0.124)	(0.082)
Controls	Y	Y	Y	Y	Y
Fixed effects	Year	Year	-	-	-
R-squared	.334	.339	.260	.398	.881
N	45150	45150	253	253	253
Regression	OLS	OLS	OLS	OLS	OLS

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: **Beliefs and Experiences.** Return is the stock market return between the two rounds of interviews. Belief is PSTK/100. Change of belief (ΔBelief) is belief in the second round of interview minus that in the first round. Participation is a dummy that equals to 1 if participating and 0 otherwise. Investment is the dollar amount invested in the stock market, conditional on investment. In (1) and (2) controls include dummy variables for wealth quartiles, income quartiles, age group (<40, 40-60, >60), level of education (from lowest to highest grade completed), the housing situation of the household (own or rent), the region of residence of the household (North, South, Northeast, Central North), gender of respondent, and political affiliation (democrat, republican, non affiliated). In (3), the first stage, the belief gap is regressed on experience gap. In (4), the second stage, participation rate gap is regressed on the linearly predicted belief gap. In (5), the second stage, the average dollar investment gap is regressed on the linearly predicted belief gap. Age groups are defined as (<40, 40-60, >60), where old is >60, young is <40. Experience is the average total stock return in the past 50 years for old and past 20 years for young. Gap is time-series difference between old and young. Controls include the difference in average wealth and average income between old and young. All differences are standardized to have a mean of 0 and standard deviation of 1. Returns on the stock market are from Amit Goyal's website.

3 The Model

In our continuous-time overlapping generations (OLG) exchange economy, agents die at rate $\nu > 0$ to be replaced by newborns at the same rate. Hence, the total population size remains constant and can be normalized to equal 1, where the time-t size of the cohort born at time $s < t$ is $\nu e^{-\nu(t-s)}$.[13]

Output Y_t evolves as follows

$$dY_t/Y_t = \mu_Y dt + \sigma_Y dz_t^Y, \tag{1}$$

with z_t^Y representing a standard Brownian motion.

Further, agents observe an additional signal as in Dumas, Kurshev, and Uppal (2009). We assume that only agents who actively trade the stock market pay attention to the signal SI_t. It evolves as

$$dSI_t = \phi dz_t^Y + \sqrt{1 - \phi^2} dz_t^{SI}, \tag{2}$$

where ϕ is the correlation between shocks to output and the signal, with z_t^{SI} representing a standard Brownian motion, uncorrelated with z_t^Y. We interpret the magnitude of ϕ as information quality. A higher magnitude of ϕ entails acquiring more precise information from observing the signal.

For experience to matter, we assume that agents observe the output and may or may not pay attention to the signal but do not know the expected growth rate, μ_Y. At time s, newborn agents start with a prior belief about expected output growth $\hat{\mu}_{s,s}$, which is dependent on the time of birth, and variance $\hat{V} > 0$, which is independent of it. Based on the perceived expected output growth $\hat{\mu}_{s,t}$ agents who do not participate in the stock market (we refer to them as nonparticipants; N) observe the dynamics of output in the following way:

$$dY_t/Y_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t}^Y, \tag{3}$$

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[13]See Blanchard (1985) or Gârleanu and Panageas (2015).

where $z_{s,t}^Y$ denotes a Brownian motion under the belief of an agent born at time s. In similar fashion, the stock market participants (P) perceive the following pair of stochastic differential equations:

$$dY_t/Y_t = \hat{\mu}_{s,t}dt + \sigma_Y dz_{s,t}^Y, \quad dSI_t = \phi dz_{s,t}^Y + \sqrt{1-\phi^2}dz_{s,t}^{SI}, \tag{4}$$

where $z_{s,t}^Y$ and $z_{s,t}^{SI}$ denote two uncorrelated Brownian motions under the belief of an agent born at time s. Here, all agents in the cohort s participate or do not participate, and the decision to become a participant or nonparticipant is endogenously determined.

Independent of whether the agents follow the additional signal or not, they update their beliefs about expected output growth through the Bayes' rule. Specifically, by standard filtering theory, at time t a cohort born at time s perceives the dynamics of the expected output growth and its posterior variance in the following way:

$$d\hat{\mu}_{s,t} = \begin{cases} \frac{\hat{V}_{s,t}}{\sigma_Y}dz_{s,t}^Y, & \text{if } N, \text{ or} \\ \frac{\hat{V}_{s,t}}{\sigma_Y}\left(dz_{s,t}^Y - \frac{\phi}{\sqrt{1-\phi^2}}dz_{s,t}^{SI}\right), & \text{if } P, \text{ where} \end{cases} \tag{5}$$

$$\frac{1}{\hat{V}_{s,t}} = \begin{cases} \frac{1}{\hat{V}_s'} + \frac{t-t_s'}{\sigma_Y^2}, & \text{if } N, \text{ or} \\ \frac{1}{\hat{V}_s'} + \frac{t-t_s'}{\sigma_Y^2(1-\phi^2)}, & \text{if } P, \end{cases} \tag{6}$$

where t_s' denotes the last time when all agents from a cohort s switched from participant to nonparticipant or vice versa. Else, $t_s' = s$. Similarly, $\hat{V}_s'$ stands for the variance of the perceived growth of output at t_s'. From Equations (5 - 6), we see that as cohorts learn about the true growth of output over time, their posterior variance decreases.

Given that agents know σ_Y and ϕ, the perceived and true shocks are linked with each other in the following manner:

$$dz_{s,t}^Y = dz_t^Y - \Delta_{s,t}dt, \quad \text{and} \tag{7}$$

$$dz_{s,t}^{SI} = dz_t^{SI} + \frac{\phi}{\sqrt{1-\phi^2}}\Delta_{s,t}dt, \tag{8}$$

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where $\Delta_{s,t} = \frac{\hat{\mu}_{s,t} - \mu_Y}{\sigma_Y}$ is the standardized estimation error. According to Equations (7 - 8) the dynamics of the expected output growth of a cohort born at time s, under the true probability measure, are

$$d\hat{\mu}_{s,t} = \begin{cases} -\frac{\hat{V}_{s,t}}{\sigma_Y}\Delta_{s,t}dt + \frac{\hat{V}_{s,t}}{\sigma_Y}dz_t^Y, & \text{if } N, \text{ or} \\ -\frac{\hat{V}_{s,t}}{\sigma_Y}\frac{1}{1-\phi^2}\Delta_{s,t}dt + \frac{\hat{V}_{s,t}}{\sigma_Y}dz_t^Y - \frac{\hat{V}_{s,t}}{\sigma_Y}\frac{\phi}{\sqrt{1-\phi^2}}dz_t^{SI}, & \text{if } P. \end{cases} \tag{9}$$

Solving the stochastic differential equations in (9) leads to the following proposition.

Proposition 1. *The estimation error at time t of the cohort born at time s is*

$$\Delta_{s,t} = \begin{cases} \frac{\sigma_Y^2}{\sigma_Y^2 + \hat{V}_s'(t-t_s')}\Delta_{s,t_s'} + \frac{\hat{V}_s'}{\sigma_Y^2 + \hat{V}_s'(t-t_s')}\left(z_t^Y - z_{t_s'}^Y\right), & \text{if } N, \text{ or} \\ \frac{\sigma_Y^2(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}_s'(t-t_s')}\Delta_{s,t_s'} + \frac{\hat{V}_s'(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}_s'(t-t_s')}\left\{\left(z_t^Y - z_{t_s'}^Y\right) - \frac{\phi}{\sqrt{1-\phi^2}}\left(z_t^{SI} - z_{t_s'}^{SI}\right)\right\}, & \text{if } P, \end{cases} \tag{10}$$

where $t_s' > s$ denotes switching from participant to nonparticipant, or vice versa, and where $\Delta_{s,t_s'} = \frac{\hat{\mu}_{s,t_s'} - \mu_Y}{\sigma_Y}$, $\lim_{t-t_s' \to \infty} \Delta_{s,t} = 0$ a.s.

According to Proposition 1, the estimation error at time t of the cohort born at time s, $\Delta_{s,t}$, is a piecewise continuous function partitioned by the collection of switching times t_s' into intervals during which the cohort either participates or not in the stock market.

Furthermore, we see that the negative coefficient in front of $\frac{\phi}{\sqrt{1-\phi^2}}$ implies that with a positive value ϕ, shocks to the output dz_t^Y and those to the signal dz_t^{SI} affect the updates to beliefs in different directions. For intuition, when dz_t^{SI} is large, an agent who pays attention to the signal as in Equation (2) tends to overestimate dz_t^Y. She thus forms lower expectations about the output growth rate, compared to if she does not pay attention.[14]

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[14]The impact of ϕ therefore differs from Dumas, Kurshev, and Uppal (2009), as in their setting output growth follows a mean-reverting process, where the signal contains information about shocks to output growth instead of output.

3.1 Security Markets and Prices

Cohorts trade in an instantaneously risk-free bond, shares of the stock market, and annuities. The instantaneously risk-free bond is in zero net supply and evolves according to

$$dB_t/B_t = r_t dt, \tag{11}$$

where r_t denotes the real short rate of interest determined in equilibrium.

Shares in the stock market are normalized to one with price S_t. All output is paid out as dividend, where stock returns R_t have dynamics

$$dR_t = (dS_t + D_t dt)/S_t = (dS_t + Y_t dt)/S_t = \mu_t^S dt + \sigma_t^S dz_t^Y = \mu_{s,t}^S dt + \sigma_t^S dz_{s,t}^Y, \tag{12}$$

where μ_t^S, $\mu_{s,t}^S$ and σ_t^S are determined in equilibrium. Cohorts agree on current prices, but disagree on their probability distribution in the future. From the relation between the perceived and actual shocks in (7), we have $\mu_{s,t}^S = \mu_t^S + \sigma_t^S \Delta_{s,t}$.

There is a competitive insurance industry that offers annuity contracts as in Yaari (1965) at the actuarily fair rate ν per unit of wealth, such that agents with positive financial wealth $W_{s,t}$ find it optimal to annuitize all wealth. This is because cohorts face mortality risk and do not derive utility from leaving financial wealth behind. The contract stipulates that the insurance industry receives the financial wealth of the dead and in return pays $\nu W_{s,t}$ to agents currently alive.

3.2 The Disagreement Process

In our framework, nonparticipants in the stock market are simply agents who cannot or do not want to sell short. Since shares in the stock market are the only source of consumption risk in the economy, agents become nonparticipants only when their perceived instantaneous price of risk $\theta_{s,t}$ is negative. Importantly, effectively binding or perceived constraints do not

affect relatively optimistic cohorts who have a positive demand for the stock. Therefore, when a cohort perceives $\theta_{s,t} \geq 0$, all agents in that cohort participate in the stock market. Coherently, when a cohort perceives $\theta_{s,t} < 0$, all wealth is invested in the bond market. To solve the model, we make use of the constrained problem with fictitious state prices as in He and Pearson (1991) and Karatzas, Lehoczky, Shreve, and Xu (1991), which is equivalent to a complete market setting. Hence, in our fictitious economy with participants and nonparticipants, the stochastic discount factor perceived by a cohort born at time s follows the process

$$d\xi_{s,t}/\xi_{s,t} = -r_t dt - \theta_{s,t}^+ dz_{s,t}^Y, \tag{13}$$

where $\theta_{s,t}^+ = \max(\theta_{s,t}, 0)$. On the true probability measure, ξ_t follows

$$d\xi_t/\xi_t = -r_t dt - \theta_t dz_t^Y, \tag{14}$$

where from Equation (12), we have that $\theta_{s,t} = \theta_t + \Delta_{s,t}$, implying that nonparticipation occurs when $\Delta_{s,t} < -\theta_t$.

Next, we define the disagreement process $\eta_{s,t}$ through the relation between objective and perceived stochastic discount factors $\xi_t = \eta_{s,t} \xi_{s,t}$, where the disagreement follows

$$d\eta_{s,t}/\eta_{s,t} = \left(\theta_{s,t}^+ - \theta_t \right) dz_t^Y. \tag{15}$$

3.3 Preferences and Individual Optimization

For the sake of variation in valuation ratios, we assume that agents differ in their time preferences within each cohort, where the distribution of types remains invariant over time. We denote the time preference of type i agents as ρ_i, and the density of type i agents as α_i, with $\int_i \alpha_i di = 1$. Since all agents of the same cohort learn from the same experience, the subjective stochastic discount factor does not differ across different agent types within the

same cohort.[15]

To finance the endowment of the newly born, we impose a tax on the consumption of all agents currently alive.[16] Specifically, a cohort born at time s with a cohort size of ν share the total consumption tax revenue of τC_s, where τ is the tax rate, and C_s denotes the aggregate level of consumption in the economy at time s. Thus, each agent in the cohort s has an initial wealth of $W_{i,s,s} = W_{s,s} = \frac{\tau}{\nu}C_s$.

Agents face constant mortality risk and since the random time of death is independent of aggregate output, we integrate it out from the expected lifetime utility, which then is given by

$$\mathbb{E}_{s,s}\left[\int_s^\infty e^{-(\rho_i+\nu)(t-s)}log(c_{i,s,t})dt\right]. \tag{16}$$

The dynamics of wealth $W_{i,s,t}$ follows

$$dW_{i,s,t} = \begin{cases} \left(r_t W_{i,s,t} + \nu W_{i,s,t} - \tau c_{i,s,t} - c_{i,s,t}\right)dt, & \text{if } N, \quad \text{or} \\ \left[r_t W_{i,s,t} + \pi_{i,s,t}\left(\mu_{s,t}^S - r_t\right) + \nu W_{i,s,t} - \tau c_{i,s,t} - c_{i,s,t}\right]dt + \pi_{i,s,t}\sigma_t^S dz_{s,t}^Y, & \text{if } P, \end{cases} \tag{17}$$

where $\pi_{i,s,t}$ denotes the dollar investment in the stock market.

3.4   Equilibrium

The fictitious unconstrained consumption portfolio choice problem, corresponding to the static problem of Cox and Huang (1989), is given by

$$\max_{c_{i,s}} \mathbb{E}_{s,s}\left[\int_s^\infty e^{-(\rho_i+\nu)(t-s)}log(c_{i,s,t})dt\right]$$

s.t.

$$\mathbb{E}_{s,s}\left[\int_s^\infty e^{-v(t-s)}\frac{\xi_{s,t}}{\xi_{s,s}}(c_{i,s,t} + \tau c_{i,s,t})dt\right] = W_{i,s,s}. \tag{18}$$

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[15]We thus omit i in the subscript for $dz_{s,t}^Y$, $\xi_{s,t}$, $\theta_{s,t}^+$, and $\eta_{s,t}$.

[16]The equilibrium with a wealth tax, which has similar structure, is summarized in Subsection A.5 in the Online Appendix.

From the first-order conditions (FOCs), we have

$$\frac{e^{-(\rho_i+\nu)(t-u)}}{c_{i,s,t}} = \kappa_{i,s}\left(1+\tau\right)e^{-\nu(t-u)}\frac{\xi_{s,t}}{\xi_{s,s}},\tag{19}$$

where $\kappa_{i,s}$ is the Lagrange multiplier associated with the static budget constraint.

Proposition 2. *At t, the optimal consumption for a living type i agent of cohort s is*

$$c_{i,s,t} = c_{i,s,s}e^{-\rho_i(t-s)}\frac{\eta_{s,t}}{\eta_{s,s}}\frac{\xi_s}{\xi_t}.\tag{20}$$

We conjecture and verify that $\frac{c_{i,s,t}}{W_{i,s,t}} = \beta_i = \frac{\rho_i+\nu}{1+\tau}$, which is consistent with the standard constant consumption-to-wealth ratio for log utility. We define the wealth share of type i agents born at time s as $f_{i,s,t}^{W} = \frac{\alpha_i\nu e^{-\nu(t-s)}W_{i,s,t}}{W_t}$, and the consumption share as $f_{i,s,t}^{c} = \frac{\alpha_i\nu e^{-\nu(t-s)}c_{i,s,t}}{Y_t}$.

Proposition 3. *In equilibrium, the real short rate of interest is*

$$r_t = \overbrace{\nu - \tau\beta}^{OLG} + \overbrace{\bar\rho_t + \mu_Y - \sigma_Y(\sigma_Y}^{Log\ utility} + \underbrace{\sigma_Y\left(\frac{1}{\bar\Phi_t}-1\right) - \bar\Delta_t}_{Beliefs\text{-}driven\ participation}) = \nu - \tau\beta + \bar\rho_t - \sigma_Y^2\frac{1}{\bar\Phi_t} + \bar\mu_t,\tag{21}$$

and the market price of risk is

$$\theta_t = \overbrace{\sigma_Y}^{Log\ utility} + \underbrace{\sigma_Y\left(\frac{1}{\bar\Phi_t}-1\right) - \bar\Delta_t}_{Beliefs\text{-}driven\ participation} = \sigma_Y\frac{1}{\bar\Phi_t} - \frac{\bar\mu_t - \mu_Y}{\sigma_Y},\tag{22}$$

where $\beta = \int_i \alpha_i\beta_i di$ is the consumption-to-wealth ratio of the newborn cohort, $\bar\rho_t = \int_i \rho_i \int_{-\infty}^{t} f_{i,s,t}^c dsdi$ is the consumption weighted average time preference, $\bar\Phi_t = \int_i \int_{-\infty}^{t} f_{i,s,t}^c \mathbb{1}_{s,t}^+ dsdi$ denotes the aggregate consumption share of the agents who invest in the stock market, where $\mathbb{1}_{s,t}^+$ is an indicator that equals to 1 if $\Delta_{s,t} \geq -\theta_t$ and equals to 0 otherwise, $\bar\Delta_t = \frac{\int_i \int_{-\infty}^{t} f_{i,s,t}^c \Delta_{s,t} \mathbb{1}_{s,t}^+ dsdi}{\int_i \int_{-\infty}^{t} f_{i,s,t}^c \mathbb{1}_{s,t}^+ dsdi} = \frac{\int_i \int_{-\infty}^{t} f_{i,s,t}^c \Delta_{s,t} \mathbb{1}_{s,t}^+ dsdi}{\bar\Phi_t}$ is the consumption weighted average standardized estimation error in the

economy conditional on stock market participation, and $\bar{\mu}_t = \frac{\int_i \int_{-\infty}^t f^c_{i,s,t} \hat{\mu}_{s,t} \mathbb{1}^+_{s,t} dsdi}{\Phi_t}$ denotes the consumption weighted average expected growth rate conditional on stock market participation. We label it the market view of the participants.

In Equation (21), we see that the real short rate of interest is given by an OLG term, a log utility term and a *beliefs-driven participation effect*. In addition, in Equation (22), the market price of risk is given by the standard log utility term and the *beliefs-driven participation effect*. Here, the experience term differs from the complete market case since it includes only investors holding the stock market.

We now inspect the numerator and denominator of the market price of risk in the proposition below.

Proposition 4. *The expected excess return on the stock market is given by*

$$\mu_t^S - r_t = \sigma_t^S \theta_t, \tag{23}$$

and the volatility of the stock market is

$$\sigma_t^S = \tilde{\Phi}_t \left(\theta_t + \tilde{\Delta}_t \right), \tag{24}$$

where $\tilde{\Phi}_t = \int_i \int_{-\infty}^t f^W_{i,s,t} \mathbb{1}^+_{s,t} dsdi$ denotes the aggregate wealth share of the stock market participants, and $\tilde{\Delta}_t = \frac{\int_i \int_{-\infty}^t f^W_{i,s,t} \Delta_{s,t} \mathbb{1}^+_{s,t} dsdi}{\int_i \int_{-\infty}^t f^W_{i,s,t} \mathbb{1}^+_{s,t} dsdi}$ is the wealth weighted average belief conditional on stock market participation.

As for the real short rate of interest and the market price of risk, the expected excess return on the stock market also depends on the *beliefs-driven participation effect*, $\frac{1}{\Phi_t} - \bar{\Delta}_t$ embedded in θ_t. Similarly, the volatility of the stock varies over time as the gap between the consumption- and the wealth-weighted belief widens and shrinks, $-\bar{\Delta}_t + \tilde{\Delta}_t$, and with the dynamics of the ratio between the aggregate wealth to consumption share of the participants, $\frac{\tilde{\Phi}_t}{\Phi_t}$. What sets Proposition 4 apart from Proposition 3, and more generally from the literature,

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is that despite log utility, consumption- and wealth-weighted equilibrium quantities differ. When they do not differ, because the time preference is homogeneous, the volatility of the stock market collapses to σ_Y. The intuition for why wealth-weighted quantities drive the stock market volatility is simple: Clearing of the stock market involves the wealth shares of the agents.

The following proposition shows the investment in the stock market.

Proposition 5. *For participants, the optimal dollar investment in the stock market, $\pi_{i,s,t}$, is*

$$\pi_{i,s,t} = \frac{\Delta_{s,t} + \theta_t}{\sigma_t^S} W_{i,s,t}. \tag{25}$$

The portfolio policy in Proposition 5 has two components: $\frac{\Delta_{s,t} - \bar{\Delta}_t}{\sigma_t^S} = \frac{\hat{\mu}_{s,t} - \bar{\mu}_t}{\sigma_Y \sigma_t^S}$, which depends on the market view, and $\frac{\theta_t + \bar{\Delta}_t}{\sigma_t^S} = \frac{\sigma_Y}{\sigma_t^S} \frac{1}{\Phi_t}$, which depends on the ratio of exogenous to endogenous volatility times the inverse of the consumption share of the participants. Instead, in the complete financial market benchmark with log utility, variations in portfolio composition arise only through the belief component. Furthermore, an agent's leverage, and therefore the participation decision itself, does not directly depend on her time preference.

An important step in identifying equilibrium is that participation and θ_t are determined jointly. Specifically, cohorts with cutoff estimation error $\Delta_{s,t}^* = \frac{\hat{\mu}_{s,t}^* - \mu_Y}{\sigma_Y} = -\theta_t$ have exactly zero demand for the stock. Agents who are more optimistic are participants, while agents who are more pessimistic are nonparticipants.

A.2 in the Online Appendix contains two propositions regarding the dynamics of equilibrium consumption of participants and nonparticipants and the dynamics of consumption shares and wealth shares by agent type; all proofs are in A.3.

3.5   Scenarios

To ground our analysis, we consider two scenarios:

Definition 1. *In the reentry scenario a cohort leaves the stock market upon the event of the perceived instantaneous price of risk being negative, that is $\theta_{s,t} < 0$, and returns to the stock market upon the event of the perceived instantaneous price of risk being positive, that is $\theta_{s,t} > 0$. This corresponds to the equilibrium described above.*

We use the *reentry* scenario mainly to present unconditional averages.

Definition 2. *In the mix scenario, the economy is populated by four groups of agents: reentry type, disappointment type,[17] designated participants, and designated nonparticipants. Here, designated participants are unconstrained and designated nonparticipants never hold the stock market.*

To simplify, we assume that designated nonparticipants as well as agents of disappointment type who have left the market do not update their beliefs, which affects neither asset pricing quantities nor cohort consumption dynamics. We use the *mix* scenario mainly to present time-series results using shocks implied by macroeconomic data. The *mix* scenario showcases that our model can accommodate a rich cross-section of agent types within a cohort.

4   Properties of the Model

We now examine the equilibrium dynamics of the model presented in Section 3.

4.1   Parameters

The model has nine parameters $\left(\nu, \mu_Y, \sigma_Y, \hat{\mu}_{s,s}, \hat{V}, \rho_a, \rho_b, \phi, \tau\right)$. We set the birth and death rate ν at 2%, and the drift and volatility of aggregate output μ_Y and σ_Y at 2% and 3.3%, respectively. Ehling, Graniero, and Heyerdahl-Larsen (2018b) connect the prior belief $\hat{\mu}_{s,s}$

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[17]The *disappointment* type leave the stock market for good upon the event of the perceived instantaneous price of risk being negative, that is $\theta_{j,s,t} < 0$. Proofs for an equilibrium with *disappointment* type are in subsection A.6 in the Online Appendix.

and the prior variance $\hat{V}$ through an initial window with n years during which an agent builds formative experience with stock returns before entering the economy, and set $\hat{\mu}_{s,s} = \mu_Y + \sigma_Y \frac{z_s^Y - z_{s-n}^Y}{n}$, and $\hat{V} = \frac{\sigma_Y^2}{n}$. Following this approach, we focus on $n = 5$, and we regard agents as 20 years old when entering the economy. For the time discount factor, we set ρ_a at 0.1% for half of the agents, and 0.5% for the other half. For the correlation between the fundamental and the signal ϕ, we focus on $\phi = 0.5$.[18] We set τ at 0.35, which implies that a-types consume $\frac{c_{a,t,t}}{C_t} = 27.2\%$ and b-types consume $\frac{c_{b,t,t}}{C_t} = 32.4\%$.[19] Therefore, an average newborn consumes 29.8% of the average consumption in the economy.

4.2 Average Participation, Entry and Exit

Our focus is on stock market participation, entry, and exit. More standard asset pricing moments are delegated to Table 8 in the Online Appendix.

Table 3 summarizes the means of the participation, entry, and exit rates in the model and the observed data along with various correlations from the model. The caption of the table also summarizes the forward-simulation that produces the model data for the unconditional averages. The observed data are from the following countries: Finland, Germany, Norway, and the United States.[20] From the table we see that the model can match low participation rates common in Asia and Europe, as well as rates that are seen in the United States.[21] Further, the model can also match entry and exit rates in the range of 1 to 6 percent consistent with what we see in the data, where both in the data and the model entry and exit rates have comparable levels.

Table 3 also speaks to the motivating facts in Section 2: First, we establish that both in the *reentry* and especially in the *mix* scenario the observable participation rate correlates

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[18]See for example Chen, Roll, and Ross (1986), which is a classical paper on the relation between fluctuations in macroeconomic state variables and the stock market; it presents absolute correlations ranging from 0 to at most 0.5.

[19]The consumption tax rate τ directly affects the consumption share of a new born agent, $\frac{c_{i,t,t}}{C_t} = \frac{\tau}{\nu}\beta_i = \frac{\tau}{\nu}\frac{\nu + \rho_i}{1+\tau} = \frac{\tau}{1+\tau}(1 + \frac{\rho_i}{\nu})$.

[20]The data are described in Subsection A.8 in the Online Appendix.

[21]By construction, the IRS data contain only direct stock holdings of dividend-paying stocks.

with the endogenous state variables $\bar{\Phi}$ and $\tilde{\Phi}$, namely the aggregate consumption and wealth shares of the participants. The positive correlations between changes in beliefs and changes in participation and changes in beliefs and changes in portfolios in the model scenarios correspond to the regression coefficient on changes in beliefs in the regressions shown in columns (3)-(4) of Table 1, and are consistent with those. Moreover, the positive correlations between Experience Gap and Belief Gap, Belief Gap and Participation Gap, and Belief Gap and Investment Gap are consistent with the regression results in columns (3) to (5) in Table 2.

	Reentry	*Mix*	Michigan	IRS	SCF	Finland	Norway
Participation rate	0.5224	0.4673	0.6753	0.2115	0.4798	0.2412	0.2282
Entry rate	0.0637	0.0101	0.0574			0.0198	0.0175
Exit rate	0.0451	0.0112	0.0603			0.0141	0.0182
Corr($\bar{\Phi}$, Participation)	0.2758	0.6532					
Corr($\tilde{\Phi}$, Participation)	0.2573	0.6384					
Corr(dBelief, dParticipation)	0.2048	0.1760					
Corr(dBelief, dInvestment)	0.5286	0.5668					
Corr(Experience Gap, Belief Gap)	0.2918	0.1811					
Corr(Belief Gap, Participation Gap)	0.8806	0.9109					
Corr(Belief Gap, Investment Gap)	0.5781	0.8332					

Table 3: **Means of Participation, Entry and Exit.** The table shows the average participation rate, entry rate and exit rate for the *reentry* scenario and the *mix* scenario, as well average correlations between the participation rate and $\bar{\Phi}$ and $\tilde{\Phi}$, changes in beliefs and changes in participation and portfolios, and Experience Gap and Belief Gap, Belief Gap and Participation Gap, and Belief Gap and Investment Gap. Model averages are based on a forward simulation of the economy, which use final values from a burn-in simulation. In the burn-in as in the forward simulation, a period equals one month and in every period one additional cohort is born. After 6000 periods in the burn-in, there are 6000 cohorts, which produce the starting values and from then on we keep the number of cohorts constant at 6000. For computational convenience in the first 240 periods of the burn-in, we use $\hat{\mu}_{s,s} = \mu_Y$, and $\hat{V} = \frac{\sigma_Y^2}{n}$. The forward simulation contains 10,000 paths, each with 6000 periods or 500 years. For parameters see Subsection 4.1 and for data see Subsection A.8 in the Online Appendix. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth in the *mix* scenario.

Because of the unusual path-dependency of the model through the experience-based learning and the endogenous attention to the signal through exit and entry, below we present

both unconditional moments as well as time-series of quantities obtained through feeding shocks into the model that are extracted from the stock market and macroeconomic signals.

4.3 Shocks to the Stock Market and the Signal

For Finland, Germany and Norway, we use the monthly international country portfolio data from Kenneth French's website, starting from year 1975. Specifically, we use the value-weighed average of the total returns in the local currency. Prior to 1975 for Finland, we use market returns constructed by Nyberg and Vaihekoski (2014), which are available from the year 1912 onward. For the US, the stock returns we use are CRSP value weighted total returns after 1926, and index returns between 1871 and 1926 from Amit Goyal's website.

For Finland, Germany and Norway, we use the exchange rate against the USD, industrial production, inflation, and unemployment rate as macroeconomic indicators. These data are seasonally adjusted monthly time-series from the OECD database. For the US, the macroeconomic indicators include changes in 12-month earnings, industrial production, Michigan consumer sentiment, and macroeconomic factors from Ludvigson and Ng (2009).

From the historical data on the stock returns and the pool of macroeconomic indicators, we construct shocks to the fundamental dz_t^Y and shocks to the signal dz_t^{SI}. Specifically, we standardize (with mean zero and standard deviation $\sqrt{dt}$) the monthly stock returns from the four countries to obtain the fundamental shocks dz_t^Y. For the shocks to the signal dz_t^{SI}, we first regress dz_t^Y on the pool of macroeconomic indicators to obtain the fitted values, which we regard as dSI_t. We then regress dSI_t on dz_t^Y, take the residual, and use the standardized residual as dz_t^{SI}.

4.4 Cohort Estimation Error

We begin discussing results by inspecting how different cohorts of agents update beliefs in response to shocks to the stock market and the macroeconomy.

In this regard, the middle top plot in Figure 3 shows the standardized estimation error

$\Delta_{s,t}$ of three cohorts, born in the years 1950, 1970 and 1990 respectively, where red solid dots represent transitions from nonparticipants to participants, and orange dots represent transitions from participants to nonparticipants. These estimation errors in the *reentry* scenario are driven by the implied joint path for the shocks to output dz_t^Y and the signal dz_t^{SI} (top plot of Figure 3) of the US economy.[22] The correlation between the shocks equals the baseline of 0.5.

From the plot, we see that each cohort updates aggressively when young, responding to shocks with large changes in $\Delta_{s,t}$, and gradually learn from experience and through that the precision of estimates about output growth increases and updates to $\Delta_{s,t}$ decline in magnitude. The plot also shows that even after collecting long time-series of data, the estimation error across the three cohorts differ considerably. Further, it is evident that cohorts switch from participation to nonparticipation at differing frequency or just stay in the market all the time such as the cohort born in 1990.[23]

In the *mix* scenario, shown in Figure 12 in the Online Appendix, differences in the estimation error across designated participants and reentry type within the same cohort arise only through entry and exit, as all agents in the same cohort share the same experience, while only those who participate pay attention to the signal. Such differences in estimation error, independently of whether they arise within or across cohorts, are at the core of our model and thus drive fluctuations in the financial market, which then drive economy-wide learning.

4.5 Endogenous Learning

Turning to the interplay between beliefs and equilibrium in more detail: the middle bottom plot in Figure 3 shows the endogenous cross-sectional distribution of estimation errors over time for participants and nonparticipants using the implied shocks of the US economy, where starting values of the simulation come from an average of burn-in simulations. The blue area

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[22]See Figure 11 in the Online Appendix for shocks to Finland, Germany, and Norway.

[23]The intricate influence of the signal on the estimation error is most evident from the differing participation of the two older cohorts with and without the macroeconomic signal. See plot (b) and (c) in Figure 12 in the Online Appendix.

Figure 3: **Estimation Error Across Joint Path of Output and Signal.** The top plot shows the implied joint path of shocks to the fundamental and to the signal for the US economy. The middle top plot shows the standardized estimation error $\Delta_{s,t}$ for cohorts born in the years 1950, 1970, and 1990, marked by dotted vertical lines. Solid time-series lines denote times when a cohort participates in the stock market, whereas the dotted lines represent times when a cohort does not participate, where red dots indicate the timing of a switch from nonparticipant to participant, and orange dots indicate the timing of a switch from participant to nonparticipant. The middle bottom plot shows the cross-sectional distribution of the estimation error over time for participants and nonparticipants, with the blue area representing the estimation error of the participants, the green area for that of the nonparticipants, where dark areas are in-between the 25th and 75th percentile by population. The cutoff estimation error separating the participants from the nonparticipants corresponds to $\Delta_{s,t} < -\theta_t$. The bottom plot shows the distribution of the estimation error for age groups. The model data are from the *reentry* scenario with $\phi = 0.5$. For parameters see Subsection 4.1. For implied shocks see Subsection 4.3.

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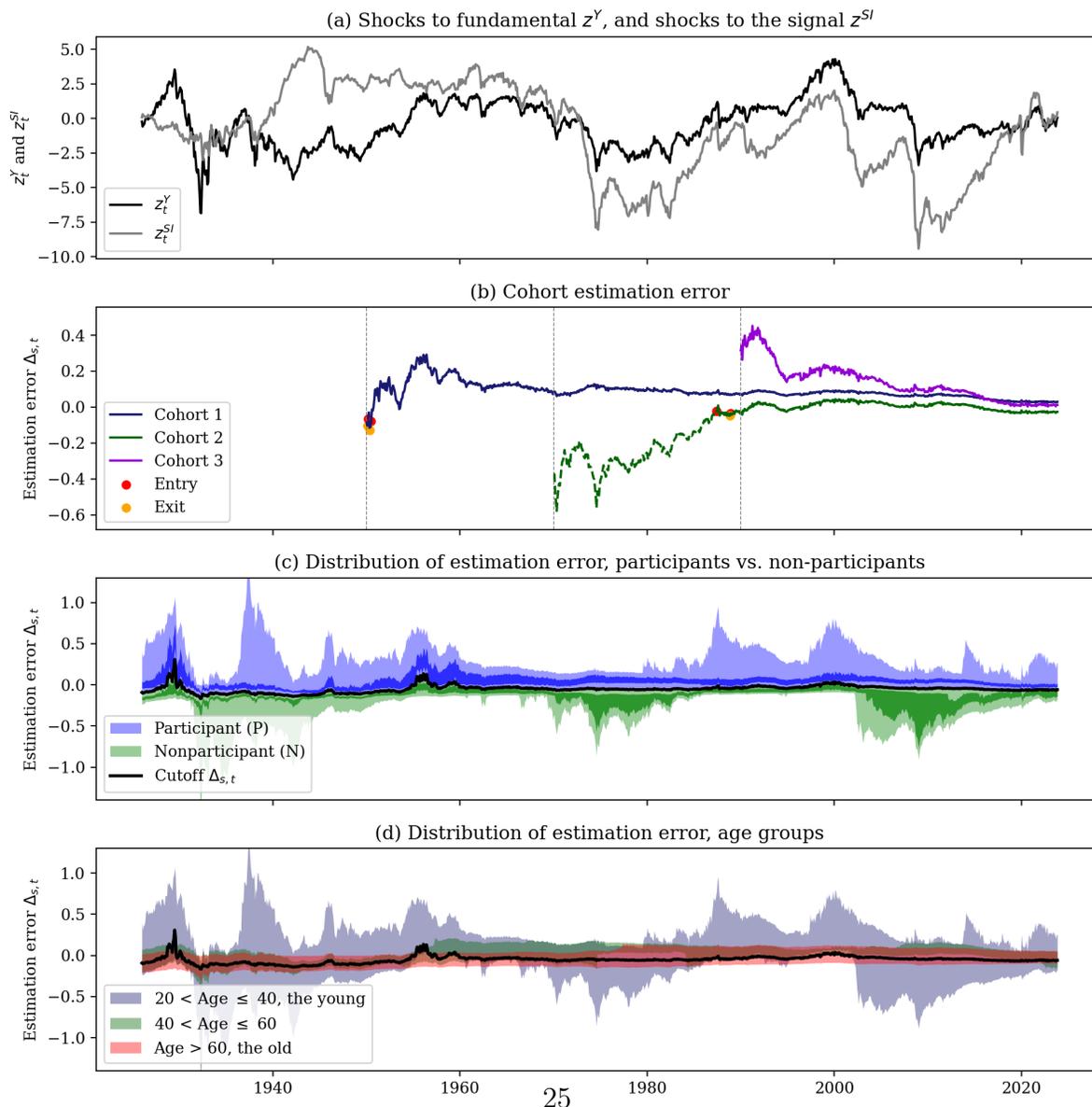
represents the distribution of estimation errors of the participants, the green area for that of the nonparticipants, and the dark areas represent the range between the 25th and 75th percentiles. The total width of the estimation error distribution maps the distance between the most optimistic and pessimistic agents. This range varies remarkably over time. Further, the relative division between P and N is largely independent of the total width, and each shows large time-series variation.

Next, we inspect the distribution of the estimation error for age groups in the bottom plot of Figure 3. The youngest group experiences large swings in their estimation error, whereas the oldest group enjoys significantly more stable beliefs that, in addition, are closer to the true parameter. As a consequence, the youngest group become the most optimistic agents in the economy following a series of positive shocks (mainly shocks to output), and the most pessimistic following a series of negative shocks.

We see that the distribution of the estimation error of the oldest group of agents is sometimes entirely above the cutoff belief or at least a large part of the distribution is above the cutoff. In contrast, the distribution of the estimation error of the youngest group of agents shows large swings and is therefore equally likely to be mainly above or mainly below the cutoff belief. This is because the cutoff belief gravitates around the beliefs of the older cohorts, as they have large wealth shares. As a result, only over three short episodes, where the young show exceptionally large exuberance in beliefs, the entire group of old leave the stock market.

5 Stock Market Participation, Entry and Exit Dynamics

5.1 Dynamics

5.1.1 Stock Market Participation

Figure 4 shows the detrended stock market participation rates in Finland, Germany, Norway, and the US.[24] Comparing these participation rates to rates implied by the model based on the *mix* scenario using $\phi = 0.0$ (shown as black solid lines) and $\phi = 0.5$ (shown as gray dotted lines), we see that although the level of the model implied participation rates are higher than in the data the dynamics are similar.[25] Out of the eight correlations only the one for Norway with $\phi = 0.0$ is statistically insignificant. This occurs likely due to the muted stock market fluctuations in the later part of the sample in Norway. In sum, the proposed beliefs-driven mechanism not only accounts for the unconditional stock market participation rate but also for its fluctuations.[26]

5.1.2 Entry and Exit

We now compare the time-series of entry and exit in the data with the time-series implied by the model by levering on the registry data from Finland and Norway. Just as for stock market participation rates, we base this exercise on the *mix* scenario using $\phi = 0.0$ (black solid lines) and $\phi = 0.5$ (gray dotted lines). Specifically, the top plots in Figure 5a show the entry rates for Finland (left) and Norway (right); here, we see that the model implied entry rates largely mimic the dynamics of the data and even match the levels of entry especially

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[24]Here, we focus on IRS data for the US as it is comprehensive. The Online Appendix contains the time-series dynamics of the Michigan and SCF surveys vis-à-vis the model implied one.

[25]For the US, where the signal has a richer structure the difference in the model based participation rates can be as high as 8%.

[26]We view the correlation between the data and the model counterpart as a relevant measure of fit because the literature has offered other important explanations for nonparticipation, entry and exit such as participation costs (see Gomes and Michaelides (2008)) or labor income uncertainty (Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2025)). Since we built the model to isolate the *beliefs-driven participation effect* it cannot account for these other important contributing factors. Hence, we do not test the hypothesis of a perfect fit.

Figure 4: **Participation Rate: Data versus Model.** The figure contains the time-series of detrended participation rate in Finland, Germany, Norway and the US (blue solid lines with dots), juxtaposed with simulated participation rates of the model based on the *mix* scenario using $\phi = 0.0$ (solid lines) and $\phi = 0.5$ (dashed lines), where n is the length of the initial window for the prior belief and T_ϕ is the t-statistics on the actual participation rate from regressing the model participation rate on a constant and the actual participation rate. For parameters, see Subsection 4.1. For implied shocks, see Subsection 4.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.

28
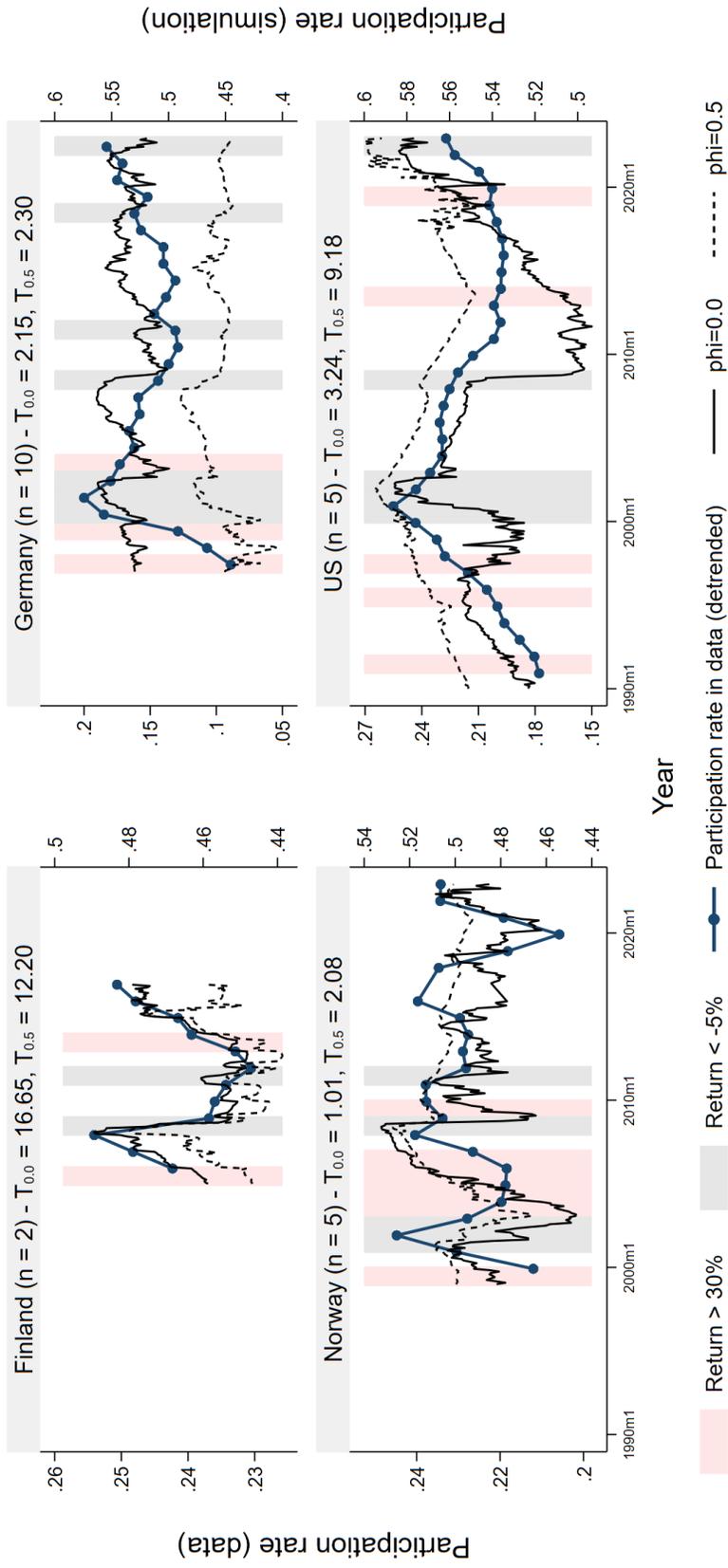
so for Finland. The picture for the exit rates is comparable as can be seen from the bottom plots in Figure 5b. Further, for Finland, three of the four correlations have a t-statistic of at least 2.33, while for Norway one correlation shows a t-statistic of 1.97 and one other 1.87.

5.2 Stylized Patterns

To strengthen the evidence in favor of the proposed model, we present two stylized patterns of stock market participation that are consistent with a learning-from-experience mechanism.

5.2.1 Lifetime Experience and Participation

Inspired by works such as Malmendier and Nagel (2011) or Knüpfer, Rantapuska, and Sarvimäki (2017), we turn to differences in participation rates and relate them to differences in experiences. We stress that these empirical contributions have inspired several theoretical works such as Collin-Dufresne, Johannes, and Lochstoer (2017) or Ehling, Graniero, and Heyerdahl-Larsen (2018b); yet, in these models agents always participate in the stock market. Moreover, without our model it appears challenging to directly attribute differences in participation to beliefs as experiences may affect preferences.

To this end, we compute model-driven and empirical differences in beliefs based on differences in experienced returns from the US stock market. Specifically, we compute the beliefs between agents that are 40 and younger (agents in the model start trading at age 20) and agents that are older than 60. These differences in implied beliefs are shown in Figure 6a using the *mix* scenario with $\phi = 0.5$ and the Michigan Survey of Consumers.[27] We see that the larger the difference in experienced returns between old and young, the larger the difference in beliefs between the old and young.

Based on the difference in the experienced returns, we then compute model implied differences in participation rates between the old and young (red arrows) to juxtapose these with the differences in participation rates based on data from Michigan (blue squares).

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[27]See also Figure 19 using SCF (Survey of Consumer Finance).

Figure 5: **Entry and Exit: Data versus Model.** The figure shows the time-series of entry and exit from the stock market in Finland and Norway (the blue lines with dots), compared to the simulated entry and exit for the *mix* scenario using $\phi = 0.0$ (solid lines) and $\phi = 0.5$ (dashed lines), where n is the length of the initial window for the prior belief and T_ϕ is the t-statistics on the actual entry (exit) rate from regressing the model entry (exit) rate on a constant and the actual entry (exit) rate. The number of individuals entering and exiting from the stock market in Finland are from Breitkopf, Knüpfer, and Rantapuska (2021), and the total population from Statistics Finland. The percentage of individuals entering and exiting from the stock market in Norway are from Statistics Norway. For parameters see Subsection 4.1. For implied shocks see Subsection 4.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.

(a) **Entry in the stock market - Finland and Norway**
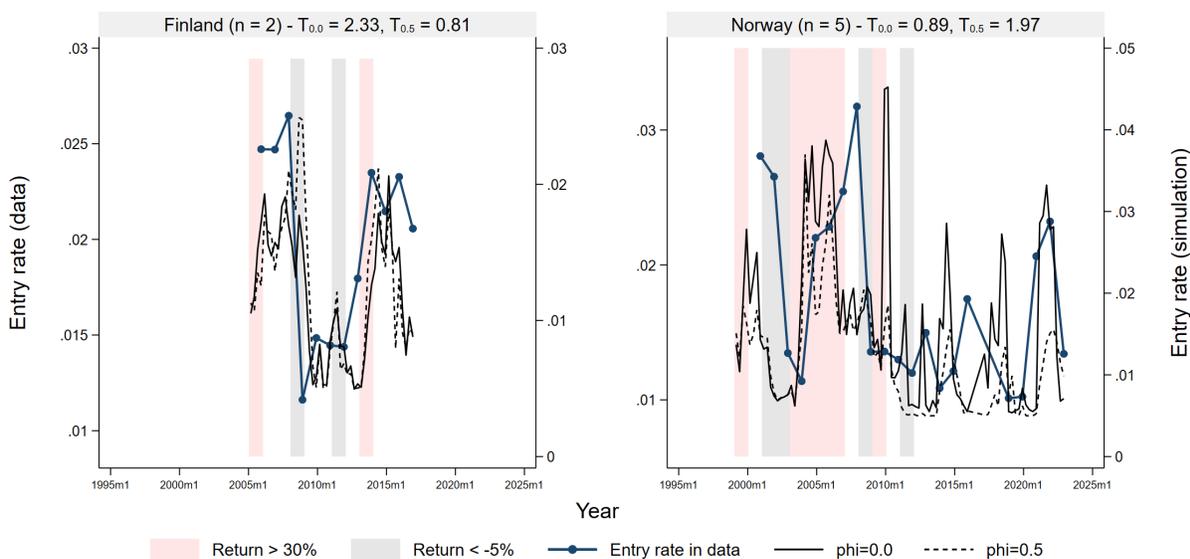
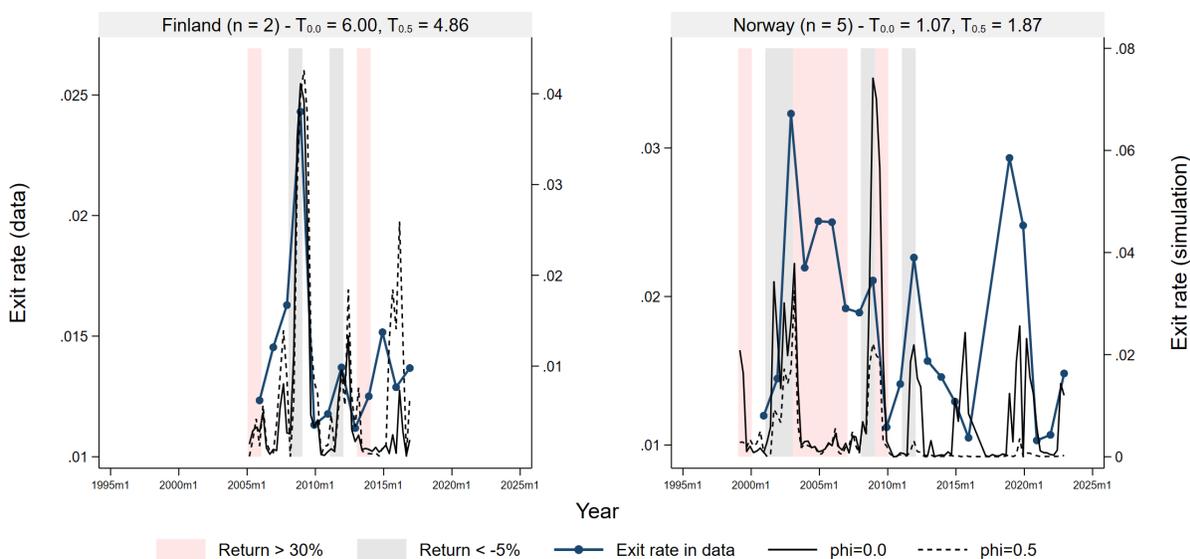

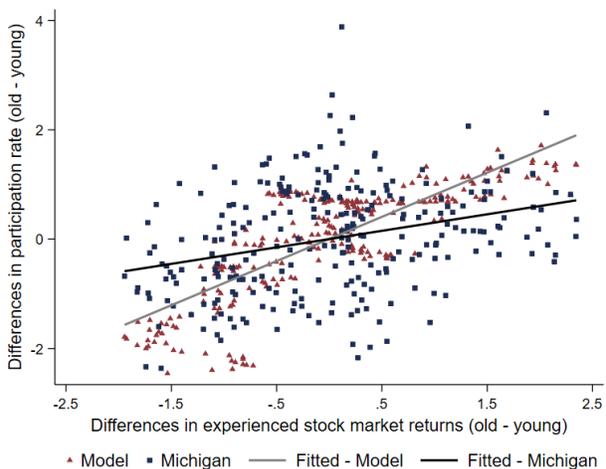(b) **Exit from the stock market - Finland and Norway**

Figure 6: **Experiences, Beliefs, and Participation: Data versus Model.** The top plot (a) shows the model implied difference in beliefs (in red) and the Michigan Survey of Consumers (Michigan, in blue) between the old and young based on experienced returns. The bottom left plot (b) shows the model implied (in red) and the Michigan (in blue) based difference in stock market participation rates between the old and young based on experienced returns. The bottom right plot (c) shows the model implied (in red) and the Michigan (in blue) based difference in stock market investment between the old and young based on experienced returns. We proxy for the difference in experienced stock market returns by taking the (time-series) average stock market return over the prior 50 years minus the return over the prior 20 years. For Michigan survey, old refers to households over 60 years old, and young refers to those below 40. In the simulation, we use the same cutoffs, where agents that are 40 years old have 20 year of experience. All the differences are standardized. For parameters see Subsection 4.1. For implied shocks see Subsection 4.3. The model data are from the *mix* scenario with $\phi = 0.5$.
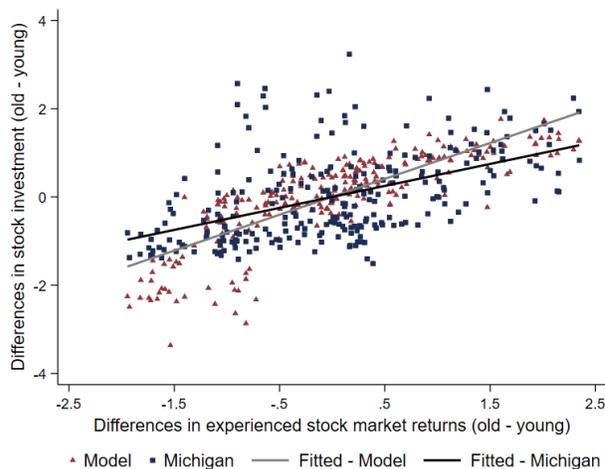
(a) **Experiences and beliefs**
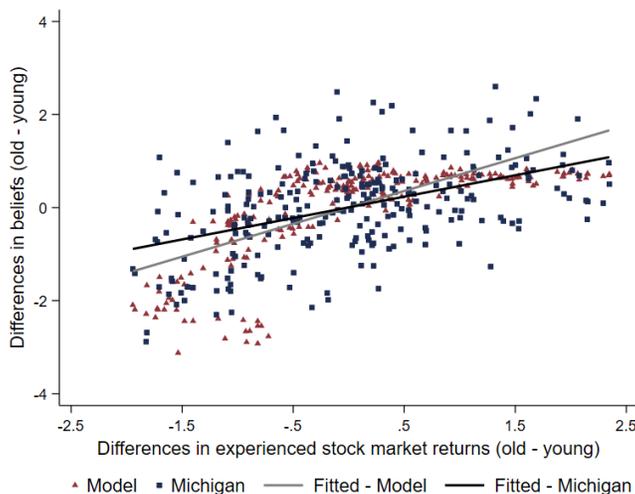
(b) **Experiences and participation rates**   (c) **Experiences and investments**

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The results support the view that the differences in experienced returns relate positively to differences in stock market participation in the model and in the observed data.

Moreover, the larger the difference in the experienced returns between old and young, the larger are the differences in stock market investments. The latter evidence further corroborates a beliefs-driven mechanism.

Each plot includes the fitted relations (dashed lines); these suggest that the belief-driven (constrained experience-based) model reproduces the differences in participation between old and young in the survey data. Specifically, the ability of the model to reproduce beliefs, participation, and investment based on experienced beliefs relative to the data is comparable as the correlation between the gap in beliefs in the model and in the Michigan data is 0.4098, between the gap in participation rates is 0.4595, and between the gap in investments is 0.4065.

5.2.2 Reentry frequencies

Next, we study reentry frequencies conditional on experienced returns. In this regard, the top plot in Figure 7a shows the fraction of investors who subsequently reenter the stock market in Finland, from the year 2005 on through 2011. According to the data, approximately 30% of the investors who exit the stock market reenter within 5 years. What stands out is that the fraction of re-entering investors among those who exited in year 2008, when the total stock return in Finland was -51%, is at the absolute bottom of re-entering investors. This pattern highlights the role of experience-based learning.

Our model replicates this pattern. Simulating our model forward, we follow the exiting cohorts for a ten-year window and calculate the proportion of agents who re-enter the first time after a given number of years. This comparison with the data is shown in Figure 7b. In the plot, the blue solid lines show the unconditional average reentry rate by time since exit. The red solid lines show the reentry rate conditional on the 1-year stock return at the time of exit being below the 10th percentile in the time-series. The takeaway from this exercise is that, as in the data, we see a large gap in the fraction of re-entering investors when investors

Figure 7: **Fraction of Investors Re-entering the Stock Market within n Years After Exiting - Data versus Model.** The left plot shows the fraction of investors re-entering the stock market within certain years since exit, using Finnish data. The red line is the fraction of re-entering investors among those that exit in 2008, and the blue line is the average of re-entering investors among those that exit in the years 2005 to 2011, excluding 2008. The right plot shows the fraction of investors returning to the stock market after exit, for the *mix* scenario, conditional on realized 1-year stock returns at the time of exit. The blue lines show the unconditional average, and the red lines show the average for cases when the 1-year stock return are within the bottom decile at the time of exit. We set $\phi = 0.5$, $\tau = 0.35$, and use a 5-year time-series for pre-entry learning. For other parameters see Subsection 4.1. For details of the forward-simulation see the caption of Table 3.

(a) **Finland** (b) **Simulation**
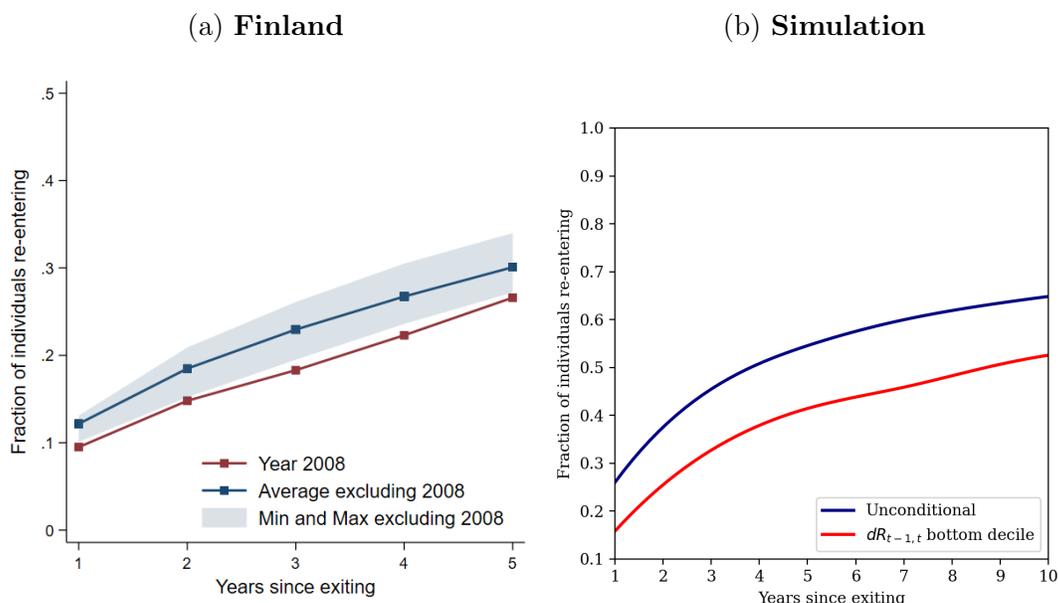
leave upon experiencing particularly negative returns.

5.3   Predictability

In our model, the participation rate negatively predicts the returns of the stock market as it is correlated with the consumption and wealth share of the participants, which are state variables for the real short rate of interest and the market price of risk. Intuitively, times of particularly high stock market participation are times of rampant exuberance, especially by the young. As over time statistically more common shocks fade away the exuberance, we

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see lower realized stock market returns. Panel A in Table 4 confirms this intuition both for the total and excess model-based returns for 12-, 24-, and 36-month horizons.

Panel B lends empirical support to the predictive power of the participation rate and constitutes another footprint of the mechanism entertained in this paper. Here, we use detrended participation rates from the four countries (Finland, Germany, Norway, and the US) to run predictive panel regressions. As we can see, all coefficients of the participation rate are consistent with the model in that they are negative. Further, they are statistically significant at the 0.01 level for 12-, 24-, and 36-month horizons for total as well as excess returns. These regressions include country fixed effects with standard errors clustered by country. Moreover, the results are robust to including the price-dividend-ratio, see Table 9 in the Online Appendix.

5.4   The Cost of Beliefs-driven Entry and Exit

Above we have established that entry and exit can be costly, as agents tend to enter at times when expected returns are low and exit at times when expected returns are high.

To quantify the welfare cost of entry and exit, we calculate the equivalent growth rate of aggregate consumption that yields the same realized average lifetime utility in an economy with complete information as in our incomplete information economy, where in the complete information economy all cohorts participate at all times.[28] This equivalent economic growth rate in the *reentry* scenario differs for agents with different time-preference and depends on the preentry learning window. According to Figure 8 the cost of entry and exit appears very large in that even for the impatient type of agent with a 25-year pre-entry learning the equivalent growth rate with complete information is only 1.3% compared to the actual 2% in our model. With a one-year pre-entry learning the equivalent consumption growth rate is as low as 0.6% for type-a agents and as low as 0.4% for type-b agents.

The documented costs of entry and exit expressed in terms of reduced consumption

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[28]For details of the derivations on the benchmark economy with complete information, refer to Subsection A.7 in the Online Appendix.

|	Panel A: Simulation results					
	Total returns			Excess returns		
	(1) 12m	(2) 24m	(3) 36m	(4) 12m	(5) 24m	(6) 36m
Participation rate	-0.022	-0.032	-0.039	-0.054	-0.076	-0.093
	Panel B: International evidence					
	Total returns			Excess returns		
	(1) 12m	(2) 24m	(3) 36m	(4) 12m	(5) 24m	(6) 36m
Participation rate	-0.350*** (0.113)	-0.374*** (0.078)	-0.323*** (0.058)	-0.378*** (0.131)	-0.406*** (0.092)	-0.361*** (0.070)
N	94	91	88	94	91	88

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: **Participation Rate and Future Returns - Data versus Model.** Panel A shows predictive regressions using the *mix scenario*, with 12-, 24-, and 36-month horizons, with model participation rate as explanatory variable, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. All variables are standardized. The coefficients estimates are averaged from the regression coefficients obtained from 10000 paths, with non-overlapping annual data samples along each path, each with 200 data points. For parameters see Subsection 4.1. For further details of the forward-simulation see the caption of Table 3. Panel B shows predictive regressions with 12-, 24-, and 36-month horizons, with detrended participation rate as explanatory variable from Finland, Germany, Norway and the US, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. Participation rates are at annual frequency. Interest rates are 12-month T-bill rates from Bloomberg for Finland, Bundesbank for Germany, Statistics Norway for Norway, and Amit Goyal's website for the US. All variables are standardized. The panel regressions include country fixed effects and use Driscoll-Kraay standard error with 2, 4, and 6 lags, respectively.

growth may be best compared to an equivalent increase in consumption volatility in the complete market benchmark. As an example, increasing aggregate consumption volatility in the frictionless benchmark economy from 3.3% to 10% yields the same expected utility as a reduction in consumption growth from 2% to 1.6%. Because even with a long 25-year pre-entry learning window the equivalent growth rate relative to the complete information is already as low as 1.3%, we conclude that the welfare cost of the return-driven entry and exit that we study cannot be understated.

Figure 8: **Equivalent Economic Growth Rate in the *Reentry* Scenario.** The blue and red lines denote the equivalent economic growth rate in a benchmark OLG economy with complete information, which gives the same realized average lifetime utility with a given number of years of pre-entry learning window, for an agent with time-preference ρ equals to 0.1% and 0.5%, respectively. The gray dash line denotes the actual growth rate in the economy with complete information. For details of the derivations on the benchmark economy with complete information, refer to Subsection A.7 in the Online Appendix. For parameters see Subsection 4.1. For details of the forward-simulation see the caption of Table 3.
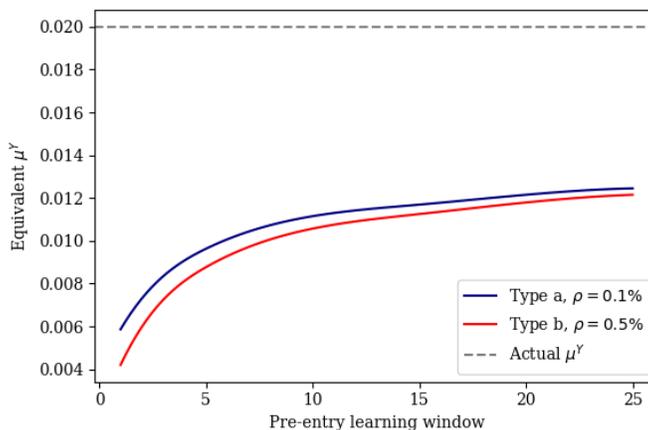
6 Conclusion

We empirically show that participants are optimists and nonparticipants are pessimists. In addition, entry and exit are associated with significant revisions in beliefs, which are associated with fluctuations in recent returns.

Based on these observations, we study the consequences of cyclical entry into and exit from the stock market in a general equilibrium model with overlapping generations. We introduce constrained rationality through learning from experience: agents are exposed to different parts of an economic path by birth, and thus form different expectations about growth. Further, we use a friction to capture the pattern that pessimists do not participate in the stock market. These agents pay less attention, and hence ignore an additional signal containing imperfect information about the fundamental. As belief and wealth dynamics determine the equilibrium market price of risk and through that participation, their joint dynamics in turn drive belief formation and wealth accumulation. Consequently, nonpartic-

ipation slows the speed of learning in the aggregate economy.

Besides the large cross-section of cohorts, our model accommodates many agent types within a cohort, including unconstrained designated participants, designated nonparticipants who never invest in the stock market, and agents who can leave the stock market permanently when they are disappointed by the experienced stock returns. With these ingredients, our model produces a large cross section of beliefs, as well as time-varying participation as cohorts enter and exit the stock market endogenously.

Leveraging on the large cross-section of agents in the model, we feed into it stock market shocks to produce simulated time-series of participation, entry, and exit. In sum, our model produces realistic dynamics of participation rates as well as entry and exit vis-à-vis data from Finland, Germany, Norway and the US. In our model, the constrained learning from experience entirely drives fluctuations in participation, entry, and exit and captures the swings that we observe in the data across these countries.

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Online Appendix:
Not for publication

A Online Appendix

A.1 Empirics using the Michigan Survey of Consumers

Here we present additional findings from our motivating analysis using the Surveys of Consumers by the University of Michigan.

A.1.1 Summary Statistics

The left plot of figure 9 shows the monthly time-series of the number of respondents who are selected for interview, as well as the number of respondents who are subsequently interviewed for a second time after 6 months. We further divide the panel sample into entry and exit samples according to whether they report participating in the stock market in the first interview. The exit (entry) sample refers to the group of respondents who report to invest (not to invest) in the first interview, and thus can potentially exit (enter) in the second interview. The right plot of figure 9 shows the proportion of the exit sample in the panel sample.

Figure 9: **Time-series of Sample Size and Sample Composition.**

(a) Whole sample and panel sample   (b) Exit sample / panel sample
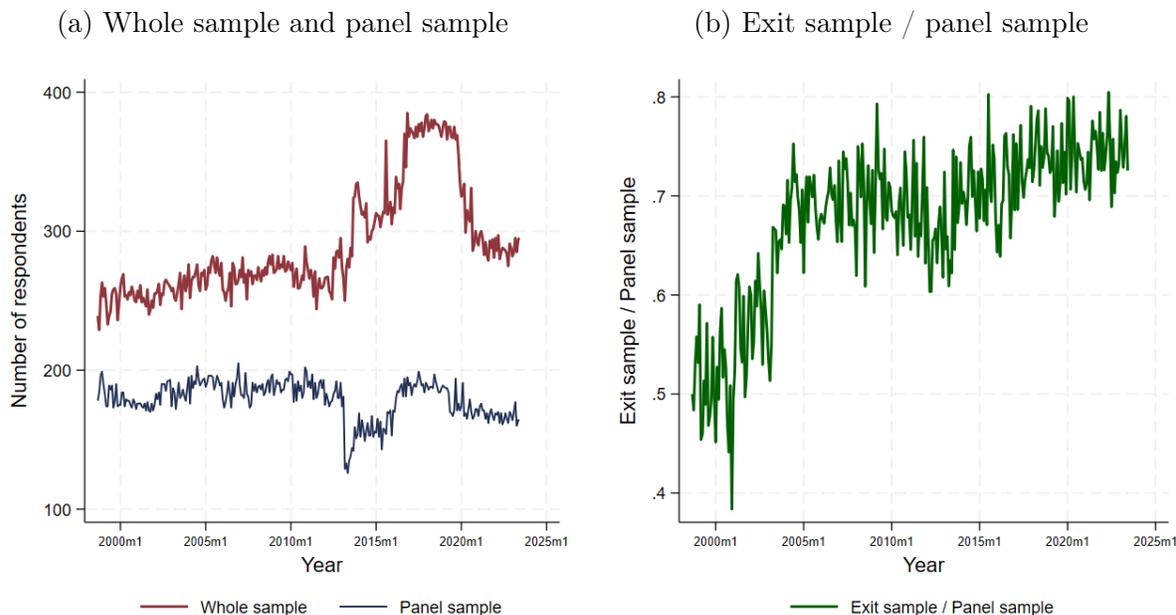
Further, we report the summary statistics for the whole sample, the panel sample, as well as the entry and exit samples in Table 5. The panel sample does not statistically differ from the whole sample, while the entry and exit samples involve endogenous sample selection. We thus use Heckman corrections for entry and exit regressions.

Panel A: Characteristics - Reported in first interview						
Whole Sample	Count	Mean	Std	p25	p50	p75
Age	85864	50	17	37	50	63
Income	85864	84199	78909	35000	62000	105000
Total Wealth	62203	471915	868790	95000	245000	510000
Investment Amount	45937	311672	772213	30000	100000	300000
Belief	74539	0.48	0.31	0.20	0.50	0.75
Panel sample - Interviewed Twice						
Age	53002	52	17	39	53	65
Income	53002	84793	77914	35000	65000	105000
Total Wealth	39794	490591	905893	100000	250000	530000
Investment Amount	30395	330547	808473	30000	100000	300000
Belief	44959	0.49	0.31	0.20	0.50	0.75
Entry Sample - Not Participating in Round 1						
Age	17212	52	18	37	53	67
Income	17212	44763	43885	19200	34000	56000
Total Wealth	6524	199233	230942	80000	145000	250000
Belief	13425	0.37	0.30	0.10	0.35	0.50
Exit Sample - Participating in Round 1						
Age	35790	52	15	40	52	64
Income	35790	104044	83202	50000	80000	125000
Total Wealth	33270	547725	975296	100000	290000	600000
Belief	31534	0.54	0.30	0.30	0.50	0.80

Panel B: Control Variables		Whole Sample	Panel Sample	Entry Sample	Exit Sample
Gender	Female	47.83	47.60	53.70	44.67
	Male	52.17	52.40	46.30	55.33
Region	North Central	25.87	26.62	25.41	27.21
	Northeast	17.11	17.16	15.66	17.88
	South	35.32	34.33	37.15	32.97
	West	21.71	21.89	21.78	21.94
Political affiliation	Democrat	12.67	12.19	10.37	13.07
	Indep / Not answered	76.18	77.34	82.04	75.09
	Republican	11.16	10.46	7.59	11.84
Education	Grade 8	1.91	1.54	4.19	0.27
	Grade 12 - no diploma	3.63	2.99	7.07	1.03
	Grade 12 - diploma	22.70	21.26	33.35	15.44
	Grade 16 - college diploma	25.55	26.65	16.61	31.48
	Grade 17 - no college diploma	18.18	20.48	8.76	26.12
	Grade 17 - no diploma	28.04	27.07	30.02	25.66
Home ownership	Own	74.00	78.18	62.28	85.83
	Rent	26.00	21.82	37.72	14.17

Panel C: Entry and Exit Frequencies				
Entry sample	3196	**Exit sample**		3040
Entry rate	6.02%	Exit rate		5.74%
Average investment of entrants	88210	Average investment before exit		110397

Table 5: **Summary Statistics, Whole Sample versus Subsamples.**

A.1.2 Monthly Average Belief

We plot the monthly average belief of participants and nonparticipants in Figure 10. The average beliefs of participants and nonparticipants are time-varying and correlated, and the average beliefs of participants are consistently more optimistic than those of nonparticipants.

Figure 10: **Stock Market Beliefs.** The plots show the average beliefs of participants and nonparticipants in each monthly sample.
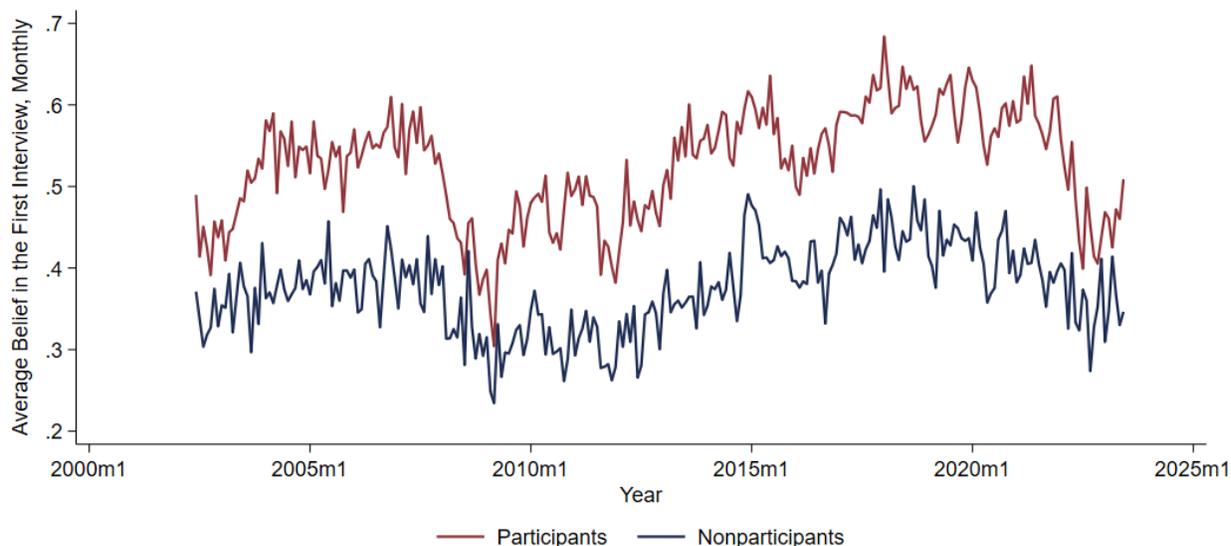
A.1.3 Experiences Matter for Beliefs

We test whether households of different age form different beliefs, and whether through that they trade differently, using regressions inspired by Malmendier and Nagel (2011). We label the households below 40 years old as young, and those above 60 years old as old. We use the average stock return of the past 20 years as a proxy for the experience of the young, and that of the past 50 years as a proxy for the experience of the old. Following this line of thought, we calculate the time-series of the difference between old and young in average belief (belief gap), participation rate (participation gap), and investment amount (investment gap). We standardize the time-series of differences to have zero mean and unit variation for easier interpretation.

We report the regression results in columns 3 to 5 of Table 2. We perform the same regressions using Newey-West standard errors with maximum 3 lags, and report the results in Table 6. Our results are robust to Newey-West correction.

Old - young	(1) First stage	(2) Participation gap	(3) Investment gap	(4) Participation gap	(5) Investment gap	(6) Participation gap	(7) Investment gap
Experience gap	0.393*** (0.064)					0.150*** (0.055)	0.143*** (0.042)
Fitted values		0.382*** (0.141)	0.363*** (0.108)				
Belief gap				0.318*** (0.053)	0.140*** (0.025)		
Controls	Y	Y	Y	Y	Y	Y	Y
N	253	253	253	253	253	253	253

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: **Difference in Participation and Investment for Old versus Young II.** Belief is PSTK/100. Change of belief (ΔBelief) is belief in the second round of interview minus that in the first round. Investment is the dollar amount invested in the stock market, conditional on investment. In the second stages, participation rate gap (average dollar investment gap) is regressed on experience gap or the linearly predicted belief gap or the belief gap. Age groups are defined as (<40, 40-60, >60), where old is >60, young is <40. Experience is the average total stock return in the past 50 years for old, and past 20 years for young. Controls include the difference in average wealth and average income between old and young. All differences are standardized to have a mean of 0 and standard deviation of 1. Returns on the stock market are from Amit Goyal's website. We report Newey-West adjusted T-statistics.

A.2 Equilibrium Consumption Dynamics

In equilibrium, the (log) consumption dynamics of participants and nonparticipants differ:

Proposition 6. *The log consumption at time t for an agent of type i born at time s has the dynamics*

$$dlog(c_{i,s,t}) = \begin{cases} (-\rho_i + r_t)\, dt, & \text{if } N, \text{ or} \\ \left(-\rho_i + r_t + \frac{1}{2}\theta_t^2 - \frac{1}{2}\Delta_{s,t}^2\right) dt + (\theta_t + \Delta_{s,t})\, dz_t^Y, & \text{if } P. \end{cases} \tag{B1}$$

Proposition 6 captures when an agent is expected to lose out in terms of lower expected log consumption growth. Nonparticipants lose simply because they do not earn the risk premium. Participants lose whenever the expected value of the squared estimation error is larger than that of the market price of risk. The last proposition summarizes the dynamics of the consumption share of a cohort:

Proposition 7. *The consumption share at time t of type i agents born at s follows the process*

$$df_{i,s,t}^c / f_{i,s,t}^c = (-\tau\beta + \bar{\rho}_t - \rho_i)\, dt + (\theta_t - \sigma_Y)\left(\theta_{s,t}^+ - \sigma_Y\right) dt + \left(\theta_{s,t}^+ - \sigma_Y\right) dz_t^Y$$

$$= \begin{cases} \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right)\right] dt - \sigma_Y dz_t^Y, & \text{if } N, \text{ or} \\ \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right) + (\theta_t + \Delta_{s,t})\left(\theta_t - \sigma_Y\right)\right] dt \\ \quad + \left(\theta_t + \Delta_{s,t} - \sigma_Y\right) dz_t^Y, & \text{if } P, \end{cases} \tag{B2}$$

and the wealth share at time t of type i agents born at s follows the process

$$df_{i,s,t}^W / f_{i,s,t}^W = \begin{cases} \left[-\nu + \tilde{\beta}_t - \rho_i - \sigma_t^S\left(\theta_t - \sigma_t^S\right)\right] dt - \sigma_t^S dz_t^Y, & \text{if } N, \text{ or} \\ \left[-\nu + \tilde{\beta}_t - \rho_i - \sigma_t^S\left(\theta_t - \sigma_t^S\right) + (\theta_t + \Delta_{s,t})\left(\theta_t - \sigma_t^S\right)\right] dt \\ \quad + \left(\theta_t + \Delta_{s,t} - \sigma_t^S\right) dz_t^Y, & \text{if } P. \end{cases} \tag{B3}$$

where $\tilde{\beta}_t \equiv \frac{C_t}{W_t}$ is the aggregate consumption-to-wealth ratio.

The significance of Proposition 7 is that although our framework contains two Brownian shocks, of which only the fundamental shock to output is priced, financial markets are complete; and that although we obtain closed-form solution throughout this entire section, we still must follow all consumption and wealth shares to simulate the economy forward.

A.3 Proofs of Propositions

A.3.1 Proof of Proposition 1

Following standard filtering theory in Liptser and Shiryayev (2013), we have that the dynamics of the expected output growth rate and the posterior variance of the estimates perceived by an agent born at s being either N or P are given by Equations (5 - 6). From the definition of the standardized estimation error $\Delta_{s,t}$, and using Ito's lemma, we have

$$
d\Delta_{s,t} = \frac{d\hat{\mu}_{s,t}}{\sigma_Y} = \begin{cases} \dfrac{\hat{V}'_s}{\sigma_Y^2 + \hat{V}'_s\left(t-t'_s\right)} dz_{s,t}^Y, & \text{if } N, \text{ or} \\[4mm] \dfrac{(1-\phi^2)\hat{V}'_s}{\sigma_Y^2(1-\phi^2) + \hat{V}'_s\left(t-t'_s\right)}\left(dz_{s,t}^Y - \dfrac{\phi}{\sqrt{1-\phi^2}} dz_{s,t}^{SI}\right), & \text{if } P. \end{cases}
$$

Solving the stochastic differential equations gives the following expression for $\Delta_{s,t}$:

$$
\Delta_{s,t} = \begin{cases} \dfrac{\sigma_Y^2}{\sigma_Y^2 + \hat{V}'_s\left(t-t'_s\right)}\Delta_{s,t'_s} + \dfrac{\hat{V}'_s}{\sigma_Y^2 + \hat{V}'_s\left(t-t'_s\right)}\left(z_t^Y - z_{t'_s}^Y\right), & \text{if } N, \text{ or} \\[4mm] \dfrac{\sigma_Y^2(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}'_s\left(t-t'_s\right)}\Delta_{s,t'_s} + \dfrac{\hat{V}'_s(1-\phi^2)}{\sigma_Y^2(1-\phi^2) + \hat{V}'_s\left(t-t'_s\right)}\left\{\left(z_t^Y - z_{t'_s}^Y\right) - \dfrac{\phi}{\sqrt{1-\phi^2}}\left(z_t^{SI} - z_{t'_s}^{SI}\right)\right\}, & \text{if } P. \end{cases}
$$

A.3.2 Proof of Proposition 2

The proof follows directly from Equation (19), when t = s, and $c_{i,s,s} = \frac{1}{\kappa_{i,s}(1+\tau)\xi_{s,s}}$. We also show that β_i is a constant through the initial budget constraint. Given that $\beta_i \equiv \frac{c_{i,s,t}}{W_{i,s,t}}$ for any $t > s$, we have

$$
\begin{aligned}
W_{i,s,s} &= E_{s,s}\left[\int_s^\infty e^{-\nu(t-s)}\frac{\xi_{s,t}}{\xi_{s,s}}\left(c_{i,s,t} + \tau c_{i,s,t}\right) dt\right] \\
&= (1+\tau)\, E_{s,s}\left[\int_s^\infty e^{-\nu(t-s)}\frac{\xi_{s,t}}{\xi_{s,s}} c_{i,s,s} e^{-\rho_i(t-s)}\frac{\xi_{s,s}}{\xi_{s,t}} dt\right] \\
&= (1+\tau)\frac{1}{\rho_i + \nu} c_{i,s,s} \\
&= \frac{c_{i,s,s}}{\beta_i}.
\end{aligned}
$$

We thus have $\beta_i = \frac{\rho_i + \nu}{1+\tau}$, which is a constant.

A.3.3 Proof of Proposition 3

From the market clearing condition for the consumption goods market, we find

$$Y_t = \int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} ds di, \quad \text{a.s., and therefore,}$$

$$dY_t = \left(\int_i \alpha_i \nu c_{i,t,t} di - \nu C_t \right) dt$$

$$+ \int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} \left[\left(-\rho_i + r_t + \theta_{s,t}^+ \theta_t \right) dt + \theta_{s,t}^+ dz_t^Y \right] ds di$$

$$= \left(\int_i \alpha_i \nu \beta_i W_{i,t,t} di - \nu C_t \right) dt + r_t C_t dt - \left(\int_i \alpha_i \rho_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} ds di \right) dt$$

$$+ \theta_t \left[\int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dt$$

$$+ \left[\int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dz_t^Y$$

$$= \left(\int_i \alpha_i \nu \beta_i \frac{\tau}{\nu} C_t di - \nu C_t \right) dt + r_t C_t dt - \left(\int_i \alpha_i \rho_i \int_{-\infty}^t f_{i,s,t}^c ds di \right) C_t dt$$

$$+ \theta_t C_t \left[\int_i \alpha_i \int_{-\infty}^t f_{i,s,t}^c \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dt$$

$$+ C_t \left[\int_i \alpha_i \int_{-\infty}^t f_{i,s,t}^c \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right] dz_t^Y$$

$$= C_t \left\{ \left(\tau \beta - \nu + r_t - \bar{\rho}_t + \theta_t^2 \bar{\Phi}_t + \theta_t \bar{\Phi}_t \bar{\Delta}_t \right) dt + \bar{\Phi}_t \left(\theta_t + \bar{\Delta}_t \right) \right\}$$

$$= Y_t \left(\mu_Y dt + \sigma_Y dz_t^Y \right),$$

using the definitions of β, $\bar{\Delta}_t$ and $\bar{\Phi}_t$. By matching the drift and diffusion terms, we have the interest rate r_t and the market price of risk θ_t.

A.3.4 Proof of Proposition 4

As the bond has zero net supply, and the stock is the only form of wealth in the economy, we have,

$$S_t = W_t = \int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} W_{i,s,t} ds di.$$

With log utility, $\frac{dW_{i,s,t}}{W_{i,s,t}} = \frac{dc_{i,s,t}}{c_{i,s,t}}$ holds for each type i. Using Proposition (20) and $\int_i \alpha_i \nu W_{i,t,t} di = \tau C_t$ from consumption tax, we have

$$
\begin{aligned}
dW_t &= d \int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} W_{i,s,t} ds di \\
&= \int_i \alpha_i \nu W_{i,t,t} di dt - \nu W_t dt \\
&\quad + \int_i \alpha_i \int_{-\infty}^t \nu e^{-\nu(t-s)} W_{i,s,t} \left[\left(-\rho_i + r_t + \theta_{s,t}^+ \theta_t \right) dt + \theta_{s,t}^+ dz_t^Y \right] ds di \\
&= W_t \left\{ \tau \tilde{\beta}_t dt - \nu dt + \int_i \alpha_i \int_{-\infty}^t f_{i,s,t}^W \left[\left(-\rho_i + r_t + \theta_{s,t}^+ \theta_t \right) dt + \theta_{s,t}^+ dz_t^Y \right] ds di \right\} \\
&= W_t \left\{ \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + r_t + \theta_t \int_i \alpha_i \int_{-\infty}^t f_{i,s,t}^W \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right\} dt \\
&\quad + W_t \left\{ \int_i \alpha_i \int_{-\infty}^t f_{i,s,t}^W \left(\Delta_{s,t} + \theta_t \right) \mid \left(\Delta_{s,t} \geq -\theta_t \right) ds di \right\} dz_t^Y \\
&= W_t \left\{ \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + r_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) \right\} dt + W_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) dz_t^Y \\
&= dS_t = S_t \left(\mu_t^S dt + \sigma_t^S dz_t^Y \right) - Y_t dt,
\end{aligned}
$$

where $\tilde{\rho}_t = \int_i \rho_i \int_{-\infty}^t f_{i,s,t}^W ds di$ is the wealth weighted average time preference, and $\tilde{\beta}_t \equiv \frac{C_t}{W_t}$ is the aggregate consumption-to-wealth ratio. Matching the drift and diffusion terms, we then have μ_t^S and σ_t^S. Specifically, with the expression of β_i from A.3.2, we have

$$
\begin{aligned}
\tilde{\beta}_t = \frac{C_t}{W_t} &= \frac{\int_i \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{i,s,t} ds di}{W_t} = \int_i \int_{-\infty}^t \frac{c_{i,s,t}}{W_{i,s,t}} \frac{W_{i,s,t}}{W_t} ds di \\
&= \int_i \beta_i \int_{-\infty}^t f_{i,s,t}^W ds di = \int_i \frac{\rho_i + \nu}{1 + \tau} \int_{-\infty}^t f_{i,s,t}^W ds di \\
&= \frac{\tilde{\rho}_i + \nu}{1 + \tau}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mu_t^S - r_t &= \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) + \frac{Y_t}{S_t} = \tau \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) + \tilde{\beta}_t \\
&= (\tau + 1) \tilde{\beta}_t - \nu - \tilde{\rho}_t + \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right) \\
&= \theta_t \tilde{\Phi}_t \left(\tilde{\Delta}_t + \theta_t \right).
\end{aligned}
$$

A.3.5 Proof of Proposition 5

The proof is obtained by matching the diffusion terms of the wealth process as in Equation (17) and the relation between wealth and consumption, as well as the fact that for participants we have, $\theta_{s,t}^+ = \theta_{s,t} = \theta_t + \Delta_{s,t}$,

$$W_{i,s,t} = \frac{c_{i,s,t}}{\beta_i} = \frac{c_{i,s,s}e^{-\rho_i(t-s)}}{\beta_i}\frac{\eta_{i,s,t}}{\eta_{i,s,s}}\frac{\xi_s}{\xi_t},$$

$$\therefore dW_{i,s,t}/W_{i,s,t} = \left[-\rho_i + r_t + \theta_t^2 - \left(\theta_{s,t}^+ - \theta_t\right)\theta_t\right]dt + \left(\theta_t + \theta_{s,t}^+ - \theta_t\right)dz_t^Y$$
$$= \left(-\rho_i + r_t + \theta_t^2 - \Delta_{s,t}\theta_t\right)dt + \left(\theta_t + \Delta_{s,t}\right)dz_t^Y,$$

and thus, $\pi_{i,s,t} = \frac{\Delta_{s,t}+\theta_t}{\sigma_t^S}W_{i,s,t}$ for stock market participants. The optimal stock investment also confirms that an agent born at s would have non-negative demand for risk at time t if $\Delta_{s,t} \geq -\theta_t$. Otherwise, she would be subject to the short-sale constraint and not participate in the stock market.

A.3.6 Proof of Proposition 6

From the expression for optimal consumption as described in Proposition 2, and using Ito's lemma, we have

$$dc_{i,s,t}/c_{i,s,t} = (-\rho_i + r_t + \theta_{s,t}^+\theta_t)dt + \theta_{s,t}^+dz_t^Y,$$

which implies that

$$d\log(c_{i,s,t}) = \left(-\rho_i + r_t + \theta_{s,t}^+\theta_t - \frac{1}{2}(\theta_{s,t}^+)^2\right)dt + \theta_{s,t}^+dz_t^Y$$

$$= \begin{cases} (-\rho_i + r_t)dt, & \text{if } N, \text{ or} \\ (-\rho_i + r_t + \frac{1}{2}\theta_t^2 - \frac{1}{2}\Delta_{s,t}^2)dt + (\theta_t + \Delta_{s,t})dz_t^Y, & \text{if } P. \end{cases}$$

A.3.7 Proof of Proposition 7

From the definition of $f_{i,s,t}$, we have

$$df_{i,s,t}^c = d\frac{\alpha_i\nu e^{-\nu(t-s)}c_{i,s,t}}{C_t}$$

$$df_{i,s,t}^c/f_{i,s,t}^c = -\nu dt + \frac{dc_{i,s,t}}{c_{i,s,t}} - \frac{dC_t}{C_t} + \frac{dC_t^2}{C_t^2} - \frac{dc_{i,s,t}dC_t}{c_{i,s,t}C_t}$$

$$= \left\{-\nu - \rho_i + r_t + \theta_{s,t}^+\theta_t - \mu_Y + \sigma_Y^2 - \theta_{s,t}^+\sigma_Y\right\}dt + \left(\theta_{s,t}^+ - \sigma_Y\right)dz_t^Y$$

$$= -\tau\beta dt + (\theta_t - \sigma_Y)(\theta_{s,t}^+ - \sigma_Y)dt + (\theta_{s,t}^+ - \sigma_Y)dz_t^Y.$$

Recall that $\theta_{s,t}^+ = 0$ for N and $\theta_{s,t}^+ = \theta_t + \Delta_{s,t}$ for P. Thus, the dynamics of consumption share follows

$$df_{i,s,t}^c / f_{i,s,t}^c = \left(-\tau\beta + \bar{\rho}_t - \rho_i\right) dt + \left(\theta_t - \sigma_Y\right)\left(\theta_{s,t}^+ - \sigma_Y\right) dt + \left(\theta_{s,t}^+ - \sigma_Y\right) dz_t^Y$$

$$= \begin{cases} \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right)\right] dt - \sigma_Y dz_t^Y, & \text{if } N, \text{ or} \\ \left[-\tau\beta + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma_Y\right)\right] dt \\ \quad + \left(\theta_t + \Delta_{s,t} - \sigma_Y\right) dz_t^Y, & \text{if } P. \end{cases}$$

For wealth share, since $df_{i,s,t}^W = d\frac{\alpha_i \nu e^{-\nu(t-s)} W_{i,s,t}}{W_t}$, we have

$$df_{i,s,t}^W / f_{i,s,t}^W = -\nu dt + \frac{dW_{i,s,t}}{W_{i,s,t}} - \frac{dW_t}{W_t} + \frac{dW_t^2}{W_t^2} - \frac{dW_{i,s,t} dW_t}{W_{i,s,t} W_t}$$

$$= -\nu dt + \frac{dc_{i,s,t}}{c_{i,s,t}} - \frac{dW_t}{W_t} + \frac{dW_t^2}{W_t^2} - \frac{dc_{i,s,t} dW_t}{c_{i,s,t} W_t}$$

$$= \left\{ -\nu - \rho_i + r_t + \theta_{s,t}^+ \theta_t - \mu_t^S + \frac{Y_t}{W_t} + \left(\sigma_t^S\right)^2 - \theta_{s,t}^+ \sigma_t^S \right\} dt + \left(\theta_{s,t}^+ - \sigma_t^S\right) dz_t^Y$$

$$= \left\{ -\nu - \rho_i + \frac{Y_t}{W_t} + \theta_{s,t}^+ \theta_t - \sigma_t^S \theta_t + \left(\sigma_t^S\right)^2 - \theta_{s,t}^+ \sigma_t^S \right\} dt + \left(\theta_{s,t}^+ - \sigma_t^S\right) dz_t^Y$$

$$= \begin{cases} \left[-\nu - \rho_i + \tilde{\beta}_t - \sigma_t^S\left(\theta_t - \sigma_t^S\right)\right] dt - \sigma_t^S dz_t^Y, & \text{if } N, \text{ or} \\ \left[-\nu - \rho_i + \tilde{\beta}_t - \sigma_t^S\left(\theta_t - \sigma_t^S\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma_t^S\right)\right] dt \\ \quad + \left(\theta_t + \Delta_{s,t} - \sigma_t^S\right) dz_t^Y, & \text{if } P. \end{cases}$$

A.4 Other Proofs

A.4.1 Proof of the Static Budget Constraint

With Equations (17) and (13), and applying Ito's lemma, we have

$$d\left(\xi_{s,t} W_{i,s,t}\right) = -\xi_{s,t} W_{i,s,t}\left(r_t dt + \theta_{s,t} dz_{s,t}^Y\right)$$

$$+ \xi_{s,t}\left\{\left[\left(r_t + \nu - \beta\right) W_{i,s,t} + \pi_{i,s,t} \sigma_t^S \theta_{s,t} - c_{i,s,t}\right] dt + \pi_{i,s,t} \sigma_t^S dz_{s,t}^Y\right\}$$

$$- \xi_{s,t} \theta_{s,t} \sigma_t^S \pi_{i,s,t} dt$$

$$= \xi_{s,t}\left(\left(\nu - \beta_i\right) W_{i,s,t} - c_{i,s,t}\right) dt + \xi_{s,t}\left(-W_{i,s,t} \theta_{s,t} + \pi_{i,s,t} \sigma_t^S\right) dz_{s,t}^Y.$$

With stochastic time of death T, we have

$$\xi_{s,t} W_{i,s,\tau} + \int_s^T \xi_{s,t} c_{i,s,t} dt + \int_s^T \xi_{s,t} \beta_i W_{i,s,t} dt - \int_s^T \xi_{s,t} \nu W_{i,s,t} dt$$

$$= \xi_{s,s} W_{i,s,s} + \int_s^T \xi_{s,t} \left(\pi_{i,s,t} \sigma_t^S - W_{i,s,t} \theta_{s,t} \right) dz_{s,t}^Y.$$

A.5 Wealth Tax

Alternatively to a consumption tax, we can impose a wealth tax on all living agents to finance the consumption of the new-born cohort. With wealth tax at rate τ^W, the dynamic budget constraint becomes

$$dW_{i,s,t} = \begin{cases} \left(r_t W_{i,s,t} + \nu W_{i,s,t} - \tau^W W_{i,s,t} - c_{i,s,t}\right) dt, & \text{if } N, \quad \text{or} \\ \left[r_t W_{i,s,t} + \pi_{i,s,t}\left(\mu^S_{s,t} - r_t\right) + \nu W_{i,s,t} - \tau^W W_{i,s,t} - c_{i,s,t}\right] dt + \pi_{i,s,t}\sigma^S_t dz^Y_{s,t}, & \text{if } P, \end{cases}$$

which implies that the static budget constraint is

$$E_{s,s}\left[\int_s^\infty e^{-\nu(t-s)}\xi_{s,t}(c_{i,s,t} + \tau^W W_{i,s,t})dt\right] = W_{i,s,s}.$$

We conjecture and verify that $\beta_i = \frac{c_{i,s,t}}{W_{i,s,t}}$ is a constant. Using (2), we have $\frac{1}{\rho_i+\nu}c_{i,s,t}\left(1 + \frac{\tau^W}{\beta_i}\right) = W_{i,s,t} = \frac{c_{i,s,t}}{\beta_i}$. Rearranging, we have $\beta_i = \rho_i + \nu - \tau^W$, which is a constant. It then follows that the equilibrium real short rate of interest and the market price of risk with wealth tax are

$$r_t = \nu - \tau\frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t + \mu_Y - \sigma_Y\left(\sigma_Y\frac{1}{\bar{\Phi}_t} - \bar{\Delta}_t\right) = \nu - \tau^W\frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t - \sigma_Y^2\frac{1}{\bar{\Phi}_t} + \bar{\mu}_t, \qquad \text{(E4)}$$

and the market price of risk is

$$\theta_t = \sigma_Y\frac{1}{\bar{\Phi}_t} - \bar{\Delta}_t = \sigma_Y\frac{1}{\bar{\Phi}_t} - \frac{\bar{\mu}_t - \mu_Y}{\sigma_Y}, \qquad \text{(E5)}$$

where $\tilde{\beta}_t \equiv \frac{Y_t}{W_t} = \int_i \beta_i \int_{-\infty}^t f^W_{i,s,t}dsdi$ is the aggregate consumption-wealth ratio at time t.[29] Imposing a wealth tax instead of a consumption tax does not change the expressions of the stock volatility σ^S_t, the risky portfolio $\pi_{i,s,t}$, as well as the dynamics of log consumption $log(c_{i,s,t})$ in Propositions (4), (5), and (6). The consumption share at time t of type i agents

---

[29]Equivalently, $\tilde{\beta}_t$ is also the wealth-weighted average consumption-wealth ratio in the economy, as $\frac{Y_t}{W_t} = \frac{\int_i \int_{-\infty}^t \alpha_i\nu e^{-\nu(t-s)}c_{i,s,t}dsdi}{W_t} = \frac{\int_i \int_{-\infty}^t \alpha_i\beta_i\nu e^{-\nu(t-s)}W_{i,s,t}dsdi}{W_t} = \int_i \beta_i \int_{-\infty}^t f^W_{i,s,t}dsdi$. Therefore, $\tilde{\beta}_t = \tilde{\rho}_t + \nu - \tau^W$.

born at s follows the process

$$df_{i,s,t}^c/f_{i,s,t}^c = \left(-\tau^W \frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t - \rho_i\right) dt + (\theta_t - \sigma_Y)\left(\theta_{s,t}^+ - \sigma_Y\right) dt + \left(\theta_{s,t}^+ - \sigma_Y\right) dz_t^Y$$

$$= \begin{cases} \left[-\tau^W \frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right)\right] dt - \sigma_Y dz_t^Y, & \text{if } N \text{, or} \\ \left[-\tau^W \frac{\beta}{\tilde{\beta}_t} + \bar{\rho}_t - \rho_i - \sigma_Y\left(\theta_t - \sigma_Y\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma_Y\right)\right] dt \\ \qquad + \left(\theta_t + \Delta_{s,t} - \sigma_Y\right) dz_t^Y, & \text{if } P, \end{cases}$$

and the wealth share at time t of type i agents born at s follows the process

$$df_{i,s,t}^W/f_{i,s,t}^W = \begin{cases} \left[-\tau^W + \tilde{\rho}_t - \rho_i - \sigma_t^S\left(\theta_t - \sigma_t^S\right)\right] dt - \sigma_t^S dz_t^Y, & \text{if } N \text{, or} \\ \left[-\tau^W + \tilde{\rho}_t - \rho_i - \sigma_t^S\left(\theta_t - \sigma_t^S\right) + \left(\theta_t + \Delta_{s,t}\right)\left(\theta_t - \sigma_t^S\right)\right] dt \\ \qquad + \left(\theta_t + \Delta_{s,t} - \sigma_t^S\right) dz_t^Y, & \text{if } P. \end{cases}$$

A.6 Equilibrium with *Disappointment* Type

Here, we consider an alternative economy, which has an identical setting with the economy described in the main model section (Section 3), except that a cohort leaves the stock market for good once the short-sale constraint binds (and to simply the exposition we turn off heterogeneity in time preferences). In this setting, the same learning mechanism continues to hold, while the disagreement process contains an indicator variable, $I_{s,t}$:

$$I_{s,t} \equiv \begin{cases} 1, & \text{if } t \le t'_s, \\ 0, & \text{otherwise,} \end{cases}$$

where t'_s denotes the first time when cohort s transitions from P to N. For a cohort to participate in the stock market, both the two conditions $I_{s,t} = 1$ and $\theta_{s,t} \ge 0$ have to be satisfied. Therefore, the disagreement process follows

$$d\eta'_{s,t}/\eta'_{s,t} = (\theta^+_{s,t} \cdot I_{s,t} - \theta'_t)dz^Y_t.$$

In equilibrium, the real short rate of interest is

$$r'_t = \nu - \tau + \rho - \sigma^2_Y \frac{1}{\Phi'_t} + \bar{\mu}'_t,$$

and the market price of risk is

$$\theta'_t = \sigma_Y \frac{1}{\Phi'_t} - \frac{\bar{\mu}'_t - \mu_Y}{\sigma_Y},$$

where $\Phi'_t \equiv \int_{-\infty}^t f_{s,t} \mid (\Delta_{s,t} \ge -\theta'_t \cap I_{s,t} = 1)ds$ is the total wealth share of the participants in the stock market, and $\bar{\mu}'_t \equiv \frac{\int_{-\infty}^t f_{s,t}\hat{\mu}_{s,t}|(\Delta_{s,t}\ge-\theta'_t \cap I_{s,t}=1)ds}{\Phi'_t}$ denotes the wealth-weighted average expected growth rate conditional on stock market participation. Except for the additional component $I_{s,t}$, the real short rate of interest and the market price of risk have the same construction as in the *reentry* scenario. Construction of the equilibrium in the *mix* scenario follows analogous steps.

A.7 Benchmark OLG Economy with Complete Information

In Subsection 5.4, we compare the welfare loss from entry to and exit from the stock market using a benchmark overlapping generations economy with complete information. Clearly, complete information does not change the form of the individual or cohort specific optimization problems. Hence, the interest rate r_t and the market price of risk θ_t can be written as,

$$r_t = \nu - \tau\beta + \bar{\rho}_t + \mu_Y - \sigma_Y^2,$$
$$\theta_t = \sigma_Y,$$

where $\bar{\rho}_t = \int_t \rho_i \int_{-\infty}^t f_{i,s,t}^c ds di$ is the consumption weighted average time preference. These two equilibrium quantities have identical structure to Proposition (3), with $\bar{\Delta}_t = 0$ and $\bar{\Phi}_t = 1$ implying zero estimation error and participation by all cohorts. Furthermore, also here we conjecture and verify that $\bar{\rho}_t = \bar{\rho}$ is a constant. That is, the consumption share of type-i agents, $f_i^c \equiv \int_{-\infty}^t f_{i,s}^c ds = \int_{-\infty}^t \frac{\alpha_i \nu e^{-\nu(t-s)} c_{i,s,t}}{C_t} ds$ is a constant for any t. For f_i^c to be a constant,

$$\begin{aligned}
df_i^c &= d \int_{-\infty}^t \frac{\alpha_i \nu e^{-\nu(t-s)} c_{i,s,t}}{C_t} ds \\
&= f_i^c \left(\bar{\rho} - \tau\beta - \rho_i \right) dt + \alpha_i \nu \frac{c_{i,t,t}}{C_t} dt \\
&= f_i^c \left(\bar{\rho} - \tau\beta - \rho_i \right) dt + \alpha_i \tau \beta_i dt = 0 \quad \text{satisfies,}
\end{aligned}$$

which implies $f_i^c = \frac{\alpha_i \tau \beta_i}{\tau\beta + \rho_i - \bar{\rho}}$ is a constant. In our simulations with two types of time preference, ρ_a and ρ_b (let $\rho_a < \rho_b$), we can further write out $\bar{\rho} = \rho_a f_a^c + \rho_b f_b^c$. Therefore, we have

$$\begin{aligned}
\bar{\rho} &= \rho_a f_a^c + \rho_b f_b^c = \rho_a f_a^c + \rho_b \left(1 - f_b^c \right) \\
&= \rho_a \frac{\alpha_a \tau \beta_a}{\tau\beta + \rho_a - \bar{\rho}} + \rho_b \left(1 - \frac{\alpha_a \tau \beta_a}{\tau\beta + \rho_a - \bar{\rho}} \right).
\end{aligned}$$

Rearranging, we have $\bar{\rho}^2 - (\tau\beta + \rho_a + \rho_b) + [\alpha_a \tau \beta_a (\rho_a - \rho_b) - \tau\beta\rho_b + \rho_a\rho_b] = 0$, for which $\bar{\rho} = \frac{(\tau\beta + \rho_a + \rho_b) - \sqrt{(\tau\beta + \rho_a + \rho_b)^2 - 4 \cdot [\alpha_a \tau \beta_a (\rho_a - \rho_b) - \tau\beta\rho_b + \rho_a\rho_b]}}{2} \in (\rho_a, \rho_b)$ is the only economically meaningful solution.

A.8   Stock Market Participation Data

The data on stock market participation rates come from the following countries: Finland,[30] Germany, Norway,[31] and the United States. A summary of the data is shown in Table 7. We define a participant in the stock market as an individual or household that has direct or indirect holdings of stocks in a given year. Using income data, a participant in the stock market is defined as an individual or household that receives dividends in a given year. The participation rates for the United States are from the Internal Revenue Services, rates from Finland and Norway are from the tax authorities covering all the residents with a tax obligation, and are recorded at the year end. We also supplement with survey-based data for the United States and use survey-based data from Germany.[32]

Country	Measure	Time-series	Frequency	Entry and exit	Data Source
Finland	Portfolio	2004 - 2016	Annual	Yes	Statistics Finland
Germany	Survey	1997 - 2022	Annual	No	Deutsches Aktieninstitut
Norway	Portfolio	1999 - 2022	Annual	Yes	Statistics Norway
US	Dividend	1990 - 2022	Annual	No	Internal Revenue Services
	Survey	1989 - 2022	Triannual	No	Survey of Consumer Finances
	Survey	1998 - 2023	Monthly	Yes	Michigan Surveys of Consumers

Table 7: **Participation Data.** Participation rates are based on registry data, dividends reported on tax returns, and surveys. Entry and exit rates are available through registry data and the Surveys of Consumers of the University of Michigan.

We detrend participation rates using country-specific time trends, to remove any potential effects due to changes in participation costs, tax treatment, and demographic compositions. This removes any significant linear time trend, while keeping the time-series mean participation rate unchanged. Specifically, we remove time-trends for Finland, Norway and the United States, while there appears to be no significant trend in Germany.[33]

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[30]Data for Finland are compiled by Breitkopf, Knüpfer, and Rantapuska (2021) using data from Statistics Finland.

[31]We commissioned Statistics Norway for annual participation rates. In the Norwegian data, we define a participant in the stock market as an individual who reports holdings in shares in equity mutual funds in a given year, to isolate the exposure to the stock market. The Norwegian tax authorities impose a wealth tax and require reporting any taxable value of the following four categories: (1) listed and unlisted Norwegian shares, bonds, equity certificate and options registered in Central Securities Depository (VPS); (2) those not registered in VPS; (3) shares in equity mutual funds; and (4) bonds and money market funds. (The first two accounts are related to direct holding of securities. However, we cannot isolate equity exposure from them.)

[32]The data for Germany are annual aggregates of 12 monthly waves, and thus we shift dating to mid-year.

[33]The original and the detrended time-series of participation rates for Finland, Germany, Norway and the United States are in Table 13 in Subsection A.12 in the Online Appendix.

For Finland and Norway,[34] we also have data on entry and exit from the stock market. Entry and exit are constructed from stock or portfolio holdings at the year-end. Investors are regarded as entering if they hold stocks or mutual funds at the end of the current year but did not hold any stocks or mutual funds at the end of the previous year. Conversely, investors are regarded as exiting if they owned stocks or mutual funds at the end of the previous year but do not own stocks or mutual funds at the end of the current year.[35]

For the United States, we rely on the dividend income tax as measure of stock market participation because it covers the general population. We supplement the data with Survey of Consumer Finances and the Michigan Survey.[36]

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[34]For Norway, entry and exit data for the year 2016 are missing due to change of tax treatment on equity and bond mutual funds.

[35]The original and the detrended time-series of entry and exit rates for Finland and Norway are Table 14 in Subsection A.12 in the Online Appendix.

[36]Survey of Consumer Finances is conducted every three years, and we use aggregate participation rate time-series by age groups from 1989 to 2022. For the Michigan Surveys of Consumers see Section 2.

A.9 Asset Pricing Moments

Table 8 summarizes the mean and standard deviation of standard asset pricing quantities and correlations between the two shocks and the market price of risk and shocks to output and the diffusion coefficient of the stock market.

| | Asset pricing quantities | | | | | |
| | Complete Market | | Reentry | | Mix | |
	Mean	Std	Mean	Std	Mean	Std
θ_t	0.0330	0.0761	-0.0029	0.0947	0.0458	0.0841
r_t	0.0354	0.0025	0.0367	0.0031	0.0350	0.0028
μ_t^S	0.0365	0.0003	0.0366	0.0001	0.0366	0.0001
σ_t^S	0.0330	0.0013	0.0329	0.0005	0.0330	0.0007
Correlations						
$\text{Corr}(dz_t^Y, \theta_t)$	-0.0812		-0.0744		-0.0793	
$\text{Corr}(dz_t^Y, \sigma_t^S)$	-0.0975		-0.0768		-0.1107	
$\text{Corr}(dz_t^{SI}, \theta_t)$	-0.0975		-0.0768		-0.1107	

Table 8: **Mean and standard deviation of the asset pricing quantities.** The Table shows the mean and standard deviation of the market price of risk θ_t, the real short rate of interest r_t, as well as the drift μ_t^S and diffusion σ_t^S of the stock return, for the complete market benchmark, the *reentry* scenario, and the *mix* scenario. The 2nd section shows the average covariance coefficients between shocks to the fundamental dz_t^Y and θ_t, dz_t^Y and σ_t^S, shocks to the signal dz_t^{SI} and θ_t, the consumption share of participants $\bar{\Phi}_t$ and the participation rate P_t, and the wealth share of participants $\tilde{\Phi}_t$ and P_t. To compute the averages, we simulate the economies to generate data based on 10,000 simulations, each with 6000 periods or 500 years. For parameters see Subsection 4.1.
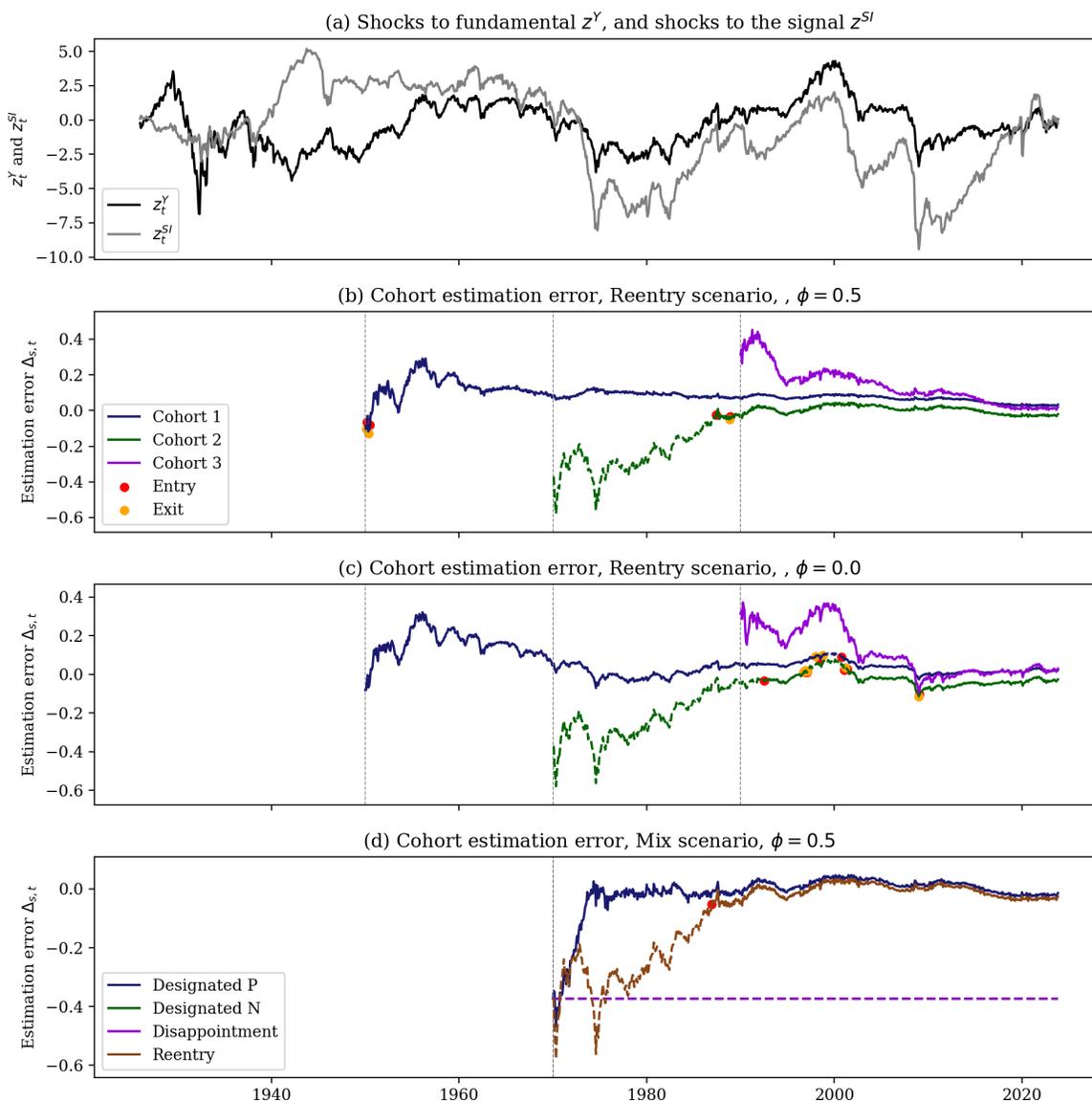
A.10 Implied Shocks

Figure 11: **Shocks to the Fundamental and to the Signal.** The plots show the implied joint path of shocks to the fundamental and to the signal for Finland, Germany, Norway and the US economy. For details on implied shocks see Subsection 4.3.
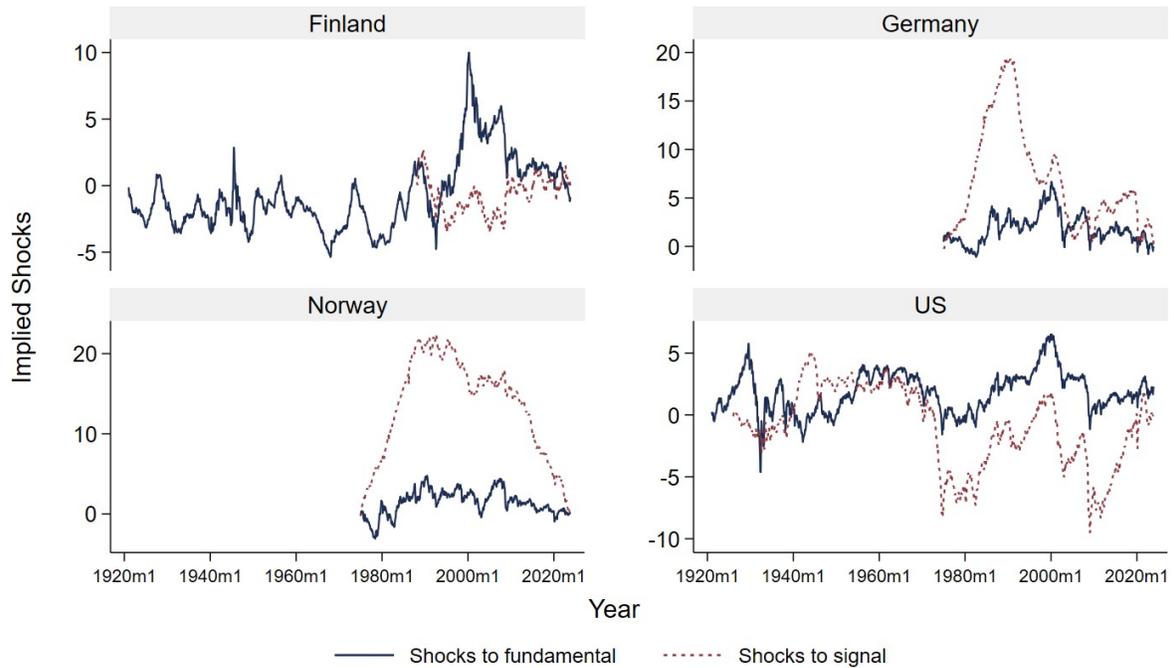
A.11 Cohort Estimation Error II

Figure 12: **Estimation Error Across Joint Path of Output and Signal II.** The figure complements Figure 3. The top plot show the implied joint path of shocks to the fundamental and to the signal for the US economy. The middle plots show the standardized estimation error $\Delta_{s,t}$ of three cohorts in the *reentry* scenario, with $\phi = 0.0$ and $\phi = 0.5$, respectively. The cohorts are born in the years 1950, 1970, and 1990, marked by dotted vertical lines. The bottom plot shows $\Delta_{j,s,t}$ for designated participants, disappointment type, and reentry type born in the year 1950 in the *mix* scenario, with $\phi = 0.5$ and equal density on types. Solid time-series lines denote times when a cohort participates in the stock market, whereas the dotted lines represent times when a cohort does not participate, where red dots indicate the timing of a switch from nonparticipant to participant, and orange dots indicate the timing of a switch from participant to nonparticipant. For parameters see Subsection 4.1. For implied shocks see Subsection 4.3.

(a) Shocks to fundamental z^Y, and shocks to the signal z^{SI}

(b) Cohort estimation error, Reentry scenario, , $\phi = 0.5$

(c) Cohort estimation error, Reentry scenario, , $\phi = 0.0$

(d) Cohort estimation error, Mix scenario, $\phi = 0.5$

A.12 Participation Rate Time-series

In Figure 13, we present the original and the detrended time-series of participation rates for Finland, Germany, Norway and the US.

And in Figure 14, we present the original and the detrended time-series of entry and exit rates for Finland and Norway.

Figure 13: **Time-series of Stock Market Participation Rate - Raw and Detrended.**
The top plots show the time-series of participation rate in Finland, Germany, Norway and the
US. The bottom plots show the detrended time-series of participation rate in these countries.
We detrend any significant country-specific time trends, and then add back the time-series
average. We remove significant time-trends in Finland, Norway and the US. The gray (red)
shaded areas are the years when the total stock returns are lower than -5% (higher than
30%).

(a) **Raw participation rate time-series**
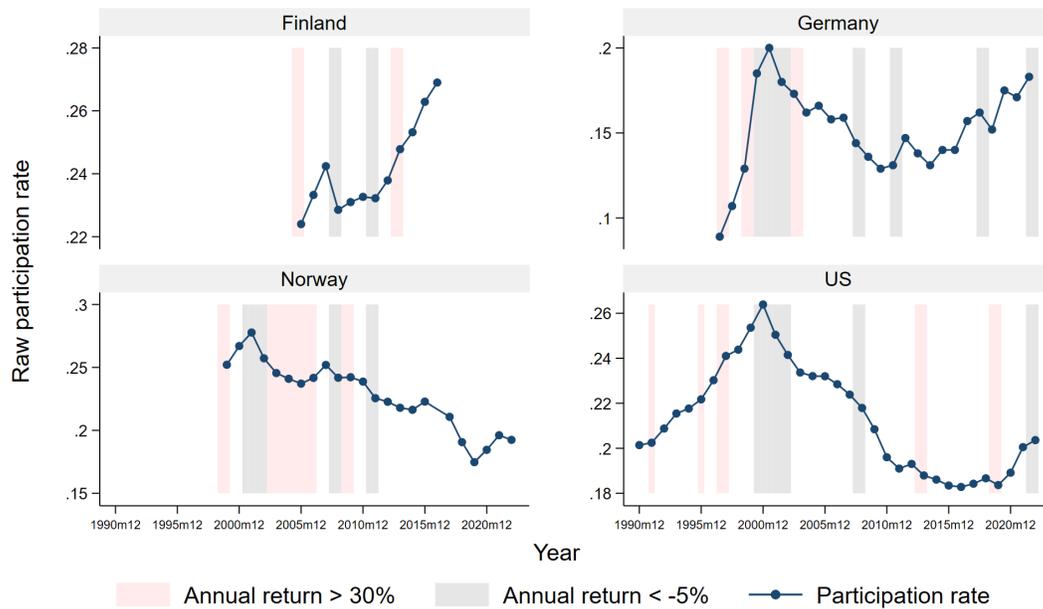
(b) **Detrended participation rate time-series**
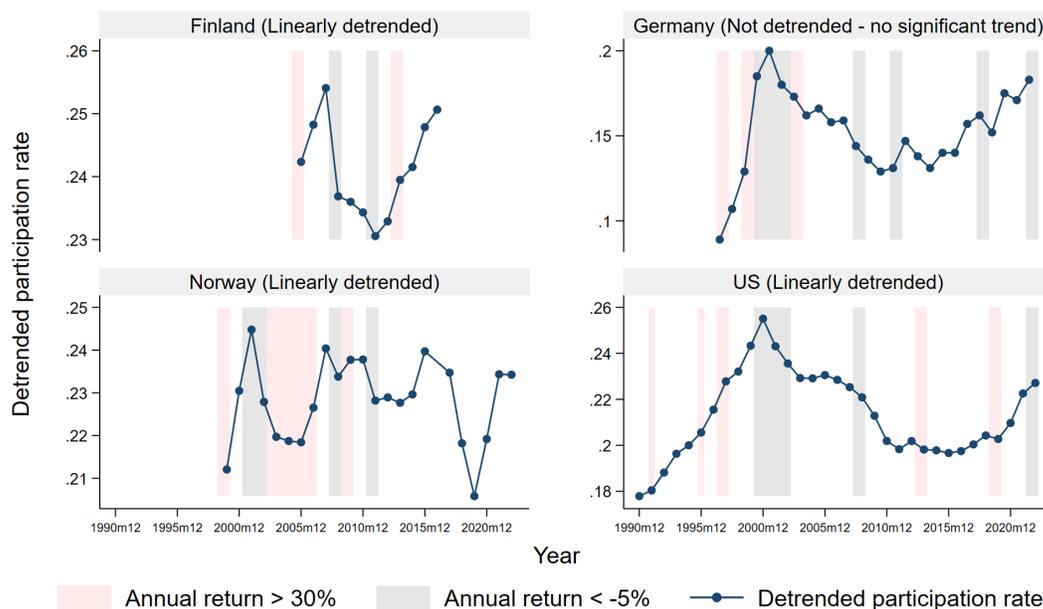
Figure 14: **Stock Market and Entry and Exit.** The top plots show the time-series of entry rate in the stock market in Finland and Norway. The bottom plots show the time series of exit rate from the stock market in Finland and Norway. The gray shaded areas are the years when the total stock returns are lower than -5%, and the red shaded areas are the years when the total stock returns are higher than 30%. Comparable entry and exit rate in Norway for the year 2016 is missing in the tax registry due to changes in the tax definition in that year.

(a) **Entry rate in Finland and Norway**
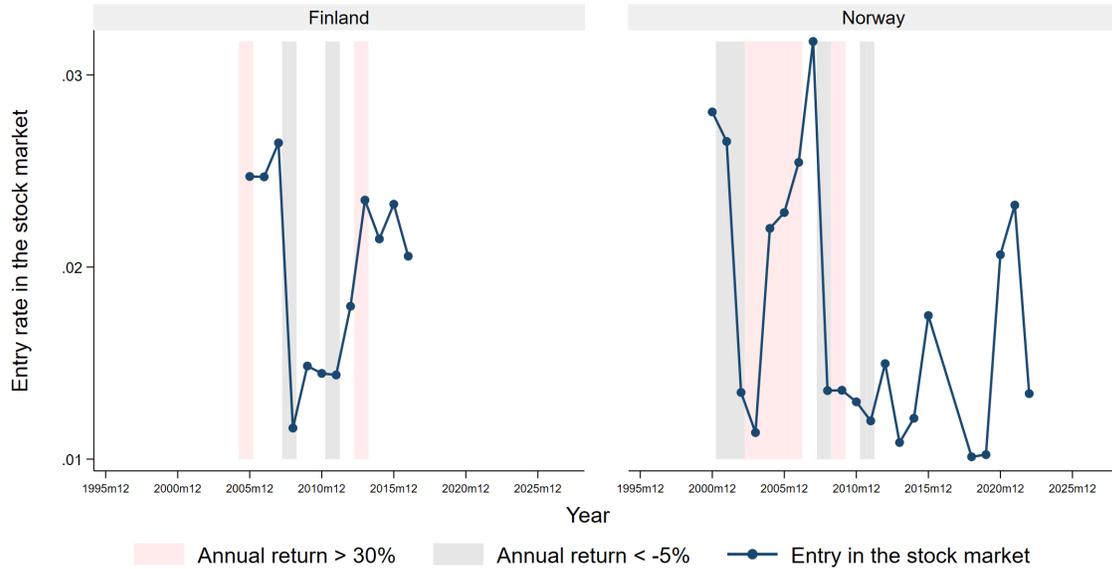
(b) **Exit rate in Finland and Norway**
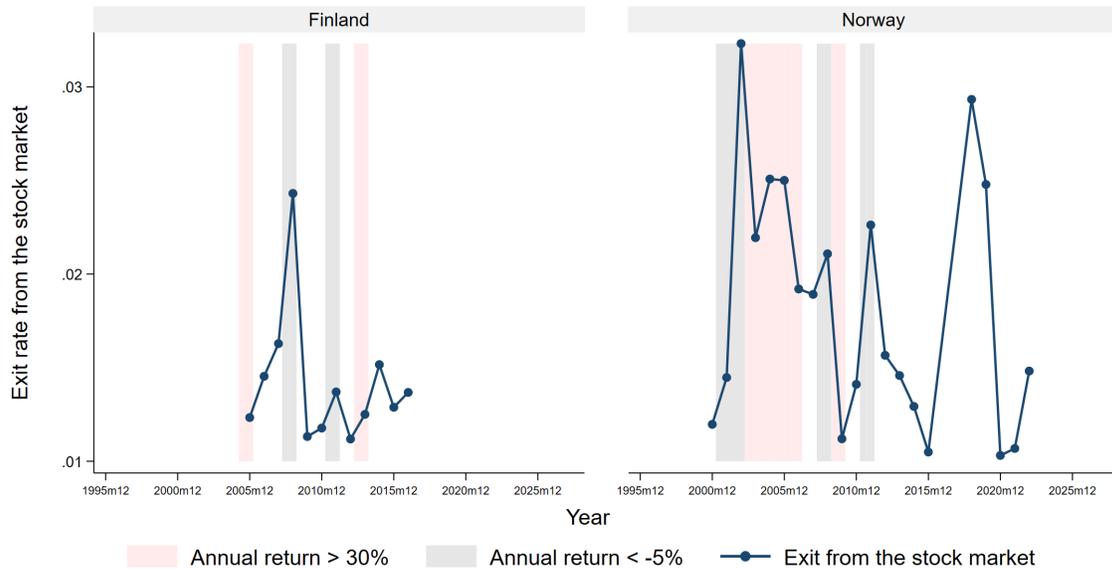
A.13 Alternative Measures of Participation Rate

Figure 15: **Alternative Measure of Participation Rate from Simulation - Annual Average.**
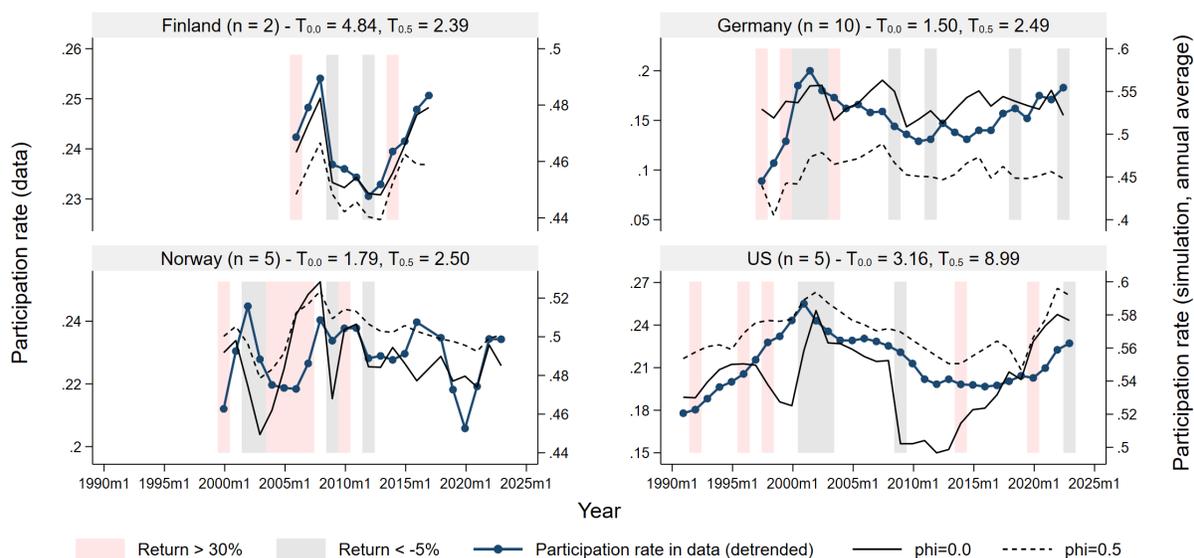
Figure 16: **Alternative Measure of Participation Rate from Simulation - Dividend-based.** In simulation, we define participants as agents who have received dividend or who have been in the stock market at any point within the past 12 months. This definition is analogous to the tax measure in the US, which is based on dividend income.
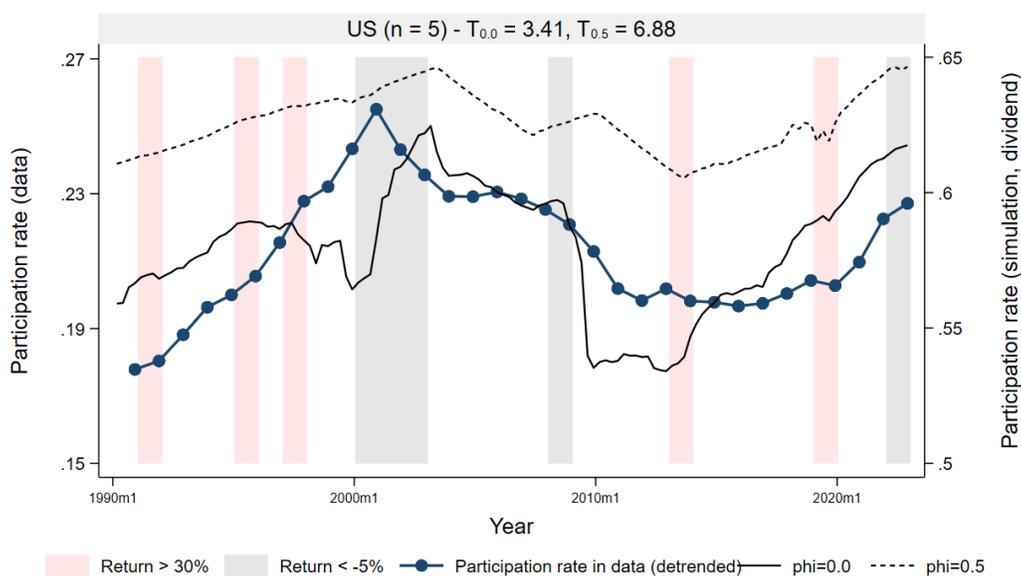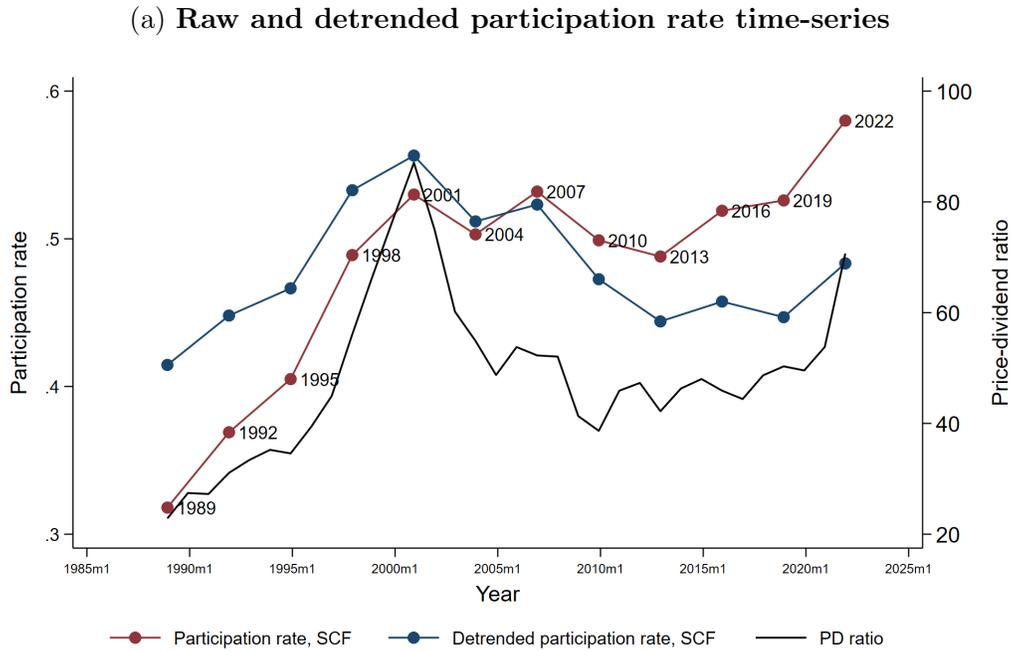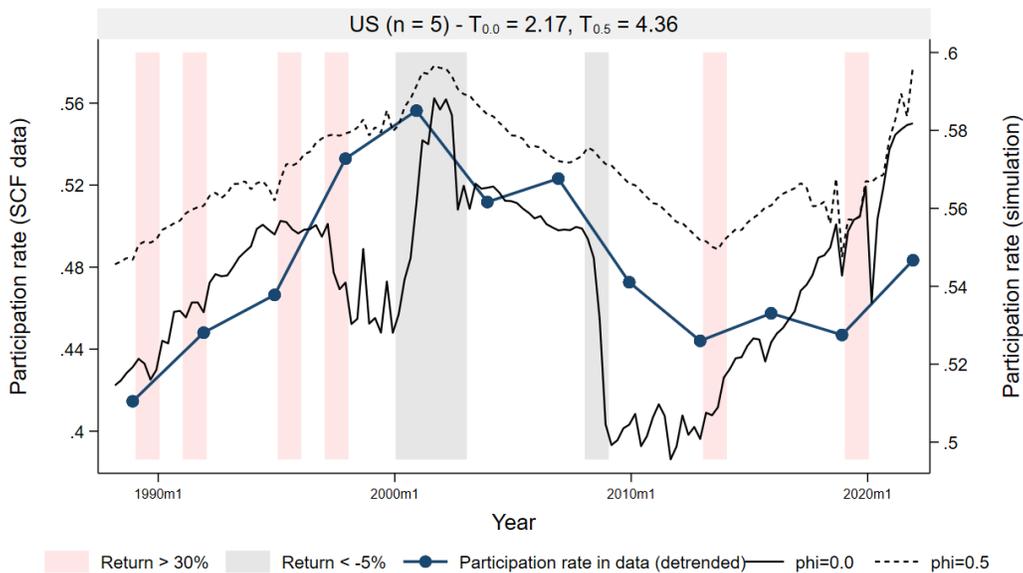
Figure 17: **Alternative Measure of Participation Rate in the US - Survey of Consumer Finances.** We use participation rate from Survey of Consumer Finances as a measure for the US.

(a) **Raw and detrended participation rate time-series**

(b) **Participation rate, Data versus Model.**

For the Michigan survey, we present the time series of net entry rate (entry rate - exit rate) and compare to model implied dynamics in Figure 18. With a perfectly balanced panel dataset, changes in participation rate are fully driven by net entry rate. In our model with

overlapping generations, net entry rate is also highly correlated with participation rate. For the Michigan survey, however, the differences between the two quantities are large: net entry rate compares the entry and exit from the same sample across two rounds of interviews, while participation rate is calculated over a different sample in each month. Therefore, we consider the net entry rate to be less subject to changes in participation driven by sampling reasons and to better represent the dynamics of participation rate described in our model.

Figure 18: **Alternative Measure of Participation Rate in the US - Michigan Survey of Consumers.** The figure shows the time-series of net-entry rate in the US (blue solid lines with dots), juxtaposed with simulated net-entry rate of the model based on the *mix* scenario using $\phi = 0.0$ (solid lines) and $\phi = 0.5$ (dashed lines), where n is the length of the initial window for the prior belief and T_ϕ is the t-statistics on the actual net-entry rate from regressing the model net-entry rate on a constant and the actual net-entry rate. For parameters, see Subsection 4.1. To account for the excess time-series volatility of net entry rate from the survey, net entry rate data are the moving average from lagged 6 months to forward 6 months, and we consider the timing of entry and exit actions at the middle of the 6-month interval between interviews. For implied shocks, see Subsection 4.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.
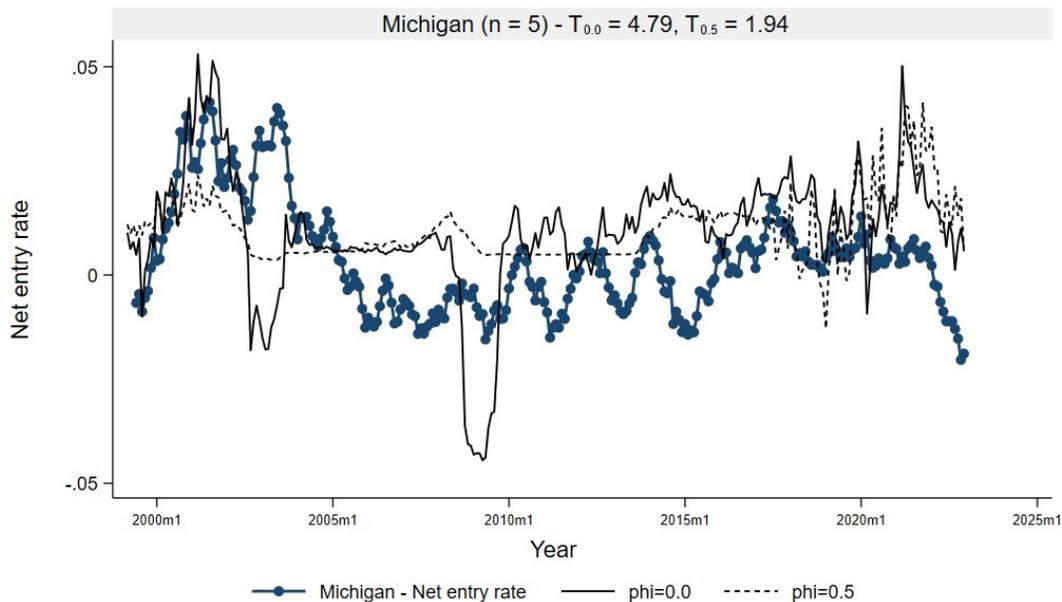
A.14   Experience and Participation

We compute model implied differences in participation rates between the old and young (red arrows) to juxtapose these with the differences in participation rates based on data from SCF (blue squares). Specifically, regressing the difference in participation rate between old and young from the simulation on that from the data produces a t-statistic of 3.84, and R-squared of 40%.

Figure 19: **Experience and Participation Rate - Data versus Model, Survey of Consumer Finances.** The plot shows the model implied (in red) and the Survey of Consumer Finances (SCF, in blue) based difference in stock market participation rates between the old and young based on experienced returns. The bottom plot also shows the fitted relation between differences in experienced returns and differences in participation rates. We proxy for the difference in experienced stock market returns by taking the (time-series) average stock market return over the prior 50 years minus the return over the prior 20 years. For SCF survey, old refers to households over 75 years old, and young refers to those below 35. In the simulation, we use the same cutoffs, where agents that are 35 years old have 15 year of experience. The calendar years refer to the respective SCF survey waves. For parameters see Subsection 4.1. For implied shocks see Subsection 4.3. Agent types (reentry, disappointment, designated participants, and designated nonparticipants) have 25% weight at birth.
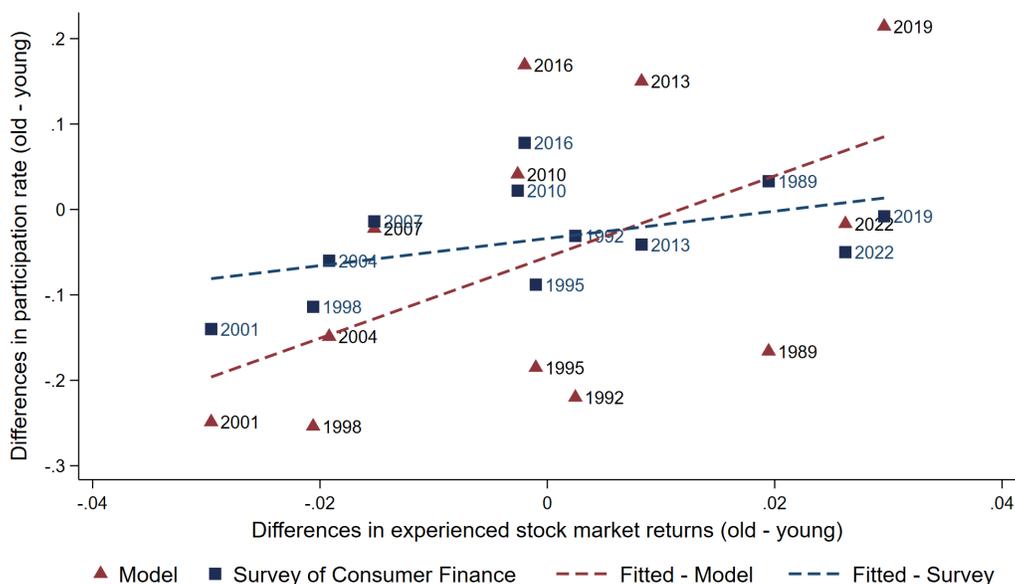
A.15   Robustness Tests

Table 9 shows predictive regressions that include the price-dividend-ratio on top of the participation rate as explanatory variables.

	Total returns			Excess returns		
	(1) 12m	(2) 24m	(3) 36m	(4) 12m	(5) 24m	(6) 36m
Participation rate	-0.340***	-0.330***	-0.268***	-0.362**	-0.351***	-0.290***
	(0.118)	(0.081)	(0.062)	(0.138)	(0.099)	(0.085)
Price-dividend ratio	-0.056	-0.209	-0.273**	-0.086	-0.262*	-0.347**
	(0.124)	(0.125)	(0.124)	(0.135)	(0.144)	(0.146)
N	94	91	88	94	91	88

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 9: **Participation Rate and Future Returns - Data versus Model.** The table shows predictive regressions with 12, 24 and 36 month horizons, with detrended participation rate and the price-dividend ratio as explanatory variables for Finland, Germany, Norway and the US, where columns (1) - (3) show regressions with total returns and columns (4) - (6) use excess returns. Interest rates are 12-month T-bill rates from Bloomberg for Finland, Bundesbank for Germany, Statistics Norway for Norway, and Amit Goyal's website for the US. Price-dividend ratio are calculated using rolling 12-month returns from Ken French's website. All the variables are standardized. The panel regressions include country fixed effects and use Driscoll-Kraay standard error with respectively 2, 4, and 6 lags.