

Too Big to Fail, Too Small to Survive

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Abstract

This paper identifies a novel bank-run externality. We show that regulations intended to enhance the safety of large national banks may disproportionately increase the likelihood of bank runs among smaller banks. A minor shock can prompt a cascading chain reaction of deposit withdrawals among many regional banks, even if those banks are not severely distressed—such simultaneous exodus from small institutions generates novel systemic vulnerabilities beyond the traditional channel through interbank connectivity. We extend global game models by incorporating realistic deposit mobility across multiple risky banks, alongside the conventional risk-free option (typically the sole outside option in models studying bank-runs). Our framework captures the interaction between strategic complementarity in run/stay decisions among multiple substitutes—a dynamic that remains underexplored in the literature.

Keywords: Bank deposit competition, Financial Stability, Global Game, Macroproductential Policy, Strategic Complementarity.

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1 Introduction

Since the Great Financial Crisis, macroprudential policies have aimed to fortify the financial system by prioritizing the stability of large, systemically important banks.¹ This priority follows a conventional logic: because national institutions play central roles in the financial system, safeguarding them is expected to stabilize the broader network of intermediaries. This paper shows that the post-GFC architecture overlooks a subtle countervailing force. Strengthening national banks—through liquidity requirements, crisis support, or a perceived Too Big To Fail (TBTF) backstop—can unintentionally raise run risk at smaller regional banks by making national institutions the *safer neighbors* during stress. When depositors can shift funds across risky banks in a crisis, a perceived-safer neighboring institution that continues to provide banking services represents a strictly more attractive outside option than withdrawing into cash. This raises depositors’ incentives to withdraw and—due to strategic complementarities in withdrawal decisions—even moderate concerns about a regional bank’s health are sufficient to justify running and reallocating their deposits, thereby increasing the propensity for runs on regional banks in downturns. We refer to this mechanism as the *safe-neighbor externality*.

Regional banks collectively serve as pillars of national economic development and financial stability. As of 2023, regional and smaller banks manage approximately 30% of total assets in the U.S. banking industry, hold 54% of total deposits, and account for more than 60% of total loans outstanding.² In addition, these banks play a critical role in local economies by providing specialized knowledge and tailored financial services that national banks often cannot replicate.³ Their decline imposes significant costs on consumers, reduces financial inclusion, and suppresses regional economic growth, disproportionately affecting underserved communities. Therefore, factors that increase their collective fragility across multiple regions can pose substantial systemic risks.

To formalize the safe-neighbor externality and evaluate its interaction with macroprudential policies, we enrich the standard global-game framework in the bank-run literature by allowing for multiple risky banks and cross-bank deposit mobility. In

¹E.g., [Hanson, Kashyap and Stein 2011](#); [Begenau 2020](#); [Elenev, Landvoigt and Van Nieuwerburgh 2021](#); [BCBS 2018b, 2010a, 2024](#).

²Source: [Banking Snapshot](#) by the American Bankers Association.

³Using a 1970–1993 sample from Italy, [Usai and Vannini \(2005\)](#) find that smaller banks positively influence local economic growth, contrasting with the insignificant or negative effects associated with national banks. Regional banks also hold approximately one-third of all small business loans provided by banks ([Covas, Gross and Tapia, 2023](#)).

contrast to the classic setting in which cash is the sole outside option at the run stage (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005), cross-bank mobility introduces an additional layer of strategic interaction: depositors coordinate not only on whether to withdraw but also on *where* to reallocate funds. When deposits are reallocated from one bank to a neighboring institution during downturns, the inflows strengthen the recipient bank's balance sheet and survival probability, endogenously generating a self-fulfilling divergence in their equilibrium survival probabilities. These feedback effects can amplify coordination frictions at smaller institutions and pose challenges for regional financial stability. Facing the additional complexity introduced by cross-bank deposit mobility, we develop new proof techniques to analyze the limiting equilibrium with multiple banks, thereby extending the scope of the standard framework.

Subsequently, our analysis demonstrates that the safe-neighbor externality can induce systemic fragility among smaller banks across regions, even when their asset holdings are uncorrelated. When a negative shock threatens one regional bank, the resulting inflows into the national institution strengthen its balance sheet and raise its survival probability, which depositors in other regions rationally anticipate. This feedback makes the national bank a focal destination across regions, and shifts run thresholds at otherwise viable regional banks, generating a cross-regional chain reaction of deposit flight, particularly among regional banks that are vulnerable but not insolvent during downturns.

To the extent that small and regional banks collectively play a critical role in nationwide financial stability, the aforementioned chain reaction emerges as a potential source of systemic risk. Indeed, following the collapse of Silicon Valley Bank (SVB) in March 2023, depositors shifted funds to larger institutions perceived as safer, prompting significant withdrawals at regional banks such as First Republic Bank, Signature Bank, PacWest Bancorp, Western Alliance Bancorporation, and Zions Bancorporation,⁴ among many others operating across diverse sectors in the United States. This cascade of withdrawals triggered liquidity pressures across regional financial markets, culminating in a nationwide wave of deposit outflows from smaller banks. For instance, Choi, Goldsmith-Pinkham and Yorulmazer (2023) observe that mid-sized banks bore the brunt of stress immediately after SVB's failure, with negative spillovers eventually becoming

⁴Signature Bank, headquartered in New York City, operated 40 private client offices across New York State. Western Alliance Bancorporation, headquartered in Phoenix, served Arizona, California, and Nevada. Zions Bancorporation, headquartered in Salt Lake City, operated across several Western states. First Republic Bank and PacWest Bancorp primarily operated in California. Of the banks that experienced runs in March 2023, First Republic was the 14th largest at the time, while SVB was the 16th, and Signature the 29th (Rose, 2023).

widespread across the banking system—except for the largest banks. Similarly, [Caglio, Dlugosz and Rezende \(2023\)](#) document that, during periods of heightened stress, large banks experienced significantly faster deposit growth than smaller regional banks without raising deposit rates, indicating a pronounced flight to safety. Their analysis further reveals that depositors’ flight to larger, perceived safer banks was not fully justified by fundamentals, consistent with our model’s prediction of a chain reaction in bank runs driven by strategic complementarities. This financial panic drew substantial media attention⁵ and necessitated government intervention, including the creation of the Bank Term Funding Program (BTFP) and a guarantee of uninsured deposits.⁶

Beyond analyzing new dynamics in bank runs, we also show that regulatory support for large national banks can distort ex-ante deposit competition beyond traditional pricing margins, weakening the competitive position of smaller institutions. While the existing literature has established that capital and liquidity regulations distort large banks’ incentives to attract deposits and expand loan portfolios ([Begenau, 2020](#); [Boissay and Collard, 2016](#); [Vives, 2016](#)), relatively little attention has been paid to how these regulations shape equilibrium deposit competition among banks of heterogeneous sizes. Our work highlights non-pecuniary externalities associated with the perceived safety of banks, a channel that is not fully captured in previous, primarily price-based analyses.

On the technical side, our modeling framework and proof techniques extend beyond traditional global-game and bank-run models through three key generalizations: First, our global-game foundation is deliberately general. We establish the most general sufficient condition in the literature on the information structure to guarantee equilibrium uniqueness, demonstrating the robustness of our analysis and enhancing the practical predictive power of our model. This flexibility also expands the applicability of global-game models to economic environments in which the natural payoff and information structures do not align with the assumptions imposed in existing global-game models. In particular, we allow a continuum of depositors with independent and heterogeneous signal and payoffs distributions that are essentially arbitrary. Our new

⁵For example, see “[Banking Crisis Hangs Over Economy, Rekindling Recession Fear](#)”, *The New York Times*, March 17, 2023; “[The 11 Days of Turmoil That Brought Down Four Banks And Left A Fifth Teetering](#)”, *Bloomberg*, March 20, 2023; “[US Experiences Biggest Bank Failures Since Global Financial Crisis](#)”, *Financial Times*, March 25, 2023.

⁶Despite the [fact that](#) about 89% of SVB’s \$175 billion in deposits were uninsured as of the end of 2022, on March 12 the FDIC received exceptional authority from the Treasury and [announced jointly](#) with other agencies that all depositors would have full access to their funds the next morning, including uninsured deposits. On the same day, the Federal Reserve [announced](#) the creation of the Bank Term Funding Program (BTFP), an emergency lending program providing loans of up to one year in length to banks, savings associations, credit unions, and other eligible depository institutions.

proof technique shows that the small-noise refinement still selects a unique limiting equilibrium. This delivers the “robust equilibrium” we emphasize throughout, while avoiding restrictive assumptions that are common in the literature.⁷ This new methodology also allows us to study liquidity requirements (e.g., regulatory liquidity ratios such as LCR or NSFR) that break the commonly assumed property of *global strategic complementarities* used in canonical proofs.

Second, classical bank-run models typically restrict depositor strategies to a one-dimensional run-or-stay choice. In contrast, our framework introduces multiple outside options by allowing depositors to choose between holding risk-free cash and transferring deposits to other risky banks during episodes of panic. This richer strategic margin better captures the dynamics observed in modern banking crises (e.g., [Baron, Schularick and Zimmermann 2023](#)), but it also breaks the one-dimensional monotonicity exploited by standard solution methods, because beliefs and fragility become mutually reinforcing across institutions. For instance, the canonical global-game benchmark predicts that depositors’ withdrawal decisions are characterized by a unique run–stay cutoff in the limiting equilibrium, determined solely by the relative payoff of remaining deposited with the bank versus holding cash. In our setting, however, banks’ relative fragility and the corresponding run cutoffs are endogenous to depositors’ joint strategies and beliefs and must be determined simultaneously. Even so, the equilibrium set is tightly structured: the economically relevant outcome preserves the benchmark fragility ordering. Whenever the national bank’s run cutoff lies below the regional bank’s in the cash-only benchmark model, this ranking is unchanged once deposit mobility is introduced, regardless of other model parameters. Unless economic conditions are very bad, depositors at smaller regional banks prefer transferring funds to a national bank rather than holding cash. This reallocation of deposits reinforces the financial strength of larger, regulated institutions while exacerbating vulnerabilities among smaller banks—a central prediction of our model.

Third, we model depositor payoffs in a bank-run crisis with greater realism. As noted by [Goldstein and Pauzner \(2005\)](#) and [Hanson, Kashyap and Stein \(2011\)](#), realistic depositor payoffs in a bank-run scenario do not satisfy the *global strategic*

⁷Besides generalizing the information structure, our proof relaxes several other assumptions commonly imposed in the literature and clearly enumerated in the seminal contribution of [Frankel, Morris and Pauzner 2003](#), including the assumptions of (a) *payoff continuity* and (b) *bounded derivatives*. These conditions are typically violated in banking crises with endogenous bank defaults, where depositors’ financial returns can drop abruptly to zero. In particular, we do not require a finite number of player types, which allows us to model an uncountably large set of heterogeneous depositors and to apply IO-style structural models to study bank deposit competition more effectively.

complementarities assumption typically imposed in global-game models. Global strategic complementarities—where the incentive to act changes monotonically with the fundamental state of the economy—are typically a key requirement in canonical proofs of equilibrium uniqueness in global-game models (e.g., [Frankel et al. 2003](#); [Morris and Shin 2001](#); [He and Xiong 2012](#)). To address this challenge, the arguments of [Goldstein and Pauzner \(2005\)](#) and [Hanson et al. \(2011\)](#) rely on the assumption that each depositor receives a private noisy signal with small error terms that are identically, independently, and *uniformly* distributed. In contrast, we only require the error terms to be independent, allowing them to follow different and flexible probability density functions.

Although the primary goal of this paper is to identify the externality and document its implications, our findings naturally point toward potential policy adjustments from the lens of our model. In our model, the severity of this externality depends on factors such as the perceived government guarantees for large banks, the regulatory constraints they face, and the scope of deposit insurance available to smaller banks. This analysis highlights the role of post-GFC banking regulations in shaping depositor behavior during financial uncertainty. These regulations, along with implicit government guarantees, have led regional banks’ depositors to increasingly perceive large U.S. banks as safer alternatives, heightening their incentives to withdraw from regional banks to large national banks during periods of stress. In fact, the severity of bank runs on regional banks has notably increased since the implementation of Basel III regulations. For instance, in 2008, Wachovia and Washington Mutual, the largest banks affected at the time, experienced what were called “massive” deposit runs of 4.4% and 10%, respectively⁸ ([Rose, 2023, 2015](#)). By contrast, Silicon Valley Bank lost 25% of its deposits in a single day, with an additional 62% scheduled for withdrawal before its shutdown. Signature Bank lost 20% of its deposits in one day, while First Republic Bank lost 14% on the first day, 23% on the next business day, and an additional 20% over the course of the run ([Gruenberg 2023](#); [BCBS 2023](#)). These trends underscore the possibility that regulations intended to stabilize large banks have inadvertently increased the systemic vulnerabilities of smaller institutions. Our results underscore the importance of developing regulatory frameworks that enhance systemic stability without undermining the competitiveness or safety of smaller institutions.

The rest of the paper is organized as follows: [Section 2](#) provides more background and discusses the relationship between our findings and prior research. [Section 3](#)

⁸Prior to 2022, the most severe run was on Continental Illinois, which lost 30% of its funding over 10 days. ([Rose, 2023](#))

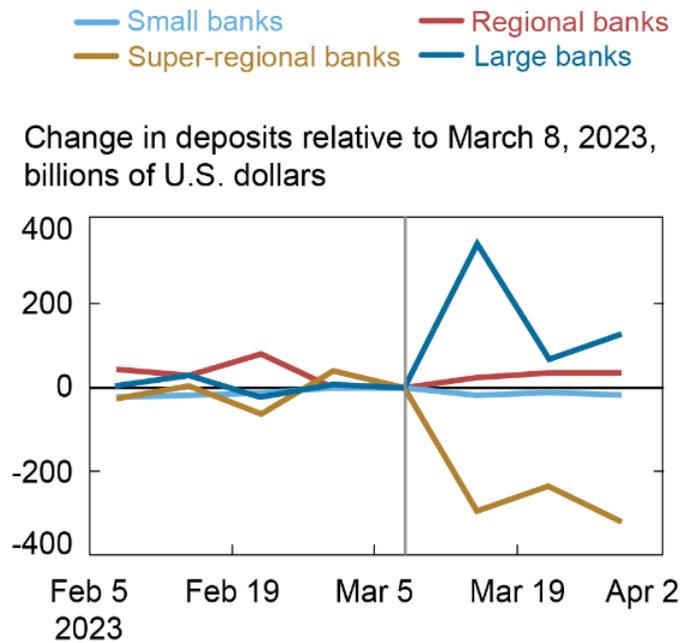


Figure 1: Deposit flow leaving regional banks to large national banks in March 2023. Source: Luck, Plosser and Younger (2023). Notes: Banks are categorized by the size of their domestic assets: small banks—less than \$5 billion; regional banks—\$5 to \$50 billion; super-regional banks—\$50 to \$250 billion; large banks—greater than \$250 billion.

introduces the model environment. [Section 4](#) documents the externality of big national banks on regional banks during a bank-run crisis. [Section 5](#) extends our analysis to deposit competition. [Section 6](#) provides the concluding remarks.

2 Related Literature

Our work is closely related to the literature on financial fragility and the policy responses to banking crises. Classic models emphasize the role of self-fulfilling prophecies in triggering bank runs ([Diamond and Dybvig, 1983](#); [Goldstein and Pauzner, 2005](#)), while subsequent research has explored how deteriorating fundamentals can initiate run dynamics ([Chari and Jagannathan, 1988](#); [Jacklin and Bhattacharya, 1988](#); [Uhlig, 2010](#); [Allen and Gale, 1998](#); [Acharya and Mora, 2015](#)). Network analyses further highlight how the failure of one institution can spread distress throughout the system ([Allen and Gale, 2000](#); [Eisenberg and Noe, 2001](#)),⁹ Although ex post government guarantees can reduce run risk ([Allen, Carletti, Goldstein and Leonello, 2018](#); [Keister, 2016](#)), they may also remove the disciplining effect of deposits and thus encourage excessive risk-taking ([Cooper and Ross, 2002](#)). Previous research has examined how capital and liquidity

⁹Some recent work demonstrates that heterogeneous asset holdings can mitigate these contagion effects ([Goldstein, Kopytov, Shen and Xiang, 2024](#)).

regulations influence large banks' incentives (Kara and Ozsoy, 2020; Gertler and Kiyotaki, 2015; Van den Heuvel, 2022; Gertler, Kiyotaki and Prestipino, 2020), noting that while such measures curb risk-taking, they sometimes create unintended consequences (Jeanne and Korinek, 2020; Hanson, Kashyap and Stein, 2011; Cooper and Ross, 2002). Relatedly, Dávila and Goldstein (2023) derive optimal deposit insurance from a sufficient-statistics perspective, and Schilling (2023) shows that aggressive interventions can be counterproductive in a global game setting.

Our model incorporates heterogeneous depositors who derive utility from both financial returns and banking services, as in Egan, Hortaçsu and Matvos (2017a). This feature allows us to examine the ex ante deposit competition. By linking deposit competition and bank-run fragility, our work connects to research on the deposit franchise (Drechsler, Savov, Schnabl and Wang, 2023; Haddad, Hartman-Glaser and Muir, 2023), as well as on deposit insurance pricing, its impact on bank portfolios, and the determination of deposit interest rates (d'Avernas, Eisfeldt, Huang, Stanton and Wallace, 2023; Kim and Rezende, 2023; Egan, Hortaçsu and Matvos, 2017b; Egan, Lewellen and Sunderam, 2022). (We abstract away from the longer-term implications of deposit allocations on bank lending and real activity, as explored in Mian and Khwaja (2008); Gilje, Loutskina and Strahan (2016); Acharya, Das, Kulkarni, Mishra and Prabhala (2022).) Although the effects of macroprudential policy on the deposit competition of smaller banks are relatively underexplored, the recent 2023 regional banking crisis has enabled researchers to gather empirical evidence in this direction. For example, Caglio et al. (2023) examine the economic implications of deposit reallocation from smaller banks to the largest national bank during this episode. In line with their findings, our model identifies a non-pecuniary distortion of macroprudential policy in bank competition: by reducing the stability of smaller institutions, regulations and implicit government guarantees that safeguard national banks may also substantially reshape the competitive landscape. This underscores the need for a more nuanced understanding of regulatory impacts (Hanson, Kashyap and Stein, 2011). Our findings enrich prior studies of bank competition that focus on pecuniary externalities operating through price adjustments (Benigno, Chen, Otrok, Rebucci and Young, 2013; Walther, 2016; Jeanne and Korinek, 2019).

Our modeling approach is also linked to the global games literature, which provides insights into equilibrium selection in coordination problems involving strategic complementarities (Carlsson and Van Damme, 1993; Morris and Shin, 1998, 2001; Goldstein and Pauzner, 2005; He and Xiong, 2012; Heider, Hoerova and Holthausen,

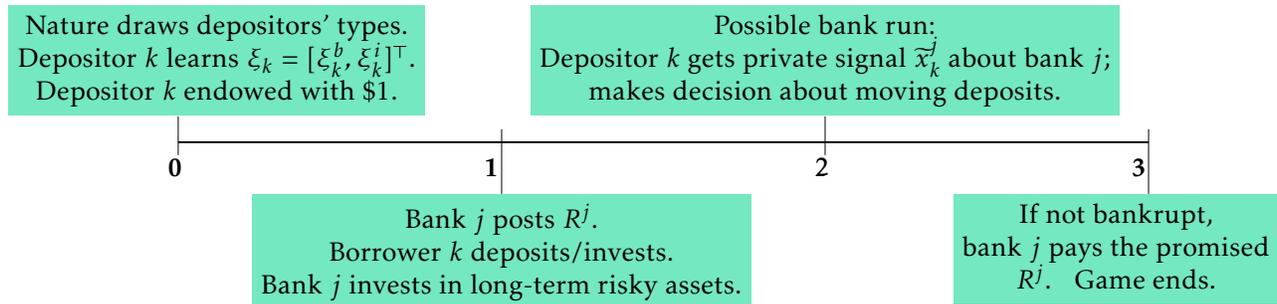


Figure 2: The timeline of the model.

2015; He, Krishnamurthy and Milbradt, 2019; Liu, 2016). Empirical studies (Chen, Goldstein, Huang and Vashishtha, 2024) support the relevance of these theoretical mechanisms, and experimental work (Heinemann, Nagel and Ockenfels, 2004, 2009) confirms that global-game equilibria can predict observed behavior in laboratory settings.

Our study also speaks to the ongoing debate surrounding the regional banking crisis of 2023 (Kim, Kundu and Purnanandam, 2024; Jiang, Matvos, Piskorski and Seru, 2024; Chang, Cheng and Hong, 2023; Caglio, Dlugosz and Rezende, 2023; Cookson et al., 2023). The collapse of SVB and the regional banking crisis exemplify how perceived differences in safety and regulatory backstops can influence depositor behavior and exacerbate fragility, as illustrated by our findings.

3 Model Setup

Time is discrete, and there is a single type of consumption good (dollar), which serves as Numéraire. Throughout the paper, we use the notation \tilde{x} to denote the random variable and x for its realization.

3.1 Agents, Timeline and Action Space

There are two types of agents: banks and depositors. The economy consists of \mathcal{N} regions, denoted by $\{1, 2, 3, \dots, \mathcal{N}\}$. Each region i hosts a regional bank and a continuum of local depositors. In region i , depositors are indexed by $k \in [0, \mathcal{M}^i]$, where \mathcal{M}^i denotes the mass of depositors in the region. Additionally, there is a national bank operating across all regions,¹⁰ while regional banks operate exclusively within their home regions. Throughout the paper, we refer to the representative regional bank operating in region i as “bank i ” and to the national bank as “bank b .” We use “bank j ” to refer to either bank i or bank b .

¹⁰We abstract from the national bank’s location decisions, as in d’Avernas et al. (2023), since this does not affect our main predictions.

Time is discrete and consists of three periods, $t = 1, 2, 3$, preceded by a pre-game period $t = 0$, as illustrated in Fig. 2. Let us focus on the representative region i . At $t = 0$, each depositor in region i receives an initial endowment of \$1 and learns their type, denoted as $\xi_k = [\xi_k^b, \xi_k^i]^\top$. Depositors' types represent their private utility from banking services, which will be detailed in Section 3.3.2. Each bank j begins with an initial asset level E^j , which is normalized to zero for analytical simplicity, as it does not affect our main results.

At $t = 1$, bank j announces a deposit rate R^j , aiming to attract deposits for investment in long-term risky projects.¹¹ Banks also provide signals regarding their default risks (which in equilibrium are truthful). Based on this information, depositors in region i choose among three investment options: depositing with the risk-free security, the regional bank i , or the national bank b . The return on risk-free securities is normalized to 1 per period, effectively making it equivalent to holding cash.

At $t = 1^+$, after raising total deposits L^j (assumed to be the sole liability for bank j), the bank allocates its total assets between long-term risky projects and risk-free securities. Let H^j represent the amount invested in risk-free securities and A^j the amount allocated to long-term risky assets.

At $t = 2$, depositor k with positive deposits at bank j receives a private signal \tilde{x}_k about the financial health of bank j . Based on these signals, depositors may withdraw some or all of their deposits. (In equilibrium, depositors either remain with bank j or withdraw all their deposits. If the latter occurs, the depositors are said to “run bank j .”) Withdrawn funds can either be held as cash or transferred to another bank, such as bank b . For simplicity, we assume that bank b invests all net incoming deposits at $t = 2$ exclusively in risk-free securities and promises a financial return of 1. The primary benefit of transferring deposits to bank b instead of holding cash is the additional banking service utility.¹² The ability of depositors to transfer funds between banks represents a key innovation of our model. This added dimension of flexibility not only reflects empirical patterns observed during banking crises but also introduces novel dynamics not previously captured. In particular, for depositors who value banking services, a safer bank within the same region provides a more convenient alternative than the less accessible risk-free security. The inflow of deposits from bank j to the safer bank

¹¹Our model assumes deposit contracts that mirror real-world practices without micro-founding their specific form. Prior literature suggests that more sophisticated contracts, such as state-contingent deposit rates, could mitigate bank-run risks (Allen and Gale, 2004).

¹²Relaxing this assumption does not affect the qualitative predictions of our model.

further strengthens the latter’s financial condition, increasing its appeal and intensifying the incentives for remaining depositors to run bank j .

At $t = 3$, the long-term risky projects initiated by banks at $t = 1$ mature. If the returns from these projects are insufficient, equity holders of bank j declares bankruptcy. Solvent banks pay the promised return R^j to depositors who stayed with the bank since $t = 1$ and a return of 1 to depositors who transferred to the bank at $t = 2$. In the event of bankruptcy, all remaining assets are liquidated and distributed pro rata among the bank’s remaining debt holders. In equilibrium, banks may also default at $t = 2$ due to bank runs and the associated fire-sale costs. Agents then consume the payoff at $t = 3$.

3.2 Regulation and Insurance

The U.S. banking sector is heavily regulated. Large national banks are subject to liquidity constraints, and all depository institutions are required to purchase Federal Deposit Insurance Corporation (FDIC) insurance. We model these regulatory requirements explicitly as follows:

- **Liquidity Regulation:** The national bank b must hold at least a fraction Γ of its total assets in risk-free securities at time $t = 1$.¹³ This requirement implies $H^b \geq \Gamma(A^b + H^b)$.
- **FDIC Insurance:** Bank j is required to pay an insurance premium φ^j for each dollar of its liabilities at time $t = 1$. Therefore, $A^j + H^j = E^j + (1 - \varphi^j)L^j$.

If bank j defaults, the FDIC guarantees a payoff of Λ per dollar deposit to the depositors of bank j .¹⁴ Here, Γ and φ^j are policy instruments that may depend on factors such as L^j , macroeconomic conditions, bank-specific productive technology, and banks’ specific risks. We will analyze the equilibrium implications of different policy configurations.

3.3 Agents’ Payoffs

Unlike [Diamond and Dybvig \(1983\)](#), agents in our model consume only at time $t = 3$, making their utility solely dependent on the payoff at $t = 3$. Early withdrawals occur only because depositors are concerned about potential bank defaults. Banks are operated by

¹³This corresponds to regulatory requirements such as the Supplementary Leverage Ratio (SLR) or the Liquidity Coverage Ratio (LCR).

¹⁴In practice, FDIC deposit insurance covers up to \$250,000 per depositor, per FDIC-insured bank, for each account ownership category. Depositors in our model are homogeneous, so we capture FDIC insurance with the parameter Λ . Our model of FDIC insurance aligns with the framework in [Dávila and Goldstein \(2023\)](#). Because regulators seldom vary protection offered to depositors across banks, we treat Λ as identical for all institutions.

their equity owners. For simplicity, we refer to the bank j 's equity holders as “banker j .”

3.3.1 Bankers' Payoffs

Suppose bank j attracts total deposits L^j at $t = 1$ and allocates H^j to risk-free securities. The bank invests the remaining A^j in long-term risky projects, which mature at $t = 3$. Early liquidation of these risky assets at $t = 2$ incurs a fire-sale loss, whereas liquidating risk-free securities at $t = 2$ does not incur such a loss; selling one unit of risk-free securities always returns exactly one dollar. If a run occurs at $t = 2$, and the total deposit outflow from bank j is O^j , the financial return on bank j 's total assets is given by¹⁵

$$(\bar{\theta} + \bar{\zeta}_j)G^j(A^j) - C^j(A^j, (O^j - H^j)^+) + (H^j - O^j)^+. \quad (1)$$

In Eq. (1), $\bar{\theta}$ represents aggregate macroeconomic uncertainty faced by all banks, such as interest rate risk, where larger realizations of θ indicate a better macroeconomic environment. The term $\bar{\zeta}_j$ captures bank-specific risk. In our baseline model, we focus on common economic shock and assume that $\bar{\zeta}_j = 0$ almost surely. The function $G^j : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ represents bank j 's production function, reflecting its investment opportunities. G^j is differentiable, with $G^{j'}(x) \geq 1$ weakly decreasing for all $x \geq 0$, capturing diminishing returns to scale as the bank's investment opportunities decline with size. When the total outflow O^j exceeds H^j , bank j must liquidate its long-term projects in the secondary market. This liquidation incurs a fire-sale cost, reducing the bank's asset value. For the baseline model, the fire-sale cost is given by an exogenous function $C^j : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$. The function C^j is twice differentiable in both arguments and satisfies $C^j(A^j, 0) = 0$ for any $A^j \geq 0$. Furthermore,¹⁶ $\partial_2 C^j(A^j, 0) > G^{j'}(A^j)$, $\partial_2 C^j(A^j, 0) > 1$ and $\partial_2^2 C^j(A^j, x)$ is weakly increasing for $x \in [0, \infty)$.¹⁷

Depositors may also transfer deposits from other banks or from their holdings of the risk-free security to bank j at $t = 2$. Let I^j denote the total deposit inflow. Bank j can utilize this inflow either to meet withdrawal demands or to purchase additional risk-free securities. For simplicity, we assume that the only investment available at time $t = 2$ is a risk-free asset with a fixed financial return of 1. Therefore, bank j offers a fixed deposit rate of 1 to depositors transferring their deposits at this stage. The final payoff for bank j at $t = 3$ is therefore given by

¹⁵For any real number x , we use the notation $(x)^+ = x\mathbb{1}_{[x \geq 0]}$ and $(x)^- = -x\mathbb{1}_{[x < 0]}$. Here, $\mathbb{1}_{[\cdot]}$ is the indicator function. $\mathbb{1}_{[\mathcal{S}]} = 1$ if statement \mathcal{S} is true, and $\mathbb{1}_{[\mathcal{S}]} = 0$ otherwise.

¹⁶For any function $f(x, y)$, let $\partial_2 f(x, y)$ denote $\frac{\partial}{\partial y} f(x, y)$, and ∂_2^2 indicates that this partial derivative is taken twice.

¹⁷For intuition of these functions, see the discussion below [Assumption 4](#).

$$\mathcal{V}_j^3 \equiv \bar{\theta} G^j(A^j) - C^j(A^j, (O^j - I^j - H^j)^+) + (H^j + I^j - O^j)^+ - R^j(L^j - O^j) - I^j. \quad (2)$$

Banker j will declare bankruptcy if and only if $\mathcal{V}_j^3 \leq 0$. Let $\mathcal{D}_j \equiv \{\mathcal{V}_j^3 \leq 0\}$ denote the event that bank j defaults at or before $t = 3$, and let $\mathcal{D}_j^c \equiv \{\mathcal{V}_j^3 > 0\}$ denote the event that bank j does not default at or before $t = 3$.

3.3.2 Depositors' Payoffs

Let $s_{k,j}$ denote the deposit made by depositor k with bank j at $t = 1$ and $s_{k,H}$ their investment in the risk-free security, i.e. cash. We assume $s_{k,H}, s_{k,i}, s_{k,b} \geq 0$. Resource constraint implies that $s_{k,H} + s_{k,i} + s_{k,b} = 1$. At time $t = 2$, depositor k receives a signal \tilde{x}_k about the payoff-relevant state and determine how to reallocate her deposits across the risky banks and cash. Suppose that depositor k successfully transfers $r_{k,\ell \rightarrow j}$ from bank ℓ to another investment option $j \in \{i, b, H\} \setminus \{\ell\}$ at time $t = 2$. Then, the ex-post payoff of depositor k at $t = 3$ is given by:

$$\begin{aligned} \mathcal{U}_k^3 &\equiv \underbrace{(s_{k,b} - r_{k,b \rightarrow i} - r_{k,b \rightarrow H}) \left((R^b + \xi_k^b) \mathbb{1}_{\mathcal{D}_b^c} + \Lambda \mathbb{1}_{\mathcal{D}_b} \right) + (s_{k,i} - r_{k,i \rightarrow b} - r_{k,i \rightarrow H}) \left((R^i + \xi_k^i) \mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i} \right)}_{\text{Payoff from deposits that stays with original bank since } t = 1} \\ &+ \underbrace{(r_{k,i \rightarrow b} + r_{k,H \rightarrow b}) \left((1 + \chi^b \xi_k^b) \mathbb{1}_{\mathcal{D}_b^c} + \Lambda \mathbb{1}_{\mathcal{D}_b} \right) + (r_{k,b \rightarrow i} + r_{k,H \rightarrow i}) \left((1 + \chi^i \xi_k^i) \mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i} \right)}_{\text{Payoff from deposits that moves to another bank at } t = 2} \\ &+ \underbrace{s_{k,H} + r_{k,i \rightarrow H} + r_{k,b \rightarrow H} - r_{k,H \rightarrow i} - r_{k,H \rightarrow b}}_{\text{Payoff from holding the risk-free security}}. \end{aligned} \quad (3)$$

Eq. (3) captures that the final payoff to depositor k depends on their initial deposits and investments at $t = 1$, their decisions regarding fund reallocations at $t = 2$, and whether banks default. If bank j defaults (i.e., event \mathcal{D}_j occurs), the depositor receives only the FDIC insurance payout of Λ^j per dollar remaining at bank j at the end of period $t = 2$, payable at $t = 3$. The term ξ_k^j in Eq. (3) represents depositor k 's private utility derived from banking services at bank j . Depositors have heterogeneous private utilities determined by their individual types: depositor k 's type $\xi_k = [\xi_k^b, \xi_k^i]^\top$ is determined at $t = 0$, and the probability distribution of depositor types across all regions is assumed to be independent and identically distributed (i.i.d.) with a common density function f_{ξ} . (While banks are assumed to know this density function, depositors need not possess such knowledge.) For each dollar deposited at bank j from time $t = 1$ through $t = 3$, depositor k receives not only the financial return R^j but also the non-monetary banking service utility ξ_k^j , provided bank j does not default (i.e., event \mathcal{D}_j^c occurs).

Suppose depositor k transfers an amount $r_{k,\ell \rightarrow j}$ of deposits from bank ℓ to bank j

at $t = 2$. If bank j does not default, it pays a rate of 1, and thus depositor k receives a financial payoff of $r_{k,\ell \rightarrow j}$. Depositors also derive non-monetary utility from banking services; however, this service utility may depreciate since depositors only transfer to the bank at stage $t = 2$. Consequently, we assume that the per-dollar service utility at $t = 2$ for depositor k who move to bank j is $\chi^j \xi_k^j$, where $\chi^j \in [0, 1]$.¹⁸

If bank j does not default, depositors retain the flexibility to withdraw all their deposits at any time. However, due to fire-sale losses incurred when banks liquidate long-term risky assets during a bank run, banks may lack sufficient liquid resources to meet all withdrawal requests at $t = 2$. As in [Goldstein and Pauzner \(2005\)](#), we assume that the bank must adhere to a **sequential service constraint**: depositors are served in the order they appear until the bank's liquid resources are depleted. Depositors running the bank arrive in a random order, making each equally likely to be served. Therefore, if the bank's maximum amount of withdrawals it can service before its resources are exhausted is O^j , and the total deposit outflow from bank j is O^j , then probability that a depositor can fully withdraw their funds is given by: $M^j \equiv \min\left(\frac{(O^j)^+}{O^j}, 1\right)$. With probability $1 - M^j$, the bank fails before the depositor is served, leaving the depositor unable to collect any funds directly from the bank. In this case, the depositor receives only the FDIC insurance payout of Λ per dollar, payable at $t = 3$.

Because depositors have linear utilities, their optimal strategies always result in a bang-bang solution, where they choose the best available option based on their information set. Linear utility also implies that depositors either remain with bank j or attempt to withdraw all their deposits at $t = 2$. Due to the sequential service constraint, if depositor k initiate transfer request $r_{k,j \rightarrow \ell} = s_{k,j}$ to investment option ℓ at $t = 2$, the following outcomes occur:

$$r_{k,j \rightarrow \ell} = \begin{cases} s_{k,j} & \text{with probability } M^j; \\ 0 & \text{with probability } 1 - M^j. \end{cases}$$

We restrict attention to parameter values for which the ex-ante default probabilities of both banks are *sufficiently small* (details provided later), so that no depositor chooses the risk-free asset at $t = 1$.

¹⁸Large national banks are valued for their convenience and standardized services, whereas regional banks provide localized support that relies on sustained relationships. In our numerical example, we set $\chi^b = 1$ and $\chi^i = 0$. Other values of χ do not qualitatively alter our results.

3.4 Agents' Information

Our information structure builds on the global game framework, and we focus on the limiting equilibrium as agents' information becomes increasingly precise. In this section, we explicitly state our assumptions regarding the information structure. This explicitness is important for two reasons: (1) To the best of our knowledge, our assumptions represent the most relaxed conditions in the global game literature that ensure the uniqueness of the limiting equilibrium. (2) By presenting these assumptions clearly, we aim to benefit researchers exploring other applications of global games, particularly in scenarios where canonical assumptions may not apply, but our framework may prove more suitable.

Bank j 's default decision depends on the realization of one random variable, $\tilde{\theta}$ which captures all the payoff-relevant uncertainty in this model. The banks' payoff structure ensures that, regardless of depositors' decisions, bank j defaults when $\tilde{\theta} \leq 0$ (even if no depositors run the bank) and does not default when $\tilde{\theta} \geq \bar{\Theta}^j$ for some $\bar{\Theta}^j > 0$ (even if all depositors run the bank). Let $\bar{\Theta} = \sup\{\bar{\Theta}^j\}$.

Assumption 1. The random variable $\tilde{\theta}$ has a probability density function $f_{\tilde{\theta}}$. We assume that $f_{\tilde{\theta}}$ is Lipschitz continuous on the interval $[0, \bar{\Theta}]$.

At time $t = 2$, each depositor k in region i gets a private signal \tilde{x}_k about the economic state $\tilde{\theta}$. The private signal \tilde{x}_k about $\tilde{\theta}$ is given by

$$\tilde{x}_k = \tilde{\theta} + \sigma \tilde{\varepsilon}_k \quad (4)$$

where $\sigma > 0$ is a constant. As in other global-game models, we are interested in studying the limiting equilibrium as σ approaches zero. However, unlike most previous models, we impose minimal restrictions on the distributions of $\tilde{\theta}$ and $\{\tilde{\varepsilon}_k\}$, allowing them to have essentially arbitrary PDFs. Without loss of generality, we assume that $\mathbb{E}[\tilde{\varepsilon}_k] = 0$ and the $\text{ess sup}_k \{\|f_{\tilde{\varepsilon}_k}\|_{\infty}\}$ is finite $F_{\tilde{\varepsilon}_k}$ -almost surely.

Assumption 2. $\tilde{\theta}$ and $\{\tilde{\varepsilon}_k\}$ are independent. All agents have common knowledge about the distributions of these random variables and the information structure.

Compared with the existing literature, our setting allows each depositor to have her own idiosyncratic private-signal distribution with density $f_{\tilde{\varepsilon}_k}$. This information structure is not only more empirically plausible but also expands the scope of global-game applications to environments in which relaxing canonical distributional assumptions is essential.

4 Subgame Equilibrium: Bank Run at $t = 2$

At time $t = 2$, depositor k in region i with existing investment in option $j \in \{i, b, H\}$ solves:

$$\begin{aligned} \max_{\substack{\{a_{k,j \rightarrow H}, a_{k,j \rightarrow l}\} \\ l \in \{i, b\}, l \neq j}} \mathcal{U}_k^2 &\equiv \mathbb{E} \left[\mathcal{U}_k^3 \mid \tilde{x}_k = x_k \right] \\ \text{subject to} \quad 0 &\leq a_{k,j \rightarrow H}, a_{k,j \rightarrow l}, \quad a_{k,j \rightarrow H} + a_{k,j \rightarrow l} \leq s_{k,j} \\ \mathbb{P}(r_{k,j \rightarrow H} = a_{k,j \rightarrow H}) &= 1 - \mathbb{P}(r_{k,j \rightarrow H} = 0) = \mathbb{M}^j \\ \mathbb{P}(r_{k,j \rightarrow l} = a_{k,j \rightarrow l}) &= 1 - \mathbb{P}(r_{k,j \rightarrow l} = 0) = \mathbb{M}^j. \end{aligned} \quad (5)$$

By Exact Law of Large Number (ELLN) stated in [Sun \(2006\)](#), the total mass of deposits intended to be withdrawn from bank regional i is

$$O^i = \mathcal{M}^i \int_{\xi_k} (a_{k,i \rightarrow H} + a_{k,i \rightarrow b}) \mathbb{1}_{[s_{k,i,1} > 0]} dF_{\bar{\xi}}(\xi_k). \quad (6)$$

The total mass of deposits intended to be withdrawn from national bank b is

$$O^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} (a_{k,b \rightarrow H} + a_{k,b \rightarrow i}) \mathbb{1}_{[s_{k,b,1} > 0]} dF_{\bar{\xi}}(\xi_k). \quad (7)$$

Bank j defaults if and only if economic state $\tilde{\theta} \in \mathcal{D}_j$ where

$$\mathcal{D}_j = \left\{ (\tilde{\theta}) G^j(A^j) - C^j(A^j, (O^j - I^j - H^j)^+) + (H^j + I^j - O^j)^+ - R^j(L^j - O^j) - I^j \leq 0 \right\}. \quad (8)$$

By ELLN, the total mass of deposits inflow to regional bank i is

$$I^i = \mathcal{M}^i \int_{\xi_k} \mathbb{M}^b a_{k,b \rightarrow i} \mathbb{1}_{[s_{k,b,1} > 0]} dF_{\bar{\xi}}(\xi_k). \quad (9)$$

The total mass of deposits inflow to regional bank b is

$$I^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \mathbb{M}^i a_{k,i \rightarrow b} \mathbb{1}_{[s_{k,i,1} > 0]} dF_{\bar{\xi}}(\xi_k). \quad (10)$$

Sequential service constraint Define bank j 's total payout capacity, $O^j(\theta)$, as the maximum amount of withdrawals it can service before its resources are exhausted—i.e.,

the unique value that solves:¹⁹

$$\theta G^j(A^j) - C^j(A^j, O^j(\theta) - I^j - H^j) = 0 \quad \forall j \in \{i, b\}. \quad (11)$$

Bank j can only pay out a total of O^j to depositors before depleting its liquid resources. Therefore, in equilibrium, the probability that a depositor at bank j can fully withdraw is given by

$$M^j = \min\left\{\frac{(O^j)^+}{O^j}, 1\right\}, \quad \forall j \in \{i, b\}. \quad (12)$$

Definition 1. For a fixed noise level $\sigma > 0$, a **noisy subgame equilibrium** consists of a strategy profile (with depositor actions $a_{k,j \rightarrow l}$ and bank default strategies \mathcal{D}_j) and a set of outcome variables (withdrawals O^j and inflows I^j) that simultaneously satisfy the core conditions in Eqs. (5) to (10). A **noisy subgame equilibrium with sequential service constraint** must also satisfy the additional friction constraints from Eqs. (11) and (12).

Definition 2. The **Benchmark Equilibrium of Connected Bank Run** is the limit of the noisy subgame equilibrium as $\sigma \downarrow 0$. The **Benchmark Equilibrium with a Sequential Service Constraint** is the limit of the noisy subgame equilibrium with a sequential service constraint as $\sigma \downarrow 0$.

Definition 2 introduces the subgame equilibrium for our **Benchmark Economy of Connected Bank Run**. A central contribution of this paper is to highlight the new equilibrium dynamics that arise from depositor mobility across banks. To underscore these novel dynamics, we juxtapose our benchmark with the classical global-games setting, where withdrawing into cash is the sole outside option (Morris and Shin, 2004; Goldstein and Pauzner, 2005). Our framework nests that setting, allowing us to conveniently formalize an **Alternative Economy of Isolated Bank Run** in which the run option is fixed to cash. Throughout the paper we compare equilibrium predictions under these two environments, thereby quantifying how cross-bank deposit reallocations amplify or dampen fragility.

Definition 3. In the subgame equilibrium of the **Alternative Economy of Isolated Bank Run** at $t = 2$, depositors are prohibited from transferring funds between banks at time $t = 2$. Formally, each depositor k faces the constraint

$$a_{k,i \rightarrow b} = a_{k,b \rightarrow i} = 0, \quad \forall k, i. \quad (13)$$

¹⁹ O^j is well defined because $\partial_2 C(A^j, \cdot) > 0$.

Hence, a depositor who runs from bank j may reallocate funds only into the risk-free asset (cash). The equilibrium of the **Alternative Economy of Isolated Bank Run** consists of the variables defined in [Definition 2](#) as $\sigma \downarrow 0$, subject to the additional constraint [Eq. \(13\)](#).

4.1 Analysis of Isolated Bank Run

4.1.1 Classic result without liquidity buffer or sequential service constraint

Consider any regional bank i . Our general framework relaxes several assumptions common in the global games literature. To establish a clear benchmark, we first analyze a baseline setting that reimposes three standard constraints: (1) there is no cross-bank deposit mobility; (2) the representative bank j holds no liquidity buffer ($H^i = 0$); and (3) the sequential service constraint is absent, such that all depositors who run believe that they will be served ($M^i = 1$). In this environment, we show that the subgame of the **Alternative Economy of Isolated Bank Run** ([Definition 3](#)) has a unique limiting equilibrium as the noise level vanishes ($\sigma \downarrow 0$). This finding confirms that our framework nests the classic predictions of the global game model.

Lemma 1. *Suppose that $\partial_2 C(A^i, 0) \geq R^i > 1$ and $H^i = 0$. For any PDF $f_{\bar{\theta}}$ and PDF family $\{f_{\bar{\theta},k}, \forall k\}$ that satisfy the previously stated assumptions, there exists a unique limiting subgame equilibrium as $\sigma \rightarrow 0$. In this limiting equilibrium:*

1. All depositors stay with bank i if realization of $\theta > \theta_B^i$ and run the bank if $\theta < \theta_B^i$.
2. $\theta_B^i \equiv \frac{C^i(A^i, O_B^i) + R^i(L^i - O_B^i)}{G^i(A^i)}$ where O_B^i is a constant given by $O_B^i \equiv \mathcal{M}^i \int_{\xi_k} \frac{(1-\Lambda)s_{k,i}}{R^i + \xi_k^i - \Lambda} dF_{\xi}^i$.
3. The equilibrium can be solved by iteratively eliminating dominated strategies.²⁰

Proofs of all results, including this lemma, are provided in [Internet Appendix 1](#). Intuitively, whenever the common state is low enough, a fraction of depositors get low enough signals that trigger them to run the bank, which then mobilize all depositors to run the bank in a feedback loop captured by [Fig. 3](#). Here, we highlight several innovations in our proof techniques which may be valuable for other applications involving games with strategic complementarities: (1) Different applications of global game models may impose natural restrictions on the probability distributions of $\bar{\theta}$ and $\bar{\xi}_k$. In our proof, we maximize flexibility for these distributions, allowing for a broader range

²⁰For a sufficiently small σ , all possible equilibria are bounded by two extreme sequential equilibria. These two extreme sequential equilibria can be derived by iteratively eliminating dominated strategies, and the distance between these two equilibria—defined as the maximum difference across their corresponding equilibrium strategies—converges to zero as $\sigma \rightarrow 0$, ensuring a unique equilibrium outcome in the limit.

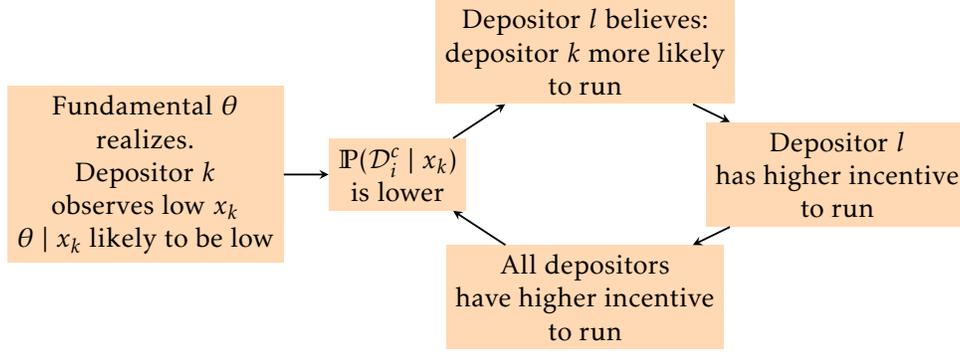


Figure 3: The feedback loop in a bank run stemming from strategic complementarity.

of applications. (2) Our proof also does not require the assumptions of *Payoff continuity* and *Bounded derivatives*, thus extending the applicability of global game models to settings with more flexible payoff structures.

4.1.2 The Effect of the Sequential Service Constraint

Starting from [Goldstein and Pauzner \(2005\)](#), recent literature highlights that realistic bank-run models do not exhibit global strategic complementarities—a condition where any one depositor’s withdrawal invariably increases the incentive for all others to run. Instead, the sequential service constraint induces a non-monotonic relationship between total outflows and the incentive to run.

Initially, as withdrawals from bank i increase, so does the perceived risk of failure, encouraging more depositors to join the run. However, this incentive reverses once the queue of depositors becomes sufficiently long. Specifically, the probability of an individual depositor being served, $M^i = \min\left(\frac{Q^i}{O^i}, 1\right)$, is decreasing in the size of the outflow, all else equal. Consequently, the expected payoff from joining a long queue diminishes, discouraging depositors to run and thereby breaking the global strategic complementarity. Our proof is robust to this feature. However, there is no closed-form characterization of the limiting equilibrium.

To build intuition and facilitate direct comparison with the existing literature, we focus on a more tractable special case. Specifically, we study a setting in which all depositors are ex-ante homogeneous before receiving their private signals: they share the same banking utility and draw private signals from an identical distribution (that is, $\xi_k^i = \xi^i$ and $f_{\tilde{\epsilon}_k} = f_{\tilde{\epsilon}}$ for all k). This simplification yields a much clearer characterization of the limiting equilibrium. (For the general result without this homogeneity assumption, see [Lemma 6](#) in [Internet Appendix 1.2](#).) This setting generalizes the environments studied by [Goldstein and Pauzner \(2005\)](#); [Elenev et al. \(2021\)](#) and others, in which the

private-signal density $f_{\bar{\epsilon}}$ is restricted to be the density of a uniform distribution.

Lemma 2. *Suppose $H^i = 0$ and $\partial_2 C(A^i, 0) \geq R^i > 1$. In addition, assume that depositors in region i are ex-ante homogeneous, so that $\xi_k^i = \xi^i$ and $f_{\bar{\epsilon}_k} = f_{\bar{\epsilon}}$ for all k , where $f_{\bar{\epsilon}}$ is an arbitrary probability density function with sufficiently large support. Then, as the signal noise vanishes, $\sigma \rightarrow 0$, there exists at least one limiting subgame equilibrium. In any such limiting equilibrium:*

1. All depositors **stay** if the fundamental $\theta > \theta_S^i$ and **run** if $\theta < \theta_S^i$. The run threshold θ_S^i is weakly less than θ_B^i .
2. The run threshold θ_S^i is given by $\theta_S^i \equiv \frac{C^i(A^i, O_S^i) + R^i(L^i - O_S^i)}{G^i(A^i)}$, where the quantities O_S^i and O_B^i are the solution to the system of equations

$$\frac{1 - \Lambda}{R^i + \xi^i - \Lambda} + \log(L^i) - \log(O_S^i) = \frac{O_S^i}{O_B^i}, \quad (14)$$

$$C^i(A^i, O_S^i) + R^i(L^i - O_S^i) = C^i(A^i, O_B^i).$$

3. The equilibrium cannot be obtained by iterative elimination of dominated strategies.
4. The limiting equilibrium is unique if Eq. (14) has a unique solution.²¹

Remarks Comparing Lemma 1 and Lemma 2, we find that the **Sequential Service Constraint** quantitatively lowers the bank-run cutoff, but the model's qualitative predictions remain unchanged. However, the equilibrium with the Sequential Service Constraint departs from the standard global-games framework in a crucial way—it is *not solvable* by iterative elimination of dominated strategies. Instead, it requires all depositors to share common knowledge of the entire strategy space and to correctly conjecture the strategies of others when the signal noise $\sigma > 0$. This represents a demanding rationality assumption, especially for a dispersed group of depositors.

From a policy perspective, the simplified model without the Sequential Service Constraint may be more prudent, as it relies on weaker rationality assumptions. Moreover, since the run cutoff under the simplified model is weakly larger than under the model with the Sequential Service Constraint, it provides a more cautious and robust basis for policy design. This motivates a simplifying assumption for the remainder of our analysis.

²¹Although this condition holds for a broad class of cost functions C^i , fully characterizing the conditions that guarantee a unique solution is beyond the scope of this paper. One sufficient condition is that $\partial_2^3 C^i$ is sufficiently large. See [Internet Appendix 1.3](#) for the proof of the lemma.

Assumption 3. For the remainder of our analysis, we assume that in games with $\sigma > 0$ depositors ignore the Sequential Service Constraint and behave as if they can always withdraw successfully (i.e., $M^i = 1$).

This choice is justified on three grounds. First, it is necessary for **tractability**, as the subgame equilibrium has no closed-form solution in Lemma 2 and becomes unwieldy in extensions with cross-bank deposit mobility, which is the main focus of this paper. Second, it may be more **behaviorally realistic**, reflecting bounded rationality. Third, the assumption is consistent with the fully rational outcome in the limiting equilibrium: at the run threshold θ_B^i , the sequential service constraint would **not** bind ($O_B^i < O(\theta_B^i)$), so $M^i = 1$ always holds when $\sigma = 0$.

4.1.3 Effects of Liquidity Regulation: $H^j > 0$

When banks must hold a stock of risk-free assets ($H^j > 0$), the analysis becomes richer: for small outflows ($O^j < H^j$) no fire-sale loss is incurred and the bank saves $(R^j - 1)O^j$ on foregone liabilities, lowering its failure probability and inducing *strategic substitutability*. To rule out degenerate cases in which the bank would prefer to induce universal early withdrawal, we assume that paying all depositors early is strictly more costly than continuing to maturity: $C^j(A^j, L^j - H^j) + H^j > R^j L^j$.²²

Our results show that strategic complementarity dominates when H^j is small, and substitutability dominates when H^j is large, but in both cases a *unique limiting sub-game equilibrium* exists. In our model, bank j is *fundamentally insolvent* when its asset payoff cannot meet promised liabilities without fire-sales; we term such episodes **fundamental runs**. Even when the bank would remain fundamentally solvent, a coordination failure among depositors can still provoke early withdrawals, trigger fire-sales, and lead to default—an outcome we label a **coordination run**. The next definition formally distinguishes these two regions, and the key result (Theorem 1) in this section shows how macro-prudential tools shift the boundary between them, thereby revealing a policy trade-off.

Definition 4 (Fundamental Default Boundary). For each bank $j \in \{b, 1, 2, \dots, N\}$ and some $H \geq 0$, the **Fundamental Default Boundary** is the unique constant $\widehat{\theta}^j(H)$ such that $\widehat{\theta}^j(H)G^j(A^j) + H^j - R^j L^j = 0$.

For each bank j , let O_B^j be the constant defined in Lemma 1. Define \widehat{H}^j to be the unique

²²As a microfoundation, when R^j is chosen endogenously at $t = 1$, the inequality always holds in equilibrium: bank j will not choose an R^j that violates it.

constant such that $C^j(A^j, O_B^j - \widehat{H}^j) + \widehat{H}^j = R^j O_B^j$. By [Assumption 4](#), \widehat{H}^j is well-defined and $\widehat{H}^j < O_B^j$.

Lemma 3. *Suppose that for bank j , $\partial_2 C(A^j, 0) \geq R^j > 1$ and $0 < H^j < L^j$. There is a unique limiting subgame equilibrium as $\sigma \rightarrow 0$.*

1. *If $H^j < \widehat{H}^j$, then in this limiting equilibrium, all depositors will stay with bank j if the realization $\theta > \theta_B^j(H^j)$, and will run the bank if $\theta < \theta_B^j(H^j)$ where $\theta_B^j(H^j) \equiv \frac{C^j(A^j, O_B^j - H^j) + R^j(L^j - O_B^j)}{G^j(A^j)}$. The constant O_B^j is defined in [Lemma 1](#) for $j \neq b$; if $j = b$, $O_B^b = \sum_i \mathcal{M}^i \int_{\xi_k} \frac{(1-\Lambda) s_{k,b}}{R^b + \xi_k^b - \Lambda} dF_{\xi}$.*
2. *If $H^j \geq \widehat{H}^j$, then in this limiting equilibrium, all depositors will stay with bank j if the realization $\theta > \widehat{\theta}^j(H^j)$, and will run the bank if $\theta < \widehat{\theta}^j(H^j)$ where $\widehat{\theta}^j(H^j)$ is the Fundamental Default Boundary for bank j .*

Although [Lemma 3](#) studies effects of banks' liquid buffer H , our framework also extends to incorporate explicit government guarantees. If depositors of bank b anticipate government capital injections amounting to H^b during a banking crisis, their equilibrium strategies mirror those characterized in [Lemma 3](#).

Our analysis shows that liquidity requirements can mitigate coordination runs among U.S. banks and, if sufficiently stringent (i.e. $H^j \geq \widehat{H}^j$), can eliminate them altogether. Mandating excessive liquidity, however, is generally inefficient because safe assets typically earn less than risky investment. Empirical evidence indicates that once Basel III liquidity and leverage constraints became binding, large U.S. banks withdrew from otherwise profitable trades, such as covered-interest-parity (CIP) arbitrage ([Du, Tepper and Verdelhan, 2018](#); [Anderson, Du and Schlusche, 2021](#)). The same constraints have been linked to a measurable contraction in loan supply and a decline in overall return on assets ([Ben Naceur, Pépy and Roulet, 2017](#); [Basel Committee on Banking Supervision, 2022](#)). By depressing profitability, an overly large liquid-asset buffer raises the fundamental solvency boundary when liabilities remain fixed. Thus, higher liquidity can—counterintuitively—increase the risk of bank failure (see [Fig. 4](#)). The next theorem formalizes this result.

Theorem 1. *Consider a national bank b that operates in isolation with zero equity and total liabilities L^b , promising a gross return R^b . Regulation requires the bank to hold safe assets of amount H^b , while the remainder $A^b = L^b - H^b$ is invested in risky assets. Suppose that*

$R^b \geq \frac{G'(O_B^b - \widehat{H}^b)}{G'(L^b - \widehat{H}^b)}$.²³ The macro-prudential policy that minimizes the overall probability of default is $H^{b*} = \widehat{H}^b < O_B^b$.

Policy debates following the 2023 banking crisis emphasize that new patterns in modern banking—marked by rapid deposit withdrawals—necessitates rethinking liquidity regulations like the Liquidity Coverage Ratio (LCR). The collapse of banks like SVB, where liquid assets evaporated in a matter of hours, challenges the LCR’s assumption of a 30-day outflow horizon. In response, many policy recommendations advocate for significantly stronger liquidity buffers or improved access to lender-of-last-resort (LoLR) facilities. Notably, a January 2024 Group of Thirty (G30) report, chaired by William Dudley, recommends that banks pre-position enough collateral at central bank discount windows to cover *all runnable liabilities*.²⁴

In our model, such a policy corresponds to setting H_B^b to be more than L_B^b , i.e., a nearly full liquidity buffer. In contrast to the simple intuition that increased liquidity buffer enhance bank stability, our [Theorem 1](#) shows that this extreme requirement is **suboptimal** for minimizing bank-run risk. The optimal required holding of liquid assets for promoting financial stability lies well below the full-liability threshold and even the projected deposit outflow O_B^b .

Since large national banks face more stringent regulatory requirements and are substantially more likely to receive liquidity injections or government support during crises, we normalize $H^i = 0$ for all regional banks—an assumption we will microfound later—while allowing policymakers to set a $H^b \leq \widehat{H}^b$ for the large national bank.²⁵

4.2 Connected Bank Run: Equilibrium

Most U.S. banks compete with other banks within the same neighborhood. This regional banking competition implies that depositors perceive these banks as substitutable providers of banking services. Our next set of results demonstrates how a neighboring national bank with regulatory support or implicit government guarantees affects the fragility of other regional banks within the same area. We first focus on the case where $\mathcal{N} = 1$: there is just one regional bank i competing with one national bank.

²³Any alternative condition that guarantees $\widehat{\theta}^b(H)$ is increasing on $(\widehat{H}^b, L^b]$ —a property that plausibly holds for large U.S. banks—would serve equally well.

²⁴Group of Thirty an international forum of central bankers, economists and private financiers. Similar proposals can be found [here](#) and [here](#).

²⁵[Theorem 5](#) microfounds the choice $H^i = 0$. In the rest of the paper, we suppress notation and write $\theta_B^i(0), \widehat{\theta}^i(0)$ as $\theta_B^i, \widehat{\theta}^i$ respectively.

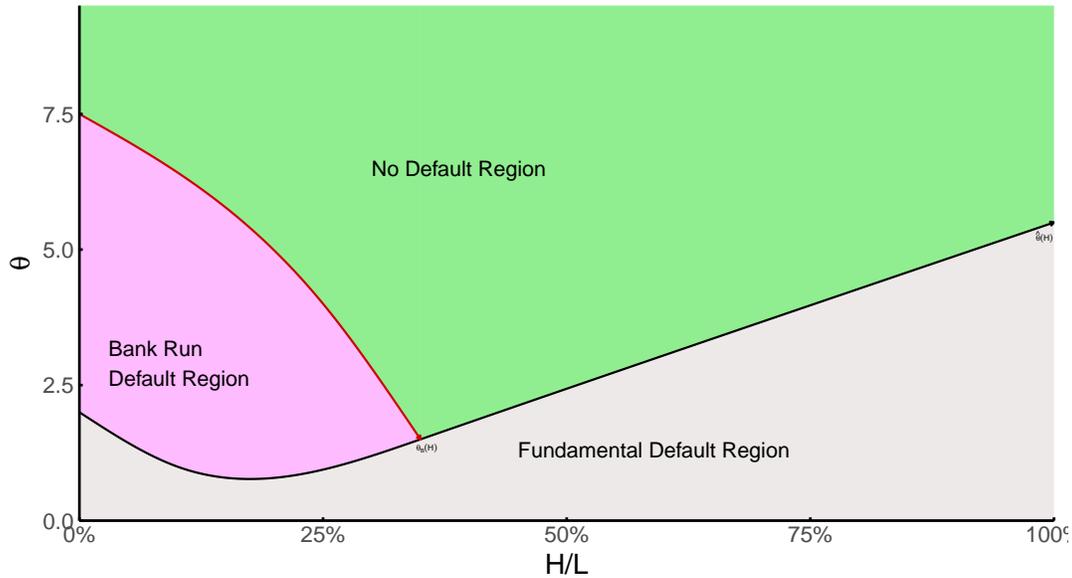


Figure 4: Fix R^b . A larger fraction of cash buffer H/L dampens **coordination run** risk but lowers lending efficiency, eventually raising **fundamental/overall** default risk.

Intuition and Complication As shown in Section 4.1, the classic global-game models guarantee a unique equilibrium by exploiting the one-dimensional structure of depositor decisions: each depositor employs a monotonic switching strategy based on their noisy private signal with a single-threshold rule. Small noise in these private signals ensures that only the self-consistent equilibrium—one that is robust to infinitesimal perturbations—survives in the limit. However, with cross-bank deposit mobility, this one-dimensional logic breaks down. In this richer setting, beliefs and fragility are mutually reinforcing across banks: if depositors flee one bank en masse and flood another with withdrawals-turned-deposits, they can inadvertently increase the stability of those other banks, entwining beliefs about multiple institutions. Equilibrium analysis lacks the monotonicity and single-crossing properties leveraged in classic models—resulting in a high-dimensional fixed-point problem that appears intractable.

It is therefore striking that, even in this enriched setting, *no more than two limiting equilibria* can arise, as we show below. We also identify conditions under which the standard global-games refinement continues to pin down a single outcome. To clarify this intuition, we first analyze a simple two-bank environment: a single regional bank (i) and a national bank (b). The regional bank holds zero safe asset ($H_i = 0$) and is more fragile than the national bank, $\theta_B^i > \theta_B^b(H^b)$. Because we focus on bank-run crises, we exclude reallocation at $t = 2$ that arise for reasons unrelated to a run—for example, situations in which a high ex-ante failure probability at the regional bank prompts depositors to place

funds with the national bank at $t = 1$ and transfer once the realized state is sufficiently high at $t = 2$. We rule out such behavior by assuming that for every bank pair j, ℓ and depositor k , $R^j + \xi_k^j \geq 1 + \chi^\ell \xi_k^\ell$, so depositors may move funds only in anticipation of a run.

Before stating the main predictions of the connected bank run model in [Theorem 2](#), the following lemma defines the key variables that characterize the equilibrium.

Lemma 4. *Assume $\mathcal{N} = 1$ and $R^j \leq \partial_2 C^j(A^j, 0)$ for every bank j . Then there exists a unique fixed point $(\bar{I}_D^b, \underline{\theta}_D^b(H^b), \bar{O}_D^i, \bar{\theta}_D^i)$ that solves*

$$\begin{aligned} \bar{I}_D^b &= \mathcal{M}^i \int_{\xi_k} \frac{\mathcal{O}^i(\underline{\theta}_D^b(H^b))}{L^i} \frac{s_{k,i} \chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda} dF_{\xi}(\xi_k), \\ \underline{\theta}_D^b(H^b) &= \max \left\{ \frac{C^b(A^b, (O_B^b - \bar{I}_D^b - H^b)^+) - (H^b + \bar{I}_D^b - O_B^b)^+ + R^b(L^b - O_B^b) + \bar{I}_D^b}{G^b(A^b)}, \widehat{\theta}^b(H^b) \right\}, \\ \bar{O}_D^i &= \mathcal{M}^i \int_{\xi_k} \frac{s_{k,i} (1 - \Lambda + \chi^b \xi_k^b)}{R^i + \xi_k^i - \Lambda} dF_{\xi}(\xi_k), \quad \bar{\theta}_D^i = \frac{C^i(A^i, \bar{O}_D^i) + R^i(L^i - \bar{O}_D^i)}{G^i(A^i)}. \end{aligned} \quad (15)$$

Moreover, there exists a unique fixed point $(\bar{I}_D^i, \underline{\theta}_D^i, \bar{O}_D^b, \bar{\theta}_D^b)$ that solves

$$\begin{aligned} \bar{I}_D^i &= \mathcal{M}^i \int_{\xi_w} \frac{\mathcal{O}^b(\underline{\theta}_D^i)}{L^b} \frac{s_{w,b} \chi^i \xi_w^i}{1 + \chi^i \xi_w^i - \Lambda} dF_{\xi}(\xi_w), \\ \underline{\theta}_D^i &= \max \left\{ \frac{C^i(A^i, (O_B^i - \bar{I}_D^i)^+) - (\bar{I}_D^i - O_B^i)^+ + R^i(L^i - O_B^i) + \bar{I}_D^i}{G^i(A^i)}, \widehat{\theta}^i \right\}, \\ \bar{O}_D^b &= \mathcal{M}^i \int_{\xi_w} \frac{s_{w,b} (1 - \Lambda + \chi^i \xi_w^i)}{R^b + \xi_w^b - \Lambda} dF_{\xi}(\xi_w), \quad \bar{\theta}_D^b = \frac{C^b(A^b, (\bar{O}_D^b - H^b)^+) - (H^b - \bar{O}_D^b)^+ + R^b(L^b - \bar{O}_D^b)}{G^b(A^b)}. \end{aligned} \quad (16)$$

Theorem 2. *Assume $\mathcal{N} = 1$ and $R^j \leq \partial_2 C^j(A^j, 0)$ for every bank j . Then there exists a unique fixed point $(\bar{I}_D^b, \underline{\theta}_D^b(H^b), \bar{O}_D^i, \bar{\theta}_D^i)$*

- (i) *If $\bar{\theta}_D^b(H^b) < \underline{\theta}_D^i$, the $t = 2$ subgame admits a unique limiting equilibrium as $\sigma \downarrow 0$. National-bank depositors stay when $\theta > \underline{\theta}_D^b(H^b)$ and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$. Regional-bank depositors stay when $\theta > \bar{\theta}_D^i$, switch to the national bank when $\underline{\theta}_D^b(H^b) < \theta < \bar{\theta}_D^i$, and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$.*
- (ii) *If $\bar{\theta}_D^b(H^b) \geq \underline{\theta}_D^i$, a second limiting equilibrium may arise in addition to the one above.*

*Regional-bank depositors stay when $\theta > \underline{\theta}_D^i$ and run to the safe asset when $\theta < \underline{\theta}_D^i$.
National-bank depositors stay when $\theta > \bar{\theta}_D^b(H^b)$, switch to the regional bank when $\underline{\theta}_D^i < \theta < \bar{\theta}_D^b(H^b)$, and run to the safe asset when $\theta < \underline{\theta}_D^i$.*

Remark Unlike the alternative economy of isolated bank run ([Definition 3](#)), the benchmark economy of connected bank run ([Definition 2](#)) admits cases where two equilibria coexist. However, we argue that only one of these equilibria is robust. The second equilibrium—in which the regional bank becomes more stable than the national bank—is a fragile, self-fulfilling prophecy. Its existence relies on perfect coordination among all depositors to generate strong inflows to the regional bank. As demonstrated in [Section A](#), this delicate coordination collapses immediately with the introduction of even an arbitrarily small degree of coordination uncertainty, causing the equilibrium to vanish. This lack of robustness suggests the equilibrium is a theoretical artifact. Because real-world depositors rarely possess such common knowledge about others’ strategies—especially at different institutions—this type of hyper-rational outcome has limited empirical relevance. Therefore, for the remainder of our analysis, we restrict our attention to the **unique, robust equilibrium**, which preserves the ordering of the run cutoffs established in the isolated economy.

4.3 The Safer Neighbor Externality

Up to this point, our specification does not exogenously differentiate national and regional banks. Because of this theoretical symmetry, a relabeled version of [Theorem 2](#) could, in principle, describe deposit flight in the opposite direction even in the first robust equilibrium. Yet runs from large national banks to small local banks are, to our knowledge, absent from modern banking crises. Market participants generally view regional banks as riskier than nationally chartered institutions under existing regulations. To capture this empirical asymmetry in our policy counterfactuals, we set $\chi^i = 0$ for the remainder of the paper. This restriction eliminates reverse flights from national to regional banks in all policy scenarios, allowing the equilibrium analysis to focus on the primary transmission mechanism of interest.

Corollary 1. *With the additional assumption that $\chi^i = 0$, there is an essentially unique robust subgame equilibrium for the **Benchmark Economy of Connected Bank Run** at $t = 2$.*

- (i) *When $\theta_B^i < \theta_B^b(H^b)$, the essentially unique subgame equilibrium is the same as the one described in [Lemma 3](#).*
- (ii) *When $\theta_B^i > \theta_B^b(H^b)$, then national-bank depositors stay when $\theta > \underline{\theta}_D^i(H^b)$ and run to the*

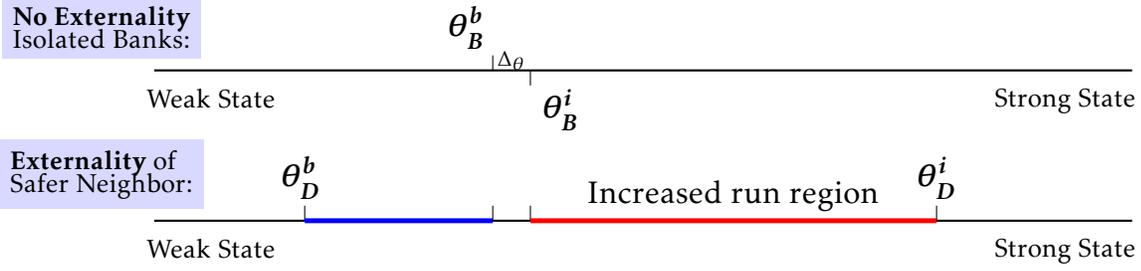


Figure 5: Matthew effect of bank fragility.

safe asset when $\theta < \underline{\theta}_D^b(H^b)$. Regional-bank depositors stay when $\theta > \bar{\theta}_D^i$, switch to the national bank when $\underline{\theta}_D^b(H^b) < \theta < \bar{\theta}_D^i$, and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$.

Corollary 1 demonstrates that the ordering of bank fragility can be conveniently determined by examining the canonical setting of the economy of isolated bank run. Therefore, we can call the national bank “safer” relative to the regional bank whenever $\theta_B^b(H^b) < \theta_B^i$. As illustrated by Fig. 5, the presence of a relatively safer neighboring bank increases the fragility of regional banks. We refer to this as the “safe neighbor externality”.

4.3.1 Contagion of Flight to National Bank

Once we consider bank competitions across more than 1 regions, a more subtle and systemic dynamic emerges. In fact, the instability created by a safe national banks can affect smaller regional banks, even for those banks which are safer than the national bank when considered in isolation. Our first main result demonstrates the intricacies of a “chain reaction” caused by the safe neighbor externality. Without loss of generality, we first rank the regional banks in the order of fragility: $\theta_B^1 \geq \theta_B^2 \geq \dots \geq \theta_B^N$. When consider in isolation, the national bank may be safer than some of the regional bank, but more fragile than other regional banks, i.e. $\theta_B^i > \theta_B^b(H^b), \forall i \leq \mathcal{N}^{cut}$ and $\theta_B^i < \theta_B^b(H^b), \forall i > \mathcal{N}^{cut}$.

Theorem 3. Suppose that $R^i \leq \partial_2 C^i(A^i, 0)$ and $R^b \leq \partial_2 C^b(A^b, 0)$. For any integer $u \in [\mathcal{N}^{cut}, \mathcal{N}]$, define functions

$$O_C^b(H_i, u) \equiv \left(\sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \left[\frac{s_{k,b}(1-\Lambda)}{R^b + \xi_k^b - \Lambda} - \mathbb{1}_{[i \leq u]} \left(\frac{O^i}{L^i} \right) \frac{s_{k,i} \chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda} \right] dF_{\xi}(\xi_k) \right)^+$$

$$\theta_C^b(H^b, u) \equiv \frac{C^b(A^b, (O_C^b(H_i, u) - H^b)^+) + (H^b - O_C^b(H_i, u))^+ + R^b(L^b - O_C^b(H_i, u))}{G^b(A^b)}.$$

For any i , let O_D^i, θ_D^i be defined as Eq. (15). Define N^{run} be the largest index of the regional bank that is relatively riskier than the national bank if depositors from all regional banks with smaller indexes move their deposits to the national bank, i.e., $N^{run} \equiv \sup\{u \leq \mathcal{N} \mid \theta_B^u \geq \theta_C^b(H^b, u - 1)\}$. Assume that $\theta_B^{N^{run}} > \theta_C^b(H^b, N^{run} - 1)$. There exists an essentially unique robust limiting subgame equilibrium as $\sigma \rightarrow 0$. In this limiting equilibrium:

1. For any regional bank $i \leq N^{run}$, all depositors stay with regional bank i if the realization $\theta > \theta_D^i$ and run the bank if $\theta < \theta_D^i$.
2. For any regional bank $i > N^{run}$, all depositors stay with regional bank i if the realization $\theta > \theta_B^i$ and run the bank if $\theta < \theta_B^i$, where O_B^i, θ_B^i are defined in Lemma 1.
3. Depositors do not run national bank b if the realization $\theta > \theta_C^b(H^b, N^{run})$; all depositors run the bank if $\theta < \theta_C^b(H^b, N^{run})$.

In addition, $N^{run} \geq N^{cut}$ and increases with H^b .

Theorem 3 highlights the fragility stemmed from the safe neighbor externality does not need one very safe national bank. Whenever there are deposits flowing into the national bank b in a banking crisis, the national bank b will be perceived as safer, which encourages depositors from other even safer regional banks to run from the regional bank. Our theory predicts that a local bank run can trigger the chain reaction and create a potential regional banking crisis, mirroring the event that took place in March 2023.

4.4 Discussion: macroprudential policy

Liquidity Regulation Since large national banks are typically considered systemically important, the importance-weighted marginal value of safety might outweigh any single regional bank. However, Lemma 4 demonstrates the adverse impact a single safe national bank can have on all regional banks. Ultimately, determining the optimal macroprudential policy that aims solely to increase the resilience of the financial system is a quantitative question and, within the framework of our model, depends on empirical estimates of the state probability distribution $f_{\bar{\theta}}$, fire-sale losses C^i , and the distribution of deposits between large and small banks. Nevertheless, our analysis underpin the message that regulations or support focused exclusively on national banks are likely to harm regional banks and may not be optimal. To formalize this, for economy \mathcal{E} , let

$$\mathcal{L}_{\mathcal{E}}^j(H^j) \equiv \left[(\bar{\theta} + \bar{\zeta}_j)G^j(A^j) - C^j(A^j, (O_{\mathcal{E}}^j - H^j)^+) + (H^j - O_{\mathcal{E}}^j)^+ - R^j(L^j - O_{\mathcal{E}}^j) \right]^-. \quad (17)$$

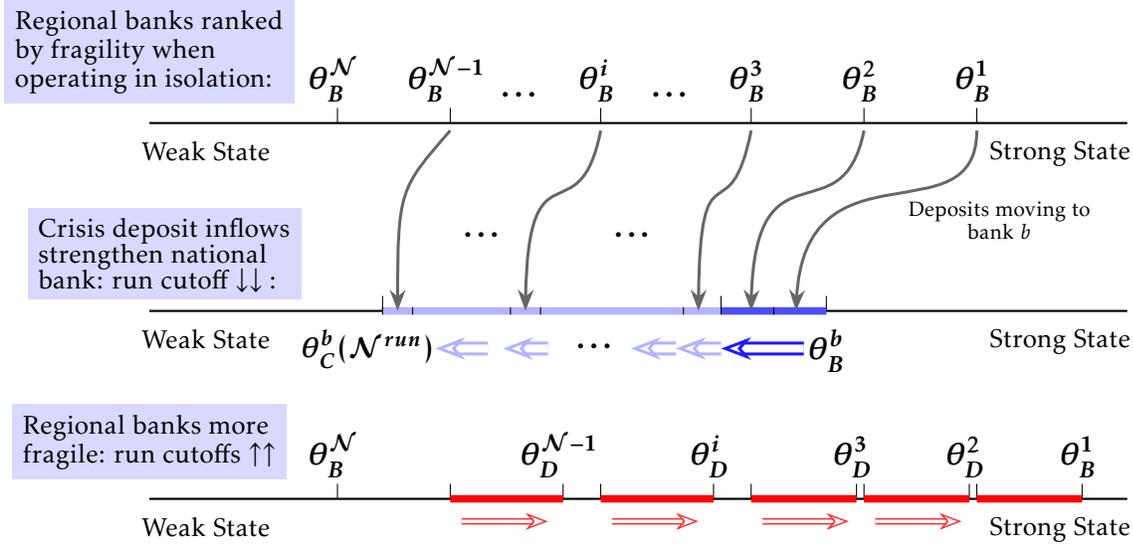


Figure 6: An example of the chain reaction of bank run. When operating in isolation, only regional banks 1 and 2 are more fragile than the national bank b . However, when the national bank b competes with each regional bank across all regions, deposits and bank runs become interconnected. If banks 1 and 2 fail, their deposits shift to and reinforce bank b , as depicted by the dark blue line segment. This reallocation increases bank b 's stability, making it safer relative to bank 3, and the process continues. Ultimately, nearly all regional banks are affected by the “safer neighbor externality,” becoming more fragile—even those that would be considered more stable than the national bank b in isolation.

to be the funding shortage for national bank j when it defaults due to a bank run. For example, \mathcal{L}_D^b is the loss to depositors of the national bank b in the Benchmark Economy of Connected Bank Run. From the literature on macroprudential policy, the social cost for bank failures is typically modeled as a monotonic function of \mathcal{L}_E . Suppose that the social cost associated with the failure of national j is $\mathcal{C}^j(\mathcal{L}_E^j)$ and the social benefit of successful bank j is $\mathcal{B}^j(\Gamma)\mathbb{1}_{[D_j^c]}$. The optimal liquidity regulation on the national bank b seeks to maximize ex ante social welfare:

$$\mathcal{W}^{t=2}(\Gamma) = \int_{\theta} \mathcal{B}^b(\Gamma)\mathbb{1}_{[D_b^c]} - \mathcal{C}^b(\mathcal{L}_E^b) + \sum_{i=1}^{\mathcal{N}} \mathcal{B}^i(0)\mathbb{1}_{[D_i^c]} - \mathcal{C}^i(\mathcal{L}_E^i) dF_{\bar{\theta}}(\theta). \quad (18)$$

The social planner who neglect the externality of safer national bank on the regional banks will set a liquidity regulation higher than optimal:

Theorem 4. Fix the following set of variables: $\{R^j, L^j, E^j, \varphi^j, \Lambda^j$ for $j \in \{b, 1, 2, \dots, \mathcal{N}\}\}$. Suppose that $\theta_B^i > \theta_B^b(0)$ for all $i \neq b$ and there exists a unique maximizer to Eq. (18) for $\mathcal{E} = B, D$. Then, the optimal liquidity regulation in the isolated bank-run economy Γ_B^* is always larger than that in the connected bank-run economy Γ_B^* : $\Gamma_B^* > \Gamma_D^*$ for any family of positive and increasing social cost function \mathcal{C}^j and strictly decreasing social benefit functions \mathcal{B}^j .

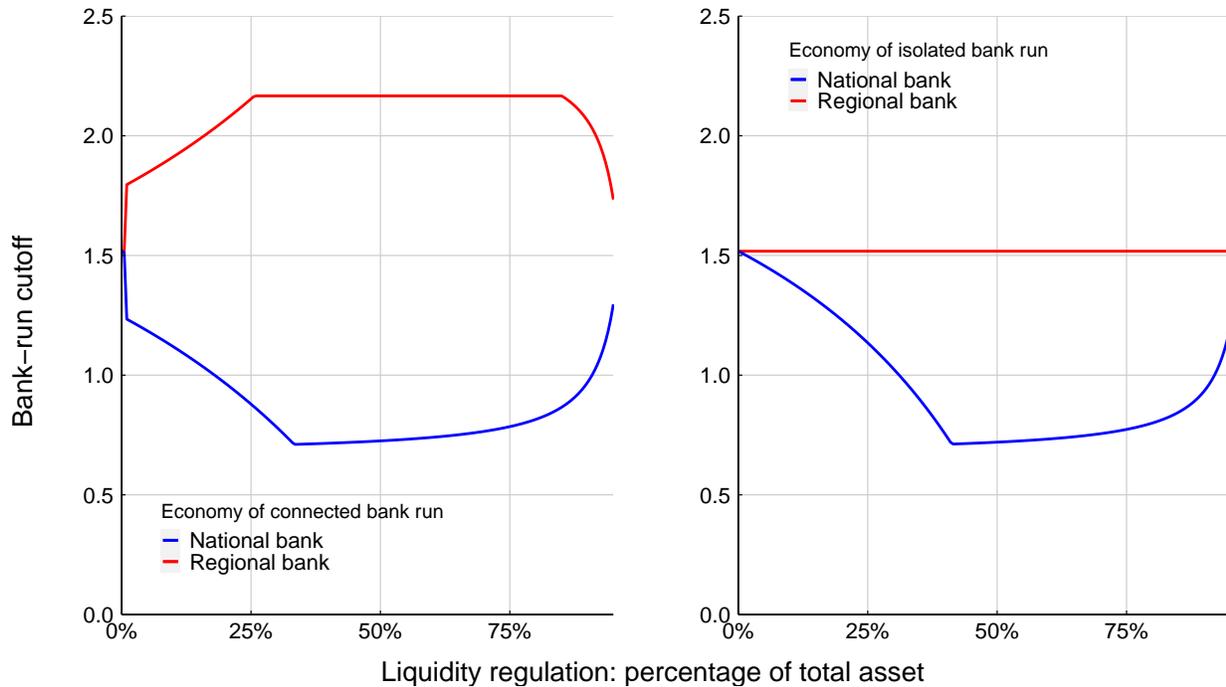


Figure 7: The x-axis is the percentage of risk-free assets of the national bank’s total assets required by regulation. The y-axis is the bank-run cutoff of each banks in two economies. When $\bar{\theta}$ is greater than the cutoff, no depositor runs, and vice verca. The national bank and the regional bank are otherwise identical, except that (1) the national bank faces liquidity regulation. The left panel shows the economy of connected bank run. The right panel shows the economy of isolated bank run.

Fig. 7 illustrates the effect of changing liquidity regulation under parametric assumptions described in Section 5.3. Higher liquidity regulation can effectively reduce the bank-run region of the large bank, but the moment when bank b becomes safer than bank i , depositors of the regional bank have higher incentive to run bank i , pushing up the run cutoff of bank i . Higher liquidity regulation does not always stabilize the national bank, as holding risk-free assets reduces banks’ productivity. When liquidity regulation is large enough, the inefficiency cause bad financial performance of bank b , and encourages depositors to run early.

Deposit Insurance After the collapse of SVB, there has been renewed discussion regarding the optimal FDIC insurance coverage ratio. On March 12, 2023, U.S. Treasury Secretary Janet Yellen, Federal Reserve Chair Jerome Powell, and FDIC Chair Martin Gruenberg issued a joint statement promising full protection for all depositors, effectively setting the coverage ratio at 100%. The literature, such as Dávila and Goldstein (2023), highlights the key trade-offs involved in determining an optimal deposit insurance coverage ratio, denoted here by Λ^j : Ex post, increasing Λ^j can reduce the likelihood of bank failures by avoiding marginal failures, but it may simultaneously elevate the social

costs of intervention and distort banks' risk-taking incentives. Our study introduces two new insights to this debate:

1. As shown in the left panel of Fig. 8, when deposit insurance coverage ratios Λ and Λ are constrained to be equal, increasing the coverage ratio can inadvertently *undermine* the stability of the regional banking system. Higher coverage ratios make large national banks unambiguously safer, thereby increasing their appeal as outside options for depositors of regional banks. This dynamic raises the incentives to run on regional banks. Importantly, this destabilizing effect persists unless the coverage ratio is high enough to entirely eliminate the risk of a bank-run at national banks. In our numerical example, this threshold occurs when $\Lambda \geq 61\%$, a level that is unlikely to be efficient or practical in most scenarios.
2. The right panel of Fig. 8 demonstrates the benefits of selectively increasing deposit insurance coverage ratios for regional banks. When the social planner accounts for the externalities created by safer national banks, increasing the coverage ratio for regional banks can simultaneously reduce the likelihood of regional bank failures and decrease the average social cost of ex-post interventions, all without distorting regional banks' risk-taking incentives. If the insurance coverage ratio for regional banks is sufficiently larger than that of national banks,²⁶ the externality posed by safer national banks on regional banks can be fully neutralized. Compared to the left panel, the right panel of Fig. 8 illustrates how better coverage for regional banks can stabilize the system even at relatively modest coverage levels.

These findings provide theoretical support for the actions taken by regulators in March 2023, which aimed to mitigate the spillover effects of regional bank runs.²⁷ Our analysis underscores the importance of nuanced deposit insurance policies that address the distinct needs and risks of regional banks versus national banks. At this point, we turn our attention to the implications of the aforementioned safe neighbor externality on banks' deposit competition at $t = 1$.

²⁶This can be reasonable, as many states have the jurisdiction to implement programs supporting local economies, and an increase in deposit insurance could be interpreted as an extension of such state-level support mechanisms.

²⁷Historically, the FDIC has provided equal insurance coverage to all depository institutions, albeit with differences in insurance premiums. However, during the 2023 regional banking crisis, the FDIC announced on March 12 that all depositors would have full access to their funds, including uninsured deposits. In the context of our model, this policy choice corresponds to setting $\Lambda = 100\%$.

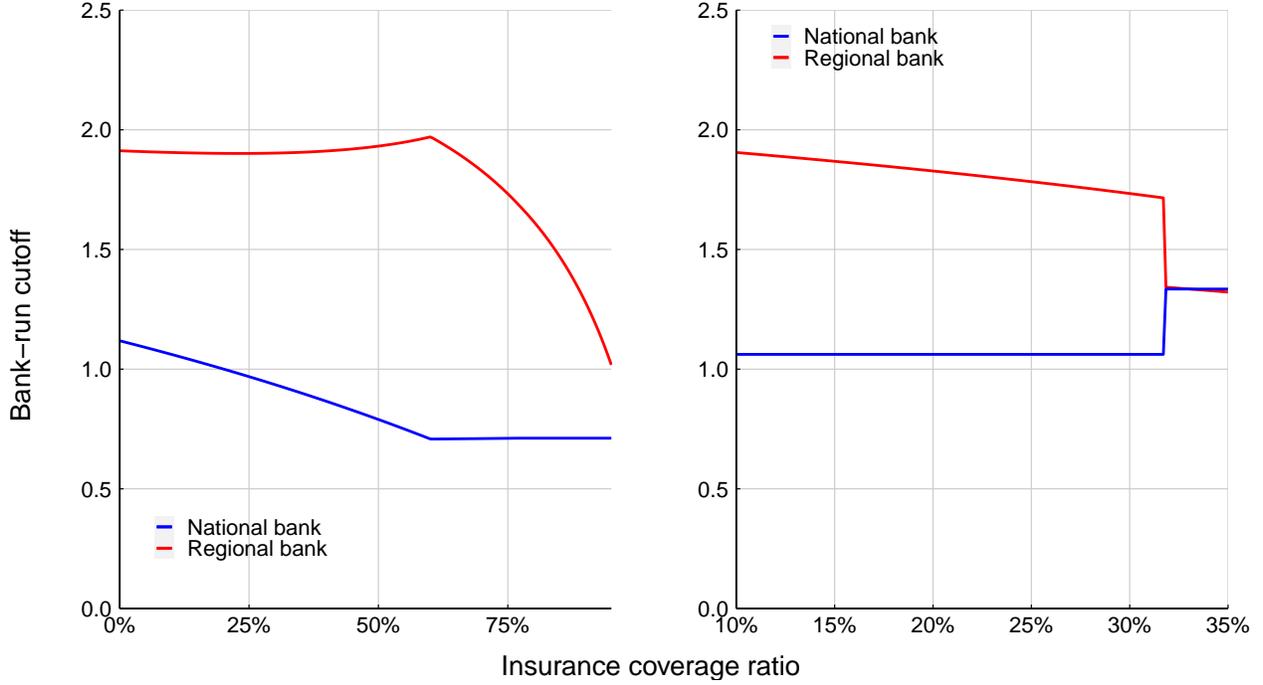


Figure 8: Both panels shows the bank-run cutoff in the economy of connected bank run. The x-axis shows the insurance coverage ratio Λ . We fix the liquidity regulation of the national bank $\Gamma = 10\%$. The left panel shows the effect of increasing insurance coverage ratio $\Lambda = \Lambda$ from 0% to 95% for both banks. The right panel shows the effect of increasing the insurance coverage ratio for just the regional bank Λ from 10% to 35%, while fixing the ratio for the national bank $\Lambda = 10\%$.

5 First Stage Equilibrium: Deposit Competition at $t = 1$

Each region i is a market of oligopolistic competition at time $t = 1$. In the following sections, we keep all assumptions outlined in [Lemma 4](#). We first study banks' asset allocation decisions.

5.1 Banks' Investment Decision

At time $t = 1^+$, after attracting total deposits of L^i , regional bank i allocates its assets between risky and risk-free investments by solving

$$\begin{aligned} & \max_{H^b} \mathbb{E} \left[\left((\bar{\theta} + \bar{\zeta}_i) G^i(A^i) - C^i(A^i, (O^i - I^i - H^i)^+) + (H^i + I^i - O^i)^+ - R^i(L^i - O^i) - I^i \right) \mathbf{1}_{\mathcal{D}_i^c} \middle| L^i \right] \\ & \text{subject to} \quad A^i = E^i + L^i(R^i) - \varphi^i L^i(R^i) - H^i \quad (\text{FDIC Premium}) \\ & \quad \quad \quad H^i \geq 0. \end{aligned} \tag{19}$$

The national bank b solves a similar problem:

$$\begin{aligned} & \max_{H^b} \mathbb{E} \left[\left((\bar{\theta} + \bar{\zeta}_b) G^b(A^b) - C^b(A^b, (O^b - I^b - H^b)^+) + (H^b + I^b - O^b)^+ - R^b(L^b - O^b) - I^b \right) \mathbb{1}_{\mathcal{D}_b^c} \Big| L^b \right] \\ \text{subject to} \quad & A^b = E^b + L^b(R^b) - \varphi^b L^b(R^b) - H^b \quad (\text{FDIC premium}) \\ & H^b \geq \Gamma (E^b + L^b(R^b) - \varphi^b L^b(R^b)) \quad (\text{Liquidity regulation}). \end{aligned} \quad (20)$$

Our first result reaffirms the well-documented insight that bankers have little incentive to hold the risk-free assets.

Theorem 5. *Without regulatory constraints, bank i holds no risk-free assets, i.e., $H^{i*} = 0$. Bank b holds the minimum amount of risk-free assets necessary to satisfy its regulatory requirements, namely,*

$$H^{b*} = \Gamma E^b + \Gamma (1 - \varphi^b) L^b(R^b).$$

Intuitively, holding the risk-free asset H^j enhances the safety of bank j , thereby increasing the value of its debt. However, without a change in the bank's total assets (i.e., $E^j + (1 - \varphi^j)L^j$), any rise in debt value transfers wealth from the bank's equity holders to its debt holders (see discussions in, e.g., [Andersen, Duffie and Song 2019](#); [Cooperman, Duffie, Luck, Wang and Yang 2025](#)). Consequently, equity holders have an incentive to minimize their holdings of safe assets.

5.2 Deposit Competition: Setup

Moving backward to the beginning of time $t = 1$, banks i and b in region i post deposit rates R^i and R^b , respectively, to attract deposits. We again analyze the equilibrium dynamics separately for the **Benchmark Economy of Connected Bank Run** and the **Alternative Economy of Isolated Bank Run**. For simplicity, we denote a generic economy by \mathcal{E} . Specifically, when $\mathcal{E} = D$, the economy corresponds to the Benchmark Economy of Connected Bank Run, and when $\mathcal{E} = B$, it corresponds to the Alternative Economy of Isolated Bank Run.

The dynamics of the deposit market, influenced by potential bank-run risks, can be quite intricate. Depositors' beliefs regarding the likelihood of future bank runs may become self-fulfilling, and banks' optimal strategies may vary according to how their actions influence these beliefs. To address this technical difficulty, we assume that, before choosing how to allocate their endowment at $t = 1$, depositors are unaware of both the distribution of other depositors' types (i.e., $f_{\bar{\zeta}}(\cdot)$) and the total market size \mathcal{M}^i , whereas $f_{\bar{\zeta}}(\cdot)$ and \mathcal{M}^i are private information among banks. This assumption may be more realistic

than presuming that depositors possess complete information about their local deposit markets. Absent this information, depositors cannot independently form accurate expectations regarding each bank's fragility from just observing the posted rates R^i and R^b .

As a result, banks i and b have an incentive to announce their fragility in order to coordinate depositors' beliefs. Specifically, both banks commit to their possible run cutoffs in the subgame at $t = 2$, denoted by $\widehat{\theta}^i$ and $\widehat{\theta}^b$, as well as to their liabilities, \mathcal{L}^i and \mathcal{L}^b , respectively. These commitments are announced publicly, bundled together with their deposit rates. Once banks i and b have attracted total deposits L^i and L^b , some sophisticated depositors may scrutinize the banks' balance sheets and thereby gain insight into the ex-post distribution of depositor types ξ associated with each bank. This information enables sophisticated depositors to compute the actual run cutoffs conditional on banks' balance sheet and investment decisions. If the calculated run cutoff exceeds bank j 's commitment, i.e., $\widehat{\theta}^j \neq \theta_{\mathcal{E}}^j$, or if the actual liabilities surpass the committed level, i.e., $L^j \neq \mathcal{L}^j$, these depositors disclose the discrepancy publicly (e.g., via the media or legal action). Consequently, bank j immediately goes bankrupt, forgoing long-term investments, and its equity holders incur a nonzero punitive cost—potentially in the form of a government-imposed fine. These dynamics ensure that, in equilibrium, bank j consistently sets $\widehat{\theta}^j = \theta_{\mathcal{E}}^j$ and $\mathcal{L}^j = L^j$. Our modeling approach thus establishes a microfoundation for the deposit market equilibrium that incorporates *rational expectations* among depositors.

With rational expectation, each regional bank i maximizes its payoff by solving the following (recall that that $H^{i*} = 0$ and $\mathcal{D}_i^c = \{\widehat{\theta} > \theta_{\mathcal{E}}^i\}$):

$$\begin{aligned} & \max_{R^i, \widehat{\theta}^i, \mathcal{L}^i} \mathbb{E} \left[\left(\widehat{\theta} G^i(A^i) - C^i(A^i, O^i) - R^i(L^i - O^i) \right) \mathbb{1}_{\mathcal{D}_i^c} \right] \\ & \text{subject to} \quad L^i = \mathcal{M}^i \int_{\xi_k} s_{k,i}^* dF_{\xi}(\xi_k) = \mathcal{L}^i, \quad 1 \leq R^i, \quad \text{and} \quad \theta_{\mathcal{E}}^i = \widehat{\theta}^i \end{aligned} \quad (21)$$

where $s_{k,i}^*$ is the strategy of depositor k in region i : due to linear payoff, $s_{k,i}^* = 1$ if bank i is the best investment option in her choice set, and $s_{k,i}^* = 0$ otherwise. Similarly, national

bank b solves²⁸

$$\begin{aligned} & \max_{R^b, \widehat{\theta}^b, \mathcal{L}^b} \mathbb{E} \left[\left(\widehat{\theta} G^b(A^b) - C^b(A^b, (O^b - H^{b*})^+) + (H^{b*} + I^b - O^b)^+ - R^b(L^b - O^b) - I^b \right) \mathbf{1}_{\mathcal{D}_b^c} \right] \\ \text{subject to} \quad & L^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} s_{k,b}^* dF_{\xi}(\xi_k) = \mathcal{L}^b \\ & H^{b*} = \Gamma E^b + \Gamma(1 - \varphi^b)L^b(R^b), \quad 1 \leq R^b, \quad \text{and} \quad \theta_{\mathcal{E}}^b = \widehat{\theta}^b. \end{aligned} \quad (22)$$

Depositors in region i take R^i and R^b as given and form rational expectations about H^{i*} and H^{b*} as well as the subgame equilibrium at $t = 2$. Specifically depositor k in region i chooses the optimal investment strategy by solving

$$\max_{\{s_{k,j}, j \in \mathcal{I}_k\}} \mathbb{E}[\mathcal{U}_k^2 \mid \theta_{\mathcal{E}}^j = \widehat{\theta}^j, R^j, \forall j \in \{i, b\}]. \quad (23)$$

Definition 5. *The deposit market equilibrium at $t = 1$ for the **benchmark economy of connected bank run** is defined as the set of strategies that include each depositor's optimal deposit allocation $s_{k,j}^*$ for depositor k in each region i , the interest rate strategy R^j set by bank j , and the investment strategy H^j of bank j . These strategies must satisfy the conditions outlined in [Eqs. \(19\) to \(23\)](#), where all agents anticipate that the subgame equilibrium follow the one outlined by [Definition 2](#).*

Definition 6. *The deposit market equilibrium at $t = 1$ for the **alternative economy of isolated bank run** is similar to [Definition 5](#), except that agents expect the subgame equilibrium to follow the one outlined by [Definition 3](#).*

5.3 Deposit Competition: Equilibrium Analysis

To lay the groundwork for our analysis of bank strategy, we characterize the optimal depositor strategy in the following lemma.

Lemma 5. *In region i , given R^i , R^b , $\widehat{\theta}^i$, $\widehat{\theta}^b$, \mathcal{L}^i , \mathcal{L}^b and other depositors' strategy profile, the best response strategy for a depositor k in region i of type $\xi_k = (\xi_k^b, \xi_k^i)^\top$ at time $t = 1$ is characterized*

²⁸The findings of [d'Avernas et al. \(2023\)](#); [Begenau and Stafford \(2022\)](#) demonstrate national banks set uniform rate across different local markets.

by

$$S_{k,i}^* = \mathbb{1} \left[\left(\mathbb{P}(\bar{\theta} > \widehat{\theta}^i)(R^i + \xi_k^i) + \mathbb{E} \left[(\Lambda + M^i(\Xi_{\mathcal{E}} - \Lambda)) \mathbb{1}_{[\bar{\theta} \leq \widehat{\theta}^i]} \right] \right) \geq \max \left(\mathbb{P}(\bar{\theta} > \widehat{\theta}^b)(R^b + \xi_k^b) + \mathbb{E} \left[(\Lambda + M^b(1 - \Lambda)) \mathbb{1}_{[\bar{\theta} \leq \widehat{\theta}^b]} \right], 1 \right) \right]$$

$$S_{k,b}^* = \mathbb{1} \left[\left(\mathbb{P}(\bar{\theta} > \widehat{\theta}^b)(R^b + \xi_k^b) + \mathbb{E} \left[(\Lambda + M^b(1 - \Lambda)) \mathbb{1}_{[\bar{\theta} \leq \widehat{\theta}^b]} \right] \right) > \max \left(\mathbb{P}(\bar{\theta} > \widehat{\theta}^i)(R^i + \xi_k^i) + \mathbb{E} \left[(\Lambda + M^i(\Xi_{\mathcal{E}} - \Lambda)) \mathbb{1}_{[\bar{\theta} \leq \widehat{\theta}^i]} \right], 1 \right) \right]$$

where

1. $\Xi_{\mathcal{E}}$ is the best outside option for depositor k if bank i defaults. In the alternative economy, $\mathcal{E} = B$ and $\Xi_B = 1$. In the benchmark economy, $\mathcal{E} = D$ and $\Xi_D = 1 + \chi^b \xi_k^b \mathbb{1}_{[D^b]}$.
2. $M^i = \min \left(\left(\frac{\mathbb{O}^i(\mathcal{L}^i, \theta)}{\mathcal{L}^i} \right)^+, 1 \right)$ in which $\mathbb{O}^i(\mathcal{L}^i, \theta)$ is the unique constant that solves $C^i(E^i + (1 - \varphi^i)\mathcal{L}^i, \mathbb{O}^i(\mathcal{L}^i, \theta)) = \theta G^i(E^i + (1 - \varphi^i)\mathcal{L}^i)$;
3. $M^b = \min \left(\left(\frac{\mathbb{O}^b(\mathcal{L}^b, \theta) + H^b}{\mathcal{L}^b} \right)^+, 1 \right)$, in which $\mathbb{O}(\mathcal{L}^b, \theta)$ is the unique constant that solves $C^b(E^b + (1 - \varphi^b)\mathcal{L}^b, \mathbb{O}(\mathcal{L}^b, \theta)) = \theta G^b(E^b + (1 - \varphi^b)\mathcal{L}^b - H^b) + H^b$.

The proof is straightforward. [Lemma 5](#) demonstrates that the equilibrium strategies of all depositors depend solely on the public signals R^i , R^b , \mathcal{L}^i , \mathcal{L}^b , $\widehat{\theta}^i$, and $\widehat{\theta}^b$. Consequently, banks can determine L^i and L^b by applying the exact law of large numbers conditional on these signals. Substituting the constraints $\widehat{\theta}^j = \theta_{\mathcal{E}}^j$ and $L^j = \mathcal{L}^j$ in banks' problem [Eq. \(21\)](#), the Lagrangian for bank i 's problem becomes:

$$\mathcal{L}^i(R^i, \widehat{\theta}^i, \Lambda) = \int_{\theta_{\mathcal{E}}^i} (\theta G^i(E^i + (1 - \varphi^i)L^i) - R^i L^i) f_{\bar{\theta}}(\theta) d\theta + \Lambda_1 (R^i - 1).$$

The first-order condition (FOC) with respect to R^i yields:

$$R^{i*} = \underbrace{(1 - \varphi^i)}_{\text{insurance fee}} \underbrace{\mathbb{E}[\bar{\theta} G^{i'}(A^i) | \bar{\theta} > \theta_{\mathcal{E}}^i]}_{\text{moral hazard}} - \underbrace{L^i (\partial L^i / \partial R^i)^{-1}}_{\text{market power}} - \underbrace{\frac{\theta_{\mathcal{E}}^i G^i(A^i) - R^i L^i}{\mathbb{P}(\bar{\theta} > \theta_{\mathcal{E}}^i)} \frac{\partial \theta_{\mathcal{E}}^i}{\partial R^i}}_{\text{bank run concern}} + \frac{\Lambda_1}{\mathbb{P}(\bar{\theta} > \theta_{\mathcal{E}}^i) \frac{\partial L^i}{\partial R^i}}.$$

Several frictions prevent bank i from setting the deposit rate at the competitive first-best level. The insurance premium creates a wedge in the bank's asset returns, while moral hazard implies that equity holders focus primarily on favorable states—where $\bar{\theta}$ is high—leading to excessive risk-taking. In addition, market power allows the bank to charge a high markup, and bank-run concerns force the equity holders to carefully manage the run cutoff. The FOC for bank b 's problem is analogous, though liquidity regulation introduces

an additional wedge:

$$R^{b*} = (1 - \varphi^b) \underbrace{\mathbb{E}[\bar{\theta}(1 - \Gamma) G^{b'}(A^b)]}_{\text{liquidity regulation}} + \Gamma \mathbb{P}[\bar{\theta} > \theta_\varepsilon^b] - L^b (\partial L^b / \partial R^b)^{-1} - \frac{\theta_\varepsilon^b G^b(A^b) + \Gamma L^b - R^b L^b \frac{\partial \theta_\varepsilon^b}{\partial R^b}}{\mathbb{P}(\bar{\theta} > \theta_\varepsilon^b) / f_{\bar{\theta}}(\theta_\varepsilon^b)} \frac{\partial \theta_\varepsilon^b}{\partial R^b} + \frac{\Lambda_1}{\mathbb{P}(\bar{\theta} > \theta_\varepsilon^b) \frac{\partial L^b}{\partial R^b}}.$$

In our model, the total deposits L^j are determined by depositors' investment decisions at $t = 1$. As illustrated by [Lemma 5](#), depositors' optimal decisions depend on the announced run cutoff $\widehat{\theta}^j$ (which equals θ_ε^i in equilibrium) and on the deposit rate R^j . However, as shown in [Lemma 3](#) and [Theorem 3](#), θ_ε^i is itself a function of R^j and L^j , with L^j being determined by depositors' optimal choices. Consequently, characterizing the equilibrium requires solving a complex fixed point problem. Thus far, we have not specified any functional forms for the production technology G^j , the firesale cost C^j , or the distribution of the random variables. This generality renders the deposit market equilibrium intractable. To better illustrate the intuition, we subsequently impose some parametric assumptions and demonstrate the properties of deposit competition equilibrium using a numerical example.

A Parametric Example In what follows, we assume that $\bar{\theta}$ is a standard exponential random variable with mean $1/\lambda_\theta$. All banks share identical production technologies and fire-sale cost functions. Specifically, for every bank $j \in \{b, 1, 2, \dots, \mathcal{N}\}$ and any $x \geq 0$, the production function is given by

$$G^j(x) = g_1 \lambda_\theta x + \frac{1 - e^{-g_2 x}}{g_2} \lambda_\theta, \quad \frac{d}{dx} G^j(x) = g_1 \lambda_\theta + e^{-g_2 x} \lambda_\theta$$

where $g_1, g_2 > 0$ are constants. Thus, in the absence of any frictions, the marginal return on banks' investments, $\mathbb{E}[\bar{\theta} G^{j'}(x)]$, equals $1 + g_1$ at $x = 0$ and asymptotically converges to 1 as $x \rightarrow \infty$. For simplicity, we assume that the fire-sale cost for regional banks is $C^i(y, x) = c^s x$, with $c^s > g^s$ for some benchmark g^s . Similarly, for the national bank, we assume $C^b(y, x) = c^b x$, with $c^b > g^b$ for a constant g^b . Given this symmetry, we also assume that regional banks share a uniform deposit insurance ratio Λ^s and insurance premium φ^s . Empirically, banks generally operate with high leverage. In our baseline analysis, we do not consider banks' equity issuance decisions; hence, we normalize $E^i = E^b = 0$. Furthermore, we assume that the depositors' type vector $[\bar{\xi}^b, \bar{\xi}^i]^\top$ is distributed as follows: $\bar{\xi}^b$ equals a constant $\xi^b > 0$ almost surely, and $\bar{\xi}^i$ is uniformly distributed on the interval $[0, \bar{\xi}]$ for some $\bar{\xi} > \xi^b$. We also normalize $\chi^b = 1$. The chosen parameter values in the baseline case are summarized in [Table 1](#).

Parameter	Value
λ_θ	0.02
g_1	1.8
$c^s = c^b$	4.5
φ^s	0.03
$\Lambda^s = \Lambda$	0.3
Γ	0.15
ξ^b	0.9
$\bar{\xi}$	1.7

Table 1: Primitive parameter values.

This example illustrates how the “safe neighbor externality” highlighted in [Section 4](#) reshapes deposit market competition. When depositors can flexibly reallocate their funds toward banks perceived as safer under connected bank-run conditions, the externality initiates a reinforcing feedback loop. Specifically, deposit movements toward a national bank perceived as safer enhance its stability, incentivizing further withdrawals from regional banks. In equilibrium, this externality disproportionately increases bank-run fragility among regional banks, while providing only a marginal stability improvement for national banks (see the right panel of [Fig. 9](#)). This asymmetric impact arises because the benefits to national banks from deposit inflows are sublinear, whereas deposit outflows affect regional banks’ run cutoffs in a superlinear manner.

Consequently, regional banks become significantly more fragile and less attractive to depositors *ex ante*, compelling them to compete more aggressively with national banks to offset their heightened vulnerability. Although increased deposits from regional banks marginally intensify local deposit-market competition, the national bank is compelled to increase its deposit rate only slightly, as its enhanced perceived stability simultaneously bolsters its market power as illustrated by the left panel of [Fig. 9](#). Ultimately, these regulations distort deposit competition beyond traditional pricing mechanisms, weakening smaller institutions’ competitive positions in the market.

6 Conclusion

This paper examines a previously overlooked externality associated with banking regulations: measures intended to safeguard large national banks—often regarded as systemically important—may inadvertently destabilize smaller regional banks. Our analysis highlights the potential role of post-GFC banking regulations and the Too-Big-To-Fail (TBTF) perception in shaping depositor behavior during periods of financial uncertainty. These regulations, together with implicit government guarantees,

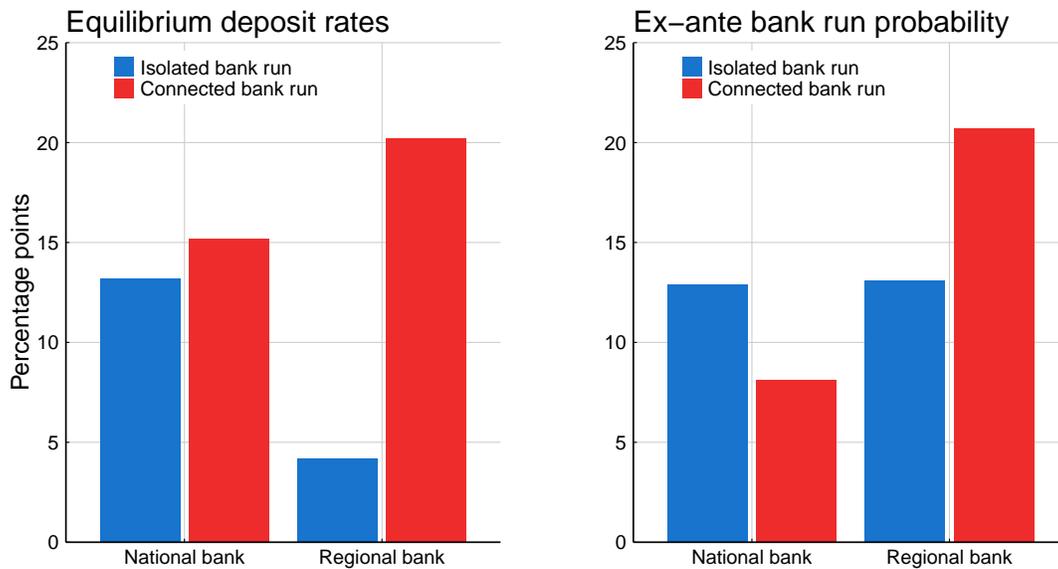


Figure 9: Equilibrium under the economy of connected/isolated bank run. The parameters are set to make the national and regional banks almost identical, with regional banks being slightly more fragile under the economy of isolated bank run. The left panel shows the equilibrium deposit rates offered by banks under the economies of isolated and connected bank runs, respectively, and the right panel shows the bank run probability. Notably, the bank run probability for the regional bank is only 0.2% higher than that for the national bank under the economy of isolated bank runs.

amplify depositor incentives to shift funds toward large national banks, thereby triggering significant withdrawals from smaller regional institutions.

Our analysis further reveals that the fragility of regional banks can become contagious, as distress at one regional institution quickly spills over to others through the externality we identified. If multiple regional banks experience simultaneous deposit runs, the cumulative impact could escalate into a systemic crisis, potentially triggering widespread financial instability. While regulatory measures aimed at stabilizing large national banks are essential, addressing their unintended consequences on smaller regional institutions is equally critical for safeguarding broader financial stability.

References

- Acharya, Viral V, "A theory of systemic risk and design of prudential bank regulation," *Journal of financial stability*, 2009, 5 (3), 224–255.
- , Abhiman Das, Nirupama Kulkarni, Prachi Mishra, and Nagpurnanand R Prabhala, "Deposit and credit reallocation in a banking panic: The role of state-owned banks," Technical Report, National Bureau of Economic Research 2022.

- **and Nada Mora**, “A crisis of banks as liquidity providers,” *The journal of Finance*, 2015, 70 (1), 1–43.
- Admati, Anat and Martin Hellwig**, *The bankers’ new clothes: What’s wrong with banking and what to do about it*, Princeton University Press, 2014.
- Adrian, Mr Tobias, Nassira Abbas, Ms Silvia Ramirez, and Gonzalo Fernandez Dionis**, *The US Banking Sector since the March 2023 Turmoil: Navigating the Aftermath*, International Monetary Fund, 2024.
- Allen, Franklin and Douglas Gale**, “Optimal financial crises,” *The journal of finance*, 1998, 53 (4), 1245–1284.
- **and** — , “Financial contagion,” *Journal of political economy*, 2000, 108 (1), 1–33.
- **and** — , “Financial intermediaries and markets,” *Econometrica*, 2004, 72 (4), 1023–1061.
- , **Elena Carletti, Itay Goldstein, and Agnese Leonello**, “Government guarantees and financial stability,” *Journal of Economic Theory*, 2018, 177, 518–557.
- Amador, Manuel and Javier Bianchi**, “Bank Runs, Fragility, and Regulation,” Technical Report, National Bureau of Economic Research 2024.
- Andersen, Leif, Darrell Duffie, and Yang Song**, “Funding value adjustments,” *The Journal of Finance*, 2019, 74 (1), 145–192.
- Anderson, Alyssa G, Wenxin Du, and Bernd Schlusche**, “Arbitrage capital of global banks,” Technical Report, National Bureau of Economic Research 2021.
- Baron, Matthew, Moritz Schularick, and Kaspar Zimmermann**, “Survival of the Biggest: Large Banks and Financial Crises,” *Available at SSRN 4189014*, 2023.
- Basel Committee on Banking Supervision**, “Evaluation of the Impact and Efficacy of the Basel III Reforms,” BCBS Working Paper 44, Bank for International Settlements July 2022.
- BCBS**, “Assessing the impact of Basel III: Evidence from macroeconomic models: literature review and simulations,” 2010.
- , “An assessment of the long-term economic impact of stronger capital and liquidity requirements,” 2010.
- , “Global systemically important banks: Updated assessment methodology and the higher loss absorbency requirement,” 2018.
- , “Stress testing principles,” 2018.
- , “Report on the 2023 banking turmoil,” 2023.
- , “Core Principles for effective banking supervision,” 2024.
- Begenau, Juliane**, “Capital requirements, risk choice, and liquidity provision in a business-cycle model,” *Journal of Financial Economics*, 2020, 136 (2), 355–378.
- **and Erik Stafford**, “Uniform rate setting and the deposit channel,” *Available at SSRN 4136858*, 2022.

- Beiseitov, Eldar**, “Small Banks, Big Impact: Community Banks and Their Role in Small Business Lending,” *The Regional Economist*, 2023.
- Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young**, “Financial crises and macro-prudential policies,” *Journal of International Economics*, 2013, 89 (2), 453–470.
- Berger, Allen N, Nathan H Miller, Mitchell A Petersen, Raghuram G Rajan, and Jeremy C Stein**, “Does function follow organizational form? Evidence from the lending practices of large and small banks,” *Journal of Financial economics*, 2005, 76 (2), 237–269.
- , **William Goulding, and Tara Rice**, “Do small businesses still prefer community banks?,” *Journal of Banking & Finance*, 2014, 44, 264–278.
- Boissay, Frédéric and Fabrice Collard**, “Macroeconomics of bank capital and liquidity regulations,” Technical Report, BIS Working Paper 2016.
- Caglio, Cecilia, Jennifer Dlugosz, and Marcelo Rezende**, “Flight to safety in the regional bank crisis of 2023,” *Available at SSRN*, 2023, 4457140.
- Carlsson, Hans and Eric Van Damme**, “Global games and equilibrium selection,” *Econometrica: Journal of the Econometric Society*, 1993, pp. 989–1018.
- Chang, Tom, Ing-Haw Cheng, and Han Hong**, “The Fundamental Role of Uninsured Depositors in the Regional Banking Crisis,” *Wharton International Finance Research Program*, 2023.
- Chari, Varadarajan V and Ravi Jagannathan**, “Banking panics, information, and rational expectations equilibrium,” *The Journal of Finance*, 1988, 43 (3), 749–761.
- Chen, Qi, Itay Goldstein, Zeqiong Huang, and Rahul Vashishtha**, “Liquidity transformation and fragility in the US banking sector,” *Journal of Finance*, 2024, *forthcoming*.
- Choi, Dong Beom**, “Heterogeneity and stability: Bolster the strong, not the weak,” *The Review of Financial Studies*, 2014, 27 (6), 1830–1867.
- , **Paul Goldsmith-Pinkham, and Tanju Yorulmazer**, “Contagion effects of the silicon valley bank run,” Technical Report, National Bureau of Economic Research 2023.
- Cookson, J. Anthony et al.**, “Social Media as Bank Run Catalyst,” *SSRN*, 2023.
- Cooper, Russell and Thomas W Ross**, “Bank runs: Deposit insurance and capital requirements,” *International Economic Review*, 2002, 43 (1), 55–72.
- Cooperman, Harry R, Darrell Duffie, Stephan Luck, Zachry Wang, and Yilin (David) Yang**, “Bank funding risk, reference rates, and credit supply,” *The Journal of Finance*, 2025.
- Covas, Francisco, Benjamin Gross, and Jose Maria Tapia**, “The Importance of Regional Banks for Small Business Lending and Economic Growth,” Technical Report, Bank Policy Institute 2023.
- Dai, Liang, Dan Luo, and Ming Yang**, “Disclosure of Bank-Specific Information and the Stability of Financial Systems,” *Available at SSRN 3762941*, 2022.
- d’Avernas, Adrien, Andrea L Eisfeldt, Can Huang, Richard Stanton, and Nancy Wallace**, “The Deposit Business at Large vs. Small Banks,” Technical Report, National Bureau of Economic Research 2023.

- Dávila, Eduardo and Itay Goldstein**, “Optimal deposit insurance,” *Journal of Political Economy*, 2023, 131 (7), 1676–1730.
- den Heuvel, Skander Van**, “The welfare effects of bank liquidity and capital requirements,” 2022.
- Diamond, Douglas W and Philip H Dybvig**, “Bank runs, deposit insurance, and liquidity,” *Journal of political economy*, 1983, 91 (3), 401–419.
- Drechsler, Itamar, Alexi Savov, Philipp Schnabl, and Olivier Wang**, “Banking on uninsured deposits,” Available at SSRN 4411127, 2023.
- Du, Wenxin, Alexander Tepper, and Adrien Verdelhan**, “Deviations from covered interest rate parity,” *The Journal of Finance*, 2018, 73 (3), 915–957.
- Egan, Mark, Ali Hortaçsu, and Gregor Matvos**, “Deposit competition and financial fragility: Evidence from the us banking sector,” *American Economic Review*, 2017, 107 (1), 169–216.
- , —, and —, “Deposit competition and financial fragility: Evidence from the us banking sector,” *American Economic Review*, 2017, 107 (1), 169–216.
- , **Stefan Lewellen, and Adi Sunderam**, “The cross-section of bank value,” *The Review of Financial Studies*, 2022, 35 (5), 2101–2143.
- Eisenberg, Larry and Thomas H Noe**, “Systemic risk in financial systems,” *Management Science*, 2001, 47 (2), 236–249.
- Elenev, Vadim, Tim Landoigt, and Stijn Van Nieuwerburgh**, “A macroeconomic model with financially constrained producers and intermediaries,” *Econometrica*, 2021, 89 (3), 1361–1418.
- Frankel, David M, Stephen Morris, and Ady Pauzner**, “Equilibrium selection in global games with strategic complementarities,” *Journal of Economic Theory*, 2003, 108 (1), 1–44.
- Gertler, Mark and Nobuhiro Kiyotaki**, “Banking, liquidity, and bank runs in an infinite horizon economy,” *American Economic Review*, 2015, 105 (7), 2011–2043.
- , —, and **Andrea Prestipino**, “A macroeconomic model with financial panics,” *The Review of Economic Studies*, 2020, 87 (1), 240–288.
- Gilje, Erik P, Elena Loutskina, and Philip E. Strahan**, “Exporting liquidity: Branch banking and financial integration,” *The Journal of Finance*, 2016, 71 (3), 1159–1184.
- Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang**, “Bank heterogeneity and financial stability,” *Journal of Financial Economics*, 2024, 162, 103934.
- and **Ady Pauzner**, “Demand–deposit contracts and the probability of bank runs,” *Journal of Finance*, 2005, 60 (3), 1293–1327.
- Gruenberg, Martin J**, “Recent bank failures and the federal regulatory response,” *Statement before the Committee on Banking, Housing and Urban Affairs (US Senate)*, 2023, 28.
- Haddad, Valentin, Barney Hartman-Glaser, and Tyler Muir**, “Bank fragility when depositors are the asset,” Available at SSRN 4412256, 2023.
- Hanson, Samuel G, Anil K Kashyap, and Jeremy C Stein**, “A macroprudential approach to financial regulation,” *Journal of economic Perspectives*, 2011, 25 (1), 3–28.

- He, Zhiguo and Wei Xiong**, “Dynamic debt runs,” *The Review of Financial Studies*, 2012, 25 (6), 1799–1843.
- , **Arvind Krishnamurthy**, and **Konstantin Milbradt**, “A model of safe asset determination,” *American Economic Review*, 2019, 109 (4), 1230–62.
- Heider, Florian, Marie Hoerova, and Cornelia Holthausen**, “Liquidity hoarding and interbank market rates: The role of counterparty risk,” *Journal of Financial Economics*, 2015, 118 (2), 336–354.
- Heinemann, Frank, Rosemarie Nagel, and Peter Ockenfels**, “The theory of global games on test: experimental analysis of coordination games with public and private information,” *Econometrica*, 2004, 72 (5), 1583–1599.
- , —, and —, “Measuring strategic uncertainty in coordination games,” *The review of economic studies*, 2009, 76 (1), 181–221.
- Iyer, Rajkamal and Manju Puri**, “Understanding bank runs: The importance of depositor-bank relationships and networks,” *American Economic Review*, 2012, 102 (4), 1414–1445.
- Jacklin, Charles J and Sudipto Bhattacharya**, “Distinguishing panics and information-based bank runs: Welfare and policy implications,” *Journal of political economy*, 1988, 96 (3), 568–592.
- Jeanne, Olivier and Anton Korinek**, “Managing credit booms and busts: A Pigouvian taxation approach,” *Journal of Monetary Economics*, 2019, 107, 2–17.
- and —, “Macroprudential regulation versus mopping up after the crash,” *The Review of Economic Studies*, 2020, 87 (3), 1470–1497.
- Jiang, Erica Xuewei, Gregor Matvos, Tomasz Piskorski, and Amit Seru**, “Monetary tightening and US bank fragility in 2023: Mark-to-market losses and uninsured depositor runs?,” *Journal of Financial Economics*, 2024, 159, 103899.
- Kara, Gazi I and S Mehmet Ozsoy**, “Bank regulation under fire sale externalities,” *The Review of Financial Studies*, 2020, 33 (6), 2554–2584.
- Kashyap, Anil K, Dimitrios P Tsomocos, and Alexandros P Vardoulakis**, “Optimal bank regulation in the presence of credit and run risk,” *Journal of Political Economy*, 2024, 132 (3), 772–823.
- Keister, Todd**, “Bailouts and financial fragility,” *The Review of Economic Studies*, 2016, 83 (2), 704–736.
- Kim, Edward T and Marcelo Rezende**, “Deposit insurance premiums and bank risk,” *The Review of Corporate Finance Studies*, 2023, 12 (2), 291–325.
- , **Shohini Kundu**, and **Amiyatosh Purnanandam**, “The Economics of Market-Based Deposit Insurance,” *Available at SSRN 4813996*, 2024.
- Kreps, David M and Robert Wilson**, “Sequential equilibria,” *Econometrica: Journal of the Econometric Society*, 1982, pp. 863–894.
- Laeven, Luc and Fabian Valencia**, “Systemic banking crises database,” *IMF Economic Review*, 2013, 61 (2), 225–270.

- Liu, Xuewen**, “Interbank market freezes and creditor runs,” *The Review of financial studies*, 2016, 29 (7), 1860–1910.
- Luo, Dan and Ming Yang**, “The Optimal Structure of Securities under Coordination Frictions,” Available at SSRN 4484914, 2023.
- Mian, Atif R. and Asim I. Khwaja**, “Tracing the impact of bank liquidity shocks: Evidence from an emerging market,” *American Economic Review*, 2008, 98 (4), 1413–1442.
- Morris, Stephen and Hyun Song Shin**, “Unique equilibrium in a model of self-fulfilling currency attacks,” *American Economic Review*, 1998, pp. 587–597.
- and —, “Global games: Theory and applications,” 2001.
- and —, “Coordination risk and the price of debt,” *European Economic Review*, 2004, 48 (1), 133–153.
- Naceur, Sami Ben, Jérémy Pépy, and Caroline Roulet**, “Basel III and Bank-Lending: Evidence from the United States and Europe,” IMF Working Paper 17/245, International Monetary Fund November 2017.
- of Thirty G30, Group**, “Bank Failures and Contagion Lender of Last Resort, Liquidity, and Risk Management,” Technical Report 2024.
- Reinhart, Carmen M**, *This time is different: Eight centuries of financial folly*, Princeton University Press, 2009.
- Rose, Jonathan**, “Old-fashioned deposit runs,” Technical Report, FEDS Working Paper 2015.
- , “Understanding the speed and size of bank runs in historical comparison,” *Economic Synopses*, 2023, 12.
- Schilling, Linda M**, “Optimal forbearance of bank resolution,” *The Journal of Finance*, 2023, 78 (6), 3621–3675.
- Sun, Yeneng**, “The exact law of large numbers via Fubini extension and characterization of insurable risks,” *Journal of Economic Theory*, 2006, 126 (1), 31–69.
- Uhlig, Harald**, “A model of a systemic bank run,” *Journal of Monetary Economics*, 2010, 57 (1), 78–96.
- Usai, Stefano and Marco Vannini**, “Banking structure and regional economic growth: lessons from Italy,” *The Annals of Regional Science*, 2005, 39, 691–714.
- Villas-Boas, J.Miguel**, “Comparative Statics of Fixed Points,” *Journal of Economic Theory*, 1997, 73 (1), 183–198.
- Vives, Xavier**, *Competition and stability in banking: The role of regulation and competition policy*, Princeton University Press, 2016.
- Walther, Ansgar**, “Jointly optimal regulation of bank capital and liquidity,” *Journal of Money, Credit and Banking*, 2016, 48 (2-3), 415–448.
- Yang, Yilin David**, “What Quantity of Reserves Is Sufficient?,” Available at SSRN 3721785, 2020.

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A Equilibrium Selection via Strategic Uncertainty

In this section, we demonstrate that the equilibrium multiplicity in the **Benchmark Economy of Connected Bank Run** is not robust. We show that introducing a vanishingly small amount of strategic uncertainty about cross-bank deposit mobility is sufficient to rule out the second equilibrium.

The source of this uncertainty is the cross-bank service depreciation parameter, χ^j . Recall that, in our main model, we assume that the per-dollar service utility at $t = 2$ for depositor k who move to bank j is $\chi^j \xi_k^j$, where $\chi^j \in [0, 1]$. The parameter χ^j thus captures how much banking service depreciate, so it governs the cross-bank deposit mobility. When $\chi^j = 0$, depositors will only move to hold the safe asset if they run from their own institutions. When $\chi^j = 1$, bank j is most appealing to other depositors, everything else equal. In this section, we also call χ^j the “cross-bank deposit mobility parameter”.

Our robustness analysis follows the setup in [Section 4.2](#) with one key modification to the information structure. We assume that while depositors at bank j know their own banking utility when moving to the other bank, they are uncertain about the mobility parameter of the depositors from the *other* bank. This captures the intuition that depositors have limited information about the preferences of depositors at different institutions.

Formally, depositors at bank j regard χ^j , as the realization of a random variable $\tilde{\chi}^j$. To save space, we assume depositors share a uniform prior, $\tilde{\chi}^j \sim U[0, 1]$, though the result holds for any continuous prior with support that includes zero.

Each depositor k at bank j receives **two** private signals at $t = 2$: the original signal \tilde{x}_k about the fundamental $\tilde{\theta}$ and a new signal \tilde{y}_k about their own bank’s uncertain mobility parameter, $\tilde{\chi}^j$. The new signal is generated as follows:

$$\tilde{y}_k = \tilde{\chi}^j + \varsigma \tilde{v}_k$$

where $\varsigma > 0$ and $\tilde{v}_k \sim U[-1, 1]$ are i.i.d. Therefore,

$$\tilde{\chi}^j | \tilde{y}_k = y_k \sim U[\max(0, y_k - \varsigma), \min(1, y_k + \varsigma)].$$

This specific distributional assumption is made for brevity; the results can be extended to general distributions that satisfy the technical assumptions in ??, using a proof analogous to the one in [Internet Appendix 1](#). For brevity, we also assume that $H^i = H^b = 0$.

We are interested in the limiting equilibrium as both sources of private information noise vanish. We analyze the sequential limit where the noise on the economic fundamental vanishes first, followed by the noise on the strategic mobility parameter. Formally, let $\mathcal{E}(\sigma, \varsigma)$ denote the noisy subgame equilibrium for any $\sigma, \varsigma > 0$. Our object of study is the limiting equilibrium defined by:

$$\lim_{\varsigma \downarrow 0} \left(\lim_{\sigma \downarrow 0} \mathcal{E}(\sigma, \varsigma) \right).$$

Theorem 6. *Suppose that χ^i is mildly smaller than χ^b . The $t = 2$ subgame admits a unique*

limiting equilibrium $\lim_{\varsigma \downarrow 0} (\lim_{\sigma \downarrow 0} \mathcal{E}(\sigma, \varsigma))$. In this unique limiting equilibrium, national-bank depositors stay when $\theta > \underline{\theta}_D^b(H^b)$ and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$. Regional-bank depositors stay when $\theta > \bar{\theta}_D^i$, switch to the national bank when $\underline{\theta}_D^b(H^b) < \theta < \bar{\theta}_D^i$, and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$.

The second equilibrium described in [Theorem 2](#) is sustained by a self-fulfilling collective belief. It arises if depositors conjecture that the regional bank will receive strong inflows from the national bank, even in relatively good economic states. If held by all, such a belief implies that the national bank is riskier (formally, $\mathcal{D}_i \subsetneq \mathcal{D}_b$). This perceived safety advantage makes the regional bank the more attractive choice for depositors, irrespective of their private signals about the fundamental $\tilde{\theta}$, thus making the initial belief self-fulfilling. Consequently, a standard global game with uncertainty *only* over the fundamental cannot rule out such an equilibrium.

However, this coordinated belief is fragile. [Theorem 6](#) proves that introducing an infinitesimal amount of private uncertainty about cross-bank mobility causes this delicate coordination to collapse. This refinement restores equilibrium uniqueness by selecting the single, robust outcome, which preserves the ordering of the run cutoffs established in the isolated economy.

Internet Appendix

1 Proofs

For clarity, we summarize the assumptions regarding banks' production technology and fire-sale losses as follows:

Assumption 4. For any $j \in \{b, 1, 2, 3, \dots, \mathcal{N}\}$:

- (I) The production function G^j is differentiable, and $G^{j'}(x) \geq 1$ is weakly decreasing in x for all $x \geq 0$.
- (II) The fire-sale loss function C^j is twice differentiable in both arguments.
- (III) For any $A^j \geq 0$, $C^j(A^j, 0) = 0$.
- (IV) For any $A^j \geq 0$, $\partial_2 C^j(A^j, 0) > G^{j'}(A^j)$ and $\partial_2 C^j(A^j, 0) > 1$.
- (V) For any $A^j \geq 0$, $\partial_2^2 C^j(A^j, x)$ is weakly increasing in $x \in [0, \infty)$.

Discussion: Parts (I) and (IV) of [Assumption 4](#) imply that banks are marginally more efficient at managing their own assets than outside buyers at $t = 2$, possibly due to market frictions such as adverse selection. A marginal amount δ invested in the bank's assets generates an internal value of $G^{j'}(A^j)\delta$, while its value to the next-best buyer in the market is reduced by a marginal fire-sale cost of $(\partial_2 C^j(A^j, 0) - G^{j'}(A^j))\delta$. Thus, if bank j liquidates δ worth of assets to meet outflows, the productive value of the bank's assets decreases by $\partial_2 C^j(A^j, 0)\delta$. Part (V) of [Assumption 4](#) indicates that greater liquidation leads to higher marginal fire-sale costs.²⁹

1.1 Proof of [Lemma 1](#)

From the premises of the lemma, we take $H^i = 0$ in the proof. The following calculation of depositor k 's posterior beliefs will be useful in the remainder of the proof. Since $\bar{\theta}$ has an atomless distribution, for any ϑ ,

$$\begin{aligned} \mathbb{P}(\bar{\theta} \geq \vartheta \mid \bar{x}_k = x_k) &= 1 - F_{\bar{\theta}|\bar{x}_k}(\vartheta \mid \bar{x}_k = x_k) = 1 - \frac{\int_{-\infty}^{\vartheta} f_{\bar{\theta}}(\theta) f_{\bar{\epsilon}_k}\left(\frac{x_k - \theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} f_{\bar{\theta}}(\theta) f_{\bar{\epsilon}_k}\left(\frac{x_k - \theta}{\sigma}\right) d\theta} \\ &= 1 - \frac{\int_{(x_k - \vartheta)/\sigma}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}{\int_{-\infty}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz} = \frac{\int_{-\infty}^{(x_k - \vartheta)/\sigma} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}{\int_{-\infty}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}. \end{aligned} \tag{24}$$

²⁹This modeling choice implicitly suggests that financial markets at $t = 1$ lack information about $\bar{\theta}$ and $\bar{\zeta}_j$, so assets are priced based on prior beliefs. While C^j could, in principle, depend on signals regarding $\bar{\theta}$ or $\bar{\zeta}_j$, incorporating such dependence would complicate the analysis without generating additional insights.

Consider the representative bank i . Let agent k be a representative depositor at bank i after time $t = 1$. Denote by \mathcal{O}^i the aggregate deposit outflow from bank i . In the **Alternative Economy of Isolated Bank Run** that is considered here, $\mathbb{I}^i = 0$. Given any agents' strategy profile, denoted as $\{r_{k,i \rightarrow H}\}$, if the true economic state is θ , then the Exact Law of Large Numbers (ELLN) yields

$$\mathcal{O}^i = \mathcal{M}^i \int_{\xi_k} \int_{x_k} r_{k,i \rightarrow H}(x_k) f_{\bar{x}_k | \bar{\theta}}(x_k | \theta) dx_k f_{\bar{\xi}}(\xi_k) d\xi_k.$$

Hence aggregate deposit outflow is a function of θ and σ (recall that private signal precision is governed by σ , so the density $f_{\bar{x}_k | \bar{\theta}}$ depends on σ)—i.e., any outflow can be written as a “deposit flow function”, $\mathcal{O}^i(\theta, \sigma)$.

Given any possible deposit flow function, depositor k 's expected payoff from *staying* with bank i after observing x_k at $t = 2$ is

$$\mathcal{U}_{k,i \rightarrow i}^2(x_k) \equiv \mathcal{U}_k^2(\mathbf{a}_{k,i \rightarrow i} = 1 | x_k) = \mathbb{P}(\mathcal{D}_i^c | x_k) (R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i | x_k) \Lambda. \quad (25)$$

If k leaves the banking system to hold the risk-free asset, her expected payoff is

$$\mathcal{U}_{k,i \rightarrow H}^2(x_k) \equiv \mathcal{U}_k^2(\mathbf{a}_{k,i \rightarrow H} = 1 | x_k) = \int_{\mathcal{D}_i} [\mathbb{M}^i + (1 - \mathbb{M}^i) \Lambda] f_{\bar{\theta} | \bar{x}_k}(\theta | x_k) d\theta + \int_{\mathcal{D}_i^c} f_{\bar{\theta} | \bar{x}_k}(\theta | x_k) d\theta. \quad (26)$$

In this lemma, we only study equilibria in which depositors believe that they can always successfully run from bank i , i.e., all depositors believe that the success probability $\mathbb{M}^i = 1$. Under the best-responding strategy, depositor k leaves bank i iff $\mathcal{U}_{k,i \rightarrow H}^2(x_k) > \mathcal{U}_{k,i \rightarrow i}^2(x_k)$ and vice versa. Hence, depositor k 's best response strategy only depends on her private signal x_k . Let $\mathcal{R}_{\text{tem}}^i(k, \sigma)$ denote the set of signal realizations for a type- ξ_k depositor who runs from bank i after observing $x_k \in \mathcal{R}_{\text{tem}}^i(k, \sigma)$. Then the equilibrium outflow must satisfy $\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\vartheta + \sigma \tilde{\epsilon}_k \in \mathcal{R}_{\text{tem}}^i(k, \sigma)) dF_{\bar{\xi}}(\xi_k)$.

1.1.1 Step 1: existence of upper and lower equilibria

Fix a value of σ , and consider the possible equilibrium outcome at time $t = 2$. Define a constant function $\overline{\mathcal{O}}^i(\theta, 0, \sigma) \equiv \mathcal{M}^i \int_{\xi_k} s_{k,i} f_{\bar{\xi}}(\xi_k) d\xi_k$ for any θ . This quantity is the total deposit at bank i right after $t = 1$, and the largest possible outflow from bank i . Define a constant function $\underline{\mathcal{O}}^i(\theta, 0, \sigma) = 0$ for all θ . It follows that any possible equilibrium outflow $\mathcal{O}^i(\bar{\theta}, \sigma)$ must almost surely satisfy

$$\underline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma) \leq \mathcal{O}^i(\bar{\theta}, \sigma) \leq \overline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma), \quad (27)$$

and this is common knowledge among all agents. Define $\mathcal{R}(k, 0, \sigma)$ to be the set of signal realizations for agent k with type ξ_k who, after observing any $x_k \in \mathcal{R}(k, 0, \sigma)$, would run away from bank i even if agent k optimistically conjectures that the total outflow at time $t = 2$ is the lowest possible one, $\underline{\mathcal{O}}^i(\theta, 0, \sigma)$. By definition, from Eqs. (34) and (35)

$$\mathcal{R}(k, 0, \sigma) = \left\{ x : \mathbb{P} \left(\bar{\theta} G^i(A^i) - C^i(A^i, (\underline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma))^+) - R^i(L^i - \underline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma)) > 0 \mid \bar{x}_k = x \right) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}.$$

Similarly, define $\mathfrak{S}(k, 0, \sigma)$ to be the set of signals for agent k with type ξ_k such that, after observing any signal $x_k \in \mathfrak{S}(k, 0, \sigma)$, agent k would stay with bank i even if she is most pessimistic by conjecturing that the total outflow at time $t = 2$ is the highest possible one $\overline{\mathcal{O}}^i(\theta, 0, \sigma)$:

$$\mathfrak{S}(k, 0, \sigma) = \left\{ x : \mathbb{P} \left(\overline{\theta} G^i(A^i) - C^i(A^i, (\overline{\mathcal{O}}^i(\overline{\theta}, 0, \sigma))^+) - R^i(L^i - (\overline{\mathcal{O}}^i(\overline{\theta}, 0, \sigma))^+) > 0 \mid \tilde{x}_k = x \right) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}.$$

Recursively, for any non-negative integer ℓ , define

$$\underline{\mathcal{O}}^i(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \in \mathfrak{R}(k, \ell, \sigma)) f_{\tilde{\xi}}(\xi_k) d\xi_k, \quad \overline{\mathcal{O}}^i(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \notin \mathfrak{S}(k, \ell, \sigma)) f_{\tilde{\xi}}(\xi_k) d\xi_k,$$

$$\mathfrak{R}(k, \ell + 1, \sigma) = \left\{ x : \mathbb{P} \left(\underline{\theta} G^i(A^i) - C^i(A^i, (\underline{\mathcal{O}}^i(\underline{\theta}, \ell + 1, \sigma))^+) - R^i(L^i - (\underline{\mathcal{O}}^i(\underline{\theta}, \ell + 1, \sigma))^+) > 0 \mid \tilde{x}_k = x \right) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\},$$

$$\mathfrak{S}(k, \ell + 1, \sigma) = \left\{ x : \mathbb{P} \left(\overline{\theta} G^i(A^i) - C^i(A^i, (\overline{\mathcal{O}}^i(\overline{\theta}, \ell + 1, \sigma))^+) - R^i(L^i - (\overline{\mathcal{O}}^i(\overline{\theta}, \ell + 1, \sigma))^+) > 0 \mid \tilde{x}_k = x \right) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}.$$

Claim 1. Fix any σ . For any non-negative integer ℓ , $\underline{\mathcal{O}}^i(\theta, \ell, \sigma)$ and $\overline{\mathcal{O}}^i(\theta, \ell, \sigma)$ are continuous in θ . In addition, $\underline{\mathcal{O}}^i(\theta, \ell, \sigma) \leq \underline{\mathcal{O}}^i(\theta, \ell + 1, \sigma) \leq \overline{\mathcal{O}}^i(\theta, \ell + 1, \sigma) \leq \overline{\mathcal{O}}^i(\theta, \ell, \sigma)$. Moreover, $\mathfrak{R}(k, \ell, \sigma) \subseteq \mathfrak{R}(k, \ell + 1, \sigma)$ and $\mathfrak{S}(k, \ell, \sigma) \supseteq \mathfrak{S}(k, \ell + 1, \sigma)$. Total outflow from bank i has to lie in the interval $[\underline{\mathcal{O}}^i(\theta, \ell, \sigma), \overline{\mathcal{O}}^i(\theta, \ell, \sigma)]$ after eliminating dominated strategies. This is common knowledge among all agents.

Proof of Claim 1. First, from [Assumption 4](#), if two functions $\mathcal{O}^1(\theta)$ and $\mathcal{O}^2(\theta)$ satisfy $\mathcal{O}^1(\theta) \leq \mathcal{O}^2(\theta)$ for all θ , then

$$\theta G^i(A^i) - C^i(A^i, (\mathcal{O}^1(\theta))^+) - R^i(L^i - (\mathcal{O}^1(\theta))^+) \geq \theta G^i(A^i) - C^i(A^i, (\mathcal{O}^2(\theta))^+) - R^i(L^i - (\mathcal{O}^2(\theta))^+).$$

By construction, $\underline{\mathcal{O}}^i(\theta, 0, \sigma) \leq \underline{\mathcal{O}}^i(\theta, 1, \sigma) \leq \overline{\mathcal{O}}^i(\theta, 1, \sigma) \leq \overline{\mathcal{O}}^i(\theta, 0, \sigma)$. Therefore,

$$\begin{aligned} \mathcal{Q}_0^l &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\underline{\mathcal{O}}^i(\vartheta, 0, \sigma))^+) - R^i(L^i - (\underline{\mathcal{O}}^i(\vartheta, 0, \sigma))^+) > 0 \right\} \supseteq \\ \mathcal{Q}_1^l &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\underline{\mathcal{O}}^i(\vartheta, 1, \sigma))^+) - R^i(L^i - (\underline{\mathcal{O}}^i(\vartheta, 1, \sigma))^+) > 0 \right\} \supseteq \\ \mathcal{Q}_1^u &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\overline{\mathcal{O}}^i(\vartheta, 1, \sigma))^+) - R^i(L^i - (\overline{\mathcal{O}}^i(\vartheta, 1, \sigma))^+) > 0 \right\} \supseteq \\ \mathcal{Q}_0^u &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\overline{\mathcal{O}}^i(\vartheta, 0, \sigma))^+) - R^i(L^i - (\overline{\mathcal{O}}^i(\vartheta, 0, \sigma))^+) > 0 \right\}. \end{aligned}$$

Thus, $\mathbb{P}(\overline{\theta} \in \mathcal{Q}_0^l \mid \tilde{x}_k = x) \geq \mathbb{P}(\overline{\theta} \in \mathcal{Q}_1^l \mid \tilde{x}_k = x) \geq \mathbb{P}(\overline{\theta} \in \mathcal{Q}_1^u \mid \tilde{x}_k = x) \geq \mathbb{P}(\overline{\theta} \in \mathcal{Q}_0^u \mid \tilde{x}_k = x)$. It follows that

$$\begin{aligned} \mathfrak{R}(k, 0, \sigma) &= \left\{ x : \mathbb{P}(\overline{\theta} \in \mathcal{Q}_0^l \mid \tilde{x}_k = x) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\} \subseteq \mathfrak{R}(k, 1, \sigma) = \left\{ x : \mathbb{P}(\overline{\theta} \in \mathcal{Q}_1^l \mid \tilde{x}_k = x) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\} \subseteq \\ \mathfrak{S}^c(k, 0, \sigma) &= \left\{ x : \mathbb{P}(\overline{\theta} \in \mathcal{Q}_1^u \mid \tilde{x}_k = x) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\} \subseteq \mathfrak{S}^c(k, 1, \sigma) = \left\{ x : \mathbb{P}(\overline{\theta} \in \mathcal{Q}_0^u \mid \tilde{x}_k = x) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}. \end{aligned}$$

Hence $\mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \in \mathfrak{R}(k, 0, \sigma)) \leq \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \in \mathfrak{R}(k, 1, \sigma)) \leq \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \notin \mathfrak{S}(k, 1, \sigma)) \leq \mathbb{P}(\theta +$

$\sigma \bar{\varepsilon}_k \notin \mathfrak{S}(k, 0, \sigma)$). Therefore, by the Exact Law of Large Numbers (Sun, 2006), for any θ ,

$$\begin{aligned} \underline{\mathcal{Q}}^i(\theta, 1, \sigma) &= \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \bar{\varepsilon}_k \in \mathfrak{R}(k, 0, \sigma)) f_{\bar{\xi}}(\xi_k) d\xi_k \leq \underline{\mathcal{Q}}^i(\theta, 2, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \bar{\varepsilon}_k \in \mathfrak{R}(k, 1, \sigma)) f_{\bar{\xi}}(\xi_k) d\xi_k \\ &\leq \overline{\mathcal{Q}}^i(\theta, 2, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \bar{\varepsilon}_k \notin \mathfrak{S}(k, 1, \sigma)) f_{\bar{\xi}}(\xi_k) d\xi_k \leq \overline{\mathcal{Q}}^i(\theta, 1, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \bar{\varepsilon}_k \notin \mathfrak{S}(k, 0, \sigma)) f_{\bar{\xi}}(\xi_k) d\xi_k. \end{aligned}$$

In addition, $\underline{\mathcal{Q}}^i(\theta, 0, \sigma)$ and $\overline{\mathcal{Q}}^i(\theta, 0, \sigma)$ are constant functions, hence continuous in θ . The functions $\vartheta \mapsto \vartheta G^i(A^i) - C^i(A^i, (\underline{\mathcal{Q}}^i(\vartheta, 0, \sigma))^+) - R^i(L^i - (\underline{\mathcal{Q}}^i(\vartheta, 0, \sigma))^+)$ and $\vartheta \mapsto \vartheta G^i(A^i) - C^i(A^i, (\overline{\mathcal{Q}}^i(\vartheta, 0, \sigma))^+) - R^i(L^i - (\overline{\mathcal{Q}}^i(\vartheta, 0, \sigma))^+)$ are continuous in ϑ . Therefore, \mathcal{Q}_0^u and \mathcal{Q}_0^l are both countable unions of disjoint open intervals with end points (ranked in decreasing order): $\infty = \bar{\vartheta}_{tem,0} > \underline{\vartheta}_{tem,0} > \bar{\vartheta}_{tem,1} > \underline{\vartheta}_{tem,1} > \dots$. By Eq. (24)

$$\mathbb{P}(\bar{\theta} \in \mathcal{Q}_0^l \mid \bar{x}_k = x_k) = \left(\int_{-\infty}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz \right)^{-1} \sum_{i=0}^{\infty} \left(\int_{(x_k - \bar{\vartheta}_{tem,i})/\sigma}^{(x_k - \underline{\vartheta}_{tem,i})/\sigma} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz \right).$$

Using Lebesgue's dominated convergence theorem, we can show that $\mathbb{P}(\bar{\theta} \in \mathcal{Q}_0^l \mid \bar{x}_k = x)$ is continuous in x . It follows that $\mathfrak{R}(k, 0, \sigma)$ is also a countable union of disjoint intervals. Hence the function $\theta \mapsto \mathbb{P}(\theta + \sigma \bar{\varepsilon}_k \in \mathfrak{R}(k, 0, \sigma))$ is continuous in θ as $F_{\bar{\varepsilon}}$ has no point mass. Thus, by the bounded convergence theorem, $\underline{\mathcal{Q}}^i(\theta, 1, \sigma)$, an integral of continuous functions, is also continuous in θ . We can repeat the above calculation and use mathematical induction to prove the claim. Note that each round is an elimination of dominated strategies, so equilibrium outflow must be in the interval $[\underline{\mathcal{Q}}^i(\theta, \ell, \sigma), \overline{\mathcal{Q}}^i(\theta, \ell, \sigma)]$ when the true state is θ . This is common knowledge among all agents. \square

In the spirit of Claim 1, we can define the following two limiting functions:

$$\underline{\mathcal{Q}}^i(\theta, \infty, \sigma) = \lim_{\ell \rightarrow \infty} \underline{\mathcal{Q}}^i(\theta, \ell, \sigma), \quad \overline{\mathcal{Q}}^i(\theta, \infty, \sigma) = \lim_{\ell \rightarrow \infty} \overline{\mathcal{Q}}^i(\theta, \ell, \sigma), \quad \forall \theta.$$

By definition, any equilibrium outflow in state θ must be in the interval $[\underline{\mathcal{Q}}^i(\theta, \infty, \sigma), \overline{\mathcal{Q}}^i(\theta, \infty, \sigma)]$. Define sets

$$\begin{aligned} \underline{\mathcal{Q}}_{\infty}^{\sigma} &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\underline{\mathcal{Q}}^i(\vartheta, \infty, \sigma))^+) - R^i(L^i - (\underline{\mathcal{Q}}^i(\vartheta, \infty, \sigma))^+) > 0 \right\}, \\ \overline{\mathcal{Q}}_{\infty}^{\sigma} &\equiv \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\overline{\mathcal{Q}}^i(\vartheta, \infty, \sigma))^+) - R^i(L^i - (\overline{\mathcal{Q}}^i(\vartheta, \infty, \sigma))^+) > 0 \right\}, \\ \mathfrak{R}(k, \infty, \sigma) &\equiv \left\{ x : \mathbb{P}(\bar{\theta} \in \underline{\mathcal{Q}}_{\infty}^{\sigma} \mid \bar{x}_k = x) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}, \\ \mathfrak{S}(k, \infty, \sigma) &\equiv \left\{ x : \mathbb{P}(\bar{\theta} \in \overline{\mathcal{Q}}_{\infty}^{\sigma} \mid \bar{x}_k = x) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}. \end{aligned}$$

It is easy to verify that $\mathfrak{R}(k, \infty, \sigma)$ and $\mathfrak{S}(k, \infty, \sigma)$ are the limiting sets of the sequences of sets described in Claim 1, so $\mathfrak{R}(k, \infty, \sigma) = \cup_{\ell=1}^{\infty} \mathfrak{R}(k, \ell, \sigma)$ and $\mathfrak{S}(k, \infty, \sigma) = \cap_{\ell=1}^{\infty} \mathfrak{S}(k, \ell, \sigma)$. It is easy to show that the boundaries of $\mathfrak{R}(k, \infty, \sigma)$ and $\mathfrak{S}^c(k, \infty, \sigma)$ are countable unions of

intervals, so both are continuity sets of the measure induced by $F_{\tilde{\epsilon}_k}$ for all k (since $F_{\tilde{\epsilon}_k}$ is absolutely continuous with respect to the Lebesgue measure on the real line). In addition,

$$\underline{\mathcal{O}}^i(\theta, \infty, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \in \mathcal{R}(k, \infty, \sigma)) f_{\tilde{\epsilon}}(\xi_k) d\xi_k \leq \overline{\mathcal{O}}^i(\theta, \infty, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta + \sigma \tilde{\epsilon}_k \notin \mathcal{S}(k, \infty, \sigma)) f_{\tilde{\epsilon}}(\xi_k) d\xi_k.$$

From this, it is easy to show that $\underline{\mathcal{O}}^i(\theta, \infty, \sigma)$ and $\overline{\mathcal{O}}^i(\theta, \infty, \sigma)$ are both equilibrium outflows in state θ in some equilibrium (the equilibrium strategy profile is characterized by $\mathcal{S}(k, \infty, \sigma)$ and $\mathcal{R}(k, \infty, \sigma)$ respectively). In addition, since $f_{\tilde{\epsilon}_k} \in \mathcal{L}^1$, both $\underline{\mathcal{O}}^i(\theta, \infty, \sigma)$ and $\overline{\mathcal{O}}^i(\theta, \infty, \sigma)$ are continuous in θ for fixed σ .

1.1.2 Step 2: necessary conditions for any candidate equilibrium

Fix a sufficiently small noise level $\sigma > 0$. Consider a candidate equilibrium where the deposit outflow at bank i is determined by the function $\mathcal{O}_{\text{cand}}^i(\theta, \sigma)$. While the form of this function is common knowledge in the candidate equilibrium, each depositor observes only a private signal \tilde{x}_k about the true state. Since depositor k 's best response strategy only depends on her private signal x_k , define the run-signal set

$$\begin{aligned} \mathcal{R}_{\text{cand}}^i(k, \sigma) &\equiv \{x_k : \mathcal{U}_{k,i \rightarrow i}^2(x_k) < \mathcal{U}_{k,i \rightarrow H}^2(x_k)\} \\ &= \left\{ x_k : \mathbb{P}(\tilde{\theta} G^i(A^i) - C^i(A^i, \mathcal{O}_{\text{cand}}^i(\tilde{\theta}, \sigma)^+) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\tilde{\theta}, \sigma)^+) > 0 \mid \tilde{x}_k = x_k) < \frac{(1 - \Lambda)}{R^i - \Lambda + \xi_k^i} \right\}. \end{aligned} \quad (28)$$

The equilibrium outflow must also satisfy

$$\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\vartheta + \sigma \tilde{\epsilon}_k \in \mathcal{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\epsilon}}(\xi_k).$$

Because $f_{\tilde{\epsilon}} \in \mathcal{L}^1$ is atomless, $\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)$ is continuous in ϑ for any fixed σ . Given these beliefs, an agent regards bank i as defaulting iff $\tilde{\theta} \in \mathcal{D}_i(\text{cand}, \sigma)$, where³⁰

$$\mathcal{D}_i(\text{cand}, \sigma) = \left\{ \theta : \theta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta, \sigma))^+) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta, \sigma)) \leq 0 \right\}.$$

Define the solvency thresholds

$$\begin{aligned} \theta_{\text{cand,up}}^i(\sigma) &= \inf \left\{ \rho : \left[\vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) \right] > 0 \quad \forall \vartheta \geq \rho \right\}, \\ \theta_{\text{cand,down}}^i(\sigma) &= \sup \left\{ \rho : \left[\vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) \right] < 0 \quad \forall \vartheta \leq \rho \right\}. \end{aligned} \quad (29)$$

Therefore,

$$(-\infty, \theta_{\text{cand,down}}^i(\sigma)] \subseteq \mathcal{D}_i(\text{cand}, \sigma) \subseteq (-\infty, \theta_{\text{cand,up}}^i(\sigma)].$$

By continuity, bankers' payoff should be zero at both thresholds:

$$\begin{aligned} 0 &= \theta_{\text{cand,up}}^i(\sigma) G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma))^+) - R^i(L^i - (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma))^+) \\ &= \theta_{\text{cand,down}}^i(\sigma) G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma))^+) - R^i(L^i - (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma))^+). \end{aligned} \quad (30)$$

³⁰Recall that $H^i = 0$.

By Eq. (29), $\theta_{\text{cand,down}}^i(\sigma) \leq \theta_{\text{cand,up}}^i(\sigma)$, so Eq. (30) implies that

$$\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \geq \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma). \quad (31)$$

For each depositor k , define:

$$\mathbb{C}_k^\epsilon \equiv \inf \left\{ x : F_{\tilde{\epsilon}_k}(x) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}. \quad (32)$$

Claim 2. Fix some small $\sigma^{\text{tem}} > 0$. Then there exists a constant $\mathbb{C}^{\text{adj}} > 0$ such that, for all $\sigma \in (0, \sigma^{\text{tem}}]$,

1. If depositor k observes $\tilde{x}_k \geq \theta_{\text{cand,up}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma + \mathbb{C}^{\text{adj}} \sigma^2$, she does not run on bank i .
2. If she observes $\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2$, she moves from bank i to hold the risk-free asset.

Proof. Let σ be small. Suppose that the realization x_k of \tilde{x}_k is weakly smaller than $\theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2$, then there is some $\mathbb{C}^{\text{tem}} \geq 0$ such that $x_k = \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2 - \mathbb{C}^{\text{tem}} \sigma$. Then

$$\mathbb{P}(\mathcal{D}_i^c | x_k) \leq \mathbb{P}(\tilde{\theta} \geq \theta_{\text{cand,down}}^i(\sigma) | x_k) = \frac{\int_{-\infty}^{(x_k - \theta_{\text{cand,down}}^i(\sigma))/\sigma} f_{\tilde{\theta}}(x_k - \sigma z) f_{\tilde{\epsilon}_k}(z) dz}{\int_{-\infty}^{\infty} f_{\tilde{\theta}}(x_k - \sigma z) f_{\tilde{\epsilon}_k}(z) dz}.$$

Because $f_{\tilde{\theta}}$ is uniformly continuous on a suitable interval, for sufficiently small σ this ratio equals

$$F_{\tilde{\epsilon}_k}(\mathbb{C}_k^\epsilon - \mathbb{C}^{\text{adj}} \sigma - \mathbb{C}^{\text{tem}}) + \mathcal{O}(\sigma) = \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} + F_{\tilde{\epsilon}_k}(\mathbb{C}_k^\epsilon - \mathbb{C}^{\text{adj}} \sigma - \mathbb{C}^{\text{tem}}) - F_{\tilde{\epsilon}_k}(\mathbb{C}_k^\epsilon) + \mathcal{O}(\sigma).$$

Because $\text{ess sup}_k \{\|f_{\tilde{\epsilon}_k}\|_\infty\}$ is finite almost surely and $\mathbb{C}^{\text{tem}} \geq 0$, one can choose \mathbb{C}^{adj} large enough such that, for any depositor k and all $\mathbb{C}^{\text{tem}} \geq 0$,

$$\frac{\mathcal{U}_{k,i \rightarrow H}^2(x_k) - \mathcal{U}_{k,i \rightarrow i}^2(x_k)}{R^i + \xi_k^i - \Lambda} = \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} - \mathbb{P}(\mathcal{D}_i^c | x_k) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} - F_{\tilde{\epsilon}_k}(\mathbb{C}_k^\epsilon - \mathbb{C}^{\text{adj}} \sigma - \mathbb{C}^{\text{tem}}) - \mathcal{O}(\sigma) > 0.$$

Hence, depositor k runs from bank i . The other direction can be proved similarly. \square

Claim 3. The limit $\lim_{\sigma \downarrow 0} (\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma)) = 0$.

Proof. Suppose that a subsequence $\{\sigma_n\}_{n \geq 1}$ with $\sigma_n \rightarrow 0$ satisfies

$$\frac{\theta_{\text{cand,up}}^i(\sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n} \xrightarrow{n \rightarrow \infty} \infty.$$

By [Claim 2](#), $(-\infty, \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2] \subset \mathfrak{R}_{\text{cand}}^i(k, \sigma) \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma + \mathbb{C}^{\text{adj}} \sigma^2]$, $\mathfrak{R}_{\text{cand}}^i(k, \sigma) \theta_{\text{cand,up}}^i(\sigma) \not\subseteq \mathfrak{R}_{\text{cand}}^i(k, \sigma) - \theta_{\text{cand,down}}^i(\sigma)$. Then

$$\begin{aligned} & \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma_n), \sigma_n) - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma_n), \sigma_n) \\ &= \mathcal{M}^i \int_{\xi_k} s_{k,i} \underbrace{\left(\mathbb{P}(\theta_{\text{cand,down}}^i(\sigma_n) + \sigma_n \tilde{\epsilon}_k \in \mathfrak{R}_{\text{cand}}^i(k, \sigma_n)) - \mathbb{P}(\theta_{\text{cand,up}}^i(\sigma_n) + \sigma_n \tilde{\epsilon}_k \in \mathfrak{R}_{\text{cand}}^i(k, \sigma_n)) \right)}_{> \omega > 0} dF_{\tilde{\xi}}(\xi_k) > \omega L^i, \end{aligned}$$

for some fixed $\omega \in (0, 1)$ and all n sufficiently large. This clearly contradicts [Eq. \(31\)](#). \square

Since at both thresholds the solvency condition equals zero, [Claim 3](#) implies that there exist constants $\mathbb{C}^{\text{adj}_3} > 0$ and $\sigma_0 > 0$ such that, for every $\sigma \in (0, \sigma_0)$,

$$\left| \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \right| < \mathbb{C}^{\text{adj}_3} \sigma.$$

By [Claim 2](#), $(-\infty, \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2] \subset \mathfrak{R}_{\text{cand}}^i(k, \sigma) \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma + \mathbb{C}^{\text{adj}} \sigma^2]$. Therefore,

$$\begin{aligned} \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\tilde{\epsilon}_k \leq \mathbb{C}_k^\epsilon - \mathbb{C}^{\text{adj}} \sigma) dF_{\tilde{\xi}}(\xi_k) &\leq \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta_{\text{cand,down}}^i(\sigma) + \sigma \tilde{\epsilon}_k \in \mathfrak{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\xi}}(\xi_k) = \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma), \\ \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\tilde{\epsilon}_k \leq \mathbb{C}_k^\epsilon + \mathbb{C}^{\text{adj}} \sigma) dF_{\tilde{\xi}}(\xi_k) &\geq \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\theta_{\text{cand,up}}^i(\sigma) + \sigma \tilde{\epsilon}_k \in \mathfrak{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\xi}}(\xi_k) = \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma). \end{aligned} \tag{33}$$

Let O_B^i denote the benchmark outflow defined in [Lemma 1](#). [Eq. \(33\)](#) implies that

$$\lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) \geq O_B^i, \quad \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \leq O_B^i.$$

Recalling [Eq. \(30\)](#) and the definition of θ_B^i in [Lemma 1](#), we obtain

$$\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^i(\sigma) \leq \theta_B^i \leq \lim_{\sigma \downarrow 0} \theta_{\text{cand,up}}^i(\sigma).$$

So θ_B^i lies between the endpoints. [Claim 3](#) further implies that

$$\lim_{\sigma \downarrow 0} \left| \theta_{\text{cand,up}}^i(\sigma) - \theta_B^i \right| + \left| \theta_B^i - \theta_{\text{cand,down}}^i(\sigma) \right| = \lim_{\sigma \downarrow 0} \left(\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma) \right) = 0.$$

Therefore, any candidate equilibrium converges to the limiting equilibrium described in [Lemma 1](#).

1.1.3 Step 3: the existence and characterization of the limiting equilibrium

In Step 1, we showed that for each σ there exists at least one equilibrium. In Step 2, we established that if there is a candidate sequence of equilibria indexed by σ , then the associated cutoffs must converge to θ_B^i as $\sigma \downarrow 0$. Therefore, the equilibrium characterized in Step 1 must converge to the limiting equilibrium described in [Lemma 1](#), and that limiting equilibrium must be unique by Step 2. \square

1.2 Sequential Service Constraint: General Result

Lemma 6. *Suppose that $H^i = 0$ and $\partial_2 C(A^i, 0) \geq R^i > 1$. Then, in the limit as the signal noise vanishes ($\sigma \rightarrow 0$), there exists at least one limiting subgame equilibrium. Each such equilibrium is characterized by a threshold, θ_S^i , which is a fixed point of a continuous map \mathcal{G} . The equilibrium strategy is for all depositors to **stay** with bank i if the fundamental $\theta > \theta_S^i$ and **run** on bank i if the fundamental $\theta < \theta_S^i$. The limiting subgame equilibrium is **unique** with some technical conditions.³¹ Furthermore, the run cutoff with sequential service constraint is weakly less than without: $\theta_S^i \leq \theta_B^i$*

The full construction of the map \mathcal{G} is quite complex, and its details are deferred below in the text of the following proof. The basic idea is similar to [Internet Appendix 1.1](#), but we need to handle additional difficulties associated with sequential service constraint.

Setting up Consider the representative bank i . Let agent k be a representative depositor at bank i after time $t = 1$. Denote by \mathcal{O}^i the aggregate deposit outflow from bank i . In the **Alternative Economy of Isolated Bank Run** that is concerned here, $\mathbb{I}^i = 0$. Given any agents' strategy profile, denoted as $\{r_{k,i \rightarrow H}\}$, if the true economic state is θ , then the Exact Law of Large Numbers (ELLN) yields

$$\mathcal{O}^i = \mathcal{M}^i \int_{\xi_k} \int_{x_k} r_{k,i \rightarrow H}(x_k) f_{\bar{x}_k | \bar{\theta}}(x_k | \theta) dx_k f_{\bar{\xi}}(\xi_k) d\xi_k.$$

Hence any candidate equilibrium outflows is a function of θ and σ (recall that private signal precision is governed by σ , so the densities $f_{\bar{x}_k | \bar{\theta}}$ and $f_{\bar{x}_w | \bar{\theta}}$ depend on σ)—i.e., any outflow can be written as a “deposit flow function”, $\mathcal{O}^i(\theta, \sigma)$.

Given any possible deposit flow functions, depositor k 's expected payoff from *staying* with bank i after observing x_k at $t = 2$ is

$$\mathcal{U}_{k,i \rightarrow i}^2(x_k) \equiv \mathcal{U}_k^2(a_{k,i \rightarrow i} = 1 | x_k) = \mathbb{P}(\mathcal{D}_i^c | x_k) (R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i | x_k) \Lambda. \quad (34)$$

If k leaves the banking system to hold the risk-free asset, her expected payoff is³²

$$\mathcal{U}_{k,i \rightarrow H}^2(x_k) \equiv \mathcal{U}_k^2(a_{k,i \rightarrow H} = 1 | x_k) = \int_{\mathcal{D}_i} [M^i + (1 - M^i)\Lambda] f_{\bar{\theta} | \bar{x}_k}(\theta | x_k) d\theta + \int_{\mathcal{D}_i^c} f_{\bar{\theta} | \bar{x}_k}(\theta | x_k) d\theta, \quad (35)$$

where $M^i = \min\left(\frac{\mathcal{O}^i(\bar{\theta})}{\mathcal{O}_{\text{cand}}^i(\bar{\theta}, \sigma)}, 1\right)$. Under the best-responding strategy, depositor k leaves bank i iff $\mathcal{U}_{k,i \rightarrow H}^2(x_k) > \mathcal{U}_{k,i \rightarrow i}^2(x_k)$ and vice versa. Hence, depositor k 's best response strategy only

³¹The equilibrium is unique under a broad range of specific modeling choices, but pinpointing the exact sufficient condition that ensures uniqueness is highly technical and would be a digression from the main focus of this paper. One condition is that $\partial_2 C(A^i, 0)$ is sufficiently large.

³²Recall that $\mathcal{O}^i(\theta)$ denotes the maximum outflow that bank i can pay out without exhausting its liquid resources, see [Eq. \(11\)](#).

depend on her private signal x_k . Define the run-signal set

$$\begin{aligned} \mathfrak{R}_{\text{cand}}^i(k, \sigma) &\equiv \{x_k : \mathcal{U}_{k,i \rightarrow i}^2(x_k) < \mathcal{U}_{k,i \rightarrow H}^2(x_k)\} \\ &= \left\{x_k : \mathbb{P}\left(\tilde{\theta} G^i(A^i) - C^i\left(A^i, \mathcal{O}_{\text{cand}}^i(\tilde{\theta}, \sigma)^+\right) - R^i\left(L^i - \mathcal{O}_{\text{cand}}^i(\tilde{\theta}, \sigma)^+\right) > 0 \mid \tilde{x}_k = x_k\right) \right. \\ &\quad \left. < \frac{(1 - \Lambda)}{R^i - \Lambda + \xi_k^i} \mathbb{E}\left[\min\left(\frac{\mathcal{O}_{\text{cand}}^i(\tilde{\theta})}{\mathcal{O}_{\text{cand}}^i(\tilde{\theta}, \sigma)}, 1\right) \mid \tilde{x}_k = x_k\right]\right\}. \end{aligned} \quad (36)$$

The equilibrium outflow must also satisfy

$$\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) = \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\vartheta + \sigma \tilde{\epsilon}_k \in \mathfrak{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\xi}}(\xi_k).$$

Necessary conditions for any candidate equilibrium Fix a sufficiently small noise level $\sigma > 0$. Consider a candidate equilibrium where the deposit outflow at bank i is determined by the function $\mathcal{O}_{\text{cand}}^i(\theta, \sigma)$. While the form of this function is common knowledge in the candidate equilibrium, each depositor observes only a private signal \tilde{x}_k about the true state. Given these beliefs, an agent regards bank b as defaulting iff $\tilde{\theta} \in \mathcal{D}_i(\text{cand}, \sigma)$, where³³

$$\mathcal{D}_i(\text{cand}, \sigma) = \left\{\theta : \theta G^i(A^i) - C^i\left(A^i, \left(\mathcal{O}_{\text{cand}}^i(\theta, \sigma)\right)^+\right) - R^i\left(L^i - \mathcal{O}_{\text{cand}}^i(\theta, \sigma)\right) \leq 0\right\}.$$

Define the solvency thresholds $\theta_{\text{cand,up}}^i(\sigma)$ and $\theta_{\text{cand,down}}^i(\sigma)$ as in Eq. (29). Therefore, $(-\infty, \theta_{\text{cand,down}}^i(\sigma)] \subseteq \mathcal{D}_i(\text{cand}, \sigma) \subseteq (-\infty, \theta_{\text{cand,up}}^i(\sigma)]$. Eq. (31) also holds.

Our next goal is to characterize the optimal strategy for depositor k . This strategy is defined by a "run region," a set $\mathfrak{R}_{\text{cand}}^i(k, \sigma)$ containing all signal values \tilde{x}_k that induce the depositor to run. An argument similar to the one in Claim 2 shows that there is a this run region is bounded by some candidate thresholds for all small enough σ :

1. The depositor **runs** on bank i (by moving funds to a safe asset) if the observed signal satisfies:

$$\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \underline{\rho}_k^\epsilon(\sigma)\sigma + \mathbb{C}^{\text{adj}}\sigma^2$$

2. The depositor does **not run** on bank i if the observed signal satisfies:

$$\tilde{x}_k \geq \theta_{\text{cand,up}}^i(\sigma) + \bar{\rho}_k^\epsilon(\sigma)\sigma - \mathbb{C}^{\text{adj}}\sigma^2$$

These inequalities hold for sets of constants $\underline{\rho}_k^\epsilon(\sigma)$ and $\bar{\rho}_k^\epsilon(\sigma)$. We define the ultimate decision thresholds, $\underline{\mathbb{C}}_k^\epsilon$ and $\bar{\mathbb{C}}_k^\epsilon$, by taking the tightest possible bounds:

$$\underline{\mathbb{C}}_k^\epsilon(\sigma) \equiv \sup\{\underline{\rho}_k^\epsilon(\sigma)\} \quad \text{and} \quad \bar{\mathbb{C}}_k^\epsilon(\sigma) \equiv \inf\{\bar{\rho}_k^\epsilon(\sigma)\}.$$

Therefore,

$$(-\infty, \theta_{\text{cand,down}}^i(\sigma) + \underline{\mathbb{C}}_k^\epsilon(\sigma)\sigma - \mathbb{C}^{\text{adj}}\sigma^2] \subset \mathfrak{R}_{\text{cand}}^i(k, \sigma) \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma) + \bar{\mathbb{C}}_k^\epsilon(\sigma)\sigma + \mathbb{C}^{\text{adj}}\sigma^2].$$

³³Recall that $H^i = 0$.

This structure implies that the key threshold constants, \underline{C}_k^ϵ and \bar{C}_k^ϵ , are also the normalized boundaries of the run region and its complement, $(\mathcal{R}_{\text{cand}}^i(k, \sigma))^c$:

$$\underline{C}_k^\epsilon(\sigma) = \frac{\inf\left((\mathcal{R}_{\text{cand}}^i(k, \sigma))^c\right) - \theta_{\text{cand,down}}^i(\sigma)}{\sigma} \quad \text{and} \quad \bar{C}_k^\epsilon(\sigma) = \frac{\sup\left(\mathcal{R}_{\text{cand}}^i(k, \sigma)\right) - \theta_{\text{cand,up}}^i(\sigma)}{\sigma}.$$

Claim 4. *Because $\{f_{\tilde{\epsilon}_k}\}$ has large enough common support, for any possible sequence of equilibria as $\sigma \rightarrow 0$,*

$$\lim_{\sigma \downarrow 0} \frac{\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma)}{\sigma} + \bar{C}_k^\epsilon(\sigma) - \underline{C}_k^\epsilon(\sigma) = 0.$$

Proof. Suppose that

$$\Delta_\sigma^{\text{tem}} \equiv \frac{\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma)}{\sigma} + \bar{C}_k^\epsilon(\sigma) - \underline{C}_k^\epsilon(\sigma) > 0.$$

The argument now proceeds by contradiction. Suppose that there exists some subsequence $\{\sigma_n\}_{n \geq 1}$ where $\sigma_n \rightarrow 0$, such that $\liminf_{n \rightarrow \infty} \Delta_{\sigma_n}^{\text{tem}} = \bar{\Delta}^{\text{tem}} > 0$. Define set translation $\frac{\mathcal{R}_{\text{cand}}^i(k, \sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n} \equiv \{x \mid x\sigma_n + \theta_{\text{cand,down}}^i(\sigma_n) \in \mathcal{R}_{\text{cand}}^i(k, \sigma_n)\}$. We define the following disjoint normalized sets:

$$\begin{aligned} \Omega^i &= \left(\frac{\mathcal{R}_{\text{cand}}^i(k, \sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n} \right) \cap \left(\underline{C}_k^\epsilon(\sigma_n), \underline{C}_k^\epsilon(\sigma_n) + \Delta_{\sigma_n}^{\text{tem}} \right) \\ \Omega^{i,c} &= \left(\left(\frac{\mathcal{R}_{\text{cand}}^i(k, \sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n} \right) \cap \left(\underline{C}_k^\epsilon(\sigma_n), \underline{C}_k^\epsilon(\sigma_n) + \Delta_{\sigma_n}^{\text{tem}} \right) \right)^c - \Delta_{\sigma_n}^{\text{tem}}. \end{aligned}$$

Then a direct calculation shows that there is some constant $\mathcal{K} > 0$ such that

$$\begin{aligned} & |\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma_n), \sigma_n) - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma_n) + \Delta_{\sigma_n}^{\text{tem}} \sigma_n, \sigma_n)| \\ &= \mathcal{M}^i \int_{\tilde{\xi}_k} s_{k,i} \left(\mathbb{P}(\tilde{\epsilon}_k \in \frac{\mathcal{R}_{\text{cand}}^i(k, \sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n}) - \mathbb{P}(\tilde{\epsilon}_k + \Delta_{\sigma_n}^{\text{tem}} \in \frac{\mathcal{R}_{\text{cand}}^i(k, \sigma_n) - \theta_{\text{cand,down}}^i(\sigma_n)}{\sigma_n}) \right) dF_{\tilde{\xi}}(\xi_k) \\ &= \mathcal{M}^i \int_{\tilde{\xi}_k} s_{k,i} \left(\mathbb{P}(\tilde{\epsilon}_k \in \Omega^i \cup \Omega^{i,c}) \right) dF_{\tilde{\xi}}(\xi_k) > \mathcal{K}(\Delta_{\sigma_n}^{\text{tem}}). \end{aligned} \quad (37)$$

This contradicts Eq. (31) for sufficiently small σ_n . \square

Claim 4 immediately implies the following characterization of the depositor strategy and its resulting outflow bounds.

Claim 5. *There exist constants $C_k^\epsilon(\sigma)$ and $C^{\text{adj}} > 0$ such that for all sufficiently small σ , the optimal strategy for depositor k is to:*

1. **Run** on bank i (by moving funds to a safe asset) if the observed signal satisfies:

$$\tilde{x}_k < \theta_{\text{cand,down}}^i(\sigma) + C_k^\epsilon(\sigma)\sigma - C^{\text{adj}}\sigma^2$$

2. **Not run** on bank i if the observed signal satisfies:

$$\tilde{x}_k \geq \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\varepsilon(\sigma)\sigma + \mathbb{C}^{\text{adj}}\sigma^2$$

This strategy profile implies the following bounds on the candidate outflow function for any fundamental state ϑ :

$$\mathbb{E}_{\varepsilon_k} \left[s_{k,i} F_{\bar{\varepsilon}} \left(\frac{\theta_{\text{cand,down}}^i(\sigma) - \vartheta}{\sigma} + \mathbb{C}_k^\varepsilon(\sigma) - \mathbb{C}^{\text{adj}}\sigma \right) \right] \leq \frac{\mathbb{O}_{\text{cand}}^i(\vartheta, \sigma)}{\mathcal{M}^i} \leq \mathbb{E}_{\varepsilon_k} \left[s_{k,i} F_{\bar{\varepsilon}} \left(\frac{\theta_{\text{cand,down}}^i(\sigma) - \vartheta}{\sigma} + \mathbb{C}_k^\varepsilon(\sigma) + \mathbb{C}^{\text{adj}}\sigma \right) \right].$$

Proof. This is trivial given [Claim 4](#). □

Characterizing the unique limiting equilibrium Our next goal is to characterize $\mathbb{C}_k^\varepsilon(\sigma)$ as $\sigma \rightarrow 0$, which will allow us to fully characterize the unique limiting equilibrium. It will be clear that $\mathbb{C}_k^\varepsilon(\sigma)$ is determined in a system of equations that characterize the equilibrium. By definition of \mathbb{O}^i ,

$$\begin{aligned} 0 &= \theta_{\text{cand,down}}^i(\sigma) G^i(A^i) - C^i(A^i, \mathbb{O}^i(\theta_{\text{cand,down}}^i(\sigma))) \\ &= \theta_{\text{cand,down}}^i(\sigma) G^i(A^i) - C^i(A^i, \mathbb{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma)) - R^i(L^i - \mathbb{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma)), \end{aligned}$$

which implies $\mathbb{O}^i(\theta_{\text{cand,down}}^i(\sigma)) > \mathbb{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma)$. Define the constant $\mathbb{C}_\sigma^{\text{cut}}$ by (it is easy to check that $\mathbb{C}_\sigma^{\text{cut}} \in \mathbb{R}$ is well defined for small enough σ)

$$\mathbb{C}_\sigma^{\text{cut}} \equiv \inf \left\{ \rho \geq 0 : \mathbb{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma) - \rho\sigma, \sigma) = \mathbb{O}^i(\theta_{\text{cand,down}}^i(\sigma) - \rho\sigma) \right\}.$$

When receiving signal $\theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\varepsilon\sigma - \mathbb{C}^{\text{adj}}\sigma^2$, depositor k estimates that

$$\begin{aligned} & \int_{\theta} \min \left[\frac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta, \sigma)}, 1 \right] f_{\bar{\theta}|\tilde{x}_k}(\theta | \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\varepsilon\sigma - \mathbb{C}^{\text{adj}}\sigma^2) d\theta \\ &= \frac{\int_{-\infty}^{\mathbb{C}_k^\varepsilon + \mathbb{C}_\sigma^{\text{cut}} - \mathbb{C}^{\text{adj}}\sigma} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz}{\int_z f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz} + \frac{\int_{\mathbb{C}_k^\varepsilon + \mathbb{C}_\sigma^{\text{cut}} - \mathbb{C}^{\text{adj}}\sigma}^{\infty} \frac{\mathbb{O}^i(x_k - \sigma z)}{\mathbb{O}^i(x_k - \sigma z, \sigma)} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz}{\int_z f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\varepsilon}_k}(z) dz} \\ &= F_{\bar{\varepsilon}_k}(\mathbb{C}_k^\varepsilon + \mathbb{C}_\sigma^{\text{cut}} - \mathbb{C}^{\text{adj}}\sigma) + \int_{\mathbb{C}_k^\varepsilon + \mathbb{C}_\sigma^{\text{cut}} - \mathbb{C}^{\text{adj}}\sigma}^{\infty} \frac{\mathbb{O}^i(\theta_{\text{cand,down}}^i(\sigma))}{\mathbb{O}^i(x_k - \sigma z, \sigma)} f_{\bar{\varepsilon}_k}(z) dz + \mathcal{O}(\sigma) \end{aligned}$$

Fix $x_k = \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\varepsilon\sigma - \mathbb{C}^{\text{adj}}\sigma^2$. [Claim 5](#) implies

$$\begin{aligned} & \frac{(1 - \Lambda)}{R^i + \xi_k^i - \Lambda} \int_{\theta} \min \left[\frac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta, \sigma)}, 1 \right] f_{\bar{\theta}|\tilde{x}_k}(\theta | x_k) d\theta > \mathbb{P}(\mathcal{D}_i^c | x_k) \\ & \frac{(1 - \Lambda)}{R^i + \xi_k^i - \Lambda} \int_{\theta} \min \left[\frac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta, \sigma)}, 1 \right] f_{\bar{\theta}|\tilde{x}_k}(\theta | x_k + 2\mathbb{C}^{\text{adj}}\sigma^2) d\theta \leq \mathbb{P}(\mathcal{D}_i^c | x_k + 2\mathbb{C}^{\text{adj}}\sigma^2). \end{aligned} \tag{38}$$

Claim 4 implies that for sufficiently small σ ,

$$\mathbb{P}(\mathcal{D}_i^c | x_k) = \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,down}}^i(\sigma) | x_k) + \mathcal{O}(\sigma) = \frac{\int_{-\infty}^{(x_k - \theta_{\text{cand,down}}^i(\sigma))/\sigma} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}{\int_{-\infty}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz} + \mathcal{O}(\sigma) = F_{\bar{\epsilon}_k}(\mathbf{C}_k^\epsilon - \mathbf{C}^{\text{adj}}\sigma) + \mathcal{O}(\sigma).$$

Similarly, $\mathbb{P}(\mathcal{D}_i^c | x_k + 2\mathbf{C}^{\text{adj}}\sigma^2) = F_{\bar{\epsilon}_k}(\mathbf{C}_k^\epsilon + \mathbf{C}^{\text{adj}}\sigma) + \mathcal{O}(\sigma)$. Taking the limits as $\sigma \rightarrow 0$, we obtain from Eq. (38)

$$\begin{aligned} \lim_{\sigma \downarrow 0} \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda} \int_{\theta} \min\left[\frac{\mathcal{O}^i(\theta)}{\mathcal{O}^i(\theta, \sigma)}, 1\right] f_{\bar{\theta}|x_k}(\theta | x_k) d\theta &= F_{\bar{\theta}}(\mathbf{C}_k^\epsilon) \Rightarrow \\ \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda} \left(F_{\bar{\epsilon}_k}(\mathbf{C}_k^\epsilon + \mathbf{C}_\sigma^{\text{cut}}) + \int_{\mathbf{C}_k^\epsilon + \mathbf{C}^{\text{cut}}}^{\infty} \frac{\mathcal{O}^i(\theta_{\text{cand,down}}^i(\sigma))}{\mathcal{M}^i \mathbb{E}_{\xi_w} [s_{w,i} F_{\bar{\epsilon}}(z + \mathbf{C}_w^\epsilon - \mathbf{C}_k^\epsilon)]} f_{\bar{\epsilon}_k}(z) dz \right) &= F_{\bar{\theta}}(\mathbf{C}_k^\epsilon). \end{aligned} \quad (39)$$

By monotonicity, $\mathcal{O}^i(\theta_{\text{cand,down}}^i(0))$ and \mathbf{C}^{cut} uniquely determine \mathbf{C}_k^ϵ . Also, by definition, $\lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma) - \mathbf{C}_\sigma^{\text{cut}}\sigma, \sigma) = \mathcal{O}^i(\theta_{\text{cand,down}}^i(\sigma))$, so

$$\mathbb{E}_{\xi_k} [s_{k,i} F_{\bar{\epsilon}}(\mathbf{C}_k^\epsilon + \mathbf{C}^{\text{cut}})] = \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))}{\mathcal{M}^i}. \quad (40)$$

Therefore, given $\{\mathbf{C}_k^\epsilon\}$ and $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))$, \mathbf{C}^{cut} is uniquely determined. The system of equations Eqs. (39) and (40) implicitly defines a mapping, T , from the space of possible cutoffs \mathbf{C}^{cut} to itself. In any limiting equilibrium, \mathbf{C}^{cut} must be a **fixed point** of this mapping.

In general, T is a contraction. Intuitively, the aggregation across heterogeneous depositors in Eq. (40) has a ‘‘smoothing’’ effect. A small change in an initial assumed cutoff, $\mathbf{C}_{\text{old}}^{\text{cut}}$, will cause a dampened, smaller change in the resulting cutoff, $\mathbf{C}_{\text{new}}^{\text{cut}}$. Formally, it can be shown that the mapping T has a Lipschitz constant less than one generically. To rule out pathological cases, we assume that $\{f_{\bar{\epsilon}_k}\}$ ensures that T is a contraction. It is easy to check that most common distribution families satisfy this assumption. By the Banach Fixed-Point Theorem, any contraction mapping on a complete metric space admits a unique fixed point. Therefore, the system has a unique equilibrium set of thresholds, $(\{\mathbf{C}_k^\epsilon\}, \mathbf{C}^{\text{cut}})$, when we fix each possible value of $\theta_{\text{cand,down}}^i(0)$. The exact value of $\theta_{\text{cand,down}}^i(0)$ in the limiting equilibrium will be determined by the following system of equations:

Aggregating Eq. (39), we obtain

$$\mathbb{E}_{\xi_k} \left[s_{k,i} \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda} \left(F_{\bar{\epsilon}_k}(\mathbf{C}_k^\epsilon + \mathbf{C}^{\text{cut}}) + \int_{\mathbf{C}_k^\epsilon + \mathbf{C}^{\text{cut}}}^{\infty} \frac{\mathcal{O}^i(\theta_{\text{cand,down}}^i(0)) f_{\bar{\epsilon}_k}(z)}{\mathcal{M}^i \mathbb{E}_{\xi_w} [s_{w,i} F_{\bar{\epsilon}}(z + \mathbf{C}_w^\epsilon - \mathbf{C}_k^\epsilon)]} dz \right) \right] = \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)}{\mathcal{M}^i} \quad (41)$$

In the limiting equilibrium, the following must also be true

$$\theta_{\text{cand,down}}^i(0)G^i(A^i) = C^i(A^i, \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)) + R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)) \quad (42)$$

$$\theta_{\text{cand,down}}^i(0)G^i(A^i) = C^i(A^i, \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))) \quad (43)$$

The limiting equilibrium is fully characterized by the system of equations in Eqs. (39) to (43), and the existence of the solution of $\theta_{\text{cand,down}}^i(0)$ is guaranteed by Brouwer's fixed-point theorem. While a unique solution is expected under generic conditions, a formal proof would be a digression from the main focus of this paper. We therefore confine to a brief argument of the tractable case where the cost function is sufficiently convex (i.e., $\partial_2 C^i$ is large).

The uniqueness proof proceeds by showing a contradiction. On one hand, Eqs. (42) and (43) imply that any increase in $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))$ necessitates a correspondingly large increase in $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))$; this required increment is bounded below by a function of $\partial_2 C^i$. On the other hand, Eq. (41) place a fixed upper bound on the possible increment of $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))$. For a sufficiently large $\partial_2 C^i$, the required increase (the lower bound) will exceed the permitted increase (the upper bound). This makes the existence of two distinct solutions impossible, thus guaranteeing uniqueness. \square

Algorithm to compute the equilibrium The above system of equations characterizing the subgame equilibrium inspires the construction of a continuous map $\mathcal{G} : \mathbb{R} \rightarrow \mathbb{R}$ whose fixed point, θ_S^i , defines the equilibrium. We build this map as a composition of four functions.

1. Define $\mathcal{G}_1(x) = \inf\{y \mid xG^i(A^i) = C^i(A^i, y) = 0\}$. The output of this function is continuously increasing in x .
2. Define the map \mathcal{G}_2 , which takes a scalar x and returns the tuple $(C^{\text{cut}}, \{C_k^\epsilon\})$. The mapping is defined such that if $x = \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))$, then Eqs. (39) and (40) hold. As established previously, for any fixed x , the map returns a unique image, and \mathcal{G}_2 is a continuous function of x .
3. The map \mathcal{G}_3 takes the tuple $(C^{\text{cut}}, \{C_k^\epsilon\})$ as input and returns the scalar outflow O_S^i that solves the following condition:

$$\mathbb{E}_{\xi_k} \left[s_{k,i} \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda} \left(F_{\bar{\epsilon}_k}(C_k^\epsilon + C^{\text{cut}}) + \int_{C_k^\epsilon + C^{\text{cut}}}^{\infty} \frac{\mathcal{O}^i(\theta_{\text{cand,down}}^i(0)) f_{\bar{\epsilon}_k}(z)}{\mathcal{M}^i \mathbb{E}_{\xi_w} [s_{w,i} F_{\bar{\epsilon}}(z + C_w^\epsilon - C_k^\epsilon)]} dz \right) \right] = \frac{O_S^i}{\mathcal{M}^i}$$

4. Define $\mathcal{G}_4(y) = \frac{C^i(A^i, y) + R^i(L^i - y)}{G^i(A^i)}$.

5. The final map is the composition of the previous four steps: $\mathcal{G} = \mathcal{G}_4 \circ \mathcal{G}_3 \circ \mathcal{G}_2 \circ \mathcal{G}_1$.

Any fixed point of the above system would be one limiting equilibrium.

1.3 Proof of Lemma 2

We prove this lemma as a corollary of Lemma 6 discussed in Internet Appendix 1.2 above. Under the assumption of ex-ante homogeneity—that is, when all depositors have the same banking utility ($\xi_k^i = \xi^i$) and draw signals from the same distribution ($f_{\bar{\epsilon}_k} = f_{\bar{\epsilon}}$)—the system of equations Eqs. (39) to (43) simplifies considerably. Specifically, all depositors adopt the same cutoff strategy, so C_k^ϵ becomes a constant C^ϵ independent of k . This allows us to simplify Eq. (40). Because C^ϵ is constant, we can separate the expectation, and since $\mathbb{E}_{\xi_k} [s_{k,i}] = L^i/M^i$, Eq. (40) simplifies to:

$$F_{\bar{\epsilon}}(C^\epsilon + C^{\text{cut}}) = \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))}{L^i}.$$

Furthermore, the homogeneity assumption simplifies the limiting expression for the outflow function from Claim 5:

$$\lim_{\sigma \downarrow 0} \mathcal{O}^i(\theta_{\text{cand,down}}^i(0) + C^\epsilon \sigma - \sigma z, \sigma) = \mathcal{M}^i \mathbb{E}_{\xi_w} [s_{w,i} F_{\bar{\epsilon}}(z + C^\epsilon - C^\epsilon)] = L^i F_{\bar{\epsilon}}(z).$$

Therefore, Eq. (41) simplifies to

$$\begin{aligned} \frac{(1-\Lambda)}{R^i + \xi^i - \Lambda} \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))}{\mathcal{M}^i} + \frac{L^i}{\mathcal{M}^i} \int_{C^\epsilon + C^{\text{cut}}}^{\infty} \frac{\mathcal{O}^i(\theta_{\text{cand,down}}^i(0))}{L^i F_{\bar{\epsilon}}(z)} f_{\bar{\epsilon}}(z) dz &= \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)}{\mathcal{M}^i} \Rightarrow \\ \frac{(1-\Lambda)}{R^i + \xi^i - \Lambda} + \log(L^i) - \log(\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))) &= \frac{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)}{\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))} \end{aligned}$$

The equilibrium conditions in Eqs. (42) and (43) establish the following key relationship between the expected deposit outflow (\mathcal{O}) and the bank's total resources (\mathcal{O}) at the run threshold:

$$C^i(A^i, \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)) + R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0), 0)) = C^i(A^i, \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(0))).$$

The above two equations imply that in any candidate limiting equilibrium, the outflow \mathcal{O}_S^i and the total resources \mathcal{O}_S^i must be the solution to the following system of two equations:

$$\left(\frac{1-\Lambda}{R^i + \xi^i - \Lambda} + \log(L^i) - \log(\mathcal{O}_S^i) \right) \mathcal{O}_S^i = \mathcal{O}_S^i \quad (44)$$

$$C^i(A^i, \mathcal{O}_S^i) + R^i(L^i - \mathcal{O}_S^i) = C^i(A^i, \mathcal{O}_S^i). \quad (45)$$

We can prove the uniqueness of this solution geometrically by treating \mathcal{O}_S^i as a function of \mathcal{O}_S^i in the $(\mathcal{O}_S^i, \mathcal{O}_S^i)$ space.

- Eq. (44) implicitly defines a **hump-shaped, concave function** that passes through the (limiting) points $(0, 0)$ and $(L^i, \frac{(1-\Lambda)L^i}{R^i + \xi^i - \Lambda})$.
- When $\partial_2^3 C^i$ is large enough, Eq. (45) implicitly defines a **convex function** that passes through the points $(\mathcal{O}_{\text{min}}^i, 0)$ and (L^i, L^i) , where $\mathcal{O}_{\text{min}}^i$ solves $C^i(A^i, \mathcal{O}_{\text{min}}^i) = R^i L^i$.

The concave function starts above the convex function near the origin and ends below it at L^i . A continuous concave and convex function pair with this configuration must intersect at a single point. This guarantees that the system has a unique solution, and therefore the limiting equilibrium is unique. Furthermore, the solution of $\mathcal{O}_S^i \leq O_B^i$, so $\theta_S^i \leq \theta_B^i$.

Finally, we note a crucial departure from the standard global games framework: the equilibrium is **not solvable** by the iterative elimination of dominated strategies. Instead, it is a coordination game that requires all agents to correctly conjecture the key parameter C^ε , which in turn determines the collective run region $\mathcal{R}_{\text{cand}}^i(k, \sigma)$. This reliance on common knowledge of the full strategy space represents a demanding rationality assumption, particularly for a dispersed group of individual bank depositors. \square

1.4 Proof of Lemma 3

We prove the lemma for $j = i$ as the case of $j = b$ follows an identical argument.

Key Challenge The liquid buffer H^i creates a non-monotonicity: counterintuitively, a larger deposit outflow \mathcal{O}^i can *reduce* default risk for bank i by shrinking its default set when the outflow remains below H^i . Bank i defaults in state θ exactly when

$$\theta \in \left\{ \vartheta \mid \vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}^i - H^i)^+) + (H^i - \mathcal{O}^i)^+ - R^i(L^i - \mathcal{O}^i) \leq 0 \right\}.$$

When $\mathcal{O}^i < H^i$, increasing \mathcal{O}^i *shrinks* the above default set, so remaining depositors have a stronger incentive to stay with bank i . This prevents a direct reuse of the $H^i = 0$ argument to prove Lemma 1 outlined in Internet Appendix 1.1. (Specifically, ?? no longer holds true.)

Proof Overview It is necessary to analyze two cases separately. **Case 1** ($H^i \leq \widehat{H}^i$). The liquid buffer is small, so in equilibrium outflows necessarily exceed H^i ; the limiting equilibrium therefore coincides with Lemma 1. **Case 2** ($H^i > \widehat{H}^i$). The equilibrium logic differs. In both cases, we first characterize the equilibrium for a fixed noise variance $\sigma > 0$ (with σ small but positive), and then take the limit as $\sigma \downarrow 0$ to obtain the noise-free outcome. We begin with results common to both cases.

Prerequisite Results Let $\theta_B^i(H^i)$ (abbrev. “ θ_B^i ”) and O_B^i be as in Lemma 3. By definition, \widehat{H}^i uniquely solves $C^i(A^i, O_B^i - \widehat{H}^i) + \widehat{H}^i = R^i O_B^i$, and by Assumption 4 it is well defined with $\widehat{H}^i < O_B^i$. For each $H \geq 0$, the **fundamental default boundary** $\widehat{\theta}^i(H)$ solves $\widehat{\theta}^i(H) G^i(A^i) + H - R^i L^i = 0$, i.e., $\widehat{\theta}^i(H)$ is the default boundary absent outflows. Hence

$$\begin{aligned} \widehat{\theta}^i(\widehat{H}^i) G^i(A^i) + (\widehat{H}^i - O_B^i)^+ - C^i(A^i, (O_B^i - \widehat{H}^i)^+) - R^i(L^i - O_B^i) &= 0, \\ \widehat{\theta}^i(H^i) G^i(A^i) + (H^i - O_B^i)^+ - C^i(A^i, (O_B^i - H^i)^+) - R^i(L^i - O_B^i) &< 0, \quad \forall H^i < \widehat{H}^i, \\ \widehat{\theta}^i(H^i) G^i(A^i) + (H^i - O_B^i)^+ - C^i(A^i, (O_B^i - H^i)^+) - R^i(L^i - O_B^i) &> 0, \quad \forall H^i > \widehat{H}^i. \end{aligned} \quad (46)$$

Fix a small $\sigma > 0$, and let $\mathcal{O}_{\text{cand}}^i(\theta, \sigma)$ be the (candidate) equilibrium outflow in state

θ . In this equilibrium, a depositor k at bank i believes the default set for bank i is

$$\mathcal{D}_{\text{cand}}^\sigma = \left\{ \vartheta : \vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) \leq 0 \right\}.$$

As in the proof of [Lemma 1](#), define the bracketing thresholds

$$\begin{aligned} \theta_{\text{cand,up}}^i(\sigma) &\equiv \inf \left\{ \rho : \vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) > 0, \forall \vartheta \geq \rho \right\}, \\ \theta_{\text{cand,down}}^i(\sigma) &\equiv \sup \left\{ \rho : \vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) < 0, \forall \vartheta \leq \rho \right\}. \end{aligned}$$

Then similarly we have $(-\infty, \theta_{\text{cand,down}}^i(\sigma)] \subset \mathcal{D}_{\text{cand}}^\sigma \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma)]$ and

$$\begin{aligned} 0 &= \theta_{\text{cand,up}}^i(\sigma) G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma)) \\ &= \theta_{\text{cand,down}}^i(\sigma) G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma)). \end{aligned} \quad (47)$$

Recall the run set for the representative depositor k and the equilibrium outflows are

$$\begin{aligned} \mathfrak{R}^{\text{cand}}(k, \sigma) &\equiv \left\{ x_k : \mathbb{P}(\tilde{\theta} \notin \mathcal{D}_{\text{cand}}^\sigma \mid \tilde{x}_k = x_k) < \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}, \\ \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) &= \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\vartheta + \sigma \tilde{\epsilon}_k \in \mathfrak{R}^{\text{cand}}(k, \sigma)) dF_{\tilde{\epsilon}}(\xi_k). \end{aligned} \quad (48)$$

Claim 6. Let \mathcal{C}_k^ϵ be defined in [Eq. \(32\)](#). There is $\mathcal{C}_k^{\text{adj}} > 0$ such that, for all sufficiently small σ :

1. If $\tilde{x}_k \geq \theta_{\text{cand,up}}^i(\sigma) + \mathcal{C}_k^\epsilon \sigma + \mathcal{C}_k^{\text{adj}} \sigma^2$, depositor k does not run on bank i .
2. If $\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \mathcal{C}_k^\epsilon \sigma - \mathcal{C}_k^{\text{adj}} \sigma^2$, depositor k runs on bank i .

Proof. Identical to [Claim 2](#). □

[Claim 6](#) implies that

$$\frac{\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)}{\mathcal{M}^i} \leq \mathbb{E}_{\xi_k} \left[s_{k,i} F_{\tilde{\epsilon}} \left(\frac{\theta_{\text{cand,up}}^i(\sigma) - \vartheta}{\sigma} + \mathcal{C}_k^\epsilon(\sigma) + \mathcal{C}_k^{\text{adj}} \sigma \right) \right]. \quad (49)$$

1.4.1 Case 1: $H^i \leq \widehat{H}^i$

[Claim 6](#) implies $(-\infty, \theta_{\text{cand,down}}^i(\sigma) + \mathcal{C}_k^\epsilon \sigma - \mathcal{C}_k^{\text{adj}} \sigma^2] \subset \mathfrak{R}_{\text{cand}}^i(k, \sigma)$. Hence by [Eq. \(48\)](#)

$$\begin{aligned} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) &\geq \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}(\tilde{\epsilon}_k \leq \mathcal{C}_k^\epsilon - \mathcal{C}_k^{\text{adj}} \sigma) dF_{\tilde{\epsilon}}(\xi_k) \Rightarrow \\ \liminf_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) &\geq \mathcal{M}^i \int_{\xi_k} s_{k,i} \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} dF_{\tilde{\epsilon}}(\xi_k) = O_B^i. \end{aligned}$$

So for all σ small enough, $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) > O_B^i - \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma > H^i$ (last inequality follows from $H^i \leq \widehat{H}^i < O_B^i$). Apply this inequality and $\partial_2 C^i(\cdot, \cdot) \geq R^i$ to Eq. (47):

$$\begin{aligned} 0 &= \theta_{\text{cand,down}}^i(\sigma)G^i(A^i) - C^i(A^i, \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) - H^i) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma)) \\ &\leq \theta_{\text{cand,down}}^i(\sigma)G^i(A^i) - C^i(A^i, O_B^i - \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma - H^i) - R^i(L^i - (O_B^i - \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma)), \end{aligned} \quad (50)$$

We claim that $\theta_{\text{cand,down}}^i(\sigma) > \widehat{\theta}^i(H^i)$. Otherwise, if $\theta_{\text{cand,down}}^i(\sigma) \leq \widehat{\theta}^i(H^i)$, then by Eq. (46) the last line of Eq. (50) < 0 for small σ , a contradiction. From Eq. (48), for all $\vartheta \leq \widehat{\theta}^i(H^i)$,

$$\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) \geq O_B^i - \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma > H^i.$$

This bound on the equilibrium outflow is obtained by eliminating dominated strategies (Claim 6). This motivates the following deterministic envelopes for equilibrium outflows:

$$\underline{\mathcal{O}}^i(\theta, 0, \sigma) \equiv (O_B^i - \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma) \mathbb{1}_{[\theta \leq \widehat{\theta}^i(H^i)]}, \quad \overline{\mathcal{O}}^i(\theta, 0, \sigma) \equiv L^i.$$

Then a.s. $\underline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma) \leq \mathcal{O}_{\text{cand}}^i(\bar{\theta}, \sigma) \leq \overline{\mathcal{O}}^i(\bar{\theta}, 0, \sigma)$. This is common knowledge among all agents. From here, the remaining steps after Eq. (27) in the proof of Lemma 1 (see Internet Appendix 1.1.1) apply verbatim. In particular, any pointwise increase in \mathcal{O}^i weakly enlarges the default set—there is strategic complementarity and no strategic substitutability. Hence, the prediction of Lemma 1 remains valid: the limiting equilibrium outflow at the run cutoff equals O_B^i , and the run cutoff is $\theta_B^i(H^i)$ as defined in Lemma 3.

1.4.2 Case 2: $H^i > \widehat{H}^i$

By Claim 6, $\mathcal{R}^{\text{cand}}(k, \sigma) \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma) + \mathcal{C}_k^\varepsilon\sigma + \mathcal{C}_k^{\text{adj}}\sigma^2]$. Hence, by Eq. (48),

$$\mathcal{O}_{\text{cand}}^i(\theta, \sigma) \leq \mathcal{M}^i \int_{\xi_k} s_{k,i} \mathbb{P}\left(\tilde{\varepsilon}_k \leq \frac{\theta_{\text{cand,up}}^i(\sigma) - \theta}{\sigma} + \mathcal{C}_k^\varepsilon + \mathcal{C}_k^{\text{adj}}\sigma\right) dF_{\tilde{\varepsilon}}(\xi_k). \quad (51)$$

Thus, for all sufficiently small σ there exists $\mathcal{C}_{\text{cand}}^{\text{tem}} > 0$ such that $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \leq O_B^i + \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma$. Thus if $\forall \sigma$ sufficiently small, $H^i \leq \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma)$, then $H^i \leq O_B^i$.

Claim 7. *Therefore there exists $\mathcal{C}_{\text{up}}^{\text{adj}} > 0$ such that $\widehat{\theta}^i(H^i) + \mathcal{C}_{\text{up}}^{\text{adj}}\sigma \leq \theta_{\text{cand,up}}^i(\sigma) \leq \widehat{\theta}^i(H^i)$.*

Proof. If $H^i \leq \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma)$, then from Eq. (47), Assumption 4 and continuity,

$$\begin{aligned} 0 &= \theta_{\text{cand,up}}^i(\sigma)G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) - H^i)) - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma)) \\ &\geq \theta_{\text{cand,up}}^i(\sigma)G^i(A^i) - C^i(A^i, (O_B^i + \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma - H^i)) - R^i(L^i - O_B^i + \mathcal{C}_{\text{cand}}^{\text{tem}}\sigma) \\ &\geq \theta_{\text{cand,up}}^i(\sigma)G^i(A^i) - C^i(A^i, (O_B^i - H^i)) - R^i(L^i - O_B^i) - \mathcal{O}(\sigma) \end{aligned}$$

Comparing this inequality with Eq. (46), we conclude that $\theta_{\text{cand,up}}^i(\sigma) < \widehat{\theta}^i(H^i)$.

If $H^i > \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma)$, then from Eq. (47),

$$0 = \theta_{\text{cand,up}}^i(\sigma)G^i(A^i) + H^i - R^iL^i + (R^i - 1)\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \geq \theta_{\text{cand,up}}^i(\sigma)G^i(A^i) + H^i - R^iL^i$$

Comparing this inequality with Eq. (46), we conclude that $\theta_{\text{cand,up}}^i(\sigma) \leq \widehat{\theta}^i(H^i)$, and equality only holds iff $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) = 0$.

For the other bound, we proceed with contradiction. Suppose instead that $\limsup_{\sigma \downarrow 0} (\widehat{\theta}^i(H^i) - \theta_{\text{cand,up}}^i(\sigma))/\sigma = \infty$. Then, by Eq. (49), for any small $\omega > 0$ there exists σ such that $\mathcal{O}_{\text{cand}}^i(\widehat{\theta}^i(H^i), \sigma) < \omega$. By continuity, we can pick $\vartheta \in (\theta_{\text{cand,up}}^i(\sigma), \widehat{\theta}^i(H^i)]$ such that

$$\vartheta G^i(A^i) - C^i(A^i, (\mathcal{O}_{\text{cand}}^i(\vartheta, \sigma) - H^i)^+) + (H^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma))^+ - R^i(L^i - \mathcal{O}_{\text{cand}}^i(\vartheta, \sigma)) \leq \widehat{\theta}^i(H^i)G^i(A^i) + H^i - R^iL^i = 0,$$

contradicting the definition of $\theta_{\text{cand,up}}^i(\sigma)$. \square

Claim 8. *There exists $\mathcal{C}_{\text{down}}^{\text{adj}} > 0$ such that*

$$\theta_{\text{cand,down}}^i(\sigma) \geq \theta_{\text{cand,up}}^i(\sigma) - \mathcal{C}_{\text{down}}^{\text{adj}}\sigma \quad \text{for all sufficiently small } \sigma.$$

Proof. We already have $\theta_{\text{cand,down}}^i(\sigma) \leq \theta_{\text{cand,up}}^i(\sigma) \leq \widehat{\theta}^i(H^i) + \mathcal{C}_{\text{up}}^{\text{adj}}\sigma$. If $\theta_{\text{cand,down}}^i(\sigma) \geq \widehat{\theta}^i(H^i)$, the claim is immediate. Otherwise, for $\theta_{\text{cand,down}}^i(\sigma) < \widehat{\theta}^i(H^i)$,

$$\theta_{\text{cand,down}}^i(\sigma)G^i(A^i) - C^i(A^i, L^i - H^i) < \widehat{\theta}^i(H^i)G^i(A^i) - R^i(L^i - H^i) = 0.$$

Now assume by contradiction that $\limsup_{\sigma \downarrow 0} (\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma))/\sigma = \infty$. Then for arbitrarily small $\omega > 0$ and some small σ , $\mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) > L^i - \omega$. By continuity and monotonicity of the solvency expression in the outflow argument,

$$\theta_{\text{cand,down}}^i(\sigma)G^i(A^i) - C^i(A^i, L^i - \omega - H^i) - R^i\omega < \widehat{\theta}^i(H^i)G^i(A^i) - R^i(L^i - H^i) = 0,$$

contradicting the defining equality at $\theta_{\text{cand,down}}^i(\sigma)$. \square

Combining the two claims,

$$\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^i(\sigma) = \lim_{\sigma \downarrow 0} \theta_{\text{cand,up}}^i(\sigma) = \widehat{\theta}^i(H^i).$$

Thus the limiting equilibrium is as stated in Lemma 3: the run cutoff converges to the fundamental boundary $\widehat{\theta}^i(H^i)$. For sufficiency, we again employ iterated elimination of dominated strategies (as in Case 1) and show that the sequence of outflow bounds converges to an outflow that, together with the cutoff $\widehat{\theta}^i(H^i)$, satisfies best responses; hence it is an equilibrium, and the sequence of equilibria converges to this limit.

1.5 Proof of Theorem 1

By Lemma 3, bank b 's overall default boundary satisfies

$$\theta_B^b(H) \geq \widehat{\theta}^b(H) \quad \text{for } H < \widehat{H}^b, \quad \theta_B^b(H) = \widehat{\theta}^b(H) \quad \text{for } H \geq \widehat{H}^b.$$

Because $\theta_B^b(H)$ is decreasing in H , the regulator must choose $H^* \geq \widehat{H}^b$. If $\widehat{\theta}^b(H)$ also increases on $(\widehat{H}^b, L^b]$, the optimum is $H^* = \widehat{H}^b$. Fix $H \in (\widehat{H}^b, L^b]$. Since

$$\widehat{\theta}^b(H) = \frac{R^b L^b - H}{G^b(L^b - H)},$$

we obtain

$$\frac{d\widehat{\theta}^b(H)}{dH} > 0 \iff R^b L^b > \frac{G^b(L^b - H)}{G^{b'}(L^b - H)} + H.$$

Because $R^b \geq G'(O_B^b - \widehat{H}^b)/G'(L^b - \widehat{H}^b)$ and $G^{b'}(L^b - \widehat{H}^b) \geq 1$,

$$R^b(L^b - O_B^b) \geq G^b(L^b - \widehat{H}^b) - G^b(O_B^b - \widehat{H}^b).$$

Using $C^b(A^b, O_B^b - \widehat{H}^b) + \widehat{H}^b = R^b O_B^b$, and $C^b(A^b, O_B^b - \widehat{H}^b) > G^b(O_B^b - \widehat{H}^b)$, we show that $R^b L^b \geq \frac{G^b(L^b - \widehat{H}^b)}{G^{b'}(L^b - \widehat{H}^b)} + \widehat{H}^b$. It is trivial to see that $x \rightarrow \frac{G^b(L^b - x)}{G^{b'}(L^b - x)} + x$ is decreasing in x . Therefore $\widehat{\theta}^b(H)$ increases on $(\widehat{H}^b, L^b]$, and the optimal liquidity requirement is $H^* = \widehat{H}^b$. \square

1.6 Proof of Lemma 4

The conclusion is a direct application of the Banach contraction mapping theorem.

1.7 Proof of Theorem 2

The main complication from deposit mobility across multiple banks arises because each agent must conjecture the strategies of depositors not only at her own bank but also at the other bank. Moreover, the fraction of depositors who move between banks affects the best-response functions of both agents.

Consider region i , where banks i and b both operate. Let agent k be a representative depositor at bank i after time $t = 1$, and let agent w be a representative depositor who resides in region i but deposits at bank b after time $t = 1$. Denote by \mathcal{O}^j and \mathcal{I}^j the aggregate deposit outflow from, and inflow to, bank j , respectively. When the true state is θ and the depositor strategy profile is $\{r_{k,i \rightarrow j}, r_{w,b \rightarrow j}\}$, the Exact Law of Large Numbers

yields

$$\begin{aligned}\mathbb{O}^i(\theta) &= \mathcal{M}^i \int_{\xi_k} \int_{x_k} [r_{k,i \rightarrow b}(x_k) + r_{k,i \rightarrow H}(x_k)] f_{\bar{x}_k|\bar{\theta}}(x_k | \theta) dx_k f_{\bar{\xi}}(\xi_k) d\xi_k, \\ \mathbb{O}^b(\theta) &= \mathcal{M}^i \int_{\xi_w} \int_{x_w} [r_{w,b \rightarrow i}(x_w) + r_{w,b \rightarrow H}(x_w)] f_{\bar{x}_w|\bar{\theta}}(x_w | \theta) dx_w f_{\bar{\xi}}(\xi_w) d\xi_w, \\ \mathbb{I}^i(\theta) &= \mathcal{M}^i \int_{\xi_w} \int_{x_w} [r_{w,b \rightarrow i}(x_w) + r_{w,H \rightarrow i}(x_w)] f_{\bar{x}_w|\bar{\theta}}(x_w | \theta) dx_w f_{\bar{\xi}}(\xi_w) d\xi_w, \\ \mathbb{I}^b(\theta) &= \sum_i \mathcal{M}^i \int_{\xi_k} \int_{x_k} [r_{k,i \rightarrow b}(x_k) + r_{k,H \rightarrow b}(x_k)] f_{\bar{x}_k|\bar{\theta}}(x_k | \theta) dx_k f_{\bar{\xi}}(\xi_k) d\xi_k.\end{aligned}$$

Recall that private signal precision is governed by σ , so the densities $f_{\bar{x}_k|\bar{\theta}}$ and $f_{\bar{x}_w|\bar{\theta}}$ depend on σ . Hence any candidate equilibrium inflows and outflows can be written as $\mathbb{O}^j(\theta, \sigma)$ and $\mathbb{I}^j(\theta, \sigma)$. Under these deposit flow functions, depositor k 's expected payoff from *staying* with bank i after observing x_k at $t = 2$ is

$$\mathcal{U}_{k,i \rightarrow i}^2(x_k) \equiv \mathcal{U}_k^2(a_{k,i \rightarrow i} = 1 | x_k) = \mathbb{P}(\mathcal{D}_i^c | x_k) (R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i | x_k) \Lambda.$$

If instead k moves to bank b , her expected payoff is (recall that depositors ignore the sequential service constraint)

$$\begin{aligned}\mathcal{U}_{k,i \rightarrow b}^2(x_k) &\equiv \mathcal{U}_k^2(a_{k,i \rightarrow b} = 1 | x_k) \\ &= \int_{\mathcal{D}_i} ((1 + \chi^b \xi_k^b) \mathbb{1}_{\mathcal{D}_b^c} + \Lambda \mathbb{1}_{\mathcal{D}_b}) f_{\bar{\theta}|\bar{x}_k}(\theta | x_k) d\theta + \int_{\mathcal{D}_i^c} ((1 + \chi^b \xi_k^b) \mathbb{1}_{\mathcal{D}_b^c} + \Lambda \mathbb{1}_{\mathcal{D}_b}) f_{\bar{\theta}|\bar{x}_k}(\theta | x_k) d\theta.\end{aligned}$$

If k leaves the banking system to hold the risk-free asset, her expected payoff is

$$\mathcal{U}_{k,i \rightarrow H}^2(x_k) \equiv \mathcal{U}_k^2(a_{k,i \rightarrow H} = 1 | x_k) = 1.$$

Similarly, for depositor w at bank b , the expected payoff from *staying* is

$$\mathcal{U}_{w,b \rightarrow b}^2(x_w) \equiv \mathcal{U}_w^2(a_{w,b \rightarrow b} = 1 | x_w) = \mathbb{P}(\mathcal{D}_b^c | x_w) (R^b + \xi_w^b) + \mathbb{P}(\mathcal{D}_b | x_w) \Lambda,$$

while the payoff from *moving* to bank i is

$$\begin{aligned}\mathcal{U}_{w,b \rightarrow i}^2(x_w) &\equiv \mathcal{U}_w^2(a_{w,b \rightarrow i} = 1 | x_w) \\ &= \int_{\mathcal{D}_b} ((1 + \chi^i \xi_w^i) \mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i}) f_{\bar{\theta}|\bar{x}_w}(\theta | x_w) d\theta + \int_{\mathcal{D}_b^c} ((1 + \chi^i \xi_w^i) \mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i}) f_{\bar{\theta}|\bar{x}_w}(\theta | x_w) d\theta,\end{aligned}$$

and the payoff from *leaving* the banking system to hold the risk-free asset is

$$\mathcal{U}_{w,b \rightarrow H}^2(x_w) \equiv \mathcal{U}_w^2(a_{w,b \rightarrow H} = 1 | x_w) = 1.$$

1.7.1 Necessary conditions for any candidate equilibrium

Fix any sufficiently small $\sigma > 0$. Let $\mathcal{O}_{\text{cand}}^j(\theta, \sigma)$ and $\mathcal{I}_{\text{cand}}^j(\theta, \sigma)$ denote the candidate equilibrium deposit outflow and inflow, respectively, for bank $j \in \{i, b\}$ in state θ . As in standard global-games models, these functions are common knowledge, whereas each agent observes only a private signal \tilde{x} about the state $\tilde{\theta}$. Given these beliefs, an agent regards bank b as defaulting iff $\tilde{\theta} \in \mathcal{D}_b(\text{cand}, \sigma)$, where

$$\begin{aligned} \mathcal{D}_b(\text{cand}, \sigma) = \left\{ \theta : \theta G^b(A^b) - C^b \left(A^b, \left(\mathcal{O}_{\text{cand}}^b(\theta, \sigma) - \mathcal{I}_{\text{cand}}^b(\theta, \sigma) - H^b \right)^+ \right) \right. \\ \left. + \left(H^b + \mathcal{I}_{\text{cand}}^b(\theta, \sigma) - \mathcal{O}_{\text{cand}}^b(\theta, \sigma) \right)^+ - \mathcal{I}_{\text{cand}}^b(\theta, \sigma) - R^b \left(L^b - \mathcal{O}_{\text{cand}}^b(\theta, \sigma) \right) \leq 0 \right\}. \end{aligned}$$

Similarly, since $H^i = 0$, bank i is deemed insolvent iff $\tilde{\theta} \in \mathcal{D}_i(\text{cand}, \sigma)$, where

$$\begin{aligned} \mathcal{D}_i(\text{cand}, \sigma) = \left\{ \theta : \theta G^i(A^i) - C^i \left(A^i, \left(\mathcal{O}_{\text{cand}}^i(\theta, \sigma) - \mathcal{I}_{\text{cand}}^i(\theta, \sigma) \right)^+ \right) \right. \\ \left. + \left(\mathcal{I}_{\text{cand}}^i(\theta, \sigma) - \mathcal{O}_{\text{cand}}^i(\theta, \sigma) \right)^+ - \mathcal{I}_{\text{cand}}^i(\theta, \sigma) - R^i \left(L^i - \mathcal{O}_{\text{cand}}^i(\theta, \sigma) \right) \leq 0 \right\}. \end{aligned}$$

For $j \in \{i, b\}$ define the solvency thresholds

$$\begin{aligned} \theta_{\text{cand,up}}^j(\sigma) &= \inf \left\{ \rho : \left[\vartheta G^j(A^j) - C^j \left(A^j, \left(\mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) - H^j - \mathcal{I}_{\text{cand}}^j(\vartheta, \sigma) \right)^+ \right) \right. \right. \\ &\quad \left. \left. + \left(H^j + \mathcal{I}_{\text{cand}}^j(\vartheta, \sigma) - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) \right)^+ - R^j \left(L^j - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) \right) \right] > 0 \quad \forall \vartheta \geq \rho \right\}, \\ \theta_{\text{cand,down}}^j(\sigma) &= \sup \left\{ \rho : \left[\vartheta G^j(A^j) - C^j \left(A^j, \left(\mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) - H^j - \mathcal{I}_{\text{cand}}^j(\vartheta, \sigma) \right)^+ \right) \right. \right. \\ &\quad \left. \left. + \left(H^j + \mathcal{I}_{\text{cand}}^j(\vartheta, \sigma) - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) \right)^+ - R^j \left(L^j - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) \right) \right] < 0 \quad \forall \vartheta \leq \rho \right\}. \end{aligned}$$

Therefore,

$$(-\infty, \theta_{\text{cand,down}}^j(\sigma)] \subseteq \mathcal{D}_j(\text{cand}, \sigma) \subseteq (-\infty, \theta_{\text{cand,up}}^j(\sigma)].$$

Claim 9. Suppose the candidate solvency thresholds satisfy

$$\theta_{\text{cand,up}}^b(\sigma) < \theta_{\text{cand,down}}^i(\sigma).$$

Then $\mathcal{I}_{\text{cand}}^i(\vartheta, \sigma) = 0$ for every ϑ . Conversely, if

$$\theta_{\text{cand,down}}^b(\sigma) > \theta_{\text{cand,up}}^i(\sigma),$$

then $\mathcal{I}_{\text{cand}}^b(\vartheta, \sigma) = 0$ for every ϑ .

Proof. The first inequality implies $\mathcal{D}_b(\text{cand}, \sigma) \subsetneq \mathcal{D}_i(\text{cand}, \sigma)$ and $\mathcal{D}_i^c(\text{cand}, \sigma) \subsetneq \mathcal{D}_b^c(\text{cand}, \sigma)$. For any signal realization x_w , the assumption that

$R^b + \xi_w^b > 1 + \chi^i \xi_w^i$ yields

$$\begin{aligned} \mathcal{U}_{w,b \rightarrow i}^2(x_w) &= \int_{\mathcal{D}_b} \Lambda f_{\theta|\tilde{x}_w}(\theta | x_w) d\theta + \int_{\mathcal{D}_b^c} \left((1 + \chi^i \xi_w^i) \mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i} \right) f_{\theta|\tilde{x}_w}(\theta | x_w) d\theta \\ &< \mathbb{P}(\mathcal{D}_b | x_w) \Lambda + \mathbb{P}(\mathcal{D}_b^c | x_w) (R^b + \xi_w^b) = \mathcal{U}_{w,b \rightarrow b}^2(x_w). \end{aligned}$$

Therefore, no depositor at bank b moves to bank i . Because no depositor holds the risk-free asset at $t = 1$, it follows that $I_{\text{cand}}^i(\vartheta, \sigma) = 0$. The converse claim is proved by symmetry. \square

Let $\mathcal{R}_{\text{cand}}^j(k, \sigma)$ be the set of signals for which depositor k withdraws from bank j . Then (for regional bank i , let $s_{k,j} = 0$ if $j \neq i$)

$$\mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} s_{k,j} \mathbb{P}(\vartheta + \sigma \tilde{\epsilon}_k \in \mathcal{R}_{\text{cand}}^j(k, \sigma)) dF_{\xi}(\xi_k),$$

so $\mathcal{O}_{\text{cand}}^j$ (and similarly for I_{cand}^j) is continuous in ϑ . Therefore, for $\vartheta = \theta_{\text{cand,up}}^j(\sigma)$ or $\vartheta = \theta_{\text{cand,down}}^j(\sigma)$,

$$\vartheta G^j(A^j) - C^j(A^j, (\mathcal{O}_{\text{cand}}^j(\vartheta, \sigma) - H^j - I_{\text{cand}}^j(\vartheta, \sigma))^+) + (H^j + I_{\text{cand}}^j(\vartheta, \sigma) - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma))^+ - R^j(L^j - \mathcal{O}_{\text{cand}}^j(\vartheta, \sigma)) = 0.$$

Definition 7. For each depositor k , define:

$$\mathbb{C}_k^\epsilon \equiv \inf \left\{ x : F_{\tilde{\epsilon}_k}(x) \geq \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}, \quad \bar{\mathbb{C}}_k^\epsilon \equiv \inf \left\{ x : F_{\tilde{\epsilon}_k}(x) \geq \frac{1 + \chi^b \xi_k^b - \Lambda}{R^i - \Lambda + \xi_k^i} \right\}.$$

Analogously, for each depositor w ,

$$\mathbb{C}_w^\epsilon \equiv \inf \left\{ x : F_{\tilde{\epsilon}_w}(x) \geq \frac{1 - \Lambda}{R^b - \Lambda + \xi_w^b} \right\}, \quad \bar{\mathbb{C}}_w^\epsilon \equiv \inf \left\{ x : F_{\tilde{\epsilon}_w}(x) \geq \frac{1 + \chi^i \xi_w^i - \Lambda}{R^b - \Lambda + \xi_w^b} \right\}.$$

In any candidate equilibrium, the strategy of the representative depositor k originally with bank i and depositor w originally with bank b is characterized as follows.

Claim 10. There exists a constant $\mathbb{C}^{\text{adj}} > 0$ such that, for all sufficiently small σ ,

1. If depositor k observes $\tilde{x}_k \geq \theta_{\text{cand,up}}^i(\sigma) + \bar{\mathbb{C}}_k^\epsilon \sigma + \mathbb{C}^{\text{adj}} \sigma^2$, depositor k does not run on bank i .
2. If depositor k observes $\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \mathbb{C}_k^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2$, she moves from bank i to bank b or to the safe asset.
3. If depositor w observes $\tilde{x}_w \geq \theta_{\text{cand,up}}^b(\sigma) + \bar{\mathbb{C}}_w^\epsilon \sigma + \mathbb{C}^{\text{adj}} \sigma^2$, depositor w does not run on bank b .
4. If she observes $\tilde{x}_w \leq \theta_{\text{cand,down}}^b(\sigma) + \mathbb{C}_w^\epsilon \sigma - \mathbb{C}^{\text{adj}} \sigma^2$, depositor w moves from bank b to bank i or to the safe asset.

Proof. Let σ be small, and write x_k for the realization of \tilde{x}_k .

(i) Suppose $x_k = \theta_{\text{cand,up}}^i(\sigma) + \bar{\mathbf{C}}_k^\epsilon \sigma + \mathbf{C}^{\text{adj}} \sigma^2 + \mathbf{C}^{\text{tem}} \sigma$ for some $\mathbf{C}^{\text{tem}} \geq 0$. Then

$$\mathbb{P}(\mathcal{D}_i^c | x_k) \geq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,up}}^i(\sigma) | x_k) = \frac{\int_{-\infty}^{(x_k - \theta_{\text{cand,up}}^i(\sigma))/\sigma} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}{\int_{-\infty}^{\infty} f_{\bar{\theta}}(x_k - \sigma z) f_{\bar{\epsilon}_k}(z) dz}.$$

By continuity of $f_{\bar{\theta}}$, for σ small this equals

$$\frac{1 + \chi^b \xi_k^b - \Lambda}{R^i - \Lambda + \xi_k^i} + F_{\bar{\theta}}(\bar{\mathbf{C}}_k^\epsilon + \mathbf{C}^{\text{adj}} \sigma + \mathbf{C}^{\text{tem}}) - F_{\bar{\theta}}(\bar{\mathbf{C}}_k^\epsilon) - \mathcal{O}(\sigma).$$

Choosing \mathbf{C}^{adj} large enough yields

$$\mathbb{P}(\mathcal{D}_i^c | x_k) (R^i - \Lambda + \xi_k^i) > 1 + \chi^b \xi_k^b - \Lambda,$$

i.e., $\mathcal{U}_{k,i \rightarrow i}^2(x_k) \geq 1 + \chi^b \xi_k^b$. Because the highest payoff from leaving bank i is bounded above by $1 + \chi^b \xi_k^b$, depositor k stays with bank i .

(ii) Suppose

$$x_k = \theta_{\text{cand,down}}^i(\sigma) + \mathbf{C}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2 - \mathbf{C}^{\text{tem}} \sigma, \quad \mathbf{C}^{\text{tem}} \geq 0.$$

Then, for σ small,

$$\mathbb{P}(\mathcal{D}_i^c | x_k) \leq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,down}}^i(\sigma) | x_k) = \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} + F_{\bar{\theta}}(\mathbf{C}_k^\epsilon - \mathbf{C}^{\text{adj}} \sigma - \mathbf{C}^{\text{tem}}) - F_{\bar{\theta}}(\mathbf{C}_k^\epsilon) + \mathcal{O}(\sigma),$$

so for \mathbf{C}^{adj} large enough we have $\mathcal{U}_{k,i \rightarrow i}^2(x_k) < 1$. Because the lowest payoff from leaving bank i is bounded below by 1, depositor k runs from bank i .

(iii)–(iv) Identical by symmetry after interchanging i and b , and k and w . \square

Definition 8. For each depositor k originally with bank i and depositor w originally with bank b , define

$$\underline{\mathbf{C}}_k^\epsilon \equiv \inf \left\{ x : F_{\bar{\epsilon}_k}(x) \geq \frac{1 - \Lambda}{1 - \Lambda + \chi^b \xi_k^b} \right\}, \quad \underline{\mathbf{C}}_w^\epsilon \equiv \inf \left\{ x : F_{\bar{\epsilon}_w}(x) \geq \frac{1 - \Lambda}{1 - \Lambda + \chi^i \xi_w^i} \right\}.$$

Claim 11. Consider a sequence of $\{\sigma\}$ such that $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^i(\sigma) - \theta_{\text{cand,up}}^b(\sigma) > 0$. There exists a constant $\mathbf{C}^{\text{adj}} > 0$ such that, for all sufficiently small σ from that sequence,

1. If $\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \bar{\mathbf{C}}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2$ and $\tilde{x}_k \geq \theta_{\text{cand,up}}^b(\sigma) + \underline{\mathbf{C}}_k^\epsilon \sigma + \mathbf{C}^{\text{adj}} \sigma^2$, then depositor k (originally at bank i) moves from bank i to bank b .

2. If $\tilde{x}_k \leq \theta_{\text{cand,down}}^b(\sigma) + \underline{C}_k^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2$, then depositor k (originally at bank i) moves from bank i to the risk-free asset H .
3. If depositor w (originally at bank b) observes $\tilde{x}_w \geq \theta_{\text{cand,up}}^b(\sigma) + \underline{C}_w^\epsilon \sigma + \mathcal{C}^{\text{adj}} \sigma^2$, she does not run on bank b .
4. If depositor w (originally at bank b) observes $\tilde{x}_w \leq \theta_{\text{cand,down}}^b(\sigma) + \underline{C}_w^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2$, she moves from bank b to the risk-free asset H .

Conversely, if $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^b(\sigma) - \theta_{\text{cand,up}}^i(\sigma) > 0$, then there exists a constant $\mathcal{C}^{\text{adj}} > 0$ such that, for all sufficiently small σ ,

1. If $\tilde{x}_w \leq \theta_{\text{cand,down}}^b(\sigma) + \bar{C}_w^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2$ and $\tilde{x}_w \geq \theta_{\text{cand,up}}^i(\sigma) + \underline{C}_w^\epsilon \sigma + \mathcal{C}^{\text{adj}} \sigma^2$, then depositor w (originally at bank b) moves from bank b to bank i .
2. If $\tilde{x}_w \leq \theta_{\text{cand,down}}^i(\sigma) + \underline{C}_w^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2$, then depositor w moves from bank b to the risk-free asset H .
3. If depositor k (originally at bank i) observes $\tilde{x}_k \geq \theta_{\text{cand,up}}^i(\sigma) + \underline{C}_k^\epsilon \sigma + \mathcal{C}^{\text{adj}} \sigma^2$, she does not run on bank i .
4. If depositor k observes $\tilde{x}_k \leq \theta_{\text{cand,down}}^i(\sigma) + \underline{C}_k^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2$, she moves from bank i to the risk-free asset H .

Proof. Suppose $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^i(\sigma) - \theta_{\text{cand,up}}^b(\sigma) > 0$. Then for sufficiently small σ , by [Claim 9](#), $\mathcal{D}_i^c(\text{cand}, \sigma) \subsetneq \mathcal{D}_b^c(\text{cand}, \sigma)$. For any signal realization x_w , the assumption that $R^b + \xi_w^b > 1 + \chi^i \xi_w^i$ yields $\mathcal{U}_{w,b \rightarrow i}^2(x_w) < \mathcal{U}_{w,b \rightarrow b}^2(x_w)$ for all x_w .

(i) Let

$$x_k \geq \theta_{\text{cand,up}}^b(\sigma) + \underline{C}_k^\epsilon \sigma + \mathcal{C}^{\text{adj}} \sigma^2.$$

Then, by Bayes' rule and continuity of $f_{\bar{\theta}}$, for small σ ,

$$\mathbb{P}(\mathcal{D}_b^c | x_k) \geq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,up}}^b(\sigma) | x_k) = F_{\bar{\epsilon}_k} \left(\frac{x_k - \theta_{\text{cand,up}}^b(\sigma)}{\sigma} \right) + \mathcal{O}(\sigma) \geq F_{\bar{\epsilon}_k}(\underline{C}_k^\epsilon + \mathcal{C}^{\text{adj}} \sigma) + \mathcal{O}(\sigma).$$

By the definition of \underline{C}_k^ϵ , $F_{\bar{\epsilon}_k}(\underline{C}_k^\epsilon) = \frac{1-\Lambda}{1-\Lambda+\chi^b \xi_k^b}$. Hence, for σ small and \mathcal{C}^{adj} large enough,

$$\mathcal{U}_{k,i \rightarrow b}^2(x_k) = \mathbb{P}(\mathcal{D}_b^c | x_k) (1 + \chi^b \xi_k^b) + \mathbb{P}(\mathcal{D}_b | x_k) \Lambda > 1 = \mathcal{U}_{k,i \rightarrow H}^2(x_k).$$

If additionally

$$x_k \leq \theta_{\text{cand,down}}^i(\sigma) + \underline{C}_k^\epsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2,$$

then

$$\mathbb{P}(\mathcal{D}_i^c | x_k) \leq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,down}}^i(\sigma) | x_k) = F_{\bar{\epsilon}_k} \left(\frac{x_k - \theta_{\text{cand,down}}^i(\sigma)}{\sigma} \right) + \mathcal{O}(\sigma) \leq F_{\bar{\epsilon}_k}(\underline{C}_k^\epsilon - \mathcal{C}^{\text{adj}} \sigma) + \mathcal{O}(\sigma).$$

Since $F_{\bar{\epsilon}_k}(\mathbf{C}_k^\epsilon) = \frac{1}{R^i - \Lambda + \xi_k^i}$, for small σ , $\mathcal{U}_{k,i \rightarrow i}^2(x_k) = \mathbb{P}(\mathcal{D}_i^c | x_k) (R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i | x_k) \Lambda < 1 < \mathcal{U}_{k,i \rightarrow b}^2(x_k)$.

Alternatively, suppose that

$$x_k \in [\theta_{\text{cand,down}}^i(\sigma) + \mathbf{C}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2, \theta_{\text{cand,down}}^i(\sigma) + \bar{\mathbf{C}}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2],$$

then

$$\mathbb{P}(\mathcal{D}_i^c | x_k) \leq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,down}}^i(\sigma) | x_k) = F_{\bar{\epsilon}_k} \left(\frac{x_k - \theta_{\text{cand,down}}^i(\sigma)}{\sigma} \right) + \mathcal{O}(\sigma) \leq F_{\bar{\epsilon}_k}(\bar{\mathbf{C}}_k^\epsilon - \mathbf{C}^{\text{adj}} \sigma) + \mathcal{O}(\sigma).$$

By the definition of $\bar{\mathbf{C}}_k^\epsilon$, $F_{\bar{\epsilon}_k}(\bar{\mathbf{C}}_k^\epsilon) = \frac{1 + \chi^b \xi_k^b - \Lambda}{R^i - \Lambda + \xi_k^i}$. Therefore, for small σ , $\mathcal{U}_{k,i \rightarrow i}^2(x_k) = \mathbb{P}(\mathcal{D}_i^c | x_k) (R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i | x_k) \Lambda < 1 + \chi^b \xi_k^b$. Also,

$$\mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,up}}^b(\sigma) | x_k) \geq F_{\bar{\epsilon}_k} \left(\frac{\theta_{\text{cand,down}}^i(\sigma) + \mathbf{C}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2 - \theta_{\text{cand,up}}^b(\sigma)}{\sigma} \right) + \mathcal{O}(\sigma).$$

As σ goes to 0, $\mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,up}}^b(\sigma) | x_k)$ goes to 1. Therefore, $\mathcal{U}_{k,i \rightarrow i}^2(x_k) < \mathcal{U}_{k,i \rightarrow b}^2(x_k)$, so k moves from i to b .

(ii) If $x_k \leq \theta_{\text{cand,down}}^b(\sigma) + \underline{\mathbf{C}}_k^\epsilon \sigma - \mathbf{C}^{\text{adj}} \sigma^2$, then

$$\mathbb{P}(\mathcal{D}_b^c | x_k) \leq \mathbb{P}(\bar{\theta} \geq \theta_{\text{cand,down}}^b(\sigma) | x_k) \leq F_{\bar{\epsilon}_k}(\underline{\mathbf{C}}_k^\epsilon - \mathbf{C}^{\text{adj}} \sigma) + \mathcal{O}(\sigma).$$

By the definition of $\underline{\mathbf{C}}_k^\epsilon$ this yields $\mathcal{U}_{k,i \rightarrow i}^2(x_k) < \mathcal{U}_{k,i \rightarrow b}^2(x_k) < 1 = \mathcal{U}_{k,i \rightarrow H}^2(x_k)$, so k prefers H .

(iii)–(iv) With [Claim 9](#), w never moves to the riskier bank i under the assumption. Her decision reduces to the single-bank case, and the thresholds follow from [Claim 10](#) (replace i by b and use \mathbf{C}_w^ϵ).

The converse statements follow by symmetry when $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^b(\sigma) - \theta_{\text{cand,up}}^i(\sigma) > 0$. □

[Claim 10](#) and [Claim 11](#) have important implications for the equilibrium deposit outflows and inflows near the run cutoffs $\theta_{\text{cand,up}}^b(\sigma)$, $\theta_{\text{cand,down}}^b(\sigma)$ and $\theta_{\text{cand,up}}^i(\sigma)$, $\theta_{\text{cand,down}}^i(\sigma)$.

Claim 12. *Let*

$$\begin{aligned} \bar{O}_D^i &= \mathcal{M}^i \int_{\xi_k}^{s_{k,i}} \frac{1 + \chi^b \xi_k^b - \Lambda}{R^i - \Lambda + \xi_k^i} dF_{\bar{\xi}}(\xi_k), & O_B^b &= \mathcal{M}^i \int_{\xi_w}^{s_{w,b}} \frac{1 - \Lambda}{R^b - \Lambda + \xi_w^b} dF_{\bar{\xi}}(\xi_w), & \bar{I}_D^b &= \mathcal{M}^i \int_{\xi_k} \frac{O^i(\theta_D^b(H^b))}{L^i} \frac{s_{k,i} \chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda} dF_{\bar{\xi}}(\xi_k), \\ \bar{O}_D^b &= \mathcal{M}^i \int_{\xi_w}^{s_{w,b}} \frac{1 - \Lambda + \chi^i \xi_w^i}{R^b + \xi_w^b - \Lambda} dF_{\bar{\xi}}(\xi_w), & O_B^i &= \mathcal{M}^i \int_{\xi_k}^{s_{k,i}} \frac{1 - \Lambda}{R^i - \Lambda + \xi_k^i} dF_{\bar{\xi}}(\xi_k), & \bar{I}_D^i &= \mathcal{M}^i \int_{\xi_w} \frac{O^b(\theta_D^i)}{L^b} \frac{s_{w,b} \chi^i \xi_w^i}{1 + \chi^i \xi_w^i - \Lambda} dF_{\bar{\xi}}(\xi_w), \end{aligned}$$

be the constants defined in [Lemmas 3](#) and [4](#). Then any candidate equilibrium outflows satisfy the following:

1. If $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^i(\sigma) - \theta_{\text{cand,up}}^b(\sigma) > 0$, then $I_{\text{cand}}^i = 0$ is a constant function, and

$$\begin{aligned} \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) &\leq \bar{O}_D^i \leq \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma), \\ \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma), \sigma) &\leq O_B^b \leq \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma), \sigma), \\ \lim_{\sigma \downarrow 0} I_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma), \sigma) &\leq \bar{I}_D^b \leq \lim_{\sigma \downarrow 0} I_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma), \sigma). \end{aligned}$$

2. If $\lim_{\sigma \downarrow 0} \theta_{\text{cand,down}}^b(\sigma) - \theta_{\text{cand,up}}^i(\sigma) > 0$, then $I_{\text{cand}}^b = 0$ is a constant function, and

$$\begin{aligned} \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma), \sigma) &\leq \bar{O}_D^b \leq \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma), \sigma), \\ \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) &\leq O_B^i \leq \lim_{\sigma \downarrow 0} \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma), \\ \lim_{\sigma \downarrow 0} I_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) &\leq \bar{I}_D^i \leq \lim_{\sigma \downarrow 0} I_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma). \end{aligned}$$

Proof. Let $\mathcal{R}_{\text{cand}}^i(k, \sigma)$ denote the run set for depositor k at bank i . Then, by [Claim 10](#) and [Claim 11](#),

$$(-\infty, \theta_{\text{cand,down}}^i(\sigma) + \bar{C}_k^\varepsilon \sigma - \mathcal{C}^{\text{adj}} \sigma^2] \subset \mathcal{R}_{\text{cand}}^i(k, \sigma) \subset (-\infty, \theta_{\text{cand,up}}^i(\sigma) + \bar{C}_k^\varepsilon \sigma + \mathcal{C}^{\text{adj}} \sigma^2],$$

which yields

$$\begin{aligned} \mathcal{M}^i \int_{\xi_k} \mathbb{1}_{\mathcal{S}_{k,i}} \mathbb{P}(\tilde{\varepsilon}_k \leq \bar{C}_k^\varepsilon - \mathcal{C}^{\text{adj}} \sigma) dF_{\tilde{\xi}}(\xi_k) &\leq \mathcal{M}^i \int_{\xi_k} \mathbb{1}_{\mathcal{S}_{k,i}} \mathbb{P}(\theta_{\text{cand,down}}^i(\sigma) + \sigma \tilde{\varepsilon}_k \in \mathcal{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\xi}}(\xi_k) \\ &= \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma), \\ \mathcal{M}^i \int_{\xi_k} \mathbb{1}_{\mathcal{S}_{k,i}} \mathbb{P}(\tilde{\varepsilon}_k \leq \bar{C}_k^\varepsilon + \mathcal{C}^{\text{adj}} \sigma) dF_{\tilde{\xi}}(\xi_k) &\geq \mathcal{M}^i \int_{\xi_k} \mathbb{1}_{\mathcal{S}_{k,i}} \mathbb{P}(\theta_{\text{cand,up}}^i(\sigma) + \sigma \tilde{\varepsilon}_k \in \mathcal{R}_{\text{cand}}^i(k, \sigma)) dF_{\tilde{\xi}}(\xi_k) \\ &= \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma). \end{aligned}$$

The left-hand sides converge (as $\sigma \downarrow 0$) to the displayed integrals, respectively, which gives the two bounds. Other inequalities can be proved similarly. \square

Suppose that $\lim_{\sigma \downarrow 0} \theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma) > 0$, then there exists $\sigma_0 > 0$ such that $\theta_{\text{cand,down}}^i(\sigma) > \theta_{\text{cand,up}}^b(\sigma)$ for all $\sigma \in (0, \sigma_0)$. By [Claim 9](#), this implies $I_{\text{cand}}^i(\vartheta, \sigma) \equiv 0$ for all ϑ and all sufficiently small σ . Since $H^i = 0$, then an argument identical to ?? shows that $\lim_{\sigma \downarrow 0} \frac{\theta_{\text{cand,up}}^i(\sigma) - \theta_{\text{cand,down}}^i(\sigma)}{\sigma}$ is bounded, and hence there exists a constant $\mathcal{C}^{\text{adj}_3} > 0$ such that, for every $\sigma \in (0, \sigma_0)$,

$$\left| \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,down}}^i(\sigma), \sigma) - \mathcal{O}_{\text{cand}}^i(\theta_{\text{cand,up}}^i(\sigma), \sigma) \right| < \mathcal{C}^{\text{adj}_3} \sigma.$$

Let \widehat{H}^b be the unique constant defined in [Lemma 3](#). Then $\widehat{H}^b < O_B^b$. In addition, let $\widehat{\theta}^b$

be the Fundamental Default Boundary of bank b defined in [Definition 4](#), i.e.,

$$\widehat{\theta}^b G^i(A^b) + H^b - R^b L^b = 0.$$

Case 1 Suppose that $H^b + \bar{I}_D^b < \widehat{H}^b$, then

$$C^b(A^b, O_B^b - H^b - \bar{I}_D^b) + H^b + \bar{I}_D^b > R^b O_B^b.$$

Hence

$$\widehat{\theta}^b G^b(A^b) + (H^b + \bar{I}_D^b - O_B^b)^+ - C^b(A^b, (O_B^b - H^b)^+) - \bar{I}_D^b - R^b(L^b - O_B^b) < \widehat{\theta}^b G^i(A^b) + H^b + \bar{I}_D^b - \bar{I}_D^b - R^b L^b = 0.$$

By [Claim 12](#), when σ is small enough, there is a constant $\mathfrak{C}_{cand}^{tem} > 0$ such that

$$\mathfrak{O}_{cand}^b(\theta_{cand,down}^b(\sigma), \sigma) \geq O_B^b - \mathfrak{C}_{cand}^{tem} \sigma; \quad \mathfrak{I}_{cand}^b(\theta_{cand,down}^b(\sigma), \sigma) \leq \bar{I}_D^b - \mathfrak{C}_{cand}^{tem} \sigma.$$

It follows that $\theta_{cand,down}^b(\sigma) > \widehat{\theta}^i(H^i)$ for all small enough σ . Otherwise, for any $\vartheta > \theta_{cand,down}^b(\sigma)$ in a small neighborhood of $\theta_{cand,down}^b(\sigma)$,

$$\begin{aligned} \vartheta G^b(A^b) - C^b(A^b, (\mathfrak{O}_{cand}^b(\vartheta, \sigma) - H^b - \mathfrak{I}_{cand}^b(\vartheta, \sigma))^+) + (H^b + \mathfrak{I}_{cand}^b(\vartheta, \sigma) - \mathfrak{O}_{cand}^b(\vartheta, \sigma))^+ - R^b(L^b - \mathfrak{O}_{cand}^b(\vartheta, \sigma)) \\ < \widehat{\theta}^b G^b(A^b) + (H^b + \bar{I}_D^b - O_B^b)^+ - C^b(A^b, (O_B^b - H^b)^+) - \bar{I}_D^b - R^b(L^b - O_B^b) < 0. \end{aligned}$$

This contradicts the definition of $\theta_{cand,down}^b(\sigma)$.

Note that whenever $\theta_{cand,up}^b(\sigma) \geq \theta_{cand,down}^b(\sigma) > \widehat{\theta}^i(H^i)$, we can apply the identical argument from ?? to show that $\lim_{\sigma \downarrow 0} \frac{\theta_{cand,up}^b(\sigma) - \theta_{cand,down}^b(\sigma)}{\sigma}$ is bounded. Therefore,

$$\lim_{\sigma \downarrow 0} \left| \mathfrak{O}_{cand}^i(\theta_{cand,down}^i(\sigma), \sigma) - \mathfrak{O}_{cand}^i(\theta_{cand,up}^i(\sigma), \sigma) \right| = 0.$$

Case 2 Suppose that $H^b + \bar{I}_D^b \geq \widehat{H}^b$, then a similar argument from [Section 1.4.2](#) shows that

$$\lim_{\sigma \downarrow 0} \theta_{cand,down}^i(\sigma) = \lim_{\sigma \downarrow 0} \theta_{cand,up}^i(\sigma) = \widehat{\theta}^i(H).$$

The above argument show that

Claim 13. Suppose $\lim_{\sigma \downarrow 0} [\theta_{cand,down}^i(\sigma) - \theta_{cand,up}^i(\sigma)] > 0$. Then the candidate limiting equilibrium as $\sigma \downarrow 0$ is the one described in (i) of [Theorem 2](#). Conversely, if $\lim_{\sigma \downarrow 0} [\theta_{cand,down}^i(\sigma) - \theta_{cand,up}^i(\sigma)] > 0$, the candidate limiting equilibrium as $\sigma \downarrow 0$ is the one described in (ii) of [Theorem 2](#).

Proof. We prove the first statement; the second follows by symmetry. Evaluating the bank- j solvency condition at the candidate thresholds and passing to the limit pins down the

limiting solvency cutoffs, which match those in case (i) of [Theorem 2](#). Finally, by [Claim 11](#), depositors' best responses converge to the move/stay regions stated in (i) of [Theorem 2](#), completing the characterization. \square

1.8 Proof of [Theorem 3](#)

Using similar argument of [Section 1.7](#), it is easy to show that that the limiting equilibrium is unique. Therefore, we focus on characterizing the bank-run cutoffs.

By assumption, $\theta_B^\ell > \theta_B^b(H^b), \forall \ell \leq \mathcal{N}^{cut}$ and $\theta_B^\ell < \theta_B^b(H^b), \forall \ell > \mathcal{N}^{cut}$. By [Lemma 4](#), when the true state θ satisfies $\theta_B^b(H^b) < \theta < \theta_B^{\mathcal{N}^{cut}}$, depositors at bank $\ell < \mathcal{N}^{cut}$ will move their deposits to bank b . Similar argument of [Section 1.7](#) shows that this influx of deposit flow will further strengthen bank b 's financial stability, pushing lower its bank-run cutoff.

Consider the strategies of depositors from region ℓ : At bank b 's potential run cutoff θ_{cand}^b , as $\sigma \downarrow 0$, the probability of depositor w with type ξ_w to run from bank b converges to $\frac{(1-\Lambda)}{R^b + \xi_w^b - \Lambda}$ and the probability of depositor k from any bank ℓ to **not** to choose bank b converges to $\frac{(1-\Lambda)}{1 + \chi^b \xi_k^b - \Lambda}$. In other words, the probability of depositor k from any bank ℓ that still moves to choose bank b converges to $1 - \frac{(1-\Lambda)}{1 + \chi^b \xi_k^b - \Lambda} = \frac{\chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda}$. There maybe deposit inflows from other regional banks, so the total outflow from bank b at θ_{cand}^b must be weakly less than

$$O_C^b(H_i, \mathcal{N}^{cut}) = \left(\sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \left[\frac{s_{k,b}(1-\Lambda)}{R^b + \xi_k^b - \Lambda} - \mathbb{1}_{[i \leq \mathcal{N}^{cut}]} \left(\frac{O^i}{L^i} \right) \frac{s_{k,i} \chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda} \right] dF_{\xi}(\xi_k) \right)^+$$

Therefore, the run cutoff is weakly less than $\theta_C^b(H^b, \mathcal{N}^{cut})$:

$$\theta_{cand}^b \leq \theta_C^b(H^b, \mathcal{N}^{cut}) = \frac{C^b(A^b, (O_C^b(H_i, \mathcal{N}^{cut}) - H^b)^+) + (H^b - O_C^b(H_i, \mathcal{N}^{cut}))^+ + R^b(L^b - O_C^b(H_i, \mathcal{N}^{cut}))}{G^b(A^b)}.$$

This is common knowledge among all agents.

Consider the strategy of depositors in region $u = \mathcal{N}^{cut} + 1$: if $\theta_C^b(H^b, \mathcal{N}^{cut}) < \theta_B^u$ then $\theta_{cand}^b < \theta_B^u$. Again, in the same spirit of [Lemma 4](#), it is easy to use similar argument of [Section 1.7](#) to show that the probability of depositor w with type ξ_w to run from bank b converges to $\frac{(1-\Lambda)}{R^b + \xi_w^b - \Lambda}$ and the probability of depositor k from any bank ℓ to **not** to choose bank b converges to $\frac{(1-\Lambda)}{1 + \chi^b \xi_k^b - \Lambda}$, so the total outflow from bank b at θ_{cand}^b must be weakly less than $O_C^b(H_i, \mathcal{N}^{cut} + 1)$. Iteratively, we can show that for all $u > \mathcal{N}^{cut}$ $\theta_C^b(H^b, u) < \theta_B^u + 1$, the deposit outflow in equilibrium must be weakly less than $O_C^b(H_i, u + 1)$ and the bank-run cutoff must satisfy $\theta_{cand}^b \leq \theta_C^b(H^b, u + 1)$.

We can repeat this process until we reach $u = \mathcal{N}$ or some positive integer $\mathcal{N}^{run} < \mathcal{N}$

such that $\theta_B^{\mathcal{N}^{run}+1} < \theta_C^b(H^b, \mathcal{N}^{run})$. By [Lemma 4](#), the bank-run equilibrium for the regional bank $\mathcal{N}^{run} + 1$ will not be affected by the national bank b . Finally it is trivial to show that when H^b gets larger, then \mathcal{N}^{run} weakly increases.

1.9 Proof of [Theorem 4](#)

WLOG, normalize $\mathcal{E}^j(0) = 0$ for all j . Also normalize $\mathcal{B}^i(0) = 0$ for $i \neq b$. By [Theorem 5](#) bank i will hold $H^i = 0$ and bank b will hold $H^b = \Gamma E^b + \Gamma(1 - \varphi^b)L^b(R^b)$ for all Γ .

Under the economy of isolated bank run the social planner seeks to solve

$$\max_{\Gamma} \mathcal{W}_B(\Gamma) = \int_{\theta > \theta_B^b} \mathcal{B}^b(\Gamma) dF_{\bar{\theta}}(\theta) - \int_{\theta \leq \theta_B^b} \mathcal{E}^b(\mathcal{L}_B^b) dF_{\bar{\theta}}(\theta) - \sum_i \int_{\theta \leq \theta_B^i} \mathcal{E}^i(\mathcal{L}_B^i) dF_{\bar{\theta}}(\theta) \quad (52)$$

subject to

$$\begin{aligned} \mathcal{L}_B^b &= (\theta + \zeta_b) G^b((1 - \Gamma)(E^b + (1 - \varphi^b)L^b)) - C^j(A^j, (O_B^b - \Gamma(E^b + (1 - \varphi^b)L^b))^+) - R^b(L^b - O_B^b) \\ \mathcal{L}_B^i &= \theta G^i(A^i) - C^i(A^i, O_B^i) - R^i(L^i - O_B^i). \end{aligned}$$

Under the economy of connected bank run the social planner seeks to solve

$$\max_{\Gamma} \mathcal{W}_D(\Gamma) = \int_{\theta > \theta_D^b} \mathcal{B}^b(\Gamma) dF_{\bar{\theta}}(\theta) - \int_{\theta \leq \theta_D^b} \mathcal{E}^b(\mathcal{L}_D^b) dF_{\bar{\theta}}(\theta) - \sum_{i=1}^{\mathcal{N}} \int_{\theta \leq \theta_D^i} \mathcal{E}^i(\mathcal{L}_D^i) dF_{\bar{\theta}}(\theta) \quad (53)$$

subject to

$$\begin{aligned} \mathcal{L}_D^b &= (\theta + \zeta_b) G^b((1 - \Gamma)(E^b + (1 - \varphi^b)L^b)) - C^j(A^j, (O_D^b - \Gamma(E^b + (1 - \varphi^b)L^b))^+) - R^b(L^b - O_D^b) \\ \mathcal{L}_D^i &= \theta G^i(A^i) - C^i(A^i, O_D^i) - R^i(L^i - O_D^i). \end{aligned}$$

By [Lemma 4](#), for any Γ , $O_D^b < O_B^b$, $\theta_D^b < \theta_B^b$ and $O_D^i > O_B^i$, $\theta_D^i > \theta_B^i$. Therefore if Γ_B^* solves [Eq. \(52\)](#), then reducing Γ_B^* by a small ζ would induce $\mathcal{W}_D(\Gamma_B^* - \zeta) > \mathcal{W}_D(\Gamma_B^*)$ as reducing ζ strictly increases θ_D^b making it more closer to θ_B^b , which improves $\int_{\theta > \theta_D^b} \mathcal{B}^b(\Gamma) dF_{\bar{\theta}}(\theta) -$

$\int_{\theta \leq \theta_D^b} \mathcal{E}^b(\mathcal{L}_D^b) dF_{\bar{\theta}}(\theta)$ by the optimality of Γ_B^* . Doing so also weakly reduces θ_D^b and \mathcal{L}_D^i ,

hence weakly improving the other components. It follows that the optimal $\Gamma_D^* < \Gamma_B^*$.

1.10 Proof of [Theorem 5](#)

We first show that in the **Alternative Economy of Isolated Bank Run**, banks do not voluntarily hold safe asset $H^j > O_B^j$ for $j \in \{1, 2, \dots, \mathcal{N}, b\}$. Suppose regulatory constraint is not binding when $H^j > O_B^j$. If bank j were to hold $H^j > O_B^j$, then reducing H^j by a small $\varepsilon > 0$ strictly improves the payoff to bank's equity holders if bank j does not default, and weakly improves $\mathbb{1}_{[D_j^c]}$ (when bank run does not occur, improving banks' financial return decreases banks' default probability). Hence holding $H^j > O_B^j$ is never optimal for bank j in the economy of isolated banks. We can use similar arguments for the **Benchmark Economy of Connected Bank Run** to show that $H^j > O_D^j$ is never optimal for bank j .

Next, we show that banks hold minimum possible risk-free assets on their balance sheets in the **Alternative Economy of Isolated Bank Run**. For regional bank i , when $H^i \leq Q_B^i$, we can rewrite bank i 's objective function in Eq. (21) as (recall the assumption that $\zeta_i = 0$ almost surely):

$$\begin{aligned} & \mathbb{E} \left[\left((\bar{\theta}) G^i(A^i) - C^i(A^i, (O^i - I^i - H^i)^+) + (H^i + I^i - O^i)^+ - R^i(L^i - O^i) - I^i \right) \mathbb{1}_{\mathcal{D}_i^c} \left| L^i \right. \right] \\ &= \int_{\theta > \theta_B^i(H^i)} \left(\theta G^i(A^i) - C^i(A^i, (O^i - I^i - H^i)^+) + (H^i + I^i - O^i)^+ - R^i(L^i - O^i) - I^i \right) f_{\bar{\theta}}(\theta) d\theta \\ &= \int_{\theta > \theta_B^i(H^i)} \left(\theta G^i(E^i + (1 - \varphi^i)L^i - H^i) + H^i - R^i L^i \right) f_{\bar{\theta}}(\theta) d\theta \end{aligned}$$

The second equality holds because by Lemma 3, when $\theta > \theta_B^i(H^i)$, all depositors will stay with bank i and the total deposit flow would be zero. Also, since $\bar{\theta}$ has atomless distribution, $\mathbb{P}(\bar{\theta} = \theta_B^i) = 0$. We can get the First Order Condition (FOC) for bank i at time $t = 1$ by differentiating the objective function with respect to H^i :

$$\begin{aligned} & \int_{\theta > \theta_B^i(H^i)} \left(\theta \frac{\partial}{\partial H^i} G^i(E^i + (1 - \varphi^i)L^i - H^i) + 1 \right) f_{\bar{\theta}}(\theta) d\theta - \frac{\partial \theta_B^i(H^i)}{\partial H^i} \cdot \left(\theta_B^i(H^i) G^i(A^i) - C^i(A^i, (O_B^i - H^i)) - R^i(L^i - O_B^i) \right) f_{\bar{\theta}}(\theta_B^i(H^i)) \\ &= \int_{\theta > \theta_B^i(H^i)} \left(-\theta (G^i)'(E^i + (1 - \varphi^i)L^i - H^i) + 1 \right) f_{\bar{\theta}}(\theta) d\theta < 0 \end{aligned}$$

The last equality holds because if $\theta_B^i(H^i) G^{i'}(A^i) \geq 1$, then $\vartheta G^{i'}(A^i) > 1 \quad \forall \vartheta \geq \theta_B^i(H^i)$ and if $\theta_B^i(H^i) G^{i'}(A^i) \leq 1$, then

$$\begin{aligned} & \int_{\theta > \theta_B^i(H^i)} \left(-\theta (G^i)'(E^i + (1 - \varphi^i)L^i - H^i) + 1 \right) f_{\bar{\theta}}(\theta) d\theta \leq \int_{\theta \in \mathbb{R}} \left(-\theta (G^i)'(E^i + (1 - \varphi^i)L^i - H^i) + 1 \right) f_{\bar{\theta}}(\theta) d\theta \\ &= 1 - (G^i)'(E^i + (1 - \varphi^i)L^i - H^i) \mathbb{E}[\bar{\theta}] = 1 - (G^i)'(E^i + (1 - \varphi^i)L^i - H^i) < 0. \end{aligned}$$

The above argument applies also to **Benchmark Economy of Connected Bank Run**. Therefore, bank i only holds as little as risk-free asset as possible. Next, the objective function for national bank b is

$$\begin{aligned} & \mathbb{E} \left[\left((\bar{\theta}) G^b(A^b) - C^b(A^b, (O^b - I^b - H^b)^+) + (H^b + I^b - O^b)^+ - R^b(L^b - O^b) - I^b \right) \mathbb{1}_{\mathcal{D}_b^c} \left| L^b \right. \right] \\ &= \iint_{\theta + \zeta_b > \theta_B^b(H^b)} \left((\theta + \zeta_b) G^b(E^b + (1 - \varphi^b)L^b - H^b) + H^b - R^b L^b \right) f_{\bar{\zeta}_b}(\zeta_b) f_{\bar{\theta}}(\theta) d\zeta_b d\theta. \end{aligned}$$

We can then apply almost identical argument to show that the derivative of bank b 's objective function is negative with respect to H^b . Therefore, bank b holds minimum H^b that meet the regulatory constraint.

1.11 Proof of Theorem 6

Consider region i where banks i (regional) and b (national) operate. Let k be a depositor at bank i and w be a depositor at bank b . With two sources of uncertainty, depositor strategies are a function of the joint signal realization (\bar{x}, \bar{y}) .

Strategy Sets Let $\mathcal{R}_{\text{cand}}^j$ be the set of signals (x, y) that cause a depositor to withdraw from bank j . The subset of $\mathcal{R}_{\text{cand}}^j$ corresponding to depositors switching banks is denoted by $\mathcal{M}_{\text{cand}}^j$. For example, $\mathcal{M}_{\text{cand}}^i(k, \sigma)$ is the set of signals causing depositor k to switch from bank i to bank b . Following the logic of ??, these sets have a monotone or interval structure in x for any fixed y in the $\sigma \downarrow 0$ limit. Formally:

$$(x_k, y_k) \in \lim_{\sigma \downarrow 0} \mathcal{R}_{\text{cand}}^j(k, \sigma) \implies (x'_k, y_k) \in \lim_{\sigma \downarrow 0} \mathcal{R}_{\text{cand}}^j(k, \sigma) \quad \forall x'_k \leq x_k$$

$$(x_k, y_k), (x'_k, y_k) \in \lim_{\sigma \downarrow 0} \mathcal{M}_{\text{cand}}^j(k, \sigma) \implies (x''_k, y_k) \in \lim_{\sigma \downarrow 0} \mathcal{M}_{\text{cand}}^j(k, \sigma) \quad \forall x''_k \in [x'_k, x_k].$$

Aggregate Flows and Default Conditions For any given noise levels $\sigma, \varsigma > 0$, let $\mathcal{O}_{\text{cand}}^j$ and $\mathcal{I}_{\text{cand}}^j$ be the candidate equilibrium deposit outflow and inflow for bank $j \in \{i, b\}$ in the state (θ, χ^i, χ^b) . The set of states in which bank b defaults, \mathcal{D}_b , is therefore defined as:

$$\mathcal{D}_b(\text{cand}, \varsigma, \sigma) = \left\{ (\theta, \chi^i, \chi^b) \mid \theta G^b(A^b) - C^b \left(A^b, (\mathcal{O}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma) - \mathcal{I}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma) - H^b)^+ \right) \right. \\ \left. + (H^b + \mathcal{I}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma) - \mathcal{O}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma))^+ - \mathcal{I}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma) - R^b(L^b - \mathcal{O}_{\text{cand}}^b(\theta, \chi^j, \varsigma, \sigma)) \leq 0 \right\}.$$

The default set for bank i , $\mathcal{D}_i(\text{cand}, \varsigma, \sigma)$, is defined analogously.

We begin by noting that when the regional bank's mobility parameter, χ^i , is sufficiently small relative to the national bank's, χ^b , the condition for multiplicity from case (ii) of [Theorem 2](#) is not met. In this straightforward scenario, the unique limiting equilibrium is the one described in case (i).

The more interesting case for equilibrium selection arises when χ^i is larger than χ^b . To build intuition, we simplify the analysis by setting $\chi^b = 0$, noting that the logic extends readily to cases where $\chi^b > 0$. With a slight abuse of notation, we can analyze the default set as a function of the realized $\chi^i = x$:

$$\mathcal{D}_j(\text{cand}, \varsigma, \sigma)(x) \equiv \{\theta \mid (\theta, x, 0) \in \mathcal{D}_j(\text{cand}, \varsigma, \sigma)\}.$$

The monotone structure of depositor strategies, as previously established, implies that in the limit, this default set takes the form of a simple cutoff rule determined by a threshold, $\theta_{\varsigma}^{\text{cand}}(x)$:

$$\lim_{\sigma \downarrow 0} \mathcal{D}_j(\text{cand}, \varsigma, \sigma)(x) = (-\infty, \theta_{j, \varsigma}^{\text{cand}}(x)).$$

Claim 14. Assume $\theta_B^b < \theta_B^i$. For a sufficiently small mobility parameter $\chi^i > 0$, there exists a $\bar{\varsigma} > 0$ such that for all $\varsigma \in (0, \bar{\varsigma})$:

$$\lim_{\sigma \downarrow 0} \mathcal{D}_i(\text{cand}, \varsigma, \sigma)(\chi^i) \subsetneq \lim_{\sigma \downarrow 0} \mathcal{D}_b(\text{cand}, \varsigma, \sigma)(\chi^i).$$

Proof. For any sufficiently small $\varsigma > 0$, we analyze the system for a small $\chi^i > 0$. The

potential inflow into bank i is bounded above:

$$I_{\text{cand}}^i \leq \mathcal{M}^i \int_{\xi_w} \frac{s_{w,b}(\chi^i + \varsigma)\xi_w^b}{1 + \chi^i \xi_w^i - \Lambda} dF_{\tilde{\xi}}(\xi_w) + \mathcal{O}(\sigma),$$

so there is some constant $C_1^{\text{tem}} > 0$, such that

$$\left(-\infty, \theta_B^i - C_1^{\text{cand}}(\chi^i + \varsigma)\right) \subset \lim_{\sigma \downarrow 0} \mathcal{D}_i(\text{cand}, \varsigma, \sigma)(\chi^i) \subset \left(-\infty, \theta_B^i\right),$$

where θ_B^i is defined in [Lemma 1](#). In parallel, the outflow from bank b due to depositors switching to bank i is

$$O_{\text{cand}}^b \leq \mathcal{M}^i \int_{\xi_k} \frac{s_{k,b}(1 - \Lambda + \chi^i \xi_k^i)}{R^b + \xi_k^b - \Lambda} dF_{\tilde{\xi}}(\xi_k) + \mathcal{O}(\sigma) = O_B^b + \mathcal{O}(\sigma) + \mathcal{O}(\chi^i),$$

where O_B^b is defined in [Lemma 3](#), taking $H^b = 0$. Thus there is some constant $C_2^{\text{cand}} > 0$:

$$\left(-\infty, \theta_B^b\right) \subset \lim_{\sigma \downarrow 0} \mathcal{D}_b(\text{cand}, \varsigma, \sigma)(\chi^i) \subset \left(-\infty, \theta_B^b + C_2^{\text{cand}} \chi^i\right).$$

Given the initial assumption that $\theta_B^b < \theta_B^i$, these bounds collectively show that $\lim_{\sigma \downarrow 0} \mathcal{D}_b \not\subseteq \lim_{\sigma \downarrow 0} \mathcal{D}_i$ for small enough $\varsigma + \chi^i$. \square

The above claim implies that when χ^i is sufficiently small, no depositors will move from bank b to bank i , regardless of their private signals in the limiting equilibrium.

We next proceed our proof by contradiction. Suppose that there is some χ^i such that $\lim_{\sigma \downarrow 0} \mathcal{D}_i(\text{cand}, \varsigma, \sigma)(\chi^i) \not\subseteq \lim_{\sigma \downarrow 0} \mathcal{D}_b(\text{cand}, \varsigma, \sigma)(\chi^i)$. Define

$$\underline{\chi}^i \equiv \inf\{\chi^i \mid \lim_{\sigma \downarrow 0} \mathcal{D}_i(\text{cand}, \varsigma, \sigma)(\chi^i) \not\subseteq \lim_{\sigma \downarrow 0} \mathcal{D}_b(\text{cand}, \varsigma, \sigma)(\chi^i)\}.$$

[Claim 14](#) implies that $\underline{\chi}^i > 0$. By continuity,

$$\lim_{\sigma \downarrow 0} \mathcal{D}_i(\text{cand}, \varsigma, \sigma)(\underline{\chi}^i) = \lim_{\sigma \downarrow 0} \mathcal{D}_b(\text{cand}, \varsigma, \sigma)(\underline{\chi}^i) = \left(-\infty, \theta_{i,\varsigma}^{\text{cand}}(x)\right).$$

Let $C_k^\varsigma = F_{\tilde{v}}^{-1}\left(\frac{1 + \chi^b \xi_k^b - \Lambda}{R^i + \xi_k^i - \Lambda}\right)$ and $C_w^\varsigma = F_{\tilde{v}}^{-1}\left(\frac{1 + \chi^i \xi_w^i - \Lambda}{R^b + \xi_k^b - \Lambda}\right)$. It is easy to verify the following claim:

Claim 15. Fix some small $\varsigma^{\text{tem}} > 0$. Suppose the true state is $\theta_{i,\varsigma}^{\text{cand}}(x)$. Then there exists a constant $C^{\text{adj}} > 0$ such that, for all small enough ς , in the limit as $\sigma \rightarrow 0$,

1. If depositor k observes $\tilde{y}_k \geq \underline{\chi}^i + C_k^\varsigma \sigma + C^{\text{adj}} \sigma^2$, she may stay with bank i .
2. If she observes $\tilde{y}_k \leq \underline{\chi}^i + C_k^\varsigma \sigma - C^{\text{adj}} \sigma^2$, she moves from bank i to bank b .

Therefore, when the true state is $(\theta_{i,\zeta}^{\text{cand}}(x), \underline{\chi}^i)$, then $I^b = \int_{\underline{\xi}_w^b} \frac{1+\underline{\chi}^b \xi_w^b - \Lambda}{R^b + \xi_w^b - \Lambda} dF_{\bar{\xi}}(\xi_w^b)$. and $I^i = \int_{\underline{\xi}_k^i} \frac{1+\underline{\chi}^i \xi_k^i - \Lambda}{R^b + \xi_k^i - \Lambda} dF_{\bar{\xi}}(\xi_k^i)$. Those constants will pin down $\underline{\chi}^i$ and $\underline{\chi}^b$. Whenever $\chi^i < \underline{\chi}^i$, then it is not possible for any depositors from bank b to run to bank i .