Endogenous Misinformation and Source Authentication

Abstract

We study endogenous information provision and source authentication when secondary senders can copy primary senders' signals, providing a microfoundation for correlation neglect. Authentication mitigates this duplication bias but hinders information diffusion, creating ambiguous effects on misinformation and welfare. Crucially, we show that policies aimed at maximizing user welfare can be fundamentally misaligned with the goal of minimizing misinformation. Non-verification can be optimal when diffusion is highly valued or primary senders hold strong bargaining power. While factors like signal quality reduce misinformation under exogenous verification, the effects are uneven when verification is endogenous. We also examine intellectual property protection and self-regulation, consistently highlighting a core trade-off between information accuracy and diffusion in shaping platform policy and welfare outcomes.

JEL Classification: C72, C81, D82, D83.

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1 Introduction

We live in an age of information explosion. Individuals are less dependent on mainstream media, less prone to content censorship, and more effective at getting their voice heard digitally by a larger audience. Meanwhile, the constant flow of information from self-proclaimed experts, charismatic influencers, digital social networks, etc., stretch our attention. Many information providers care little about truth or intellectual properties and copy contents and insights created by others with twists in writing and presentation style to appeal to the masses. Social media platforms and regulators also seem to face a critical dilemma: When the source and quality of the information are not carefully authenticated, in the name of freedom of speech and digital inclusion, it can lead to duplicated information, misinformation from copycats' noise, and misaligned incentives in content creation.

Against this backdrop, this paper studies information provision and optimal authentication of information sources in the presence of second-hand information manufacturers and data copycats. An information receiver — be it an econometrician or a fund manager — may obtain information from multiple senders. The decision of how much to authenticate the source of information with commitment is fundamentally an information design problem. Verifying data or news can help the receiver properly weigh the information and avoid duplication bias. However, it may also reduce copycat senders' incentive for searching for information and relaying it to a broader audience. Intuitively, the receiver does not always prefer committing to authenticating information sources. Our model essentially underscore a key, underexplored tension in this debate: how does the ability of information to spread through a digital network interact with a platform's incentive to verify its source? We show that, paradoxically, when information is expected to 'go viral,' a platform may have less incentive to authenticate it, leading to a greater prevalence of misinformation.

The tradeoff is not centered on how informative the senders' signals are (e.g., Habu, 2023)—since the receiver cannot determine this without verification—but rather on balancing the avoidance of duplication bias with the benefits of receiving more signals in the aggregation process. This economic trade-off immediately gives rise to what appears ex post as a behavioral bias but can ex ante rational — correlation neglect. It also bears relevance in the amount of misinformation in aggregate, which the World Economic Forum (2024) identifies as the biggest short-term risk for the next decade. Does more verification always reduce misinformation? How does intellectual property (IP) protection in the digital age affect total welfare and societal misinformation? We address these questions by developing a new, parsimonious model in which receivers rationally authenticate information sources when primary data providers and second-hand data manufacturers coexist to endogenize data quality.

Specifically, our baseline setup features two senders (S1 and S2) and two receivers (R1 and R2). It is commonly known that S1 is endowed with an informative signal and thus serves as a primary source. With a positive probability, S2 does not have a signal and can exert costly efforts to search for information and produce second-hand information. S2 cares about receiver subscriptions and breaks even if and only if both receivers subscribe to him. To illustrate the main economic forces, we consider a simple network in which S1 and R1 are frictionlessly connected, reflecting how some agents traditionally enjoy advantageous access to information, while both receivers can potentially subscribe to S2's information, representing emerging digital media with broader outreach at low costs.

We also introduce a regulator, R_o, whose payoff equals the sum of both receivers' payoffs in the baseline setup and who can verify the authenticity of information sources. The interaction unfolds in sequential stages. First, the representative receiver R_0 selects the optimal verification rule. Second, primary senders choose signal quality, secondary senders decide whether to replicate signals, and receivers choose whether to subscribe. Third, receivers act upon the information they receive. Finally, all payoffs are realized. To capture the endogenous choice of signal quality, we incorporate multilateral bargaining to model the pricing between S1 and R1.

We find that non-verification can be optimal, allowing correlation neglect to emerge as an equilibrium outcome in a fully rational environment. First, *diffusion-driven non-verification* arises when the demand for information diffusion is sufficiently high, access to primary sources is limited, and the noise in second-hand information is small. Intuitively, when R2's decision carries more weight (e.g., due to a larger investment opportunity) and S2 is likely a secondary source that preserves information content during duplication, verifying authenticity reduces the reach of information and is thus suboptimal. Second, *bargaining-driven non-verification* occurs when receivers strategically avoid verification to counter the market power of primary senders. Retaining more senders through non-verification enhances competition and improves receivers' bargaining positions. When S_1 holds significant bargaining power, non-verification becomes optimal regardless of the importance of diffusion. Moreover, increases in sender bargaining power always expand the range of conditions under which non-verification is preferred. Third, concerns about *information quality* encourage more frequent verification to incentivize investment in higher quality signals from primary sources. However, this effect arises only at intermediate levels of investment cost. When the cost is too high or too low, senders' incentive to improve signal quality becomes insensitive to the verification rule.

We identify two key mechanisms driving this result: a 'bargaining-driven' effect where the platform uses non-verification to extract rents from the sender, and a 'diffusion-driven' effect where non-verification can act as a commitment to wider information sharing.

These findings culminate in a key, policy-relevant takeaway: the objective of maximizing receiver welfare is not synonymous with minimizing misinformation. Our framework reveals the precise conditions under which a platform, acting in the best interest of its users' utility, would rationally choose a policy that permits more misinformation to circulate. This speaks directly to the ongoing debate about platform regulation, cautioning that a narrow focus on user engagement metrics may come at the cost of information quality.

Our equilibrium analysis provides key insights into welfare and misinformation. First, total welfare is defined as the sum of the regulator-weighted receiver payoffs and the senders' payoffs. The optimal verification rule may become socially suboptimal due to externalities in sender utilities and endogenous signal quality. While perfect verification improves precision and benefits primary senders, it may discourage participation of secondary senders, resulting in either overly strict or overly lenient verification. This generates a non-monotonic welfare response to parameters, depending on the utilities generated for both senders. In particular, stronger verification can reduce welfare by suppressing valuable duplication, especially when access to primary sources is limited and the value of diffusion is high. These tensions complicate the design of optimal verification policies and highlight trade-offs among accuracy, diffusion, and market power.

Second, aggregate misinformation is defined as the average belief error between receivers' inferred beliefs and the true economic state. Consistent with empirical findings (Pennycook et al., 2021), higher signal quality, better access to primary sources, and less noise in data duplication reduce misinformation under exogenous verification. However, when verification is endogenous, its effect on misinformation is ambiguous and uneven across receivers: it reduces misinformation for R1, who can better assess authenticity, but increases it for R2, who lacks access to high-quality data. Notably, misinformation does not align with total welfare or the regulator's objective. Non-verification may increase misinformation when S2 is likely to be a primary source or when duplication noise is high. As diffusion becomes more important, the optimal rule often shifts toward non-verification, but the resulting level of misinformation may rise or fall depending on key parameters such as the probability of S2 being a primary source and the degree of noise in duplication.

Our analysis extends to sender authentication and self-regulation. We first study IP protection in which primary senders can sue copycats at a litigation cost. Since copying only occurs under non-verification, IP enforcement and verification policy are closely intertwined. When information cost is high, IP protection influences litigation and verification behavior but not signal quality; moderate protection leads to wasteful litigation, while excessive protection triggers perfect verification, reducing diffusion and potentially harming welfare. With moderate information costs, stronger IP protection shifts bargaining power to primary sources, incentivizing investment in higher-quality signals. However, it also limits access to duplicated content, narrowing user choice and potentially reducing welfare. Overall, the welfare effect of IP protection is non-monotonic, and its impact on misinformation is ambiguous.

Furthermore, under sender self-regulation, a sender-side regulator chooses whether to verify secondary sources, balancing the interests of primary and secondary senders. When diffusion is valuable and information costs are either too high or too low, non-verification is optimal, particularly when primary senders have weak bargaining power. With moderate costs and endogenous signal quality, verification may be used to support quality investment—provided the benefits exceed the diffusion loss. Overall, the same trade-off arises between curbing duplication bias and promoting diffusion, with non-verification favored when it supports secondary senders' revenues and broader information access. In addition, we also study a general regulator who assigns welfare weights to all senders and receivers. Non-verification is optimal when grassroots users (i.e., secondary senders like S2 and peripheral receivers like R2) receive sufficiently high welfare weights compared to elites (i.e., primary senders like S1 and core receivers like R1).

Finally, our results are robust when we introduce general information design and large networks. First, we study a general verification design where R0 can flexibly choose posterior beliefs, introducing a positive subscription cost for R1. Under mild assumptions, R1's subscription follows a cutoff rule, and the optimal verification generally involves partial verification to balance R1 and R2's incentives. Perfect verification is never optimal when information diffusion is sufficiently important. Second, in large networks, when senders are abundant relative to receivers, when each receiver has relatively a small number of neighbors, and when connections are formed randomly, the duplication bias becomes less of a concern, and the optimal verification strategy is not to verify. However, if one of these conditions fails to hold, duplication bias becomes significant, and we must solve a trade-off problem similar to the one in the benchmark model. In addition, two real-world applications are discussed regarding expert vs. charlatans in policy advising and information design transmission on digital platforms.

Literature. Our paper is closely related to studies on information design and strategic communication, especially the multiple-sender framework (DeMarzo et al., 2003b; Kamenica and Gentzkow, 2011, 2017; Li and Norman, 2021). Kamenica and Gentzkow (2017) explore how competing or complementary senders strategically disclose information to influence a receiver's decision, while Li and Norman (2021) examine sequential information disclosure in a dynamic setting. In contrast, we consider multiple senders with different levels of authenticity. As such, our paper adds to the literature on credibility, reputation, and source reliability (Ottaviani and Sørensen, 2006; Kartik et al., 2007; Kartik, 2009; Sobel, 2013; Ben-Porath et al., 2014). Specifically, Ottaviani and Sørensen (2006) explore how senders may provide redundant information to signal their credibility. Kartik et al. (2007) provide insights into how receiver beliefs might depend on sender authenticity, and Kartik (2009) explore how lying costs affect strategic communication. Sobel (2013) discusses how receivers might evaluate the credibility of advisors and aggregate advice from multiple sources, while Ben-Porath et al. (2014) explore the idea of costly verification in a multi-agent setting. We differ primarily by allowing for costless verification and multiple senders. The idea of avoiding duplication bias by verifying sender authenticity is related to the literature on information aggregation and redundancy in multi-sender settings (Prendergast, 1993; Morris and Shin, 2002; Veldkamp, 2006; Golub and Jackson, 2010b, 2012). Golub and Jackson (2010b) study how redundancy and correlation across sources affect information aggregation in a network setting. Veldkamp (2006) highlight the role of information duplication and explore how information aggregation in financial markets can lead to correlated asset prices. To avoid duplication bias, receivers must optimally weight information from different sources (DeGroot, 1974b; Acemoglu et al., 2011), and improper weighting of redundant information can drive herding behavior (Banerjee, 1992; Bikhchandani et al., 1992). Unlike these papers, we model the authentication of information sources.

Our paper also relates to the literature on correlation neglect, a bias where individuals treat correlated signals as independent, leading to distorted beliefs and suboptimal decisions in environments with redundant information. DeGroot (1974a) introduced early ideas on belief aggregation, and DeMarzo et al. (2003a) formalized the persuasion bias from repetition. Recent work models correlation neglect in social networks (Golub and Jackson, 2010a; Ortoleva and Snowberg, 2015; Levy and Razin, 2015, 2018), with Levy and Razin (2015) showing that, under some conditions, bias may improve group outcomes despite individual voting errors. Levy et al. (2022) show how senders can exploit correlation neglect to influence beliefs, while Klümper and Kräkel (2020) study similar dynamics in a principal-agent context. Furthermore, Enke and Zimmermann (2019) provide experimental evidence that people double-count repeated information. Epstein and Halevy (2019) study correlation ambiguity, where individuals are uncertain about event dependencies, extended by Levy and Razin (2022) in complex settings. Additional experimental work includes Bolte and Fan (2024), Hossain and Okui (2024), and Exley and Kessler (2024). The bias has been applied to school choice (Rees-Jones et al., 2020; Jiang et al., 2024), patenting disparities (Cong and Yang, 2025) and financial investment (Eyster and Weizsacker, 2016; Fedyk and Hodson, 2023). It also helps explain echo chambers, where repeated like-minded messages are mistakenly treated as independent, reinforcing, and polarizing beliefs (Levy and Razin, 2019). Unlike the existing literature that treats correlation neglect as a behavioral bias, our model features rational agents who fully anticipate correlated information ex post and deliberately avoid source authentication to enhance information availability. In this way, we add to the literature a rare rational micro-foundation for correlation neglect.

Finally, we complement empirical work on misinformation along the dimensions of verification, market power, and data quality (Tucker et al., 2018; Benkler, 2018). First, Allcott and Gentzkow (2017) examines how misinformation spreads on social media and highlights the role of verification (or lack thereof) in enabling the dissemination of false information. Pennycook and Rand (2019) show that crowd-sourced verification of news sources can reduce the spread of misinformation. Second, Gentzkow and Shapiro (2008) explores how competition among news providers affects the accuracy of information and Mullainathan and Shleifer (2005) discuss how media bias arises from the preferences of consumers and the incentives of news providers. Third, Vosoughi et al. (2018) finds that false news spreads faster and more widely than true news, in part due to lower quality standards for false information, and Lazer et al. (2018) calls for research on how to improve information quality to combat misinformation. Our paper provides a formal analysis of how information provision and authentication affect misinformation, complementing empirical studies on the role of verification, market power, IP protection, and data quality in reducing false information.[1]

2 A Model of Source Authentication

While more general network and information structure are considered in Section 5, our parsimonious baseline model highlights core economic tradeoffs arising from the following fact: Only a subset of agents in the economy can receive information from primary sources; others acquire information indirectly.

2.1 The Baseline Model

There are two information sources (senders), S1 and S2, two information users (receivers), R1 and R2, and a regulator, R0, all risk-neutral. S1 is a commonly known primary (a.k.a. first-hand, "FH") information source, while S2 is an FH information source with probability $\lambda \in (0, 1)$, and otherwise a second-hand ("SH") information source.

States of the economy and signals. The economic state θ is drawn from the state space $\Theta = \{0, 1\}$ with $\Pr(\theta = 1) = \Pr(\theta = 0) = 1/2$. FH sender S1 is endowed with a signal $s_1 \in \{h, l\}$ with probability $\Pr(s_1 = h|\theta = 1) = \Pr(s_1 = l|\theta = 0) = \rho \in (1/2, 1)$.

Sender S2, if FH, independently draws a signal s_2, also symmetric with precision ρ. Otherwise, an SH S2 searches for information about s_1 and replicates with success probability α:

$$s_2 = \begin{cases} s_1, & w.p. \quad \alpha \\ h, & w.p. \quad \frac{1-\alpha}{2} \\ l, & w.p. \quad \frac{1-\alpha}{2} \end{cases} . \tag{1}$$

Here, when $\alpha = 1$, s_2 perfectly copies s_1; when $\alpha < 1$, s_2 adds noise with probability $(1-\alpha)$.

[1] Another closely related reference, Habu (2023), models strategic ignorance of information sources, that is, receivers may not always want to verify information sources. See Golman et al. (2017) for a survey on information avoidance. However, the mechanism is different. In our model, verification is not based on how informative the sender's signal is, and the trade-off is really between avoiding duplication bias in information aggregation versus receiving less signals for information aggregation.

Actions and payoffs. Each receiver takes an action $a \in [0, 1]$ with a quadratic payoff:

$$u_R(a_R, \theta) = -(a - \theta)^2. \tag{2}$$

The quadratic utility function implies that the optimal action is:

$$a^*(\mathcal{I}) = \mathbb{E}_\pi[\theta|\mathcal{I}] = \pi(1|\mathcal{I}), \tag{3}$$

where $\mathcal{I}$ is the receiver's information set and π is the belief of the event $\{\theta = 1\}$.

Sender S2 cares about receiver subscriptions and each subscription generates a revenue of 1. An SH S2 chooses whether to replicate at a cost $c_S \in (1, 2)$ or not.[2] His payoff is:

$$u_{S2}(a_{S2}) = \#\text{subscriptions } - c_S. \tag{4}$$

This captures the fact that an SH information source generates content only if he can at least break even, i.e., is sufficiently popular.

Sender S1 can incur a cost $Q > 0$ to improve the signal precision ρ from $\underline{\rho}$ to $\overline{\rho}$, where $\frac{1}{2} < \underline{\rho} < \overline{\rho} < 1$. The meaning of the parameter ρ will be clear and determined by the context.

Regulator R0 cares about both receivers' utilities.

$$u_{R0} = u_{R1}(a_{R1}, \theta) + \beta_R \times u_{R2}(a_{R2}, \theta) \tag{5}$$

The weight $\beta_R \geq 0$ has a natural micro-founded interpretation as the relative market size or investment scale of the two types of receivers. It also represents the social planner's normative preference for information diffusion (benefiting the peripheral R2) over information accuracy (benefiting the core R1). R0 only cares about R1 when $\beta_R \to 0$ and (only) R2 when $\beta_R \to \infty$.

Notation. Let u and U denote the Bernoulli utility and the expected utility/payoff.

Bargaining and endogenous pricing. We explicitly use a bargaining process to model endogenous pricing between S1 and R1, assuming that S2 cares only about the number of subscriptions. This helps isolate the incentive for signal quality. Specifically:

[2] We discuss S1's costly information production, as opposed to direct endowment, in Section 3.1. Moreover, the assumption that $c_S \in (1, 2)$ can be generalized such that a second-hand sender (S2) can only break even when endorsed by core receivers (like R1) with access to primary senders.

When bargaining with S1, R1 claims that her outside option is to work with S2; therefore, they negotiate only over the incremental profit generated by S1's participation. Likewise, when bargaining with S2, R1 claims her outside option is to work with S1, and negotiates only over any additional profit S2 might bring. Let $\gamma \in (0, 1)$ and $1 - \gamma$ denote the (bilateral) Nash bargaining powers of S1 and R1, respectively.

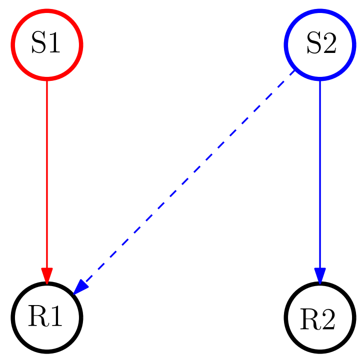


Figure 1: The Network

Network structure. Senders {S1, S2} and receivers {R1, R2} are connected in a network (see Figure 1). Specifically, S1 is directly related to R1, potentially through the trust established in a prior family, social, or professional network. S2, being new in the information market, can reach both R1 and R2. Note that for S2, a wider audience does not immediately generate an interaction. Furthermore, since R1 can always access the primary source from S1, she cares more about the authenticity of S2's signal (duplication bias concern). In contrast, R2 cares about the availability of informative signals (information diffusion concern) in addition to its authenticity. Therefore, $\beta_R \geq 0$ also reflects the relative importance of information diffusion versus duplication bias to the user regulator.

Verification. R0 can verify, to some extent, the type of S2, i.e., the authenticity of the information S2 produces. Absent verification costs, R1 always prefers authenticating S2, but R2 has concerns that verification may ex ante discourage a SH S2 from providing information. Thus, R0 has to balance these competing forces.

Specifically, R0 can publicly generate an authenticity signal $m \in M$ about S2, which is informative about his type $t \in \{\text{FH}, \text{SH}\}$. Given a prior $\lambda \in (0, 1)$ of S2 being FH, verification adjusts the posterior belief of Sender 2's type. Let $\nu(t|m)$ denote the posterior

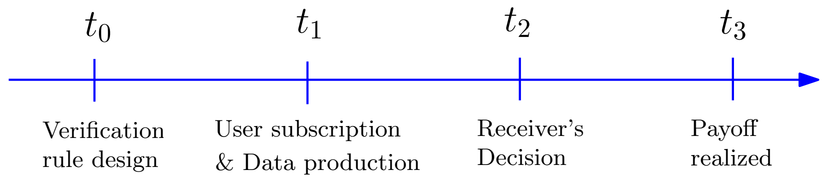


Figure 2: Timeline

belief that S2 is of type t conditional on m. Given binary types, it is without loss of generality to focus on binary signals, i.e., $M = \{m_1, m_2\}$. To enhance tractability, we focus on a specific information structure in the baseline to illustrate the key economic trade-offs. Specifically, R0 can choose $\Pr(m = m_1|t = FH) = \Pr(m = m_2|t = SH) = q$ with $q \geq 1/2$. Information control manifests itself as a higher q leads to a more extreme posterior distribution. Here, $q = 1/2$ means "non-verification" and $q = 1$ means "perfect verification." We analyze the more general information design problem in Section 5.1, only imposing Bayes' rule:

$$\sum_{m \in M} \Pr(m) \times \nu(FH|m) = \lambda. \tag{6}$$

Timeline. Figure 2 represents the four stages of the game. In stage t_0, R0 sets the verification rule with commitment, an information design problem. The stage t_1 corresponds to the user subscription and the production of information by the senders, including S1 endogenously choosing the precision of the signal and S2 choosing whether to copy the information. In stage t_2, receivers make decisions based on the signals offered by the data providers. Finally, all payoffs are realized in Stage t_3.

Equilibrium concept. We use backward induction to solve the equilibrium. The proper equilibrium concept is the perfect Bayesian equilibrium:

(i) Given their information sets, the receivers optimally choose actions according to Equation (3).

(ii) Given the verification rule set by R0, R1 and R2 optimally decide their subscription, anticipating their subsequent optimal choice of actions a^*_{R1} and a^*_{R2}.

(iii) Given the verification rule of R0 and the receiver strategies for subscriptions and subsequent actions, Sender S2 optimally chooses whether to copy S1's signal, and Sender

S1 optimally selects signal quality.

(iv) Given the optimal strategies of the senders and receivers, R0 optimally sets the verification rule q.

(v) Both $\nu(FH|m)$ and $\mathbb{E}[\theta|\mathcal{I}]$ are updated according to Bayes' rule.

2.2 An Illustration

We illustrate the main idea with a simple example where $\alpha = 1$ (i.e., zero noise in the duplication process). We shut down the endogenous signal quality by S1 and consider a discrete verification rule design for two extreme cases of R0's welfare weight.

(i) $\beta_R = 0$. In this case, R0 only cares about Receiver R1's payoff. It is then optimal to always verify the authenticity of S2's signal. By doing so, Receiver R1 perfectly avoids duplication bias, as she knows exactly when the two signals s_1 and s_2 are independent and when they are perfectly correlated.

(ii) $\beta_R = \infty$. In this case, R0 only cares about Receiver R2's payoff. It is then suboptimal to verify the authenticity of S2's signal. Non-verification leads to subscriptions from both receivers, since R1 still receives an informative signal s_2 whenever $\lambda > 0$. In other words, non-verification can help Sender S2 break even, incentivizing information duplication. As a result, Receiver R2 always receives an informative signal, although sometimes duplicated, which represents her best-case scenario.

3 Equilibrium, Welfare, and Misinformation

This section presents equilibrium solutions, aggregate misinformation, and welfare.

3.1 Equilibirum Analysis

This section characterizes the equilibrium analysis when the information cost is high. We solve, in sequence, the receivers' actions and subscription decisions, the senders' optimal pricing and choices of signal quality, and finally, the regulator R0's optimal verification rule design. The case with a relatively low information cost is analyzed in Section 3.1.

Receivers' optimal actions and subscription choices. We begin by analyzing the incentive of Receiver R1 to subscribe to S2. To do so, we compute her expected payoff in two scenarios: when she subscribes only to S1 and when she subscribes to both senders.

First, if Receiver R1 subscribes only to Sender S1 (that is, $\mathcal{I}_{R1} = \{s_1\}$), her posterior belief can be derived using Bayes' rule: $\pi(1|s_1) = \rho \times \mathbb{1}(s_1 = h) + (1-\rho) \times \mathbb{1}(s_1 = l)$. By Equation (3), this leads to an expected utility given by:

$$U_{R1}(\{S1\}) = -\rho(1-\rho). \tag{7}$$

Second, if Receiver R1 subscribes to both senders S2 and S1 (that is, $\mathcal{I}_{R1} = \{s_1, s_2\}$), Equation (8) below outlines how Receiver R1 updates her belief when information sources may be duplicated: she first evaluates the authenticity of the sources, then revises her belief about the economic state. Specifically:

$$\begin{aligned}\pi(1|s_1, s_2) = &\Pr(\text{S2 is FH}|s_1, s_2) \times \Pr(1|s_1, s_2, \text{S2 is FH}) \\ &+ \Pr(\text{S2 is SH}|s_1, s_2) \times \Pr(1|s_1, s_2, \text{S2 is SH})\end{aligned} \tag{8}$$

Define: $\mu_1 = \frac{\lambda\left(\rho^2+(1-\rho)^2\right)}{\lambda(\rho^2+(1-\rho)^2)+(1-\lambda)\times\frac{(1+\alpha)}{2}}$ and $\mu_2 = \frac{\lambda\rho(1-\rho)}{\lambda\rho(1-\rho)+(1-\lambda)\times\frac{(1-\alpha)}{4}}$. Then, a direct application of Bayes' rule implies:

$$\Pr(\text{S2 is FH}|h, h) = \Pr(\text{S2 is FH}|l, l) = \mu_1,$$
$$\Pr(\text{S2 is FH}|h, l) = \Pr(\text{S2 is FH}|l, h) = \mu_2.$$

and $\Pr(\text{S2 is SH}|s_1, s_2) = 1 - \Pr(\text{S2 is FH}|s_1, s_2)$. Here, the posterior probability that Sender S2 is FH depends solely on whether s_1 agrees with s_2, not on the realized value of s_1. This symmetry stems from the symmetric prior over the state and the symmetric noise structure.

Furthermore, the following intuitive observation simplifies the second step of updating the economic state: conditional on S2 being an SH source, the signal s_2 carries no additional information. Formally, as derived in Appendix A.2:

$$\Pr(1|s_1, s_2, \text{S2 is SH}) = \pi(1|s_1). \tag{9}$$

In contrast, when Sender S2 is an FH source, the two signals are independent. Thus:

$$\Pr(1|s_1, s_2, \text{S2 is FH}) = \frac{\Pr(s_1|1)\Pr(s_2|1)}{\Pr(s_1|1)\Pr(s_2|1) + \Pr(s_1|0)\Pr(s_2|0)}.$$

Then, we can plug $\pi(1|s_1, s_2)$ into Equation (3) to compute the expected payoff of Receiver R1 when she subscribes to both senders. Formally, as shown in Appendix A.3:

$$U_{R1}(\{S1, S2\}) = -\frac{(1-\rho)\rho\left(1-\alpha^2(1-\lambda)^2 - 2\alpha\lambda(1-\lambda)(1-2\rho)^2 - \lambda^2(1-2\rho)^2\right)}{1-\left(\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)^2} \tag{10}$$

Finally, using equations (7) and (10), we compute the change in R1's payoff from subscribing to both sources versus only S1:

$$\Delta U_{R1} := U_{R1}(\{S1, S2\}) - U_{R1}(\{S1\}) = \frac{4\lambda^2\rho^2(1-\rho)^2(1-2\rho)^2}{1-\left(\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)^2} \tag{11}$$

which is strictly positive whenever $\lambda > 0$ and $\rho \in (1/2, 1)$.

We summarize this in Lemma 1 below.

Lemma 1 (R1's Decision)**.** *When there is no subscription cost,*

(i) R1 always subscribes to both S1 and S2 whenever $\lambda > 0$ and $\rho \in (1/2, 1)$;

(ii) Furthermore, R1 subscribes only to S1 when $\lambda = 0$.

Lemma 1(ii) follows from Equation (9), which shows that a duplicated signal s_2 contains no additional information when R2 already observes the original signal s_1. Thus, R1 has no incentive to subscribe to S2. This can be viewed as a tie-breaking rule under an infinitesimal subscription cost. We also allow for a positive subscription cost in Section 5.1.

Receivers' utility (before pricing). Let U_i^v denote the expected utility of Receiver $i \in \{\text{R1}, \text{R2}\}$ under the verification rule $v \in \{\text{NV}, \text{PV}\}$, before the utility is transferred to S1 through endogenous pricing. Here, "NV" and "PV" refer to "Non-verification" and "Perfect Verification," respectively. Lemma 2 below computes and compares these utilities.

Lemma 2. *Fix $\lambda > 0$. Define:*

$$
\begin{aligned}
U_{R1}^{NV} &= \frac{(\rho-1)\rho\left(\alpha^2(\lambda-1)^2 - 2\alpha\lambda(\lambda-1)(1-2\rho)^2 + \lambda^2(1-2\rho)^2 - 1\right)}{\left(\alpha(\lambda-1) - \lambda(1-2\rho)^2 - 1\right)\left(\alpha(\lambda-1) - \lambda(1-2\rho)^2 + 1\right)}, \\
U_{R2}^{NV} &= \frac{1}{4}\left(\alpha^2(\lambda-1)^2(1-2\rho)^2 - 2\alpha(\lambda-1)\lambda(1-2\rho)^2 + \lambda^2(1-2\rho)^2 - 1\right), \\
U_{R1}^{PV} &= -\frac{\lambda\rho(1-\rho)}{1+(1-2\rho)^2} - (1-\lambda)\rho(1-\rho), \\
U_{R2}^{PV} &= -\lambda\rho(1-\rho) - (1-\lambda)/4.
\end{aligned}
\tag{12}
$$

Then: (i) $U_{R1}^{PV} > U_{R1}^{NV}$; and (ii) $U_{R2}^{PV} \geq U_{R2}^{NV}$ if and only if $\alpha^2 - \lambda(1-\alpha)^2 \leq 0$ holds.

Lemma 2 is intuitive. First, in the absence of bargaining, R1 always values perfect verification to mitigate duplication bias, as she already has sufficient access to the first-hand source S1. Second, R2, with limited access to the first-hand information source, prefers non-verification when the copied data are sufficiently informative (i.e., $\alpha \uparrow$). Specifically, when $\alpha \geq \frac{1}{2}$ (indicating that there is more than a 50% chance that the second-hand sender successfully replicates the first-hand information), R2 always prefers non-verification.

Information pricing. Next, we analyze endogenous pricing of information. Specifically, we consider the following two scenarios. First, under non-verification, the joint surplus generated for R1 when working solely with S2 coincides with U_{R2}^{NV}, which serves as R1's reservation utility. When R1 uses signals from both senders, the total surplus is U_{R1}^{NV}. Therefore, the price charged by S1 is given by:

$$
U_{S1}^{NV}(\rho_{NV}, \alpha, \lambda) = \gamma\left(U_{R1}^{NV} - U_{R2}^{NV}\right). \tag{13}
$$

Second, under perfect verification, the authenticity of S2 is fully known. Analogously, the price charged by S1 is given by:

$$
U_{S1}^{PV}(\rho_{PV}, \alpha, \lambda) = \gamma\left(U_{R1}^{PV} - U_{R2}^{PV}\right). \tag{14}
$$

Note that the signal quality ρ_v may depend on the verification rule $v \in \{NV, PV\}$.

S1's choice of signal quality. S1 can incur a fixed cost $Q > 0$ to improve signal quality from $\underline{\rho}$ to $\overline{\rho}$. Given the verification rule $v \in \{NV, PV\}$, S1 choose ρ_v^* to solve:

$$\rho_v^\star \in \underset{\rho \in \{\underline{\rho}, \overline{\rho}\}}{\text{argmax}} \; U_{S1}^v(\rho) - Q * \mathbb{1}\{\rho = \overline{\rho}\} \tag{15}$$

Here, we suppress the parameters α and λ. For ease of reference, we further define:

$$\overline{Q} = U_{S1}^{PV}(\overline{\rho}) - U_{S1}^{PV}(\underline{\rho}), \quad \text{and} \quad \underline{Q} = U_{S1}^{NV}(\overline{\rho}) - U_{S1}^{NV}(\underline{\rho}). \tag{16}$$

Note that only $\underline{Q}$ depends on the degree of noise α in signal duplication.

R0's optimal verification design. Finally, we investigate the optimal verification design by R0, which can be stated as follows: if R0 designs a verification rule, the prior belief λ is altered. Specifically, the baseline model considers discrete verification design between perfect verification and non-verification. The main trade-off involves comparing:

- **Non-verification:** $\nu(FH|m) = \lambda$. The posterior remains inaccurate, so Sender S2 will replicate the original signal if she is an SH source. This results in greater information diffusion, but potentially less accuracy.
- **Perfect verification:** $\nu(FH|m) \in \{0, 1\}$. The posterior is accurate and, thus, an SH S2 will not replicate data. This leads to less information diffusion but higher accuracy.

Intuitively, the optimal verification rule varies with the relative importance of information diffusion versus concerns about duplication bias, as captured by the welfare weight parameter β_R. Then, Regulator R0 solves the following problem:

$$v^* = \underset{v \in \{NV, PV\}}{\text{argmax}} \; U_{R0}^v := (U_{R1}^v - U_{S1}^v) + \beta_R \times U_{R2}^v. \tag{17}$$

Based on Equations (13) and (14), perfect verification is optimal when:

$$(1-\gamma)\left(U_{R1}^{PV} - U_{R1}^{NV}\right) - \gamma\left(U_{R2}^{NV} - U_{R2}^{PV}\right) \geq \beta_R\left(U_{R2}^{NV} - U_{R2}^{PV}\right). \tag{18}$$

Define:

$$\widehat{\beta}_R(\rho_{NV}, \rho_{PV}) = \frac{(1-\gamma)\left(U_{R1}^{PV}(\rho_{PV}) - U_{R1}^{NV}(\rho_{NV})\right) - \gamma\left(U_{R2}^{NV}(\rho_{NV}) - U_{R2}^{PV}(\rho_{PV})\right)}{U_{R2}^{NV}(\rho_{NV}) - U_{R2}^{PV}(\rho_{PV})}. \tag{19}$$

Now, the equilibrium is fully characterized in Proposition 1 below.

Proposition 1 (Equilibrium Characterization)**.** *Suppose that* $Q > \max\{\overline{Q}, \underline{Q}\}$.

Case (I). *Suppose: (1)* $\alpha^2 - \lambda(1-\alpha)^2 > 0$*; and (2) one of the following two conditions is satisfied: (a)* $(1-\gamma)\left(U_{R1}^{PV}(\underline{\rho}) - U_{R1}^{NV}(\underline{\rho})\right) \leq \gamma\left(U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\underline{\rho})\right)$*; or (b)* $\beta_R \geq \widehat{\beta}_R(\underline{\rho}, \underline{\rho})$*, given that* $(1-\gamma)\left(U_{R1}^{PV}(\underline{\rho}) - U_{R1}^{NV}(\underline{\rho})\right) - \gamma\left(U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\underline{\rho})\right) > 0$*. Then:*

(i) R0: $v^* = NV$*, that is, R0 chooses non-verification;*

(ii) S1: $\rho_{NV}^* = \underline{\rho}$*, that is, S1 chooses low signal quality;*

(iii) S2: an SH S2 chooses to copy s_1*;*

(iv) R1: R1 always chooses to subscribe to both S1 and S2.

Case (II). *Suppose the conditions in Case (I) do not hold. Then:*

(i) R0: $v^* = PV$*, that is, R0 chooses perfect verification;*

(ii) S1: $\rho_{PV}^* = \underline{\rho}$*, that is, S1 chooses low signal quality;*

(iii) S2: an SH S2 chooses not to copy s_1*;*

(iv) R1: R1 chooses to subscribe only to an FH S2.

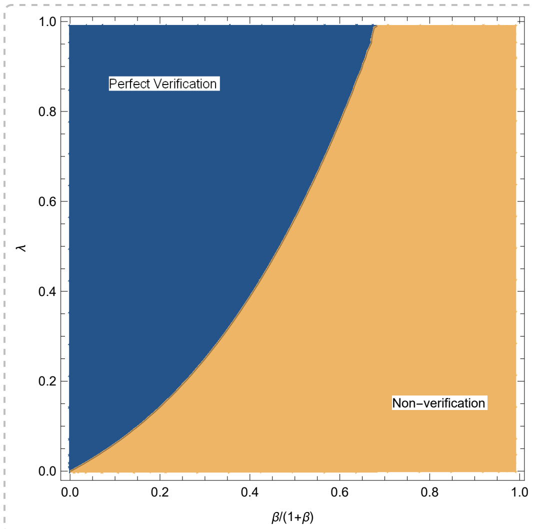
Proposition 1 shows that non-verification can be optimal and thus correlation neglect can arise as an equilibrium phenomenon ex post in a fully rational environment. To fully understand the content of Proposition 1, we decompose this non-verification result into two parts: *diffusion-driven non-verification* and *bargaining-driven non-verification.*

Diffusion-driven non-verification. We first present a corollary of Proposition 1 to illustrate diffusion-driven non-verification. To this end, we remove the influence of endogenous pricing by assuming $\gamma = 0$, which also applies under fixed pricing.

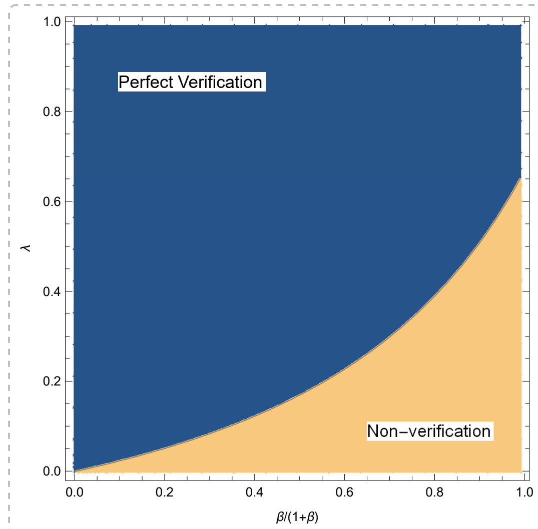
Corollary 1 (Optimal Non-Verification)**.** *Assume* $\gamma = 0$*. Define*

$$\overline{\beta}_R = \frac{U_{R1}^{PV}(\underline{\rho}) - U_{R1}^{NV}(\underline{\rho})}{U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\underline{\rho})}.$$

Then, non-verification is optimal if $\beta_R > \overline{\beta}_R$ *and* $\alpha^2 - \lambda(1-\alpha)^2 > 0$*; otherwise, perfect verification is optimal.*



(a) Small Noise



(b) Large Noise

Figure 3: Optimal Verification Rule

Parameters: (left) $\alpha = 0.7$, $\rho = 0.6$; (right) $\alpha = 0.45$, $\rho = 0.6$.

Non-verification is optimal when information diffusion is critical (i.e., $\beta_R > \overline{\beta}_R$), the noise in data duplication is low (i.e., α is high), and access to first-hand (FH) information sources is limited (i.e., λ is small). Figure 3 depicts the optimal verification rule. Specifically, Figure 3a (left) considers the case of relatively precise duplication ($\alpha = 0.7$), while Figure 3a (right) examines the case of more noisy duplication ($\alpha = 0.4$). In both cases, Regulator R0 prefers perfect verification in the upper-left region and non-verification in the bottom-right region. Furthermore, both figures reveal a common pattern: stronger concerns about duplication bias and greater access to first-hand sources lead to more frequent verification. Conversely, a greater demand for information diffusion and limited access to first-hand sources result in non-verification. The two figures differ in that, when second-hand information is sufficiently noisy, Receiver R2 always prefers perfect verification with easy access to FH sources (λ is large), regardless of the importance of diffusion.

Bargaining-driven non-verification. Why can bargaining create additional incentives for non-verification beyond the need for information diffusion? Introducing endogenous pricing between S1 and R1 can substantially affect the optimal verification rule. The key insight is that expanding the set of information sources intensifies the competition among them. Consequently, Regulator R0 may strategically use non-verification to retain second-hand providers and strengthen bargaining power against the primary source.

Theorem 1 establishes that endogenous pricing can incentivize non-verification through its effect on bargaining power. Specifically, when Sender S1 possesses substantial bargaining power, Regulator R0 prefers non-verification. This results in greater duplication, which Receiver R1 can leverage to counteract S1's market power in information pricing. Holding all other parameters fixed, there exists a threshold $\overline{\gamma} < 1$ such that non-verification is optimal for all $\gamma > \overline{\gamma}$, regardless of the welfare weight β_R.[3] Moreover, when $(U_{R2}^{NV} - U_{R2}^{PV}) \geq 0$, the threshold $\widehat{\beta}_R$ under endogenous pricing is strictly lower than $\overline{\beta}_R$, the corresponding threshold without endogenous pricing. Finally, we verify that $\frac{\partial \widehat{\beta}_R}{\partial \gamma} < 0$, implying that greater bargaining power for S1 expands the range of β_R for which non-verification is optimal.

Decrypting incentives for (non-)verification. We next decompose the incentives for (non-)verification. Consistent with Lemma 2, Receiver R1's expected payoff is convex in the prior probability λ that S2 is a primary source. This convexity creates an incentive for perfect verification, as illustrated in Figure 4. Specifically, the red dashed line and the blue solid line depict R1's payoff under perfect verification and non-verification, respectively.

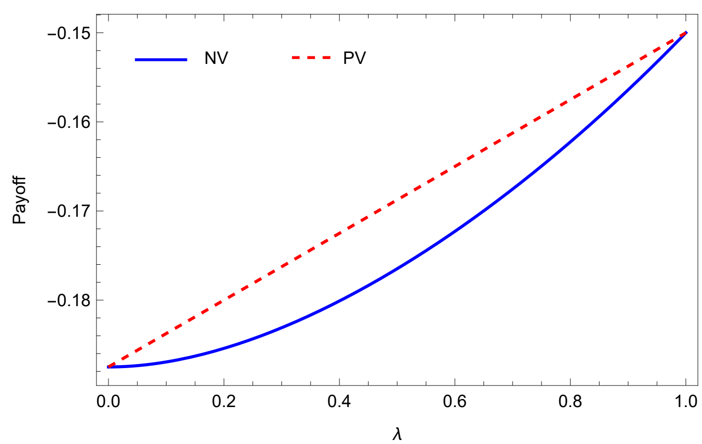
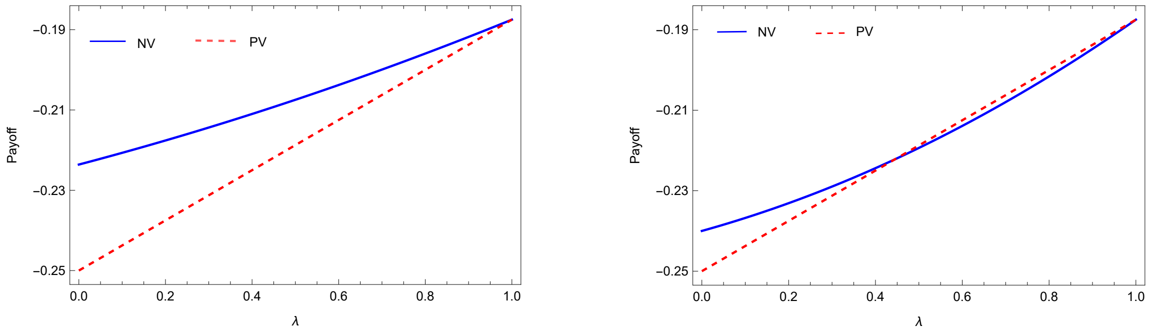


Figure 4: R1's Incentive for Verification

Figure 5 illustrates Receiver R2's incentive for verification, where the blue solid line and the red dashed line represent the payoffs under non-verification and perfect verification, respectively. When the noise in replicated information is relatively low, R2 strictly prefers non-verification, as shown in Figure 5a. However, when the replicated information is highly noisy, R2 prefers non-verification only when S2 is very unlikely to be a primary source — emphasizing the value of information diffusion. In summary, Figures 4 and 5 highlight the inherent trade-off between controlling duplication bias and enhancing information diffusion for receivers overall.

[3] Specifically, $\overline{\gamma} = (U_{R1}^{PV} - U_{R1}^{NV}) / \big((U_{R1}^{PV} - U_{R1}^{NV}) + (U_{R2}^{NV} - U_{R2}^{PV})\big)$.



(a) Small Noise (b) Large Noise

Figure 5: R2's Incentive for Verification

Parameters: (left) $\alpha = 0.40$, $\rho = 0.75$; (right) $\alpha = 0.65$, $\rho = 0.75$.

Small and intermediate information cost. We consider an intermediate level of information costs, where signal quality depends on the verification rule. The key insight is that concerns about information quality can induce more frequent verification of information sources. Lemma 3 characterizes the equilibrium.

Lemma 3 (Intermediate Information Cost and Variable Signal Quality). *Assume: (i) $\alpha = 1$ (i.e., perfect duplication) and $\lambda \leq \frac{1}{3}(5 - \sqrt{13}) \approx 0.46$. Then:*

(i) If $Q > \overline{Q}$, S1 always chooses $\rho_v^* = \underline{\rho}$ for all $v \in \{NV, PV\}$;

(ii) If $Q \leq \underline{Q}$, S1 always chooses $\rho_v^* = \overline{\rho}$ for all $v \in \{NV, PV\}$; and

(iii) If $\underline{Q} < Q \leq \overline{Q}$, then S1 chooses $\rho_{PV}^* = \overline{\rho}$ and $\rho_{NV}^* = \underline{\rho}$.

One clarifying remark is in order. Sender S1's optimal choice of signal quality, ρ^*, depends on the verification rule only within an intermediate range of the investment cost Q. When Q is sufficiently large, S1 does not improve information quality, even under perfect verification; when Q is small, S1 improves information quality even without verification.[4]

Proposition 2 (Signal Quality-Driven Verification). *Fix $\alpha = 1$, $\lambda \leq \frac{1}{3}(5 - \sqrt{13})$, and $\underline{Q}(\overline{\rho}, \underline{\rho}) < Q \leq \overline{Q}(\overline{\rho}, \underline{\rho})$. Compared to the case with high information cost, perfect verification is more likely to dominate non-verification.*

Proposition 2 shows that concerns about information quality lead to more frequent verification of source authenticity, serving to incentivize primary sources to invest in high-quality signals. However, this effect arises only at intermediate cost levels, as senders no longer respond to verification rules when the cost is either too high or too low.

Theorem 1 summarizes the key drivers of (non-)verification.

[4]It is possible that $\underline{Q}(\overline{\rho}, \underline{\rho}) \leq 0$, as an increase in data quality may raise R1's outside option more rapidly, particularly when $\underline{\rho}$ is large, thereby reducing the price S1 can charge.

Theorem 1 (Optimal Verification Design)**.** *The need for information diffusion and concerns about sender bargaining power drive non-verification, while concerns about duplication bias and signal quality encourage more frequent verification. Specifically, non-verification is optimal for receivers when: (i) the sender holds large bargaining power; (ii) signal quality is insensitive to verification rule; and (iii) diffusion is more important than precision.*

3.2 Welfare and Misinformation

We first examine total welfare W, defined as the sum of receiver payoffs (weighted by Regulator R0's welfare weight β_R) and sender payoffs. Specifically:

$$W = \begin{cases} U_{R1}^{NV}(\rho_{NV}) + \beta_R U_{R2}^{NV}(\rho_{NV}) + 2 - c_S, & \text{if } v = NV; \\ U_{R1}^{PV}(\rho_{PV}) + \beta_R U_{R2}^{PV}(\rho_{PV}) - Q * \mathbb{1}\{\rho_{PV} = \overline{\rho}\}, & \text{if } v = PV. \end{cases} \tag{20}$$

Total welfare differs from Regulator R0's utility by the inclusion of both senders' payoffs. Thus, verification rule design generates an externality, leading to two key consequences:

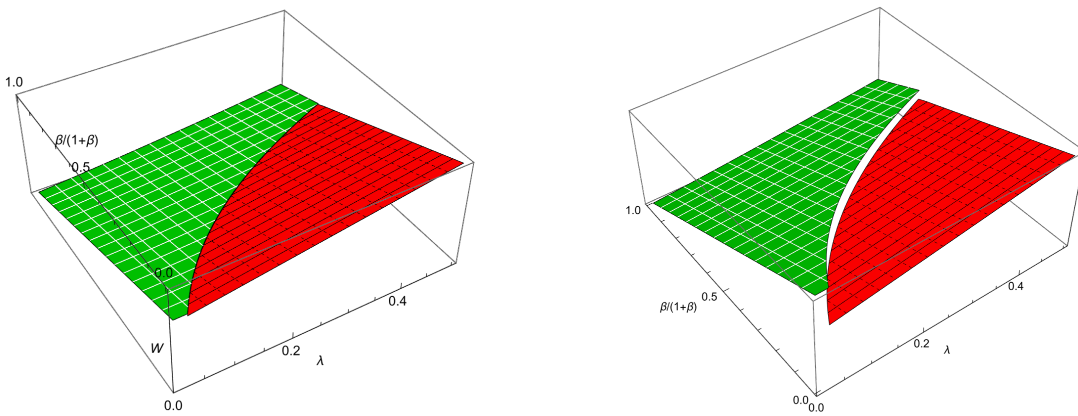
- **Suboptimal verification:** The verification rule becomes socially inefficient due to the divergence between total welfare and Regulator R0's objective. Depending on the relative utilities of S1 and S2, it may result in either excessive or insufficient verification. Specifically, ignoring S1's utility leads to insufficient verification, while ignoring S2's utility leads to excessive verification.

- **Non-monotonic welfare:** Total welfare can vary non-monotonically with underlying parameters. This arises from an externality in Regulator R0's verification design—unless R0's utility fully aligns with total welfare.

To illustrate this, consider parameter values where R0 is indifferent between perfect verification and non-verification and the information cost Q is large. A slight change in parameters (with Q fixed) can shift the optimal rule, causing a welfare jump driven by changes in the senders' utility: $(U_{S1}^{NV} - U_{S2}^{PV}) + (2 - c_S)$. If $(U_{S1}^{NV} - U_{S2}^{PV}) < 0$, the direction of the welfare jump depends on the size of $(2 - c_S)$:

- If $2 - c_S$ is small, the net effect is an upward jump (Figure 6a).

- If $2 - c_S$ is large, the net effect is a downward jump (Figure 6b).



(a) c_S is large. (b) c_S is small.

Figure 6: Total Welfare

The externality created by verification rule design introduces an indeterminacy in welfare jumps, breaking the monotonic relationship with the underlying parameters. Figure 6 illustrates this, where the red and green areas represent total welfare under perfect verification and non-verification, respectively.

Misinformation. We next examine how verification affects the level of aggregate misinformation, defined as the average misinformation experienced by the two receivers. This measure can be interpreted as a forward-looking metric for a newcomer who randomly encounters one of them. Let M_i^v denote the misinformation faced by Receiver $i \in \{R_1, R_2\}$ under verification rule $v \in \{NV, PV\}$. Then, we define:

$$M^v = \frac{M_{R1}^v + M_{R2}^v}{2}, \tag{21}$$

where $M_i^v = \mathbb{E}\left[\left|\mathbb{E}[\theta \mid \mathcal{I}_i^v] - \theta\right|\right]$ and $\mathcal{I}_i^v$ denotes the information set for Receiver i.

We first present some preliminary results for the misinformation measures M_i^v.

Lemma 4 (Aggregate Misinformation)**.**

(i) The misinformation measures satisfy:

$$M_{R1}^{NV} = \frac{2(1-\rho)\rho\left(1-\alpha^2(1-\lambda)^2 - 2\alpha\lambda(1-\lambda)(1-2\rho)^2 - \lambda^2(1-2\rho)^2\right)}{\left(1+\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)\left(1-\alpha(1-\lambda)-\lambda(1-2\rho)^2\right)},$$
$$M_{R2}^{NV} = \frac{1}{2}\left(1-\alpha^2(1-\lambda)^2(1-2\rho)^2 + 2\alpha(1-\lambda)\lambda(1-2\rho)^2 - \lambda^2(1-2\rho)^2\right),$$

$$M_{R1}^{PV} = \frac{(1-\rho)\rho\left(1+(1-\lambda)(1-2\rho)^2\right)}{2\rho^2-2\rho+1}, \quad \textit{and} \quad M_{R2}^{PV} = \frac{1}{2}\left(1-\lambda(1-2\rho)^2\right).$$

(ii) Monotonicity: $\frac{\partial M_i^v}{\partial \lambda} < 0$, $\frac{\partial M_i^v}{\partial \rho} < 0$, *and* $\frac{\partial M_i^{NV}}{\partial \alpha} < 0$*;*

(iii) Misinformation comparison: (a) $M_{R1}^{NV} > M_{R1}^{PV}$*; and (b)* $M_{R2}^{NV} > M_{R2}^{PV}$ *if and only if* $\lambda(1-\alpha)^2 - \alpha^2 > 0$ *holds.*

Our theory offers policymakers new insights into the design of verification systems to combat misinformation. Lemma 4 (ii) shows that higher information quality ($\rho \uparrow$), better access to primary sources ($\lambda \uparrow$), and reduced noise in duplicated information ($\alpha \uparrow$) all help to reduce aggregate misinformation, regardless of the verification rule, which is consistent with the literature on misinformation that emphasizes improvements in information quality and shifts focus to accuracy (Vosoughi et al., 2018; Lazer et al., 2018; Pennycook et al., 2021). Figure 7 depicts the aggregate misinformation, as defined in Equation (21), confirming this pattern under a fixed verification rule. The red and green areas correspond to aggregate misinformation under perfect verification and non-verification by R0, respectively.

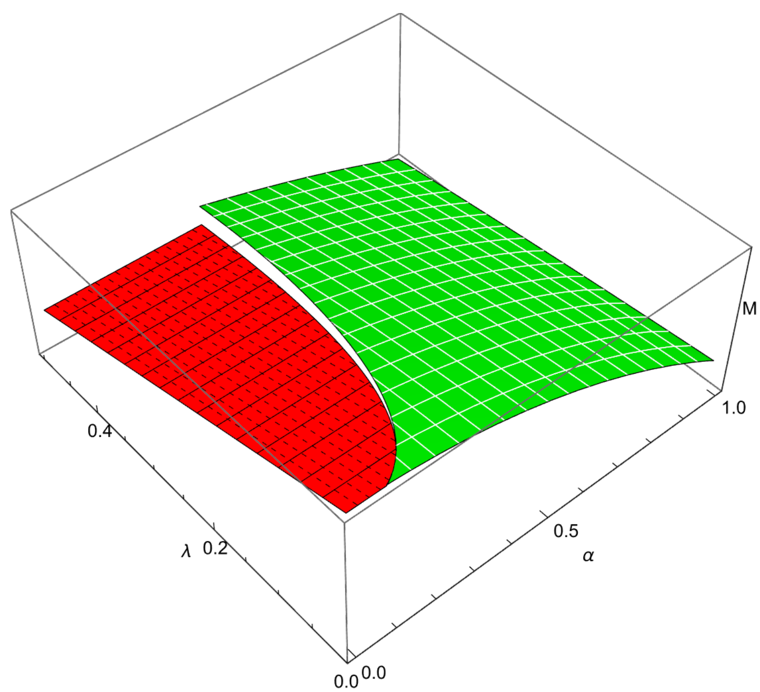


Figure 7: Endogenous verification and Aggregate Information

However, under endogenous verification design, the effect of key parameters on aggregate misinformation can be non-monotonic. For example, consider a decrease in noise ($\alpha \uparrow$) in Figure 7. Initially, in the red region, the aggregate misinformation is invariant to α because the authenticity of S2 is perfectly verified. As noise further decreases, R0 switches to

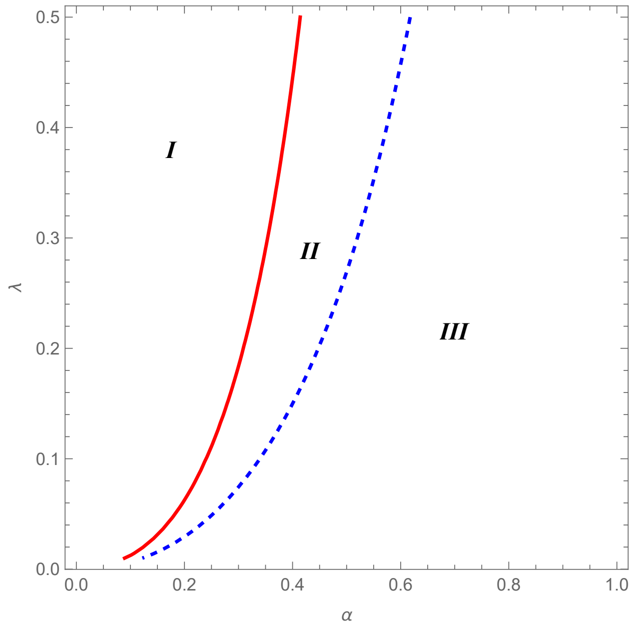


Figure 8: Misinformation Comparison

Notes: R0 chooses perfect verification in Region I, and non-verification in Regions II and III. Perfect verification reduces misinformation in Regions I and II, while non-verification is more effective in Region III.

non-verification, resulting in a discrete increase in aggregate misinformation. Subsequently, continued noise reduction lowers aggregate misinformation under non-verification, consistent with the monotonicity in Lemma 4(ii). Thus, our theory can offer additional deeper insights to complement existing empirical research on aggregate misinformation.

Second, Lemma 4 (iii) reveals how verification rules affect aggregate misinformation. On the one hand, non-verification increases misinformation for R1, who already has extensive access to primary information sources. For R1, knowing the authenticity of the information sources allows optimal weighting of signals from multiple sources. On the other hand, non-verification may reduce misinformation for R2 when access to primary information is limited ($\lambda \downarrow$) or when duplicated information is relatively accurate (low noise, $\alpha \uparrow$). However, if R2 can easily access primary sources or if duplicated information is highly noisy, non-verification generates more misinformation compared to perfect verification.

Third, aggregate misinformation may increase or decrease when we compare non-verification to perfect verification. Specifically, Figure 8 shows that non-verification leads to more misinformation in Regions I and II, but less in Region III. As the importance of information diffusion increases ($\beta_R \uparrow$), the optimal verification rule shifts from perfect verification to non-verification. This transition results in higher misinformation in Regions I and II, but lower misinformation in Region III. Furthermore, the red line indicates the optimal verification rule for R0 (with β_R fixed), highlighting a mismatch between minimizing misinformation and optimal verification. In Region II, perfect verification minimizes misinformation, yet non-verification is optimal for R0.

In summary, we provide a theoretical foundation for understanding how verification policies balance accuracy and diffusion, thereby shaping misinformation outcomes. Our results highlight the complexity of designing effective interventions: verification has an ambiguous effect on aggregate misinformation. Moreover, the strategic use of non-verification by information users to counteract market power is consistent with the literature on user-driven responses in the misinformation ecosystem (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2008).

4 Sender Authentication and Regulation

This section explores sender authentication and self-regulation, with our main results continuing to hold. We first analyze intellectual property (IP) protection, then sender self-regulation, and finally a general regulatory problem.

4.1 Intellectual Property Protection

We first examine how IP protection affects source authentication, welfare, and misinformation. Specifically, suppose that the primary sender S1 can sue Sender S2 at a fixed cost $L > 0$ to detect IP infringement. Conditional an IP theft, the probability of successful detection is $\kappa > 0$, and if convicted, a fine $F > 0$ is imposed and transferred to S1. The strength of IP protection can be measured as the expected penalty $\widehat{F} = \kappa F$. Note that IP violations occur only under non-verification in our framework.

Large information cost. We first analyze the case with a high information cost $Q > 0$ such that signal quality is independent of the verification rule.

Lemma 5 (Equilibrium Characteriztion with IP Protection)**.** *Let $\underline{\alpha}$ denote the solution to $U_{R0}^{NV}(\underline{\alpha}, \underline{\rho}) = U_{R0}^{PV}(\underline{\rho})$. Assume that: (1) $\frac{L}{(1-\lambda)} < 2 - c_S$; (2) $U_{R0}^{NV}(\alpha, \underline{\rho}) > U_{R0}^{PV}(\underline{\rho})$; (3) $\frac{\alpha L}{(2-c_S)} > \underline{\alpha}$; and (4) $Q > \max\{\overline{Q}, \underline{Q}\}$. Then:*

(i) Litiqation strategy. There are four cases:

(1) If $\widehat{F} \leq \frac{L}{1-\lambda}$, S1 does not sue S2, and an SH S2 copies S1's signal;

(2) If $2 - c_S > \widehat{F} > \frac{L}{1-\lambda}$, S1 sues S2, and an SH S2 copies S1's signal;

(3) If $\frac{\alpha L}{(1-\lambda)\underline{\alpha}} > \widehat{F} \geq 2 - c_S$, there is a mixed-strategy equilibrium in which S1 sues S2 with probability $\frac{2-c_S}{\widehat{F}}$, and an SH S2 copies S1's signal with probability $\frac{L}{(1-\lambda)\widehat{F}}$;

(4) If $\widehat{F} \geq \frac{\alpha L}{(1-\lambda)\underline{\alpha}}$, S1 does not sue S2, and an SH S2 does not copy S1's signal since R0 switches to perfect verification.

(ii) R0's verification design. R0 chooses non-verification when $\widehat{F} < \frac{\alpha L}{(1-\lambda)\underline{\alpha}}$, and switches to perfect verification when $\widehat{F} \geq \frac{\alpha L}{(1-\lambda)\underline{\alpha}}$.

(iii) S1's signal quality. S1 selects $\rho_v^* = \underline{\rho}$ under perfect verification and non-verification.

(iv) R1's subscription strategy. R1 only subscribes to an SH S2 under non-verification.

First, we clarify the conditions in Lemma 5. Condition (1) simply reduces the number of cases to be considered. Condition (2) ensures that R0 initially prefers non-verification when IP protection is minimal. Condition (3) guarantees that R0 does not immediately switch to perfect verification when $\widehat{F}$ just exceeds $(2 - c_S)$.

Second, we interpret the equilibrium described in Lemma 5. Initially, when IP protection is minimal, the expected penalty $\widehat{F}$ has no impact on information providers or users. As IP protection strengthens, primary sources begin to sue second-hand providers, leading to a monetary transfer from the copycat S2 to the primary source S1. This generates a deadweight loss L in the form of the litigation cost. Furthermore, when IP protection becomes very strong, both the probability of litigation and that of IP thefts decrease. Eventually, if IP protection becomes excessive, Regulator R0 switches to perfect verification. Surprisingly, this may reduce overall welfare when information diffusion is important ($\beta_R \uparrow$) and access to FH sources is limited ($\lambda \downarrow$). Excessive IP protection hinders information diffusion, leading to under-use of information.

Intermediate information cost. Next, we analyze the case with intermediate information cost Q, where strong IP protection incentivizes high-quality signals.[5]

Lemma 6 (IP Protection and Equilibrium Signal Quality)**.** *Let $\widetilde{\alpha}$ solve $U_{R0}^{NV}(\widetilde{\alpha}, \overline{\rho}) = U_{R0}^{PV}(\overline{\rho})$. Assume: (1) $\frac{L}{1-\lambda} < 2 - c_S$; (2) $\lambda \leq 1/3$; (3) $U_{R0}^{NV}(\alpha, \underline{\rho}) > U_{R0}^{PV}(\overline{\rho})$; (4) $\max\{\underline{Q}(\alpha), 0\} < Q < \max\{\overline{Q}, \underline{Q}(\widetilde{\alpha})\}$; (5) $\widetilde{\alpha} < \frac{\alpha L}{2-c_S}$. Then:*

[5] Recall that $\underline{Q}(\alpha)$ measures the price change charged by S1 between high-precision $\overline{\rho}$ and low-precision $\underline{\rho}$ signals under non-verification and thus depends on α.

(i) Litigation strategy. There are four cases:

(1) If $\widehat{F} \leq \frac{L}{1-\lambda}$*, S1 does not sue S2, and an SH S2 copies S1's signal;*

(2) If $2 - c_S > \widehat{F} > \frac{L}{1-\lambda}$*, S1 sues S2, and an SH S2 copies S1's signal;*

(3) If $\frac{\alpha L}{\widetilde{\alpha}(1-\lambda)} > \widehat{F} \geq 2 - c_S$*, a mixed-strategy equilibrium exists in which S1 sues S2 with probability* $\frac{2-c_S}{\widehat{F}}$*, and an SH S2 copies with probability* $\frac{L}{(1-\lambda)\widehat{F}}$*;*

(4) If $\widehat{F} \geq \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}$*, S1 does not sue S2, and an SH S2 does not copy S1's signal since R0 switches to perfect verification.*

(ii) R0's verification design. R0 chooses non-verification when $\widehat{F} < \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}$*, and switches to perfect verification when* $\widehat{F} \geq \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}$*.*

(iii) S1's signal quality. S1 selects $\rho^*_{PV} = \overline{\rho}$ *and there exists a unique* $\widetilde{F} \in \left[\frac{\alpha L}{2-c_S}, \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}\right)$ *such that:*

$$\rho^*_{NV} = \begin{cases} \overline{\rho}, & \text{if} \quad \widehat{F} \geq \widetilde{F}; \\ \underline{\rho}, & \text{if} \quad \widehat{F} < \widetilde{F}. \end{cases}$$

(iv) R1's subscription strategy. R1 only subscribes to an SH S2 under non-verification.

First, we interpret the conditions in Lemma 6. Condition (2) supports Lemma A.1, which shows that the incentive of S1 to invest in signal quality improves when S2 is less likely to copy S1's signal. Condition (3) ensures that R0 prefers non-verification when IP penality is small. Condition (4) has two parts: the first inequality implies $\rho^*_{NV}(\alpha) = \underline{\rho}$, and the second implies $\rho^*_{NV}(\widetilde{\alpha}) = \overline{\rho}$ (for $\widetilde{\alpha} < \alpha$) and $\rho^*_{PV} = \overline{\rho}$. This condition is non-empty, as shown in Lemma 6 by setting $\alpha = 1$. Condition (5) helps reduce the number of cases.[6]

Second, compared to the case with large information costs, the equilibrium in Lemma 6 differs due to strategic pricing and endogenous signal quality. Specifically, when IP protection is weak, the equilibrium aligns with that of Lemma 5. In contrast, strong IP protection motivates investment in signal quality. Large IP theft penalties reduce the availability of duplicated information, weakening the receivers' outside option in bargaining with primary senders. This improves the return on strategic pricing and motivates primary senders to improve signal quality. Lemma 6 also identifies a penalty threshold above which S1 invests

[6] Without it, if $\widetilde{\alpha} \geq \frac{\alpha L}{2-c_S}$, R0 would switch to perfect verification at $\widehat{F} = 2 - c_S$.

in high-quality signals, provided that the investment costs are not large. These investments improve overall welfare, as receivers also benefit, as long as S1 does not hold all the bargaining power. Finally, excessive IP protection limits access to second-hand information, prompting R0 to switch to perfect verification.

Ambiguous welfare impact of IP protection. To analyze the impact of IP protection on total welfare, we compute the total welfare W based on the equilibrium characterized in Lemma 6 as below:

$$W = \begin{cases} U_{R1}^{NV}(\alpha, \underline{\rho}) + \beta_R * U_{R2}^{NV}(\alpha, \underline{\rho}) + (2 - c_S), & \text{if } \widehat{F} \leq \frac{L}{(1-\lambda)}; \\ U_{R1}^{NV}(\alpha, \underline{\rho}) + \beta_R * U_{R2}^{NV}(\alpha, \underline{\rho}) + (2 - c_S) - L, & \text{if } 2 - c_S > \widehat{F} \geq \frac{L}{(1-\lambda)}; \\ U_{R1}^{NV}(\frac{\alpha L}{(1-\lambda)\widehat{F}}, \underline{\rho}) + \beta_R * U_{R2}^{NV}(\frac{\alpha L}{(1-\lambda)\widehat{F}}, \underline{\rho}), & \text{if } \widetilde{F} > \widehat{F} > 2 - c_S; \\ U_{R1}^{NV}(\frac{\alpha L}{(1-\lambda)\widehat{F}}, \overline{\rho}) + \beta_R * U_{R2}^{NV}(\frac{\alpha L}{(1-\lambda)\widehat{F}}, \overline{\rho}) - Q, & \text{if } \frac{\alpha L}{\underline{\alpha}(1-\lambda)} > \widehat{F} \geq \widetilde{F}; \\ U_{R1}^{PV}(\overline{\rho}) + \beta_R * U_{R2}^{PV}(\overline{\rho}) - Q, & \text{if } \widehat{F} \geq \frac{\alpha L}{\underline{\alpha}(1-\lambda)}. \end{cases} \tag{22}$$

Figure 9 illustrates how IP protection, measured by $\widehat{F}$, affects total welfare.[7] First, when $\widehat{F} \leq L/(1-\lambda)$, total welfare remains unchanged, as S1 has no incentive to sue copycats. Second, when $\widehat{F} \in [L/(1-\lambda), 2-c_S)$, the penalty is high enough to cover the litigation cost of S1 but not enough to discourage S2 from copying S1's signal. This results in a deadweight loss equal to the cost of litigation L. These correspond to the regions $\widehat{F} \leq 1/8$ and $\widehat{F} \in (1/8, 1/5]$ in Figure 9.

Third, for $\widehat{F} \in (2 - c_S, \widetilde{F})$, the IP penalty exceeds the maximum benefit for information copycats, and total welfare drops discontinuously as $\widehat{F}$ crosses the threshold $2 - c_S$. Within this range, total welfare strictly decreases as the IP penalty increases. Intuitively, a higher penalty reduces duplicated information, which is still valuable, thereby lowering the overall surplus from investment.However, the diminished outside option for R1 motivates S1 to invest in high-quality signal. This corresponds to the range $\widehat{F} \in (1/5, 1/4)$.

Fourth, when the penalty is high enough, Sender S1 can break even by investing in high quality signals, leading to a welfare jump at $\widehat{F} = \widetilde{F}$ due to improved signal quality. However, as $\widehat{F}$ further increases, overall information quality decreases again because of reduced access to duplicated information, further lowering total welfare. This corresponds to the range

[7] We use the following parameter values: $\alpha = 1$, $\lambda = 0.2$, $c_S = 1.8$, $L = 0.1$, $\overline{\rho} = 0.85$, $\underline{\rho} = 0.7$, $\gamma = 0.75$, $\beta_R = 5$, $Q = 0.04$, and $\widetilde{\alpha} = 0.4$.

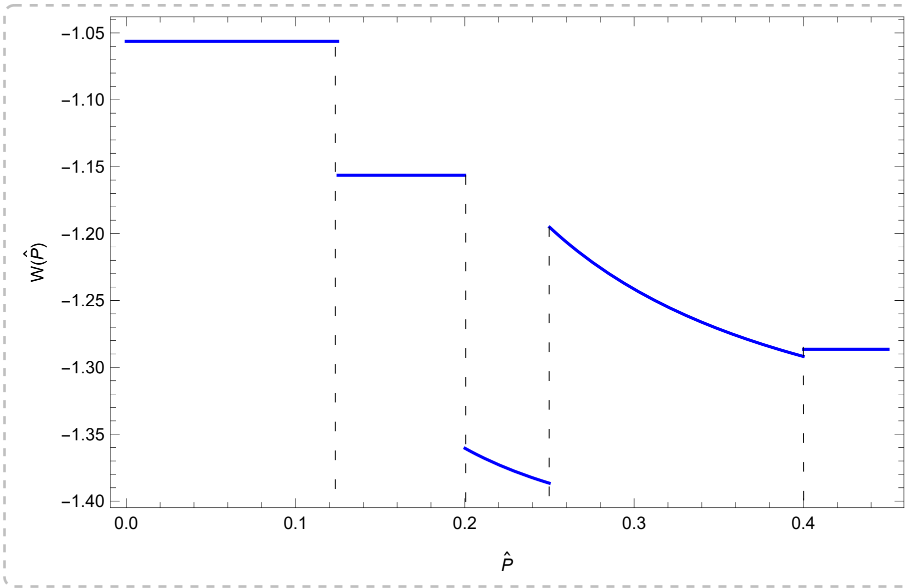


Figure 9: IP Protection and Welfare

$\widehat{F} \in (1/4, 2/5)$.

Finally, when $\widehat{F} \geq 2/5$, Regulator R0 chooses perfect verification. Extreme IP protection reduces receivers' payoffs by eliminating secondary signals. Thus, strong IP protection may harm welfare by making non-verification infeasible when information diffusion is important. These insights are summarized in Proposition 3 below.

Proposition 3 (Intellectual Property Protection)**.**

(i) IP protection incentivizes investment in high-quality signals; and

(ii) The impact of IP protection on total welfare is non-monotonic, and excessive protection can reduce it.

We conclude this section with two remarks.

Remark 1 (Ambiguous Effects of IP Protection on Misinformation)**.** *On the one hand, strong IP protection incentivizes primary sources to invest in high-quality signals, thereby reducing misinformation. On the other hand, it discourages secondary sources from producing duplicated, but still informative, signals. The reduced availability of such duplicated information can increase aggregate misinformation. These opposing forces create an ambiguous overall effect. For example, when the information cost is high, as discussed in Lemma 5, strong IP protection increases aggregate misinformation.*

Remark 2 (Why IP Protection Can Harm Welfare)**.** *Proposition 3 shows that while strong IP protection can improve information quality, it does not necessarily improve total welfare.*

Why does IP protection harm welfare even when it works? First, perfect verification can provide strong incentives for investing in information quality, so IP protection mainly improves quality under non-verification. However, as shown in Proposition 1, non-verification is optimal when primary information providers hold significant market power or when information diffusion is important. In such cases, excessive IP protection can worsen market concentration or limit information access, ultimately reducing welfare.

4.2 Sender Self-Regulation

The key trade-off between avoiding duplication bias and enhancing information diffusion also applies when senders choose whether to verify the authenticity of secondary sources. To illustrate this, we introduce a (sender) regulator, S0, who aggregates the payoffs of the senders S1 and S2 with a relative welfare weight β_S, that is,

$$u_{S0} = u_{S1} + \beta_S \times u_{S2}.$$

Then, non-verification is preferred by S0 when:

$$U_{S1}^{NV}(\rho_{NV}) - Q * \mathbb{1}\{\rho_{NV} = \overline{\rho}\} + \beta_S \times (2 - c_S) \geq U_{S1}^{PV}(\rho_{PV}) - Q * \mathbb{1}\{\rho_{PV} = \overline{\rho}\} \tag{23}$$

where U_{S1}^{NV} and U_{S1}^{PV} are given in Equations (13) and (14).

Define:

$$\widehat{\beta}_S(\rho_{NV}, \rho_{PV}) = \frac{U_{S1}^{PV}(\rho_{PV}) - U_{S1}^{NV}(\rho_{NV}) - Q * (\mathbb{1}\{\rho_{PV} = \overline{\rho}\} - \mathbb{1}\{\rho_{NV} = \overline{\rho}\})}{(2 - c_S)}$$

Proposition 4 (Optimal Self-Authentication for Senders)**.** *Non-verification dominates perfect verification if and only if $\beta \geq \widehat{\beta}_S(\rho_{NV}, \rho_{PV})$.*

Proposition 4 shows that senders, like receivers, face a trade-off between duplication bias and information diffusion in the optimal verification design.

Constant signal quality. For a large information cost Q, constant signal quality arises (i.e., $\rho_{PV} = \rho_{NV} = \underline{\rho}$). Then:

$$\widehat{\beta}_S = \frac{U_{S1}^{PV}(\underline{\rho}) - U_{S1}^{NV}(\underline{\rho})}{2 - c_S} = \frac{\gamma \left(U_{R1}^{PV}(\underline{\rho}) - U_{R1}^{NV}(\underline{\rho}) + U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\underline{\rho})\right)}{2 - c_S}.$$

Thus, senders never verify the authenticity of senders when information diffusion is sufficiently important (that is, when β_S is large). The regulator S0 balances the payoffs of the primary and secondary senders, and strictly prefers non-verification when SH senders generate high payoffs. Furthermore, when the primary source S1 has weaker bargaining power with Receiver R1, S0 is more likely to choose non-verification (i.e., $\partial\widehat{\beta}_S/\partial\gamma > 0$).

Variable signal quality. Variable signal quality emerges for an intermediate information cost. For example, suppose that the conditions in Lemma 3 hold. Then:

$$\widehat{\beta}_S = \frac{U_{S1}^{PV}(\overline{\rho}) - Q - U_{S1}^{NV}(\underline{\rho})}{2 - c_S} > \frac{U_{S1}^{PV}(\underline{\rho}) - U_{S1}^{NV}(\underline{\rho})}{2 - c_S}.$$

Therefore, when information diffusion is important, senders still abstain from verifying the authenticity of sources to increase revenue benefits related to enhanced information diffusion. In addition, senders verify the sources of the information more frequently for an intermediate information cost due to the concern related to signal quality.

4.3 General Regulator: Elite VS. Grassroot

Consider a general regulator problem in which a social planner assigns welfare weights to all senders and receivers. The social planner's problem solves:

$$\max_{v\in\{NV,PV\}} U_{SP}^v = \beta_{S1}U_{S1}^v + \beta_{S2}U_{S2}^v + \beta_{R1}U_{R1}^v + \beta_{R2}U_{R2}^v.$$

Note that receiver verification, sender verification, and total welfare maximization can be interpreted as special cases: (1) Regulator R0's preference is given by a welfare weight vector $(0, 0, 1, \beta_R)$; (2) Regulator S0's preference is $(1, \beta_s, 0, 0)$; and (3) total welfare is $(1, 1, 1, \beta_R)$.

Proposition 5. *Assume: (1)* $0 < \frac{1}{k} \leq \frac{\beta_{R2}}{\beta_{S2}} \leq k$*; (2)* $\alpha^2 - \lambda(1-\alpha)^2 > 0$*; and (3)* $\rho_{NV} = \rho_{PV}$*. Then, non-verification is optimal when* $\frac{(\beta_{R2}+\beta_{S2})}{(\beta_{R1}+\beta_{S1})} \geq (1 + k) * \max\{\widehat{\beta}_S, \widehat{\beta}_R\}$ *holds.*

We first clarify the conditions in Proposition 5. Condition (1) requires that β_{S2} and β_{R2} are comparable. Condition (2) ensures $U_{R2}^{NV} > U_{R2}^{PV}$. Condition (3) assumes constant signal quality, which holds when $Q > \max\{\overline{Q}, \underline{Q}\}$. These conditions can be relaxed. For Condition (1), it suffices that β_{R2} or β_{S2} dominates the combined weights of the other three weights. For Condition (3), variable signal quality can be considered, though at the cost of more complex algebra and equilibrium characterization.

Proposition 5 shows that diffusion-driven non-verification is generally optimal if the regulator places high welfare weights on senders and receivers with strong demand for information diffusion. In practice, S1 and R1 can be viewed as elite players, such as established media outlets and educated subscribers (e.g., those who subscribe to Elsevier), who are more concerned with duplication bias. In contrast, S2 and R2 represent grassroots participants, such as influencers and individuals who cannot afford costly information sources.

5 General Theory and Real-World Applications

5.1 General Verification Rules

This section analyzes a general verification design problem, where R0 can choose any posterior distribution subject only to the Bayes' rule constraint described by Equation (6). We begin by relaxing the assumption of zero subscription cost. Instead, we introduce a positive subscription cost $c_R > 0$ for R1 to subscribe to S2. For simplicity, we still assume zero subscription costs in all other interactions. Furthermore, we assume:

Assumption 1 (Subscription Cost and Tie-breaking Rule)**.**
(i) $c_R < \bar{c}_R := \frac{(1-2\rho)^2\left(1-\alpha(1-2\rho)^2\right)}{8(2\rho^2-2\rho+1)^2}$*; and*
(ii) R1 subscribes to S2 when indifferent between subscribing and not subscribing (to R2).

Under Assumption 1, R1's subscription decision follows a cutoff rule: there exists $\bar{\lambda}(c_R)$ such that R1 subscribes to S2 (and S1) if and only if $\lambda > \bar{\lambda}(c_R)$.[8] Furthermore, the cutoff $\bar{\lambda}(c_R)$ increases strictly in c_R because $\frac{d(\Delta U_{R1})}{d\lambda}$. Finally, the tie-breaking rule in Assumption 1 ensures that R2's payoff is right-continuous at upward jumps; otherwise, the optimal verification rule is only asymptotically optimal.

[8]First, if $c_R \geq \bar{c}_R$, R1 never subscribes to S2, making the case trivial. Second, recall that $\Delta U_{R1} = U_{R1}(\{S1, S2\}) - U_{R1}(\{S1\})$ and Equation (11) implies:

$$\frac{d(\Delta U_{R1})}{d\lambda} = \frac{8\lambda\rho^2(2\rho^2-3\rho+1)^2\left(\alpha^2(\lambda-1)-\alpha\lambda(1-2\rho)^2+1\right)}{(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1)^2(\alpha(1-\lambda)+\lambda(1-2\rho)^2+1)^2} > 0$$

for all $\lambda > 0$ and $\rho \in (1/2, 1)$. Thus, ΔU_{R1} increases strictly in λ and reaches its maximum $\bar{c}_R$ at $\lambda = 1$. By the Intermediate Value Theorem and strict monotonicity, there exists a unique $\bar{\lambda}(c_R) \in (0,1)$ such that R1 subscribes to S2 if and only if $\lambda > \bar{\lambda}(c_R)$. Formally: $\bar{\lambda}(c_R) = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where $a = -\left(\alpha-(1-2\rho)^2\right)^2 c_R - 4\rho^2(2\rho^2-3\rho+1)^2 < 0$, $b = 2\alpha c_R\left(\alpha-(1-2\rho)^2\right)$, and $c = (1-\alpha^2)c_R$.

We first calculate the receivers' payoff with a positive subscription cost c_R. For R1:

$$U_{R1}(\lambda|c_R) = \begin{cases} U_{R1}^{NV} - c_R, & \text{if } \lambda \geq \bar{\lambda}(c_R) \\ -\rho(1-\rho), & \text{if } \lambda < \bar{\lambda}(c_R) \end{cases} \tag{24}$$

Similarly, for R2:

$$U_{R2}(\lambda|c_R) = \begin{cases} \frac{\lambda^2(1-2\rho)^2-1}{4}, & \text{if } \lambda < \bar{\lambda}(c_R); \\ U_{R2}^{NV}, & \text{if } \lambda \geq \bar{\lambda}(c_R) \end{cases} \tag{25}$$

where U_{R2}^{NV} and U_{R1}^{NV} are displayed in Equation (12).

Then, we investigate the incentives for verification. By the concavification result (Kamenica and Gentzkow, 2011), the highest payoffs achievable for R1 and R2 are given by:

$$V_{R1}(\lambda) = \sup\{U|(\lambda, U) \in co(\{(\lambda, U) \in \mathbb{R}^2 : U = U_{R1}(\lambda|c_R)\})\},$$
$$V_{R2}(\lambda) = \sup\{U|(\lambda, U) \in co(\{(\lambda, U) \in \mathbb{R}^2 : U = U_{R2}(\lambda|c_R)\})\}.$$

Proposition 6 (Receivers' Preferences for Verification)**.** *Assume Assumption 1 holds.*

(i) R1 always prefers perfect verification.

(ii) For R2: (a) If $\bar{\lambda}(c_R)(1-\alpha)^2 \geq \alpha^2$, R2 also prefers perfect verification; and (b) If $\bar{\lambda}(c_R)(1-\alpha)^2 < \alpha^2$, R2 prefers partial verification. Specifically, there exists $m \in \{m_1, m_2\}$ such that $\nu(FH \mid m) = \bar{\lambda}(c_R)$ for any $\lambda \in (0,1)$.

Figure 10 illustrates Proposition 6 (ii). The solid blue line and dashed red line represent R2's payoff U_{R2} without information design and its concavification $co(U_{R2})$ under the optimal information design, respectively. Fix an arbitrary prior $\lambda \in (0, \bar{\lambda}(c_R))$. The optimal verification rule satisfies $\nu(FH \mid m) \in \{0, \bar{\lambda}(c_R)\}$. When $\nu(FH \mid m) = 0$, the authenticity of the secondary source S2 is fully revealed (with certain signals). When $\nu(FH \mid m) = \bar{\lambda}(c_R)$, it generalizes the idea of non-verification in the presence of a positive subscription cost ($c_R > 0$), as this belief makes R1 indifferent between subscribing and not subscribing to S2. Similarly, for $\lambda \in (\bar{\lambda}(c_R), 1)$, the optimal verification rule satisfies $\nu(FH \mid m) \in \{\bar{\lambda}(c_R), 1\}$. Therefore, for any prior belief $\lambda \in (0,1)$, R2 strictly prefers partial verification.

Proposition 6 extends the key insights of the benchmark model to a more general information design framework. The key findings remain valid: R1 always prefers perfect verification,

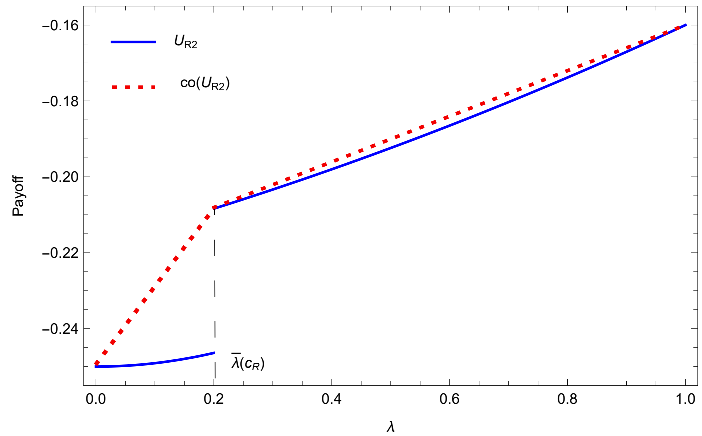


Figure 10: General Information Design

while R2 benefits from some degree of non-verification. Corollary 2 below further shows that for any prior $\lambda \in (0,1)$ (representing the belief that S2 is a primary source), perfect verification is suboptimal whenever information diffusion is sufficiently important.

Corollary 2 (Optimal Partial Verification)**.** *Suppose that* $\bar{\lambda}(c_B)(1-\alpha)^2 < \alpha^2$ *and Assumption 1 holds. Then, for any* $\lambda \in (0,1)$*, there exists* $\widetilde{\beta}_R(\lambda)$ *such that partial verification is optimal for* $\beta_R \geq \widetilde{\beta}_R(\lambda)$*.*

5.2 General Networks

We then extend our baseline model to general networks.

Senders. Let $\mathcal{S}$ denote the set of senders. Among these senders, there are two categories: $\mathcal{S}_1$ and $\mathcal{S}_2$. First, $\mathcal{S}_1 = \{S_1, \ldots, S_k\}$ denotes the set of publicly known first-hand information providers (Type-I senders). These senders receive independent signals from a common DGP. Second, $\mathcal{S}_2 = \mathcal{S} \backslash \mathcal{S}_1$ denotes the set of potentially second-hand information providers (Type-II senders). Each sender in $\mathcal{S}_2$ can copy information from at most one sender in $\mathcal{S}_1$, and

$$\mathcal{S}_2 = \{S_{11}, \ldots, S_{1M}, \ldots, S_{k1}, \ldots, S_{kM}\},$$

where: (i) senders $S_{i1}, \ldots, S_{iM}$ can copy information only from sender S_i, for $i = 1, 2, \ldots, k$; and (ii) $M < \infty$ describes the relative abundance of Type-I and Type-II senders. Suppose that for any sender in $\mathcal{S} \backslash \mathcal{S}^*$, the probability of being a first-hand information provider is λ, where $\lambda \in (0,1)$.

Receivers. Let $\mathcal{R} = \{R_1, \ldots, R_n\}$ denote the set of receivers, who choose which senders to listen to. Each receiver R_i draws uniformly a fixed number $d_i > 0$ of senders from the set $\mathcal{S}$. These senders, denoted by $\mathcal{N}_i$, are referred to as the potential information providers of R_i, and d_i is referred to as the degree of receiver R_i. For simplicity, assume that all receivers have the same degree, denoted by d, where $d < k$, although the analysis can be readily extended to the case of heterogeneous degrees. Note that linking to a sender only implies that receiver R_i is aware of their existence; whether she can actually subscribe depends on the verification policy. Furthermore, parameters are set such that without verification, all senders are willing to produce information, whereas with verification, only first-hand information providers are willing to do so.

Regulator. A regulator observes the realized social network and then chooses a verification policy.[9] As in the benchmark model, the regulator faces a trade-off between duplication bias and the spread of information. One key difference from the benchmark model is that when a receiver is linked to both a first-hand and a second-hand information provider, it is not necessarily optimal to verify. This is because even if the receiver subscribes to a second-hand provider, that provider may not be copying from the first-hand provider she is connected to. In such cases, the second-hand provider may offer independent information and thus not contribute to duplication bias.

5.2.1 Optimal Verification in Large Networks

To obtain a sharp characterization, we focus on situations where the network size is very large. This section discusses the prevalence of duplication bias in large networks and its implications for the optimal verification rule.

A benchmark: non-verification in large networks. To build intuition, we first consider a simple case in which only k, the number of Type-I senders, becomes very large, while other aspects of the network remain fixed. We state the following lemma:

Lemma 7. *As $k \to +\infty$, the optimal verification policy converges to non-verification.*

The idea behind this lemma is straightforward. Since each receiver has d neighbors, the probability that all neighbors' information comes from independent sources approaches 1 as

[9] This corresponds to ex post regulation. One could alternatively model an ex ante perspective in which the regulator sets the policy without observing the network.

$k \to +\infty$. Specifically, this probability is:

$$P \equiv \frac{\binom{k}{d}(M+1)^d}{\binom{k(M+1)}{d}} \approx 1 - \frac{C}{k} \quad \text{when } k \text{ is large,}$$

where $C > 0$ is a constant. Hence, $P \to 1$ as $k \to +\infty$. Note that the number of receivers is fixed at n, so the probability that all receivers obtain signals from independent sources is:

$$\Psi \equiv P^n \approx \left(1 - \frac{C}{k}\right)^n \quad \text{when } k \text{ is large,}$$

which also approaches 1 as $k \to +\infty$. As a result, duplication bias gradually vanishes, and the optimal strategy is **not** to verify. The intuition, as noted earlier, is that when the number of independent sources grows large, duplication bias becomes negligible, and verification is no longer necessary.

General characterizations. It is worth noting that Proposition 7 is built on several assumptions: (i) the size of the receiver set, n, is fixed; (ii) each receiver has a fixed number of neighbors, d; and (iii) the ratio of Type-I to Type-II senders, captured by M, is also fixed. The following theorem relaxes these assumptions:

Theorem 2. *Suppose that $k \to +\infty$, and let $n = O(k^\alpha)$, $d = O(k^\beta)$, and $M = O(k^\gamma)$, where $\alpha, \gamma \geq 0$ and $0 \leq \beta < 1$.*[10] *Then, as $k \to +\infty$, we have:*

1. *If $\alpha + 2\beta < 1$, duplication bias disappears almost surely, and the optimal verification policy converges almost surely to non-verification.*

2. *If $\alpha + 2\beta = 1$, duplication bias occurs with positive probability, and the optimal verification policy converges to non-verification with positive probability.*

3. *If $\alpha + 2\beta > 1$, duplication bias occurs almost surely for some receivers, and the optimal policy depends on the trade-off between duplication bias and information diffusion.*

Theorem 2 shows that the persistence of duplication bias depends on the magnitude of $\alpha + 2\beta$, which reflects the number of receivers n and the degree of the network d. In contrast, the ratio of Type-I to Type-II senders (captured by M) does not affect the result.

[10] We require $\beta < 1$ because if $\beta > 1$, then d would exceed the number of groups k; the case $\beta = 1$ is analytically more complex.

This theorem also provides a simple criterion to determine whether duplication bias persists in large networks. Define the index:

$$E = \frac{\text{Number of receivers} \times (\text{Degree of the network})^2}{\text{Number of senders}}.$$

Then, if E is very small, duplication bias is of little concern, and the regulator should not verify the authenticity of information sources. Conversely, if E is very large, the duplication bias arises with high probability. In this case, verification helps eliminate duplication bias, and optimal policy depends on a careful trade-off between information diffusion and bias mitigation. We formally state this result below:

Corollary 3. *Suppose that the assumptions of Theorem 2 hold and that M is fixed. Then, as $k \to +\infty$, we have:*

1. *If $E \to 0$, then the duplication bias disappears almost surely.*
2. *If $E \to c \in (0, \infty)$, then the duplication bias occurs with a positive probability.*
3. *If $E \to +\infty$, then the duplication bias occurs almost surely.*

Intuitively, the index E captures the effective "pressure" placed on the information network. When this load is light (i.e., E is small), receivers are unlikely to converge on the same information source. As a result, the information environment remains naturally diversified and robust to redundancy, even without regulatory intervention. However, as E grows large, the likelihood that multiple receivers unknowingly rely on the same underlying source increases sharply, especially when many neighbors are indirectly connected through overlapping second-hand senders. In this regime, duplication bias becomes a systemic concern: Without verification, the network may falsely appear to convey a diverse set of opinions, while in reality it echoes the same information multiple times. Thus, the index E serves as a useful proxy for predicting when regulatory intervention is necessary — by source verification — to preserve the integrity of information aggregation in large-scale systems.

5.3 Equivalent Finite Networks and Real-World Applications

First, we present an equivalent network structure in Figure 11, which may appear more natural. Specifically, two senders, S1 (primary) and S2 (potentially second hand), each randomly interact with separate groups of followers, while also sharing a common receiver, R1. Note that R3's payoff is unaffected by the verification rule in the baseline model.

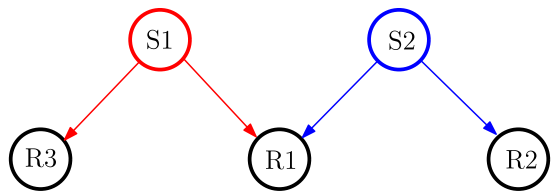


Figure 11: An Equivalent Network

Second, we examine two alternative network structures to show the robustness and practical relevance of our theory: one with randomly formed connections and another with an influencer–follower structure.

Advisory by experts versus charlatans. Figure 12 presents a network with randomly formed connections. Two information sources, S1 (primary) and S2 (potentially secondary), interact with a single receiver R (or a continuum of infinitesimal receivers). Receiver R meets S1 with probability $p_1 \geq 0$, S2 with $p_2 > 0$, and both with $p_{1,2} > 0$. Including a noncontact probability $p_{\varnothing}$ does not affect the analysis.

A second-hand S2 faces an information production cost $c_S > 0$, which is too high to justify when interacting with R alone but becomes worthwhile S2 is also rewarded when interacting with R and S1 together. By setting $\beta_R = p_2/p_{1,2}$, the model reduces to our baseline framework.

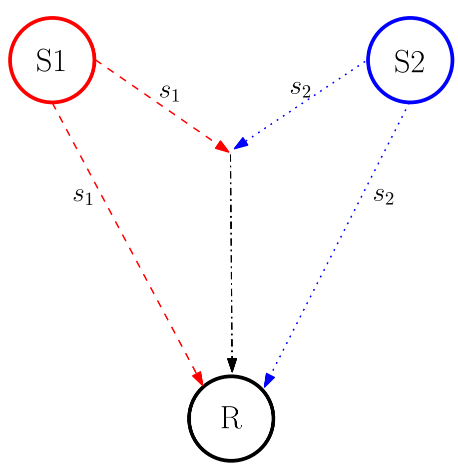


Figure 12: A Network with Random Contact

This network structure can also be used to describe policy advising by experts vs. charlatans. Here, S1 is an expert and S2 can potentially be a charlatan. Is it always a good idea for the receiver to debunk the charlatans? Our theory posits that removing charlatans can help make more informed decisions when multiple information sources are available, but it reduces the availability of valuable information when it is likely to meet charlatans.

Information transmission on digital platforms. Our main insight extends to more general and realistic network structures. A direct application is the influencer and follower networks commonly found on digital social media platforms, such as SeekingAlpha and Snowball, which are the two largest investing communities in the United States and China.

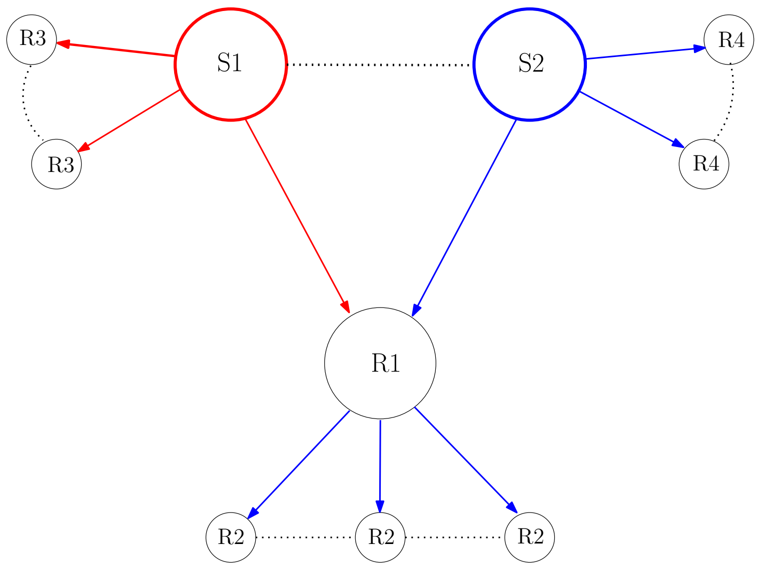


Figure 13: An Influencer-follower Network

Figure 13 presents a network with two information sources or senders, S1 (primary) and S2 (potentially secondary). R1 acts as an influencer, also known as a Key Opinion Leader (KOL) on social platforms, or can be interpreted as a large receiver with n_{R1} smaller follower receivers, denoted R2. In addition, a group of n_{R2} small receivers R4 directly follow S2, while another group of n_{R3} small receivers R3 directly follow S1.

Beyond their authenticity, S1 and S2 also differ in their outreach ability. For example, S2 can reach R2 through R1, whereas S1 cannot. This asymmetry may arise because S1's information is restricted from being shared on social platforms due to intellectual property protection, is too costly for R2 to access, or requires specialized expertise to interpret. In contrast, R3 may possess the wealth or expertise needed to process S1's signals, while R2 does not. R1 can use S2's information for free, provided that she is willing to share it with her followers, the R2 group, by posting it on the platform. S2 incurs a fixed cost c_S to copy S1's information. This cost is greater than the payoff from reaching only R4, but smaller than the payoff from also reaching a portion of R2.

Let I_B and I_S denote the investment sizes of the large receiver R1 and the small receivers, respectively. If the regulator aims to maximize the total payoff of all receivers, the setting is equivalent to the baseline model with $\beta_R = (n_{R4} + n_{R2}) \cdot I_S / I_B$.

6 Conclusion

We study the optimal verification of information sources when secondary providers can copy from primary ones. A novel trade-off emerges because perfect verification reduces duplication bias for decision makers but may discourage information diffusion. Our theory yields several key insights. First, non-verification can outperform perfect verification when the demand for information diffusion is high, access to primary information is limited, noise in duplicated information is low, or information providers have strong bargaining power. Second, information users tend to rely more on non-verification to counter the market power of providers, while concerns about information quality lead to more verification. Third, strong IP protection encourages improvements in information quality, but affects total welfare in a non-monotonic way and may even reduce it. Fourth, higher information quality, better access to primary sources, and less noise in the duplication process help reduce misinformation, but the effects of verification and IP protection on aggregate misinformation are ambiguous.

Overall, this paper challenges the conventional view of correlation neglect as a behavioral bias by providing a new, ex-ante rational microfoundation. We demonstrate that the core trade-off between mitigating duplication bias and encouraging information diffusion can lead a regulator or platform to strategically commit to not verifying its sources. Our results thus highlight a fundamental misalignment between two common regulatory goals: maximizing user welfare and minimizing misinformation. In fact, for strategic reasons—such as extracting bargaining surplus or committing to broader information diffusion—the welfare-maximizing policy for a platform may be one that tolerates a significant, and perhaps even greater, level of aggregate misinformation. Promising directions for future research include designing optimal verification rules in networks and testing our theory using data from social media platforms.

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Appendix

A Derivations and Proofs

A.1 Auxilliary Results

Lemma A.1. *Assume that* $\lambda \leq \frac{1}{3}$*. Then:*
(i) $U_{S1}^{NV}(0, \overline{\rho}) - U_{S1}^{NV}(0, \underline{\rho}) > 0$*; and*
(ii) $\frac{\partial^2}{\partial\alpha\partial\rho} U_{S1}^{NV}(\alpha, \rho|\lambda) < 0$.

Proof. See the Internet Appendix B.1. □

Lemma A.2. *(i)* $\frac{\partial U_{R1}^{NV}}{\partial\alpha} > 0$, $\frac{\partial U_{R2}^{NV}}{\partial\alpha} > 0$, $\frac{\partial U_{R2}^{NV}}{\partial\rho} > 0$*; and (ii) Assume* $\lambda \leq \frac{1}{2}$, $\frac{\partial U_{R1}^{NV}}{\partial\rho} > 0$

Proof. From Equation (12),

$$\frac{\partial U_{R1}^{NV}}{\partial\alpha} = \frac{8(1-\lambda)(\rho-1)^2\rho^2(\lambda-2\lambda\rho)^2\left(\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2+1\right)^2},$$
$$\frac{\partial U_{R2}^{NV}}{\partial\alpha} = \frac{1}{2}(1-\lambda)(1-2\rho)^2(\alpha(1-\lambda)+\lambda),$$
$$\frac{\partial U_{R2}^{NV}}{\partial\rho} = (2\rho-1)(\alpha(-\lambda)+\alpha+\lambda)^2.$$

Obviously, all three partial derivatives are positive.

Note that the proof of $\frac{\partial U_{R1}^{NV}}{\partial\rho} > 0$ follows an approach similar to that of Lemma A.1 and the formal proof is relegated to Appendix B.2. □

A.2 Proof of Equation (9)

We can verify equation (9) using Bayes' rule. Specifically:

$$\Pr\left(1|s_1, s_2, \text{S2 is SH}\right) = \frac{\Pr(s_1, s_2, \text{S2 is SH}, \theta=1)}{\Pr(s_1, s_2, \text{S2 is SH}, \theta=1) + \Pr(s_1, s_2, \text{S2 is SH}, \theta=0)}$$

where

$$\Pr(s_1, s_2, \text{S2 is SH}, \theta=1) = p(1)p\left(s_1|1\right)p(\text{S2 is SH}|1, s_1)p\left(s_2|1, s_1, \text{S2 is SH}\right)$$
$$\Pr(s_1, s_2, \text{S2 is SH}, \theta=1) = p(0)p\left(s_1|0\right)p(\text{S2 is SH}|1, s_1)p\left(s_2|0, s_1, \text{S2 is SH}\right)$$

Thus,

$$\Pr\left(1|s_1, s_2, \text{S2 is SH}\right) = \begin{cases} \frac{p(s_1|1)*\frac{(\alpha+1)}{2}}{p(s_1|1)*\frac{(\alpha+1)}{2}+p(s_1|0)*\frac{(\alpha+1)}{2}}, & \text{if } s_1 = s_2 \\ \frac{p(s_1|1)*\frac{(1-\alpha)}{2}}{p(s_1|1)*\frac{(1-\alpha)}{2}+p(s_1|0)*\frac{(1-\alpha)}{2}}, & \text{if } s_1 \neq s_2 \end{cases}$$
$$= \Pr\left(1|s_1\right)$$

A.3 Proof of Equation (10)

Proof. First, we can compute the optimal decision rule for R1, that is,

$$a^*_{R_1}(s_1, s_2) = \mathbb{E}[\theta|s_1, s_2] = \pi(1|s_1, s_2)$$

which can be further expressed as:

$$\begin{aligned} \pi(1|h,h) &= \mu_1 \times \frac{\rho^2}{\rho^2 + (1-\rho)^2} + (1-\mu_1)\rho \\ \pi(1|h,l) &= \mu_2 \times \frac{1}{2} + (1-\mu_2)\rho \\ \pi(1|l,h) &= \mu_2 \times \frac{1}{2} + (1-\mu_2)(1-\rho) \\ \pi(1|l,l) &= \mu_1 \times \frac{(1-\rho)^2}{\rho^2 + (1-\rho)^2} + (1-\mu_1)(1-\rho) \end{aligned}$$

Second, we need to compute the probabilities $\Pr(s_1, s_2, \theta)$ as below.

$$\begin{aligned} \Pr(h,h,1) &= \frac{1}{2} \times \left(\lambda\rho^2 + (1-\lambda)\rho * \frac{(1+\alpha)}{2}\right), \\ \Pr(h,l,1) &= \frac{1}{2} \times \left(\lambda\rho(1-\rho) + (1-\lambda)\rho * \frac{(1-\alpha)}{2}\right) \\ \Pr(l,h,1) &= \frac{1}{2} \times \left(\lambda\rho(1-\rho) + (1-\lambda)(1-\rho) * \frac{(1-\alpha)}{2}\right), \\ \Pr(l,l,1) &= \frac{1}{2} \times \left(\lambda(1-\rho)^2 + (1-\lambda)(1-\rho) * \frac{(1+\alpha)}{2}\right) \end{aligned}$$

and

$$\begin{aligned} \Pr(h,h,0) = \Pr(l,l,1), \quad \Pr(h,l,0) = \Pr(l,h,1) \\ \Pr(l,h,0) = \Pr(h,l,1), \quad \Pr(l,l,0) = \Pr(h,h,1) \end{aligned}$$

Finally, we can compute the expected payoff using the following equation:

$$U_{R1}(\{S1, S2\}) = \sum_{\theta \in \Theta} \sum_{(s_1,s_2) \in h,l^2} -(\theta - a^*_{R1}(s_1, s_2))^2 \times \Pr(s_1, s_2, \theta).$$

where $U_{R1}(\{S1, S2\})$ is the expected utility for R1 when she subscribes to both S1 and S2. The proof concludes. □

A.4 Proof of Lemma 2

Proof. The proof consists of two steps.

Step (1): Calculating U_i^j where $i \in \{R1, R2\}$ and $j \in \{NV, PV\}$.

(i) U_{R1}^{NV}. Given non-verification, it follows from Equation (10).

(ii) U_{R1}^{PV}. Given perfect verification: (a) with probability λ, both S1 and S2 are first-hand, and the expected utility can be obtained by plugging $\lambda = 1$ in U_{R1}^{NV}; and (b) with probability $(1-\lambda)$, S2 is second-hand, and the expected utility can be computed by plugging $\lambda = 0$ in U_{R1}^{NV}.

(iii) U_{R2}^{PV}. Given perfect verification: (a) with probability λ, S2 is a first-hand data provider, and the expected utility given by equation (7); and (b) with probability $(1-\lambda)$, S2 is a second-hand data provider. Note that given Lemma 1 (ii), S2 has no incentive to copy data, and thus $\mathbb{E}[\theta|s_2 = \varnothing] = \frac{1}{2}$ and an expected utility of $-\frac{1}{4}$.

(iv) U_{R2}^{NV}. We can mimic the derivation of $U_{R1}(\{S1, S2\})$. First, we can compute the optimal decision rule for R2, that is, $a_{R_2}^*(s_2) = \mathbb{E}[\theta|s_2] = \pi(1|s_2)$. Furthermore,

$$\pi(1|s_2) = \Pr(\text{S2 is FH}|s_2) \times p(1|s_2, \text{S2 is FH}) + \Pr(\text{S2 is SH}|s_2) \times p(1|s_2, \text{S2 is SH}) \quad (A.1)$$

Note that $\Pr(\text{S2 is FH}|s_2) = \lambda$ and, $\Pr(\text{S2 is FH}|s_2) = (1-\lambda)$.[11]

Now, we need to compute $\Pr(1|s_2, \text{S2 is FH})$ and $\Pr(1|s_2, \text{S2 is SH})$. Obviously,

$$\Pr(1|s_2, \text{S2 is FH}) = \begin{cases} \rho, & \text{if } s_2 = h \\ 1-\rho, & \text{if } s_2 = l \end{cases}$$

[11]This can be derived using Bayes' rule. Specifically:

$$\begin{aligned}\Pr(\text{S2 is FH}|s_2 = h) &= \frac{\Pr(\text{S2 is FH}, s_2 = h)}{\Pr(\text{S2 is FH}, s_2 = h) + \Pr(\text{S2 is SH}, s_2 = h)} \\ &= \frac{\Pr(\text{S2 is FH})\Pr(s_2 = h|\text{S2 is FH})}{\Pr(\text{S2 is FH})\Pr(s_2 = h|\text{S2 is FH}) + \Pr(\text{S2 is SH})\Pr(s_2 = h|\text{S2 is SH})}\end{aligned}$$

Furthermore, $\Pr(s_2 = h|\text{S2 is FH}) = \sum_{\theta \in \{0,1\}} \Pr(s_2 = h|\theta)\Pr(\theta) = \rho * \frac{1}{2} + (1-\rho) * \frac{1}{2} = \frac{1}{2}$ and

$$\begin{aligned}&\Pr(s_2 = h|\text{S2 is SH}) = \Pr(\theta = 1)\Pr(s_2 = h|\theta = 1, \text{S2 is SH}) + \Pr(\theta = 0)\Pr(s_2 = h|\theta = 0, \text{S2 is SH}) \\ &= \frac{1}{2} \times \left(\rho \times \frac{(1+\alpha)}{2} + (1-\rho) \times \frac{(1-\alpha)}{2}\right) + \frac{1}{2} \times \left(\rho \times \frac{(1-\alpha)}{2} + (1-\rho) \times \frac{(1+\alpha)}{2}\right) = \frac{1}{2}\end{aligned}$$

Thus, $\Pr(\text{S2 is FH}|s_2 = h) = \lambda$. Similarly, we can show $\Pr(\text{S2 is FH}|s_2 = l) = \lambda$.

Second, we can compute $\Pr(1|s_2, \text{S2 is SH})$ as below. By Bayes' rule, we have:

$$\Pr(1|s_2, \text{S2 is SH}) = \frac{\Pr(\theta = 1, s_2|\text{S2 is SH})}{\Pr(\theta = 1, s_2|\text{S2 is SH}) + \Pr(\theta = 0, s_2|\text{S2 is SH})}$$

We can also directly compute:

$$\Pr(\theta = 1, s_2 = h|\text{S2 is SH}) = \frac{1}{2} \times \left(\rho \times \frac{(1+\alpha)}{2} + (1-\rho) \times \frac{(1-\alpha)}{2}\right)$$
$$\Pr(\theta = 0, s_2 = h|\text{S2 is SH}) = \frac{1}{2} \times \left((1-\rho) \times \frac{(1+\alpha)}{2} + \rho \times \frac{(1-\alpha)}{2}\right)$$

Thus, $\Pr(1|s_2 = h, \text{S2 is SH}) = \frac{(1-\alpha)}{2} + \alpha\rho$. Analogously, we can show that

$$\Pr(1|s_2 = l, \text{S2 is SH}) = \frac{(1+\alpha)}{2} - \alpha\rho.$$

Therefore, using equation (A.1), we get:

$$\pi(1|s_2) = \begin{cases} \lambda\rho + (1-\lambda) \times \left(\frac{(1-\alpha)}{2} + \alpha\rho\right), & \text{if } s_2 = h \\ \lambda(1-\rho) + (1-\lambda) \times \left(\frac{(1+\alpha)}{2} - \alpha\rho\right), & \text{if } s_2 = l \end{cases}$$

Third, we need to compute the probabilities $\Pr(s_2, \theta) = \frac{1}{2}\Pr(s_2|\theta)$ for $s_2 \in \{h, l\}$ and $\theta \in \{0, 1\}$. Specifically:

$$\begin{aligned} \Pr(s_2 = h|1) &= \Pr(\text{S2 is FH})\Pr(s_2 = h|1) + \Pr(\text{S2 is SH}) \\ &\quad \times (\Pr(s_1 = h|1)\Pr(s_2 = h|s_1 = h, 1) + \Pr(s_1 = l|1)\Pr(s_2 = h|s_1 = l, 1)) \\ &= \lambda\rho + (1-\lambda) \times \frac{\rho(1+\alpha) + (1-\rho)(1-\alpha)}{2} \end{aligned}$$

Similarly, we can show that

$$\Pr(s_2 = h|0) = \lambda(1-\rho) + (1-\lambda) \times \frac{(1-\rho)(1+\alpha) + \rho(1-\alpha)}{2},$$

and $\Pr(s_2 = l|1) = 1 - \Pr(s_2 = h|1)$, $\quad \Pr(s_2 = l|0) = 1 - \Pr(s_2 = h|0)$.

Finally, we can compute the expected payoff using the following equation:

$$U_{R2}^{NV} = \sum_{\theta\in\Theta}\sum_{s_2\in h,l} -(\theta - a_{R2}^*(s_2))^2 \times \Pr(s_2, \theta).$$

Step (2): Comparing utilities. First, we consider R1. Note that

$$\frac{\partial U_{R1}^{NV}}{\partial\alpha} = \frac{8(1-\lambda)(\rho-1)^2\rho^2(\lambda-2\lambda\rho)^2\left(\alpha(1-\lambda) + \lambda(1-2\rho)^2\right)}{\left(\alpha(\lambda-1) - \lambda(1-2\rho)^2+1\right)^2\left(\alpha(1-\lambda) + \lambda(1-2\rho)^2+1\right)^2} > 0$$

Thus, U_{R1}^{NV} is maximized when $\alpha = 1$, that is,

$$U_{R1}^{NV} \leq \lim_{\alpha \to 1} U_{R1}^{NV} = -\frac{(2-\lambda)(1-\rho)\rho}{4\lambda(\rho-1)\rho+2}$$

Furthermore, note that

$$U_{R1}^{PV} - U_{R1}^{NV} \geq U_{R1}^{PV} - \lim_{\alpha \to 1} U_{R1}^{NV} = \frac{(1-\lambda)\lambda\rho^2\left(2\rho^2-3\rho+1\right)^2}{(2(\rho-1)\rho+1)(2\lambda(\rho-1)\rho+1)} > 0$$

Second, we consider R2. We can directly compute

$$U_{R2}^{NV} - U_{R2}^{PV} = \frac{1}{4}(1-\lambda)(1-2\rho)^2\left(\alpha^2 - \lambda(1-\alpha)^2\right)$$

Thus, $U_{R2}^{NV} - U_{R2}^{PV} \geq 0$ if and only if $(\alpha^2 - \lambda(1-\alpha)^2) \geq 0$. The proof concludes. □

A.5 Proof of Proposition 1

Proof. The condition $Q > \max\{\overline{Q}, \underline{Q}\}$ ensures that $\rho_{NV}^* = \rho_{PV}^* = \underline{\rho}$. Anticipating that $\rho_{NV}^* = \rho_{PV}^* = \underline{\rho}$, the argument preceding Proposition 1 (see Equations (18) and (19)) implies that the conditions in case (I) guarantee that the regulator R0 optimally selects $v^* = NV$. Under non-verification, R1 subscribes to S2, and S2 subsequently copies S1's signal.

Similarly, when the conditions in case (II) hold, the regulator R0 optimally selects $v^* = PV$. Under perfect verification, R1 does not subscribe to S2, and hence S2 does not copy S1's signal. □

A.6 Proof of Corollary 1

Proof. Note that under fixed pricing or when S1 has no bargaining power (i.e., $\gamma = 0$), we have $\rho^* = \underline{\rho}$. The corollary then follows directly from Equations (18) and (19). □

A.7 Proof of Lemma 3

Proof. We first show that $\frac{\partial U_{S1}^{PV}}{\partial \rho} > 0$ and $\frac{\partial}{\partial \rho}(U_{S1}^{PV} - U_{S1}^{NV}) > 0$.

Using Equation (13), we compute:

$$\frac{\partial U_{S1}^{NV}}{\partial \rho} = \frac{\lambda(2\rho-1)\left(-8\lambda\rho^2(1-\rho)^2 - 8\rho^2 + 8\rho - 1\right)}{2(2\lambda(\rho-1)\rho+1)^2},$$
$$\frac{\partial U_{S1}^{PV}}{\partial \rho} = \frac{(2\rho-1)\left(2(2\rho^2-2\rho+1)^2 - \lambda(1-2\rho)^2(4\rho^2-4\rho+3)\right)}{2(2\rho^2-2\rho+1)^2}.$$

Note that $\frac{\partial U_{S1}^{PV}}{\partial \rho}$ strictly decreases in λ, and equals $\frac{4\rho(1-\rho)(2\rho-1)(1-\rho+\rho^2)}{3(2\rho^2-2\rho+1)^2} > 0$ when $\lambda = \frac{2}{3}$.

Next, we compute:

$$\frac{\partial}{\partial\rho}(U_{S1}^{PV} - U_{S1}^{NV}) = \frac{1}{2}(2\rho - 1)\left(2 - \frac{\lambda(4\rho^2 - 4\rho + 3)(1 - 2\rho)^2}{(2\rho^2 - 2\rho + 1)^2} + \frac{\lambda(8\lambda\rho^4 - 16\lambda\rho^3 + 8(\lambda + 1)\rho^2 - 8\rho + 1)}{(2\lambda(\rho - 1)\rho + 1)^2}\right).$$

We can show that

$$\frac{\lambda(4\rho^2 - 4\rho + 3)(1 - 2\rho)^2}{(2\rho^2 - 2\rho + 1)^2} \leq 3\lambda,$$

as the ratio strictly increases in ρ and attains its maximum of 3 at $\rho = 1$.

Define

$$g(\lambda, \rho) := \frac{\lambda(8\lambda\rho^4 - 16\lambda\rho^3 + 8(\lambda + 1)\rho^2 - 8\rho + 1)}{(2\lambda(\rho - 1)\rho + 1)^2},$$

and note that $g(0) = 0$. Moreover,

$$\frac{\partial g}{\partial\lambda} = \frac{-2(\lambda - 4)\rho^2 + 2(\lambda - 4)\rho + 1}{(2\lambda(\rho - 1)\rho + 1)^3} \geq -\frac{1}{(1 - \lambda/2)^2},$$

so integrating gives:

$$g(\lambda) - g(0) \geq \int_0^\lambda -\frac{dt}{(1 - t/2)^2} = -\frac{\lambda}{1 - \lambda/2}.$$

Thus,

$$\frac{\partial}{\partial\rho}(U_{S1}^{PV} - U_{S1}^{NV}) \geq \frac{1}{2}(2\rho - 1)\left(2 - 3\lambda - \frac{\lambda}{1 - \lambda/2}\right),$$

which is positive when $\lambda \leq \frac{1}{3}(5 - \sqrt{13}) \approx 0.46$.

Next, note that

$$\overline{Q} = \int_{\underline{\rho}}^{\overline{\rho}} \frac{\partial U_{S1}^{PV}(\rho)}{\partial\rho} d\rho, \quad \underline{Q} = \int_{\underline{\rho}}^{\overline{\rho}} \frac{\partial U_{S1}^{NV}(\rho)}{\partial\rho} d\rho.$$

The derivatives imply $\overline{Q}(\overline{\rho}, \underline{\rho}) > 0$ and $\overline{Q}(\overline{\rho}, \underline{\rho}) > \underline{Q}(\overline{\rho}, \underline{\rho})$, while $\underline{Q}(\overline{\rho}, \underline{\rho}) < 0$ may occur.

Now, consider three cases:

(1) If $Q > \overline{Q}(\overline{\rho}, \underline{\rho})$, then S1 always chooses $\rho^* = \underline{\rho}$;

(2) If $Q \leq \underline{Q}(\overline{\rho}, \underline{\rho})$, then S1 always chooses $\rho^* = \overline{\rho}$;

(3) If $\underline{Q}(\overline{\rho}, \underline{\rho}) < Q \leq \overline{Q}(\overline{\rho}, \underline{\rho})$, then S1 chooses $\rho^*_{PV} = \overline{\rho}$ and $\rho^*_{NV} = \underline{\rho}$.

The proof concludes. □

A.8 Proof of Proposition 2

Proof. The proof utilizes the necessary and sufficient conditions in Proposition 1.

We first examine condition (i) in Proposition 1. The non-verification condition requires:

$$(1-\gamma)\left(U_{R1}^{PV}(\overline{\rho}) - U_{R1}^{NV}(\underline{\rho})\right) - \gamma\left(U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\overline{\rho})\right) \leq 0 \tag{A.2}$$

Note that all functions, including U_{R1}^{PV}, U_{R1}^{NV}, U_{R2}^{NV}, and V_1, strictly increase in ρ, that is,

$$\begin{aligned}
\frac{\partial U_{R1}^{PV}}{\partial \rho} &= \frac{(2\rho-1)}{2\left(2\rho^2-2\rho+1\right)^2}\left\{2\left(2\rho^2-2\rho+1\right)^2(1-\lambda)+\lambda\right\}, \\
\frac{\partial U_{R2}^{PV}}{\partial \rho} &= \lambda(2\rho-1), \\
\frac{\partial U_{R1}^{NV}}{\partial \rho} &= \frac{(2-\lambda)(2\rho-1)}{2\left(2\lambda(\rho-1)\rho+1\right)^2}, \\
\frac{\partial U_{R2}^{NV}}{\partial \rho} &= (2\rho-1)\left(\alpha(1-\lambda)+\lambda\right)^2.
\end{aligned}$$

Since the coefficients of the terms involving $\overline{\rho}$ in equation (A.2) are positive, the inequality is less likely to be satisfied.

Next, we analyze condition (ii) in Proposition 1. By definition of $\widehat{\beta}_R$,

$$\widehat{\beta}_R(\underline{\rho},\overline{\rho}) = \frac{(1-\gamma)\left(U_{R1}^{PV}(\overline{\rho}) - U_{R1}^{NV}(\underline{\rho})\right) - \gamma\left(U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\overline{\rho})\right)}{U_{R2}^{NV}(\underline{\rho}) - U_{R2}^{PV}(\overline{\rho})}.$$

Observe that all terms involving $\overline{\rho}$ increase the value of $\widehat{\beta}_R$, since they appear with positive coefficients in the numerator and negative coefficients in the denominator. Thus,

$$\widehat{\beta}_R(\underline{\rho},\overline{\rho}) \geq \widehat{\beta}_R(\underline{\rho},\underline{\rho}).$$

This completes the proof. □

A.9 Proof of Lemma 4

Proof. **Part (i).** Deriving misinformation measures.

First, consider *non-verification.*

(1) For R1:

$$M_{R1}^{NV} = \sum_{\theta}\sum_{(s_1,s_2)\in\{h,l\}^2} |\theta - \mathbb{E}[\theta|s_1,s_2]| \times \Pr(s_1,s_2,\theta).$$

Note that both $\Pr(s_1,s_2,\theta)$ and $\mathbb{E}[\theta|s_1,s_2]$ are computed in the proof of Equation (9).

(2) For R2:

$$M_{R2}^{NV} = \sum_{\theta}\sum_{s_2\in\{h,l\}} |\theta - \mathbb{E}[\theta|s_2]| \times \Pr(s_2,\theta).$$

These quantities are derived in the proof of Lemma 2 (see, e.g., step (1)).

Next, consider *perfect verification.*

(1) For R1:

$$M_{R1}^{PV} = \lambda \left(\sum_{\theta} \sum_{(s_1,s_2)\in\{h,l\}^2} |\theta - \mathbb{E}[\theta|s_1,s_2]| \times \Pr(s_1,s_2,\theta) \right) + (1-\lambda) \left(\sum_{\theta} \sum_{s_1\in\{h,l\}} |\theta - \mathbb{E}[\theta|s_1]| \times \Pr(s_1,\theta) \right)$$

Note that s_1 and s_2 are independently realized. Furthermore,

$$\Pr(h,h,1) = \Pr(l,l,0) = \rho^2/2, \quad \Pr(h,l,1) = \Pr(l,h,1) = \rho(1-\rho)/2,$$
$$\Pr(l,l,1) = \Pr(h,h,0) = (1-\rho)^2/2,$$

$$\mathbb{E}[\theta|h] = \rho, \quad \mathbb{E}[\theta|l] = 1-\rho,$$

$$\Pr(h,1) = \Pr(l,0) = \rho/2, \quad \Pr(h,0) = \Pr(l,1) = (1-\rho)/2.$$

$\Pr(s_1,s_2,\theta)$ and $\mathbb{E}[\theta|s_1,s_2]$ are reused from the proof of Equation (9).

(2) For R2:

$$M_{R2}^{PV} = \lambda \left(\sum_{\theta} \sum_{s_2\in\{h,l\}} |\theta - \mathbb{E}[\theta|\text{S2 is FH}, s_2]| \times \Pr(s_2,\theta) \right) + (1-\lambda) \left(\sum_{\theta} |\theta - \mathbb{E}[\theta|s_2 = \varnothing]| \times \Pr(\theta) \right)$$

With probability λ, S2 is a first-hand data provider, which corresponds to the R1 case. With probability $1-\lambda$, S2 has no information, and $|\theta - \mathbb{E}[\theta|s_2 = \varnothing]| = \frac{1}{2}$.

Part (ii). Monotonicity.

First, we compute the partial derivatives of M_{R1}^{NV}:

$$\frac{\partial M_{R1}^{NV}}{\partial \alpha} = -\frac{16(1-\lambda)(\rho-1)^2\rho^2(\lambda-2\lambda\rho)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2\right)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2+1\right)^2}$$
$$\frac{\partial M_{R1}^{NV}}{\partial \lambda} = -\frac{16\lambda(1-2\rho)^2(\rho-1)^2\rho^2\left(1-(1-\lambda)\alpha^2-\alpha\lambda(1-2\rho)^2\right)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2+1\right)^2}$$
$$\frac{\partial M_{R1}^{NV}}{\partial \rho} = -\frac{2(2\rho-1)(k_0+k_1\rho(1-\rho)+k_2\rho^2(1-\rho)^2)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2+1\right)^2}$$

where

$$k_0 = (1-\alpha)^2(1-\lambda)^2(-\alpha\lambda+\alpha+\lambda+1)^2$$
$$k_1 = -8(1-\alpha)(1-\lambda)\lambda(1+\lambda+\alpha(1-\lambda))(\lambda+2\alpha(1-\lambda))$$
$$k_2 = 16\lambda^2(3\alpha^2(\lambda-1)^2+4\alpha\lambda(1-\lambda)+\lambda^2+1)$$

We can verify:

$$\Delta = k_1^2 - 4k_0k_2 = -64(\alpha-1)^2(\lambda-1)^2\lambda^2\left(1-\alpha^2(\lambda-1)^2\right)(-\alpha\lambda+\alpha+\lambda+1)^2 < 0,$$

so all partial derivatives are negative.

Next, we compute the partial derivatives of M_{R1}^{PV}:

$$\frac{\partial M_{R1}^{PV}}{\partial \rho} = -\frac{(2\rho-1)\left(\lambda+2(1-\lambda)(2\rho^2-2\rho+1)^2\right)}{(2\rho^2-2\rho+1)^2}$$
$$\frac{\partial M_{R1}^{PV}}{\partial \lambda} = -\frac{(1-2\rho)^2(1-\rho)\rho}{2\rho^2-2\rho+1}$$

Partial derivatives of M_{R2}^{NV}*:*

$$\frac{\partial M_{R2}^{NV}}{\partial \alpha} = -(1-\lambda)(1-2\rho)^2(\alpha+\lambda(1-\alpha))$$
$$\frac{\partial M_{R2}^{NV}}{\partial \rho} = -2(2\rho-1)(\alpha(1-\lambda)+\lambda)^2$$
$$\frac{\partial M_{R2}^{NV}}{\partial \lambda} = -(1-\alpha)(1-2\rho)^2(\alpha(1-\lambda)+\lambda)$$

Partial derivatives of M_{R2}^{PV}*:*

$$\frac{\partial M_{R2}^{PV}}{\partial \rho} = -\lambda(2\rho-1), \quad \frac{\partial M_{R2}^{PV}}{\partial \lambda} = -\frac{1}{2}(1-2\rho)^2$$

Part (iii). Misinformation comparison.

$$M_{R1}^{NV} - M_{R1}^{PV} = \frac{\lambda(1-\lambda)\rho(1-\rho)(1-2\rho)^2\left(1-\alpha^2+\lambda((1-2\rho)^2-\alpha)^2\right)}{\left(1+\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)\left(1-\alpha(1-\lambda)-\lambda(1-2\rho)^2\right)(2\rho^2-2\rho+1)}$$
$$M_{R2}^{NV} - M_{R2}^{PV} = \frac{1}{2}(1-\lambda)(1-2\rho)^2\left(\lambda(1-2\alpha)-(1-\lambda)\alpha^2\right)$$

Clearly, the first expression is always positive, while the second is non-negative if and only if $\lambda(1-2\alpha)-(1-\lambda)\alpha^2 \geq 0$. This concludes the proof. □

A.10 Proof of Lemma 5

Proof. First, note that the condition $Q > \max\{\overline{Q}, \underline{Q}\}$ implies $\rho_{NV}^* = \rho_{PV}^* = \underline{\rho}$.

Second, we verify the litigation strategy. There are four cases:

Case (i). If S1 sues S2, then with probability $(1-\lambda)$, S2 is second-hand, and the expected return from suing is $(1-\lambda)\widehat{F}$, which is less than the litigation cost L. Thus, S1 does not sue. Moreover, since $L/(1-\lambda) < 2 - c_S$, a second-hand S2 earns a positive rent from copying S1's data.

Case (ii). Since $(1-\lambda)\widehat{F} > L$, S1 sues. Furthermore, as $2 - c_S > \widehat{F}$, S2 optimally copies S1's data, earning rent $2 - c_S - \widehat{F} > 0$.

Case (iii). When $\widehat{F} \geq 2 - c_S$, there exists a mixed-strategy equilibrium where S1 sues with probability $\frac{2-c_S}{\widehat{F}}$, and a second-hand S2 copies with probability $\frac{L}{(1-\lambda)\widehat{F}}$. No pure-strategy equilibrium exists. Let r and t denote the probabilities that S1 sues and that a second-hand S2 copies, respectively.

S1's strategy. Given $t \in [0,1]$, S1 is indifferent if:

$$t \cdot ((1-\lambda)\widehat{F} - L) + (1-t)(-L) + U_{S1}^{NV}(\alpha t, \rho^*) = U_{S1}^{NV}(\alpha t, \rho^*),$$

which implies $t = \frac{L}{(1-\lambda)\widehat{F}}$.

S2's strategy. Given litigation probability $r \in [0,1]$, S2 is indifferent if:

$$r \cdot (2 - c_S - \widehat{F}) + (1-r)(2 - c_S) = 0,$$

which yields $r = \frac{2-c_S}{\widehat{F}}$.

Case (iv). By the definition of $\underline{\alpha}$, $U_{R0}^{NV}(\underline{\alpha}, \rho) = U_{R0}^{PV}(\rho)$. Since $U_{R0}^{NV} = (1-\gamma)U_{R1}^{NV} + (\beta_R + \gamma)U_{R2}^{NV}$ and U_{R0}^{NV} decreases in α (by Lemma A.2), R0 switches to perfect verification when $\widehat{F} \geq \frac{\alpha L}{\underline{\alpha}(1-\lambda)}$, as the probability that S2 copies under non-verification declines with $\widehat{F}$.

Third, the optimality of R0's verification rule follows from Case (iv). Meanwhile, R1's subscription strategy is trivial: under non-verification, R1 subscribes to a second-hand S2.

The proof concludes. □

A.11 Proof of Lemma 6

Proof. We first verify the litigation strategy. There are four cases:

Case (1). The result follows directly from Lemma 5.

Case (2). Again, the result follows from Lemma 5. Note that a second-hand sender S2 always receives a positive payoff from copying S1's data.

Case (3). There is no pure-strategy equilibrium. Let r and t denote the probabilities that S1 sues S2 and that a second-hand S2 copies S1's data. S2's indifference condition is identical to that in Lemma 5. S1 is indifferent if:

$$t(1-\lambda)\widehat{F} - L + U_{S1}^{NV}(\alpha t, \rho^*) = U_{S1}^{NV}(\alpha t, \rho^*),$$

which implies $t = \frac{L}{(1-\lambda)\widehat{F}}$. Given S2's copying strategy, S1's choice of ρ^* is independent of its own litigation strategy.

Next, we characterize S1's optimal data quality ρ^* before analyzing Case (4). For any $\alpha_1 > \alpha_2$ and $\rho_1 > \rho_2$, Lemma A.1 implies:

$$U_{S1}^{NV}(\alpha_1, \rho_1) - U_{S1}^{NV}(\alpha_1, \rho_2) < U_{S1}^{NV}(\alpha_2, \rho_1) - U_{S1}^{NV}(\alpha_2, \rho_2). \tag{A.3}$$

To see this, note that by the Mean Value Theorem:

$$\begin{aligned}
&\left[U_{S1}^{NV}(\alpha_1, \rho_1) - U_{S1}^{NV}(\alpha_1, \rho_2)\right] - \left[U_{S1}^{NV}(\alpha_2, \rho_1) - U_{S1}^{NV}(\alpha_2, \rho_2)\right] \\
&= \int_{\rho_2}^{\rho_1} \left(\frac{\partial U_{S1}^{NV}(\alpha_1, \rho)}{\partial \rho} - \frac{\partial U_{S1}^{NV}(\alpha_2, \rho)}{\partial \rho} \right) d\rho \\
&= \int_{\alpha_2}^{\alpha_1} \int_{\rho_2}^{\rho_1} \frac{\partial^2 U_{S1}^{NV}(\alpha, \rho)}{\partial \rho \partial \alpha} d\rho d\alpha < 0.
\end{aligned}$$

Taking $\alpha_1 = \alpha$, $\alpha_2 = \widetilde{\alpha}$, $\rho_1 = \overline{\rho}$, and $\rho_2 = \underline{\rho}$ yields:

$$U_{S1}^{NV}(\alpha, \overline{\rho}) - U_{S1}^{NV}(\alpha, \underline{\rho}) < U_{S1}^{NV}(\widetilde{\alpha}, \overline{\rho}) - U_{S1}^{NV}(\widetilde{\alpha}, \underline{\rho}).$$

Thus, for any $Q \in \left(\max\left\{U_{S1}^{NV}(\alpha, \overline{\rho}) - U_{S1}^{NV}(\alpha, \underline{\rho}), 0\right\}, U_{S1}^{NV}(\widetilde{\alpha}, \overline{\rho}) - U_{S1}^{NV}(\widetilde{\alpha}, \underline{\rho})\right)$, the intermediate value theorem guarantees a unique $\widetilde{\alpha} \in (\underline{\rho}, \rho)$ such that:

$$U_{S1}^{NV}(\widetilde{\alpha}, \overline{\rho}) - U_{S1}^{NV}(\widetilde{\alpha}, \underline{\rho}) = Q.$$

Since $\widetilde{\alpha} = \frac{\alpha L}{(1-\lambda)\widetilde{F}}$, we have $\widetilde{F} = \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}$. Hence, if $\widehat{F} > \widetilde{F}$, then $\frac{\alpha L}{(1-\lambda)\widehat{F}} < \widetilde{\alpha}$, and from Equation (A.3):

$$Q < U_{S1}^{NV}\left(\alpha L/(1-\lambda)\widehat{F}, \overline{\rho}\right) - U_{S1}^{NV}\left(\alpha L/(1-\lambda)\widehat{F}, \underline{\rho}\right),$$

so it is optimal to increase signal quality from $\underline{\rho}$ to $\overline{\rho}$, i.e., $\rho^* = \overline{\rho}$. Otherwise, when $\widehat{F} \leq \widetilde{F}$, $\rho^* = \underline{\rho}$.

Case (4). By the definition of $\underline{\rho}$, we have $U_{R0}^{NV}(\widetilde{\alpha}, \overline{\rho}) = U_{R0}^{PV}(\overline{\rho})$. Thus, R0 switches to perfect verification whenever $\widehat{F} \geq \frac{\alpha L}{(1-\lambda)\widetilde{\alpha}}$, since this leads to a lower payoff under non-verification.

The proof concludes. □

A.12 Proof of Proposition 5

Proof. The conditions $\frac{1}{k} \leq \frac{\beta_{R2}}{\beta_{S2}} \leq k$ and $\frac{\beta_{R2}+\beta_{S2}}{\beta_{R1}+\beta_{S1}} \geq (1+k)\widehat{\beta}_S$ imply:

$$(1+k)\beta_{S2} \geq \beta_{R2} + \beta_{S2} > (\beta_{R1} + \beta_{S1})(1+k)\widehat{\beta}_S = (1+k)\beta_{S1}\widehat{\beta}_S.$$

By Proposition 4, non-verification is optimal for senders.

Similarly, the conditions $\frac{1}{k} \leq \frac{\beta_{R2}}{\beta_{S2}} \leq k$ and $\frac{\beta_{R2}+\beta_{S2}}{\beta_{R1}+\beta_{S1}} \geq (1+k)\widehat{\beta}_R$ imply:

$$(1+k)\beta_{R2} \geq \beta_{R2} + \beta_{S2} > (\beta_{R1} + \beta_{S1})(1+k)\widehat{\beta}_R = (1+k)\beta_{R1}\widehat{\beta}_R.$$

Thus, non-verification is also optimal for receivers. In summary, under the stated conditions, non-verification is optimal for the general regulator. □

A.13 Proof of Proposition 6

Proof. **Part (i). Receiver R1.** From Equation (12), we obtain:

$$\frac{\partial(U_{R1}^{NV})}{\partial\lambda} = \frac{8\lambda(1-2\rho)^2(\rho-1)^2\rho^2\left(1-\alpha^2(1-\lambda)-\alpha\lambda(1-2\rho)^2\right)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^2\left(-\alpha\lambda+\alpha+\lambda(1-2\rho)^2+1\right)^2}$$

It is clear that $\frac{\partial(U_{R1}^{NV})}{\partial\lambda} > 0$ whenever $\rho \in (1/2, 1)$. Furthermore,

$$\frac{\partial^2(U_{R1}^{NV})}{\partial\lambda^2} = \frac{8(1-2\rho)^2(\rho-1)^2\rho^2 \cdot (a_0 + \lambda^2(a_2 + a_3\lambda))}{\left(1+\alpha(1-\lambda)+\lambda(1-2\rho)^2\right)^3\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^3}$$

where $a_0 = (1-\alpha^2)^2 > 0$, $a_2 = 3(1-\alpha^2)(\alpha-(1-2\rho)^2)^2 > 0$, and $a_3 = 2\alpha(\alpha-(1-2\rho)^2)^3$. Note that a_3 may be negative, but $a_2 + a_3 = (\alpha-(1-2\rho)^2)^2(3-\alpha^2-2\alpha(1-2\rho)^2) > 0$ since $\alpha \in (0,1)$. Thus, $\frac{\partial^2(U_{R1}^{NV})}{\partial\lambda^2} > 0$. By the standard concavification result in Bayesian persuasion (Kamenica and Gentzkow, 2011), R1 prefers perfect verification.

Part (ii). Receiver R2. We compute R2's payoff as follows:

$$U_{R2}(\lambda \mid c_R) = \begin{cases} \frac{-1+\lambda^2(1-2\rho)^2}{4}, & \text{if } \lambda < \bar{\lambda}(c_R); \\ U_{R2}^{NV}, & \text{if } \lambda \geq \bar{\lambda}(c_R), \end{cases}$$

where U_{R2}^{NV} is defined in Equation (12). Note that there is a downward jump at $\lambda = \bar{\lambda}(c_R)$ because $\frac{\partial U_{R2}^{NV}}{\partial\alpha} > 0$ for all $\lambda \in (0,1)$. We verify that U_{R2}^{NV} is continuous, strictly increasing, and strictly convex in λ for $\lambda \geq \bar{\lambda}(c_R)$, as:

$$\frac{\partial(U_{R2}^{NV})}{\partial\lambda} = \frac{1}{2}(1-\alpha)(1-2\rho)^2(\alpha(1-\lambda)+\lambda) > 0, \quad \frac{\partial^2(U_{R2}^{NV})}{\partial\lambda^2} = \frac{1}{2}(1-\alpha)^2(1-2\rho)^2 > 0.$$

Similarly, $U_{R2}(\lambda)$ is strictly increasing and convex in λ for $\lambda < \bar{\lambda}(c_R)$.

Case (a): $\bar{\lambda}(c_R)(1-\alpha)^2 \geq \alpha^2$.

The upper envelope of the convex hull of $\left(\lambda, \frac{-1+\lambda^2(1-2\rho)^2}{4}\right)$ over $\lambda \in [0,1]$ is given by $U_{R2}^{PV} = \lambda(-\rho(1-\rho)) + (1-\lambda)(-1/4)$. Furthermore, when $\bar{\lambda}(c_R)(1-\alpha)^2 \geq \alpha^2$, R2 strictly prefers perfect verification because $U_{R2}^{PV} \geq U_{R2}^{NV}$. Thus, the upper envelope of the convex hull of $(\lambda, U_{R2}(\lambda \mid c_R))$ lies below U_{R2}^{PV}, and R2 prefers perfect verification.

Case (b): $\bar{\lambda}(c_R)(1-\alpha)^2 < \alpha^2$.

Consider first $\lambda \in (0, \bar{\lambda}(c_R))$. By Lemma 2, $U_{R2}^{NV} > U_{R2}^{PV}$ at $\lambda = \bar{\lambda}(c_R)$. After concavification, R2's payoff is less than $U_{R2}^{PV} = -\lambda\rho(1-\rho) - \frac{1-\lambda}{4}$.

- If $\lambda \leq \nu(FH \mid m_1) < \bar{\lambda}(c_R)$ (and $\nu(FH \mid m_2) < \lambda$), R2's payoff after concavification is strictly below $U_{R2}^{PV}(\lambda)$. However, for $\lambda \in (0, \bar{\lambda}(c_R))$, U_{R2}^{PV} is strictly dominated by the segment connecting $(0, -1/4)$ and $(\bar{\lambda}(c_R), U_{R2}^{NV}(\bar{\lambda}(c_R)))$ since $U_{R2}(\lambda \mid c_R) > U_{R2}^{PV}$ at $\bar{\lambda}(c_R)$.

- If $\nu(FH \mid m_1) > \bar{\lambda}(c_R) > \nu(FH \mid m_2) > 0$, R2's payoff can be increased by first revising $\nu(FH \mid m_2)$ to 0 and then updating $\nu(FH \mid m_1)$ to $\widehat{\nu}(FH \mid m_1) = \bar{\lambda}(c_R)$.

Now consider $\lambda \in (\bar{\lambda}(c_R), 1)$. Since $U_{R2}^{NV}(\lambda)$ is convex, the convex hull is the segment between $(\bar{\lambda}(c_R), U_{R2}^{NV}(\bar{\lambda}(c_R)))$ and $(1, U_{R2}^{PV}(1))$. For any design with $\nu(FH \mid m_1) < 1$, R2's expected payoff increases by setting $\nu(FH \mid m_1) = 1$. Similarly, if $\nu(FH \mid m_2) < \bar{\lambda}(c_R)$, R2's payoff improves by updating to $\nu(FH \mid m_2) = \bar{\lambda}(c_R)$. Thus, $(\bar{\lambda}(c_R), U_{R2}^{NV}(\bar{\lambda}(c_R)))$ is an extreme point of $co(\lambda, U_{R2}(\lambda))$, and the optimal structure satisfies $\nu(FH \mid m) = \bar{\lambda}(c_R)$ for all $m \in \{m_1, m_2\}$. This concludes the proof. □

A.14 Proof of Corollary 2

Proof. First, note that R1's payoff under the optimal information structure is given by $U_{R1}^{PV}(\lambda)$. Second, R2's payoff is:

$$\widehat{U}_{R2}(\lambda) = \begin{cases} \frac{U_{R2}^{NV}(\bar{\lambda}(c_R))}{\bar{\lambda}(c_R)} \cdot \lambda, & \text{if } \lambda \in (0, \bar{\lambda}(c_R)); \\ \frac{-\rho(1-\rho) - U_{R2}^{NV}(\bar{\lambda}(c_R))}{1-\bar{\lambda}(c_R)} \cdot (\lambda - \bar{\lambda}(c_R)) + U_{R2}^{NV}(\bar{\lambda}(c_R)), & \text{if } \lambda \in (\bar{\lambda}(c_R), 1). \end{cases}$$

In contrast, R2's payoff under perfect verification is U_{R2}^{PV}. Hence, partial verification is optimal whenever:

$$U_{R1}^{PV} + \beta_R \cdot U_{R2}^{PV} < U_{R1}^{NV} + \beta_R \cdot \widehat{U}_{R2}.$$

Solving it yields:

$$\beta_R > \frac{U_{R1}^{PV} - U_{R1}^{NV}}{\widehat{U}_{R2} - U_{R2}^{PV}} =: \widetilde{\beta}_R(\lambda).$$

This concludes the proof. □

A.15 Proof of Theorem 2

Proof. We first observe that

$$\begin{aligned} P = \prod_{i=0}^{d-1} \frac{(k-i)(M+1)}{k(M+1)-i} &= \prod_{i=0}^{d-1} \left(1 - \frac{i\left(1 - \frac{1}{M+1}\right)}{k - \frac{i}{M+1}}\right) \\ &\approx \prod_{i=0}^{d-1} \left(1 - \left(1 - \frac{1}{M+1}\right) \cdot \frac{i}{k}\right) \quad \text{when } k \text{ is large,} \end{aligned}$$

where the approximation uses the fact that $i < d = o(k)$.

Since $M = O(k^\gamma)$, we have:

$$P \approx \prod_{i=0}^{d-1} \left(1 - C \cdot \frac{i}{k}\right), \quad \text{where } C = \begin{cases} 1 & \text{if } \gamma > 1, \\ 1 - \frac{1}{M+1} & \text{if } \gamma = 0 \text{ and } M \text{ is fixed,} \end{cases}$$

and in all cases $C > 0$, with $C = 1$ if $\gamma > 0$.

When k is large, we can write:

$$\begin{aligned} \log P &\approx \sum_{i=0}^{d-1} \log\left(1 - C \cdot \frac{i}{k}\right) \\ &\approx -C \sum_{i=0}^{d-1} \frac{i}{k} = -\frac{C}{k} \cdot \frac{(d-1)d}{2} \approx -C \cdot \frac{d^2}{2k}, \end{aligned}$$

which implies:

$$P \approx \exp\left(-C \cdot \frac{d^2}{2k}\right).$$

Hence,

$$\Psi = P^n \approx \exp\left(-n \cdot C \cdot \frac{d^2}{2k}\right) \approx \exp\left(-D \cdot k^{\alpha+2\beta-1}\right) \quad \text{for some } D > 0,$$

where the last approximation uses $n = O(k^\alpha)$ and $d = O(k^\beta)$. Therefore, we conclude:

$$\text{As } k \to \infty: \quad \Psi \to \begin{cases} 1 & \text{if } \alpha + 2\beta < 1, \\ \exp(-D) & \text{if } \alpha + 2\beta = 1, \\ 0 & \text{if } \alpha + 2\beta > 1. \end{cases}$$

Note that

$$\mathbb{P}(\text{All receivers' neighbors are Type-I}) \geq \Psi.$$

Thus, when $\alpha + 2\beta < 1$, this probability approaches 1 as $k \to \infty$, implying that duplication bias vanishes asymptotically, and the optimal verification policy almost surely converges to non-verification.

Similarly, if $\alpha + 2\beta = 1$, then with positive probability all receivers' neighbors are Type-I senders. In this case, duplication bias disappears, and it is optimal not to verify.

However, when $\alpha + 2\beta > 1$, some receivers must experience duplication bias. □

B Internet Appendix

B.1 Proof of Lemma A.1

Proof. **Case (i).** $U_{S1}^{NV}(0,\overline{\rho}) - U_{S1}^{NV}(0,\underline{\rho}) > 0$.

By the Leibniz integral rule,

$$U_{S1}^{NV}(0,\overline{\rho}) - U_{S1}^{NV}(0,\underline{\rho}) = \int_{\underline{\rho}}^{\overline{\rho}} \frac{\partial U_{S1}^{NV}(0,\rho)}{\partial \rho}\, d\rho.$$

Thus, it suffices to verify that $\frac{\partial U_{S1}^{NV}(0,\rho)}{\partial \rho} > 0$. From Equation (13), we have:

$$\frac{\partial U_{S1}^{NV}(0,\rho)}{\partial \rho} = \frac{(2\rho-1)\left(-\lambda^6(1-2\rho)^8 + 3\lambda^4(1-2\rho)^4 + \lambda^2(4\rho^2-4\rho-3)(1-2\rho)^2 + 1\right)}{\left(\lambda(1-2\rho)^2-1\right)^2\left(\lambda(1-2\rho)^2+1\right)^2}.$$

Now observe:

$$\begin{aligned}
&-\lambda^6(1-2\rho)^8 + 3\lambda^4(1-2\rho)^4 + \lambda^2(4\rho^2-4\rho-3)(1-2\rho)^2 + 1 \\
&= 1 - \lambda^2(3+4\rho-4\rho^2)(1-2\rho)^2 + \lambda^4(1-2\rho)^4(3-\lambda^2(1-2\rho)^4) \\
&> 1 - \frac{1}{9}(3+4\rho-4\rho^2)(1-2\rho)^2 + \lambda^4(1-2\rho)^4\left(3-\lambda^2(1-2\rho)^4\right) > 0,
\end{aligned}$$

where we used the assumption that $\lambda \le \frac{1}{3}$.

Case (ii). $\frac{\partial^2}{\partial\alpha\,\partial\rho} U_{S1}^{NV}(\alpha,\rho \mid \lambda) < 0$.

The idea is straightforward: we aim to verify that the second-order cross-partial derivative is negative. Although the algebra involved is quite complex, we outline the procedure to handle it.

$$\begin{aligned}
\frac{\partial^2}{\partial\alpha\,\partial\rho} U_{S1}^{NV}(\alpha,\rho \mid \lambda) = \frac{1}{2}(\lambda-1)\Bigg(&-\frac{2(\alpha-1)^2(\lambda-1)^2(2\rho-1)(\alpha(\lambda-1)+1)}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2+1\right)^3} \\
&+ \frac{(\alpha-1)(\lambda-1)(2\rho-1)(3\alpha(\lambda-1)-\lambda+3)}{(\alpha(-\lambda)+\alpha+4\lambda(\rho-1)\rho+\lambda-1)^2} \\
&- \frac{(2\rho-1)(\alpha(\lambda-1)-\lambda-1)(3\alpha(\lambda-1)-\lambda-3)}{(\alpha(-\lambda)+\alpha+4\lambda(\rho-1)\rho+\lambda+1)^2} \\
&+ \frac{2(2\rho-1)(\alpha(\lambda-1)-1)(\alpha(-\lambda)+\alpha+\lambda+1)^2}{\left(\alpha(\lambda-1)-\lambda(1-2\rho)^2-1\right)^3} \\
&+ 8\rho(\alpha(-\lambda)+\alpha+\lambda) + 4\alpha(\lambda-1) - 4\lambda\Bigg)
\end{aligned}$$

By reducing to a common denominator, it suffices to show that the resulting polynomial is negative when $\lambda \leq \frac{1}{3}$. That is, we must verify: $Z = \sum_{k=0}^{9} n_k \cdot \lambda^k < 0$, where

$$n_0 = 4(\alpha-1)^3\alpha(\alpha+1)^5,$$
$$n_1 = -4(\alpha-1)^2(\alpha+1)^4\left(9\alpha^3+\alpha^2\left(-32\rho^2+32\rho-11\right)+\alpha\left(8\rho^2-8\rho+1\right)+1\right),$$
$$\begin{aligned}n_2 = 8(\alpha-1)(\alpha+1)^3\Big(&18\alpha^5-4\alpha^4\left(32\rho^2-32\rho+11\right)+(1-2\rho)^2\\&+\alpha^3\left(224\rho^4-448\rho^3+412\rho^2-188\rho+29\right)\\&+\alpha^2\left(-112\rho^4+224\rho^3-148\rho^2+36\rho+3\right)+\alpha\left(-40\rho^4+80\rho^3-72\rho^2+32\rho-7\right)\Big),\end{aligned}$$
$$\begin{aligned}n_3 = -8(\alpha+1)^2\Big(&42\alpha^7-14\alpha^6\left(32\rho^2-32\rho+11\right)+7\alpha^5\left(224\rho^4-448\rho^3+396\rho^2-172\rho+27\right)\\&+\alpha^4\left(-1792\rho^6+5376\rho^5-8064\rho^4+7168\rho^3-3560\rho^2+872\rho-53\right)\\&+4\alpha^3\left(336\rho^6-1008\rho^5+1126\rho^4-572\rho^3+71\rho^2+47\rho-16\right)\\&+12\alpha^2(1-2\rho)^2\left(16\rho^4-32\rho^3+33\rho^2-17\rho+4\right)\\&+\alpha\left(-384\rho^6+1152\rho^5-1448\rho^4+976\rho^3-376\rho^2+80\rho-7\right)\\&-(1-2\rho)^2\left(32\rho^4-64\rho^3+40\rho^2-8\rho+1\right)\Big),\end{aligned}$$
$$\begin{aligned}n_4 = 8(\alpha+1)\Big(&63\alpha^8-7\alpha^7\left(128\rho^2-128\rho+35\right)+28\alpha^6\left(168\rho^4-336\rho^3+261\rho^2-93\rho+11\right)\\&-3\alpha^5\left(3584\rho^6-10752\rho^5+13776\rho^4-9632\rho^3+3648\rho^2-624\rho+15\right)\\&+2\alpha^4\left(4480\rho^8-17920\rho^7+32032\rho^6-33376\rho^5+20464\rho^4-6208\rho^3+2\rho^2+526\rho-101\right)\\&+\alpha^3\left(7552\rho^6-22656\rho^5+28976\rho^4-20192\rho^3+8024\rho^2-1704\rho+145\right)\\&-4\alpha^2\left(1536\rho^8-6144\rho^7+10960\rho^6-11376\rho^5+7420\rho^4-3048\rho^3+743\rho^2-91\rho+3\right)\\&-3\alpha(1-2\rho)^2\left(80\rho^4-160\rho^3+116\rho^2-36\rho+5\right)\\&+(1-2\rho)^4\left(64\rho^4-128\rho^3+84\rho^2-20\rho+3\right)\Big),\end{aligned}$$
$$\begin{aligned}n_5 = -8\left(\alpha-(1-2\rho)^2\right)^2\Big(&63\alpha^7-7\alpha^6\left(88\rho^2-88\rho+19\right)+28\alpha^5\rho\left(68\rho^3-136\rho^2+93\rho-25\right)\\&+\alpha^4\left(-1792\rho^6+5376\rho^5-5712\rho^4+2464\rho^3+304\rho^2-640\rho+150\right)+\\&\alpha^3\left(-1344\rho^6+4032\rho^5-6368\rho^4+6016\rho^3-3136\rho^2+800\rho-65\right)\\&+\alpha^2\left(768\rho^6-2304\rho^5+2512\rho^4-1184\rho^3+96\rho^2+112\rho-33\right)\\&+2\alpha(1-2\rho)^2\left(48\rho^4-96\rho^3+106\rho^2-58\rho+9\right)-32\rho^2\left(2\rho^2-3\rho+1\right)^2\Big),\end{aligned}$$
$$\begin{aligned}n_6 = 8\left(\alpha-(1-2\rho)^2\right)^3\Big(&42\alpha^6-98\alpha^5(1-2\rho)^2+7\alpha^4\left(160\rho^4-320\rho^3+236\rho^2-76\rho+5\right)\\&-4\alpha^3\left(224\rho^6-672\rho^5+784\rho^4-448\rho^3+69\rho^2+43\rho-15\right)\\&-8\alpha^2\left(56\rho^6-168\rho^5+285\rho^4-290\rho^3+163\rho^2-46\rho+5\right)\\&+2\alpha\left(80\rho^6-240\rho^5+316\rho^4-232\rho^3+88\rho^2-12\rho-1\right)+(1-2\rho)^2\left(20\rho^2-20\rho+3\right)\Big),\end{aligned}$$

$$n_7 = -8\left(\alpha - (1-2\rho)^2\right)^4 \Big(18\alpha^5 - 2\alpha^4\left(80\rho^2 - 80\rho + 23\right)$$
$$+ \alpha^3\left(416\rho^4 - 832\rho^3 + 668\rho^2 - 252\rho + 29\right)$$
$$+ \alpha^2\left(-256\rho^6 + 768\rho^5 - 1024\rho^4 + 768\rho^3 - 272\rho^2 + 16\rho + 9\right)$$
$$+ \alpha\left(-64\rho^6 + 192\rho^5 - 392\rho^4 + 464\rho^3 - 288\rho^2 + 88\rho - 11\right) + (1-2\rho)^2\left(8\rho^2 - 8\rho + 1\right)\Big),$$
$$n_8 = 4\left(\alpha - (1-2\rho)^2\right)^7\left(9\alpha^2 + \alpha\left(-4\rho^2 + 4\rho - 7\right) - 2\right),$$
$$n_9 = -4(\alpha - 1)\left(\alpha - (1-2\rho)^2\right)^8.$$

- The procedure to verify $Z < 0$ is as follows.

1. First, we verify $n_0 < 0$ (all other coefficients are indeterminate).

2. Second, we consider n_1, and distinguish two cases:

   (a) If $n_1 < 0$, then $n_0 + \lambda n_1 < 0$.

   (b) If $n_1 > 0$, then $n_0 + \lambda n_1 < n_0 + \frac{1}{3}n_1$.

   Hence, if we can show that $n_0 + \frac{1}{3}n_1 < 0$, then $n_0 + \lambda n_1 < 0$ for any $\lambda \le \frac{1}{3}$, holding other parameters fixed.

3. Third, we analyze n_2, and consider several cases:

   (a) If $n_1 < 0$ and $n_2 < 0$, then clearly $n_0 + \lambda n_1 + \lambda^2 n_2 < 0$.

   (b) If $n_1 < 0$ and $n_2 > 0$, then:

      - If $n_1 + \frac{1}{3}n_2 < 0$, we are done.
      - If $n_1 + \frac{1}{3}n_2 > 0$, then:

   $$n_0 + \lambda(n_1 + \lambda n_2) < n_0 + \frac{1}{3}\left(n_1 + \frac{1}{3}n_2\right) = n_0 + \frac{1}{3}n_1 + \left(\frac{1}{3}\right)^2 n_2.$$

   Therefore, to verify $n_0 + \lambda n_1 + \lambda^2 n_2 < 0$ for all $\lambda \le \frac{1}{3}$, $\rho \in \left(\frac{1}{2}, 1\right)$, and $\alpha \in (0,1)$, it suffices to show that:

   $$n_0 + \frac{1}{3}n_1 + \left(\frac{1}{3}\right)^2 n_2 < 0.$$

4. Continuing in this fashion, it suffices to show:

$$\sum_{j=0}^{k} \lambda^j n_j < 0 \quad \text{for all } k \in \{1, 2, \ldots, 9\}. \tag{B.1}$$

We now sketch how to verify Equation (B.1).

- The procedure to verify Equation (B.1) is as follows.

1. $n_0 < 0$: This holds trivially since $\alpha \in (0,1)$.
2. $n_0 + \frac{1}{3}n_1 < 0$:
   (a) We simplify the expression as:
   $$\frac{1}{3}(-4)(\alpha-1)^2(\alpha+1)^4 \cdot Z_1,$$
   where
   $$Z_1 = (8\alpha - 32\alpha^2)\rho^2 + (32\alpha^2 - 8\alpha)\rho + 6\alpha^3 - 11\alpha^2 + 4\alpha + 1.$$
   It suffices to show that $Z_1 > 0$.
   (b) Decompose Z_1 as a polynomial in $\rho(1-\rho)$:
   $$Z_1 = \underbrace{-(8\alpha - 32\alpha^2)}_{a_1} \cdot \rho(1-\rho) + \underbrace{(\alpha-1)^2(6\alpha+1)}_{a_0}.$$
   (c) Since $\rho(1-\rho) \in (0, 1/4)$, it suffices to show:
   $$a_0 > 0 \quad \text{and} \quad a_0 + \frac{1}{4}a_1 > 0.$$
   Obviously, $a_0 > 0$, and
   $$a_0 + \frac{1}{4}a_1 = 1 - \alpha^2 + 2\alpha(1-\alpha) + 6\alpha^3 > 0.$$
3. $n_0 + \frac{1}{3}n_1 + \left(\frac{1}{3}\right)^2 n_2 < 0$:
   (a) We simplify it as:
   $$\frac{4}{9}(\alpha-1)(\alpha+1)^3 \cdot Z_2,$$
   where
   $$\begin{aligned} Z_2 = {} & (448\alpha^3 - 224\alpha^2 - 80\alpha)\rho^4 + (-896\alpha^3 + 448\alpha^2 + 160\alpha)\rho^3 \\ & + (-160\alpha^4 + 800\alpha^3 - 392\alpha^2 - 120\alpha + 8)\rho^2 \\ & + (160\alpha^4 - 352\alpha^3 + 168\alpha^2 + 40\alpha - 8)\rho \\ & + (18\alpha^5 - 55\alpha^4 + 64\alpha^3 - 30\alpha^2 - 2\alpha + 5). \end{aligned}$$
   (b) Decompose Z_2 as a polynomial in $\rho(1-\rho)$. Let $a_2 = 448\alpha^3 - 224\alpha^2 - 80\alpha$, then

subtract $a_2(\rho(1-\rho))^2$ to isolate the lower-order terms:

$$Z_2 - a_2(\rho(1-\rho))^2 = \underbrace{(\alpha-1)^2(18\alpha^3 - 19\alpha^2 + 8\alpha + 5)}_{a_0} + \underbrace{(160\alpha^4 - 352\alpha^3 + 168\alpha^2 + 40\alpha - 8)}_{a_1} \cdot \rho(1-\rho).$$

So, $Z_2 = a_0 + a_1 \cdot \rho(1-\rho) + a_2 \cdot (\rho(1-\rho))^2$.

(c) To verify $Z_2 > 0$, it suffices to check:

$$a_0 > 0, \quad a_0 + \frac{1}{4}a_1 > 0, \quad a_0 + \frac{1}{4}a_1 + \left(\frac{1}{4}\right)^2 a_2 > 0.$$

4. Continuing in this manner, we can verify Equation (B.1). The remainder of the proof follows similar logic, and the more involved algebra is omitted here, but is available upon request.

The proof of the lemma is complete.

□

B.2 Proof of Lemma A.2 (ii)

Proof. First, we use Equation (12) to derive:

$$\frac{\partial U_{R1}^{NV}}{\partial \rho} = \frac{(2\rho-1)\cdot Y}{\left(\alpha(\lambda-1) - \lambda(1-2\rho)^2 + 1\right)^2 \left(-\alpha\lambda + \alpha + \lambda(1-2\rho)^2 + 1\right)^2},$$

where $Y = \sum_{k=0}^{4} \lambda^k \cdot m_k$, and

$$\begin{aligned}
m_0 &= (\alpha^2-1)^2,\\
m_1 &= -4\alpha(\alpha^2-1)(\alpha-(1-2\rho)^2),\\
m_2 &= 2\Big(3\alpha^4 - 6\alpha^3(1-2\rho)^2 + \alpha^2(24\rho^4 - 48\rho^3 + 44\rho^2 - 20\rho + 2)\\
&\qquad + 2\alpha(1-2\rho)^2 + 8\rho^4 - 16\rho^3 + 4\rho^2 + 4\rho - 1\Big),\\
m_3 &= 4\alpha\Big(-\alpha^3 + 3\alpha^2(1-2\rho)^2 + (1-2\rho)^4\\
&\qquad + \alpha(-24\rho^4 + 48\rho^3 - 44\rho^2 + 20\rho - 3)\Big),\\
m_4 &= \alpha^4 - 4\alpha^3(1-2\rho)^2 + \alpha^2(48\rho^4 - 96\rho^3 + 88\rho^2 - 40\rho + 6)\\
&\qquad - 4\alpha(1-2\rho)^4 + (1-2\rho)^4.
\end{aligned}$$

Given that $\lambda \leq \frac{1}{2}$ (and following the proof of Lemma A.1), it suffices to show that $Y_k = \sum_{i=0}^{k}(1/2)^i m_i > 0$ for $k \in \{0,1,2,3,4\}$. Clearly,

- $Y_0 = m_0$ is positive.
- $Y_1 = m_0 + m_1 \cdot \frac{1}{2} = (\alpha^2 - 2\alpha(1-2\rho)^2 + 1)(1-\alpha^2) > 0$.
- $Y_2 = \frac{1}{4}\left(d_{20} + d_{21} \cdot \rho(1-\rho) + d_{22} \cdot \rho^2(1-\rho)^2\right)$, where
  $d_{20} = 2(\alpha-1)^2(\alpha^2+1), \quad d_{21} = 8(1-\alpha)(1+3\alpha-2\alpha^2), \quad d_{22} = 48\alpha^2 + 16.$
  Obviously, $d_{20} > 0, d_{21} > 0, d_{22} > 0$.
- $Y_3 = \frac{1}{8}\left(d_{30} + d_{31} \cdot \rho(1-\rho) + d_{32} \cdot \rho^2(1-\rho)^2\right)$, where
  $d_{30} = 4(\alpha-1)^2(\alpha+1), \quad d_{31} = 16(1-\alpha)(\alpha^2+\alpha+1), \quad d_{32} = 64\alpha + 32.$
  Clearly, $d_{30} > 0, d_{31} > 0, d_{32} > 0$.
- $Y_4 = \frac{1}{16}\left(d_{40} + d_{41} \cdot \rho(1-\rho) + d_{42} \cdot \rho^2(1-\rho)^2\right)$, where
  $d_{40} = (\alpha-1)^2(\alpha+3)^2, \quad d_{41} = 8(1-\alpha)(\alpha+3)(2\alpha+1), \quad d_{42} = 48\alpha^2 + 64\alpha + 80.$
  Obviously, $d_{40} > 0, d_{41} > 0, d_{42} > 0$.

The proof is complete. □