Too Big to Fail, Too Small to Survive*

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Abstract

This paper identifies a novel bank-run externality: when depositors can shift funds across risky banks in a crisis, a large national bank perceived as safer may disproportionately raise the likelihood of runs among smaller regional banks. A minor shock can prompt a cascading chain reaction of deposit withdrawals among many regional banks, generating systemic vulnerabilities beyond traditional interbank-contagion channels. We extend global game models by allowing realistic deposit mobility across multiple risky banks, alongside the conventional risk-free option. Our framework captures the interaction between strategic complementarity in run/stay decisions among multiple substitutes—a dynamic that remains underexplored in the literature.

Keywords: Bank deposit competition, Financial Stability, Global Game, Macroprudential Policy, Strategic Complementarity.

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1 Introduction

Since the Great Financial Crisis, macroprudential policies have aimed to fortify the financial system by prioritizing the stability of large, systemically important banks.[1] The conventional logic is that national institutions play central roles in the financial system, so safeguarding them stabilizes the broader network of intermediaries. This paper shows that the post-GFC architecture overlooks a countervailing force. Strengthening national banks through liquidity requirements, crisis support, or a perceived Too Big To Fail (TBTF) backstop can unintentionally raise run risk at smaller regional banks by making national institutions the *safer neighbors* during stress. When depositors can shift funds across risky banks in a crisis, a perceived-safer neighboring institution that continues to provide banking services represents a strictly more attractive outside option than withdrawing into cash. This raises depositors' incentives to withdraw and, because withdrawal decisions exhibit strategic complementarities, even moderate concerns about a regional bank's health can trigger runs and deposit reallocations. We refer to this mechanism as the *safe-neighbor externality*. Furthermore, the externality reshapes optimal policy even when the regulator's sole objective is the stability of large institutions. Anticipated deposit inflows from smaller banks endogenously strengthen the national bank during stress, so a regulator who ignores this channel may over-tighten liquidity requirements relative to the constrained optimum.

To formalize the safe-neighbor externality and evaluate its interaction with macroprudential policies, we enrich the standard global-game framework in the bank-run literature by allowing for multiple risky banks and cross-bank deposit mobility. In contrast to the classic setting in which cash is the sole outside option at the run stage (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005), cross-bank mobility introduces an additional layer of strategic interaction: depositors coordinate not only on whether to withdraw but also on *where* to reallocate funds. When deposits are reallocated from one bank to a neighboring institution during downturns, the inflows strengthen the recipient bank's balance sheet and survival probability, endogenously generating a self-fulfilling divergence in their equilibrium survival probabilities. These feedback effects can amplify coordination frictions at smaller institutions and pose challenges for regional financial stability. Facing the additional complexity introduced by cross-bank deposit mobility, we develop new proof techniques to analyze the limiting equilibrium with multiple banks, thereby extending the scope of the standard framework.

Subsequently, our analysis demonstrates that the safe-neighbor externality can

[1] E.g., Hanson, Kashyap and Stein 2011; Kashyap, Tsomocos and Vardoulakis 2024; Begenau 2020; Elenev, Landvoigt and Van Nieuwerburgh 2021; BCBS 2018b, 2010a, 2024.

induce systemic fragility among smaller banks across regions, even when their asset holdings are uncorrelated. When a negative shock threatens one regional bank, the resulting inflows into the national institution strengthen its balance sheet and raise its survival probability, which depositors in other regions rationally anticipate. This feedback makes the national bank a focal destination across regions, and shifts run thresholds at otherwise viable regional banks, generating a cross-regional chain reaction of deposit flight, particularly among regional banks that are vulnerable but not insolvent during downturns.

To the extent that small and regional banks collectively play a critical role in nationwide financial stability, the aforementioned chain reaction emerges as a potential source of systemic risk. As of 2023, regional and smaller banks manage approximately 30% of total assets in the U.S. banking industry, hold 54% of total deposits, and account for more than 60% of total loans outstanding.[2] In addition, these banks play a critical role in local economies by providing specialized knowledge and tailored financial services that national banks often cannot replicate.[3] Their decline imposes significant costs on consumers, reduces financial inclusion, and suppresses regional economic growth, disproportionately affecting underserved communities (Nguyen, 2018; Bonfim, Nogueira and Ongena, 2020). Factors that simultaneously increase the fragility of many regional institutions can therefore pose substantial systemic risk.

Indeed, following the collapse of Silicon Valley Bank (SVB) in March 2023, depositors shifted funds to larger institutions perceived as safer, prompting significant withdrawals at regional banks such as First Republic Bank, Signature Bank, PacWest Bancorp, Western Alliance Bancorporation, and Zions Bancorporation,[4] among many others operating across diverse sectors in the United States. This cascade of withdrawals triggered liquidity pressures across regional financial markets, culminating in a nationwide wave of deposit outflows from smaller banks. Choi, Goldsmith-Pinkham and Yorulmazer (2023) show that mid-sized banks experienced the most severe stress in the immediate aftermath of SVB's failure, while negative spillovers later propagated throughout the banking system, sparing only the largest institutions. Caglio, Dlugosz and Rezende (2023) document that,

[2]Source: Banking Snapshot by the American Bankers Association.

[3]Using a 1970–1993 sample from Italy, Usai and Vannini (2005) find that smaller banks positively influence local economic growth, contrasting with the insignificant or negative effects associated with national banks. Regional banks also hold approximately one-third of all small business loans provided by banks (Covas, Gross and Tapia, 2023).

[4]Signature Bank, headquartered in New York City, operated 40 private client offices across New York State. Western Alliance Bancorporation, headquartered in Phoenix, served Arizona, California, and Nevada. Zions Bancorporation, headquartered in Salt Lake City, operated across several Western states. First Republic Bank and PacWest Bancorp primarily operated in California. Of the banks that experienced runs in March 2023, First Republic was the 14th largest at the time, while SVB was the 16th, and Signature the 29th (Rose, 2023).

during periods of heightened stress, large banks experienced significantly faster deposit growth than smaller regional banks without raising deposit rates, indicating a pronounced flight to safety. Their analysis further reveals that depositors' flight to larger, perceived safer banks was not fully justified by fundamentals, consistent with our model's prediction of a chain reaction in bank runs driven by strategic complementarities. This financial panic drew substantial media attention[5] and necessitated government intervention, including the creation of the Bank Term Funding Program (BTFP) and a guarantee of uninsured deposits.[6]

Beyond analyzing new dynamics in bank runs, we also show that regulatory support for large national banks affects ex-ante deposit competition. While the existing literature has established that capital and liquidity regulations distort large banks' incentives to attract deposits and expand loan portfolios (Begenau, 2020; Boissay and Collard, 2016; Vives, 2016), our work highlights the non-pecuniary externalities associated with the perceived safety of banks, a channel that is not fully captured in previous, primarily price-based analyses.

On the technical side, our modeling framework and proof techniques extend beyond traditional global-game and bank-run models through two key generalizations: First, our global-game foundation is deliberately general. We establish the most general sufficient condition in the literature on the information structure to guarantee equilibrium uniqueness, demonstrating the robustness of our analysis and enhancing the practical predictive power of our model. This flexibility also expands the applicability of global game models to economic environments in which the natural payoff and information structures do not align with the assumptions imposed in existing global game models. In particular, we allow a continuum of depositors with independent and heterogeneous signal and payoff distributions that are essentially arbitrary. Our new proof technique shows that the small-noise refinement still selects a unique limiting equilibrium. This delivers the "robust equilibrium" we emphasize throughout, while avoiding restrictive assumptions that are common in the literature.[7] This new methodology also allows us to

[5]For example, see "Banking Crisis Hangs Over Economy, Rekindling Recession Fear", *The New York Times*, March 17, 2023; "The 11 Days of Turmoil That Brought Down Four Banks And Left A Fifth Teetering", *Bloomberg*, March 20, 2023; "US Experiences Biggest Bank Failures Since Global Financial Crisis", *Financial Times*, March 25, 2023.

[6]Despite the fact that about 89% of SVB's $175 billion in deposits were uninsured as of the end of 2022, on March 12 the FDIC received exceptional authority from the Treasury and announced jointly with other agencies that all depositors would have full access to their funds the next morning, including uninsured deposits. On the same day, the Federal Reserve announced the creation of the Bank Term Funding Program (BTFP), an emergency lending program providing loans of up to one year in length to banks, savings associations, credit unions, and other eligible depository institutions.

[7]Besides generalizing the information structure, our proof relaxes several other assumptions commonly imposed in the literature and clearly enumerated in the seminal contribution of Frankel, Morris and Pauzner

study liquidity requirements (e.g., regulatory liquidity ratios such as LCR or NSFR) that break the commonly assumed property of *global strategic complementarities* required by canonical proofs. Our framework nests several prominent global game models—including Goldstein and Pauzner 2005 and Kashyap, Tsomocos and Vardoulakis 2024—as special cases.

Second, classical bank-run models typically restrict depositor strategies to a one-dimensional run-or-stay choice. In contrast, our framework introduces multiple outside options by allowing depositors to choose between holding risk-free cash and transferring deposits to other risky banks during episodes of panic. This richer strategic margin better captures the dynamics observed in modern banking crises (e.g., Baron, Schularick and Zimmermann 2023), but it also breaks the one-dimensional monotonicity exploited by standard solution methods, because beliefs and fragility become mutually reinforcing across institutions. In the canonical global-games benchmark, depositors' withdrawal decisions are characterized by a unique run–stay cutoff in the limiting equilibrium, determined solely by the relative payoff of remaining deposited with the bank versus holding cash. In our setting, by contrast, banks' relative fragility and the associated run cutoffs are endogenous to depositors' joint strategies and beliefs and must be determined simultaneously. Nevertheless, the equilibrium remains tightly structured: the economically relevant outcome preserves the benchmark fragility ordering. Whenever the national bank's run cutoff lies below that of the regional bank in the cash-only benchmark, this ranking is preserved once deposit mobility is introduced, regardless of other parameter values. Except in sufficiently adverse macroeconomic conditions, depositors at smaller regional banks prefer transferring funds to the national bank rather than holding cash. This reallocation strengthens large, regulated institutions while increasing the vulnerability of smaller banks, which is a central prediction of our model.

Although the primary goal of this paper is to identify the externality and document its implications, our findings naturally point toward potential policy adjustments from the lens of our model. In our model, the severity of this externality depends on factors such as the perceived government guarantees for large banks, the regulatory constraints they face, and the scope of deposit insurance available to smaller banks. This analysis highlights the role of post-GFC banking regulations in shaping depositor behavior during financial uncertainty. These regulations, along with implicit government guarantees, have led regional banks' depositors to increasingly perceive large U.S. banks as safer

2003, including the assumptions of (a) *payoff continuity* and (b) *bounded derivatives*. These conditions are typically violated in banking crises with endogenous bank defaults, where depositors' financial returns can drop abruptly. In particular, we do not require a finite number of player types, which allows us to model an uncountably large set of heterogeneous depositors and to apply IO-style structural models to study bank deposit competition more effectively.

alternatives, heightening their incentives to withdraw from regional banks to large national banks during periods of stress. In fact, the severity of bank runs on regional banks has notably increased since the implementation of Basel III regulations. For instance, in 2008, Wachovia and Washington Mutual, the largest banks affected at the time, experienced what were called "massive" deposit runs of 4.4% and 10%, respectively[8] (Rose, 2023, 2015). By contrast, Silicon Valley Bank lost 25% of its deposits in a single day, with an additional 62% scheduled for withdrawal before its shutdown. Signature Bank lost 20% of its deposits in one day, while First Republic Bank lost 14% on the first day, 23% on the next business day, and an additional 20% over the course of the run (Gruenberg 2023; BCBS 2023). These trends underscore the possibility that regulations intended to stabilize large banks have inadvertently increased the systemic vulnerabilities of smaller institutions. Our results underscore the importance of developing regulatory frameworks that enhance systemic stability without undermining the competitiveness or safety of smaller institutions.

The rest of the paper is organized as follows: Section 2 provides more background and discusses the relationship between our findings and prior research. Section 3 introduces the model environment. Section 4 documents the externality of big national banks on regional banks during a bank-run crisis. Section 5 extends our analysis to deposit competition. Section 6 provides the concluding remarks.

2 Related Literature

Our work is closely related to the literature on financial fragility and the policy responses to banking crises. Classic models emphasize the role of self-fulfilling prophecies in triggering bank runs (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005), while subsequent research has explored how deteriorating fundamentals can initiate run dynamics (Chari and Jagannathan, 1988; Jacklin and Bhattacharya, 1988; Uhlig, 2010; Allen and Gale, 1998; Acharya and Mora, 2015). Network analyses further highlight how the failure of one institution can spread distress throughout the system (Allen and Gale, 2000; Eisenberg and Noe, 2001),[9] Although ex post government guarantees can reduce run risk (Allen, Carletti, Goldstein and Leonello, 2018; Keister, 2016), they may also remove the disciplining effect of deposits and thus encourage excessive risk-taking (Cooper and Ross, 2002). Previous research has examined how capital and liquidity regulations influence large banks' incentives (Kara and Ozsoy, 2020; Gertler and Kiyotaki, 2015; Van den Heuvel, 2022; Gertler, Kiyotaki and Prestipino, 2020), noting that while

[8]Prior to 2022, the most severe run was on Continental Illinois, which lost 30% of its funding over 10 days. (Rose, 2023)

[9]Some recent work demonstrates that heterogeneous asset holdings can mitigate these contagion effects (Goldstein, Kopytov, Shen and Xiang, 2024).

such measures curb risk-taking, they sometimes create unintended consequences (Kashyap, Tsomocos and Vardoulakis, 2024; Jeanne and Korinek, 2020; Hanson, Kashyap and Stein, 2011; Cooper and Ross, 2002). Relatedly, Dávila and Goldstein (2023) derive optimal deposit insurance from a sufficient-statistics perspective, and Schilling (2023) shows that aggressive interventions can be counterproductive in a global game setting.

Our model incorporates heterogeneous depositors who derive utility from both financial returns and banking services, as in Egan, Hortaçsu and Matvos (2017a). This feature allows us to examine the ex ante deposit competition. By linking deposit competition and bank-run fragility, our work connects to research on deposit franchise (Drechsler, Savov, Schnabl and Wang, 2023; Haddad, Hartman-Glaser and Muir, 2023), as well as on deposit insurance pricing, its impact on bank portfolios, and the determination of deposit interest rates (d'Avernas, Eisfeldt, Huang, Stanton and Wallace, 2023; Kim and Rezende, 2023; Egan, Hortaçsu and Matvos, 2017b; Egan, Lewellen and Sunderam, 2022). (We abstract from the longer-term implications of deposit allocations on bank lending and real activity, as explored in Mian and Khwaja (2008); Gilje, Loutskina and Strahan (2016); Acharya, Das, Kulkarni, Mishra and Prabhala (2022).) Although the effects of macroprudential policy on the deposit competition of smaller banks are relatively underexplored, the recent 2023 regional banking crisis has allowed researchers to gather empirical evidence in this direction. For example, Caglio et al. (2023) examine the economic implications of transferring deposits from smaller banks to the largest national bank during this episode. In line with their findings, our model identifies a non-pecuniary distortion of macroprudential policy in bank competition: by reducing the stability of smaller institutions, regulations and implicit government guarantees that safeguard national banks may also substantially reshape the competitive landscape. This underscores the need for a more nuanced understanding of regulatory impacts (Kashyap, Tsomocos and Vardoulakis, 2024; Hanson, Kashyap and Stein, 2011). Our findings enrich prior studies of bank competition that focus on pecuniary externalities operating through price adjustments (Benigno, Chen, Otrok, Rebucci and Young, 2013; Walther, 2016; Jeanne and Korinek, 2019).

Our modeling approach draws on the global-games literature on equilibrium selection in coordination problems, which is commonly used to study bank runs and other withdrawal decisions with strategic complementarities (Carlsson and Van Damme, 1993; Morris and Shin, 1998, 2001; Goldstein and Pauzner, 2005; He and Xiong, 2012; Heider, Hoerova and Holthausen, 2015; He, Krishnamurthy and Milbradt, 2019; Liu, 2016). Empirical evidence supports the relevance of these coordination mechanisms in liquidity stress episodes (Chen, Goldstein, Huang and Vashishtha, 2024), and laboratory experiments show that global-games predictions align well with observed behavior

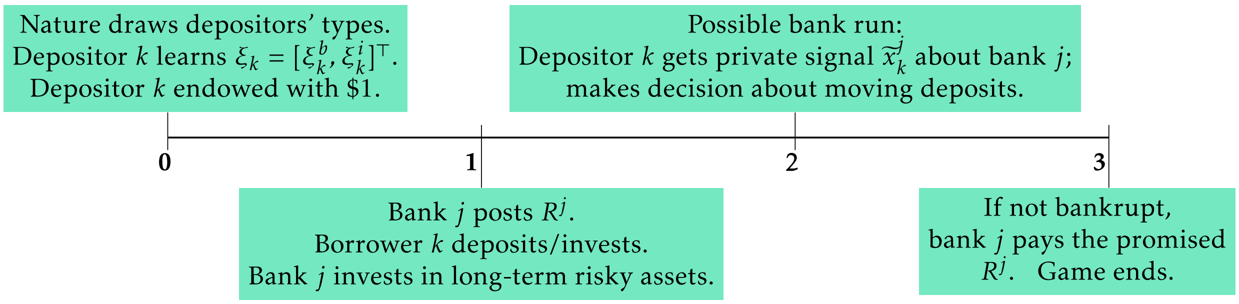

Figure 1: The timeline of the model.

(Heinemann, Nagel and Ockenfels, 2004, 2009).

Our study also speaks to the ongoing debate surrounding the regional banking crisis of 2023 (Kim, Kundu and Purnanandam, 2024; Jiang, Matvos, Piskorski and Seru, 2024; Chang, Cheng and Hong, 2023; Caglio, Dlugosz and Rezende, 2023; Cookson et al., 2023). The collapse of SVB and the regional banking crisis exemplify how perceived differences in safety and regulatory backstops can influence depositor behavior and exacerbate fragility, as illustrated by our findings.

3 Model Setup

Time is discrete, and there is a single type of a dollar-denominated consumption good, which serves as Numéraire. Throughout the paper, we use the notation $\widetilde{x}$ to denote the random variable and x for its realization.

3.1 Agents, Timeline and Action Space

There are two types of agents: banks and depositors. The economy consists of $\mathcal{N}$ regions, denoted by $\{1, 2, 3, \cdots, \mathcal{N}\}$. Each region i hosts a regional bank and a continuum of local depositors. In region i, depositors are indexed by $k \in [0, \mathcal{M}^i]$, where $\mathcal{M}^i$ denotes the mass of depositors in the region. Additionally, there is a national bank operating across all regions,[10] while regional banks operate exclusively within their home regions. Throughout the paper, we refer to the representative regional bank operating in region i as "bank i" and to the national bank as "bank b." We use "bank j" to refer to either bank i or bank b.

Time is discrete and consists of three periods, $t = 1, 2, 3$, preceded by a pre-game period $t = 0$, as illustrated in Fig. 1. We focus on the representative region i. At $t = 0$, each depositor in region i receives an initial endowment of \$1 and learns their type, denoted as $\xi_k = [\xi_k^b, \xi_k^i]^\top$. Depositors' types represent their private utility from banking services, which will be detailed in Section 3.3.2. Each bank j begins with an initial asset level E^j, which is normalized to zero for analytical simplicity, as it does not affect our main results.

[10]We abstract from the national bank's location decisions, as in d'Avernas et al. (2023), since this does not affect our main predictions.

At $t = 1$, bank j announces a deposit rate R^j, aiming to attract deposits for investment in long-term risky projects.[11] Banks also provide signals regarding their default risks (which in equilibrium are truthful). Based on this information, depositors in region i choose among three investment options: depositing with the risk-free security, the regional bank i, or the national bank b. The return on risk-free securities is normalized to 1 per period, effectively making it equivalent to holding cash.

At $t = 1^+$, after raising total deposits L^j (assumed to be the sole liability for bank j), the bank allocates its total assets between long-term risky projects and risk-free securities. Let H^j represent the amount invested in risk-free securities and A^j the amount allocated to long-term risky assets.

At $t = 2$, depositor k with positive deposits at bank j receives a private signal about the financial health of bank j. Based on these signals, depositors may withdraw some or all of their deposits. (In equilibrium, depositors either remain with bank j or withdraw all their deposits. If the latter occurs, the depositors are said to "run bank j.") Withdrawn funds can either be held as cash or transferred to another bank, such as bank b. For simplicity, we assume that bank b invests all net incoming deposits at $t = 2$ exclusively in risk-free securities and promises a financial return of 1. The primary benefit of transferring deposits to bank b instead of holding cash is the additional banking service utility.[12] The ability of depositors to transfer funds between banks represents a key innovation of our model. This added dimension of flexibility not only reflects empirical patterns observed during banking crises but also introduces a novel dynamic not previously captured in the literature. In particular, for depositors who value banking services, a safer bank within the same region provides a more convenient alternative than the less accessible risk-free security. The inflow of deposits from bank j to the safer bank further strengthens the latter's financial condition, increasing its appeal and intensifying the incentives for remaining depositors to run bank j.

At $t = 3$, the long-term risky projects initiated by banks at $t = 1$ mature. If the returns from these projects are insufficient, equity holders of bank j declares bankruptcy. Solvent banks pay the promised return R^j to depositors who stayed with the bank since $t = 1$ and a return of 1 to depositors who transferred to the bank at $t = 2$. In the event of bankruptcy, all remaining assets are liquidated and distributed pro rata among the bank's remaining debt holders. In equilibrium, banks may also default at $t = 2$ due to bank runs and the associated fire-sale costs. Agents then consume the payoff at $t = 3$.

[11]Our model assumes deposit contracts that mirror real-world practices without micro-founding their specific form. Prior literature suggests that more sophisticated contracts, such as state-contingent deposit rates, could mitigate bank-run risks (Allen and Gale, 2004).

[12]Relaxing this assumption does not affect the qualitative predictions of our model.

3.2 Regulation and Insurance

The U.S. banking sector is heavily regulated. Large national banks are subject to liquidity constraints, and all depository institutions are required to purchase Federal Deposit Insurance Corporation (FDIC) insurance. We model these regulatory requirements explicitly as follows:

- **Liquidity Regulation**: The national bank b must hold at least a fraction Γ of its total assets in risk-free securities at time $t = 1$.[13] This requirement implies $H^b \geq \Gamma(A^b + H^b)$.
- **FDIC Insurance**: Bank j is required to pay an insurance premium φ^j for each dollar of its liabilities at time $t = 1$. Therefore, $A^j + H^j = E^j + (1 - \varphi^j)L^j$.

If bank j defaults, the FDIC guarantees a payoff of Λ per dollar deposit to the depositors of bank j.[14] Here, Γ and φ^j are policy instruments that may depend on factors such as L^j, macroeconomic conditions, bank-specific productive technology, and banks' specific risks. We will analyze the equilibrium implications of different policy configurations.

3.3 Agents' Payoffs

Unlike Diamond and Dybvig (1983), agents in our model consume only at time $t = 3$, making their utility solely dependent on the payoff at $t = 3$. Early withdrawals occur only because depositors are concerned about potential bank defaults. Banks are operated by their equity owners. For simplicity, we refer to the bank j's equity holders as "banker j."

3.3.1 Bankers' Payoffs

Suppose bank j attracts total deposits L^j at $t = 1$ and allocates H^j to risk-free securities. The bank invests the remaining A^j in long-term risky projects, which mature at $t = 3$. Early liquidation of these risky assets at $t = 2$ incurs a fire-sale loss, whereas liquidating risk-free securities at $t = 2$ does not incur such a loss; selling one unit of risk-free securities always returns exactly one dollar. If a run occurs at $t = 2$, and the total deposit outflow

[13] This corresponds to regulatory requirements such as the Supplementary Leverage Ratio (SLR) or the Liquidity Coverage Ratio (LCR).

[14] In practice, FDIC deposit insurance covers up to $250,000 per depositor, per FDIC-insured bank, for each account ownership category. Depositors in our model are homogeneous, so we capture FDIC insurance with the parameter Λ. Our model of FDIC insurance aligns with the framework in Dávila and Goldstein (2023). Because regulators seldom vary protection offered to depositors across banks, we treat Λ as identical for all institutions.

from bank j is O^j, the financial return on bank j's total assets is given by[15]

$$(\widetilde{\theta} + \widetilde{\zeta}_j)G^j\left(A^j\right) - C^j\left(A^j, (O^j - H^j)^+\right) + (H^j - O^j)^+. \tag{1}$$

In Eq. (1), $\widetilde{\theta}$ represents aggregate macroeconomic uncertainty faced by all banks, such as interest rate risk, where larger realizations of θ indicate a better macroeconomic environment. The term $\widetilde{\zeta}_j$ captures bank-specific risk. In our baseline model, we focus on common economic shock and assume that $\widetilde{\zeta}_j = 0$ almost surely. The function $G^j : \mathbb{R}^+ \to \mathbb{R}^+$ represents bank j's production function, reflecting its investment opportunities. G^j is differentiable, with $\mathbb{E}[\widetilde{\theta}]G^{j\prime}(x) \geq 1$ weakly decreasing for all $x \geq 0$, capturing diminishing returns to scale as the bank's investment opportunities decline with size. When the total outflow O^j exceeds H^j, bank j must liquidate its long-term projects in the secondary market. This liquidation incurs a fire-sale cost, reducing the bank's asset value. The fire-sale cost is given by an exogenous function $C^j : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$. The function C^j is twice differentiable in both arguments and satisfies $C^j(A^j, 0) = 0$ for any $A^j \geq 0$. Furthermore,[16] $\partial_2 C^j\left(A^j, 0\right) > \mathbb{E}[(\widetilde{\theta})^+]G^{j\prime}(A^j)$, $\partial_2 C^j\left(A^j, 0\right) > 1$ and $\partial_2^2 C^j\left(A^j, x\right)$ is weakly increasing for $x \in [0, \infty)$.[17]

Depositors may also transfer deposits from other banks or from their holdings of the risk-free security to bank j at $t = 2$. Let I^j denote the total deposit inflow. Bank j can utilize this inflow either to meet withdrawal demands or to purchase additional risk-free securities. For simplicity, we assume that the only investment available at time $t = 2$ is a risk-free security with a fixed financial return of 1. Therefore, bank j offers a fixed deposit rate of 1 to depositors transferring their deposits at this stage. The final payoff for bank j at $t = 3$ is therefore given by

$$\mathcal{V}_j^3 \equiv \widetilde{\theta}G^j\left(A^j\right) - C^j\left(A^j, (O^j - I^j - H^j)^+\right) + (H^j + I^j - O^j)^+ - R^j(L^j - O^j) - I^j. \tag{2}$$

Banker j will declare bankruptcy if and only if $\mathcal{V}_j^3 \leq 0$. Let $\mathcal{D}_j \equiv \{\mathcal{V}_j^3 \leq 0\}$ denote the event that bank j defaults at or before $t = 3$, and let $\mathcal{D}_j^c \equiv \{\mathcal{V}_j^3 > 0\}$ denote the event that bank j does not default at or before $t = 3$.

3.3.2 Depositors' Payoffs

Let $\mathsf{s}_{k,j}$ denote the deposit made by depositor k with bank j at $t = 1$ and $\mathsf{s}_{k,H}$ their investment in the risk-free security, i.e. cash. We assume $\mathsf{s}_{k,H}, \mathsf{s}_{k,i}, \mathsf{s}_{k,b} \geq 0$. Resource constraint implies that $\mathsf{s}_{k,H} + \mathsf{s}_{k,i} + \mathsf{s}_{k,b} = 1$. At time $t = 2$, depositor k receives a signal $\widetilde{x}_k$

[15]For any real number x, we use the notation $(x)^+ = x\mathbb{1}_{[x\geq 0]}$ and $(x)^- = -x\mathbb{1}_{[x<0]}$. Here, $\mathbb{1}_{[\cdot]}$ is the indicator function. $\mathbb{1}_{[\mathscr{S}]} = 1$ if statement $\mathscr{S}$ is true, and $\mathbb{1}_{[\mathscr{S}]} = 0$ otherwise.

[16]For any function $f(x, y)$, let $\partial_2 f(x, y)$ denote $\frac{\partial}{\partial y}f(x, y)$, and ∂_2^2 indicates that this partial derivative is taken twice.

[17]For intuition of these functions, see the discussion below Assumption 4.

about the payoff-relevant state and determine how to reallocate her deposits across the risky banks and cash. Suppose that depositor k successfully transfers $\mathsf{r}_{k,\ell\to j}$ from bank ℓ to another investment option $j \in \{i, b, H\}/\{\ell\}$ at time $t = 2$. Then, the ex-post payoff of depositor k at $t = 3$ is given by:

$$
\begin{aligned}
\mathcal{U}_k^3 \equiv\ & \underbrace{(\mathsf{s}_{k,b} - \mathsf{r}_{k,b\to i} - \mathsf{r}_{k,b\to H})\Big((R^b + \xi_k^b)\mathbb{1}_{\mathcal{D}_b^c} + \Lambda\mathbb{1}_{\mathcal{D}_b}\Big) + (\mathsf{s}_{k,i} - \mathsf{r}_{k,i\to b} - \mathsf{r}_{k,i\to H})\Big((R^i + \xi_k^i)\mathbb{1}_{\mathcal{D}_i^c} + \Lambda\mathbb{1}_{\mathcal{D}_i}\Big)}_{\text{Payoff from deposits that stays with original bank since } t=1} \\
& + \underbrace{(\mathsf{r}_{k,i\to b} + \mathsf{r}_{k,H\to b})((1 + \chi^b\xi_k^b)\mathbb{1}_{\mathcal{D}_b^c} + \Lambda\mathbb{1}_{\mathcal{D}_b}) + (\mathsf{r}_{k,b\to i} + \mathsf{r}_{k,H\to i})((1 + \chi^i\xi_k^i)\mathbb{1}_{\mathcal{D}_i^c} + \Lambda\mathbb{1}_{\mathcal{D}_i})}_{\text{Payoff from deposits that moves to another bank at } t=2} \\
& + \underbrace{\mathsf{s}_{k,H} + \mathsf{r}_{k,i\to H} + \mathsf{r}_{k,b\to H} - \mathsf{r}_{k,H\to i} - \mathsf{r}_{k,H\to b}}_{\text{Payoff from holding the risk-free security}}. \qquad (3)
\end{aligned}
$$

Eq. (3) captures that the final payoff to depositor k depends on their initial deposits and investments at $t = 1$, their decisions regarding fund reallocations at $t = 2$, and whether banks default. If bank j defaults (i.e., event $\mathcal{D}_j$ occurs), the depositor receives only the FDIC insurance payout of Λ^j per dollar remaining at bank j at the end of period $t = 2$, payable at $t = 3$. The term ξ_k^j in Eq. (3) represents depositor k's private utility derived from banking services at bank j. Depositors have heterogeneous private utilities determined by their individual types: depositor k's type $\xi_k = [\xi_k^b, \xi_k^i]^\top$ is determined at $t = 0$, and the probability distribution of depositor types across all regions is assumed to be independent and identically distributed (i.i.d.) with a common density function f_ξ. (While banks are assumed to know this density function, depositors need not possess such knowledge.) For each dollar deposited at bank j from time $t = 1$ through $t = 3$, depositor k receives not only the financial return R^j but also the non-monetary banking service utility ξ_k^j, provided bank j does not default (i.e., event $\mathcal{D}_j^c$ occurs).

Suppose depositor k transfers an amount $\mathsf{r}_{k,\ell\to j}$ of deposits from bank ℓ to bank j at $t = 2$. If bank j does not default, it pays a rate of 1, and thus depositor k receives a financial payoff of $\mathsf{r}_{k,\ell\to j}$. Depositors also derive non-monetary utility from banking services; however, this service utility may depreciate since depositors only transfer to the bank at stage $t = 2$. Consequently, we assume that the per-dollar service utility at $t = 2$ for depositor k who move to bank j is $\chi^j\xi_k^j$, where $\chi^j \in [0, 1]$.[18]

If bank j does not default, depositors retain the flexibility to withdraw all their deposits at any time. However, due to fire-sale losses incurred when banks liquidate long-term risky assets during a bank run, banks may lack sufficient liquid resources to

[18] Large national banks are valued for their convenience and standardized services, whereas regional banks provide localized support that relies on sustained relationships. In our numerical example, we set $\chi^b = 1$ and $\chi^i = 0$. Other values of χ do not qualitatively alter our results.

meet all withdrawal requests at $t = 2$. As in Goldstein and Pauzner (2005), we assume that the bank must adhere to a **sequential service constraint**: depositors are served in the order they appear until the bank's liquid resources are depleted. Depositors running the bank arrive in a random order, making each depositor equally likely to be served. Therefore, if the bank's maximum amount of withdrawals it can service before its resources are exhausted is $\mathbb{O}^j$, and the total deposit outflow from bank j is O^j, then probability that a depositor can fully withdraw their funds is given by: $\mathtt{M}^j \equiv \min\left(\frac{(\mathbb{O}^j)^+}{O^j}, 1\right)$. With probability $1 - \mathtt{M}^j$, the bank fails before the depositor is served, leaving the depositor unable to collect any funds directly from the bank. In this case, the depositor receives only the FDIC insurance payout of Λ per dollar, payable at $t = 3$.

Because depositors have linear utilities, their optimal strategies always result in a bang-bang solution, where they choose the best available option based on their information set. Linear utility also implies that depositors either remain with bank j or attempt to withdraw all their deposits at $t = 2$. Due to the sequential service constraint, if depositor k initiate transfer request $\mathsf{r}_{k,j\to\ell} = \mathsf{s}_{k,j}$ to investment option ℓ at $t = 2$, the following outcomes occur:

$$\mathsf{r}_{k,j\to\ell} = \begin{cases} \mathsf{s}_{k,j} & \text{with probability } \mathtt{M}^j; \\ 0 & \text{with probability } 1 - \mathtt{M}^j. \end{cases}$$

We restrict attention to parameter values for which the ex-ante default probabilities of both banks are *sufficiently small* (details provided later), so that no depositor chooses the risk-free asset at $t = 1$.

3.4 Agents' Information

Our information structure builds on the global game framework, and we focus on the limiting equilibrium as agents' information becomes increasingly precise. In this section, we explicitly state our assumptions regarding the information structure. This explicitness is important for two reasons: (1) To the best of our knowledge, our assumptions represent the most relaxed conditions in the global game literature that ensure the uniqueness of the limiting equilibrium. (2) By presenting these assumptions clearly, we aim to benefit researchers exploring other applications of global games, particularly in scenarios where canonical assumptions may not apply, but our framework may prove more suitable.

Bank j's default decision depends on the realization of one random variable, $\widetilde{\theta}$ which captures all the payoff-relevant uncertainty in this model. The banks' payoff structure ensures that, regardless of depositors' decisions, bank j defaults when $\widetilde{\theta} \le 0$ (even if no depositors run the bank) and does not default when $\widetilde{\theta} \ge \overline{\Theta}^j$ for some $\overline{\Theta}^j > 0$ (even if all depositors run the bank). Let $\overline{\Theta} = \sup\{\overline{\Theta}^j\}$.

Assumption 1. The random variable $\widetilde{\theta}$ has a probability density function $f_{\widetilde{\theta}}$. We assume that $f_{\widetilde{\theta}}$ is Lipschitz continuous on the interval $[0, \overline{\Theta}]$.

At time $t = 2$, each depositor k in region i gets a private signal $\widetilde{x}_k$ about the economic state $\widetilde{\theta}$. The private signal $\widetilde{x}_k$ about $\widetilde{\theta}$ is given by

$$\widetilde{x}_k = \widetilde{\theta} + \sigma \widetilde{\epsilon}_k \tag{4}$$

where $\sigma > 0$ is a constant. As in other global game models, we are interested in studying the limiting equilibrium as σ approaches zero. However, unlike most previous models, we impose minimal restrictions on the distributions of $\widetilde{\theta}$ and $\{\widetilde{\epsilon}_k\}$, allowing them to have essentially arbitrary PDFs. Without loss of generality, we assume that $\mathbb{E}[\widetilde{\epsilon}_k] = 0$. Also, the $\operatorname{ess\,sup}_k\{\|f_{\widetilde{\epsilon}_k}\|_\infty\}$ is finite $F_{\widetilde{\xi}_k}$–almost surely.

Assumption 2. $\widetilde{\theta}$ and $\{\widetilde{\epsilon}_k\}$ are independent. All agents have common knowledge about the distributions of these random variables and the information structure.

Compared with the existing literature, our setting allows each depositor to have her own idiosyncratic private-signal distribution with density $f_{\widetilde{\epsilon}_k}$. This information structure is not only more empirically plausible but also expands the scope of global-game applications to environments in which relaxing canonical distributional assumptions is essential.

4 Subgame Equilibrium: Bank Run at $t = 2$

At time $t = 2$, depositor k in region i with existing investment in option $j \in \{i, b, H\}$ solves:

$$\begin{aligned}
\max_{\left\{\substack{\mathsf{a}_{k,j\to H},\, \mathsf{a}_{k,j\to l},\\ l\in\{i,b\}, l\neq j}\right\}} \mathcal{U}_k^2 \equiv & \mathbb{E}\left[\mathcal{U}_k^3 \,\middle|\, \widetilde{x}_k = x_k\right] \\
\text{subject to} \quad & 0 \le \mathsf{a}_{k,j\to H},\ \mathsf{a}_{k,j\to l}, \qquad \mathsf{a}_{k,j\to H} + \mathsf{a}_{k,j\to l} \le \mathsf{s}_{k,j} \\
& \mathbb{P}(\mathsf{r}_{k,j\to H} = \mathsf{a}_{k,j\to H}) = 1 - \mathbb{P}(\mathsf{r}_{k,j\to H} = 0) = \mathsf{M}^j \\
& \mathbb{P}(\mathsf{r}_{k,j\to l} = \mathsf{a}_{k,j\to l}) = 1 - \mathbb{P}(\mathsf{r}_{k,j\to l} = 0) = \mathsf{M}^j.
\end{aligned} \tag{5}$$

By Exact Law of Large Number (ELLN) stated in Sun (2006), the total mass of deposits intended to be withdrawn from regional bank i is

$$O^i = \mathcal{M}^i \int_{\xi_k} (\mathsf{a}_{k,i\to H} + \mathsf{a}_{k,i\to b}) \mathbb{1}_{[\mathsf{s}_{k,i,1} > 0]} dF_{\widetilde{\xi}}(\xi_k). \tag{6}$$

The total mass of deposits intended to be withdrawn from national bank b is

$$O^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} (\mathsf{a}_{k,b\to H} + \mathsf{a}_{k,b\to i}) \mathbb{1}_{[\mathsf{s}_{k,b,1}>0]} dF_{\widetilde{\xi}}(\xi_k). \tag{7}$$

Bank j defaults if and only if economic state $\widetilde{\theta} \in \mathcal{D}_j$ where

$$\mathcal{D}_j = \left\{ (\widetilde{\theta}) G^j\left(A^j\right) - C^j\left(A^j, (O^j - I^j - H^j)^+\right) + (H^j + I^j - O^j)^+ - R^j(L^j - O^j) - I^j \le 0 \right\}. \tag{8}$$

By ELLN, the total mass of deposits inflow to regional bank i is

$$I^i = \mathcal{M}^i \int_{\xi_k} \mathtt{M}^b \mathsf{a}_{k,b\to i} \mathbb{1}_{[\mathsf{s}_{k,b,1}>0]} dF_{\widetilde{\xi}}(\xi_k). \tag{9}$$

The total mass of deposits inflow to regional bank b is

$$I^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \mathtt{M}^i \mathsf{a}_{k,i\to b} \mathbb{1}_{[\mathsf{s}_{k,i,1}>0]} dF_{\widetilde{\xi}}(\xi_k). \tag{10}$$

Sequential service constraint. Define bank j's total payout capacity, $\mathbb{O}^j(\theta)$, as the maximum amount of withdrawals it can service before its resources are exhausted—i.e., the unique value that solves:[19]

$$\theta G^j(A^j) - C^j(A^j, \mathbb{O}^j(\theta) - I^j - H^j) = 0 \qquad \forall j \in \{i, b\}. \tag{11}$$

Bank j can only pay out a total of $\mathbb{O}^j$ to depositors before depleting its liquid resources. Therefore, in equilibrium, the probability that a depositor at bank j can fully withdraw is given by

$$\mathtt{M}^j = \min\{\frac{(\mathbb{O}^j)^+}{O^j}, 1\}, \qquad \forall j \in \{i, b\}. \tag{12}$$

Definition 1. *For a fixed noise level $\sigma > 0$, a **noisy subgame equilibrium** consists of a strategy profile (with depositor actions $\mathsf{a}_{k,j\to l}$ and bank default strategies $\mathcal{D}_j$) and a set of outcome variables (withdrawals O^j and inflows I^j) that simultaneously satisfy the core conditions in Eqs. (5) to (10). A **noisy subgame equilibrium with sequential service constraint** must also satisfy the additional friction constraints from Eqs. (11) and (12).*

Definition 2. *The **Benchmark Equilibrium of Connected Bank Run** is an equilibrium of*

[19] $\mathbb{O}^j$ is well defined because $\partial_2 C(A^j, \cdot) > 0$.

*the complete-information game (i.e., $\sigma = 0$) that is selected by taking the limit of a sequence of noisy subgame equilibria as the signal noise vanishes, $\sigma \downarrow 0$. The **Benchmark Equilibrium with a Sequential Service Constraint** is defined analogously as the selected equilibrium induced by the limit of a sequence of noisy subgame equilibria with the sequential service constraint as $\sigma \downarrow 0$.*

Definition 2 introduces the subgame equilibrium for our **Benchmark Economy of Connected Bank Run**. A central contribution of this paper is to highlight the new equilibrium dynamics that arise from depositor mobility across banks. To underscore these novel dynamics, we juxtapose our benchmark with the classical global-games setting, where withdrawing into cash is the sole outside option (Morris and Shin, 2004; Goldstein and Pauzner, 2005). Our framework nests that setting, allowing us to conveniently formalize an **Alternative Economy of Isolated Bank Run** in which the run option is fixed to cash. Throughout the paper we compare equilibrium predictions under these two environments, thereby quantifying how cross-bank deposit reallocations amplify or dampen fragility.

Definition 3. *In the $t = 2$ subgame of the **Alternative Economy of Isolated Bank Run**, depositors are* prohibited *from transferring funds between banks. Formally, each depositor k faces the constraint*

$$\mathrm{a}_{k,i\to b} = \mathrm{a}_{k,b\to i} = 0, \quad \forall\, k, i. \tag{13}$$

*The equilibrium of the **Alternative Economy of Isolated Bank Run** consists of the variables defined in Definition 2, subject to the additional constraint Eq. (13).*

4.1 Analysis of Isolated Bank Run

4.1.1 Classic result without liquidity buffer or sequential service constraint

Consider any regional bank i. To establish a clear benchmark, we first analyze a baseline setting that reimposes three standard constraints: (1) there is no cross-bank deposit mobility; (2) the representative bank j holds no liquidity buffer ($H^i = 0$); and (3) the sequential service constraint is absent, such that all depositors who run believe that they will be served ($\mathtt{M}^i = 1$). In this environment, we show that the subgame of the **Alternative Economy of Isolated Bank Run** (Definition 3) has a unique limiting equilibrium as the noise level vanishes ($\sigma \downarrow 0$). This finding confirms that our framework nests the classic predictions of the global game model.

Lemma 1. *Suppose that $\partial_2 C(A^i, 0) \geq R^i > 1$ and $H^i = 0$. For any PDF $f_{\overline{\theta}}$ and PDF family*

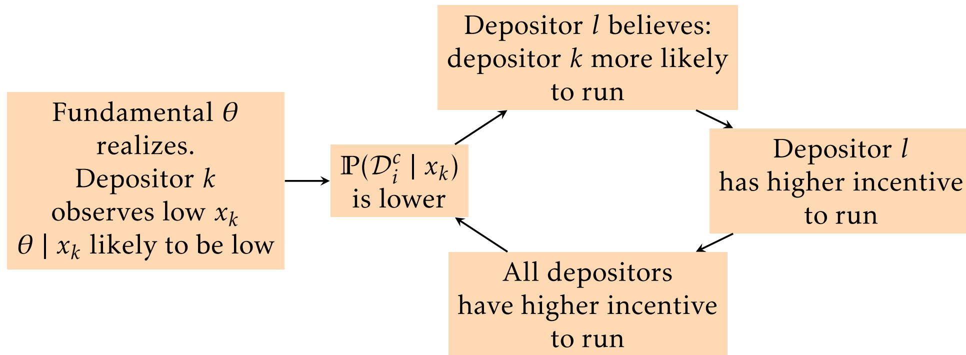

Figure 2: The feedback loop in a bank run stemming from strategic complementarity.

$\{f_{\widetilde{\epsilon},k}, \forall k\}$ that satisfy the previously stated assumptions, there exists a unique limiting subgame equilibrium as $\sigma \to 0$. In this limiting equilibrium:

1. *All depositors stay with bank i if realization of $\theta > \theta_B^i$ and run the bank if $\theta < \theta_B^i$.*
2. *$\theta_B^i \equiv \frac{C^i\left(A^i, O_B^i\right) + R^i(L^i - O_B^i)}{G^i\left(A^i\right)}$ where O_B^i is a constant given by $O_B^i \equiv \mathcal{M}^i \int_{\xi_k} \frac{(1-\Lambda)s_{k,i}}{R^i + \xi_k^i - \Lambda} dF_{\widetilde{\xi}}$.*
3. *The equilibrium can be solved by iteratively eliminating dominated strategies.*[20]

Proofs of all results, including this lemma, are provided in Internet Appendix 1.[21] Intuitively, whenever the common state is low enough, a fraction of depositors get low enough signals that trigger them to run the bank, which then mobilize all depositors to run the bank in a feedback loop captured by Fig. 2. Here, we highlight several innovations in our proof techniques which may be valuable for other applications involving games with strategic complementarities: (1) Different applications of global game models may impose natural restrictions on the probability distributions of $\widetilde{\theta}$ and $\widetilde{\epsilon}_k$. In our proof, we maximize flexibility for these distributions, allowing for a broader range of applications. (2) Our proof also does not require the assumptions of *Payoff continuity* and *Bounded derivatives*, thus extending the applicability of global game models to settings with more flexible payoff structures.

4.1.2 The Effect of the Sequential Service Constraint

Starting from Goldstein and Pauzner (2005), recent literature highlights that realistic bank-run models do not exhibit global strategic complementarities. This is a condition where any one depositor's withdrawal invariably increases the incentive for all others to run.

[20] For a sufficiently small σ, all possible equilibria are bounded by two extreme sequential equilibria. These two extreme sequential equilibria can be derived through iterative elimination of dominated strategies. The distance between them, defined as the maximal difference across their corresponding equilibrium strategies, converges to zero as $\sigma \to 0$, ensuring uniqueness of the equilibrium outcome in the limit.

[21] Please find the online appendix here

Instead, the sequential service constraint induces a non-monotonic relationship between total outflows and the incentive to run.

Initially, as withdrawals from bank i increase, so does the perceived risk of failure, encouraging more depositors to join the run. However, this incentive reverses once the queue of depositors becomes sufficiently long. Specifically, the probability of an individual depositor being served, $\mathtt{M}^i = \min\left(\frac{\mathbb{O}^i}{O^i}, 1\right)$, is decreasing in the size of the outflow, all else equal. Consequently, the expected payoff from joining a long queue diminishes, discouraging depositors to run and thereby breaking the global strategic complementarity. Our proof is robust to this feature. However, there is no closed-form characterization of the limiting equilibrium.

To build intuition and facilitate direct comparison with the existing literature, we focus on a more tractable special case. Specifically, we study a setting in which all depositors are ex-ante homogeneous before receiving their private signals: they share the same banking utility and draw private signals from an identical distribution (that is, $\xi_k^i = \xi^i$ and $f_{\widetilde{\epsilon}_k} = f_{\widetilde{\epsilon}}$ for all k). This simplification yields a much clearer characterization of the limiting equilibrium. (For the general result without this homogeneity assumption, see Lemma 6 in Internet Appendix 1.2.) This setting generalizes the environments studied by Goldstein and Pauzner (2005); Elenev et al. (2021) and others, in which the private-signal density $f_{\widetilde{\epsilon}}$ is restricted to be the density of a uniform distribution.

Lemma 2. *Suppose $H^i = 0$ and $\partial_2 C(A^i, 0) \geq R^i > 1$. In addition, assume that depositors in region i are ex-ante homogeneous, so that $\xi_k^i = \xi^i$ and $f_{\widetilde{\epsilon}_k} = f_{\widetilde{\epsilon}}$ for all k, where $f_{\widetilde{\epsilon}}$ is an arbitrary probability density function with sufficiently large support. Then, as the signal noise vanishes, $\sigma \to 0$, there exists at least one limiting subgame equilibrium. In any such limiting equilibrium:*

1. *All depositors **stay** if the fundamental $\theta > \theta_S^i$ and **run** if $\theta < \theta_S^i$. The run threshold θ_S^i is weakly less than θ_B^i.*

2. *The run threshold θ_S^i is given by $\theta_S^i \equiv \frac{C^i\left(A^i, O_S^i\right) + R^i\left(L^i - O_S^i\right)}{G^i\left(A^i\right)}$, where the quantities O_S^i and $\mathbb{O}_S^i$ are the solution to the system of equations*

$$\frac{1-\Lambda}{R^i + \xi^i - \Lambda} + \log(L^i) - \log\left(\mathbb{O}_S^i\right) = \frac{O_S^i}{\mathbb{O}_S^i},$$
$$C^i\left(A^i, O_S^i\right) + R^i\left(L^i - O_S^i\right) = C^i\left(A^i, \mathbb{O}_S^i\right). \tag{14}$$

3. *The equilibrium cannot be obtained by iterative elimination of dominated strategies.*

4. *The limiting equilibrium is unique if Eq. (14) has a unique solution.*[22]

[22]Although this condition holds for a broad class of cost functions C^i, fully characterizing the conditions

Remark. Comparing Lemma 1 and Lemma 2, we find that the Sequential Service Constraint quantitatively lowers the bank-run cutoff, but the model's qualitative predictions remain unchanged. However, the equilibrium with the Sequential Service Constraint departs from the standard global-games framework in a crucial way—it is *not solvable* by iterative elimination of dominated strategies. Instead, it requires all depositors to share common knowledge of the entire strategy space and to correctly conjecture the strategies of others when the signal noise $\sigma > 0$. This represents a demanding rationality assumption, especially for a dispersed group of depositors.

From a policy perspective, the simplified model without the Sequential Service Constraint may be more prudent, as it relies on weaker rationality assumptions. Moreover, since the run cutoff under the simplified model is weakly larger than under the model with the Sequential Service Constraint, the former provides a more cautious and robust basis for policy design. This motivates a simplifying assumption for the remainder of our analysis.

Assumption 3. For the remainder of our analysis, we assume that in games with $\sigma > 0$ depositors ignore the Sequential Service Constraint and behave as if they can always withdraw successfully (i.e., $\mathtt{M}^i = 1$).

This choice is justified on three grounds. First, it is necessary for **tractability**, as the subgame equilibrium has no closed-form solution in Lemma 2 and becomes unwieldy in extensions with cross-bank deposit mobility, which is the main focus of this paper. Second, it may be more **behaviorally realistic**, reflecting bounded rationality. Third, the assumption is consistent with the fully rational outcome in the limiting equilibrium: at the run threshold θ_B^i, the sequential service constraint would **not** bind ($O_B^i < \mathbb{O}(\theta_B^i)$), so $\mathtt{M}^i = 1$ always holds when $\sigma = 0$.

4.1.3 Effects of Liquidity Regulation: $H^j > 0$

When banks must hold a stock of risk-free assets ($H^j > 0$), the analysis becomes richer: for small outflows ($O^j < H^j$) no fire-sale loss is incurred and the bank saves $(R^j - 1)O^j$ on foregone liabilities, lowering its failure probability and inducing *strategic substitutability*. To rule out degenerate cases in which the bank would prefer to induce universal early withdrawal, we assume that paying all depositors early is strictly more costly than continuing to maturity: $C^j(A^j, L^j - H^j) + H^j > R^j L^j$.[23] Our results show that strategic complementarity dominates when H^j is small, and substitutability dominates when H^j is large, but in both cases a *unique limiting sub-game equilibrium* exists.

that guarantee a unique solution is beyond the scope of this paper. One sufficient condition is that $\partial_2^3 C^i$ is sufficiently large. See Internet Appendix 1.3 for the proof of the lemma.

[23]As a microfoundation, when R^j is chosen endogenously at $t = 1$, the inequality always holds in equilibrium: bank j will not choose an R^j that violates it.

In our model, bank j is *fundamentally insolvent* when its asset payoff cannot meet promised liabilities without fire sales; we term such an episode **fundamental default**. Definition 4 formalizes the upper bound on the set of fundamental-run states. Even when the bank would remain fundamentally solvent, a coordination failure among depositors can still provoke early withdrawals, trigger fire sales, and lead to default; we label this outcome a **coordination default** or a **bank-run default**. Theorem 1 in this section shows how macroprudential tools shift the fundamental-run and coordination-run boundaries, thereby revealing a policy trade-off.

Definition 4 (**Fundamental Default Boundary**). *For each bank $j \in \{b, 1, 2, \cdots, \mathcal{N}\}$ and some $H \geq 0$, the* ***Fundamental Default Boundary*** *is the unique constant $\widehat{\theta^j}(H)$ such that $\widehat{\theta^j}(H)G^j(A^j) + H^j - R^j L^j = 0$.*

For each bank j, let O_B^j be the constant defined in Lemma 1. Define $\widehat{H^j}$ to be the unique constant such that $C^j(A^j, O_B^j - \widehat{H^j}) + \widehat{H^j} = R^j O_B^j$. By Assumption 4, $\widehat{H^j}$ is well-defined and $\widehat{H^j} < O_B^j$.

Lemma 3. *Suppose that for bank j, $\partial_2 C(A^j, 0) \geq R^j > 1$ and $0 < H^j < L^j$. There is a unique limiting subgame equilibrium as $\sigma \to 0$.*

1. *If $H^j < \widehat{H^j}$, then in this limiting equilibrium, all depositors will stay with bank j if the realization $\theta > \theta_B^j(H^i)$, and will run the bank if $\theta < \theta_B^j(H^i)$ where $\theta_B^j(H^j) \equiv \frac{C^j\left(A^i, O_B^j - H^j\right) + R^j(L^j - O_B^j)}{G^j\left(A^j\right)}$. The constant O_B^j is defined in Lemma 1 for $j \neq b$; if $j = b$, $O_B^b = \sum_i \mathcal{M}^i \int_{\xi_k} \frac{(1-\Lambda)\, s_{k,b}}{R^b + \xi_k^b - \Lambda}\, dF_{\xi}$.*
2. *If $H^j \geq \widehat{H^j}$, then in this limiting equilibrium,*[24] *all depositors will stay with bank j if the realization $\theta > \widehat{\theta^j}(H^j)$, and will run the bank if $\theta < \widehat{\theta^j}(H^j)$ where $\widehat{\theta^j}(H^j)$ is the Fundamental Default Boundary for bank j.*

Although Lemma 3 studies effects of banks' liquid buffer H, our framework also extends to incorporate explicit government guarantees. If depositors of bank b anticipate government capital injections amounting to H^b during a banking crisis, their equilibrium strategies mirror those characterized in Lemma 3.

Our analysis shows that liquidity requirements can mitigate coordination runs and, if sufficiently stringent (i.e., $H^j \geq \widehat{H}^j$), can eliminate them altogether. However, mandating very high liquidity is generally inefficient because safe assets typically earn less than risky

[24] In this case, the limit is taken so that, for every state realization, except possibly on a measure-zero subset of $(-\infty, \widehat{\theta^j})$, the equilibrium strategy profile converges to the noiseless equilibrium strategy profile.

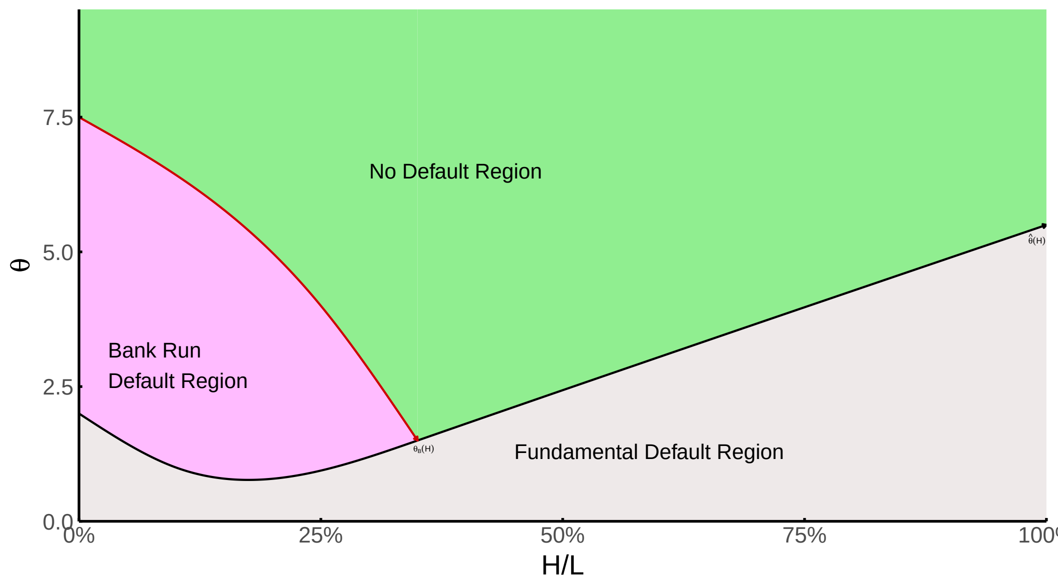

Figure 3: Fix R^b. A larger fraction of cash buffer H/L dampens **coordination default risk** but lowers lending efficiency, eventually raising **fundamental/overall default risk**.

investments. Empirical evidence indicates that once Basel III liquidity and leverage constraints became binding, large U.S. banks withdrew from otherwise profitable trades, such as covered–interest–parity (CIP) arbitrage (Du, Tepper and Verdelhan, 2018; Anderson, Du and Schlusche, 2021). The same constraints have been linked to a measurable contraction in loan supply and a decline in overall return on assets (Ben Naceur, Pépy and Roulet, 2017; Basel Committee on Banking Supervision, 2022). By depressing profitability, an overly large liquid-asset buffer raises the fundamental solvency boundary for a given level of liabilities. Thus, higher liquidity can *counterintuitively* increase the risk of bank failure (see Fig. 3). To formalize this channel, we consider parameter values under which fire-sale costs are sufficiently large that the liquidity threshold $\widehat{H}^j$ is high and, at the corresponding fundamental default boundary $\widehat{\theta}^j(\widehat{H}^j)$, bank j is marginally more productive when investing in the risky asset than when holding cash, that is, $\widehat{\theta}^j(\widehat{H}^j)G^{j\prime}(L^j - \widehat{H}^j) > 1$ in the next theorem.

Theorem 1. *Consider a national bank b that operates in isolation with zero equity and total liabilities L^b, promising a gross return R^b. Regulation requires the bank to hold safe assets of amount H^b, while the remainder $A^b = L^b - H^b$ is invested in risky assets. The macro-prudential policy that minimizes the overall probability of default is weakly less than $\widehat{H^b} < O_B^b$.*

Policy discussions following the 2023 banking crisis have underscored how rapid and large-scale deposit withdrawals in modern banking challenge the adequacy of existing liquidity regulations, including the Liquidity Coverage Ratio (LCR). The collapse of banks like SVB, where liquid assets evaporated in a matter of hours, challenges the LCR's assumption of a 30-day outflow horizon. In response, many policy recommendations advocate for significantly stronger liquidity buffers or improved access to lender-of-last-resort (LoLR) facilities. Notably, a January 2024 Group of Thirty (G30) report recommends that banks pre-position enough collateral at central bank discount windows to cover *all runnable liabilities*.[25] In our model, such a policy corresponds to setting H_B^b to be close to L_B^b, i.e., a nearly full liquidity buffer. In contrast to the simple intuition that increased liquidity buffer enhance bank stability, our Theorem 1 shows that this extreme requirement is **suboptimal** for minimizing bank-run risk. The optimal required holding of liquid assets for promoting financial stability lies well below the full-liability threshold and even the projected deposit outflow O_B^b.

Since large national banks face more stringent regulatory requirements and are substantially more likely to receive liquidity injections or government support during crises, we normalize $H^i = 0$ for all regional banks, an assumption that will be microfounded later, while allowing policymakers to set a H^b for the large national bank.[26]

4.2 Connected Bank Run: Equilibrium

Most U.S. banks compete with other banks within the same geographic area. This local competition implies that depositors perceive nearby banks as partial substitutes. Our next set of results demonstrates how the presence of a national bank—benefiting from regulatory support or implicit government guarantees—affects the fragility of regional banks operating in the same market. We begin with the case $\mathcal{N} = 1$: a single regional bank i competing with one national bank b. The regional bank holds zero safe assets ($H^i = 0$) and is more fragile than the national bank in the isolated economy, $\theta_B^i > \theta_B^b(H^b)$.

As shown in Section 4.1, classic global game models guarantee a unique equilibrium by exploiting the one-dimensional structure of depositor decisions: each depositor employs a (nearly) monotone switching strategy governed by a single-threshold rule on the noisy private signal. Small noise in these private signals ensures that only the one that is robust to infinitesimal perturbations survives in the limit. However, with cross-bank deposit mobility, this one-dimensional logic breaks down. In this richer setting, beliefs and fragility are mutually reinforcing across banks: if depositors flee one bank en masse and redirect their funds to another, they inadvertently strengthen the recipient bank, entwining beliefs about solvency across institutions. The resulting equilibrium problem is a high-dimensional fixed-point system that lacks the monotonicity or single-crossing

[25]Group of Thirty an international forum of central bankers, economists and private financiers. Similar proposals can be found here and here.

[26]Theorem 5 microfounds the choice $H^i = 0$. In the rest of the paper, we suppress notation and write $\theta_B^i(0), \widehat{\theta^i}(0)$ as $\theta_B^i, \widehat{\theta^i}$ respectively.

properties exploited in classic models. It is therefore striking that, even in this enriched setting, *no more than three limiting equilibria* can arise, as we show below. We also identify conditions under which the standard global-games refinement continues to pin down a single outcome. Because we focus on bank-run crises, we exclude reallocations at $t = 2$ that arise for reasons unrelated to a run—for example, if a high ex-ante failure probability combined with a low deposit rate at the regional bank prompts depositors to place funds with the national bank at $t = 1$ and transfer back once the realized state is sufficiently high at $t = 2$. Formally, for every bank pair j, ℓ and depositor k, we assume $R^j + \xi_k^j \geq 1 + \chi^\ell \xi_k^\ell$, so that depositors move funds only in anticipation of a run. Before stating the main result in Theorem 2, we define the key equilibrium objects.

Lemma 4. *There exists a unique fixed point* $\left(\overline{I}_D^b, \underline{\theta}_D^b(H^b), \overline{O}_D^i, \overline{\theta}_D^i\right)$ *that solves*

$$\begin{aligned}
\overline{I}_D^b &= \mathcal{M}^i \int_{\xi_k} \frac{\mathbb{O}^i\left(\underline{\theta}_D^b(H^b)\right)}{L^i} \frac{\mathsf{s}_{k,i}\chi^b \xi_k^b}{1+\chi^b\xi_k^b - \Lambda}\, dF_{\bar{\xi}}(\xi_k), \\
\underline{\theta}_D^b(H^b) &= \max\left\{ \frac{C^b\left(A^b, (O_B^b - \overline{I}_D^b - H^b)^+\right) - (H^b + \overline{I}_D^b - O_B^b)^+ + R^b(L^b - O_B^b) + \overline{I}_D^b}{G^b(A^b)}, \widehat{\theta}^b(H^b)\right\}, \qquad (15)\\
\overline{O}_D^i &= \mathcal{M}^i \int_{\xi_k} \frac{\mathsf{s}_{k,i}\,(1-\Lambda+\chi^b\xi_k^b)}{R^i+\xi_k^i-\Lambda}\, dF_{\bar{\xi}}(\xi_k), \qquad \overline{\theta}_D^i = \frac{C^i\left(A^i, \overline{O}_D^i\right) + R^i(L^i - \overline{O}_D^i)}{G^i(A^i)}.
\end{aligned}$$

Moreover, there exists a unique fixed point $\left(\overline{I}_D^i, \underline{\theta}_D^i, \overline{O}_D^b, \overline{\theta}_D^b\right)$ *that solves*

$$\begin{aligned}
\overline{I}_D^i &= \mathcal{M}^i \int_{\xi_w} \frac{\mathbb{O}^b\left(\underline{\theta}_D^i\right)}{L^b} \frac{\mathsf{s}_{w,b}\chi^i \xi_w^i}{1+\chi^i\xi_w^i - \Lambda}\, dF_{\bar{\xi}}(\xi_w), \\
\underline{\theta}_D^i &= \max\left\{ \frac{C^i\left(A^i, (O_B^i - \overline{I}_D^i)^+\right) - (\overline{I}_D^i - O_B^i)^+ + R^i(L^i - O_B^i) + \overline{I}_D^i}{G^i(A^i)}, \widehat{\theta}^i\right\}, \qquad (16)\\
\overline{O}_D^b &= \mathcal{M}^i \int_{\xi_w} \frac{\mathsf{s}_{w,b}\,(1-\Lambda+\chi^i\xi_w^i)}{R^b+\xi_w^b-\Lambda}\, dF_{\bar{\xi}}(\xi_w), \qquad \overline{\theta}_D^b = \frac{C^b\left(A^b, (\overline{O}_D^b - H^b)^+\right) - (H^b - \overline{O}_D^b)^+ + R^b(L^b - \overline{O}_D^b)}{G^b(A^b)}.
\end{aligned}$$

Theorem 2. *Assume* $\mathcal{N} = 1$ *and* $R^j \leq \partial_2 C^j(A^j, 0)$ *for every bank* j*. Let* $\left(\overline{I}_D^b, \underline{\theta}_D^b(H^b), \overline{O}_D^i, \overline{\theta}_D^i\right)$ *denote the unique fixed point that solves Eq. (15), and let* $\left(\overline{I}_D^i, \underline{\theta}_D^i, \overline{O}_D^b, \overline{\theta}_D^b(H^b)\right)$ *denote the unique fixed point that solves Eq. (16).*

(i) ***Baseline limiting equilibrium (regional-to-national flight).*** *The* $t = 2$ *subgame* ***always*** *admits a limiting equilibrium as* $\sigma \downarrow 0$*. In the limit, national-bank depositors stay when* $\theta > \underline{\theta}_D^b(H^b)$ *and run to the safe asset when* $\theta < \underline{\theta}_D^b(H^b)$*. Regional-bank*

depositors stay when $\theta > \overline{\theta}^i_D$, switch to the national bank when $\underline{\theta}^b_D(H^b) < \theta < \overline{\theta}^i_D$, and run to the safe asset when $\theta < \underline{\theta}^b_D(H^b)$.

(ii) ***Possible second limiting equilibrium (national-to-regional flight).*** *If $\overline{\theta}^b_D(H^b) \geq \underline{\theta}^i_D$, there exists an additional limiting equilibrium in which regional-bank depositors stay when $\theta > \underline{\theta}^i_D$ and run to the safe asset when $\theta < \underline{\theta}^i_D$, while national-bank depositors stay when $\theta > \overline{\theta}^b_D(H^b)$, switch to the regional bank when $\underline{\theta}^i_D < \theta < \overline{\theta}^b_D(H^b)$, and run to the safe asset when $\theta < \underline{\theta}^i_D$. If $\overline{\theta}^b_D(H^b) < \underline{\theta}^i_D$, this second limiting equilibrium does not arise.*

(iii) ***Possible knife-edge limiting equilibrium.*** *If $\overline{\theta}^b_D(H^b) \geq \underline{\theta}^i_D$ and for every bank-b depositor type ξ_w, the c.d.f. $F_{\tilde{\epsilon}_w}$ is log-concave, then there may exist a knife-edge limiting equilibrium where both banks share the same run/stay cutoff $\theta^\star$ such that $\theta^i_B > \theta^\star > \theta^b_B(H^b)$. If $\overline{\theta}^b_D(H^b) < \underline{\theta}^i_D$, this knife-edge limiting equilibrium does not arise.*

Remark. With deposit mobility, depositors at each bank face multiple outside options whose attractiveness is endogenous to depositors' coordination, giving rise to multiple limiting equilibria, unlike in canonical global game models. However, only the baseline equilibrium (i) is robust. The second and third equilibria, in which the regional bank is weakly more stable than the national bank, constitute fragile, self-fulfilling outcomes. Their existence relies on perfect coordination among all depositors to generate strong inflows to the regional bank. As demonstrated in Appendix 2, this delicate coordination collapses upon the introduction of even an arbitrarily small degree of strategic uncertainty, causing these equilibria to vanish. Because real-world depositors do not possess common knowledge of others' strategies—especially across institutions—such highly coordinated outcomes are unlikely to be empirically relevant. Therefore, for the remainder of our analysis, we restrict our attention to the baseline equilibrium, which preserves the ordering of the run cutoffs established in the isolated economy. Therefore, we can call the bank j "safer" relative to bank l whenever $\theta^j_B < \theta^l_B$.

Up to this point, our specification does not exogenously differentiate national and regional banks. Because of this theoretical symmetry, a relabeled version of Theorem 2 could, in principle, describe deposit flight from the national bank to the regional bank. Yet runs from large national banks to small regional banks are, to our knowledge, absent from modern banking crises. Depositors generally perceive regional banks as riskier than large national institutions during periods of stress. To capture this empirical asymmetry in our policy counterfactuals, we set $\chi^i = 0$ for the remainder of the paper. This restriction eliminates any deposit movement from the national bank to regional banks—even if $\theta^b_B > \theta^i_B$—allowing us to isolate the contagious flight to the national bank that characterizes regional banking crises.

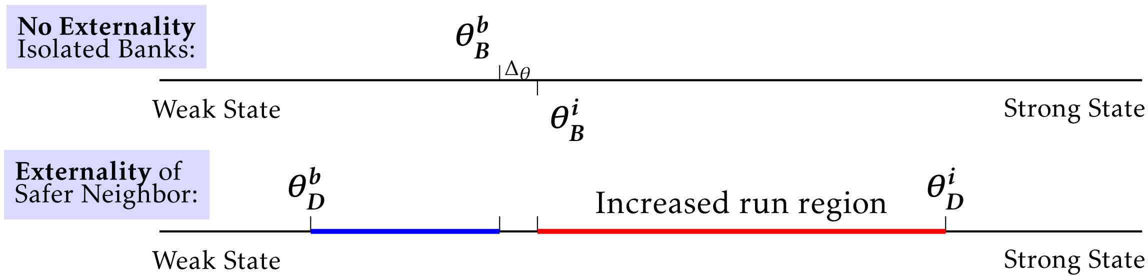

Figure 4: Matthew effect of bank fragility.

Corollary 1. *Under the additional assumption that $\chi^i = 0$, there is an essentially unique robust subgame equilibrium for the* ***Benchmark Economy of Connected Bank Run*** *at $t = 2$.*

(i) *When $\theta_B^i < \theta_B^b(H^b)$, the essentially unique subgame equilibrium is the same as the one described in Lemma 3.*

(ii) *When $\theta_B^i > \theta_B^b(H^b)$, then national-bank depositors stay when $\theta > \underline{\theta}_D^b(H^b)$ and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$. Regional-bank depositors stay when $\theta > \overline{\theta}_D^i$, switch to the national bank when $\underline{\theta}_D^b(H^b) < \theta < \overline{\theta}_D^i$, and run to the safe asset when $\theta < \underline{\theta}_D^b(H^b)$.*

As illustrated by Fig. 4, the presence of a relatively safer neighboring national bank increases the fragility of regional banks. We refer to this as the "**safe neighbor externality**".

4.3 Contagion of Flight to National Bank

Once we consider bank competition across more than one region, a more subtle and systemic dynamic emerges. The instability created by a safe national bank can affect smaller regional banks, even those that are *safer* than the national bank in isolation. The following result formalizes the chain reaction caused by the safe-neighbor externality. Without loss of generality, we rank the regional banks in decreasing order of fragility: $\theta_B^1 \geq \theta_B^2 \geq \cdots \geq \theta_B^{\mathcal{N}}$. When considered in isolation, the national bank may be safer than some regional banks but more fragile than others, that is, $\theta_B^i > \theta_B^b(H^b)$ for all $i \leq \mathcal{N}^{cut}$ and $\theta_B^i < \theta_B^b(H^b)$ for all $i > \mathcal{N}^{cut}$.

Theorem 3. *Suppose that $R^i \leq \partial_2 C^i(A^i, 0)$ and $R^b \leq \partial_2 C^b(A^b, 0)$. For each bank i, let O_D^i and θ_D^i be the connected-economy outflow and run cutoff defined in Eq. (15). For any integer*

$\mathsf{u} \in \{0, 1, \dots, \mathcal{N}\}$, *define the pair* $\left(I_C^b(H^b, \mathsf{u}),\ \theta_C^b(H^b, \mathsf{u})\right)$ *as the unique fixed point of the system*

$$
\begin{aligned}
I_C^b(H^b, \mathsf{u}) &= \sum_{i=1}^{\mathsf{u}} \mathcal{M}^i \int_{\xi_k} \frac{\mathbb{O}^i\!\left(\theta_C^b(H^b, \mathsf{u})\right)}{L^i} \frac{\mathsf{s}_{k,i}\, \chi^b \xi_k^b}{1 + \chi^b \xi_k^b - \Lambda}\, dF_{\widetilde{\xi}}(\xi_k), \\
\theta_C^b(H^b, \mathsf{u}) &= \max\left\{ \frac{C^b\!\left(A^b, (O_B^b - I_C^b - H^b)^+\right) - \left(H^b + I_C^b - O_B^b\right)^+ + R^b(L^b - O_B^b) + I_C^b}{G^b(A^b)},\ \widehat{\theta}^b(H^b) \right\},
\end{aligned}
\tag{17}
$$

where O_B^b *is the isolated-economy gross outflow from bank b defined in Lemma 3.*[27] *Define*

$$
\mathcal{N}^{run} \ \equiv\ \sup\left\{\mathsf{u} \le \mathcal{N} :\ \theta_B^{\mathsf{u}}\ >\ \theta_C^b(H^b, \mathsf{u} - 1)\right\}.
$$

Assume $\theta_B^{\mathcal{N}^{run}+1} \neq \theta_C^b(H^b, \mathcal{N}^{run})$ *whenever* $\mathcal{N}^{run} < \mathcal{N}$ *(a genericity condition that rules out knife-edge cases). Then there exists an essentially unique robust limiting equilibrium as* $\sigma \to 0$*:*

1. *For any regional bank* $i \le \mathcal{N}^{run}$*, all depositors stay with bank i if* $\theta > \theta_D^i$ *and run the bank if* $\theta < \theta_D^i$*.*
2. *For any regional bank* $i > \mathcal{N}^{run}$*, all depositors stay with bank i if* $\theta > \theta_B^i$ *and run the bank if* $\theta < \theta_B^i$*, where* O_B^i *and* θ_B^i *are defined in Lemma 1.*
3. *Depositors stay with national bank b if* $\theta > \theta_C^b(H^b, \mathcal{N}^{run})$ *and run the bank if* $\theta < \theta_C^b(H^b, \mathcal{N}^{run})$*.*

In addition, $\mathcal{N}^{run} \ge \mathcal{N}^{cut}$ *and, all else equal, increases with* H^b*.*

Theorem 3 highlights that the fragility arising from the safe-neighbor externality does not require the national bank to be exceptionally safe. Whenever deposits flow into national bank b during a banking crisis, the national bank is perceived as safer, which encourages depositors at other, even safer, regional banks to run. Our theory predicts that a local bank run can trigger a chain reaction, culminating in a regional banking crisis—a dynamic consistent with the events of the 2023 regional banking crisis.

4.4 Discussion: Liquidity Regulation

Since large national banks are typically considered systemically important, the welfare-weighted marginal value of their safety may outweigh that associated with any single regional bank. However, Theorem 3 demonstrates that a single safe national bank can have adverse effects on *many* regional banks through the chain-reaction mechanism. Ultimately, determining the optimal macroprudential policy aimed solely at increasing the resilience of the financial system is a quantitative question. Within the framework of

[27] When $\mathsf{u} = 0$ the sum is empty, so $I_C^b(H^b, 0) = 0$ and $\theta_C^b(H^b, 0) = \theta_B^b(H^b)$.

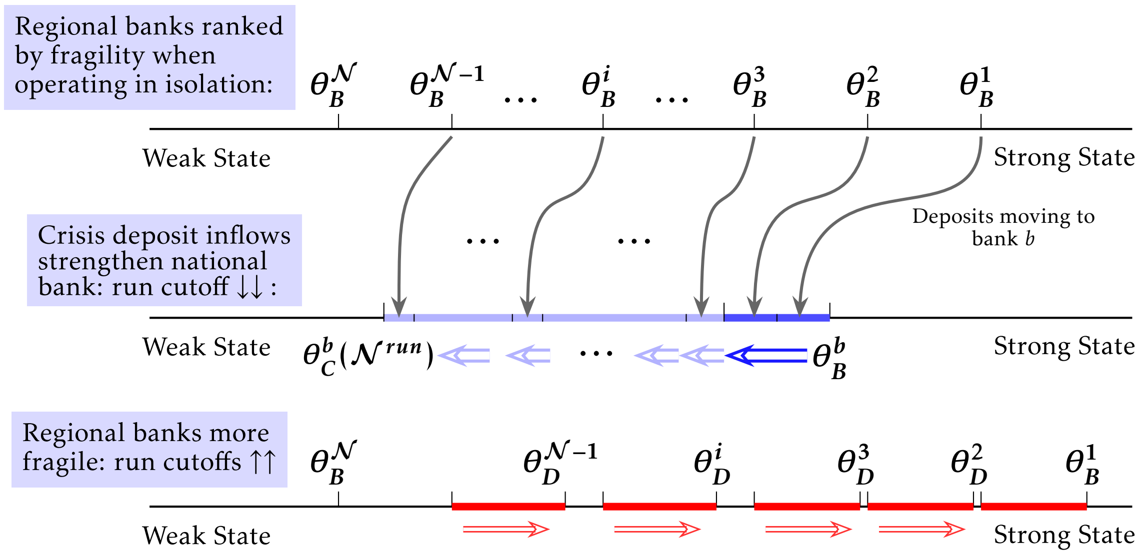

Figure 5: **An illustration of the chain reaction of Theorem 3.** When operating in isolation, only regional banks 1 and 2 are more fragile than national bank b. However, when national bank b competes with regional banks across all regions, deposit flows and bank runs become interconnected. If banks 1 and 2 experience runs, their deposits shift to bank b, reinforcing its stability (dark blue segment). This reallocation makes bank b safer relative to bank 3, which in turn becomes vulnerable, and the process cascades further (light blue segment). Ultimately, nearly all regional banks are affected by the safe-neighbor externality, becoming more fragile—including those that are more stable than national bank b in isolation.

our model, it depends on empirical estimates of the fundamental distribution $f_{\widetilde{\theta}}$, fire-sale losses C^j, and the distribution of deposits between large and small banks. Nevertheless, our analysis supports the conclusion that regulations focused exclusively on national banks may not be socially optimal. To formalize this, for economy $\mathcal{E} \in \{B, D\}$, let $O^j_{\mathcal{E}}(\theta)$ and $I^j_{\mathcal{E}}(\theta)$ denote the total deposit outflow from and inflow to bank j in state θ. Define the funding shortfall of bank j upon default as

$$\mathscr{L}^j_{\mathcal{E}} \equiv \left[\widetilde{\theta}\, G^j(A^j) - C^j\Big(A^j, (O^j_{\mathcal{E}}(\widetilde{\theta}) - I^j_{\mathcal{E}}(\widetilde{\theta}) - H^j)^+\Big) + (H^j + I^j_{\mathcal{E}}(\widetilde{\theta}) - O^j_{\mathcal{E}}(\widetilde{\theta}))^+ - I^j_{\mathcal{E}}(\widetilde{\theta}) - R^j(L^j - O^j_{\mathcal{E}}(\widetilde{\theta}))\right]^-.$$

For example, $\mathscr{L}^b_D$ is the loss to depositors of national bank b in the **Benchmark Economy of Connected Bank Run**. Following the macroprudential policy literature, we model the social cost of a bank failure as a monotonically increasing, convex function $\mathscr{C}^j(\mathbb{E}[\mathscr{L}^j_{\mathcal{E}}])$ of the expected funding shortfall.[28] It is widely recognized that regulation may distort institutions' incentives. We capture this through a social cost of regulation $\mathscr{B}^b(\Gamma)$, which is increasing and convex in the regulatory requirement Γ. Focusing on the regulation of the national bank, the optimal liquidity requirement on bank b minimizes the ex-ante social cost:

[28] Convexity captures the widely documented observation that larger bank failures cause disproportionately greater social damage through systemic risk amplification, contagion, and disruption to the payment system.

$$\min_{\Gamma} \mathscr{B}^b(\Gamma) + \mathscr{C}^b(\mathbb{E}[\mathscr{L}^b_{\mathcal{E}}]) + \sum_{i=1}^{\mathcal{N}} \Big(\mathscr{B}^i + \mathscr{C}^i(\mathbb{E}[\mathscr{L}^i_{\mathcal{E}}]) \Big). \tag{18}$$

Although the regulator does not impose any direct requirement on regional banks, the optimal regulation on bank b affects them indirectly through the safe-neighbor externality. For the following theorem, we fix $\{R^j, L^j, E^j, \varphi^j, \Lambda^j\}$ for $j \in \{b, 1, 2, \ldots, \mathcal{N}\}$ and assume that banks hold the minimum safe asset: $H^i = 0$ for all i and $H^b = \Gamma(E^b + (1-\varphi^b)L^b)$.

Theorem 4. *Suppose that $\mathcal{W}_{\mathcal{E}}(\Gamma)$ is convex in Γ for each $\mathcal{E} \in \{B, D\}$, so that there exists a unique positive minimizer (Γ^*_B and Γ^*_D, respectively) of Eq. (18). If $\mathscr{C}^b$ is sufficiently convex,then $\Gamma^*_B > \Gamma^*_D$.*

This theorem abstracts from the regulation's general-equilibrium effects on deposit-rate competition, which are considered in greater detail in Section 5.8.

5 First Stage Equilibrium: Deposit Competition at $t = 1$

Each region i constitutes an oligopolistic deposit market at time $t = 1$. We begin by studying banks' asset allocation decisions.

5.1 Banks' Investment Decision

At time $t = 1^+$, having attracted total deposits L^i, regional bank i allocates its assets between risky and risk-free investments by solving

$$\begin{aligned} &\max_{H^i} \mathbb{E}\bigg[\Big(\widetilde{\theta} G^i\big(A^i\big) - C^i\big(A^i, (O^i - I^i - H^i)^+\big) + (H^i + I^i - O^i)^+ - R^i(L^i - O^i) - I^i\Big)\mathbb{1}_{\mathcal{D}^c_i}\Big| L^i\bigg] \\ &\text{subject to} \qquad A^i = E^i + L^i(R^i) - \varphi^i L^i(R^i) - H^i \qquad\qquad \text{(FDIC Premium)} \\ &\qquad\qquad\qquad\; H^i \geq 0. \end{aligned} \tag{19}$$

National bank b solves an analogous problem:

$$\begin{aligned} &\max_{H^b} \mathbb{E}\bigg[\Big(\widetilde{\theta} G^b\big(A^b\big) - C^b\big(A^b, (O^b - I^b - H^b)^+\big) + (H^b + I^b - O^b)^+ - R^b(L^b - O^b) - I^b\Big)\mathbb{1}_{\mathcal{D}^c_b}\Big| L^b\bigg] \\ &\text{subject to} \qquad A^b = E^b + L^b(R^b) - \varphi^b L^b(R^b) - H^b \qquad\qquad \text{(FDIC premium)} \\ &\qquad\qquad\qquad\; H^b \geq \Gamma\left(E^b + L^b(R^b) - \varphi^b L^b(R^b)\right) \qquad \text{(Liquidity regulation).} \end{aligned} \tag{20}$$

Our first result reaffirms the well-documented insight that banks have little incentive to hold risk-free assets.

Theorem 5. *Suppose that ex-ante default probabilities are sufficiently small.*[29] *Then, without regulatory constraints, bank i holds no risk-free assets: $H^{i^*} = 0$. Bank b holds the minimum*

[29] Formally, the sufficient condition is $\left|\frac{d\theta^j_B}{dH^j}\right| \cdot \left[\theta^j_B G^j(A^j) + H^j - R^j L^j\right] \cdot f_{\widetilde{\theta}}(\theta^j_B) \;<\; \left|\mathbb{E}\left[\widetilde{\theta}(G^j)'(A^j) - 1 \,\middle|\, \widetilde{\theta} > \theta^j_B\right]\right| \cdot \mathbb{P}(\widetilde{\theta} > \theta^j_B)$, which holds whenever the run cutoff θ^j_B lies sufficiently deep in the left tail of $f_{\widetilde{\theta}}$, because

amount of risk-free assets necessary to satisfy its regulatory requirements, namely, $H^{b^*} = \Gamma\, E^b + \Gamma\,(1 - \varphi^b)\, L^b(R^b)$.

Intuitively, holding risk-free assets H^j reduces fire-sale losses, thereby increasing the value of the bank's debt. All else equal, any increase in debt value transfers wealth from equity holders to debt holders (see, e.g., Andersen, Duffie and Song 2019; Cooperman, Duffie, Luck, Wang and Yang 2025). Consequently, banks' equity holders have an incentive to minimize their holdings of safe assets.

5.2 Deposit Competition: Setup

Moving backward to time $t = 1$, banks i and b in region i set deposit rates R^i and R^b, respectively, to attract deposits. For simplicity, we denote a generic economy by $\mathcal{E}$. We again analyze the equilibrium dynamics separately for the **Benchmark Economy of Connected Bank Run** ($\mathcal{E} = D$) and the **Alternative Economy of Isolated Bank Run** ($\mathcal{E} = B$). The dynamics of the deposit market, influenced by potential bank-run risk, can be quite intricate. Depositors' beliefs regarding the likelihood of future bank runs may become self-fulfilling, and banks' optimal strategies may vary according to how their actions influence these beliefs. To address this technical difficulty, we assume that, before choosing how to allocate their endowment at $t = 1$, depositors are unaware of both the distribution of other depositors' types (i.e., $f_{\widetilde{\zeta}}(\cdot)$) and the total market size $\mathcal{M}^i$, whereas $f_{\widetilde{\zeta}}(\cdot)$ and $\mathcal{M}^i$ are private information of the banks.[30] Absent this information, depositors cannot independently form accurate expectations about each bank's fragility solely from the posted rates R^i and R^b. As a result, banks i and b have an incentive to disclose their fragility in order to coordinate depositors' beliefs. Specifically, both banks commit to their possible run cutoffs in the subgame at $t = 2$, denoted by $\widehat{\theta}^i$ and $\widehat{\theta}^b$, as well as to their liabilities, $\mathcal{L}^i$ and $\mathcal{L}^b$, respectively. These commitments are announced publicly, bundled together with their deposit rates. Once banks i and b have attracted total deposits L^i and L^b, some sophisticated depositors may scrutinize the banks' balance sheets and thereby gain insight into the ex-post distribution of depositor types ξ associated with each bank. This information enables sophisticated depositors to compute the actual run cutoffs conditional on the banks' balance sheets and investment decisions. If the calculated run cutoff differs from bank j's commitment, i.e., $\widehat{\theta}^j \neq \theta^j_{\mathcal{E}}$, or if the actual liabilities deviate from the committed level, i.e., $L^j \neq \mathcal{L}^j$, these depositors disclose the discrepancy publicly (e.g., via the media or legal action). Consequently, bank j immediately goes bankrupt,

the left-hand side scales with $f_{\widetilde{\theta}}(\theta^j_B) \to 0$ while the right-hand side approaches a strictly positive constant. This assumption is largely satisfied in the data for the U.S. banking industry.

[30] This assumption may be more realistic than presuming that depositors possess complete information about their local deposit markets.

forfeiting its long-term investments, and its equity holders incur a nonzero punitive cost—potentially in the form of a government-imposed fine. These dynamics ensure that, in equilibrium, bank j sets $\widehat{\theta}^j = \theta^j_{\mathcal{E}}$ and $\mathcal{L}^j = L^j$. Our modeling approach thus establishes a microfoundation for the deposit market equilibrium that incorporates *rational expectations* among depositors.

Under rational expectations, each regional bank i maximizes its payoff by solving the following (recall that $H^{i^*} = 0$ and $\mathcal{D}^c_i = \{\widetilde{\theta} > \theta^i_{\mathcal{E}}\}$):

$$\begin{aligned}
&\max_{R^i,\widehat{\theta}^i,\mathcal{L}^i} \mathbb{E}\Big[\Big(\widetilde{\theta}G^i\big(A^i\big) - C^i\big(A^i, O^i\big) - R^i(L^i - O^i)\Big)\mathbb{1}_{\mathcal{D}^c_i}\Big] \\
&\text{subject to} \qquad L^i = \mathcal{M}^i \int_{\xi_k} \mathrm{s}^*_{k,i}\, dF_{\widetilde{\xi}}(\xi_k) = \mathcal{L}^i, \qquad 1 \le R^i, \quad \text{and} \quad \theta^i_{\mathcal{E}} = \widehat{\theta}^i
\end{aligned} \tag{21}$$

where $\mathrm{s}^*_{k,i}$ is the strategy of depositor k in region i: due to the linearity of payoffs, $\mathrm{s}^*_{k,i} = 1$ if bank i is the best investment option in the depositor's choice set, and $\mathrm{s}^*_{k,i} = 0$ otherwise. Similarly, national bank b solves[31]

$$\begin{aligned}
&\max_{R^b,\widehat{\theta}^b,\mathcal{L}^b} \mathbb{E}\Big[\Big(\widetilde{\theta}G^b\big(A^b\big) - C^b\big(A^b, (O^b - H^{b^*})^+\big) + (H^{b^*} + I^b - O^b)^+ - R^b(L^b - O^b) - I^b\Big)\mathbb{1}_{\mathcal{D}^c_b}\Big] \\
&\text{subject to} \qquad L^b = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \mathrm{s}^*_{k,b}\, dF_{\widetilde{\xi}}(\xi_k) = \mathcal{L}^b \\
&\qquad\qquad\qquad H^{b^*} = \Gamma E^b + \Gamma(1-\varphi^b)L^b(R^b), \quad 1 \le R^b, \quad \text{and} \quad \theta^b_{\mathcal{E}} = \widehat{\theta}^b.
\end{aligned} \tag{22}$$

Depositors in region i take R^i and R^b as given and form rational expectations about H^{i^*} and H^{b^*} as well as the subgame equilibrium at $t = 2$. Specifically, depositor k in region i chooses the optimal investment strategy by solving

$$\max_{\{\mathrm{s}_{k,j},\ j \in \mathrm{I}_k\}} \mathbb{E}\Big[\mathcal{U}^2_k \;\Big|\; \theta^j_{\mathcal{E}} = \widehat{\theta}^j, R^j, \ \forall j \in \{i, b\}\Big]. \tag{23}$$

Definition 5. *The deposit market equilibrium at $t = 1$ for the **benchmark economy of connected bank run** is defined as the set of strategies that include each depositor's optimal deposit allocation $\mathrm{s}^*_{k,j}$ for depositor k in each region i, the interest rate strategy R^j set by bank j, and the investment strategy H^j of bank j. These strategies must satisfy the conditions outlined in Eqs. (19) to (23), where all agents anticipate that the subgame equilibrium follows the one outlined by Definition 2.*

Definition 6. *The deposit market equilibrium at $t = 1$ for the **alternative economy of***

[31] The findings of d'Avernas et al. (2023); Begenau and Stafford (2022) demonstrate that national banks set uniform rates across different local markets.

isolated bank run *is similar to Definition 5, except that agents expect the subgame equilibrium to follow the one outlined by Definition 3.*

5.3 Deposit Competition: Equilibrium Analysis

To lay the groundwork for our analysis of bank strategy, we characterize the optimal depositor strategy in the following lemma.

Lemma 5. *In region i, given R^i, R^b, $\widehat{\theta^i}$, $\widehat{\theta^b}$, $\mathcal{L}^i$, $\mathcal{L}^b$ and other depositors' strategy profile, the best response strategy for a depositor k in region i of type $\xi_k = (\xi_k^b, \xi_k^i)^\top$ at time $t = 1$ is characterized by*

$$
\begin{aligned}
\mathsf{s}_{k,i}^* =& \mathbb{1}\left[\left(\mathbb{P}(\widetilde{\theta}>\widehat{\theta^i})(R^i+\xi_k^i)+\mathbb{E}\left[(\Lambda+\mathsf{M}^i(\Xi_{\mathcal{E}}-\Lambda))\mathbb{1}_{\left[\widetilde{\theta}\leq\widehat{\theta^i}\right]}\right]\right)\geq\max\left(\mathbb{P}(\widetilde{\theta}>\widehat{\theta^b})(R^b+\xi_k^b)+\mathbb{E}\left[(\Lambda+\mathsf{M}^b(1-\Lambda))\mathbb{1}_{\left[\widetilde{\theta}\leq\widehat{\theta^b}\right]})\right], 1\right)\right] \\
\mathsf{s}_{k,b}^* =& \mathbb{1}\left[\left(\mathbb{P}(\widetilde{\theta}>\widehat{\theta^b})(R^b+\xi_k^b)+\mathbb{E}\left[(\Lambda+\mathsf{M}^b(1-\Lambda))\mathbb{1}_{\left[\widetilde{\theta}\leq\widehat{\theta^b}\right]})\right]\right)>\max\left(\mathbb{P}(\widetilde{\theta}>\widehat{\theta^i})(R^i+\xi_k^i)+\mathbb{E}\left[(\Lambda+\mathsf{M}^i(\Xi_{\mathcal{E}}-\Lambda))\mathbb{1}_{\left[\widetilde{\theta}\leq\widehat{\theta^i}\right]}\right], 1\right)\right],
\end{aligned}
$$

where

1. *$\Xi_{\mathcal{E}}$ is the best outside option for depositor k if bank i defaults. In the alternative economy, $\mathcal{E} = B$ and $\Xi_B = 1$. In the benchmark economy, $\mathcal{E} = D$ and $\Xi_D = 1 + \chi^b \xi_k^b \mathbb{1}_{[\mathcal{D}_b^c]}$.*
2. *$\mathsf{M}^i = \min\left((\frac{\mathbb{O}^i(\mathcal{L}^i,\theta)}{\mathcal{L}^i})^+, 1\right)$ in which $\mathbb{O}^i(\mathcal{L}^i, \theta)$ is the unique constant that solves $C^i(A^i, \mathbb{O}^i(\mathcal{L}^i, \theta)) = \theta G^i(A^i)$;*
3. *$\mathsf{M}^b = \min\left((\frac{\mathbb{O}^b(\mathcal{L}^b,\theta)}{\mathcal{L}^b})^+, 1\right)$, in which $\mathbb{O}(\mathcal{L}^b, \theta)$ is the unique constant that solves $C^b(A^b, \mathbb{O}(\mathcal{L}^b, \theta)) = \theta G^b(A^b) + H^b$.*

The proof is straightforward. Lemma 5 demonstrates that the equilibrium strategies of all depositors depend solely on the public signals R^i, R^b, $\mathcal{L}^i$, $\mathcal{L}^b$, $\widehat{\theta^i}$, and $\widehat{\theta^b}$. Consequently, banks can determine L^i and L^b by applying the exact law of large numbers conditional on these signals. Substituting the constraints $\widehat{\theta^j} = \theta_{\mathcal{E}}^j$ and $L^j = \mathcal{L}^j$ in banks' problem Eq. (21), the Lagrangian for bank i's problem is:

$$
\mathscr{L}^i(R^i, \widehat{\theta^i}, \Lambda) = \int_{\theta_{\mathcal{E}}^i} \left(\theta G^i(E^i + (1-\varphi^i)L^i) - R^i L^i\right) f_{\widetilde{\theta}}(\theta) d\theta + \Lambda_1(R^i - 1).
$$

The first-order condition (FOC) with respect to R^i yields:

$$
R^{i^*} = \underbrace{(1-\varphi^i)}_{\text{insurance fee}} \underbrace{\mathbb{E}[\widetilde{\theta} G^{i\prime}\left(A^i\right) \mid \widetilde{\theta} > \theta_{\mathcal{E}}^i]}_{\text{moral hazard}} - \underbrace{L^i \left(\partial L^i / \partial R^i\right)^{-1}}_{\text{market power}} - \underbrace{\frac{\theta_{\mathcal{E}}^i G^i(A^i) - R^i L^i}{\mathbb{P}(\widetilde{\theta} > \theta_{\mathcal{E}}^i)/f_{\widetilde{\theta}}(\theta_{\mathcal{E}}^i)} \frac{\frac{\partial \theta_{\mathcal{E}}^i}{\partial R^i}}{\frac{\partial L^i}{\partial R^i}}}_{\text{bank-run concern}} + \frac{\Lambda_1}{\mathbb{P}(\widetilde{\theta} > \theta_{\mathcal{E}}^i)\frac{\partial L^i}{\partial R^i}}.
$$

Several frictions prevent bank i from setting the deposit rate at the competitive first-best level. The insurance premium creates a wedge in the bank's asset returns, while moral hazard implies that equity holders focus primarily on favorable states—where $\widetilde{\theta}$ is high—leading to excessive risk-taking. In addition, market power allows the bank to charge a high markup, and bank-run concerns force equity holders to carefully manage the run cutoff. The FOC for bank b's problem is analogous, though liquidity regulation introduces an additional wedge:

$$R^{b^*} = (1-\varphi^b)\mathbb{E}[\widetilde{\theta}\underbrace{(1-\Gamma)}_{\text{liquidity regulation}} G^{b'}\left(A^b\right) + \Gamma \mid \widetilde{\theta} > \theta^b_{\mathcal{E}}] - L^b\left(\partial L^b/\partial R^b\right)^{-1} - \frac{\theta^b_{\mathcal{E}} G^b(A^b) + \Gamma L^b - R^b L^b}{\mathbb{P}(\widetilde{\theta} > \theta^b_{\mathcal{E}})/f_{\widetilde{\theta}}(\theta^b_{\mathcal{E}})} \frac{\frac{\partial \theta^b_{\mathcal{E}}}{\partial R^b}}{\frac{\partial L^b}{\partial R^b}} + \frac{\Lambda_1}{\mathbb{P}(\widetilde{\theta} > \theta^b_{\mathcal{E}})\frac{\partial L^b}{\partial R^b}}.$$

In our model, total deposits L^j are determined by depositors' investment decisions at $t = 1$. As illustrated by Lemma 5, depositors' optimal decisions depend on the announced run cutoff $\widehat{\theta}^j$ (which equals $\theta^j_{\mathcal{E}}$ in equilibrium) and on the deposit rate R^j. However, as shown in Lemma 3 and Theorem 3, $\theta^j_{\mathcal{E}}$ is itself a function of R^j and L^j, with L^j being determined by depositors' optimal choices. Consequently, characterizing the equilibrium requires solving a complex fixed-point problem. This complexity precludes a closed-form characterization of the deposit market equilibrium. To better illustrate the intuition, we subsequently impose parametric assumptions and illustrate the properties of the deposit competition equilibrium through a numerical example.

5.4 Parametric Specification

We now impose specific functional forms to obtain a tractable numerical characterization of the deposit market equilibrium. These parametric choices enable us to solve the fixed-point problem in Eqs. (21) and (22) and to quantify the safe-neighbor externality documented in Section 4.

Fundamentals. For simplicity, we assume that there is a single representative regional bank i and one representative national bank b operating in the same region. The aggregate state $\widetilde{\theta}$ follows a normal distribution, $\widetilde{\theta} \sim \mathcal{N}\left(\mu_\theta, \sigma^2_\theta\right)$. All banks share identical production technologies. For every bank $j \in \{b, i\}$ and $x \geq 0$,

$$G^j(x) = \frac{1}{\mu_\theta}\left[x + g_1 \log\left(1 + g_2\, x\right)\right], \qquad G^{j\prime}(x) = \frac{1}{\mu_\theta}\left(1 + \frac{g_1\, g_2}{1 + g_2\, x}\right),$$

where $g_1, g_2 > 0$ govern the curvature of diminishing returns. The normalization by μ_θ ensures that $\mathbb{E}[\widetilde{\theta}]\, G^{j\prime}(x) \to 1$ as $x \to \infty$: in the frictionless limit with large asset bases, the expected marginal return on bank lending converges to the risk-free rate, normalized to 1. At small scale ($x \approx 0$), the expected marginal return is $1 + g_1 g_2$, capturing the excess return from banks' informational advantage in originating loans.

Fire-sale costs. We assume the same linear fire-sale losses for both the regional and the national banks: $C^i(y,x) = C^b(y,x) = (1+c)\,x$, where $c > 0$ captures the haircut during fire sales when bank assets are liquidated at $t = 2$.

Depositor heterogeneity. For each depositor k in region i, the service-utility type $(\widetilde{\xi}^b_k, \widetilde{\xi}^i_k)$ is drawn independently, with both components uniformly distributed on $[0, \bar{\xi}]$. We set $\chi^i = 0$ and $\chi^b \in (0,1]$: depositors who flee a failing regional bank can partially recover service utility at the national bank, but there is no reverse channel.

Other assumptions. Equity is normalized to zero ($E^i = E^b = 0$), so bank assets are funded entirely by deposits. Both bank types share common deposit insurance parameters Λ and φ.

5.5 Calibration

Our calibration strategy is designed to make the two bank types *nearly identical* in the isolated economy, so that any differences in the connected economy can be attributed entirely to the safe-neighbor externality. The only ex-ante asymmetry is the liquidity regulation $\Gamma > 0$ imposed on the national bank, which requires it to hold a fraction of its investable funds in risk-free reserves. We emphasize that the goal of this exercise is *illustrative* rather than a serious quantification of the U.S. banking industry. All results presented are *stylized*.

Calibration targets. We target four equilibrium moments in the isolated-economy Nash equilibrium: the deposit rate spreads $R^i - 1$ and $R^b - 1$, and the default probabilities p^i_{def} and p^b_{def}. Because bank b holds a liquidity buffer that sacrifices return for safety, its equilibrium deposit rate is slightly lower than bank i's in the isolated economy, and its default probability is slightly lower. The targets are chosen so that default probabilities are small (around 5%), matching the order of magnitude of annual bank-distress frequencies (e.g., being added to the FDIC's "Problem Bank List") observed in the data, and low deposit rates are in a range (around 1%) consistent with those observed in the early 2010s.

Fixed parameters. We fix $g_3 = 1$ (risk-free return normalization), $\chi^b = 0.9$ (service-utility recovery at $t = 2$) and $\varphi = 0.03$ (FDIC premium). Deposit insurance coverage is set to $\Lambda = 0.55$.[32] The liquidity regulation parameter is set to $\Gamma = 0.15$. (In practice, U.S. banks subject to the LCR typically hold HQLA equal to 12–20% of total assets.) The fire-sale cost is set to $c = 0.45$, meaning that each dollar of assets liquidated at $t = 2$ yields only

[32] As of 2023:Q4, estimated uninsured deposits were approximately 40% of total domestic deposits system-wide. For mid-tier regional banks in the $50–100 billion asset range, the uninsured share typically falls in the 35–55% range. See FDIC Quarterly Banking Profile, Fourth Quarter 2023.

$1/(1 + c) \approx 69$ cents in cash, broadly consistent with the empirical literature on distressed-asset sales. Shleifer and Vishny (2011) survey fire-sale discounts across asset classes, reporting evidence of discounts ranging from 10% on commercial aircraft to 27% on foreclosed homes. The FDIC's historical experience with failed-bank resolutions documents average losses on asset disposition of 17–26% of total assets, with total resolution costs (including receivership expenses) ranging from 23% to 40% depending on the regulatory period (Bennett and Unal, 2015). These estimates imply fire-sale parameters in the range $c \in [0.20, 0.65]$.

Scale and units. We normalize the market size per region to $\mathcal{M}^i = 1$, which we interpret as \$100 billion in total regional deposits. Under this normalization, each bank's equilibrium deposit base of approximately $L^j \approx 0.50$ corresponds to roughly \$50 billion—consistent with the deposit size of mid-tier regional banks such as Synovus Bank (\$48.9 billion), Valley National Bank (\$49.2 billion), or Western Alliance Bank (\$53.6 billion) as of 2022. All welfare quantities (deadweight losses, FDIC payouts, profits) reported in subsequent figures are expressed in billions of dollars under this scaling.

Free parameters. We search over five parameters: $\rho \equiv \mu_\theta/\sigma_\theta$ (which determines default probability levels), the product $g_1 g_2$ (excess return at small scale), g_2 (speed of diminishing returns), $\bar{\xi}$ (service-utility range, controlling deposit elasticity and externality strength), and c (fire-sale haircut). The parameter σ_θ is normalized to 1 without loss of generality, since all equilibrium quantities depend only on ρ, not on μ_θ and σ_θ separately.[33] The resulting parameter values are reported in Table Appendix.1.1.

5.6 Baseline Equilibrium

Fig. 6 displays the Nash equilibrium outcomes in both economies. The left panel of Fig. 6 reveals that both banks dramatically raise their deposit rates in the connected economy: the regional bank's rate spread jumps from under 2% to nearly 7%, while the national bank's spread rises from near zero to about 5%. This escalation is driven by a self-reinforcing feedback loop. In the connected economy, depositors who run bank i can flee to bank b, increasing bank i's fragility and default risk. To compensate, bank i must raise R^i to retain market share, forcing bank b to raise R^b in response. This competitive escalation continues until both rates settle at a new, higher equilibrium. Interestingly, the deposit rates in the connected equilibrium are consistent with estimates of the all-in return on checking accounts in recent years. For example, Benetton, Hébert and McQuade (2025) estimate that the total return on a typical large-bank checking account—including cash-like incentives, rewards, and fee waivers—is approximately 4%.

[33] This scale invariance follows from the production function's $1/\mu_\theta$ normalization: every σ_θ that enters through the distribution of $\widetilde{\theta}$ is cancelled by a corresponding σ_θ from G or the recovery rate.

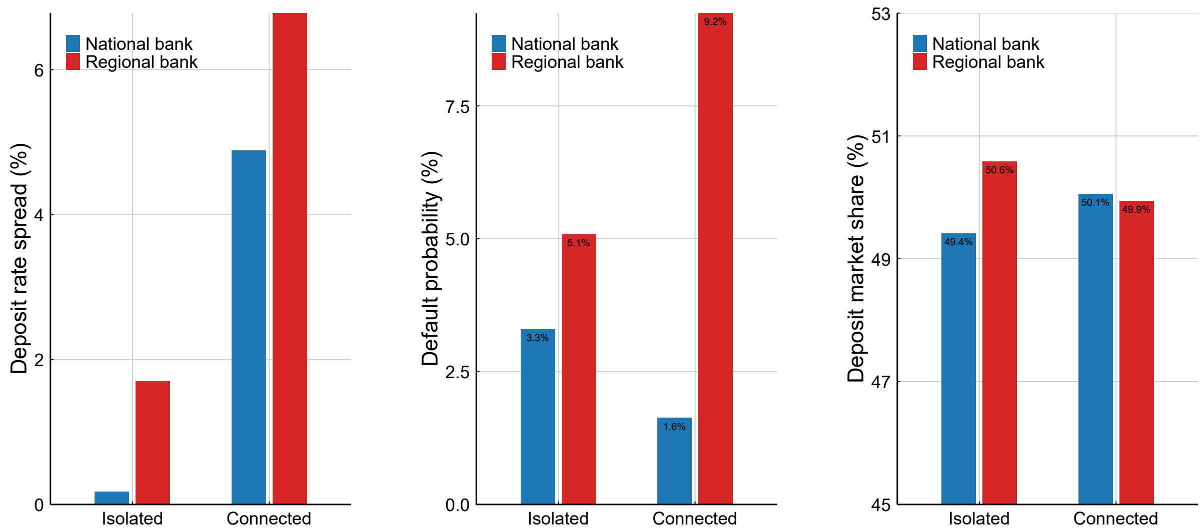

Figure 6: **Baseline equilibrium comparison.** Left panel: deposit rate spread above the risk-free rate. Center panel: equilibrium default probabilities (annotated). Right panel: deposit market shares. In each panel, bars are grouped by economy (isolated vs. connected) with red bars for the regional bank and blue bars for the national bank.

The center panel of Fig. 6 shows the starkly asymmetric consequences for financial stability. The regional bank's default probability nearly doubles (from 5.1% to 9.2%), while the national bank's default probability falls by only 1.7 percentage points. This asymmetry is the hallmark of the safe-neighbor externality: the deposit-flight channel amplifies regional bank fragility while simultaneously stabilizing the national bank through inflows, and the former effect quantitatively dominates. A 4-percentage-point increase in default probability represents a substantial deterioration in financial stability, equivalent under our $100 billion normalization to a roughly $700 million increase in expected FDIC payouts.

The right panel of Fig. 6 shows that deposit market shares barely move between economies, remaining close to a 50–50 split. This rigidity reflects the dominant role of non-rate characteristics in depositors' bank choices, a pattern well documented in the banking literature.[34]

5.7 Deadweight Loss from Coordination Failure

Beyond rates and default probabilities, the safe-neighbor externality generates significant deadweight losses (DWL) from coordination failure. In the coordination-failure zone—the set of states $\theta \in (\widehat{\theta}^j, \theta^j_\varepsilon]$ where bank j is fundamentally solvent but depositor panic

[34] Egan, Hortaçsu and Matvos (2017b) estimate a structural model of the U.S. banking sector and find that non-rate vertical differentiation—branch networks, convenience, service quality—accounts for the bulk of depositor utility

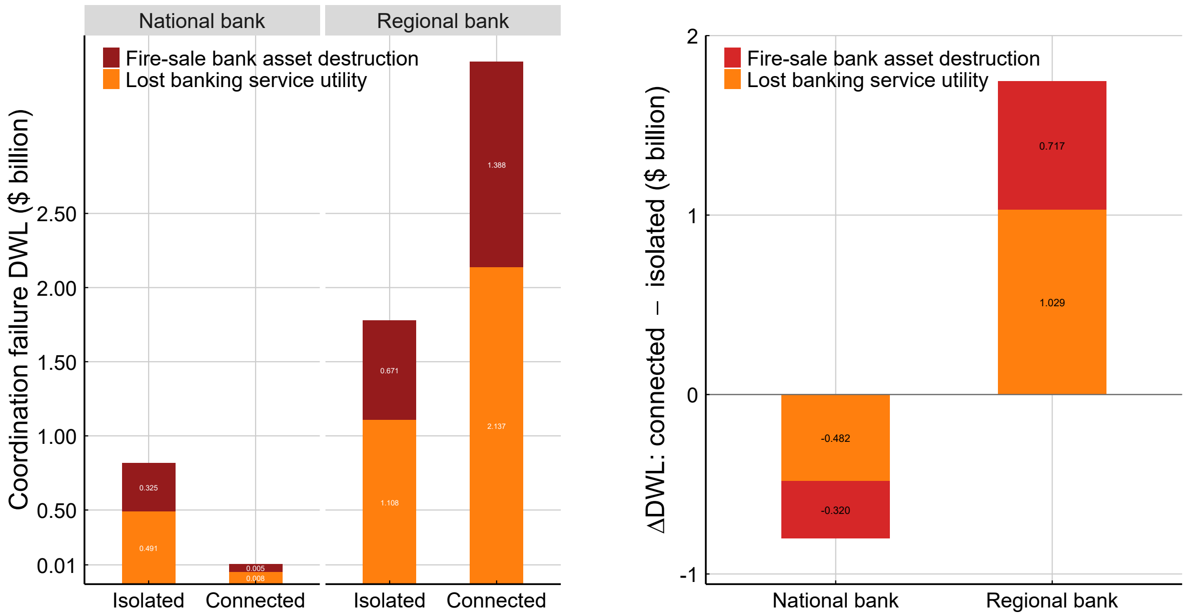

Figure 7: **Deadweight loss from coordination failure.** Left panel: DWL decomposition by bank and economy, with fire-sale asset destruction (dark) and lost banking service utility (light) stacked. The national bank's connected-economy bar is magnified 10× (dashed outline) for visibility. Right panel: change in DWL from isolated to connected economy (ΔDWL), decomposed by component. All quantities are in billions of dollars under the $100 billion normalization.

triggers default—the following three sources of social value destruction arise:

1. **Fire-sale asset destruction.** When all depositors run, the bank liquidates its entire risky portfolio. At state θ, the bank can serve at most $\mathbb{O}(\theta) = \theta G(A)/(1+c)$ dollars of withdrawals, so the fire-sale asset destruction is $\theta G(A) - \mathbb{O}(\theta) = c \cdot \theta G(A)/(1+c)$.

2. **Lost banking service utility.** All L^j depositors lose their banking relationship, forfeiting expected service utility $\mathbb{E}[\xi_k^j \mid \text{chose } j] \cdot L^j$ per state.

3. **Partial recovery at bank b (connected economy only).** Of bank i's depositors who run, the fraction $\mathbb{O}(\theta)/L^i$ served under the sequential-service constraint can transfer to bank b, recovering $\chi^b \mathbb{E}[\xi_k^b \mid \text{chose } i]$ per dollar.

Integrating over the coordination-failure zone, the expected DWL for bank i decomposes as follows (the DWL for bank b is analogous, absent the last term):

$$\text{DWL}^i = \underbrace{c\,\frac{G(A^i)}{1+c}\,\mathbb{E}\Big[\widetilde{\theta}\cdot \mathbb{1}_{[(\widehat{\theta^i}<\widetilde{\theta}\le\theta^i_{\varepsilon})]}\Big]}_{\text{fire-sale destruction}} + \underbrace{\mathbb{E}[\xi^i \mid i]\cdot L^i \cdot \mathbb{P}(\widehat{\theta^i} < \widetilde{\theta} \le \theta^i_{\varepsilon})}_{\text{service utility lost}} - \underbrace{\chi^b \mathbb{E}[\xi^b \mid i]\cdot \frac{G(A^i)}{1+c}\,\mathbb{E}\Big[\widetilde{\theta}\cdot \mathbb{1}_{[((\widehat{\theta^i}<\widetilde{\theta}\le\theta^i_{\varepsilon}))]}\Big]}_{\text{recovery at bank } b}.$$

In the isolated economy, there is no deposit flight, so the last recovery term vanishes.

Fig. 7 displays the DWL decomposition. The left panel reveals a stark compositional shift across economies. In the isolated economy, both banks contribute meaningfully to total DWL (bank i: \$1.78 billion; bank b: \$0.82 billion). In the connected economy, bank b's DWL collapses to near zero (so small that the bar must be magnified 10× for visibility), because its coordination-failure zone virtually disappears. Bank i's DWL, however, increases sharply to \$3.53 billion—a 98% increase. Crucially, the right panel of Fig. 7 shows that *total* DWL increases in the connected economy. The reduction in bank b's DWL (\$0.80 billion) is more than offset by the increase in bank i's DWL (\$1.75 billion), yielding a net increase of approximately \$0.94 billion. In this numerical example, the safe-neighbor externality destroys social value on net, even though it benefits the national bank individually.

Sublinear benefits, superlinear costs. The net increase in total DWL reflects a fundamental asymmetry in how deposit reallocation affects the two banks. For the national bank, inflows help offset outflows dollar-for-dollar up to a point, but beyond that additional inflows provide diminishing marginal benefit. In particular, when bank b's default threshold is already pinned at its fundamental lower bound $\widehat{\theta}^b$, further inflows cannot reduce default risk at all. The stability benefit to bank b is therefore *sublinear* in the size of deposit flight. For the regional bank, the relationship is reversed. Fire-sale losses are convex in the volume of assets liquidated: each additional dollar of outflow forces the bank deeper into distressed-asset markets. The fragility cost to bank i is thus *superlinear* in the deposit drain. Taken together, these asymmetries imply that, when the regional and national banks are of *comparable size*, the safe-neighbor externality is welfare-destroying in aggregate, even though it benefits the national bank individually.

A caution on interpretation. In practice, the net welfare effect is more nuanced. National banks play an outsized role in payment systems, interbank lending, and derivatives markets, so the social return to stabilizing a single large institution may exceed what our symmetric two-bank model captures. At the same time, regional and community banks collectively hold roughly 35% of U.S. domestic deposits and originate a disproportionate share of small-business and agricultural lending,[35] so the cumulative fragility cost across many regional institutions may be substantial. A full quantification of these competing forces—accounting for network centrality, lending specialization, and the distribution of bank sizes—is an important direction for future research.

[35]As of year-end 2023, community banks (assets <\$10 billion) held approximately 15% of domestic deposits while originating 35% of small-business loans and nearly 70% of agricultural loans; including mid-tier regional banks, the collective share of small-business lending exceeds 60%. (See FDIC Quarterly Banking Profile, 2023 .)

5.8 Liquidity Regulation

We now examine how the national bank's liquidity requirement Γ shapes equilibrium default risk. Fig. 8 reports default probabilities in both economies as Γ varies from 1% to 19%, decomposed into fundamental default risk and coordination-failure risk. As shown by the left panel of Fig. 8, in the isolated economy, the national bank benefits directly from the larger buffer, so its default probability declines steadily with Γ. This is the pattern a regulator would expect from a standard national-bank-only analysis. The right panel of Fig. 8 tells a markedly different story. In the connected economy, the effect on bank i is negligible, because the deposit-flight channel is fully activated once $\Gamma > 0$, which is sufficient for bank b to become marginally safer than bank i. The blue bars in the right panel of Fig. 8 reveal a non-monotone pattern in bank b's default probability.

Implications for macroprudential policy. The contrast between the two panels of Fig. 8 carries an important lesson for macroprudential regulators. A regulator who observes only the national bank's balance sheet and calibrates Γ using the isolated-economy pattern (left panel) would expect increasing Γ to reduce default probabilities steadily. But the connected-economy pattern (right panel) shows this expectation is false: the default probability for the national bank is uniformly lower in the connected economy than in the isolated economy, so a minimum level of Γ may already achieve the policy's calibrated "optimal" default risk. A regulator who ignores the deposit-flight channel will therefore overshoot—setting Γ higher than socially optimal in pursuit of stability benefits that have already been achieved at a lower Γ. This is consistent with the result formalized in Theorem 4: the optimal Γ in the connected economy is strictly lower than in the isolated economy.

6 Conclusion

This paper identifies and analyzes the safe-neighbor externality of banking regulation. Measures designed to safeguard large national banks—liquidity requirements, capital regulation, and implicit government guarantees—endogenously make these institutions more attractive destinations for fleeing depositors, amplifying coordination failures at smaller regional banks. We formalize this mechanism by extending the global-game framework to incorporate multiple banks with cross-bank deposit mobility.

Our analysis yields three results with direct policy relevance. First, the externality generates a starkly asymmetric fragility shift: regional bank default probabilities increase substantially while national banks gain only modest stability. In our stylized two-bank calibration illustration, the associated deadweight losses increase on net, because the fragility cost to regional banks is superlinear in the deposit drain, whereas the stability

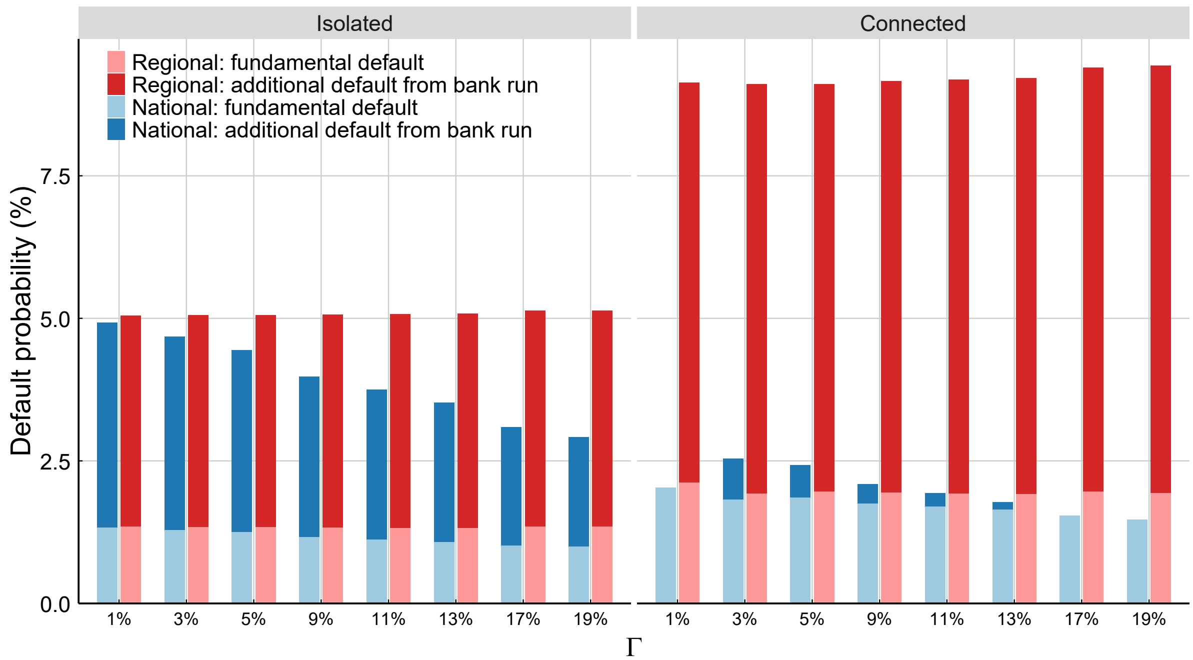

Figure 8: **Default probability decomposition over Γ.** Left panel: isolated economy. Right panel: connected economy. At each value of Γ, two dodged bars show the regional bank (red, right) and national bank (blue, left). Light shading represents fundamental default probability; the darker extension above represents the additional default probability from coordination failure. Total bar height equals the equilibrium default probability.

benefit to national banks is sublinear. Second, the optimal liquidity requirement for national banks in the connected economy is strictly lower than what a regulator would choose analyzing each bank in isolation. Our framework further reveals that regional bank fragility can become contagious across regions through the national bank's balance sheet, simultaneously destabilizing otherwise viable institutions even when their asset portfolios are uncorrelated. Given that regional and community banks collectively originate a disproportionate share of small-business and agricultural lending, such correlated fragility represents a potential source of systemic risk that existing macroprudential frameworks do not fully address.

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A 1 Additional Tables and Figures

Parameter	Symbol	Value
Mean fundamentals / s.d.	μ_θ $(=\rho)$	5.273
Std. dev. of fundamentals	σ_θ	1
Production curvature	g_1	0.624
Diminishing-return speed	g_2	1.825
Risk-free return normalization	g_3	1.0
Fire-sale haircut	c	0.45
FDIC premium	φ	0.03
Deposit insurance coverage	Λ	0.55
Liquidity regulation (bank b)	Γ	0.15
Service-utility range	$\bar{\xi}$	0.877
Service recovery at $t = 2$	χ^b	0.9
Market size per region	$\mathcal{M}^i$	1
Number of regions	$\mathcal{N}$	1

Table Appendix.1.1: Calibrated parameter values. The parameters are chosen so that both banks are nearly identical in the isolated economy, with the only asymmetry arising from the liquidity regulation $\Gamma = 0.15$ on bank b.

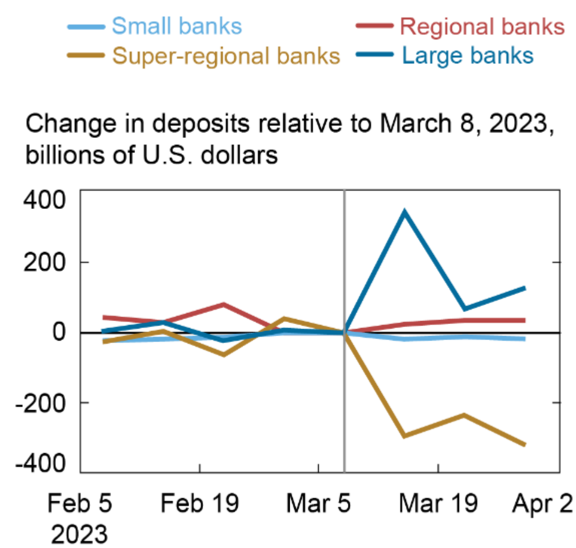

Figure Appendix.1.1: **Deposit flow leaving regional banks to large national banks in March 2023.** Source: Luck, Plosser and Younger (2023). Notes: Banks are categorized by the size of their domestic assets: small banks—less than $5 billion; regional banks—$5 to $50 billion; super-regional banks—$50 to $250 billion; large banks—greater than $250 billion.

A 2 Equilibrium Selection via Strategic Uncertainty

Theorem 2 shows that when $\overline{\theta}^b_D(H^b) \geq \underline{\theta}^i_D$, the **Benchmark Economy of Connected Bank Run** admits a second limiting equilibrium in which deposits flow from the national bank to the regional bank. In this section, we demonstrate that this second equilibrium is not robust to incomplete information about other depositors' cross-bank mobility, and is therefore eliminated by a vanishingly small amount of strategic uncertainty.

Our refinement is motivated by the concept of robustness to incomplete information developed in Kajii and Morris (1997) for finite-player, finite-action games with complete information. Because our setting features a continuum of players, we cannot apply their framework directly. Instead, we introduce a related "elaboration" of the complete-information game by perturbing beliefs about the service-depreciation parameter χ^i, which governs the attractiveness of bank i as a destination for fleeing depositors. In reality, although depositors may have precise knowledge of their own banking service utility and the associated depreciation factor, they plausibly lack equal information about other depositors' switching costs and service compatibility—especially at a different institution. We show that this natural form of strategic uncertainty is sufficient to eliminate the reverse-flight equilibrium.

Setup. Our robustness analysis maintains all assumptions of Section 4.2 with one modification to the information structure. At t=2, each depositor w at bank ℓ faces two sources of uncertainty:

1. **Fundamental uncertainty.** Depositor w receives the private signal $\widetilde{x}_w = \widetilde{\theta} + \sigma\widetilde{\epsilon}_w$ about the macroeconomic fundamental $\widetilde{\theta}$, as in the baseline model.
2. **Strategic uncertainty.** Depositor w at bank ℓ observes her own depreciation parameter $\chi^j_w = \chi^j$ perfectly, but believes that the depreciation parameter for each other depositor $w' \neq w$ is an independent draw $\widetilde{\chi}^j_{w'}(\varsigma) \sim F_{\widetilde{\chi}|\chi^j,\varsigma}$, i.i.d. across w'. This distribution satisfies: (a) $\widetilde{\chi}^j_{w'}(\varsigma) \to \chi^j$ in probability as $\varsigma \downarrow 0$, and (b) $\widetilde{\chi}^j_{w'}(\varsigma) \in [0,1]$ almost surely.

Equilibrium robustness. We study the sequential limit in which the strategic uncertainty vanishes first, followed by the fundamental-signal noise:

$$\lim_{\sigma\downarrow 0}\left(\lim_{\varsigma\downarrow 0}\mathscr{E}(\sigma,\varsigma)\right).$$

Definition 7. *A limiting equilibrium $\mathscr{E}$ of the original (ς=0, $\sigma\downarrow 0$) game is* ***robust to strategic uncertainty*** *if, for every family of distributions $\{F_{\widetilde{\chi}|\chi^j,\varsigma}\}_{\varsigma>0}$ satisfying conditions (a) and (b) above, there exists a sequence of equilibria $\mathscr{E}(\sigma,\varsigma)$ of the ς-elaboration game such that $\lim_{\sigma\downarrow 0}\lim_{\varsigma\downarrow 0}\mathscr{E}(\sigma,\varsigma) = \mathscr{E}$. Otherwise, $\mathscr{E}$ is* ***not robust to strategic uncertainty****.*

Theorem 6. *Suppose $\theta^b_B(H^b) < \theta^i_B$, $R^j + \xi^j_k > 1 + \chi^\ell \xi^\ell_k$ for all j, ℓ, k (the no-non-run-reallocation assumption), and that $\overline{\theta}^b_D(H^b) \geq \underline{\theta}^i_D$ (so that the limiting game of Theorem 2 admits both the baseline and the reverse-flight equilibria). Then:*

(i) The reverse-flight equilibrium described in Theorem 2(ii) *is* ***not robust*** *to strategic uncertainty.*

(ii) The baseline equilibrium described in Theorem 2(i) *is* ***the only equilibrium robust*** *to strategic uncertainty.*

Discussion. The second equilibrium of Theorem 2 is sustained by a self-fulfilling collective belief. It arises when depositors conjecture that the regional bank will receive strong inflows from the national bank, even in relatively favorable economic states. If held by all, such a belief implies that the national bank is riskier (formally, $\mathcal{D}_i \subsetneq \mathcal{D}_b$). This perceived safety advantage makes the regional bank the more attractive destination for depositors, irrespective of their private signals about $\widetilde{\theta}$, thus rendering the initial belief self-fulfilling. Consequently, a standard global game with uncertainty *only* over the fundamental cannot rule out this equilibrium.

However, this coordinated belief is fragile. Theorem 6 proves that introducing vanishing uncertainty about cross-bank mobility causes this coordination to collapse. The mechanism is an *infection* argument. For sufficiently low values of χ^i, the regional bank offers negligible service utility to incoming depositors, so no rational depositor at bank b would transfer funds there. This creates a "dominance region" in which the baseline ordering (bank i riskier) necessarily holds. At the boundary of this dominance region, each depositor at bank b recognizes that a positive fraction of other depositors have low mobility parameters and will not move to bank i. Without sufficient inflows, bank i cannot become safer, sustaining the incentive to stay at bank b and enlarging the dominance region. Iterating, the dominance region infects the entire parameter space, eliminating the reverse-flight equilibrium for all values of χ^i. Since the baseline equilibrium does not depend on χ^i—no deposits ever flow from bank b to bank i in the baseline—it is robust to any perturbation of this parameter.

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Internet Appendix

1 Proofs

For clarity, we summarize the assumptions regarding banks' production technology and fire-sale losses as follows:

Assumption 4. For any $j \in \{b, 1, 2, 3, \cdots, \mathcal{N}\}$:

(I) The production function G^j is differentiable, and $\mathbb{E}[\widetilde{\theta}]G^{j\prime}(x) \geq 1$ is weakly decreasing in x for all $x \geq 0$.

(II) The fire-sale loss function C^j is twice differentiable in both arguments.

(III) For any $A^j \geq 0$, $C^j(A^j, 0) = 0$.

(IV) For any $A^j \geq 0$, $\partial_2 C^j\big(A^j, 0\big) > \mathbb{E}[(\widetilde{\theta})^+]G^{j\prime}(A^j)$ and $\partial_2 C^j\big(A^j, 0\big) > 1$.

(V) For any $A^j \geq 0$, $\partial_2^2 C^j\big(A^j, x\big)$ is weakly increasing in $x \in [0, \infty)$.

Discussion: Parts **(I)** and **(IV)** of Assumption 4 imply that banks are marginally more efficient at managing their own assets than outside buyers at $t = 2$, possibly due to market frictions such as adverse selection. A marginal amount δ invested in the bank's assets generates an internal value of $G^{j\prime}(A^j)\delta$, while its value to the next-best buyer in the market is reduced by a marginal fire-sale cost of $\big(\partial_2 C^j(A^j, 0) - G^{j\prime}(A^j)\big)\delta$. Thus, if bank j liquidates δ worth of assets to meet outflows, the productive value of the bank's assets decreases by $\partial_2 C^j(A^j, 0)\delta$. Part **(V)** of Assumption 4 indicates that greater liquidation leads to higher marginal fire-sale costs.[36]

1.1 Proof of Lemma 1

From the premises of the lemma, we take $H^i = 0$ in the proof. The following calculation of depositor k's posterior beliefs will be useful in the remainder of the proof. Since $\widetilde{\theta}$ has an atomless distribution, for any ϑ,

$$\begin{aligned}
\mathbb{P}\Big(\widetilde{\theta} \geq \vartheta \,\Big|\, \widetilde{x_k} = x_k\Big) =& 1 - F_{\widetilde{\theta}|\widetilde{x_k}}\Big(\vartheta \mid \widetilde{x_k} = x_k\Big) = 1 - \frac{\int_{-\infty}^{\vartheta} f_{\widetilde{\theta}}(\theta) f_{\widetilde{\epsilon}_k}\Big(\frac{x_k - \theta}{\sigma}\Big)\, d\theta}{\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(\theta) f_{\widetilde{\epsilon}_k}\Big(\frac{x_k - \theta}{\sigma}\Big)\, d\theta} \\
=& 1 - \frac{\int_{(x_k - \vartheta)/\sigma}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z) f_{\widetilde{\epsilon}_k}(z)\, dz}{\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z) f_{\widetilde{\epsilon}_k}(z)\, dz} = \frac{\int_{-\infty}^{(x_k - \vartheta)/\sigma} f_{\widetilde{\theta}}(x_k - \sigma z) f_{\widetilde{\epsilon}_k}(z)\, dz}{\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z) f_{\widetilde{\epsilon}_k}(z)\, dz}.
\end{aligned} \tag{24}$$

[36] This modeling choice implicitly suggests that financial markets at $t = 1$ lack information about $\widetilde{\theta}$ and $\widetilde{\zeta}_j$, so assets are priced based on prior beliefs. While C^j could, in principle, depend on signals regarding $\widetilde{\theta}$ or $\widetilde{\zeta}_j$, incorporating such dependence would complicate the analysis without generating additional insights.

Consider the representative bank i. Let agent k be a representative depositor at bank i after time $t=1$. Denote by $\mathfrak{O}^i$ the aggregate deposit outflow from bank i. In the **Alternative Economy of Isolated Bank Run** that is considered here, $\mathtt{I}^i = 0$. Given any agents' strategy profile, denoted as $\{\mathsf{r}_{k,i\to H}\}$, if the true economic state is θ, then the Exact Law of Large Numbers (ELLN) yields

$$\mathfrak{O}^i = \mathcal{M}^i \int\limits_{\xi_k}\int\limits_{x_k} \mathsf{r}_{k,i\to H}(x_k) f_{\widetilde{x}_k|\widetilde{\theta}}(x_k \mid \theta)\, dx_k\, f_{\widetilde{\xi}}(\xi_k)\, d\xi_k.$$

Hence aggregate deposit outflow is a function of θ and σ (recall that private signal precision is governed by σ, so the density $f_{\widetilde{x}_k|\widetilde{\theta}}$ depends on σ)—i.e., any outflow can be written as a "deposit flow function", $\mathfrak{O}^i(\theta,\sigma)$.

Given any possible deposit flow function, depositor k's expected payoff from *staying* with bank i after observing x_k at $t=2$ is

$$\mathcal{U}^2_{k,i\to i}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to i} = 1 \mid x_k\Big) = \mathbb{P}\Big(\mathcal{D}_i^c \mid x_k\Big)(R^i + \xi_k^i) + \mathbb{P}\Big(\mathcal{D}_i \mid x_k\Big)\Lambda. \tag{25}$$

If k *leaves* the banking system to hold the risk-free asset, her expected payoff is

$$\mathcal{U}^2_{k,i\to H}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to H} = 1 \mid x_k\Big) = \int\limits_{\mathcal{D}_i}\Big[\mathtt{M}^i + \Big(1-\mathtt{M}^i\Big)\Lambda\Big] f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta + \int\limits_{\mathcal{D}_i^c} f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta. \tag{26}$$

In this lemma, we only study equilibria in which depositors believe that they can always successfully run from bank i, i.e., all depositors believe that the success probability $\mathtt{M}^i = 1$. Under the best-responding strategy, depositor k leaves bank i iff $\mathcal{U}^2_{k,i\to H}(x_k) > \mathcal{U}^2_{k,i\to i}(x_k)$ and vice versa. Hence, depositor k's best response strategy only depends on her private signal x_k. Let $\mathfrak{R}^i_{\text{tem}}(k,\sigma)$ denote the set of signal realizations for a type-ξ_k depositor who runs from bank i after observing $x_k \in \mathfrak{R}^i_{\text{tem}}(k,\sigma)$. Then the equilibrium outflow must satisfy $\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma) = \mathcal{M}^i \int\limits_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\vartheta + \sigma\tilde{\epsilon}_k \in \mathfrak{R}^i_{\text{tem}}(k,\sigma)\Big)\, dF_{\widetilde{\xi}}(\xi_k)$.

1.1.1 Step 1: existence of upper and lower equilibria

Fix a value of σ, and consider the possible equilibrium outcome at time $t=2$. Define a constant function $\overline{\mathfrak{O}^i}(\theta,0,\sigma) \equiv \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} f_{\widetilde{\xi}}(\xi_k)\, d\xi_k$ for any θ. This quantity is the total deposit at bank i right after $t=1$, and the largest possible outflow from bank i. Define a constant function $\underline{\mathfrak{O}^i}(\theta,0,\sigma) = 0$ for all θ. It follows that any possible equilibrium outflow $\mathfrak{O}^i(\widetilde{\theta},\sigma)$ must almost surely satisfy

$$\underline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma) \le \mathfrak{O}^i(\widetilde{\theta},\sigma) \le \overline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma), \tag{27}$$

and this is common knowledge among all agents. Define $\mathfrak{R}(k,0,\sigma)$ to be the set of signal realizations for agent k with type ξ_k who, after observing any $x_k \in \mathfrak{R}(k,0,\sigma)$, would run away from bank i even if agent k optimistically conjectures that the total outflow at time $t=2$ is the lowest possible one, $\underline{\mathfrak{O}^i}(\theta,0,\sigma)$. By definition, from Eqs. (25) and (26)

$$\mathfrak{R}(k,0,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta}G^i\Big(A^i\Big) - C^i\Big(A^i, (\underline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma))^+\Big) - R^i\Big(L^i - \underline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma)\Big) > 0 \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\}.$$

Similarly, define $\mathfrak{S}(k,0,\sigma)$ to be the set of signals for agent k with type ξ_k such that, after observing any signal $x_k \in \mathfrak{S}(k,0,\sigma)$, agent k would stay with bank i even if she is most pessimistic by conjecturing that the total outflow at time $t=2$ is the highest possible one $\overline{\mathfrak{O}^i}(\theta,0,\sigma)$:

$$\mathfrak{S}(k,0,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta}G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\widetilde{\theta},0,\sigma))^+) > 0 \,\Big|\, \widetilde{x_k} = x\Big) \geq \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\}.$$

Recursively, for any non-negative integer ℓ, define

$$\underline{\mathfrak{O}^i}(\theta,\ell+1,\sigma) \equiv \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,\ell,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k, \qquad \overline{\mathfrak{O}^i}(\theta,\ell+1,\sigma) \equiv \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \notin \mathfrak{S}(k,\ell,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k,$$

$$\mathfrak{R}(k,\ell+1,\sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta}G^i\big(A^i\big) - C^i\big(A^i, (\underline{\mathfrak{O}^i}(\widetilde{\theta},\ell+1,\sigma))^+\big) - R^i(L^i - (\underline{\mathfrak{O}^i}(\widetilde{\theta},\ell+1,\sigma))^+) > 0 \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\},$$

$$\mathfrak{S}(k,\ell+1,\sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta}G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\widetilde{\theta},\ell+1,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\widetilde{\theta},\ell+1,\sigma))^+) > 0 \,\Big|\, \widetilde{x_k} = x\Big) \geq \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\}.$$

Claim 1. *Fix any σ. For any non-negative integer ℓ, $\underline{\mathfrak{O}^i}(\theta,\ell,\sigma)$ and $\overline{\mathfrak{O}^i}(\theta,\ell,\sigma)$ are continuous in θ. In addition, $\underline{\mathfrak{O}^i}(\theta,\ell,\sigma) \leq \underline{\mathfrak{O}^i}(\theta,\ell+1,\sigma) \leq \overline{\mathfrak{O}^i}(\theta,\ell+1,\sigma) \leq \overline{\mathfrak{O}^i}(\theta,\ell,\sigma)$. Moreover, $\mathfrak{R}(k,\ell,\sigma) \subseteq \mathfrak{R}(k,\ell+1,\sigma)$ and $\mathfrak{S}(k,\ell,\sigma) \supseteq \mathfrak{S}(k,\ell+1,\sigma)$. Total outflow from bank i has to lie in the interval $[\underline{\mathfrak{O}^i}(\theta,\ell,\sigma), \overline{\mathfrak{O}^i}(\theta,\ell,\sigma)]$ after eliminating dominated strategies. This is common knowledge among all agents.*

Proof of Claim 1. First, from Assumption 4, if two functions $\mathfrak{O}^1(\theta)$ and $\mathfrak{O}^2(\theta)$ satisfy $\mathfrak{O}^1(\theta) \leq \mathfrak{O}^2(\theta)$ for all θ, then

$$\theta G^i\big(A^i\big) - C^i\big(A^i, (\mathfrak{O}^1(\theta))^+\big) - R^i(L^i - (\mathfrak{O}^1(\theta))^+) \geq \theta G^i\big(A^i\big) - C^i\big(A^i, (\mathfrak{O}^2(\theta))^+\big) - R^i(L^i - (\mathfrak{O}^2(\theta))^+).$$

By construction, $\underline{\mathfrak{O}^i}(\theta,0,\sigma) \leq \underline{\mathfrak{O}^i}(\theta,1,\sigma) \leq \overline{\mathfrak{O}^i}(\theta,1,\sigma) \leq \overline{\mathfrak{O}^i}(\theta,0,\sigma)$. Therefore,

$$\begin{aligned}
\mathcal{Q}_0^l &\equiv \left\{\vartheta \,:\, \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\underline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+\big) - R^i(L^i - (\underline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+) > 0\right\} \supseteq \\
\mathcal{Q}_1^l &\equiv \left\{\vartheta \,:\, \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\underline{\mathfrak{O}^i}(\vartheta,1,\sigma))^+\big) - R^i(L^i - (\underline{\mathfrak{O}^i}(\vartheta,1,\sigma))^+) > 0\right\} \supseteq \\
\mathcal{Q}_1^u &\equiv \left\{\vartheta \,:\, \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\vartheta,1,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\vartheta,1,\sigma))^+) > 0\right\} \supseteq \\
\mathcal{Q}_0^u &\equiv \left\{\vartheta \,:\, \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+) > 0\right\}.
\end{aligned}$$

Thus, $\mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_0^l \,\Big|\, \widetilde{x_k} = x\Big) \geq \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_1^l \,\Big|\, \widetilde{x_k} = x\Big) \geq \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_1^u \,\Big|\, \widetilde{x_k} = x\Big) \geq \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_0^u \,\Big|\, \widetilde{x_k} = x\Big)$. It follows that

$$\begin{aligned}
&\mathfrak{R}(k,0,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_0^l \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\} \subseteq \mathfrak{R}(k,1,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_1^l \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\} \subseteq \\
&\mathfrak{S}^c(k,0,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_1^u \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\} \subseteq \mathfrak{S}^c(k,1,\sigma) = \left\{x : \mathbb{P}\Big(\widetilde{\theta} \in \mathcal{Q}_0^u \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\}.
\end{aligned}$$

Hence $\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,0,\sigma)) \leq \mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,1,\sigma)) \leq \mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \notin \mathfrak{S}(k,1,\sigma)) \leq \mathbb{P}(\theta+$

$\sigma\widetilde{\epsilon_k} \notin \mathfrak{S}(k,0,\sigma)$). Therefore, by the Exact Law of Large Numbers (Sun, 2006), for any θ,

$$\underline{\mathfrak{O}}^i(\theta,1,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,0,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k \le \underline{\mathfrak{O}}^i(\theta,2,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,1,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k$$

$$\le \overline{\mathfrak{O}^i}(\theta,2,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \notin \mathfrak{S}(k,1,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k \le \overline{\mathfrak{O}^i}(\theta,1,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon_k} \notin \mathfrak{S}(k,0,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k.$$

In addition, $\underline{\mathfrak{O}}^i(\theta,0,\sigma)$ and $\overline{\mathfrak{O}^i}(\theta,0,\sigma)$ are constant functions, hence continuous in θ. The functions $\vartheta \mapsto \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\underline{\mathfrak{O}}^i(\vartheta,0,\sigma))^+\big) - R^i(L^i - (\underline{\mathfrak{O}}^i(\vartheta,0,\sigma))^+)$ and $\vartheta \mapsto \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\vartheta,0,\sigma))^+)$ are continuous in ϑ. Therefore, $\mathscr{Q}_0^u$ and $\mathscr{Q}_0^l$ are both countable unions of disjoint open intervals with end points (ranked in decreasing order): $\infty = \overline{\vartheta}_{tem,0} > \underline{\vartheta}_{tem,0} > \overline{\vartheta}_{tem,1} > \underline{\vartheta}_{tem,1} > \cdots$. By Eq. (24)

$$\mathbb{P}\Big(\widetilde{\theta} \in \mathscr{Q}_0^l \,\Big|\, \widetilde{x_k} = x_k\Big) = \left(\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z) f_{\widetilde{\epsilon}_k}(z)dz\right)^{-1} \sum_{\iota=0}^{\infty}\left(\int_{(x_k-\overline{\vartheta}_{tem,\iota})/\sigma}^{(x_k-\underline{\vartheta}_{tem,\iota})/\sigma} f_{\widetilde{\theta}}(x_k-\sigma z) f_{\widetilde{\epsilon}_k}(z)dz\right).$$

Using Lebesgue's dominated convergence theorem, we can show that $\mathbb{P}\Big(\widetilde{\theta} \in \mathscr{Q}_0^l \,\Big|\, \widetilde{x_k} = x\Big)$ is continuous in x. It follows that $\mathfrak{R}(k,0,\sigma)$ is also a countable union of disjoint intervals. Hence the function $\theta \mapsto \mathbb{P}(\theta + \sigma\widetilde{\epsilon_k} \in \mathfrak{R}(k,0,\sigma))$ is continuous in θ as $F_{\widetilde{\epsilon}}$ has no point mass. Thus, by the bounded convergence theorem, $\underline{\mathfrak{O}}^i(\theta,1,\sigma)$, an integral of continuous functions, is also continuous in θ. We can repeat the above calculation and use mathematical induction to prove the claim. Note that each round is an elimination of dominated strategies, so equilibrium outflow must be in the interval $[\underline{\mathfrak{O}}^i(\theta,\ell,\sigma), \overline{\mathfrak{O}^i}(\theta,\ell,\sigma)]$ when the true state is θ. This is common knowledge among all agents. □

In the spirit of Claim 1, we can define the following two limiting functions:

$$\underline{\mathfrak{O}}^i(\theta,\infty,\sigma) = \lim_{\ell\to\infty} \underline{\mathfrak{O}}^i(\theta,\ell,\sigma), \qquad \overline{\mathfrak{O}^i}(\theta,\infty,\sigma) = \lim_{\ell\to\infty} \overline{\mathfrak{O}^i}(\theta,\ell,\sigma), \qquad \forall\theta.$$

By definition, any equilibrium outflow in state θ must be in the interval $[\underline{\mathfrak{O}}^i(\theta,\infty,\sigma), \overline{\mathfrak{O}^i}(\theta,\infty,\sigma)]$. Define sets

$$\begin{aligned}
\underline{\mathscr{Q}}_\infty^\sigma &\equiv \Big\{\vartheta : \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\underline{\mathfrak{O}}^i(\vartheta,\infty,\sigma))^+\big) - R^i(L^i - (\underline{\mathfrak{O}}^i(\vartheta,\infty,\sigma))^+) > 0\Big\},\\
\overline{\mathscr{Q}}_\infty^\sigma &\equiv \Big\{\vartheta : \vartheta G^i\big(A^i\big) - C^i\big(A^i, (\overline{\mathfrak{O}^i}(\vartheta,\infty,\sigma))^+\big) - R^i(L^i - (\overline{\mathfrak{O}^i}(\vartheta,\infty,\sigma))^+) > 0\Big\},\\
\mathfrak{R}(k,\infty,\sigma) &\equiv \Big\{x : \mathbb{P}\Big(\widetilde{\theta} \in \underline{\mathscr{Q}}_\infty^\sigma \,\Big|\, \widetilde{x_k} = x\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\Big\},\\
\mathfrak{S}(k,\infty,\sigma) &\equiv \Big\{x : \mathbb{P}\Big(\widetilde{\theta} \in \overline{\mathscr{Q}}_\infty^\sigma \,\Big|\, \widetilde{x_k} = x\Big) \ge \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\Big\}.
\end{aligned}$$

It is easy to verify that $\mathfrak{R}(k,\infty,\sigma)$ and $\mathfrak{S}(k,\infty,\sigma)$ are the limiting sets of the sequences of sets described in Claim 1, so $\mathfrak{R}(k,\infty,\sigma) = \cup_{\ell=1}^\infty \mathfrak{R}(k,\ell,\sigma)$ and $\mathfrak{S}(k,\infty,\sigma) = \cap_{\ell=1}^\infty \mathfrak{S}(k,\ell,\sigma)$. It is easy to show that the boundaries of $\mathfrak{R}(k,\infty,\sigma)$ and $\mathfrak{S}^c(k,\infty,\sigma)$ are countable unions of intervals, so both are continuity sets of the measure induced by $F_{\widetilde{\epsilon}_k}$ for all k (since $F_{\widetilde{\epsilon}_k}$ is absolutely continuous with respect to the Lebesgue measure on the real line). In addition,

$$\underline{\mathfrak{O}^i}(\theta,\infty,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon}_k \in \mathfrak{R}(k,\infty,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k \leq \overline{\mathfrak{O}^i}(\theta,\infty,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\mathbb{P}(\theta+\sigma\widetilde{\epsilon}_k \notin \mathfrak{S}(k,\infty,\sigma)) f_{\widetilde{\xi}}(\xi_k)d\xi_k.$$

From this, it is easy to show that $\underline{\mathfrak{O}^i}(\theta,\infty,\sigma)$ and $\overline{\mathfrak{O}^i}(\theta,\infty,\sigma)$ are both equilibrium outflows in state θ in some equilibrium (the equilibrium strategy profile is characterized by $\mathfrak{S}(k,\infty,\sigma)$ and $\mathfrak{R}(k,\infty,\sigma)$ respectively). In addition, since $f_{\widetilde{\epsilon}_k} \in \mathcal{L}^1$, both $\underline{\mathfrak{O}^i}(\theta,\infty,\sigma)$ and $\overline{\mathfrak{O}^i}(\theta,\infty,\sigma)$ are continuous in θ for fixed σ.

1.1.2 Step 2: necessary conditions for any candidate equilibrium

Fix a sufficiently small noise level $\sigma > 0$. Consider a candidate equilibrium where the deposit outflow at bank i is determined by the function $\mathcal{O}^i_{\text{cand}}(\theta,\sigma)$. While the form of this function is common knowledge in the candidate equilibrium, each depositor observes only a private signal $\widetilde{x}_k$ about the true state. Since depositor k's best response strategy only depends on her private signal x_k, define the run-signal set

$$\begin{aligned}&\mathfrak{R}^i_{\text{cand}}(k,\sigma) \equiv \left\{x_k : \mathcal{U}^2_{k,i\to i}(x_k) < \mathcal{U}^2_{k,i\to H}(x_k)\right\}\\ =&\left\{x_k:\ \mathbb{P}\Big(\tilde{\theta}\, G^i(A^i) - C^i\Big(A^i, \mathfrak{O}^i_{\text{cand}}(\tilde{\theta},\sigma)^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\tilde{\theta},\sigma)^+\Big) > 0 \,\Big|\, \tilde{x}_k = x_k\Big) < \frac{(1-\Lambda)}{R^i - \Lambda + \xi^i_k}\right\}.\end{aligned} \tag{28}$$

The equilibrium outflow must also satisfy

$$\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\vartheta + \sigma\tilde{\epsilon}_k \in \mathfrak{R}^i_{\text{cand}}(k,\sigma)\Big)\, dF_{\widetilde{\xi}}(\xi_k).$$

Because $f_{\widetilde{\epsilon}} \in \mathcal{L}^1$ is atomless, $\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma)$ is continuous in ϑ for any fixed σ. Given these beliefs, an agent regards bank i as defaulting iff $\widetilde{\theta} \in \mathcal{D}_i(\text{cand},\sigma)$, where[37]

$$\mathcal{D}_i(\text{cand},\sigma) = \left\{\theta:\ \theta G^i(A^i) - C^i\Big(A^i, \Big(\mathfrak{O}^i_{\text{cand}}(\theta,\sigma)\Big)^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta,\sigma)\Big) \leq 0\right\}.$$

Define the solvency thresholds

$$\begin{aligned}\theta^i_{\text{cand,up}}(\sigma) &= \inf\left\{\varrho:\ \Big[\vartheta G^i(A^i) - C^i\Big(A^i, (\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma))^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma)\Big)\Big] > 0 \ \ \forall\vartheta \geq \varrho\right\},\\ \theta^i_{\text{cand,down}}(\sigma) &= \sup\left\{\varrho:\ \Big[\vartheta G^i(A^i) - C^i\Big(A^i, (\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma))^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma)\Big)\Big] < 0 \ \ \forall\vartheta \leq \varrho\right\}.\end{aligned} \tag{29}$$

Therefore,

$$(-\infty, \theta^i_{\text{cand,down}}(\sigma)] \subseteqq \mathcal{D}_i(\text{cand},\sigma) \subseteqq (-\infty, \theta^i_{\text{cand,up}}(\sigma)].$$

By continuity, bankers' payoff should be zero at both thresholds:

$$\begin{aligned}0 &= \theta^i_{\text{cand,up}}(\sigma) G^i(A^i) - C^i\Big(A^i, (\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma),\sigma))^+\Big) - R^i\Big(L^i - (\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma),\sigma))^+\Big)\\ &= \theta^i_{\text{cand,down}}(\sigma) G^i(A^i) - C^i\Big(A^i, (\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma))^+\Big) - R^i\Big(L^i - (\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma))^+\Big).\end{aligned} \tag{30}$$

[37] Recall that $H^i = 0$.

By Eq. (29), $\theta^i_{\text{cand,down}}(\sigma) \le \theta^i_{\text{cand,up}}(\sigma)$, so Eq. (30) implies that

$$\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma) \ge \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma), \sigma). \tag{31}$$

For each depositor k, define:

$$\mathsf{C}^\epsilon_k \equiv \inf\left\{x :\ F_{\widetilde{\epsilon}_k}(x) \ge \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k}\right\}. \tag{32}$$

Claim 2. *Fix some small* $\sigma^{\text{tem}} > 0$*. Then there exists a constant* $\mathsf{C}^{\text{adj}} > 0$ *such that, for all* $\sigma \in (0, \sigma^{\text{tem}}]$,

1. *If depositor k observes* $\widetilde{x}_k \ge \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^\epsilon_k\sigma + \mathsf{C}^{\text{adj}}\sigma^2$*, she* does not *run on bank i.*
2. *If she observes* $\widetilde{x}_k \le \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^\epsilon_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2$*, she* moves *from bank i to hold the risk-free asset.*

Proof. Let σ be small. Suppose that the realization x_k of $\widetilde{x}_k$ is weakly smaller than $\theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^\epsilon_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2$, then there is some $\mathsf{C}^{\text{tem}} \ge 0$ such that $x_k = \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^\epsilon_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2 - \mathsf{C}^{\text{tem}}\sigma$. Then

$$\mathbb{P}\left(\mathcal{D}^c_i \mid x_k\right) \le \mathbb{P}\left(\widetilde{\theta} \ge \theta^i_{\text{cand,down}}(\sigma) \mid x_k\right) = \frac{\displaystyle\int_{-\infty}^{(x_k-\theta^i_{\text{cand,down}}(\sigma))/\sigma} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz}{\displaystyle\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz}.$$

Because $f_{\widetilde{\theta}}$ is uniformly continuous on a suitable interval, for sufficiently small σ this ratio equals

$$F_{\widetilde{\epsilon}_k}\left(\mathsf{C}^\epsilon_k - \mathsf{C}^{\text{adj}}\sigma - \mathsf{C}^{\text{tem}}\right) + \mathcal{O}(\sigma) = \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k} + F_{\widetilde{\epsilon}_k}\left(\mathsf{C}^\epsilon_k - \mathsf{C}^{\text{adj}}\sigma - \mathsf{C}^{\text{tem}}\right) - F_{\widetilde{\epsilon}_k}\left(\mathsf{C}^\epsilon_k\right) + \mathcal{O}(\sigma).$$

Because $\operatorname{ess\,sup}_k\{\|f_{\widetilde{\epsilon}_k}\|_\infty\}$ is finite almost surely and $\mathsf{C}^{\text{tem}} \ge 0$, one can choose C^{adj} large enough such that, for any depositor k and all $\mathsf{C}^{\text{tem}} \ge 0$,

$$\frac{\mathcal{U}^2_{k,i\to H}(x_k) - \mathcal{U}^2_{k,i\to i}(x_k)}{R^i + \xi^i_k - \Lambda} = \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k} - \mathbb{P}\left(\mathcal{D}^c_i \mid x_k\right) \ge \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k} - F_{\widetilde{\epsilon}_k}\left(\mathsf{C}^\epsilon_k - \mathsf{C}^{\text{adj}}\sigma - \mathsf{C}^{\text{tem}}\right) - \mathcal{O}(\sigma) > 0.$$

Hence, depositor k runs from bank i. The other direction can be proved similarly. □

Claim 3. $\lim_{\sigma\downarrow 0}\left(\theta^i_{\text{cand,up}}(\sigma) - \theta^i_{\text{cand,down}}(\sigma)\right) = 0.$

Proof. Suppose that a subsequence $\{\sigma_n\}_{n\ge 1}$ with $\sigma_n \to 0$ satisfies

$$\frac{\theta^i_{\text{cand,up}}(\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n} \xrightarrow[n\to\infty]{} \infty.$$

By Claim 2, $(-\infty,\ \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2] \subset \mathcal{R}^i_{\text{cand}}(k,\sigma) \subset (-\infty,\ \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}\sigma^2]$, $\mathcal{R}^i_{\text{cand}}(k,\sigma)\theta^i_{\text{cand,up}}(\sigma) \subsetneqq \mathcal{R}^i_{\text{cand}}(k,\sigma) - \theta^i_{\text{cand,down}}(\sigma)$. Then

$$\begin{aligned}&\mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma_n),\sigma_n\Big) - \mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma_n),\sigma_n\Big)\\ =&\mathcal{M}^i\int\limits_{\xi_k} \mathsf{s}_{k,i}\Bigg(\underbrace{\mathbb{P}\Big(\theta^i_{\text{cand,down}}(\sigma_n) + \sigma_n\widetilde{\epsilon}_k \in \mathcal{R}^i_{\text{cand}}(k,\sigma_n)\Big) - \mathbb{P}\Big(\theta^i_{\text{cand,up}}(\sigma_n) + \sigma_n\widetilde{\epsilon}_k \in \mathcal{R}^i_{\text{cand}}(k,\sigma_n)\Big)}_{>\varpi>0}\Bigg)\,dF_{\widetilde{\xi}}(\xi_k) > \varpi L^i,\end{aligned}$$

for some fixed $\varpi \in (0,1)$ and all n sufficiently large. This clearly contradicts Eq. (31). □

Since at both thresholds the solvency condition equals zero, Claim 3 implies that there exist constants $\mathsf{C}^{\text{adj}_3} > 0$ and $\sigma_0 > 0$ such that, for every $\sigma \in (0,\sigma_0)$,

$$\left|\mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma),\sigma\Big) - \mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma),\sigma\Big)\right| < \mathsf{C}^{\text{adj}_3}\,\sigma.$$

By Claim 2, $(-\infty,\ \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2] \subset \mathcal{R}^i_{\text{cand}}(k,\sigma) \subset (-\infty,\ \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}\sigma^2]$. Therefore,

$$\begin{aligned}&\mathcal{M}^i\int\limits_{\xi_k}\mathsf{s}_{k,i}\,\mathbb{P}\Big(\widetilde{\epsilon}_k \le \mathsf{C}^{\epsilon}_k - \mathsf{C}^{\text{adj}}\sigma\Big)\,dF_{\widetilde{\xi}}(\xi_k) \le \mathcal{M}^i\int\limits_{\xi_k}\mathsf{s}_{k,i}\,\mathbb{P}\Big(\theta^i_{\text{cand,down}}(\sigma) + \sigma\widetilde{\epsilon}_k \in \mathcal{R}^i_{\text{cand}}(k,\sigma)\Big)\,dF_{\widetilde{\xi}}(\xi_k) = \mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma),\sigma\Big),\\ &\mathcal{M}^i\int\limits_{\xi_k}\mathsf{s}_{k,i}\,\mathbb{P}\Big(\widetilde{\epsilon}_k \le \mathsf{C}^{\epsilon}_k + \mathsf{C}^{\text{adj}}\sigma\Big)\,dF_{\widetilde{\xi}}(\xi_k) \ge \mathcal{M}^i\int\limits_{\xi_k}\mathsf{s}_{k,i}\,\mathbb{P}\Big(\theta^i_{\text{cand,up}}(\sigma) + \sigma\widetilde{\epsilon}_k \in \mathcal{R}^i_{\text{cand}}(k,\sigma)\Big)\,dF_{\widetilde{\xi}}(\xi_k) = \mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma),\sigma\Big).\end{aligned} \tag{33}$$

Let O^i_B denote the benchmark outflow defined in Lemma 1. Eq. (33) implies that

$$\lim_{\sigma\downarrow 0}\mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma),\sigma\Big) \ge O^i_B, \qquad \lim_{\sigma\downarrow 0}\mathcal{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma),\sigma\Big) \le O^i_B.$$

Recalling Eq. (30) and the definition of θ^i_B in Lemma 1, we obtain

$$\lim_{\sigma\downarrow 0}\theta^i_{\text{cand,down}}(\sigma) \ \le\ \theta^i_B \ \le\ \lim_{\sigma\downarrow 0}\theta^i_{\text{cand,up}}(\sigma).$$

So θ^i_B lies between the endpoints. Claim 3 further implies that

$$\lim_{\sigma\downarrow 0}\left|\theta^i_{\text{cand,up}}(\sigma) - \theta^i_B\right| + \left|\theta^i_B - \theta^i_{\text{cand,down}}(\sigma)\right| = \lim_{\sigma\downarrow 0}\Big(\theta^i_{\text{cand,up}}(\sigma) - \theta^i_{\text{cand,down}}(\sigma)\Big) = 0.$$

Therefore, any candidate equilibrium converges to the limiting equilibrium described in Lemma 1.

1.1.3 Step 3: the existence and characterization of the limiting equilibrium

In Step 1, we showed that for each σ there exists at least one equilibrium. In Step 2, we established that if there is a candidate sequence of equilibria indexed by σ, then the associated cutoffs must converge to θ^i_B as $\sigma \downarrow 0$. Therefore, the equilibrium characterized in Step 1 must converge to the limiting equilibrium described in Lemma 1, and that limiting equilibrium must be unique by Step 2. □

1.2 Sequential Service Constraint: General Result

Lemma 6. *Suppose that $H^i = 0$ and $\partial_2 C(A^i, 0) \geq R^i > 1$. Then, in the limit as the signal noise vanishes ($\sigma \to 0$), there exists at least one limiting subgame equilibrium. Each such equilibrium is characterized by a threshold, θ^i_S, which is a fixed point of a continuous map $\mathscr{G}$. The equilibrium strategy is for all depositors to* ***stay*** *with bank i if the fundamental $\theta > \theta^i_S$ and* ***run*** *on bank i if the fundamental $\theta < \theta^i_S$. The limiting subgame equilibrium is* ***unique*** *with some technical conditions.*[38] *Furthermore, the run cutoff with sequential service constraint is weakly less thant without: $\theta^i_S \leq \theta^i_B$*

The full construction of the map $\mathscr{G}$ is quite complex, and its details are deferred below in the text of the following proof. The basic idea is similar to Internet Appendix 1.1, but we need to handle additional difficulties associated with sequential service constraint.

Setting up Consider the representative bank i. Let agent k be a representative depositor at bank i after time $t = 1$. Denote by $\mathbb{O}^i$ the aggregate deposit outflow from bank i. In the **Alternative Economy of Isolated Bank Run** that is concerned here, $\mathtt{I}^i = 0$. Given any agents' strategy profile, denoted as $\{\mathsf{r}_{k,i\to H}\}$, if the true economic state is θ, then the Exact Law of Large Numbers (ELLN) yields

$$\mathbb{O}^i = \mathcal{M}^i \int_{\xi_k} \int_{x_k} \mathsf{r}_{k,j\to H}(x_k) f_{\widetilde{x}_k|\widetilde{\theta}}(x_k \mid \theta)\, dx_k\, f_{\widetilde{\xi}}(\xi_k)\, d\xi_k.$$

Hence any candidate equilibrium outflows is a function of θ and σ (recall that private signal precision is governed by σ, so the densities $f_{\widetilde{x}_k|\widetilde{\theta}}$ and $f_{\widetilde{x}_w|\widetilde{\theta}}$ depend on σ)—i.e., any outflow can be written as a "deposit flow function", $\mathbb{O}^i(\theta, \sigma)$.

Given any possible deposit flow functions, depositor k's expected payoff from *staying* with bank i after observing x_k at $t = 2$ is

$$\mathcal{U}^2_{k,i\to i}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to i} = 1 \mid x_k\Big) = \mathbb{P}\Big(\mathcal{D}^c_i \mid x_k\Big)(R^i + \xi^i_k) + \mathbb{P}\Big(\mathcal{D}_i \mid x_k\Big)\Lambda. \tag{34}$$

If k *leaves* the banking system to hold the risk-free asset, her expected payoff is[39]

$$\mathcal{U}^2_{k,i\to H}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to H} = 1 \mid x_k\Big) = \int_{\mathcal{D}_i} \Big[\mathtt{M}^i + \Big(1 - \mathtt{M}^i\Big)\Lambda\Big] f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta + \int_{\mathcal{D}^c_i} f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta, \tag{35}$$

where $\mathtt{M}^i = \min\Big(\frac{\mathbb{O}^i(\tilde{\theta})}{\mathbb{O}^i_{\text{cand}}(\tilde{\theta},\sigma)}, 1\Big)$. Under the best-responding strategy, depositor k leaves bank i iff $\mathcal{U}^2_{k,i\to H}(x_k) > \mathcal{U}^2_{k,i\to i}(x_k)$ and vice versa. Hence, depositor k's best response strategy only

[38] The equilibrium is unique under a broad range of specific modeling choices, but pinpointing the exact sufficient condition that ensures uniqueness is highly technical and would be a digression from the main focus of this paper. One condition is that $\partial_2 C(A^i, 0)$ is sufficiently large.

[39] Recall that $\mathbb{O}^i(\theta)$ denotes the maximum outflow that bank i can pay out without exhausting its liquid resources, see Eq. (11).

depend on her private signal x_k. Define the run-signal set

$$\mathfrak{R}^i_{\text{cand}}(k,\sigma) \equiv \left\{x_k : \mathcal{U}^2_{k,i\to i}(x_k) < \mathcal{U}^2_{k,i\to H}(x_k)\right\}$$
$$= \Big\{x_k :\ \mathbb{P}\Big(\tilde{\theta}\, G^i(A^i) - C^i\Big(A^i, \mathfrak{O}^i_{\text{cand}}(\tilde{\theta},\sigma)^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\tilde{\theta},\sigma)^+\Big) > 0 \,\Big|\, \tilde{x}_k = x_k\Big)$$
$$< \frac{(1-\Lambda)}{R^i - \Lambda + \xi^i_k}\mathbb{E}\Big[\min\Big(\tfrac{\mathfrak{O}^i(\tilde{\theta})}{\mathfrak{O}^i_{\text{cand}}(\tilde{\theta},\sigma)}, 1\Big) \,\Big|\, \tilde{x}_k = x_k\Big]\Big\}. \tag{36}$$

The equilibrium outflow must also satisfy

$$\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma) = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\vartheta + \sigma\tilde{\epsilon}_k \in \mathfrak{R}^i_{\text{cand}}(k,\sigma)\Big)\, dF_{\tilde{\xi}}(\xi_k).$$

Necessary conditions for any candidate equilibrium Fix a sufficiently small noise level $\sigma > 0$. Consider a candidate equilibrium where the deposit outflow at bank i is determined by the function $\mathcal{O}^i_{\text{cand}}(\theta,\sigma)$. While the form of this function is common knowledge in the candidate equilibrium, each depositor observes only a private signal $\widetilde{x}_k$ about the true state. Given these beliefs, an agent regards bank b as defaulting iff $\widetilde{\theta} \in \mathcal{D}_i(\text{cand},\sigma)$, where[40]

$$\mathcal{D}_i(\text{cand},\sigma) = \Big\{\theta :\ \theta G^i(A^i) - C^i\Big(A^i, \Big(\mathfrak{O}^i_{\text{cand}}(\theta,\sigma)\Big)^+\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta,\sigma)\Big) \le 0\Big\}.$$

Define the solvency thresholds $\theta^i_{\text{cand,up}}(\sigma)$ and $\theta^i_{\text{cand,down}}(\sigma)$ as in Eq. (29). Therefore, $(-\infty, \theta^i_{\text{cand,down}}(\sigma)] \subseteqq \mathcal{D}_i(\text{cand},\sigma) \subseteqq (-\infty, \theta^i_{\text{cand,up}}(\sigma)]$. Eq. (31) also holds.

Our next goal is to characterize the optimal strategy for depositor k. This strategy is defined by a "run region," a set $\mathfrak{R}^i_{\text{cand}}(k,\sigma)$ containing all signal values $\widetilde{x}_k$ that induce the depositor to run. An argument similar to the one in Claim 2 shows that there is a this run region is bounded by some candidate thresholds for all small enough σ:

1. The depositor **runs** on bank i (by moving funds to a safe asset) if the observed signal satisfies:
$$\widetilde{x}_k \le \theta^i_{\text{cand,down}}(\sigma) + \underline{\varrho}^\epsilon_k(\sigma)\sigma + \mathsf{C}^{\text{adj}}\sigma^2$$
2. The depositor does **not run** on bank i if the observed signal satisfies:
$$\widetilde{x}_k \ge \theta^i_{\text{cand,up}}(\sigma) + \overline{\varrho}^\epsilon_k(\sigma)\sigma - \mathsf{C}^{\text{adj}}\sigma^2$$

These inequalities hold for sets of constants $\underline{\varrho}^\epsilon_k(\sigma)$ and $\overline{\varrho}^\epsilon_k(\sigma)$. We define the ultimate decision thresholds, $\underline{\mathsf{C}}^\epsilon_k$ and $\overline{\mathsf{C}}^\epsilon_k$, by taking the tightest possible bounds:

$$\underline{\mathsf{C}}^\epsilon_k(\sigma) \equiv \sup\{\underline{\varrho}^\epsilon_k(\sigma)\} \quad \text{and} \quad \overline{\mathsf{C}}^\epsilon_k(\sigma) \equiv \inf\{\overline{\varrho}^\epsilon_k(\sigma)\}.$$

Therefore,

$$(-\infty, \theta^i_{\text{cand,down}}(\sigma) + \underline{\mathsf{C}}^\epsilon_k(\sigma)\sigma - \mathsf{C}^{\text{adj}}\sigma^2] \subset \mathfrak{R}^i_{\text{cand}}(k,\sigma) \subset (-\infty, \theta^i_{\text{cand,up}}(\sigma) + \overline{\mathsf{C}}^\epsilon_k(\sigma)\sigma + \mathsf{C}^{\text{adj}}\sigma^2].$$

[40] Recall that $H^i = 0$.

This structure implies that the key threshold constants, $\underline{\mathsf{C}}_k^\epsilon$ and $\overline{\mathsf{C}}_k^\epsilon$, are also the normalized boundaries of the run region and its complement, $(\mathcal{R}^i_{\text{cand}}(k,\sigma))^c$:

$$\underline{\mathsf{C}}_k^\epsilon(\sigma) = \frac{\inf\left((\mathcal{R}^i_{\text{cand}}(k,\sigma))^c\right) - \theta^i_{\text{cand,down}}(\sigma)}{\sigma} \quad \text{and} \quad \overline{\mathsf{C}}_k^\epsilon(\sigma) = \frac{\sup\left(\mathcal{R}^i_{\text{cand}}(k,\sigma)\right) - \theta^i_{\text{cand,up}}(\sigma)}{\sigma}.$$

Claim 4. *Because $\{f_{\tilde{\epsilon}_k}\}$ has large enough common support, for any possible sequence of equilibria as $\sigma \to 0$,*

$$\lim_{\sigma\downarrow 0} \frac{\theta^i_{\text{cand,up}}(\sigma) - \theta^i_{\text{cand,down}}(\sigma)}{\sigma} + \overline{\mathsf{C}}_k^\epsilon(\sigma) - \underline{\mathsf{C}}_k^\epsilon(\sigma) = 0.$$

Proof. Suppose that

$$\Delta^{tem}_\sigma \equiv \frac{\theta^i_{\text{cand,up}}(\sigma) - \theta^i_{\text{cand,down}}(\sigma)}{\sigma} + \overline{\mathsf{C}}_k^\epsilon(\sigma) - \underline{\mathsf{C}}_k^\epsilon(\sigma) > 0.$$

The argument now proceeds by contradiction. Suppose that there exists some subsequence $\{\sigma_n\}_{n\geq 1}$ where $\sigma_n \to 0$, such that $\liminf_{n\to\infty} \Delta^{tem}_{\sigma_n} = \bar{\Delta}^{tem} > 0$. Define set translation $\frac{\mathcal{R}^i_{\text{cand}}(k,\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n} \equiv \{x \mid x\sigma_n + \theta^i_{\text{cand,down}}(\sigma_n) \in \mathcal{R}^i_{\text{cand}}(k,\sigma_n)\}$. We define the following disjoint normalized sets:

$$\begin{aligned}
\Omega^i &= \left(\frac{\mathcal{R}^i_{\text{cand}}(k,\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n}\right) \bigcap (\underline{\mathsf{C}}_k^\epsilon(\sigma_n), \underline{\mathsf{C}}_k^\epsilon(\sigma_n) + \Delta^{tem}_{\sigma_n}) \\
\Omega^{i,c} &= \left(\left(\frac{\mathcal{R}^i_{\text{cand}}(k,\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n}\right) \bigcap (\underline{\mathsf{C}}_k^\epsilon(\sigma_n), \underline{\mathsf{C}}_k^\epsilon(\sigma_n) + \Delta^{tem}_{\sigma_n})\right)^c - \Delta^{tem}_{\sigma_n}.
\end{aligned}$$

Then a direct calculation shows that there is some constant $\mathcal{K} > 0$ such that

$$\begin{aligned}
&|\mho^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma_n), \sigma_n) - \mho^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma_n) + \Delta^{tem}_{\sigma_n}\sigma_n, \sigma_n)| \\
=&\mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \left(\mathbb{P}\Big(\tilde{\epsilon}_k \in \frac{\mathcal{R}^i_{\text{cand}}(k,\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n}\Big) - \mathbb{P}\Big(\tilde{\epsilon}_k + \Delta^{tem}_{\sigma_n} \in \frac{\mathcal{R}^i_{\text{cand}}(k,\sigma_n) - \theta^i_{\text{cand,down}}(\sigma_n)}{\sigma_n}\Big)\right) dF_{\tilde{\xi}}(\xi_k) \\
=&\mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \left(\mathbb{P}\Big(\tilde{\epsilon}_k \in \Omega^i \cup \Omega^{i,c}\Big)\right) dF_{\tilde{\xi}}(\xi_k) > \mathcal{K}(\Delta^{tem}_{\sigma_n}).
\end{aligned} \tag{37}$$

This contradicts Eq. (31) for sufficiently small σ_n. □

Claim 4 immediately implies the following characterization of the depositor strategy and its resulting outflow bounds.

Claim 5. *There exist constants $\mathsf{C}_k^\epsilon(\sigma)$ and $\mathsf{C}^{\text{adj}} > 0$ such that for all sufficiently small σ, the optimal strategy for depositor k is to:*

1. ***Run** on bank i (by moving funds to a safe asset) if the observed signal satisfies:*

$$\widetilde{x}_k < \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}_k^\epsilon(\sigma)\sigma - \mathsf{C}^{\text{adj}}\sigma^2$$

2. ***Not run*** *on bank i if the observed signal satisfies:*

$$\widetilde{x}_k \geq \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k(\sigma)\sigma + \mathsf{C}^{\text{adj}}\sigma^2$$

This strategy profile implies the following bounds on the candidate outflow function for any fundamental state ϑ:

$$\mathbb{E}_{\xi_k}\left[\mathsf{s}_{k,i}\, F_{\widetilde{\epsilon}}\left(\frac{\theta^i_{\text{cand,down}}(\sigma)-\vartheta}{\sigma}+\mathsf{C}^{\epsilon}_k(\sigma)-\mathsf{C}^{\text{adj}}\sigma\right)\right] \leq \frac{\mathbb{O}^i_{\text{cand}}(\vartheta,\sigma)}{\mathcal{M}^i} \leq \mathbb{E}_{\xi_k}\left[\mathsf{s}_{k,i}\, F_{\widetilde{\epsilon}}\left(\frac{\theta^i_{\text{cand,down}}(\sigma)-\vartheta}{\sigma}+\mathsf{C}^{\epsilon}_k(\sigma)+\mathsf{C}^{\text{adj}}\sigma\right)\right].$$

Proof. This is trivial given Claim 4. □

Characterizing the unique limiting equilibrium Our next goal is to characterize $\mathsf{C}^{\epsilon}_k(\sigma)$ as $\sigma \to 0$, which will allow us to fully characterize the unique limiting equilibrium. It will be clear that $\mathsf{C}^{\epsilon}_k(\sigma)$ is determined in a system of equations that characterize the equilibrium. By definition of $\mathbb{O}^i$,

$$\begin{aligned}0 &= \theta^i_{\text{cand,down}}(\sigma)\, G^i(A^i) - C^i\Big(A^i, \mathbb{O}^i\big(\theta^i_{\text{cand,down}}(\sigma)\big)\Big)\\ &= \theta^i_{\text{cand,down}}(\sigma)\, G^i(A^i) - C^i\Big(A^i, \mathbb{O}^i_{\text{cand}}\big(\theta^i_{\text{cand,down}}(\sigma),\sigma\big)\Big) - R^i\Big(L^i - \mathbb{O}^i_{\text{cand}}\big(\theta^i_{\text{cand,down}}(\sigma),\sigma\big)\Big),\end{aligned}$$

which implies $\mathbb{O}^i\big(\theta^i_{\text{cand,down}}(\sigma)\big) > \mathbb{O}^i_{\text{cand}}\big(\theta^i_{\text{cand,down}}(\sigma),\sigma\big)$. Define the constant $\mathsf{C}^{\text{cut}}_{\sigma}$ by (it is easy to check that $\mathsf{C}^{\text{cut}}_{\sigma} \in \mathbb{R}$ is well defined for small enough σ)

$$\mathsf{C}^{\text{cut}}_{\sigma} \equiv \inf\Big\{\varrho \geq 0:\ \mathbb{O}^i_{\text{cand}}\big(\theta^i_{\text{cand,down}}(\sigma)-\varrho\sigma,\sigma\big) = \mathbb{O}^i\big(\theta^i_{\text{cand,down}}(\sigma)-\varrho\sigma\big)\Big\}.$$

When receiving signal $\theta^i_{\text{cand,down}}(\sigma)+\mathsf{C}^{\epsilon}_k\sigma-\mathsf{C}^{\text{adj}}\sigma^2$, depositor k estimates that

$$\begin{aligned}&\int_{\theta} \min\Big[\tfrac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta,\sigma)},1\Big] f_{\widetilde{\theta}|\widetilde{x}_k}\big(\theta \mid \theta^i_{\text{cand,down}}(\sigma)+\mathsf{C}^{\epsilon}_k\sigma-\mathsf{C}^{\text{adj}}\sigma^2\big)\, d\theta\\ &= \frac{\int_{-\infty}^{\mathsf{C}^{\epsilon}_k+\mathsf{C}^{\text{cut}}_{\sigma}-\mathsf{C}^{\text{adj}}\sigma} f_{\widetilde{\theta}}(x_k-\sigma z) f_{\widetilde{\epsilon}_k}(z)dz}{\int_z f_{\widetilde{\theta}}(x_k-\sigma z) f_{\widetilde{\epsilon}_k}(z)dz} + \frac{\int_{\mathsf{C}^{\epsilon}_k+\mathsf{C}^{\text{cut}}_{\sigma}-\mathsf{C}^{\text{adj}}\sigma}^{\infty} \frac{\mathbb{O}^i(x_k-\sigma z)}{\mathbb{O}^i(x_k-\sigma z,\sigma)} f_{\widetilde{\theta}}(x_k-\sigma z) f_{\widetilde{\epsilon}_k}(z)dz}{\int_z f_{\widetilde{\theta}}(x_k-\sigma z) f_{\widetilde{\epsilon}_k}(z)dz}\\ &= F_{\widetilde{\epsilon}_k}(\mathsf{C}^{\epsilon}_k+\mathsf{C}^{\text{cut}}_{\sigma}-\mathsf{C}^{\text{adj}}\sigma) + \int_{\mathsf{C}^{\epsilon}_k+\mathsf{C}^{\text{cut}}_{\sigma}-\mathsf{C}^{\text{adj}}\sigma}^{\infty} \frac{\mathbb{O}^i(\theta^i_{\text{cand,down}}(\sigma))}{\mathbb{O}^i(x_k-\sigma z,\sigma)} f_{\widetilde{\epsilon}_k}(z)dz + \mathcal{O}(\sigma)\end{aligned}$$

Fix $x_k = \theta^i_{\text{cand,down}}(\sigma)+\mathsf{C}^{\epsilon}_k\sigma-\mathsf{C}^{\text{adj}}\sigma^2$. Claim 5 implies

$$\begin{aligned}\frac{(1-\Lambda)}{R^i+\xi^i_k-\Lambda}\int_{\theta} \min\Big[\tfrac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta,\sigma)},1\Big] f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta &> \mathbb{P}\big(\mathcal{D}^c_i \mid x_k\big)\\ \frac{(1-\Lambda)}{R^i+\xi^i_k-\Lambda}\int_{\theta} \min\Big[\tfrac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta,\sigma)},1\Big] f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k+2\mathsf{C}^{\text{adj}}\sigma^2)\, d\theta &\leq \mathbb{P}\big(\mathcal{D}^c_i \mid x_k+2\mathsf{C}^{\text{adj}}\sigma^2\big).\end{aligned} \tag{38}$$

Claim 4 implies that for sufficiently small σ,

$$\mathbb{P}\Big(\mathcal{D}_i^c \mid x_k\Big) = \mathbb{P}\Big(\widetilde{\theta} \geq \theta^i_{\text{cand,down}}(\sigma) \mid x_k\Big) + \mathcal{O}(\sigma) = \frac{\int_{-\infty}^{(x_k-\theta^i_{\text{cand,down}}(\sigma))/\sigma} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz}{\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz} + \mathcal{O}(\sigma) = F_{\widetilde{\epsilon}_k}\Big(\mathsf{C}_k^{\epsilon} - \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma).$$

Similarly, $\mathbb{P}\Big(\mathcal{D}_i^c \mid x_k + 2\mathsf{C}^{\text{adj}}\sigma^2\Big) = F_{\widetilde{\epsilon}_k}\Big(\mathsf{C}_k^{\epsilon} + \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma)$. Taking the limits as $\sigma \to 0$, we obtain from Eq. (38)

$$\lim_{\sigma\downarrow 0} \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda} \int_{\theta} \min[\tfrac{\mathbb{O}^i(\theta)}{\mathbb{O}^i(\theta,\sigma)}, 1] f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta = F_{\widetilde{\theta}}\Big(\mathsf{C}_k^{\epsilon}\Big) \Rightarrow$$

$$\frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda}\Bigg(F_{\widetilde{\epsilon}_k}(\mathsf{C}_k^{\epsilon} + \mathsf{C}_\sigma^{\text{cut}}) + \int_{\mathsf{C}_k^{\epsilon}+\mathsf{C}^{\text{cut}}}^{\infty} \frac{\mathbb{O}^i(\theta^i_{\text{cand,down}}(\sigma))}{\mathcal{M}^i \mathbb{E}_{\xi_w}\big[\mathsf{s}_{w,i}\, F_{\widetilde{\epsilon}}\big(z + \mathsf{C}_w^{\epsilon} - \mathsf{C}_k^{\epsilon}\big)\big]} f_{\widetilde{\epsilon}_k}(z) dz\Bigg) = F_{\widetilde{\theta}}\Big(\mathsf{C}_k^{\epsilon}\Big). \tag{39}$$

By monotonicity, $\mathbb{O}^i(\theta^i_{\text{cand,down}}(0))$ and C^{cut} uniquely determine C_k^{ϵ}. Also, by definition, $\lim_{\sigma\downarrow} \mathbb{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma) - \mathsf{C}_\sigma^{\text{cut}}\sigma, \sigma\Big) = \mathbb{O}^i\Big(\theta^i_{\text{cand,down}}(\sigma)\Big)$, so

$$\mathbb{E}_{\xi_k}\Big[\mathsf{s}_{k,i}\, F_{\widetilde{\epsilon}}\Big(\mathsf{C}_k^{\epsilon} + \mathsf{C}^{\text{cut}}\Big)\Big] = \frac{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))}{\mathcal{M}^i}. \tag{40}$$

Therefore, given $\{\mathsf{C}_k^{\epsilon}\}$ and $\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))$, C^{cut} is uniquely determined. The system of equations Eqs. (39) and (40) implicitly defines a mapping, T, from the space of possible cutoffs C^{cut} to itself. In any limiting equilibrium, C^{cut} must be a **fixed point** of this mapping.

In general, T is a contraction. Intuitively, the aggregation across heterogeneous depositors in Eq. (40) has a "smoothing" effect. A small change in an initial assumed cutoff, $\mathsf{C}^{\text{cut}}_{\text{old}}$, will cause a dampened, smaller change in the resulting cutoff, $\mathsf{C}^{\text{cut}}_{\text{new}}$. Formally, it can be shown that the mapping T has a Lipschitz constant less than one generically. To rule out pathological cases, we assume that $\{f_{\widetilde{\epsilon}_k}\}$ ensures that T is a contraction. It is easy to check that most common distribution families satisfy this assumption. By the Banach Fixed-Point Theorem, any contraction mapping on a complete metric space admits a unique fixed point. Therefore, the system has a unique equilibrium set of thresholds, $(\{\mathsf{C}_k^{\epsilon}\}, \mathsf{C}^{\text{cut}})$, when we fix each possible value of $\theta^i_{\text{cand,down}}(0)$. The exact value of $\theta^i_{\text{cand,down}}(0)$ in the limiting equilibrium will be determined by the following system of equations:

Aggregating Eq. (39), we obtain

$$\mathbb{E}_{\xi_k}\left[\mathsf{s}_{k,i}\, \frac{(1-\Lambda)}{R^i + \xi_k^i - \Lambda}\Bigg(F_{\widetilde{\epsilon}_k}(\mathsf{C}_k^{\epsilon} + \mathsf{C}^{\text{cut}}) + \int_{\mathsf{C}_k^{\epsilon}+\mathsf{C}^{\text{cut}}}^{\infty} \frac{\mathbb{O}^i(\theta^i_{\text{cand,down}}(0)) f_{\widetilde{\epsilon}_k}(z)}{\mathcal{M}^i \mathbb{E}_{\xi_w}\big[\mathsf{s}_{w,i}\, F_{\widetilde{\epsilon}}\big(z + \mathsf{C}_w^{\epsilon} - \mathsf{C}_k^{\epsilon}\big)\big]} dz\Bigg)\right] = \frac{\mathbb{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(0), 0\Big)}{\mathcal{M}^i} \tag{41}$$

In the limiting equilibrium, the following must also be true

$$\theta^i_{\text{cand,down}}(0)G^i(A^i) = C^i\left(A^i, \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0), 0\right)\right) + R^i\left(L^i - \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0), 0\right)\right) \tag{42}$$

$$\theta^i_{\text{cand,down}}(0)G^i(A^i) = C^i\left(A^i, \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0)\right)\right) \tag{43}$$

The limiting equilibrium is fully characterized by the system of equations in Eqs. (39) to (43), and the existence of the solution of $\theta^i_{\text{cand,down}}(0)$ is guaranteed by Brouwer's fixed-point theorem. While a unique solution is expected under generic conditions, a formal proof would be a digression from the main focus of this paper. We therefore confine to a brief argument of the tractable case where the cost function is sufficiently convex (i.e., $\partial_2 C^i$ is large).

The uniqueness proof proceeds by showing a contradiction. On one hand, Eqs. (42) and (43) imply that any increase in $\mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0)\right)$ necessitates a correspondingly large increase in $\mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0)\right)$; this required increment is bounded below by a function of $\partial_2 C^i$. On the other hand, Eq. (41) place a fixed upper bound on the possible increment of $\mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0)\right)$. For a sufficiently large $\partial_2 C^i$, the required increase (the lower bound) will exceed the permitted increase (the upper bound). This makes the existence of two distinct solutions impossible, thus guaranteeing uniqueness. □

Algorithm to compute the equilibrium The above system of equations characterizing the subgame equilibrium inspires the construction of a continuous map $\mathcal{G} : \mathbb{R} \to \mathbb{R}$ whose fixed point, θ^i_S, defines the equilibrium. We build this map as a composition of four functions.

1. Define $\mathcal{G}_1(x) = \inf\{y \mid xG^i(A^i) = C^i\left(A^i, y\right) = 0\}$. The output of this function is continuously increasing in x.

2. Define the map $\mathcal{G}_2$, which takes a scalar x and returns the tuple $(\mathtt{C}^{cut}, \{\mathtt{C}^{\epsilon}_k\})$. The mapping is defined such that if $x = \mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))$, then Eqs. (39) and (40) hold. As established previously, for any fixed x, the map returns a unique image, and $\mathcal{G}_2$ is a continuous function of x.

3. The map $\mathcal{G}_3$ takes the tuple $(\mathtt{C}^{cut}, \{\mathtt{C}^{\epsilon}_k\})$ as input and returns the scalar outflow O^i_S that solves the following condition:

$$\mathbb{E}_{\xi_k}\left[\mathsf{s}_{k,i}\,\frac{(1-\Lambda)}{R^i + \xi^i_k - \Lambda}\left(F_{\tilde{\epsilon}_k}(\mathtt{C}^{\epsilon}_k + \mathtt{C}^{\text{cut}}) + \int_{\mathtt{C}^{\epsilon}_k + \mathtt{C}^{\text{cut}}}^{\infty} \frac{\mathbb{O}^i(\theta^i_{\text{cand,down}}(0)) f_{\tilde{\epsilon}_k}(z)}{\mathcal{M}^i \mathbb{E}_{\xi_w}\left[\mathsf{s}_{w,i}\, F_{\tilde{\epsilon}}\left(z + \mathtt{C}^{\epsilon}_w - \mathtt{C}^{\epsilon}_k\right)\right]} dz\right)\right] = \frac{O^i_S}{\mathcal{M}^i}$$

4. Define $\mathcal{G}_4(y) = \frac{C^i\left(A^i, y\right) + R^i\left(L^i - y\right)}{G^i(A^i)}$.

5. The final map is the composition of the previous four steps: $\mathcal{G} = \mathcal{G}_4 \circ \mathcal{G}_3 \circ \mathcal{G}_2 \circ \mathcal{G}_1$.

Any fixed point of the above system would be one limiting equilibrium.

1.3 Proof of Lemma 2

We prove this lemma as a corollary of Lemma 6 discussed in Internet Appendix 1.2 above. Under the assumption of ex-ante homogeneity—that is, when all depositors have the same banking utility ($\xi_k^i = \xi^i$) and draw signals from the same distribution ($f_{\widetilde{\epsilon}_k} = f_{\widetilde{\epsilon}}$)—the system of equations Eqs. (39) to (43) simplifies considerably. Specifically, all depositors adopt the same cutoff strategy, so C_k^ϵ becomes a constant C^ϵ independent of k. This allows us to simplify Eq. (40). Because C^ϵ is constant, we can separate the expectation, and since $\mathbb{E}_{\xi_k}[\mathsf{s}_{k,i}] = L^i/\mathcal{M}^i$, Eq. (40) simplifies to:

$$F_{\widetilde{\epsilon}}\left(\mathsf{C}^\epsilon + \mathsf{C}^{\text{cut}}\right) = \frac{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))}{L^i}.$$

Furthermore, the homogeneity assumption simplifies the limiting expression for the outflow function from Claim 5:

$$\lim_{\sigma\downarrow 0} \mathbb{O}^i(\theta^i_{\text{cand,down}}(0) + \mathsf{C}^\epsilon\sigma - \sigma z, \sigma) = \mathcal{M}^i\mathbb{E}_{\xi_w}\left[\mathsf{s}_{w,i}\, F_{\widetilde{\epsilon}}(z + \mathsf{C}^\epsilon - \mathsf{C}^\epsilon)\right] = L^i F_{\widetilde{\epsilon}}(z).$$

Therefore, Eq. (41) simplifies to

$$\frac{(1-\Lambda)}{R^i+\xi^i-\Lambda}\frac{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))}{\mathcal{M}^i} + \frac{L^i}{\mathcal{M}^i}\int_{\mathsf{C}^\epsilon+\mathsf{C}^{\text{cut}}}^{\infty}\frac{\mathbb{O}^i(\theta^i_{\text{cand,down}}(0))}{L^iF_{\widetilde{\epsilon}}(z)} f_{\widetilde{\epsilon}_k}(z)dz = \frac{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0),0)}{\mathcal{M}^i} \Rightarrow$$

$$\frac{(1-\Lambda)}{R^i+\xi^i-\Lambda} + \log(L^i) - \log(\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))) = \frac{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0),0)}{\mathbb{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(0))}$$

The equilibrium conditions in Eqs. (42) and (43) establish the following key relationship between the expected deposit outflow ($\mathbb{O}$) and the bank's total resources ($\mathbb{O}$) at the run threshold:

$$C^i\left(A^i, \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0),0\right)\right) + R^i\left(L^i - \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0),0\right)\right) = C^i\left(A^i, \mathbb{O}^i_{\text{cand}}\left(\theta^i_{\text{cand,down}}(0)\right)\right).$$

The above two equations imply that in any candidate limiting equilibrium, the outflow $\mathbb{O}^i_S$ and the total resources $\mathbb{O}^i_S$ must be the solution to the following system of two equations:

$$\left(\frac{1-\Lambda}{R^i+\xi^i-\Lambda} + \log(L^i) - \log(\mathbb{O}^i_S)\right)\mathbb{O}^i_S = \mathbb{O}^i_S \quad (44)$$

$$C^i\left(A^i, \mathbb{O}^i_S\right) + R^i\left(L^i - \mathbb{O}^i_S\right) = C^i\left(A^i, \mathbb{O}^i_S\right). \quad (45)$$

We can prove the uniqueness of this solution geometrically by treating $\mathbb{O}^i_S$ as a function of $\mathbb{O}^i_S$ in the $(\mathbb{O}^i_S, \mathbb{O}^i_S)$ space.

- **Eq. (44)** implicitly defines a **hump-shaped, concave function** that passes through the (limiting) points $(0,0)$ and $(L^i, \frac{(1-\Lambda)L^i}{R^i+\xi^i-\Lambda})$.
- When $\partial_2^3 C^i$ is large enough, **Eq. (45)** implicitly defines a **convex function** that passes through the points $(\mathbb{O}^i_{min}, 0)$ and (L^i, L^i), where $\mathbb{O}^i_{min}$ solves $C^i(A^i, \mathbb{O}^i_{min}) = R^i L^i$.

The concave function starts above the convex function near the origin and ends below it at L^i. A continuous concave and convex function pair with this configuration must intersect at a single point. This guarantees that the system has a unique solution, and therefore the limiting equilibrium is unique. Furthermore, the solution of $\mathfrak{O}^i_S \le O^i_B$, so $\theta^i_S \le \theta^i_B$.

Finally, we note a crucial departure from the standard global games framework: the equilibrium is **not solvable** by the iterative elimination of dominated strategies. Instead, it is a coordination game that requires all agents to correctly conjecture the key parameter C^ϵ, which in turn determines the collective run region $\mathfrak{R}^i_{\text{cand}}(k,\sigma)$. This reliance on common knowledge of the full strategy space represents a demanding rationality assumption, particularly for a dispersed group of individual bank depositors. □

1.4 Proof of Lemma 3

We prove the lemma for $j = i$ as the case of $j = b$ follows an identical argument.

Key Challenge and Proof Overview The liquid buffer H^i creates a non-monotonicity: counterintuitively, a larger deposit outflow $\mathfrak{O}^i$ can *reduce* default risk for bank i by shrinking its default set when the outflow remains below H^i. Bank i defaults in state θ exactly when

$$\theta \in \left\{\vartheta \;\middle|\; \vartheta\, G^i(A^i) - C^i\Big(A^i, (\mathfrak{O}^i - H^i)^+\Big) + (H^i - \mathfrak{O}^i)^+ - R^i\Big(L^i - \mathfrak{O}^i\Big) \le 0\right\}.$$

When $\mathfrak{O}^i < H^i$, increasing $\mathfrak{O}^i$ *shrinks* the above default set, so remaining depositors have a stronger incentive to stay with bank i. This prevents a direct reuse of the $H^i = 0$ argument to prove Lemma 1 outlined in Internet Appendix 1.1. (Specifically, Claim 3 no longer holds true.)

It is necessary to analyze two cases separately. **Case 1 ($H^i \le \widehat{H^i}$).** The liquid buffer is small, so in equilibrium outflows necessarily exceed H^i; the limiting equilibrium therefore coincides with Lemma 1. **Case 2 ($H^i > \widehat{H^i}$).** The equilibrium logic differs. In both cases, we first characterize the equilibrium for a fixed noise variance $\sigma > 0$ (with σ small but positive), and then take the limit as $\sigma \downarrow 0$ to obtain the noise-free outcome. We begin with results common to both cases.

Prerequisite Results Let $\theta^i_B(H^i)$ (abbrev. "θ^i_B") and O^i_B be as in Lemma 3. By definition, $\widehat{H^i}$ uniquely solves $C^i\Big(A^i,\, O^i_B - \widehat{H^i}\Big) + \widehat{H^i} \;=\; R^i O^i_B$, and by Assumption 4 it is well defined with $\widehat{H^i} < O^i_B$. For each $H \ge 0$, the **fundamental default boundary** $\widehat{\theta^i}(H)$ solves $\widehat{\theta^i}(H)\, G^i(A^i) + H - R^i L^i \;=\; 0$, i.e., $\widehat{\theta^i}(H)$ is the default boundary absent outflows. Hence

$$\begin{aligned}
&\widehat{\theta^i}(\widehat{H^i})\, G^i(A^i) + (\widehat{H^i} - O^i_B)^+ - C^i\Big(A^i, (O^i_B - \widehat{H^i})^+\Big) - R^i(L^i - O^i_B) = 0,\\
&\widehat{\theta^i}(H^i)\, G^i(A^i) + (H^i - O^i_B)^+ - C^i\Big(A^i, (O^i_B - H^i)^+\Big) - R^i(L^i - O^i_B) < 0, \quad \forall\, H^i < \widehat{H^i},\\
&\widehat{\theta^i}(H^i)\, G^i(A^i) + (H^i - O^i_B)^+ - C^i\Big(A^i, (O^i_B - H^i)^+\Big) - R^i(L^i - O^i_B) > 0, \quad \forall\, H^i > \widehat{H^i}.
\end{aligned} \tag{46}$$

Fix a small $\sigma > 0$, and let $\mathfrak{O}^i_{\text{cand}}(\theta,\sigma)$ be the (candidate) equilibrium outflow in state

θ. In this equilibrium, a depositor k at bank i believes the default set for bank i is

$$\mathcal{D}^{\sigma}_{\text{cand}} = \left\{ \vartheta :\ \vartheta G^i(A^i) - C^i\Big(A^i, \big(\mho^i_{\text{cand}}(\vartheta,\sigma) - H^i\big)^+\Big) + \big(H^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big)^+ - R^i\big(L^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big) \le 0 \right\}.$$

As in the proof of Lemma 1, define the bracketing thresholds

$$\theta^i_{\text{cand,up}}(\sigma) \equiv \inf\left\{ \varrho :\ \vartheta G^i(A^i) - C^i\Big(A^i, \big(\mho^i_{\text{cand}}(\vartheta,\sigma) - H^i\big)^+\Big) + \big(H^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big)^+ - R^i\big(L^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big) > 0,\ \forall\, \vartheta \ge \varrho \right\},$$
$$\theta^i_{\text{cand,down}}(\sigma) \equiv \sup\left\{ \varrho :\ \vartheta G^i(A^i) - C^i\Big(A^i, \big(\mho^i_{\text{cand}}(\vartheta,\sigma) - H^i\big)^+\Big) + \big(H^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big)^+ - R^i\big(L^i - \mho^i_{\text{cand}}(\vartheta,\sigma)\big) < 0,\ \forall\, \vartheta \le \varrho \right\}.$$

Then similarly we have $(-\infty, \theta^i_{\text{cand,down}}(\sigma)] \subset \mathcal{D}^{\sigma}_{\text{cand}} \subset (-\infty, \theta^i_{\text{cand,up}}(\sigma)]$ and

$$\begin{aligned} 0 &= \theta^i_{\text{cand,up}}(\sigma) G^i(A^i) - C^i\Big(A^i, \big(\mho^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma),\sigma) - H^i\big)^+\Big) + \big(H^i - \mho^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma),\sigma)\big)^+ - R^i\big(L^i - \mho^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma),\sigma)\big) \\ &= \theta^i_{\text{cand,down}}(\sigma) G^i(A^i) - C^i\Big(A^i, \big(\mho^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma) - H^i\big)^+\Big) + \big(H^i - \mho^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma)\big)^+ - R^i\big(L^i - \mho^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma)\big). \end{aligned} \tag{47}$$

Recall the run set for the representative depositor k and the equilibrium outflows are

$$\begin{aligned} \mathfrak{R}^{\text{cand}}(k,\sigma) &\equiv \left\{ x_k :\ \mathbb{P}\Big(\widetilde{\theta} \notin \mathcal{D}^{\sigma}_{\text{cand}} \mid \widetilde{x}_k = x_k\Big) < \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k} \right\}, \\ \mho^i_{\text{cand}}(\vartheta,\sigma) &= \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\vartheta + \sigma\widetilde{\epsilon}_k \in \mathfrak{R}^{\text{cand}}(k,\sigma)\Big)\, dF_{\widetilde{\xi}}(\xi_k). \end{aligned} \tag{48}$$

Claim 6. *Let C^{ϵ}_k be defined in Eq. (32). There is $\mathsf{C}^{\text{adj}}_k > 0$ such that, for all sufficiently small σ:*

1. *If $\widetilde{x}_k \ge \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}_k\sigma^2$, depositor k does not run on bank i.*
2. *If $\widetilde{x}_k \le \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}_k\sigma^2$, depositor k runs on bank i.*

Proof. Identical to Claim 2. □

Claim 6 implies that

$$\frac{\mho^i_{\text{cand}}(\vartheta,\sigma)}{\mathcal{M}^i} \le \mathbb{E}_{\xi_k}\left[\mathsf{s}_{k,i}\, F_{\widetilde{\epsilon}}\left(\frac{\theta^i_{\text{cand,up}}(\sigma) - \vartheta}{\sigma} + \mathsf{C}^{\epsilon}_k(\sigma) + \mathsf{C}^{\text{adj}}\sigma \right) \right]. \tag{49}$$

1.4.1 Case 1: $H^i \le \widehat{H^i}$

Claim 6 implies $(-\infty,\ \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}_k\sigma^2] \subset \mathfrak{R}^i_{\text{cand}}(k,\sigma)$. Hence by Eq. (48)

$$\begin{aligned} \mho^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma),\sigma\Big) &\ge \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\widetilde{\epsilon}_k \le \mathsf{C}^{\epsilon}_k - \mathsf{C}^{\text{adj}}_k\sigma\Big)\, dF_{\xi}(\xi_k) \ \Rightarrow \\ \liminf_{\sigma\downarrow 0}\ \mho^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma),\sigma\Big) &\ge \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \frac{1-\Lambda}{R^i - \Lambda + \xi^i_k}\, dF_{\xi}(\xi_k) = O^i_B. \end{aligned}$$

So for all σ small enough, $\mathfrak{O}^i_{\text{cand}}\big(\theta^i_{\text{cand,down}}(\sigma), \sigma\big) > O^i_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma > H^i$ (last inequality follows from $H^i \le \widehat{H^i} < O^i_B$). Apply this inequality and $\partial_2 C^i(\cdot,\cdot) \ge R^i$ to Eq. (47):

$$\begin{aligned} 0 &= \theta^i_{\text{cand,down}}(\sigma)G^i(A^i) - C^i\Big(A^i, \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma) - H^i\Big) - R^i\Big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,down}}(\sigma),\sigma)\Big) \\ &\le \theta^i_{\text{cand,down}}(\sigma)G^i(A^i) - C^i\Big(A^i, O^i_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma - H^i\Big) - R^i\Big(L^i - (O^i_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma)\Big), \end{aligned} \tag{50}$$

We claim that $\theta^i_{\text{cand,down}}(\sigma) > \widehat{\theta^i}(H^i)$. Otherwise, if $\theta^i_{\text{cand,down}}(\sigma) \le \widehat{\theta^i}(H^i)$, then by Eq. (46) the last line of Eq. (50) < 0 for small σ, a contradiction. From Eq. (48), for all $\vartheta \le \widehat{\theta^i}(H^i)$,

$$\mathfrak{O}^i_{\text{cand}}(\vartheta,\sigma) \;\ge\; O^i_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma \;>\; H^i.$$

This bound on the equilibrium outflow is obtained by eliminating dominated strategies (Claim 6). This motivates the following deterministic envelopes for equilibrium outflows:

$$\underline{\mathfrak{O}}^i(\theta,0,\sigma) \equiv \Big(O^i_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma\Big)\mathbb{1}_{\left[\theta \le \widehat{\theta^i}(H^i)\right]}, \qquad \overline{\mathfrak{O}}^i(\theta,0,\sigma) \equiv L^i.$$

Then a.s. $\underline{\mathfrak{O}}^i(\widetilde{\theta},0,\sigma) \le \mathfrak{O}^i_{\text{cand}}(\widetilde{\theta},\sigma) \le \overline{\mathfrak{O}}^i(\widetilde{\theta},0,\sigma)$. This is common knowledge among all agents. From here, the remaining steps after Eq. (27) in the proof of Lemma 1 (see Internet Appendix 1.1.1) apply verbatim. In particular, any pointwise increase in $\mathfrak{O}^i$ weakly enlarges the default set—there is strategic complementarity and no strategic substitutability. Hence, the prediction of Lemma 1 remains valid: the limiting equilibrium outflow at the run cutoff equals O^i_B, and the run cutoff is $\theta^i_B(H^i)$ as defined in Lemma 3.

1.4.2 Case 2: $H^i > \widehat{H^i}$

By Claim 6,

$$\mathfrak{K}^{\text{cand}}(k,\sigma) \subset \Big(-\infty,\ \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}_k\sigma^2\Big].$$

Hence, by Eq. (48),

$$\mathfrak{O}^i_{\text{cand}}(\theta,\sigma) \le \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i}\, \mathbb{P}\Big(\widetilde{\epsilon}_k \le \frac{\theta^i_{\text{cand,up}}(\sigma) - \theta}{\sigma} + \mathsf{C}^{\epsilon}_k + \mathsf{C}^{\text{adj}}_k\sigma\Big)\, dF_{\widetilde{\xi}}(\xi_k). \tag{51}$$

In particular, evaluating at $\theta = \theta^i_{\text{cand,up}}(\sigma)$ and using continuity (and boundedness) of the CDFs $\{F_{\epsilon_k}\}$ around C^{ϵ}_k, there exists a constant $\mathsf{C}^{\text{tem}}_{\text{cand}} > 0$ such that for all sufficiently small σ,

$$\mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma),\sigma\Big) \le O^i_B + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma. \tag{52}$$

Claim 7. *There exists $\mathsf{C}^{\text{adj}}_{\text{up}} > 0$ such that, for all sufficiently small σ, $\widehat{\theta^i}(H^i) - \mathsf{C}^{\text{adj}}_{\text{up}}\sigma \le \theta^i_{\text{cand,up}}(\sigma) \le \widehat{\theta^i}(H^i)$. Consequently, $\theta^i_{\text{cand,up}}(\sigma) \to \widehat{\theta^i}(H^i)$ as $\sigma \downarrow 0$.*

Proof. **Step 1: Show bound $\theta^i_{\text{cand,up}}(\sigma) \le \widehat{\theta^i}(H^i)$.** Recall from Eq. (47) that at $\theta = \theta^i_{\text{cand,up}}(\sigma)$ the solvency expression equals 0:

$$0 = \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) - C^i\Big(A^i, \big(\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma) - H^i\big)^+\Big) + \big(H^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)\big)^+ - R^i\big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)\big). \tag{53}$$

If $H^i > \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)$, then the liquidation cost C^i is zero and

$$\begin{aligned} 0 &= \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) + H^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma) - R^i\big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)\big) \\ &= \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) + H^i - R^iL^i + (R^i - 1)\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma) \;\ge\; \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) + H^i - R^iL^i. \end{aligned}$$

Since $\widehat{\theta^i}(H^i)$ is defined by $\widehat{\theta^i}(H^i)G^i(A^i) + H^i - R^iL^i = 0$, we conclude $\theta^i_{\text{cand,up}}(\sigma) \le \widehat{\theta^i}(H^i)$.

If instead $H^i \le \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)$, then Eq. (53) becomes

$$0 = \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) - C^i\Big(A^i, \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma) - H^i\Big) - R^i\big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)\big).$$

Using the upper bound Eq. (52) and monotonicity of C^i in its second argument,

$$\begin{aligned} 0 &\ge \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) - C^i\Big(A^i, O^i_B + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma - H^i\Big) - R^i\big(L^i - (O^i_B + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma)\big) \\ &= \theta^i_{\text{cand,up}}(\sigma)G^i(A^i) - C^i\Big(A^i, (O^i_B - H^i)^+\Big) - R^i(L^i - O^i_B) - \mathcal{O}(\sigma), \end{aligned}$$

where the $\mathcal{O}(\sigma)$ term collects the (bounded) first-order variations from replacing $\mathfrak{O}^i_{\text{cand}}(\theta^i_{\text{cand,up}}(\sigma), \sigma)$ by $O^i_B + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma$. Because $H^i > \widehat{H^i}$, Eq. (46) implies

$$\widehat{\theta^i}(H^i)G^i(A^i) - C^i\Big(A^i, (O^i_B - H^i)^+\Big) - R^i(L^i - O^i_B) \;>\; 0.$$

Thus, for all sufficiently small σ, the preceding inequality forces $\theta^i_{\text{cand,up}}(\sigma) \le \widehat{\theta^i}(H^i)$. This establishes the desired upper bound in all cases.

Step 2: Show bound $\theta^i_{\text{cand,up}}(\sigma) \ge \widehat{\theta^i}(H^i) - \mathsf{C}^{\text{adj}}_{\text{up}}\sigma$. Suppose for contradiction that

$$\limsup_{\sigma \downarrow 0} \frac{\widehat{\theta^i}(H^i) - \theta^i_{\text{cand,up}}(\sigma)}{\sigma} = +\infty.$$

Then there exists a sequence $\sigma_n \downarrow 0$ such that $\theta^i_{\text{cand,up}}(\sigma_n) \le \widehat{\theta^i}(H^i) - n\sigma_n$. Fix $\delta > 0$ and let $\theta^\delta \equiv \widehat{\theta^i}(H^i) - \delta$. For n large, $\theta^\delta > \theta^i_{\text{cand,up}}(\sigma_n)$ and

$$\frac{\theta^i_{\text{cand,up}}(\sigma_n) - \theta^\delta}{\sigma_n} \le -\frac{\delta}{\sigma_n} - n \to -\infty.$$

Therefore, by Eq. (51), $\mathfrak{O}^i_{\text{cand}}(\theta^\delta, \sigma_n) \to 0$. Hence, for n large we have both $\mathfrak{O}^i_{\text{cand}}(\theta^\delta, \sigma_n) <$

H^i and $(R^i-1)\mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n) \le \frac{\delta\, G^i(A^i)}{2}$. Evaluating the solvency expression at (θ^δ,σ_n) in the no-liquidation branch yields

$$\begin{aligned}
&\theta^\delta G^i(A^i) - C^i\Big(A^i, \big(\mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n) - H^i\big)^+\Big) + \big(H^i - \mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n)\big)^+ - R^i\big(L^i - \mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n)\big)\\
&= \theta^\delta G^i(A^i) + H^i - R^i L^i + (R^i-1)\mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n)\\
&= -\delta\, G^i(A^i) + (R^i-1)\mathfrak{O}^i_{\text{cand}}(\theta^\delta,\sigma_n) \;\le\; -\frac{\delta\, G^i(A^i)}{2} \;<\; 0,
\end{aligned}$$

where we used $\widehat{\theta^i}(H^i)G^i(A^i) + H^i - R^i L^i = 0$. Thus the bank would be insolvent at $\theta^\delta > \theta^i_{\text{cand,up}}(\sigma_n)$, contradicting the definition of $\theta^i_{\text{cand,up}}(\sigma_n)$ as an upper boundary of the candidate default region. This contradiction proves the desired lower bound. □

Claim 8. *In any equilibrium in the noiseless limit $\sigma = 0$, bank i must default whenever $\theta < \widehat{\theta^i}(H^i)$.*

Proof. By definition, any valid limiting equilibrium must also be an equilibrium of the complete-information game in which $\sigma = 0$. Fix $\theta < \widehat{\theta^i}(H^i)$ and suppose for contradiction that bank i does *not* default at θ in some complete-information equilibrium. Because depositors obtain at most 1 from the risk-free asset, while staying with a solvent bank yields $R^i + \xi^i_k > 1$, no depositor prefers to withdraw from a solvent bank in state θ. Hence equilibrium outflow at bank i must satisfy $O^i(\theta) = 0$. Evaluating bank i's payoff at θ gives

$$V_i^3(\theta) = \theta G^i(A^i) + H^i - R^i L^i < \widehat{\theta^i}(H^i)G^i(A^i) + H^i - R^i L^i = 0,$$

a contradiction to solvency. □

By the arguments above, any limiting equilibrium must satisfy the same limiting cutoff characterization: depositors run if and only if $\theta < \widehat{\theta^i}(H^i)$ and stay if and only if $\theta > \widehat{\theta^i}(H^i)$, and the bank defaults if and only if $\theta < \widehat{\theta^i}(H^i)$. It follows that all limits coincide (up to behavior at the knife-edge state $\theta = \widehat{\theta^i}(H^i)$), and hence the limiting equilibrium as $\sigma \downarrow 0$ is unique. (For the existence of a sequence of converging equilibria as $\sigma \downarrow 0$, see discussion in Internet Appendix 1.7.2 under a more general case allowing for deposit inflows.) □

1.5 Proof of Theorem 1

By Lemma 3, bank b's overall default boundary satisfies

$$\theta^b_B(H) \ge \widehat{\theta}^b(H) \quad \text{for } H < \widehat{H^b}, \qquad \theta^b_B(H) = \widehat{\theta}^b(H) \quad \text{for } H \ge \widehat{H^b}.$$

Suppose that $\theta^b_B(H) > \theta^b_B(\widehat{H^b})$ for all $H < \widehat{H^b}$ (otherwise, the theorem is trivial by the same argument below), then the regulator must choose $H^* \ge \widehat{H^b}$. If $\widehat{\theta}^b(H)$ also increases on $(\widehat{H^b}, L^b]$, the optimum is $H^* = \widehat{H^b}$. Fix $H \in (\widehat{H^b}, L^b]$. Since

$$\widehat{\theta}^b(H) = \frac{R^b L^b - H}{G^b(L^b - H)},$$

we obtain

$$\frac{d\widehat{\theta}^b(H)}{dH} > 0 \iff R^b L^b > \frac{G^b(L^b - H)}{G^{b\prime}(L^b - H)} + H\,\frac{d\widehat{\theta}^b(H)}{dH} > 0 \iff \widehat{\theta}^b(H) G^{b\prime}(L^b - H) > 1.$$

Define $\phi(H) \equiv \widehat{\theta}^b(H)\, G^{b\prime}(L^b - H)$. By assumption, $\phi(\widehat{H^b}) > 1$, and by continuity of ϕ there exists $\delta > 0$ such that $\phi(H) > 1$ for all $H \in [\widehat{H^b}, \widehat{H^b} + \delta]$. Let

$$H^{\text{tem2}} \equiv \sup\Big\{H \in [\widehat{H^b}, L^b] : \phi(\widetilde{H}) > 1 \text{ for all } \widetilde{H} \in [\widehat{H^b}, H)\Big\}.$$

By construction, $\phi(H) > 1$ for all $H \in [\widehat{H^b}, H^{\text{tem2}})$. Hence $\widehat{\theta}^b$ is strictly increasing on $[\widehat{H^b}, H^{\text{tem2}})$. Moreover, since G^b is concave, $G^{b\prime}$ is weakly decreasing in its argument, so $H \mapsto G^{b\prime}(L^b - H)$ is weakly increasing in H. Combining these facts yields

$$\phi(H^{\text{tem2}}) = \widehat{\theta}^b(H^{\text{tem2}})\, G^{b\prime}(L^b - H^{\text{tem2}}) \geq \widehat{\theta}^b(\widehat{H^b})\, G^{b\prime}(L^b - \widehat{H^b}) > 1.$$

By continuity of ϕ, this implies that $\phi(H) > 1$ on a right-neighborhood of H^{tem2}, contradicting the definition of H^{tem2} unless $H^{\text{tem2}} = L^b$. Therefore, $\phi(H) > 1$ for all $H \in (\widehat{H^b}, L^b]$, so $\widehat{\theta}^b(H)$ is strictly increasing on $(\widehat{H^b}, L^b]$. □

1.6 Proof of Lemma 4

The conclusion is a direct application of the Banach contraction mapping theorem.

1.7 Proof of Theorem 2

The main complication from deposit mobility across multiple banks arises because each agent must conjecture the strategies of depositors not only at her own bank but also at the other bank. Moreover, the fraction of depositors who move between banks affects the best-response functions of both agents.

Consider region i, where banks i and b both operate. Let agent k be a representative depositor at bank i after time $t = 1$, and let agent w be a representative depositor who resides in region i but deposits at bank b after time $t = 1$. Denote by $\mathfrak{O}^j$ and $\mathfrak{I}^j$ the aggregate deposit outflow from, and inflow to, bank j, respectively. When the true state is θ and the depositor strategy profile is $\{\mathsf{r}_{k,i\to j}, \mathsf{r}_{w,b\to j}\}$, the Exact Law of Large Numbers yields

$$\mathbb{O}^i(\theta) = \mathcal{M}^i \int_{\xi_k}\int_{x_k} \Big[\mathsf{r}_{k,i\to b}(x_k) + \mathsf{r}_{k,i\to H}(x_k)\Big] f_{\widetilde{x}_k|\widetilde{\theta}}(x_k \mid \theta)\, dx_k\, f_{\widetilde{\xi}}(\xi_k)\, d\xi_k,$$

$$\mathbb{O}^b(\theta) = \mathcal{M}^i \int_{\xi_w}\int_{x_w} \Big[\mathsf{r}_{w,b\to i}(x_w) + \mathsf{r}_{w,b\to H}(x_w)\Big] f_{\widetilde{x}_w|\widetilde{\theta}}(x_w \mid \theta)\, dx_w\, f_{\widetilde{\xi}}(\xi_w)\, d\xi_w,$$

$$\mathbb{I}^i(\theta) = \mathcal{M}^i \int_{\xi_w}\int_{x_w} \Big[\mathsf{r}_{w,b\to i}(x_w) + \mathsf{r}_{w,H\to i}(x_w)\Big] f_{\widetilde{x}_w|\widetilde{\theta}}(x_w \mid \theta)\, dx_w\, f_{\widetilde{\xi}}(\xi_w)\, d\xi_w,$$

$$\mathbb{I}^b(\theta) = \sum_i \mathcal{M}^i \int_{\xi_k}\int_{x_k} \Big[\mathsf{r}_{k,i\to b}(x_k) + \mathsf{r}_{k,H\to b}(x_k)\Big] f_{\widetilde{x}_k|\widetilde{\theta}}(x_k \mid \theta)\, dx_k\, f_{\widetilde{\xi}}(\xi_k)\, d\xi_k.$$

Recall that private signal precision is governed by σ, so the densities $f_{\widetilde{x}_k|\widetilde{\theta}}$ and $f_{\widetilde{x}_w|\widetilde{\theta}}$ depend on σ. Hence any candidate equilibrium inflows and outflows can be written as $\mathbb{O}^j(\theta, \sigma)$ and $\mathbb{I}^j(\theta, \sigma)$. Under these deposit flow functions, depositor k's expected payoff from *staying* with bank i after observing x_k at $t = 2$ is

$$\mathcal{U}^2_{k,i\to i}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to i} = 1 \mid x_k\Big) = \mathbb{P}\Big(\mathcal{D}_i^c \mid x_k\Big)(R^i + \xi_k^i) + \mathbb{P}\Big(\mathcal{D}_i \mid x_k\Big)\Lambda.$$

If instead k *moves* to bank b, her expected payoff is (recall that depositors ignore the sequential service constraint)

$$\begin{aligned}\mathcal{U}^2_{k,i\to b}(x_k) &\equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to b} = 1 \mid x_k\Big)\\ &= \int_{\mathcal{D}_i}\Big((1+\chi^b\xi_k^b)\,\mathbb{1}_{\mathcal{D}_b^c} + \Lambda\,\mathbb{1}_{\mathcal{D}_b}\Big) f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta + \int_{\mathcal{D}_i^c}\Big((1+\chi^b\xi_k^b)\,\mathbb{1}_{\mathcal{D}_b^c} + \Lambda\,\mathbb{1}_{\mathcal{D}_b}\Big) f_{\widetilde{\theta}|\widetilde{x}_k}(\theta \mid x_k)\, d\theta.\end{aligned}$$

If k *leaves* the banking system to hold the risk-free asset, her expected payoff is

$$\mathcal{U}^2_{k,i\to H}(x_k) \equiv \mathcal{U}^2_k\Big(\mathsf{a}_{k,i\to H} = 1 \mid x_k\Big) = 1.$$

Similarly, for depositor w at bank b, the expected payoff from *staying* is

$$\mathcal{U}^2_{w,b\to b}(x_w) \equiv \mathcal{U}^2_w\Big(\mathsf{a}_{w,b\to b} = 1 \mid x_w\Big) = \mathbb{P}\Big(\mathcal{D}_b^c \mid x_w\Big)(R^b + \xi_w^b) + \mathbb{P}\Big(\mathcal{D}_b \mid x_w\Big)\Lambda,$$

while the payoff from *moving* to bank i is

$$\begin{aligned}\mathcal{U}^2_{w,b\to i}(x_w) &\equiv \mathcal{U}^2_w\Big(\mathsf{a}_{w,b\to i} = 1 \mid x_w\Big)\\ &= \int_{\mathcal{D}_b}\Big((1+\chi^i\xi_w^i)\,\mathbb{1}_{\mathcal{D}_i^c} + \Lambda\,\mathbb{1}_{\mathcal{D}_i}\Big) f_{\widetilde{\theta}|\widetilde{x}_w}(\theta \mid x_w)\, d\theta + \int_{\mathcal{D}_b^c}\Big((1+\chi^i\xi_w^i)\,\mathbb{1}_{\mathcal{D}_i^c} + \Lambda\,\mathbb{1}_{\mathcal{D}_i}\Big) f_{\widetilde{\theta}|\widetilde{x}_w}(\theta \mid x_w)\, d\theta,\end{aligned}$$

and the payoff from *leaving* the banking system to hold the risk-free asset is

$$\mathcal{U}^2_{w,b\to H}(x_w) \equiv \mathcal{U}^2_w\Big(\mathsf{a}_{w,b\to H} = 1 \mid x_w\Big) = 1.$$

1.7.1 Necessary conditions for any candidate equilibrium

Fix any sufficiently small $\sigma > 0$. Let $\mathbb{O}^j_{\text{cand}}(\theta,\sigma)$ and $\mathbb{I}^j_{\text{cand}}(\theta,\sigma)$ denote the candidate equilibrium deposit outflow and inflow, respectively, for bank $j \in \{i,b\}$ in state θ. As in standard global-games models, these functions are common knowledge, whereas each agent observes only a private signal $\widetilde{x}$ about the state $\widetilde{\theta}$. Given these beliefs, an agent regards bank b as defaulting iff $\widetilde{\theta} \in \mathcal{D}_b(\text{cand},\sigma)$, where

$$\mathcal{D}_b(\text{cand},\sigma) = \Big\{\theta :\ \theta G^b(A^b) - C^b\Big(A^b, \big(\mathbb{O}^b_{\text{cand}}(\theta,\sigma) - \mathbb{I}^b_{\text{cand}}(\theta,\sigma) - H^b\big)^+\Big) + \big(H^b + \mathbb{I}^b_{\text{cand}}(\theta,\sigma) - \mathbb{O}^b_{\text{cand}}(\theta,\sigma)\big)^+ - \mathbb{I}^b_{\text{cand}}(\theta,\sigma) - R^b\big(L^b - \mathbb{O}^b_{\text{cand}}(\theta,\sigma)\big) \leq 0\Big\}.$$

Similarly, since $H^i = 0$, bank i is deemed insolvent iff $\widetilde{\theta} \in \mathcal{D}_i(\text{cand},\sigma)$, where

$$\mathcal{D}_i(\text{cand},\sigma) = \Big\{\theta :\ \theta G^i(A^i) - C^i\Big(A^i, \big(\mathbb{O}^i_{\text{cand}}(\theta,\sigma) - \mathbb{I}^i_{\text{cand}}(\theta,\sigma)\big)^+\Big) + \big(\mathbb{I}^i_{\text{cand}}(\theta,\sigma) - \mathbb{O}^i_{\text{cand}}(\theta,\sigma)\big)^+ - \mathbb{I}^i_{\text{cand}}(\theta,\sigma) - R^i\big(L^i - \mathbb{O}^i_{\text{cand}}(\theta,\sigma)\big) \leq 0\Big\}.$$

For $j \in \{i,b\}$ define the solvency thresholds

$$\theta^j_{\text{cand,up}}(\sigma) = \inf\Big\{\varrho :\ \Big[\vartheta G^j(A^j) - C^j\Big(A^j, (\mathbb{O}^j_{\text{cand}}(\vartheta,\sigma) - H^j - \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma))^+\Big) + (H^j + \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma) - \mathbb{O}^j_{\text{cand}}(\vartheta,\sigma))^+ - \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma) - R^j\big(L^j - \mathbb{O}^j_{\text{cand}}(\vartheta,\sigma)\big)\Big] > 0 \ \ \forall \vartheta \geq \varrho\Big\}, \tag{54}$$

$$\theta^j_{\text{cand,down}}(\sigma) = \sup\Big\{\varrho :\ \Big[\vartheta G^j(A^j) - C^j\Big(A^j, (\mathbb{O}^j_{\text{cand}}(\vartheta,\sigma) - H^j - \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma))^+\Big) + (H^j + \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma) - \mathbb{O}^j_{\text{cand}}(\vartheta,\sigma))^+ - \mathbb{I}^j_{\text{cand}}(\vartheta,\sigma) - R^j\big(L^j - \mathbb{O}^j_{\text{cand}}(\vartheta,\sigma)\big)\Big] < 0 \ \ \forall \vartheta \leq \varrho\Big\}. \tag{55}$$

Therefore,

$$(-\infty, \theta^j_{\text{cand,down}}(\sigma)] \subseteqq \mathcal{D}_j(\text{cand},\sigma) \subseteqq (-\infty, \theta^j_{\text{cand,up}}(\sigma)].$$

Claim 9. *Suppose the candidate solvency thresholds satisfy*

$$\theta^b_{\text{cand,up}}(\sigma) < \theta^i_{\text{cand,down}}(\sigma).$$

Then $\mathbb{I}^i_{\text{cand}}(\vartheta,\sigma) = 0$ *for every* ϑ*. Conversely, if*

$$\theta^b_{\text{cand,down}}(\sigma) > \theta^i_{\text{cand,up}}(\sigma),$$

then $\mathbb{I}^b_{\text{cand}}(\vartheta,\sigma) = 0$ *for every* ϑ*.*

Proof. The first inequality implies $\mathcal{D}_b(\text{cand},\sigma) \subsetneqq \mathcal{D}_i(\text{cand},\sigma)$ and $\mathcal{D}^c_i(\text{cand},\sigma) \subsetneqq \mathcal{D}^c_b(\text{cand},\sigma)$. For any signal realization x_w, the assumption that

$R^b + \xi_w^b > 1 + \chi^i \xi_w^i$ yields

$$\mathcal{U}^2_{w,b\to i}(x_w) = \int_{\mathcal{D}_b} \Lambda\, f_{\widetilde{\theta}|\widetilde{x}_w}(\theta \mid x_w)\, d\theta + \int_{\mathcal{D}_b^c} \Big((1+\chi^i\xi_w^i)\mathbb{1}_{\mathcal{D}_i^c} + \Lambda \mathbb{1}_{\mathcal{D}_i}\Big) f_{\widetilde{\theta}|\widetilde{x}_w}(\theta \mid x_w)\, d\theta$$
$$< \mathbb{P}(\mathcal{D}_b \mid x_w)\,\Lambda + \mathbb{P}(\mathcal{D}_b^c \mid x_w)(R^b + \xi_w^b) = \mathcal{U}^2_{w,b\to b}(x_w).$$

Therefore, no depositor at bank b moves to bank i. Because no depositor holds the risk-free asset at $t = 1$, it follows that $\mathrm{I}^i_{\text{cand}}(\vartheta, \sigma) = 0$. The converse claim is proved by symmetry. □

Let $\mathcal{R}^j_{\text{cand}}(k, \sigma)$ be the set of signals for which depositor k withdraws from bank j. Then (for regional bank i, let $\mathsf{s}_{k,j} = 0$ if $j \neq i$)

$$\mathcal{O}^j_{\text{cand}}(\vartheta, \sigma) = \sum_{i=1}^{\mathcal{N}} \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,j}\, \mathbb{P}\Big(\vartheta + \sigma\widetilde{\epsilon}_k \in \mathcal{R}^j_{\text{cand}}(k, \sigma)\Big)\, dF_{\widetilde{\xi}}(\xi_k),$$

so $\mathcal{O}^j_{\text{cand}}$ (and similarly for $\mathrm{I}^j_{\text{cand}}$) is continuous in ϑ. Therefore, for $\vartheta = \theta^j_{\text{cand,up}}(\sigma)$ or $\vartheta = \theta^j_{\text{cand,down}}(\sigma)$,

$$\vartheta G^j(A^j) - C^j\Big(A^j, (\mathcal{O}^j_{\text{cand}}(\vartheta,\sigma) - H^j - \mathrm{I}^j_{\text{cand}}(\vartheta,\sigma))^+\Big) + (H^j + \mathrm{I}^j_{\text{cand}}(\vartheta,\sigma) - \mathcal{O}^j_{\text{cand}}(\vartheta,\sigma))^+ - \mathrm{I}^j_{\text{cand}}(\vartheta,\sigma) - R^j\Big(L^j - \mathcal{O}^j_{\text{cand}}(\vartheta,\sigma)\Big) = 0. \quad (56)$$

Definition 8. *For each depositor k, define:*

$$\mathsf{C}_k^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_k}(x) \geq \frac{1-\Lambda}{R^i - \Lambda + \xi_k^i}\right\}, \qquad \overline{\mathsf{C}}_k^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_k}(x) \geq \frac{1 + \chi^b\xi_k^b - \Lambda}{R^i - \Lambda + \xi_k^i}\right\},$$
$$\underline{\mathsf{C}}_k^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_k}(x) \geq \frac{1-\Lambda}{1 - \Lambda + \chi^b\xi_k^b}\right\}.$$

Analogously, for each depositor w, define:

$$\mathsf{C}_w^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_w}(x) \geq \frac{1-\Lambda}{R^b - \Lambda + \xi_w^b}\right\}, \qquad \overline{\mathsf{C}}_w^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_w}(x) \geq \frac{1 + \chi^i\xi_w^i - \Lambda}{R^b - \Lambda + \xi_w^b}\right\},$$
$$\underline{\mathsf{C}}_w^\epsilon \equiv \inf\left\{x : F_{\widetilde{\epsilon}_w}(x) \geq \frac{1-\Lambda}{1 - \Lambda + \chi^i\xi_w^i}\right\}.$$

In any candidate equilibrium, the strategy of the representative depositor k originally with bank i and depositor w originally with bank b is characterized as follows.

Claim 10. *There exists a constant $\mathsf{C}^{\text{adj}} > 0$ such that, for all sufficiently small σ,*

1. *If depositor k observes $\widetilde{x}_k \geq \theta^i_{\text{cand,up}}(\sigma) + \overline{\mathsf{C}}_k^\epsilon\sigma + \mathsf{C}^{\text{adj}}\sigma^2$, depositor k does* not *run on bank i.*

2. *If depositor k observes $\widetilde{x}_k \leq \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}_k^\epsilon\sigma - \mathsf{C}^{\text{adj}}\sigma^2$, she* moves *from bank i to bank b or to the safe asset.*

3. *If depositor w observes $\widetilde{x}_w \geq \theta^b_{\text{cand,up}}(\sigma) + \overline{\mathsf{C}}^{\epsilon}_w \sigma + \mathsf{C}^{\text{adj}}\sigma^2$, depositor w does* not *run on bank b.*

4. *If she observes $\widetilde{x}_w \leq \theta^b_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_w \sigma - \mathsf{C}^{\text{adj}}\sigma^2$, depositor w* moves *from bank b to bank i or to the safe asset.*

Proof. Let σ be small, and write x_k for the realization of $\widetilde{x}_k$.

(i) Suppose $x_k = \theta^i_{\text{cand,up}}(\sigma) + \overline{\mathsf{C}}^{\epsilon}_k \sigma + \mathsf{C}^{\text{adj}}\sigma^2 + \mathsf{C}^{\text{tem}}\sigma \quad$ for some $\mathsf{C}^{\text{tem}} \geq 0$. Then

$$\mathbb{P}\Big(\mathcal{D}^c_i \mid x_k\Big) \geq \mathbb{P}\Big(\widetilde{\theta} \geq \theta^i_{\text{cand,up}}(\sigma) \mid x_k\Big) = \frac{\int_{-\infty}^{(x_k - \theta^i_{\text{cand,up}}(\sigma))/\sigma} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz}{\int_{-\infty}^{\infty} f_{\widetilde{\theta}}(x_k - \sigma z)\, f_{\widetilde{\epsilon}_k}(z)\, dz}.$$

By continuity of $f_{\widetilde{\theta}}$, for σ small this equals

$$\frac{1 + \chi^b \xi^b_k - \Lambda}{R^i - \Lambda + \xi^i_k} + F_{\widetilde{\epsilon}}\Big(\overline{\mathsf{C}}^{\epsilon}_k + \mathsf{C}^{\text{adj}}\sigma + \mathsf{C}^{\text{tem}}\Big) - F_{\widetilde{\epsilon}}\Big(\overline{\mathsf{C}}^{\epsilon}_k\Big) - \mathcal{O}(\sigma).$$

Choosing C^{adj} large enough yields

$$\mathbb{P}\Big(\mathcal{D}^c_i \mid x_k\Big)\Big(R^i - \Lambda + \xi^i_k\Big) > 1 + \chi^b \xi^b_k - \Lambda,$$

i.e., $\mathcal{U}^2_{k,i\to i}(x_k) \geq 1 + \chi^b \xi^b_k$. Because the highest payoff from leaving bank i is bounded above by $1 + \chi^b \xi^b_k$, depositor k stays with bank i.

(ii) Suppose

$$x_k = \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k \sigma - \mathsf{C}^{\text{adj}}\sigma^2 - \mathsf{C}^{\text{tem}}\sigma, \quad \mathsf{C}^{\text{tem}} \geq 0.$$

Then, for σ small,

$$\mathbb{P}\Big(\mathcal{D}^c_i \mid x_k\Big) \leq \mathbb{P}\Big(\widetilde{\theta} \geq \theta^i_{\text{cand,down}}(\sigma) \mid x_k\Big) = \frac{1 - \Lambda}{R^i - \Lambda + \xi^i_k} + F_{\widetilde{\epsilon}}\Big(\mathsf{C}^{\epsilon}_k - \mathsf{C}^{\text{adj}}\sigma - \mathsf{C}^{\text{tem}}\Big) - F_{\widetilde{\epsilon}}\Big(\mathsf{C}^{\epsilon}_k\Big) + \mathcal{O}(\sigma),$$

so for C^{adj} large enough we have $\mathcal{U}^2_{k,i\to i}(x_k) < 1$. Because the payoff from leaving bank i to H is 1, depositor k runs from bank i.

(iii)–(iv) Identical by symmetry after interchanging i and b, and k and w. □

Claim 11. *Consider a sequence of $\{\sigma\}$ such that $\lim_{\sigma\downarrow 0} \theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$. There exists a constant $\mathsf{C}^{\text{adj}} > 0$ such that, for all sufficiently small σ from that sequence,*

1. *If $\widetilde{x}_k \leq \theta^i_{\text{cand,down}}(\sigma) + \overline{\mathsf{C}}^{\epsilon}_k \sigma - \mathsf{C}^{\text{adj}}\sigma^2$ and $\widetilde{x}_k \geq \theta^b_{\text{cand,up}}(\sigma) + \underline{\mathsf{C}}^{\epsilon}_k \sigma + \mathsf{C}^{\text{adj}}\sigma^2$, then depositor k (originally at bank i)* moves *from bank i to bank b.*

2. *If* $\widetilde{x}_k \le \theta^b_{\text{cand,down}}(\sigma) + \underline{\mathsf{C}}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2$*, then depositor* k *(originally at bank* i*)* moves *from bank* i *to the risk-free asset* H*.*

3. *If depositor* w *(originally at bank* b*) observes* $\widetilde{x}_w \ge \theta^b_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_w\sigma + \mathsf{C}^{\text{adj}}\sigma^2$*, she* does not *run on bank* b*.*

4. *If depositor* w *(originally at bank* b*) observes* $\widetilde{x}_w \le \theta^b_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_w\sigma - \mathsf{C}^{\text{adj}}\sigma^2$*, she* moves *from bank* b *to the risk-free asset* H*.*

Conversely, if $\lim_{\sigma\downarrow 0} \theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma) > 0$*, then there exists a constant* $\mathsf{C}^{\text{adj}} > 0$ *such that, for all sufficiently small* σ*,*

1. *If* $\widetilde{x}_w \le \theta^b_{\text{cand,down}}(\sigma) + \overline{\mathsf{C}}^{\epsilon}_w\sigma - \mathsf{C}^{\text{adj}}\sigma^2$ *and* $\widetilde{x}_w \ge \theta^i_{\text{cand,up}}(\sigma) + \underline{\mathsf{C}}^{\epsilon}_w\sigma + \mathsf{C}^{\text{adj}}\sigma^2$*, then depositor* w *(originally at bank* b*)* moves *from bank* b *to bank* i*.*

2. *If* $\widetilde{x}_w \le \theta^i_{\text{cand,down}}(\sigma) + \underline{\mathsf{C}}^{\epsilon}_w\sigma - \mathsf{C}^{\text{adj}}\sigma^2$*, then depositor* w moves *from bank* b *to the risk-free asset* H*.*

3. *If depositor* k *(originally at bank* i*) observes* $\widetilde{x}_k \ge \theta^i_{\text{cand,up}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}\sigma^2$*, she* does not *run on bank* i*.*

4. *If depositor* k *observes* $\widetilde{x}_k \le \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2$*, she* moves *from bank* i *to the risk-free asset* H*.*

Proof. Suppose $\lim_{\sigma\downarrow 0} \theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$. Then for sufficiently small σ, by Claim 9, $\mathcal{D}^c_i(\text{cand}, \sigma) \subsetneqq \mathcal{D}^c_b(\text{cand}, \sigma)$. For any signal realization x_w, the assumption that $R^b + \xi^b_w > 1 + \chi^i \xi^i_w$ yields $\mathcal{U}^2_{w,b\to i}(x_w) < \mathcal{U}^2_{w,b\to b}(x_w)$ for all x_w.

(i) Let

$$x_k \;\ge\; \theta^b_{\text{cand,up}}(\sigma) + \underline{\mathsf{C}}^{\epsilon}_k\sigma + \mathsf{C}^{\text{adj}}\sigma^2.$$

Then, by Bayes' rule and continuity of $f_{\widetilde{\theta}}$, for small σ,

$$\mathbb{P}\Big(\mathcal{D}^c_b \mid x_k\Big) \ge \mathbb{P}\Big(\widetilde{\theta} \ge \theta^b_{\text{cand,up}}(\sigma) \mid x_k\Big) = F_{\widetilde{\epsilon}_k}\Big(\frac{x_k - \theta^b_{\text{cand,up}}(\sigma)}{\sigma}\Big) + \mathcal{O}(\sigma) \;\ge\; F_{\widetilde{\epsilon}_k}\Big(\underline{\mathsf{C}}^{\epsilon}_k + \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma).$$

By the definition of $\underline{\mathsf{C}}^{\epsilon}_k$, $F_{\widetilde{\epsilon}_k}(\underline{\mathsf{C}}^{\epsilon}_k) = \frac{1-\Lambda}{1-\Lambda+\chi^b\xi^b_k}$. Hence, for σ small and C^{adj} large enough,

$$\mathcal{U}^2_{k,i\to b}(x_k) = \mathbb{P}(\mathcal{D}^c_b \mid x_k)\,(1 + \chi^b \xi^b_k) + \mathbb{P}(\mathcal{D}_b \mid x_k)\,\Lambda > 1 = \mathcal{U}^2_{k,i\to H}(x_k).$$

If additionally

$$x_k \;\le\; \theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}^{\epsilon}_k\sigma - \mathsf{C}^{\text{adj}}\sigma^2,$$

then

$$\mathbb{P}\Big(\mathcal{D}^c_i \mid x_k\Big) \le \mathbb{P}\Big(\widetilde{\theta} \ge \theta^i_{\text{cand,down}}(\sigma) \mid x_k\Big) = F_{\widetilde{\epsilon}_k}\Big(\frac{x_k - \theta^i_{\text{cand,down}}(\sigma)}{\sigma}\Big) + \mathcal{O}(\sigma) \;\le\; F_{\widetilde{\epsilon}_k}\Big(\mathsf{C}^{\epsilon}_k - \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma).$$

Since $F_{\widetilde{\epsilon}_k}(\mathsf{C}_k^\epsilon) = \frac{1-\Lambda}{R^i-\Lambda+\xi_k^i}$, for small σ, $\mathcal{U}_{k,i\to i}^2(x_k) = \mathbb{P}(\mathcal{D}_i^c \mid x_k)\,(R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i \mid x_k)\,\Lambda < 1 < \mathcal{U}_{k,i\to b}^2(x_k)$.

Alternatively, suppose that

$$x_k \in [\theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}_k^\epsilon\sigma - \mathsf{C}^{\text{adj}}\sigma^2, \theta^i_{\text{cand,down}}(\sigma) + \overline{\mathsf{C}}_k^\epsilon\sigma - \mathsf{C}^{\text{adj}}\sigma^2),$$

then

$$\mathbb{P}\Big(\mathcal{D}_i^c \mid x_k\Big) \le \mathbb{P}\Big(\widetilde{\theta} \ge \theta^i_{\text{cand,down}}(\sigma) \mid x_k\Big) = F_{\widetilde{\epsilon}_k}\Big(\frac{x_k - \theta^i_{\text{cand,down}}(\sigma)}{\sigma}\Big) + \mathcal{O}(\sigma) \le F_{\widetilde{\epsilon}_k}\Big(\overline{\mathsf{C}}_k^\epsilon - \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma).$$

By the definition of $\overline{\mathsf{C}}_k^\epsilon$, $F_{\widetilde{\epsilon}_k}(\overline{\mathsf{C}}_k^\epsilon) = \frac{1+\chi^b\xi_k^b-\Lambda}{R^i-\Lambda+\xi_k^i}$. Therefore, for small σ, $\mathcal{U}_{k,i\to i}^2(x_k) = \mathbb{P}(\mathcal{D}_i^c \mid x_k)\,(R^i + \xi_k^i) + \mathbb{P}(\mathcal{D}_i \mid x_k)\,\Lambda < 1 + \chi^b\xi_k^b$. Also,

$$\mathbb{P}\Big(\widetilde{\theta} \ge \theta^b_{\text{cand,up}}(\sigma) \mid x_k\Big) \ge F_{\widetilde{\epsilon}_k}\Big(\frac{\theta^i_{\text{cand,down}}(\sigma) + \mathsf{C}_k^\epsilon\sigma - \mathsf{C}^{\text{adj}}\sigma^2 - \theta^b_{\text{cand,up}}(\sigma)}{\sigma}\Big) + \mathcal{O}(\sigma).$$

As σ goes to 0, $\mathbb{P}\Big(\widetilde{\theta} \ge \theta^b_{\text{cand,up}}(\sigma) \mid x_k\Big)$ goes to 1. Therefore, $\mathcal{U}_{k,i\to i}^2(x_k) < \mathcal{U}_{k,i\to b}^2(x_k)$, so k moves from i to b.

(ii) If $x_k \le \theta^b_{\text{cand,down}}(\sigma) + \underline{\mathsf{C}}_k^\epsilon\sigma - \mathsf{C}^{\text{adj}}\sigma^2$, then

$$\mathbb{P}\Big(\mathcal{D}_b^c \mid x_k\Big) \le \mathbb{P}\Big(\widetilde{\theta} \ge \theta^b_{\text{cand,down}}(\sigma) \mid x_k\Big) \le F_{\widetilde{\epsilon}_k}\Big(\underline{\mathsf{C}}_k^\epsilon - \mathsf{C}^{\text{adj}}\sigma\Big) + \mathcal{O}(\sigma).$$

By the definition of $\underline{\mathsf{C}}_k^\epsilon$ this yields $\mathcal{U}_{k,i\to i}^2(x_k) < \mathcal{U}_{k,i\to b}^2(x_k) < 1 = \mathcal{U}_{k,i\to H}^2(x_k)$, so k prefers H.

(iii)–(iv) With Claim 9, w never moves to the riskier bank i under the assumption. Her decision reduces to the single-bank case, and the thresholds follow from Claim 10 (replace i by b and use C_w^ϵ).

The converse statements follow by symmetry when $\lim_{\sigma\downarrow 0} \theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma) > 0$. □

Claim 10 and Claim 11 have important implications for the equilibrium deposit outflows and inflows near the run cutoffs $\theta^b_{\text{cand,up}}(\sigma)$, $\theta^b_{\text{cand,down}}(\sigma)$ and $\theta^i_{\text{cand,up}}(\sigma)$, $\theta^i_{\text{cand,down}}(\sigma)$.

Claim 12. *Let*

$$\overline{O}_D^i = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \frac{1+\chi^b\xi_k^b-\Lambda}{R^i-\Lambda+\xi_k^i}\, dF_{\widetilde{\xi}}(\xi_k), \quad O_B^b = \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b} \frac{1-\Lambda}{R^b-\Lambda+\xi_w^b}\, dF_{\widetilde{\xi}}(\xi_w), \quad \overline{I}_D^b = \mathcal{M}^i \int_{\xi_k} \frac{\mathbb{O}^i\big(\underline{\theta}_D^b(H^b)\big)}{L^i} \frac{\mathsf{s}_{k,i}\chi^b\xi_k^b}{1+\chi^b\xi_k^b-\Lambda}\, dF_{\tilde{\xi}}(\xi_k),$$

$$\overline{O}_D^b = \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b} \frac{1-\Lambda+\chi^i\xi_w^i}{R^b+\xi_w^b-\Lambda}\, dF_{\tilde{\xi}}(\xi_w), \quad O_B^i = \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \frac{1-\Lambda}{R^i-\Lambda+\xi_k^i}\, dF_{\widetilde{\xi}}(\xi_k), \quad \overline{I}_D^i = \mathcal{M}^i \int_{\xi_w} \frac{\mathbb{O}^b\big(\underline{\theta}_D^i\big)}{L^b} \frac{\mathsf{s}_{w,b}\chi^i\xi_w^i}{1+\chi^i\xi_w^i-\Lambda}\, dF_{\tilde{\xi}}(\xi_w),$$

be the constants defined in Lemmas 3 and 4. Then any candidate equilibrium outflows satisfy the following:

1. *If* $\lim_{\sigma\downarrow 0} \theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$*, then* $\mathrm{I}^i_{\text{cand}} = 0$ *is a constant function, and*

$$\lim_{\sigma\downarrow 0} \mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma), \sigma\Big) \le \overline{O}^i_D \le \lim_{\sigma\downarrow 0} \mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma), \sigma\Big),$$
$$\lim_{\sigma\downarrow 0} \mathfrak{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,up}}(\sigma), \sigma\Big) \le O^b_B \le \lim_{\sigma\downarrow 0} \mathfrak{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma), \sigma\Big),$$
$$\lim_{\sigma\downarrow 0} \mathrm{I}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma), \sigma\Big) \le \overline{I}^b_D \le \lim_{\sigma\downarrow 0} \mathrm{I}^b_{\text{cand}}\Big(\theta^b_{\text{cand,up}}(\sigma), \sigma\Big).$$

2. *If* $\lim_{\sigma\downarrow 0} \theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma) > 0$*, then* $\mathrm{I}^b_{\text{cand}} = 0$ *is a constant function, and*

$$\lim_{\sigma\downarrow 0} \mathfrak{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,up}}(\sigma), \sigma\Big) \le \overline{O}^b_D \le \lim_{\sigma\downarrow 0} \mathfrak{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma), \sigma\Big),$$
$$\lim_{\sigma\downarrow 0} \mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma), \sigma\Big) \le O^i_B \le \lim_{\sigma\downarrow 0} \mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma), \sigma\Big),$$
$$\lim_{\sigma\downarrow 0} \mathrm{I}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma), \sigma\Big) \le \overline{I}^i_D \le \lim_{\sigma\downarrow 0} \mathrm{I}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma), \sigma\Big).$$

Proof. We prove (1); the proof of (2) is symmetric. Since $\lim_{\sigma\downarrow 0}\Big(\theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma)\Big) > 0$, for all sufficiently small σ we have $\theta^b_{\text{cand,up}}(\sigma) < \theta^i_{\text{cand,down}}(\sigma)$. Thus $\mathrm{I}^i_{\text{cand}}(\vartheta, \sigma) = 0$ for every ϑ by Claim 9. Let $\mathfrak{R}^i_{\text{cand}}(k, \sigma)$ denote the run set for depositor k at bank i. By Claim 10,

$$\mathfrak{R}^i_{\text{cand}}(k, \sigma) \subset (-\infty,\ \theta^i_{\text{cand,up}}(\sigma) + \overline{\mathrm{C}}^{\epsilon}_k \sigma + \mathrm{C}^{\text{adj}} \sigma^2].$$

Therefore,

$$\mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,up}}(\sigma), \sigma\Big) = \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\, \mathbb{P}\Big(\theta^i_{\text{cand,up}}(\sigma) + \sigma\widetilde{\epsilon}_k \in \mathfrak{R}^i_{\text{cand}}(k, \sigma)\Big)\, dF_{\widetilde{\xi}}(\xi_k) \le \mathcal{M}^i \int_{\xi_k} \mathrm{s}_{k,i}\, \mathbb{P}\Big(\widetilde{\epsilon}_k \le \overline{\mathrm{C}}^{\epsilon}_k + \mathrm{C}^{\text{adj}}\sigma\Big)\, dF_{\widetilde{\xi}}(\xi_k),$$

and the right-hand side converges to $\overline{O}^i_D$ as $\sigma \downarrow 0$. For the lower bound at $\theta^i_{\text{cand,down}}(\sigma)$, Claim 11 implies that depositor k leaves bank i for bank b whenever

$$\widetilde{x}_k \le \theta^i_{\text{cand,down}}(\sigma) + \overline{\mathrm{C}}^{\epsilon}_k \sigma - \mathrm{C}^{\text{adj}}\sigma^2 \quad \text{and} \quad \widetilde{x}_k \ge \theta^b_{\text{cand,up}}(\sigma) + \underline{\mathrm{C}}^{\epsilon}_k \sigma + \mathrm{C}^{\text{adj}}\sigma^2.$$

Evaluating at $\widetilde{x}_k = \theta^i_{\text{cand,down}}(\sigma) + \sigma\widetilde{\epsilon}_k$, the second inequality fails only if

$$\widetilde{\epsilon}_k < \frac{\theta^b_{\text{cand,up}}(\sigma) - \theta^i_{\text{cand,down}}(\sigma)}{\sigma} + \underline{\mathrm{C}}^{\epsilon}_k + \mathrm{C}^{\text{adj}}\sigma,$$

whose probability converges to 0 because the right-hand side tends to $-\infty$. Hence,

$$\mathbb{P}\Big(\theta^i_{\text{cand,down}}(\sigma) + \sigma\widetilde{\epsilon}_k \in \mathfrak{R}^i_{\text{cand}}(k, \sigma)\Big) \ge \mathbb{P}\Big(\widetilde{\epsilon}_k \le \overline{\mathrm{C}}^{\epsilon}_k - \mathrm{C}^{\text{adj}}\sigma\Big) - o(1).$$

Integrating over ξ_k yields $\liminf_{\sigma\downarrow 0} \mathfrak{O}^i_{\text{cand}}\Big(\theta^i_{\text{cand,down}}(\sigma), \sigma\Big) \ge \overline{O}^i_D$. Under the maintained payoff dominance in Claim 9, depositors at b do not move to i. Thus their decision reduces to the single-bank comparison of staying at b versus H, and the same argument as in Lemma 1 gives

$$\limsup_{\sigma\downarrow 0} \mathbb{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,up}}(\sigma),\sigma\Big) \le O^b_B \le \liminf_{\sigma\downarrow 0} \mathbb{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma),\sigma\Big).$$

At $\theta = \theta^b_{\text{cand,up}}(\sigma)$, depositors originally at bank i are in a region where they (attempt to) leave bank i, and among those who are served by bank i the choice between b and H is governed by the cutoff condition $\mathbb{P}(\mathcal{D}^c_b \mid x_k) > (1-\Lambda)/(1-\Lambda+\chi^b\xi^b_k)$, equivalently $\widetilde{\epsilon}_k > \underline{\mathsf{C}}^{\epsilon}_k$ up to an $\mathcal{O}(\sigma)$ error by the same posterior approximation used earlier. Therefore, the fraction of served withdrawals from bank i that are redeposited at bank b equals $\chi^b\xi^b_k/(1+\chi^b\xi^b_k-\Lambda)$ in the limit, yielding the stated representation with $\mathbb{O}^i(\theta^b_{\text{cand,up}}(\sigma))/L^i$. The argument at $\theta^b_{\text{cand,down}}(\sigma)$ is identical. □

In the following, fix that $\lim_{\sigma\downarrow 0}\theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$ (The other direction $\lim_{\sigma\downarrow 0}\theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) < 0$ are similar by symmetry.) Consider the following two cases:

Case 1. Suppose that $H^b + \overline{I}^b_D < \widehat{H^b}$. Since $\partial_2 C^b(A^b,0) > 1$, we have $C^b\Big(A^b, O^b_B - H^b - \overline{I}^b_D\Big) + H^b + \overline{I}^b_D > R^b O^b_B$. Hence

$$\widehat{\theta}^b(H^b)\,G^b(A^b) + (H^b + \overline{I}^b_D - O^b_B)^+ - C^b\Big(A^b,(O^b_B - \overline{I}^b_D - H^b)^+\Big) - \overline{I}^b_D - R^b(L^b - O^b_B) < \widehat{\theta}^b(H^b)\,G^b(A^b) + H^b + \overline{I}^b_D - \overline{I}^b_D - R^b L^b = 0. \quad (57)$$

By Claim 12, for all sufficiently small σ there exists $\mathsf{C}^{\text{tem}}_{\text{cand}} > 0$ such that

$$\mathbb{O}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma),\sigma\Big) \ge O^b_B - \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma, \qquad \mathbb{I}^b_{\text{cand}}\Big(\theta^b_{\text{cand,down}}(\sigma),\sigma\Big) \le \overline{I}^b_D + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma.$$

It follows that, for all sufficiently small σ,

$$\theta^b_{\text{cand,down}}(\sigma) > \widehat{\theta}^b(H^b). \quad (58)$$

Otherwise, take any $\vartheta > \theta^b_{\text{cand,down}}(\sigma)$ in a small neighborhood of $\theta^b_{\text{cand,down}}(\sigma)$. By continuity of $\mathbb{O}^b_{\text{cand}}(\cdot,\sigma)$ and $\mathbb{I}^b_{\text{cand}}(\cdot,\sigma)$ and the bounds above, we obtain

$$\begin{aligned}
&\vartheta G^b(A^b) - C^b\Big(A^b,(\mathbb{O}^b_{\text{cand}}(\vartheta,\sigma) - H^b - \mathbb{I}^b_{\text{cand}}(\vartheta,\sigma))^+\Big) + (H^b + \mathbb{I}^b_{\text{cand}}(\vartheta,\sigma) - \mathbb{O}^b_{\text{cand}}(\vartheta,\sigma))^+ - \mathbb{I}^b_{\text{cand}}(\vartheta,\sigma) - R^b\Big(L^b - \mathbb{O}^b_{\text{cand}}(\vartheta,\sigma)\Big)\\
&\le \widehat{\theta}^b(H^b)\,G^b(A^b) + (H^b + \overline{I}^b_D - O^b_B)^+ - C^b\Big(A^b,(O^b_B - \overline{I}^b_D - H^b)^+\Big) - \overline{I}^b_D - R^b(L^b - O^b_B) + o(1) < 0,
\end{aligned}$$

contradicting the definition of $\theta^b_{\text{cand,down}}(\sigma)$. Thus $\theta^b_{\text{cand,up}}(\sigma) \ge \theta^b_{\text{cand,down}}(\sigma) > \widehat{\theta}^b(H^b)$. In particular, at both cutoffs we must have

$$H^b + \mathbb{I}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma) \le \mathbb{O}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma), \qquad \theta^b_{\text{cand,cut}}(\sigma) \in \{\theta^b_{\text{cand,down}}(\sigma), \theta^b_{\text{cand,up}}(\sigma)\}.$$

Otherwise, if $H^b + \mathbb{I}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma) > \mathbb{O}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma)$, then Eq. (56) implies

$$0 = \theta^b_{\text{cand,cut}}(\sigma)G^b(A^b) + H^b - \mathbb{O}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma) - R^b\Big(L^b - \mathbb{O}^b_{\text{cand}}(\theta^b_{\text{cand,cut}}(\sigma),\sigma)\Big) > \widehat{\theta}^b(H^b)G^b(A^b) + H^b - R^bL^b = 0,$$

a contradiction. In addition, since $\widehat{H^b} < O_B^b$, it follows that $H^b + \bar{I}_D^b < O_B^b$ and for all small enough σ,

$$H^b + \mathrm{I}_{\text{cand}}^b(\theta,\sigma) \le \mathfrak{O}_{\text{cand}}^b(\theta,\sigma), \qquad \forall \theta \le \theta_{\text{cand,down}}^b(\sigma). \tag{59}$$

Since by definition $\theta_{\text{cand,down}}^b(\sigma) \le \theta_{\text{cand,up}}^b(\sigma)$, applying Eq. (56) at $\vartheta = \theta_{\text{cand,down}}^b(\sigma)$ and $\vartheta = \theta_{\text{cand,up}}^b(\sigma)$ and using $H^b + \mathrm{I}_{\text{cand}}^b(\cdot,\sigma) \le \mathfrak{O}_{\text{cand}}^b(\cdot,\sigma)$ at both cutoffs yields

$$\begin{aligned} &C^b\Big(A^b, \mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma),\sigma) - H^b - \mathrm{I}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma),\sigma)\Big) + R^b\Big(L^b - \mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma),\sigma)\Big) + \mathrm{I}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma),\sigma) \\ &= \theta_{\text{cand,down}}^b(\sigma)G^b(A^b) \quad \le \quad \theta_{\text{cand,up}}^b(\sigma)G^b(A^b) \\ &= \mathrm{I}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma),\sigma) + C^b\Big(A^b, \mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma),\sigma) - H^b - \mathrm{I}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma),\sigma)\Big) + R^b\Big(L^b - \mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma),\sigma)\Big). \end{aligned} \tag{60}$$

At this point, the same cutoff-squeeze argument as in Claim 3 applies: Claim 10 and 11 imply that if $\theta_{\text{cand,up}}^b(\sigma) - \theta_{\text{cand,down}}^b(\sigma)$ does not vanish as $\sigma \downarrow 0$, then $\mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,down}}^b(\sigma),\sigma) - \mathfrak{O}_{\text{cand}}^b(\theta_{\text{cand,up}}^b(\sigma),\sigma)$ is bounded away from zero, contradicting Eq. (60). Therefore,

$$\lim_{\sigma\downarrow 0}\Big(\theta_{\text{cand,up}}^b(\sigma) - \theta_{\text{cand,down}}^b(\sigma)\Big) = 0.$$

Combining this with Claim 12 yields

$$\begin{aligned} \lim_{\sigma\downarrow 0} \mathfrak{O}_{\text{cand}}^b\Big(\theta_{\text{cand,down}}^b(\sigma),\sigma\Big) &= \lim_{\sigma\downarrow 0} \mathfrak{O}_{\text{cand}}^b\Big(\theta_{\text{cand,up}}^b(\sigma),\sigma\Big) = O_B^b, \\ \lim_{\sigma\downarrow 0} \mathrm{I}_{\text{cand}}^b\Big(\theta_{\text{cand,down}}^b(\sigma),\sigma\Big) &= \lim_{\sigma\downarrow 0} \mathrm{I}_{\text{cand}}^b\Big(\theta_{\text{cand,up}}^b(\sigma),\sigma\Big) = \bar{I}_D^b. \end{aligned}$$

Case 2. Suppose that $H^b + \bar{I}_D^b \ge \widehat{H^b}$. We adapt the argument in Internet Appendix 1.4.2. By Claim 12, there exists $\mathsf{C}_{\text{cand}}^{\text{tem}} > 0$ such that for all sufficiently small σ,

$$\mathfrak{O}_{\text{cand}}^b\Big(\theta_{\text{cand,up}}^b(\sigma),\sigma\Big) \le O_B^b + \mathsf{C}_{\text{cand}}^{\text{tem}}\sigma, \qquad \mathrm{I}_{\text{cand}}^b\Big(\theta_{\text{cand,up}}^b(\sigma),\sigma\Big) \ge \bar{I}_D^b - \mathsf{C}_{\text{cand}}^{\text{tem}}\sigma.$$

Claim 13. *There exists $\mathsf{C}_{\text{up}}^{\text{adj}} > 0$ such that, for all sufficiently small σ,*

$$\widehat{\theta}^b(H^b) - \mathsf{C}_{\text{up}}^{\text{adj}}\sigma \le \theta_{\text{cand,up}}^b(\sigma) \le \widehat{\theta}^b(H^b) + \mathsf{C}_{\text{up}}^{\text{adj}}\sigma.$$

Consequently, $\theta_{\text{cand,up}}^b(\sigma) \to \widehat{\theta}^b(H^b)$ as $\sigma \downarrow 0$.

Proof. Write $\theta_{\text{up}}^b(\sigma) \equiv \theta_{\text{cand,up}}^b(\sigma)$ and abbreviate $O_{\text{up}}(\sigma) \equiv \mathfrak{O}_{\text{cand}}^b(\theta_{\text{up}}^b(\sigma),\sigma)$ and $I_{\text{up}}(\sigma) \equiv \mathrm{I}_{\text{cand}}^b(\theta_{\text{up}}^b(\sigma),\sigma)$. If $H^b + I_{\text{up}}(\sigma) \ge O_{\text{up}}(\sigma)$, then at $\vartheta = \theta_{\text{up}}^b(\sigma)$ the cutoff identity Eq. (56) becomes

$$0 = \theta_{\text{up}}^b(\sigma)G^b(A^b) + H^b - R^bL^b + (R^b-1)O_{\text{up}}(\sigma) \;\ge\; \theta_{\text{up}}^b(\sigma)G^b(A^b) + H^b - R^bL^b,$$

so $\theta_{\text{up}}^b(\sigma) \le \widehat{\theta}^b(H^b)$ from the definition of $\widehat{\theta}^b(H^b)$.

If instead $H^b + I_{\text{up}}(\sigma) \le O_{\text{up}}(\sigma)$, then Eq. (56) becomes

$$0 = \theta^b_{\text{up}}(\sigma)G^b(A^b) - C^b\Big(A^b, O_{\text{up}}(\sigma) - I_{\text{up}}(\sigma) - H^b\Big) - I_{\text{up}}(\sigma) - R^b\Big(L^b - O_{\text{up}}(\sigma)\Big) \Rightarrow$$
$$0 \ge \theta^b_{\text{up}}(\sigma)G^b(A^b) - C^b\Big(A^b, (O^b_B - I_{\text{up}}(\sigma) - H^b)^+\Big) - I_{\text{up}}(\sigma) - R^b(L^b - O^b_B) - \mathcal{O}(\sigma)$$

using $O_{\text{up}}(\sigma) \le O^b_B + \mathsf{C}^{\text{tem}}_{\text{cand}}\sigma$. Moreover, in this case we have $H^b + I_{\text{up}}(\sigma) < O_{\text{up}}(\sigma) \le O^b_B + \mathcal{O}(\sigma)$, so for σ small, $H^b + I_{\text{up}}(\sigma) \le O^b_B$ and thus $(O^b_B - I_{\text{up}}(\sigma) - H^b)^+ = O^b_B - I_{\text{up}}(\sigma) - H^b$. Since $H^b + \overline{I}^b_D \ge \widehat{H}^b$ and $I_{\text{up}}(\sigma) \ge \overline{I}^b_D - \mathcal{O}(\sigma)$, we have $H^b + I_{\text{up}}(\sigma) \ge \widehat{H}^b - \mathcal{O}(\sigma)$, hence for small σ, by Assumption 4,

$$C^b\Big(A^b, O^b_B - I_{\text{up}}(\sigma) - H^b\Big) + I_{\text{up}}(\sigma) + H^b \le R^b O^b_B + \mathcal{O}(\sigma).$$

Plugging into the previous inequality yields

$$0 \ge \theta^b_{\text{up}}(\sigma)G^b(A^b) + H^b - R^b L^b - \mathcal{O}(\sigma),$$

so $\theta^b_{\text{up}}(\sigma) \le \widehat{\theta}^b(H^b) + \mathcal{O}(\sigma)$. This proves the upper bound.

Lower bound. Suppose for contradiction that $\limsup_{\sigma\downarrow 0} \frac{\widehat{\theta}^b(H^b) - \theta^b_{\text{up}}(\sigma)}{\sigma} = +\infty$. Then there exists a sequence $\sigma_n \downarrow 0$ such that $\theta^b_{\text{up}}(\sigma_n) \le \widehat{\theta}^b(H^b) - n\sigma_n$. Fix any $\delta > 0$ and let $\theta^\delta \equiv \widehat{\theta}^b(H^b) - \delta$. For n large, $\theta^\delta > \theta^b_{\text{up}}(\sigma_n)$ and $\frac{\theta^b_{\text{up}}(\sigma_n) - \theta^\delta}{\sigma_n} \to -\infty$.

Let $\mathfrak{R}^b_{\text{cand}}(w, \sigma)$ be the run set of a depositor w at bank b. By Claim 10,

$$\mathfrak{R}^b_{\text{cand}}(w, \sigma) \subset \Big(-\infty,\ \theta^b_{\text{up}}(\sigma) + \overline{\mathsf{C}}^{\epsilon}_w\sigma + \mathsf{C}^{\text{adj}}\sigma^2\Big].$$

Therefore,

$$\begin{aligned}
\mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n) &= \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b}\, \mathbb{P}\Big(\theta^\delta + \sigma_n \widetilde{\epsilon}_w \in \mathfrak{R}^b_{\text{cand}}(w, \sigma_n)\Big)\, dF_{\widetilde{\xi}}(\xi_w) \\
&\le \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b}\, \mathbb{P}\Big(\widetilde{\epsilon}_w \le \frac{\theta^b_{\text{up}}(\sigma_n) - \theta^\delta}{\sigma_n} + \overline{\mathsf{C}}^{\epsilon}_w + \mathsf{C}^{\text{adj}}\sigma_n\Big)\, dF_{\widetilde{\xi}}(\xi_w) \xrightarrow[n\to\infty]{} 0.
\end{aligned}$$

Thus for n large, $\mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n)$ is arbitrarily small, so

$$\begin{aligned}
&\theta^\delta G^b(A^b) - C^b\Big(A^b, \big(\mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n) - \mathrm{I}^b_{\text{cand}}(\theta^\delta, \sigma_n) - H^b\big)^+\Big) + \big(H^b + \mathrm{I}^b_{\text{cand}}(\theta^\delta, \sigma_n) - \mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n)\big)^+ \\
&\quad - \mathrm{I}^b_{\text{cand}}(\theta^\delta, \sigma_n) - R^b\Big(L^b - \mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n)\Big) \\
&= \theta^\delta G^b(A^b) + H^b - R^b L^b + (R^b - 1)\mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n) = -\delta\, G^b(A^b) + (R^b - 1)\mathfrak{O}^b_{\text{cand}}(\theta^\delta, \sigma_n) < 0
\end{aligned}$$

for n large, where we used $\widehat{\theta}^b(H^b)G^b(A^b) + H^b - R^b L^b = 0$. This implies the bank is insolvent at $\theta^{\delta} > \theta^{b}_{\text{up}}(\sigma_n)$, contradicting the definition of $\theta^{b}_{\text{up}}(\sigma_n)$. Hence $\theta^{b}_{\text{up}}(\sigma) \geq \widehat{\theta}^{b}(H^b) - \mathcal{O}(\sigma)$, proving the lower bound. □

Claim 14. *In any equilibrium in the noiseless limit, bank b must default whenever* $\theta < \widehat{\theta}^{b}(H^b)$.

Proof. Similar to Claim 8. □

We now return to the noisy subgames. By Claim 13, any candidate equilibrium satisfies $\theta^{b}_{\text{cand,up}}(\sigma) \to \widehat{\theta}^{b}(H^b)$ as $\sigma \downarrow 0$. Combining this with Claim 14 implies that any limiting equilibrium (i.e., any limit of noisy equilibria along $\sigma_n \downarrow 0$) must inherit bank b's default set $(-\infty, \widehat{\theta}^{b}(H^b)]$.

Claim 15. *Suppose* $\lim_{\sigma\downarrow 0}\left[\theta^{i}_{\text{cand,down}}(\sigma) - \theta^{b}_{\text{cand,up}}(\sigma)\right] > 0$, *then any candidate limiting equilibrium as* $\sigma \downarrow 0$ *is the one described in* (i) *of Theorem 2. Conversely, if* $\lim_{\sigma\downarrow 0}\left[\theta^{b}_{\text{cand,down}}(\sigma) - \theta^{i}_{\text{cand,up}}(\sigma)\right] > 0$, *then any candidate limiting equilibrium as* $\sigma \downarrow 0$ *is the one described in* (ii) *of Theorem 2.*

Remark Claim 15 provides necessary conditions for the limiting equilibrium under the separation assumption $\lim_{\sigma\downarrow 0}\left[\theta^{i}_{\text{cand,down}}(\sigma) - \theta^{b}_{\text{cand,up}}(\sigma)\right] > 0$ (resp. < 0). What remains is twofold. First, we must show *existence* of a sequence of noisy candidate equilibria satisfying the relevant strict ordering for all sufficiently small σ. Second, we need to study the knife-edge case $\lim_{\sigma\downarrow 0}\left[\theta^{i}_{\text{cand}}(\sigma) - \theta^{b}_{\text{cand}}(\sigma)\right] = 0$.

1.7.2 Existence of candidate limiting equilibrium

One assumption for Section 4.2 is that $H^i = 0$ and $H^b \leq \widehat{H^b}$. Here we only show the existence of the limiting equilibrium where bank i deposits run to the national bank. The other equilibrium, which may exist under some parameters, can be constructed similarly.

Constructing converging equilibria when $H^b + \overline{I}^{b}_{D} < \widehat{H^b}$**.**Define

$$\underline{\mathbb{O}}^{b}(\theta, 0, \sigma) \equiv \left(O^{b}_{B} - \mathtt{C}^{\text{tem}}_{\text{cand}}\sigma\right)\mathbb{1}_{\left[\theta \leq \widehat{\theta}^{b}(H^b)\right]}, \qquad \overline{\mathbb{O}}^{i}(\theta, 0, \sigma) \equiv L^i,$$

$$\overline{\mathbb{I}}^{b}(\theta, 0, \sigma) \equiv \left(\overline{I}^{b}_{D} + \mathtt{C}^{\text{tem}}_{\text{cand}}\sigma\right)\mathbb{1}_{\left[\theta \leq \widehat{\theta}^{b}(H^b)\right]} + L^i\,\mathbb{1}_{\left[\theta > \widehat{\theta}^{b}(H^b)\right]}, \qquad \underline{\mathbb{I}}^{i}(\theta, 0, \sigma) \equiv 0.$$

where $\mathtt{C}^{\text{tem}}_{\text{cand}}$ is chosen so that, it is common knoweledge for all depositors that for every candidate equilibrium flow profile $\left(\mathbb{O}^{b}_{\text{cand}}, \mathbb{O}^{i}_{\text{cand}}, \mathbb{I}^{b}_{\text{cand}}, \mathbb{I}^{i}_{\text{cand}}\right)$ associated with the case $\lim_{\sigma\downarrow 0}\left[\theta^{i}_{\text{cand,down}}(\sigma) - \theta^{b}_{\text{cand,up}}(\sigma)\right] > 0$, the following pointwise bounds hold for all θ:

$$\underline{\mathbb{O}}^{b}(\theta, 0, \sigma) \leq \mathbb{O}^{b}_{\text{cand}}(\theta, \sigma), \qquad \mathbb{O}^{i}_{\text{cand}}(\theta, \sigma) \leq \overline{\mathbb{O}}^{i}(\theta, 0, \sigma),$$

$$\underline{\mathbb{I}}^{i}(\theta, 0, \sigma) \leq \mathbb{I}^{i}_{\text{cand}}(\theta, \sigma), \qquad \mathbb{I}^{b}_{\text{cand}}(\theta, \sigma) \leq \overline{\mathbb{I}}^{b}(\theta, 0, \sigma).$$

Such a constant exists by the outflow/inflow bounds in Claim 12 Claim 10 to 12 and Eqs. (58) and (59) together with the Case 1 restriction $H^b + \overline{I}^{b}_{D} < \widehat{H}^{b} < O^{b}_{B}$, which implies

$O_B^b - H^b - \overline{I}_D^b > 0$ and hence enforces liquidation for bank b near and below $\widehat{\theta}^b(H^b)$ for all sufficiently small σ. Since $H^b + \overline{I}_D^b < \widehat{H^b} < O_B^b$, for all sufficiently small σ,

$$\underline{\mathfrak{O}}^b(\theta, 0, \sigma) - H^b - \overline{\mathfrak{I}}^b(\theta, 0, \sigma) \geq \left(O_B^b - H^b - \overline{I}_D^b\right) - 2\mathfrak{C}_{\text{cand}}^{\text{tem}}\sigma > 0, \qquad \forall \theta \leq \widehat{\theta}^b(H^b).$$

Given any outflow and inflow function $\mathfrak{O}(\cdot)$ and $\mathfrak{I}(\cdot)$, define functional

$$\mathfrak{V}_j(\theta, \mathfrak{O}(\cdot), \mathfrak{I}(\cdot)) \equiv \theta G^j(A^j) + (\mathfrak{I}(\theta) + H^j - \mathfrak{O}(\theta))^+ - C^j\Big(A^j, (\mathfrak{O}(\theta) - \mathfrak{I}(\theta) - H^j)^+\Big) - \mathfrak{I}(\theta) - R^j\Big(L^j - \mathfrak{O}(\theta)\Big), \quad j \in \{i, b\},$$

to represent banker's payoff if the outflow and inflow is characterized by $\mathfrak{O}(\cdot)$ and $\mathfrak{I}(\cdot)$, respectively, when the state is θ. For $\ell = 0, 1, \ldots,$ iteratively define

$$\mathfrak{D}_b(\ell) \equiv \{\theta : \mathfrak{V}_b(\theta, \underline{\mathfrak{O}}^b(\cdot, \ell, \sigma), \overline{\mathfrak{I}}^b(\cdot, \ell, \sigma)) \leq 0\}, \qquad \mathfrak{D}_i(\ell) \equiv \{\theta : \mathfrak{V}_i(\theta, \overline{\mathfrak{O}}^i(\cdot, \ell, \sigma), \underline{\mathfrak{I}}^i(\cdot, \ell, \sigma)) \leq 0\}.$$

$$\mathfrak{R}^i(k, \ell, \sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell) \,\Big|\, \widetilde{x_k} = x\Big) < \frac{\max\Big(1 - \Lambda, \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell) \,\Big|\, \widetilde{x_k} = x\Big)(1 + \chi^b \xi_k^b - \Lambda)\Big)}{R^i - \Lambda + \xi_k^i}\right\},$$

$$\mathfrak{M}^b(k, \ell, \sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell) \,\Big|\, \widetilde{x_k} = x\Big) > \frac{\max\Big(1 - \Lambda, \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell) \,\Big|\, \widetilde{x_k} = x\Big)(R^i + \xi_k^i - \Lambda)\Big)}{1 + \chi^b \xi_k^b - \Lambda}\right\},$$

$$\mathfrak{R}^b(w, \ell, \sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell) \,\Big|\, \widetilde{x_w} = x\Big) < \frac{\max\Big(1 - \Lambda, \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell) \,\Big|\, \widetilde{x_w} = x\Big)(1 + \chi^i \xi_w^i - \Lambda)\Big)}{R^b - \Lambda + \xi_w^b}\right\},$$

$$\mathfrak{M}^i(w, \ell, \sigma) \equiv \left\{x : \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell) \,\Big|\, \widetilde{x_w} = x\Big) > \frac{\max\Big(1 - \Lambda, \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell) \,\Big|\, \widetilde{x_w} = x\Big)(R^b + \xi_w^b - \Lambda)\Big)}{1 + \chi^i \xi_w^i - \Lambda}\right\}.$$

$$\underline{\mathfrak{O}}^b(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b} \mathbb{P}(\theta + \sigma \widetilde{\epsilon_w} \in \mathfrak{R}^b(w, \ell, \sigma)) f_{\widetilde{\xi}}(\xi_w) d\xi_w,$$

$$\overline{\mathfrak{O}}^i(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \mathbb{P}(\theta + \sigma \widetilde{\epsilon_k} \in \mathfrak{R}^i(k, \ell, \sigma)) f_{\widetilde{\xi}}(\xi_k) d\xi_k,$$

$$\underline{\mathfrak{I}}^i(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_w} \mathsf{s}_{w,b} \mathbb{P}(\theta + \sigma \widetilde{\epsilon_w} \in \mathfrak{M}^i(w, \ell, \sigma)) f_{\widetilde{\xi}}(\xi_w) d\xi_w,$$

$$\overline{\mathfrak{I}}^b(\theta, \ell + 1, \sigma) \equiv \mathcal{M}^i \int_{\xi_k} \mathsf{s}_{k,i} \mathbb{P}(\theta + \sigma \widetilde{\epsilon_k} \in \mathfrak{M}^b(k, \ell, \sigma)) f_{\widetilde{\xi}}(\xi_k) d\xi_k.$$

Here, $\mathfrak{R}^i(k, \ell, \sigma)$ is the set of signals for a depositor k initially at bank i under which *leaving* bank i (to bank b or to H) is a best response, given the default sets $\mathfrak{D}_i(\ell)$ and $\mathfrak{D}_b(\ell)$. Similarly, $\mathfrak{M}^b(k, \ell, \sigma)$ is the set of signals under which moving specifically to bank b is optimal (i.e., bank b weakly dominates both staying at i and running to H). The sets $\mathfrak{R}^b(w, \ell, \sigma)$ and $\mathfrak{M}^i(w, \ell, \sigma)$ have the analogous interpretations for a depositor w initially at bank b.

Claim 16. *Fix $\ell \geq 0$ and $\sigma > 0$. Let $\theta^j_{\ell,\mathrm{up}} \equiv \sup \mathfrak{D}_j(\ell)$ and $\theta^j_{\ell,\mathrm{down}} \equiv \inf \mathfrak{D}_j(\ell)^c$ for $j \in \{i, b\}$. Then there exists a constant $\mathsf{C}^{\mathrm{adj}} > 0$ (independent of ℓ) such that, for all sufficiently small σ, the same signal-dominance implications as in Claim 10 and 11 hold after replacing $\theta^j_{\mathrm{cand,up}}(\sigma), \theta^j_{\mathrm{cand,down}}(\sigma)$ by $\theta^j_{\ell,\mathrm{up}}, \theta^j_{\ell,\mathrm{down}}$ throughout.*

Proof. The proof is identical to Claim 10 and 11. It only uses Bayes' rule, uniform continuity of $f_{\widetilde{\theta}}$ on a compact interval containing the relevant cutoffs, and an almost-sure bound on $\operatorname{ess\,sup}_k \|f_{\widetilde{\epsilon}_k}\|_\infty$, none of which relies on equilibrium consistency. □

Let $\underline{\theta}^b_D(H^b), \overline{\theta}^i_D$ be the run cutoffs described in Lemma 4 and Theorem 2.

Claim 17. *For all σ sufficiently small, there exists a constant $\mathsf{C}^{\mathrm{tem2}} > 0$ such that for every $\ell \geq 0$:*

$$(-\infty, \widehat{\theta}^b(H^b)] \subsetneq \mathfrak{D}_b(\ell) \subset \mathfrak{D}_b(\ell+1) \subsetneq (-\infty, \underline{\theta}^b_D(H^b) + \mathsf{C}^{\mathrm{tem2}}\sigma] \subsetneq (-\infty, \overline{\theta}^i_D - \mathsf{C}^{\mathrm{tem2}}\sigma] \quad \subsetneq \mathfrak{D}_i(\ell+1) \subset \mathfrak{D}_i(\ell).$$

$$\mathfrak{M}^i(w, \ell, \sigma) = \emptyset \qquad \text{and} \qquad \underline{\mathfrak{I}}^i(\theta, \ell, \sigma) \equiv 0.$$

For all θ, $\underline{\mathfrak{O}}^b(\theta, \ell+1, \sigma) \geq \underline{\mathfrak{O}}^b(\theta, \ell, \sigma), \overline{\mathfrak{O}}^i(\theta, \ell+1, \sigma) \leq \overline{\mathfrak{O}}^i(\theta, \ell, \sigma)$, and for all $\theta \in \mathfrak{D}_b(\ell)$, $\overline{\mathfrak{I}}^b(\theta, \ell+1, \sigma) \leq \overline{\mathfrak{I}}^b(\theta, \ell, \sigma)$.

Proof. We proceed by induction on ℓ. The base case $\ell = 0$ follows directly from the Step-0 bounds and the definitions of $\mathfrak{D}_b(0)$ and $\mathfrak{D}_i(0)$. Fix $\ell \geq 0$ and assume the stated inclusions and monotonicity properties hold up to round ℓ. In particular, for any $\theta \in \mathfrak{D}_b(\ell)$,

$$\underline{\mathfrak{O}}^b(\theta, \ell, \sigma) - H^b - \overline{\mathfrak{I}}^b(\theta, \ell, \sigma) > 0,$$

(denoted as "liquidation branch") and hence for any signal realization x,

$$\mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell) \mid \widetilde{x} = x\Big) \leq \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_b(\ell-1) \mid \widetilde{x} = x\Big), \qquad \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell) \mid \widetilde{x} = x\Big) \geq \mathbb{P}\Big(\widetilde{\theta} \notin \mathfrak{D}_i(\ell-1) \mid \widetilde{x} = x\Big).$$

Step 1: monotone updates of bounds. By Claim 16, the same signal-dominance implications used in Claim 10 and 11 apply at iteration ℓ after replacing the candidate cutoffs by the endpoints of $\mathfrak{D}_b(\ell)$ and $\mathfrak{D}_i(\ell)$. Therefore, the expansion of $\mathfrak{D}_b(\ell)$ relative to $\mathfrak{D}_b(\ell-1)$ weakly expands the leave-b set: $\mathfrak{K}^b(w, \ell-1, \sigma) \subseteq \mathfrak{K}^b(w, \ell, \sigma)$, and weakly shrinks the move-to-b set: $\mathfrak{M}^b(k, \ell, \sigma) \subseteq \mathfrak{M}^b(k, \ell-1, \sigma)$. Consequently, by the update formulas,

$$\underline{\mathfrak{O}}^b(\theta, \ell+1, \sigma) \geq \underline{\mathfrak{O}}^b(\theta, \ell, \sigma) \qquad \text{and} \qquad \overline{\mathfrak{I}}^b(\theta, \ell+1, \sigma) \leq \overline{\mathfrak{I}}^b(\theta, \ell, \sigma)$$

for all θ, and in particular pointwise on $\mathfrak{D}_b(\ell)$. Similarly, we have $\overline{\mathfrak{O}}^i(\theta, \ell+1, \sigma) \leq \overline{\mathfrak{O}}^i(\theta, \ell, \sigma) \quad \forall \theta$. Finally, under the maintained ordering $\mathfrak{D}_b(\ell) \subset \mathfrak{D}_i(\ell)$ from the induction hypothesis and the payoff condition $R^b + \xi^b_w > 1 + \chi^i \xi^i_w$, the same comparison as in Claim 9 implies $\mathfrak{M}^i(w, \ell, \sigma) = \emptyset$. Hence $\underline{\mathfrak{I}}^i(\cdot, \ell+1, \sigma) \equiv 0$ as well.

Step 2: monotone updates of default sets. On $\mathfrak{D}_b(\ell)$ we are on the liquidation branch (by the displayed inequality above). Under $R^b \leq \partial_2 C^b(A^b, 0)$ and Assumption 4, the banker payoff

$\mathfrak{V}_b(\theta, \mathfrak{O}(\cdot), \mathfrak{I}(\cdot))$ is weakly decreasing in $\mathfrak{O}(\theta)$ and weakly increasing in $\mathfrak{I}(\theta)$ on this branch. Therefore, increasing $\underline{\mathfrak{O}}^b(\cdot, \ell, \sigma)$ and decreasing $\overline{\mathfrak{I}}^b(\cdot, \ell, \sigma)$ can only reduce $\mathfrak{V}_b$, which yields

$$\mathfrak{D}_b(\ell) \subset \mathfrak{D}_b(\ell+1).$$

Likewise, since $\overline{\mathfrak{O}}^i(\cdot, \ell+1, \sigma) \leq \overline{\mathfrak{O}}^i(\cdot, \ell, \sigma)$ and $\underline{\mathfrak{I}}^i \equiv 0$, monotonicity of $\mathfrak{V}_i$ in $\mathfrak{O}$ implies

$$\mathfrak{D}_i(\ell+1) \subset \mathfrak{D}_i(\ell).$$

Step 3: uniform bounds near the moving cutoffs. By the induction hypothesis, for all $\ell \geq 0$ we have the uniform bracket

$$\sup \mathfrak{D}_b(\ell) \ \leq \ \underline{\theta}_D^b(H^b) + \mathtt{C}^{\text{tem2}}\sigma, \qquad \inf \mathfrak{D}_i(\ell)^c \ \geq \ \overline{\theta}_D^i - \mathtt{C}^{\text{tem2}}\sigma.$$

Since $\overline{\theta}_D^i > \underline{\theta}_D^b(H^b)$, so $\frac{\inf \mathfrak{D}_i(\ell)^c - \sup \mathfrak{D}_b(\ell)}{\sigma}$ is uniformly large (diverging like $1/\sigma$) as $\sigma \downarrow 0$.

Applying Claim 16 at iteration ℓ, the same run/move dominance bounds as in Claim 12 imply that, when evaluating the induced outflow/inflow maps at the boundary points $\sup \mathfrak{D}_b(\ell)$ and $\inf \mathfrak{D}_i(\ell)^c$, the relevant posteriors place probability $1 - \mathcal{O}(\sigma)$ on the other bank being in its solvency region because of the uniform gap in σ-units. Therefore, repeating the argument in Claim 12 (with the candidate cutoffs replaced by $\sup \mathfrak{D}_b(\ell)$ and $\inf \mathfrak{D}_i(\ell)^c$) yields a constant $\mathtt{C}^{\text{tem3}}$ independent of ℓ such that

$$\left|\underline{\mathfrak{O}}^b\Big(\sup \mathfrak{D}_b(\ell), \ell+1, \sigma\Big) - O_B^b\right| \leq \mathtt{C}^{\text{tem3}}\sigma, \qquad \left|\overline{\mathfrak{I}}^b(\sup \mathfrak{D}_b(\ell), \ell+1, \sigma) - \overline{I}_D^b\right| \leq \mathtt{C}^{\text{tem3}}\sigma,$$
$$\left|\overline{\mathfrak{O}}^i\Big(\inf \mathfrak{D}_i(\ell)^c, \ell+1, \sigma\Big) - \overline{O}_D^i\right| \leq \mathtt{C}^{\text{tem3}}\sigma, \qquad \underline{\mathfrak{I}}^i\Big(\inf \mathfrak{D}_i(\ell)^c, \ell+1, \sigma\Big) = 0.$$

Because $\sup \mathfrak{D}_b(\ell+1) \geq \sup \mathfrak{D}_b(\ell)$,

$$\underline{\mathfrak{O}}^b\Big(\sup \mathfrak{D}_b(\ell+1), \ell+1, \sigma\Big) \leq O_B^b + \mathtt{C}^{\text{tem3}}\sigma, \qquad \overline{\mathfrak{I}}^b\Big(\sup \mathfrak{D}_b(\ell+1), \ell+1, \sigma\Big) \leq \overline{I}_D^b + \mathtt{C}^{\text{tem3}}\sigma.$$

Comparing these inequalities with Eq. (15) implies that, for $\mathtt{C}^{\text{tem2}}$ chosen large enough (independent of ℓ),

$$\sup \mathfrak{D}_b(\ell+1) < \underline{\theta}_D^b(H^b) + \mathtt{C}^{\text{tem2}}\sigma.$$

The remaining inclusions are proved analogously.

□

For fixed $\sigma > 0$, the sequences $\{\underline{\mathfrak{O}}^b(\cdot, \ell, \sigma)\}_{\ell \geq 0}$ and $\{\overline{\mathfrak{O}}^i(\cdot, \ell, \sigma)\}_{\ell \geq 0}$ are monotone and uniformly bounded, and so is $\{\overline{\mathfrak{I}}^b(\cdot, \ell, \sigma)\}_{\ell \geq 0}$ on $\mathfrak{D}_b(\ell)$ by the previous claim. Hence the pointwise limits

$$\underline{\mathfrak{O}}^b(\theta, \infty, \sigma) \equiv \lim_{\ell \to \infty} \underline{\mathfrak{O}}^b(\theta, \ell, \sigma), \qquad \overline{\mathfrak{O}}^i(\theta, \infty, \sigma) \equiv \lim_{\ell \to \infty} \overline{\mathfrak{O}}^i(\theta, \ell, \sigma),$$
$$\overline{\mathfrak{I}}^b(\theta, \infty, \sigma) \equiv \lim_{\ell \to \infty} \overline{\mathfrak{I}}^b(\theta, \ell, \sigma), \qquad \underline{\mathfrak{I}}^i(\theta, \infty, \sigma) \equiv 0$$

are well defined for all θ. Define the limiting default sets

$$\mathfrak{D}_b(\infty) \equiv \bigcup_{\ell \geq 0} \mathfrak{D}_b(\ell), \qquad \mathfrak{D}_i(\infty) \equiv \bigcap_{\ell \geq 0} \mathfrak{D}_i(\ell),$$

and the limiting signal sets

$$\mathfrak{R}^b(w,\infty,\sigma) \equiv \bigcup_{\ell \geq 0} \mathfrak{R}^b(w,\ell,\sigma), \qquad \mathfrak{M}^b(k,\infty,\sigma) \equiv \bigcap_{\ell \geq 0} \mathfrak{M}^b(k,\ell,\sigma),$$

$$\mathfrak{R}^i(k,\infty,\sigma) \equiv \bigcup_{\ell \geq 0} \mathfrak{R}^i(k,\ell,\sigma), \qquad \mathfrak{M}^i(w,\infty,\sigma) \equiv \bigcap_{\ell \geq 0} \mathfrak{M}^i(w,\ell,\sigma).$$

By construction, these limits inherit the same dominance interpretation as in each round.

Claim 18. *Fix $\sigma > 0$ sufficiently small. Consider the strategy profile in which depositors at bank b withdraw iff $\tilde{x}_w \in \mathfrak{R}^b(w,\infty,\sigma)$ and move into b iff $\tilde{x}_k \in \mathfrak{M}^b(k,\infty,\sigma)$ (and analogously for bank i using $\mathfrak{R}^i(k,\infty,\sigma)$ and $\mathfrak{M}^i(w,\infty,\sigma)$). Let the induced flows be $\underline{\mathfrak{O}}^b(\cdot,\infty,\sigma)$, $\overline{\mathfrak{O}}^i(\cdot,\infty,\sigma)$, $\overline{\mathfrak{I}}^b(\cdot,\infty,\sigma)$, and $\underline{\mathfrak{I}}^i(\cdot,\infty,\sigma) \equiv 0$. Then this strategy profile is a (Bayesian) equilibrium of the $t = 2$ subgame for noise level σ.*

Proof. By the definitions of $\mathfrak{R}^b(w,\ell,\sigma)$ and $\mathfrak{M}^b(k,\ell,\sigma)$ and the ELLN, the updating rules defining $\underline{\mathfrak{O}}^b(\cdot,\ell+1,\sigma)$ and $\overline{\mathfrak{I}}^b(\cdot,\ell+1,\sigma)$ are continuous in the indicator events $\{\theta + \sigma\epsilon \in \mathfrak{R}^b(w,\ell,\sigma)\}$ and $\{\theta + \sigma\epsilon \in \mathfrak{M}^b(k,\ell,\sigma)\}$. Because $\mathfrak{R}^b(w,\ell,\sigma) \uparrow \mathfrak{R}^b(w,\infty,\sigma)$ and $\mathfrak{M}^b(k,\ell,\sigma) \downarrow \mathfrak{M}^b(k,\infty,\sigma)$, bounded convergence yields that the induced flows coincide with the limits $\underline{\mathfrak{O}}^b(\cdot,\infty,\sigma)$ and $\overline{\mathfrak{I}}^b(\cdot,\infty,\sigma)$. The same argument applies to bank i (with $\underline{\mathfrak{I}}^i \equiv 0$).

Fix a depositor and type. At each round ℓ, any action inconsistent with membership in $\mathfrak{R}^j(\cdot,\ell,\sigma)$ or $\mathfrak{M}^j(\cdot,\ell,\sigma)$ is eliminated as dominated under the round-ℓ default sets $\mathfrak{D}_b(\ell)$ and $\mathfrak{D}_i(\ell)$. Taking limits and using Claim 16, the same strict dominance comparisons continue to hold under $\mathfrak{D}_b(\infty)$ and $\mathfrak{D}_i(\infty)$. Hence, under the limiting beliefs, the prescribed action is a best response for every signal realization, and any deviation is weakly dominated.

By definition of $\mathfrak{D}_b(\ell)$ and monotone convergence of the flow bounds, $\mathfrak{D}_b(\infty)$ is precisely the set of θ for which $\mathfrak{V}_b(\theta, \underline{\mathfrak{O}}^b(\cdot,\infty,\sigma), \overline{\mathfrak{I}}^b(\cdot,\infty,\sigma)) \leq 0$. Likewise, $\mathfrak{D}_i(\infty)$ is the set where $\mathfrak{V}_i(\theta, \overline{\mathfrak{O}}^i(\cdot,\infty,\sigma), 0) \leq 0$. Therefore beliefs about default are consistent with the induced flows. Combining (i)–(iii) gives a $t = 2$ equilibrium for noise level σ. □

Let $\theta^b_\sigma \equiv \sup \mathfrak{D}_b(\infty), \theta^i_\sigma \equiv \inf \mathfrak{D}_i(\infty)^c$. By construction, $\theta^b_\sigma \leq \underline{\theta}^b_D(H^b) + \mathfrak{C}^{\text{tem2}}\sigma$ and $\theta^i_\sigma \geq \overline{\theta}^i_D - \mathfrak{C}^{\text{tem2}}\sigma$, so $\theta^i_\sigma - \theta^b_\sigma \geq \frac{1}{2}(\overline{\theta}^i_D - \underline{\theta}^b_D(H^b)) > 0$ for all sufficiently small σ. Thus the equilibrium constructed above satisfies the strict separation $\theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$ for all small σ, and Claim 9, 11 and 12 apply.

Passing to the limit $\sigma \downarrow 0$ in the cutoff identities Eq. (56) using Claim 12 pins down the limiting cutoffs and limiting flows as the unique fixed point $\left(\overline{I}^b_D, \underline{\theta}^b_D(H^b), \overline{O}^i_D, \overline{\theta}^i_D\right)$ from

Eq. (15). Finally, by Claim 11, the equilibrium strategies converge to the cutoff structure stated in (i) of Theorem 2. ☐

Case 2: Existence of converging equilibria when $H^b + \overline{I}_D^b > \widehat{H^b}$. Unlike Case 1, there is no direct construction of the $t = 2$ equilibrium by iterated elimination of dominated strategies (IEODS) once the national bank has a sufficiently large effective liquidity buffer. Intuitively, when $H^b + \overline{I}_D^b > \widehat{H}^b$, bank b has no "coordination-run default region" (Section 4.1.3), and depositor incentives are no longer monotone in conjectured aggregate outflows. As a result, in a noisy candidate equilibrium the default region for bank b need not be an interval: there can exist θ values in the bracket $[\theta^b_{\text{cand,down}}(\sigma), \theta^b_{\text{cand,up}}(\sigma)]$ for which bank b is solvent and others for which it is insolvent, and the normalized width $\left(\theta^b_{\text{cand,up}}(\sigma) - \theta^b_{\text{cand,down}}(\sigma)\right)/\sigma$ need not be bounded as $\sigma \downarrow 0$. Accordingly, IEODS does not generally pin down a σ-equilibrium in this case. Nevertheless, for each fixed $\sigma > 0$ a Bayesian equilibrium exists by a standard fixed-point argument (applying Schauder's fixed-point theorem to the induced best-response map; we omit the functional-analytic details, as they are orthogonal to the economic mechanism).

Although the noisy equilibrium may be complex, its limiting behavior is tightly constrained. Claim 19 below shows that every interval of solvency below the fundamental default boundary must shrink to zero width as $\sigma \downarrow 0$, so that bank b defaults almost surely on $(-\infty, \widehat{\theta}^b(H^b))$ in the limit of any convergent sequence of equilibria. This almost-sure default property, combined with the necessary conditions on any candidate limiting equilibrium established in Claim 15, pins down the limiting equilibrium, which is exactly as characterized by Theorem 2.

Claim 19. *In the limit of any convergent sequence of noisy subgame equilibria as $\sigma \downarrow 0$, bank b defaults almost surely conditional on $\widetilde{\theta} < \widehat{\theta}^b(H^b)$.*

Proof. The proof proceeds in two steps. *Step 1. Every solvency interval below $\widehat{\theta}^b(H^b)$ shrinks to zero width.* Fix $\theta_0 < \widehat{\theta}^b(H^b)$ and suppose that along some sequence $\sigma_n \downarrow 0$ the corresponding noisy subgame equilibria satisfy that bank b is solvent at θ_0; that is, for all sufficiently large n,

$$\mathcal{V}_b^3(\theta_0, n) \equiv \theta_0\, G^b(A^b) - C^b\!\left(A^b, (\mathbb{O}_n^b(\theta_0) - \mathrm{I}_n^b(\theta_0) - H^b)^+\right) + (H^b + \mathrm{I}_n^b(\theta_0) - \mathbb{O}_n^b(\theta_0))^+ - R^b\!\left(L^b - \mathbb{O}_n^b(\theta_0)\right) - \mathrm{I}_n^b(\theta_0) \;>\; 0, \tag{61}$$

where $\mathbb{O}_n^b(\theta_0)$ and $\mathrm{I}_n^b(\theta_0)$ denote the equilibrium gross outflow from, and gross inflow to, bank b at state θ_0 under noise level σ_n.

For fixed $\sigma_n > 0$, the map $\theta \mapsto \mathbb{O}_n^b(\theta)$ is continuous (by dominated convergence: depositor strategies are bounded, and shifting θ shifts the signal distribution continuously), and likewise for I_n^b. Together with the continuity of $\mathcal{V}_b^3$ in θ, solvency at θ_0 implies solvency on an open neighborhood. Define

$$\delta_n \;\equiv\; \sup\!\left\{\delta > 0 : \mathcal{V}_b^3(\theta, n) > 0 \;\text{ for all } \theta \in (\theta_0 - \delta,\; \theta_0 + \delta)\right\}.$$

We claim that $\delta_n \to 0$. Suppose not: then $\limsup_n \delta_n > 0$, so along a subsequence (still indexed by n) there exists a constant $\delta > 0$ with $\delta_n \geq \delta$ for all n. Fix any depositor k and condition on $\widetilde{\theta} = \theta_0$. Since $\widetilde{x}_k = \theta_0 + \sigma_n \widetilde{\epsilon}_k$ and $\sigma_n \downarrow 0$,

$$\mathbb{P}\Big(|\widetilde{x}_k - \theta_0| \le \delta/2 \,\Big|\, \widetilde{\theta} = \theta_0\Big) \;\to\; 1.$$

For all sufficiently large n, whenever $|x_k - \theta_0| \le \delta/2$, depositor k's posterior places arbitrarily high probability on $\theta \in (\theta_0 - \delta,\ \theta_0 + \delta)$, throughout which bank b is strictly solvent. Because $R^b + \xi_k^b \ge 1 + \chi^i \xi_k^i$ (the no-non-run-reallocation assumption), staying with bank b strictly dominates every outside option for large n. Because this holds for every type k, the equilibrium outflow satisfies $\mathbb{O}_n^b(\theta_0) \to 0$.

Now evaluate the bank's payoff in this limit. Gross inflows $\mathbb{I}_n^b(\theta_0) \ge 0$ are bounded, so writing $I_0 \equiv \lim_n \mathbb{I}_n^b(\theta_0)$ (passing to a further subsequence if necessary),

$$\mathcal{V}_b^3(\theta_0, n) \;\longrightarrow\; \theta_0\, G^b(A^b) + (H^b + I_0) - R^b L^b - I_0 \;=\; \theta_0\, G^b(A^b) + H^b - R^b L^b \;<\; 0,$$

where the last inequality holds because $\theta_0 < \widehat{\theta}^b(H^b)$ and $\widehat{\theta}^b(H^b)\, G^b(A^b) + H^b - R^b L^b = 0$ by Definition 4. This contradicts (61), establishing $\delta_n \to 0$.

Step 2. The non-default set below $\widehat{\theta}^b(H^b)$ has Lebesgue measure zero in the limit. Along any convergent sequence of equilibria as $\sigma_n \downarrow 0$, fix n and consider the non-default set $\mathcal{S}_n \equiv \Big\{\theta < \widehat{\theta}^b(H^b) : \mathcal{V}_b^3(\theta, n) > 0\Big\}$. Because $\mathcal{V}_b^3(\cdot, n)$ is continuous in θ, $\mathcal{S}_n$ is open and hence is a countable union of disjoint open intervals. Step 1 shows that every such interval that contains a fixed point $\theta_0 < \widehat{\theta}^b(H^b)$ has length converging to 0. Because $\widetilde{\theta}$ admits a probability density function (Assumption 1), any Lebesgue-null set has $\mathbb{P}$-measure zero, so bank b defaults $\mathbb{P}$-almost surely on $\{\widetilde{\theta} < \widehat{\theta}^b(H^b)\}$. □

By extracting appropriate subsequences, we have shown that there exists a sequence of noisy subgame equilibria that converges ($\mathbb{P}$-almost surely in $\widetilde{\theta}$) to the noiseless equilibrium described in Theorem 2. By construction, $\underline{\theta}_D^b(H^b) \le \theta_B^b(H^b) < \theta_B^i \le \overline{\theta}_D^i$. Hence, by Claim 15, the baseline regional-to-national flight limiting equilibrium exists for all parameter values under our assumption that $\theta_B^b(H^b) < \theta_B^i$.

1.7.3 Reverse-Flight Limiting Equilibrium

When $\overline{\theta}_D^b(H^b) \ge \underline{\theta}_D^i$, the argument above applies with the roles of banks b and i interchanged, establishing the existence of a second limiting equilibrium in which deposits flow from the national bank to the regional bank.

When $\overline{\theta}_D^b(H^b) < \underline{\theta}_D^i$, however, no such reverse-flight equilibrium can arise. A candidate with run cutoff $\overline{\theta}_D^b(H^b)$ for bank b and $\underline{\theta}_D^i$ for bank i would require $\lim_{\sigma \downarrow 0}\Big[\theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma)\Big] > 0$ (so that the national bank is weakly more fragile than the regional bank), by the necessary conditions established in Claim 15. This is inconsistent with $\overline{\theta}_D^b(H^b) < \underline{\theta}_D^i$.

1.7.4 Knife-Edge Case: Restrictions and Existence

We study the knife-edge possibility that the two cutoff regions become asymptotically adjacent as $\sigma \downarrow 0$:

$$\lim_{\sigma\downarrow 0}\Big[\theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma)\Big] = 0. \tag{62}$$

Under the maintained ordering $\theta^i_B > \theta^b_B(H^b)$, we restrict attention to candidate equilibria for which $\theta^b_{\text{cand,down}}(\sigma) \geq \theta^i_{\text{cand,up}}(\sigma)$ for all sufficiently small σ. Indeed, if instead $\theta^b_{\text{cand,down}}(\sigma) < \theta^i_{\text{cand,up}}(\sigma)$, then Claim 9 would give $\mathbb{I}^i_{\text{cand}}(\vartheta,\sigma) = 0$ for all ϑ, so that $\theta^i_{\text{cand,down}}(\sigma) \geq \theta^i_B > \theta^b_B(H^b) \geq \theta^b_{\text{cand,down}}(\sigma)$, contradicting Eq. (62). Define the normalized gap

$$\Delta_\sigma \equiv \frac{\theta^b_{\text{cand,down}}(\sigma) - \theta^i_{\text{cand,up}}(\sigma)}{\sigma} \geq 0. \tag{63}$$

As in Claim 3 and 4, the within-bank gaps are $O(\sigma)$:

$$\limsup_{\sigma\downarrow 0} \frac{\theta^j_{\text{cand,up}}(\sigma) - \theta^j_{\text{cand,down}}(\sigma)}{\sigma} < \infty, \qquad j \in \{i, b\}.$$

Therefore, along any sequence satisfying (62), there exists a subsequence $\{\sigma_n\}_{n\geq 1}$ with $\sigma_n \downarrow 0$ such that

$$\theta^b_{\text{cand,down}}(\sigma_n) \to \theta^\star, \qquad \theta^i_{\text{cand,up}}(\sigma_n) \to \theta^\star, \qquad \Delta_{\sigma_n} \to \Delta^\star \in [0,\infty). \tag{64}$$

We abuse notation and write σ for σ_n below.

Assumption 5. For every depositor type ξ_w, the c.d.f. $F_{\widetilde{\epsilon}_w}$ is log-concave; equivalently, its reverse hazard rate $f_{\widetilde{\epsilon}_w}(z)/F_{\widetilde{\epsilon}_w}(z)$ is strictly decreasing in z.

Fix a depositor w (originally at bank b) of type ξ_w and a signal of the form $x_w = \theta^b_{\text{cand,down}}(\sigma) + z\sigma$ for fixed $z \in \mathbb{R}$. By Bayes' rule and the change of variables $\theta = x_w - \sigma\zeta$:

$$\begin{aligned}
\mathbb{P}\Big(\widetilde{\theta} \notin \mathcal{D}_b(\text{cand},\sigma) \;\Big|\; \widetilde{x}_w = x_w\Big) &= F_{\widetilde{\epsilon}_w}(z) + o(1),\\
\mathbb{P}\Big(\widetilde{\theta} \notin \mathcal{D}_i(\text{cand},\sigma) \;\Big|\; \widetilde{x}_w = x_w\Big) &= F_{\widetilde{\epsilon}_w}(z+\Delta_\sigma) + o(1),
\end{aligned} \tag{65}$$

where the second line uses $\theta^i_{\text{cand,up}}(\sigma) = \theta^b_{\text{cand,down}}(\sigma) - \Delta_\sigma\sigma$. The comparison between staying at b and moving $b \to i$ for depositor w reduces, at leading order, to

$$F_{\widetilde{\epsilon}_w}(z)\,(R^b + \xi^b_w - \Lambda) \;\gtrless\; F_{\widetilde{\epsilon}_w}(z+\Delta_\sigma)\,(1 + \chi^i\xi^i_w - \Lambda). \tag{66}$$

Under Assumption 5, the ratio $F_{\widetilde{\epsilon}_w}(z+\Delta)/F_{\widetilde{\epsilon}_w}(z)$ is strictly decreasing in z for every $\Delta > 0$, so Eq. (66) admits a unique cutoff $z_w^*(\Delta_\sigma;\xi_w)\in\mathbb{R}\cup\{\pm\infty\}$ such that

$$z\geq z_w^*(\Delta_\sigma;\xi_w)\quad\Longleftrightarrow\quad F_{\widetilde{\epsilon}_w}(z)\,(R^b+\xi_w^b-\Lambda)\geq F_{\widetilde{\epsilon}_w}(z+\Delta_\sigma)\,(1+\chi^i\xi_w^i-\Lambda).\tag{67}$$

Moreover, $z_w^*(\Delta;\xi_w)$ is increasing in Δ. In the knife-edge limit, depositors at bank i never move $i\to b$ (by the maintained payoff ordering), so $\mathrm{I}^b_{\text{cand}}=0$. Let $I^{i,*}(\Delta)$ denote the limiting inflow into bank i generated by depositors originally at b who choose $b\to i$, and let $O^{b,*}(\Delta)$ denote the limiting outflow from bank b at its cutoff. These take the form

$$\begin{aligned}I^{i,*}(\Delta)&\equiv\mathcal{M}^i\int_{\xi_w}\mathsf{s}_{w,b}\Big[F_{\widetilde{\epsilon}_w}\Big(z_w^*(\Delta;\xi_w)+\Delta\Big)-F_{\widetilde{\epsilon}_w}\Big(\underline{\mathsf{C}}_w^\epsilon\Big)\Big]^+dF_{\widetilde{\xi}}(\xi_w)\\&=\mathcal{M}^i\int_{\xi_w}\mathsf{s}_{w,b}\Big[F_{\widetilde{\epsilon}_w}\Big(z_w^*(\Delta;\xi_w)\Big)\frac{R^b+\xi_w^b-\Lambda}{1+\chi^i\xi_w^i-\Lambda}-\frac{1-\Lambda}{1+\chi^i\xi_w^i-\Lambda}\Big]dF_{\widetilde{\xi}}(\xi_w)\\O^{b,*}(\Delta)&\equiv\mathcal{M}^i\int_{\xi_w}\mathsf{s}_{w,b}\,F_{\widetilde{\epsilon}_w}\Big(\max\Big(z_w^*(\Delta;\xi_w),\,\mathsf{C}_w^\epsilon\Big)\Big)\,dF_{\widetilde{\xi}}(\xi_w).\end{aligned}\tag{68}$$

Clearly, if $z_w^*(\Delta;\xi_w)<\mathsf{C}_w^\epsilon$ then the knif-edge case will not exist. In this case, $O^{b,*}(\Delta)=O_B^b$ and $I^{i,*}(\Delta)=0$: the limiting cutoffs of the two banks will never converge. Define the implied cutoff fundamentals as functions of Δ:

$$\begin{aligned}\Theta^i(\Delta)&\equiv\frac{C^i\Big(A^i,(O_B^i-I^{i,*}(\Delta))^+\Big)-(I^{i,*}(\Delta)-O_B^i)^++R^i(L^i-O_B^i)}{G^i(A^i)},\\\Theta^b(\Delta)&\equiv\frac{C^b\Big(A^b,(O^{b,*}(\Delta)-H^b)^+\Big)-(H^b-O^{b,*}(\Delta))^++R^b(L^b-O^{b,*}(\Delta))}{G^b(A^b)}.\end{aligned}\tag{69}$$

Any knife-edge limit $(\theta^\star,\Delta^\star)$ must satisfy

$$\theta_B^i>\theta^\star=\Theta^i(\Delta^\star)=\Theta^b(\Delta^\star)>\theta_B^b(H^b).\tag{70}$$

Existence and uniqueness. Under the maintained ordering $\theta_B^i>\theta_B^b(H^b)$, we have $\Theta^i(0)=\theta_B^i$ and $\Theta^b(0)=\theta_B^b(H^b)$, hence $\Theta^i(0)>\Theta^b(0)$. If, in addition, $\Theta^i(\Delta)$ is weakly decreasing in Δ and $\Theta^b(\Delta)$ is weakly increasing in Δ (which holds when $I^{i,*}(\Delta)$ is increasing and $O^{b,*}(\Delta)$ is increasing, and the payoff primitives imply the corresponding monotonicity of the solvency maps), then (70) admits a solution $\Delta^\star\geq0$ if and only if

$$\lim_{\Delta\to\infty}\Theta^i(\Delta)\leq\lim_{\Delta\to\infty}\Theta^b(\Delta).\tag{71}$$

When a solution exists, it is unique by monotonicity. Condition (71) is equivalent to the parameter restriction under which the second limiting equilibrium in Theorem 2 exists. Therefore, the knife-edge case arises if and only if that second equilibrium is feasible, and the associated $(\theta^\star,\Delta^\star)$ is uniquely pinned down by (70).

1.8 Proof of Theorem 3

We characterize the limiting cutoffs. Because $\chi^i = 0$, Claim 9 implies that no depositor at bank b ever moves to a regional bank, so each regional bank's depositors face only the choice among staying, moving to bank b, or holding cash. This reduces the multi-bank problem to a sequence of two-bank sub-problems (each regional bank i versus bank b), coupled only through the inflows that bank b receives. In each sub-problem, the machinery of Internet Appendix 1.7 applies.

Fixed-point existence and uniqueness. Fix $\mathsf{u} \in \{0, \dots, \mathcal{N}\}$. For a given candidate θ, the first equation of Eq. (17) determines $I_C^b(\theta, \mathsf{u})$ directly (no inner fixed point is needed). Since $\mathbb{O}^i(\theta)$ is increasing in θ, $\theta \mapsto I_C^b(\theta, \mathsf{u})$ is increasing. Define $\Phi(\theta)$ to be the right-hand side of the θ_C^b equation in Eq. (17) with $I_C^b = I_C^b(\theta, \mathsf{u})$ substituted. In the fire-sale regime $\left(O_B^b - I_C^b(\theta, \mathsf{u}) - H^b > 0\right)$, the derivative of the numerator with respect to θ is $\left(-\partial_2 C^b(A^b, \cdot) + 1\right) dI_C^b/d\theta < 0$, since $\partial_2 C^b(A^b, 0) > 1$ and $dI_C^b/d\theta > 0$. In the no-fire-sale regime the numerator is constant and the max with $\widehat{\theta}^b(H^b)$ binds. Hence Φ is weakly decreasing overall. Since Φ is continuous and bounded while the identity map $\theta \mapsto \theta$ is unbounded, the intermediate value theorem delivers a fixed point. The fixed point is unique because Φ is weakly decreasing and the identity is strictly increasing.

Necessary conditions via induction. We show that in any robust limiting equilibrium, banks $\{1, \dots, \mathcal{N}^{run}\}$ contribute inflows to bank b and banks $\{\mathcal{N}^{run} + 1, \dots, \mathcal{N}\}$ do not.

Fix any sufficiently small $\sigma > 0$ and let $\mathbb{O}^j_{\text{cand}}(\theta, \sigma)$, $\mathbb{I}^j_{\text{cand}}(\theta, \sigma)$ denote the candidate equilibrium flows, and $\theta^j_{\text{cand,down}}(\sigma)$, $\theta^j_{\text{cand,up}}(\sigma)$ the solvency brackets for bank j.

Base case ($\mathsf{u} = \mathcal{N}^{cut}$). By definition of $\mathcal{N}^{cut}$, $\theta_B^\ell > \theta_B^b(H^b)$ for all $\ell \le \mathcal{N}^{cut}$. Because $\chi^i = 0$, each regional bank ℓ versus bank b is a self-contained two-bank sub-problem. A direct application of Corollary 1 to each pair (ℓ, b) establishes that, in any robust limiting equilibrium, depositors at bank ℓ who withdraw transfer to bank b (rather than cash) in the intermediate region $(\theta_D^b, \theta_D^\ell)$. Hence all banks $\{1, \dots, \mathcal{N}^{cut}\}$ contribute inflows, and the similar arguments to those of Claim 12 and 15 yield

$$\mathbb{I}^b_{\text{cand}}(\theta, \sigma) \ \ge\ I_C^b(H^b, \mathcal{N}^{cut}) - \mathcal{O}(\sigma), \qquad \theta^b_{\text{cand,up}}(\sigma) \ \le\ \theta_C^b(H^b, \mathcal{N}^{cut}) + \mathcal{O}(\sigma).$$

Note that $\theta_C^b(H^b, \mathcal{N}^{cut}) \le \theta_C^b(H^b, 0) = \theta_B^b(H^b)$, since additional inflows shift the map Φ weakly downward.

Inductive step. Suppose that at step $\mathsf{u} \ge \mathcal{N}^{cut} + 1$, banks $\{1, \dots, \mathsf{u} - 1\}$ contribute inflows to bank b, so that

$$\mathbb{I}^b_{\text{cand}}(\theta, \sigma) \ \ge\ I_C^b(H^b, \mathsf{u} - 1) - \mathcal{O}(\sigma), \qquad \theta^b_{\text{cand,up}}(\sigma) \ \le\ \theta_C^b(H^b, \mathsf{u} - 1) + \mathcal{O}(\sigma).$$

Now consider bank u. Because $\chi^i = 0$, no depositor at bank b moves to bank u, so bank u's problem against bank b is a self-contained two-bank sub-problem.

If $\theta_B^{\mathsf{u}} > \theta_C^b(H^b, \mathsf{u} - 1)$: bank u is more fragile than the (strengthened) national bank.

For σ small enough,

$$\theta^{\mathsf{u}}_{\text{cand,down}}(\sigma) \;\geq\; \theta^{\mathsf{u}}_B - \mathcal{O}(\sigma) \;>\; \theta^b_C(H^b, \mathsf{u}-1) + \mathcal{O}(\sigma) \;\geq\; \theta^b_{\text{cand,up}}(\sigma),$$

where the first inequality uses the fact that the safe-neighbor externality can only raise the regional bank's cutoff above its isolated value, and the strict middle inequality holds because $\theta^{\mathsf{u}}_B - \theta^b_C(H^b, \mathsf{u}-1)$ is bounded away from zero. This separation places us in the regime of Claim 15: depositors from bank u who withdraw transfer to bank b in the intermediate region, and the limiting cutoff satisfies $\lim_{\sigma\downarrow 0} \theta^{\mathsf{u}}_{\text{cand,down}}(\sigma) = \theta^{\mathsf{u}}_D$.

Adding bank u to the contributing set yields $I^b_C(\theta, \mathsf{u}) \geq I^b_C(\theta, \mathsf{u}-1)$ for all θ, which shifts Φ weakly downward. Because Φ is weakly decreasing and the identity is strictly increasing, a downward shift moves the unique intersection to the left: $\theta^b_C(H^b, \mathsf{u}) \leq \theta^b_C(H^b, \mathsf{u}-1)$. By a direct application of Corollary 1 to bank pair (u, b) establishes that the induction hypothesis is maintained: $\mathrm{I}^b_{\text{cand}}(\theta, \sigma) \geq I^b_C(H^b, \mathsf{u}) - \mathcal{O}(\sigma)$.

If instead $\theta^{\mathsf{u}}_B < \theta^b_C(H^b, \mathsf{u}-1)$ (strict inequality by the genericity assumption): bank u is safer than the strengthened national bank. At every state where bank u defaults in isolation ($\theta < \theta^{\mathsf{u}}_B$), bank b is also in default ($\theta < \theta^{\mathsf{u}}_B < \theta^b_C(H^b, \mathsf{u}-1)$). Hence the outside option for bank u's depositors is cash alone, and bank u's cutoff remains θ^{u}_B. Bank u does not contribute inflows. The induction terminates.

The process stops at the first index $\mathcal{N}^{run}$ such that $\theta^{\mathcal{N}^{run}+1}_B < \theta^b_C(H^b, \mathcal{N}^{run})$ (or at $\mathcal{N}$ if no such index exists). Banks $\{1, \ldots, \mathcal{N}^{run}\}$ contribute inflows and have connected-economy cutoffs θ^i_D; banks $\{\mathcal{N}^{run}+1, \ldots, \mathcal{N}\}$ do not contribute and retain isolated cutoffs θ^i_B. The national bank's limiting cutoff is $\theta^b_C(H^b, \mathcal{N}^{run})$.

Existence of $\sigma > 0$ equilibria and convergence. When $H^b + I^b_C(H^b, \mathcal{N}^{run}) < \widehat{H^b}$: the IEODS construction from Internet Appendix 1.7.2 extends to the multi-region setting. The monotonicity properties of Claim 17 carry through because the one-directional coupling (regional $\to$ national only) preserves the monotone inclusion of default sets at every IEODS round. The resulting σ-equilibria satisfy $\theta^i_{\text{cand,down}}(\sigma) - \theta^b_{\text{cand,up}}(\sigma) > 0$ for all sufficiently small σ and all contributing banks $i \leq \mathcal{N}^{run}$. Passing to $\sigma \downarrow 0$ using similar arguments of Claim 12 and 15 pins down the limiting cutoffs.

When $H^b + I^b_C(H^b, \mathcal{N}^{run}) \geq \widehat{H^b}$: IEODS may fail for bank b. For each fixed $\sigma > 0$, existence follows from Schauder's fixed-point theorem (identical to Case 2 of Internet Appendix 1.7.2). Convergence follows from Claim 19: bank b defaults $\mathbb{P}$-almost surely on $\{\widetilde{\theta} < \widehat{\theta}^b(H^b)\}$ in any limit of noisy equilibria. Each regional bank's sub-problem is resolved by the two-bank machinery, and the national bank's limiting cutoff is $\theta^b_C(H^b, \mathcal{N}^{run}) = \widehat{\theta}^b(H^b)$ (the max in Eq. (17) binds).

Equilibrium verification. At bank b's candidate cutoff $\theta^b_C(H^b, \mathcal{N}^{run})$:

(a) An incumbent depositor w at bank b of type ξ^b_w runs with limiting probability $(1 - \Lambda)/(R^b + \xi^b_w - \Lambda)$. Aggregating yields gross outflow O^b_B, exactly as in the isolated economy. This outflow is independent of the regional banks' fragility ordering.

(b) A depositor k from regional bank $i \leq \mathcal{N}^{run}$ who has successfully withdrawn (served with probability $\mathbb{O}^i(\theta_C^b)/L^i)$ compares depositing at b versus holding cash. At the cutoff, the Laplacian posterior property implies she deposits at b with limiting probability $\chi^b\xi_k^b/(1+\chi^b\xi_k^b-\Lambda)$. Aggregating over types and summing over $i\in\{1,\ldots,\mathcal{N}^{run}\}$ yields inflow I_C^b.

Substituting $O^b = O_B^b$ and $I^b = I_C^b$ into bank b's solvency condition (Eq. (2)) and setting $\mathcal{V}_b^3 = 0$ recovers the second equation of Eq. (17), verifying that the described strategy profile is a limiting equilibrium.

$\mathcal{N}^{run} \geq \mathcal{N}^{cut}$**.** This is immediate from the base case: every bank $\ell \leq \mathcal{N}^{cut}$ satisfies $\theta_B^\ell > \theta_B^b(H^b) = \theta_C^b(H^b,0) \geq \theta_C^b(H^b,\ell-1)$ (adding contributing banks weakly lowers θ_C^b), so each passes the induction criterion.

Monotonicity of $\mathcal{N}^{run}$ in H^b. For each fixed u and fixed (L^j,A^j,R^j), $\theta_C^b(H^b,\mathsf{u})$ is weakly decreasing in H^b: a larger liquidity buffer relaxes the $(O_B^b - I_C^b - H^b)^+$ term in Eq. (17).[41] Hence the induction criterion $\theta_B^{\mathsf{u}} > \theta_C^b(H^b,\mathsf{u}-1)$ becomes weakly easier to satisfy as H^b increases, so $\mathcal{N}^{run}$ weakly increases with H^b. □

1.9 Proof of Theorem 4

Write $K \equiv E^b + (1-\varphi^b)L^b$ and $A^b(\Gamma) \equiv (1-\Gamma)K$, so $H^b = \Gamma K$. In both economies, below the respective run cutoff of bank j, all depositors run: $O^j = L^j$ and $I^j = 0$. The funding shortfall conditional on full withdrawal is

$$\ell^b(\theta,\Gamma) \;=\; \Big[C^b(A^b(\Gamma),\, L^b - \Gamma K) - \theta\, G^b(A^b(\Gamma))\Big]^+.$$

Define the expected losses

$$L_B(\Gamma) \;\equiv\; \mathbb{E}[\mathscr{L}_B^b] \;=\; \int_{-\infty}^{\theta_B^b(\Gamma)} \ell^b(\theta,\Gamma)\, f(\theta)\, d\theta, \qquad L_D(\Gamma) \;\equiv\; \mathbb{E}[\mathscr{L}_D^b] \;=\; \int_{-\infty}^{\underline{\theta}_D^b(\Gamma)} \ell^b(\theta,\Gamma)\, f(\theta)\, d\theta.$$

Since $\underline{\theta}_D^b(\Gamma) \leq \theta_B^b(\Gamma)$ by Corollary 1 and $\ell^b \geq 0$:

$$0 \;<\; L_D(\Gamma) \;\leq\; L_B(\Gamma) \qquad \forall\, \Gamma. \tag{72}$$

In the isolated economy, θ_B^i is independent of Γ, so $\mathcal{W}_B^i$ is constant. In the connected economy, by Theorem 3, $\theta_D^i(\Gamma) = \overline{\theta}_D^i$ when $\theta_B^b(\Gamma) < \theta_B^i$ and $\theta_D^i(\Gamma) = \theta_B^i$ otherwise; both values are independent of Γ. Reducing Γ from Γ_B^* can only trigger the regime switch to $\theta_D^i = \theta_B^i < \overline{\theta}_D^i$, weakly reducing $\mathbb{E}[\mathscr{L}_D^i]$ and hence $\mathcal{W}_D^i$. It therefore suffices to show $\Gamma_D^* < \Gamma_B^*$ using only $\mathcal{W}_\mathcal{E}^b(\Gamma) = \mathscr{B}^b(\Gamma) + \mathscr{C}^b(L_\mathcal{E}(\Gamma))$. At the unique minimizer Γ_B^*:

$$(\mathcal{W}_B^b)'(\Gamma_B^*) \;=\; (\mathscr{B}^b)'(\Gamma_B^*) + (\mathscr{C}^b)'(L_B)\, L_B' \;=\; 0. \tag{73}$$

[41] When A^b is held fixed, increasing H^b unambiguously reduces θ_C^b. When $A^b = L^b(1-\varphi^b) - H^b$ varies with H^b, the claim holds provided the direct liquidity effect dominates the productivity loss, which is the case in the empirically relevant regime with small default probabilities.

Since $(\mathscr{B}^b)' > 0$ and $(\mathscr{C}^b)' > 0$, the FOC requires $L_B'(\Gamma_B^*) < 0$, giving

$$(\mathscr{B}^b)'(\Gamma_B^*) \;=\; (\mathscr{C}^b)'(L_B)\,|L_B'|. \tag{74}$$

Evaluating at Γ_B^* and substituting Eq. (74):

$$(\mathcal{W}_D^b)'(\Gamma_B^*) = (\mathscr{B}^b)'(\Gamma_B^*) + (\mathscr{C}^b)'(L_D)\,L_D' = (\mathscr{C}^b)'(L_D)\,L_D'(\Gamma_B^*) - (\mathscr{C}^b)'(L_B)\,L_B'(\Gamma_B^*). \tag{75}$$

Because $L_B' < 0$, this is strictly positive if $L_D'(\Gamma_B^*) \geq 0$ (this is likely the case when bank b is in fundamental default region) or if

$$\frac{(\mathscr{C}^b)'(L_B)}{(\mathscr{C}^b)'(L_D)} \;>\; \frac{|L_D'|}{|L_B'|}. \tag{76}$$

The right-hand side is a fixed positive constant determined by the bank's balance sheet, the fire-sale technology C^b, and the fundamental distribution $f_{\widetilde{\theta}}$. LHS ≥ 1, because $L_B \geq L_D$ by Eq. (72) and $(\mathscr{C}^b)'$ is non-decreasing (convexity of $\mathscr{C}^b$). LHS $\to \infty$ as the curvature $(\mathscr{C}^b)''$ increases, because $L_B > L_D$ implies that the derivative ratio grows without bound. Therefore, for $\mathscr{C}^b$ sufficiently convex, Eq. (76) holds and $(\mathcal{W}_D^b)'(\Gamma_B^*) > 0$. $\mathcal{W}_D^b$ is convex with unique minimizer Γ_D^*, so it is decreasing on $(-\infty, \Gamma_D^*)$ and increasing on (Γ_D^*, ∞). Since $(\mathcal{W}_D^b)'(\Gamma_B^*) > 0$, we have $\Gamma_B^* > \Gamma_D^*$. □

1.10 Proof of Theorem 5

We prove the result for regional bank i in the **Alternative Economy of Isolated Bank Run**; the argument for bank b and for the connected economy is identical after replacing i by b and adjusting for the liquidity constraint.

By Lemma 3, when $\theta > \theta_B^i(H^i)$ no depositor runs and deposit flows are zero. Furthermore, when $\theta \leq \theta_B^i$, bank i always defaults. Bank i's expected equity therefore reduces to

$$V(H^i) = \int_{\theta_B^i(H^i)}^{\infty} \Big[\theta\, G^i(A^i) + H^i - R^i L^i\Big]\, f_{\widetilde{\theta}}(\theta)\, d\theta, \qquad A^i = E^i + (1-\varphi^i)L^i - H^i.$$

Differentiating via Leibniz's rule (noting that the integrand is continuously differentiable in H^i for $\theta > \theta_B^i$):

$$V'(H^i) = \underbrace{\int_{\theta_B^i}^{\infty} \Big[1 - \theta\,(G^i)'(A^i)\Big]\, f_{\widetilde{\theta}}(\theta)\, d\theta}_{\text{Term A}} \;-\; \underbrace{\left(\frac{d\theta_B^i}{dH^i}\right)\Big[\theta_B^i G^i(A^i) + H^i - R^i L^i\Big]\, f_{\widetilde{\theta}}(\theta_B^i)}_{\text{Term B}}. \tag{77}$$

Term A is strictly negative. If $1 - \theta_B^i\,(G^i)'(A^i) < 0$, then Term A is clearly strictly negative. If $1 - \theta_B^i\,(G^i)'(A^i) > 0$, then $1 - \theta (G^i)'$ is strictly decreasing in θ and positive for all $\theta < \theta_B^i$, so

$$\int_{\theta_B^i}^{\infty}\Big[1-\theta(G^i)'\Big]f\,d\theta \;<\; \int_{-\infty}^{\infty}\Big[1-\theta(G^i)'\Big]f\,d\theta \;=\; 1-(G^i)'\,\mathbb{E}[\widetilde{\theta}] \;\le\; 0.$$

It is never optimal to choose $H^i \geq O_B^i$. When $H^i > O_B^i$, θ_B^i equals to the fundamental default boundary by Lemma 3. Hence Term B is zero. It follows that $V'(H^i) < 0$, and H^i is not optimal. Next, we consider the case when $H^i < O_B^i$.

Term B may be non-negative. The coordination premium at the cutoff,

$$\Pi_0 \equiv \theta_B^i G^i(A^i) + H^i - R^i L^i = C^i(A^i, O_B^i - H^i) + H^i - R^i O_B^i,$$

is strictly positive when $H^i \in (0, \widehat{H^i})$ (because $C^i(A^i, O_B^i - H^i) > R^i(O_B^i - H^i)$ by convexity and $\partial_2 C^i > R^i$). When $d\theta_B^i/dH^i < 0$ and $f_{\widetilde{\theta}}(\theta_B^i) > 0$, Term B is strictly positive. One sufficient condition for $V' < 0$ is that

$$\Pi_0 \cdot \left|\frac{d\theta_B^i}{dH^i}\right| \cdot f_{\widetilde{\theta}}(\theta_B^i) \;<\; \mathbb{P}(\widetilde{\theta} > \theta_B^i) \cdot \Big|\mathbb{E}\Big[\widetilde{\theta}(G^i)' - 1 \,\Big|\, \widetilde{\theta} > \theta_B^i\Big]\Big|.$$

This ensures |Term A| > |Term B| and hence $V'(H^i) < 0$ for all $H^i \in (0, O_B^i]$. The condition is satisfied whenever θ_B^i lies sufficiently deep in the left tail of $f_{\widetilde{\theta}}$: Π_0 and $|d\theta_B^i/dH^i|$ are bounded, $f(\theta_B^i) \to 0$, while $\mathbb{P}(\theta > \theta_B^i) \to 1$ and $\mathbb{E}[\theta(G^i)' - 1 \mid \theta > \theta_B^i]$ approaches the strictly positive constant $(G^i)'\mathbb{E}[\widetilde{\theta}] - 1$. Therefore, bank i's equity-maximizing choice is $H^i = 0$. For bank b, the identical argument shows that the equity holders prefer the smallest feasible H^b; the regulatory constraint $H^b \geq \Gamma(E^b + (1-\varphi^b)L^b)$ then binds, yielding $H^{b^*} = \Gamma\, E^b + \Gamma(1-\varphi^b)L^b(R^b)$. □

1.11 Proof of Theorem 6

We prove both parts of the theorem. Let $\mho^j_{\text{cand}}(\theta, \chi^i, \chi^b, \sigma, \varsigma)$ and $\mathrm{I}^j_{\text{cand}}(\theta, \chi^i, \chi^b, \sigma, \varsigma)$ be the equilibrium deposit outflow and inflow functions for bank j in any candidate equilibrium $\mathscr{E}(\sigma, \varsigma)$, where θ is the realized fundamental and χ^j is the realized mobility parameter for bank-j depositors. The default sets are

$$\mathcal{D}_{\text{cand},b}(\chi^i, \chi^b, \sigma, \varsigma) = \Big\{\theta \;\Big|\; \theta G^b(A^b) - C^b\Big(A^b, (\mho^b_{\text{cand}} - \mathrm{I}^b_{\text{cand}} - H^b)^+\Big) + (H^b + \mathrm{I}^b_{\text{cand}} - \mho^b_{\text{cand}})^+ - \mathrm{I}^b_{\text{cand}} - R^b(L^b - \mho^b_{\text{cand}}) \leq 0\Big\},$$

$$\mathcal{D}_{\text{cand},i}(\chi^i, \chi^b, \sigma, \varsigma) = \Big\{\theta \;\Big|\; \theta G^i(A^i) - C^i\Big(A^i, (\mho^i_{\text{cand}} - \mathrm{I}^i_{\text{cand}})^+\Big) + (\mathrm{I}^i_{\text{cand}} - \mho^i_{\text{cand}})^+ - \mathrm{I}^i_{\text{cand}} - R^i(L^i - \mho^i_{\text{cand}}) \leq 0\Big\},$$

where all flow functions are evaluated at $(\theta, \chi^i, \chi^b, \sigma, \varsigma)$. Define

$$\theta^j_{\text{cand,down}}(\chi^i, \chi^b, \sigma, \varsigma) = \inf\Big(\mathcal{D}_{\text{cand},j}(\chi^i, \chi^b, \sigma, \varsigma)\Big)^c, \qquad \theta^j_{\text{cand,up}}(\chi^i, \chi^b, \sigma, \varsigma) \;= \sup\, \mathcal{D}_{\text{cand},j}(\chi^i, \chi^b, \sigma, \varsigma).$$

Then $(-\infty, \theta^j_{\text{cand,down}}) \subset \mathcal{D}_{\text{cand},j} \subset (-\infty, \theta^j_{\text{cand,up}})$.

Part (i): The reverse-flight equilibrium is not robust. We exhibit a family of distributions $\{F_{\widetilde{\chi}|\chi^j,\varsigma}\}$ satisfying conditions (a) and (b) of the setup such that no sequence of (σ, ς)-elaboration equilibria converges to the reverse-flight equilibrium. We choose the (truncated) uniform distribution: $\widetilde{\chi}^i_{w'}(\varsigma) \mid \chi^i \sim U[\chi^i - \varsigma,\ \chi^i]$ restricted to $[0,1]$. In the reverse-flight equilibrium, no depositor at bank i moves to bank b, so the distribution of $\widetilde{\chi}^b_{w'}(\varsigma)$ does not affect the analysis.

Step 1: Dominance region.

Claim 20. *There exists $x_0 > 0$ (independent of ς) such that for all $\chi^i \in [0, x_0)$, the unique inner limiting equilibrium as $\sigma \downarrow 0$ of the ς-elaboration game is the baseline: bank i is strictly riskier than bank b, and no depositor at bank b moves to bank i.*

Proof. Fix small enough ς and consider χ^i near zero. By a similar argument to Claim 11, for all small enough σ, the maximum possible inflow to bank i at its run cutoff $\left(\theta^i_{\text{cand,down}}(\chi^i, \chi^b, \sigma, \varsigma)\right)$ is bounded above by

$$\mathcal{M}^i \int_{\xi_w} \frac{\mathsf{s}_{w,b}\, \chi^i\, \xi^i_w}{1 + \chi^i \xi^i_w - \Lambda}\, dF_{\widetilde{\xi}}(\xi_w) \;+\; \mathcal{O}(\sigma).$$

The factor χ^i arises because depositor w's belief about other depositors' mobility parameter has support contained in $[\chi^i - \varsigma,\ \chi^i] \subseteq [0,\ \chi^i]$, and the equilibrium inflow is bounded by the upper envelope evaluated at the highest possible realization.[42] Similarly, the additional outflow from bank b at its run cutoff $\left(\theta^b_{\text{cand,up}}(\chi^i, \chi^b, \sigma, \varsigma)\right)$ beyond the isolated-economy level O^b_B is bounded above by

$$\mathfrak{O}^b_{\text{cand}}(\theta, \chi^i, \chi^b, \sigma, \varsigma) - O^b_B \;\le\; \mathcal{M}^i \int_{\xi_w} \frac{\mathsf{s}_{w,b}\, \chi^i\, \xi^i_w}{R^b + \xi^b_w - \Lambda}\, dF_{\widetilde{\xi}}(\xi_w) \;+\; \mathcal{O}(\sigma).$$

Taking $\sigma \downarrow 0$, the run cutoffs satisfy

$$\left|\theta^i_{\text{cand,down}}(\chi^i, \chi^b, \sigma, \varsigma) - \theta^i_B\right| \le \mathcal{O}(\chi^i), \qquad \left|\theta^b_{\text{cand,up}}(\chi^i, \chi^b, \sigma, \varsigma) - \theta^b_B\right| \le \mathcal{O}(\chi^i).$$

Because $\theta^i_B > \theta^b_B$, there exist $x_0 > 0$ and $\bar{\varsigma} > 0$ such that for all $\chi^i < x_0$ and $\varsigma < \bar{\varsigma}$,

$$\mathcal{D}_{\text{cand},b}(\chi^i, \chi^b, \sigma, \varsigma) \;\subsetneq\; \mathcal{D}_{\text{cand},i}(\chi^i, \chi^b, \sigma, \varsigma)$$

almost surely. That is, bank i is strictly riskier than bank b. Since this is common knowledge among all depositors (every depositor w at bank b knows that $\chi^i < x_0$ and can compute the same bounds), no depositor moves to bank i, and the inflow to bank i is zero. The existence condition for the reverse-flight equilibrium ($\overline{\theta}^b_D \ge \underline{\theta}^i_D$) fails. Only the baseline equilibrium exists. □

Step 2: Infection argument. Suppose, toward a contradiction, that for some $\chi^i_0 > 0$ and some $\sigma, \varsigma > 0$, the ς-elaboration game admits a candidate equilibrium in which the reverse-flight

[42] Each depositor's own service utility from moving to bank i is $\chi^i \xi^i_w$, but her willingness to move also depends on her belief about whether others move (which determines bank i's solvency). The bound replaces this belief-dependent term with its upper envelope, evaluated at the highest possible $\widetilde{\chi}^i_{w'}$ in the support.

ordering holds:

$$\theta^b_{\text{cand,down}}(\chi^i_0, \chi^b, \sigma, \varsigma) \;\geq\; \theta^i_{\text{cand,up}}(\chi^i_0, \chi^b, \sigma, \varsigma).$$

Define, for this candidate equilibrium,

$$\underline{\chi}(\sigma, \varsigma) \;\equiv\; \inf\Big\{x \in [0,1] : \; \theta^b_{\text{cand,down}}(x, \chi^b, \sigma, \varsigma) \;\geq\; \theta^i_{\text{cand,up}}(x, \chi^b, \sigma, \varsigma)\Big\}.$$

By Step 1, $\underline{\chi}(\sigma, \varsigma) \geq x_0 > 0$. The set is nonempty (it contains χ^i_0), so the infimum is well-defined and $\underline{\chi} \leq \chi^i_0$.

Now consider the equilibrium at parameter $\chi^i = \underline{\chi}$. Any depositor w at bank b who observes $\chi^i_w = \underline{\chi}$ believes that each other depositor w' has parameter $\widetilde{\chi}^i_{w'} \sim U[\underline{\chi} - \varsigma,\; \underline{\chi}]$, so $\widetilde{\chi}^i_{w'} < \underline{\chi}$ almost surely. By definition of $\underline{\chi}$, for every $x < \underline{\chi}$ the candidate equilibrium satisfies

$$\theta^b_{\text{cand,down}}(x, \chi^b, \sigma, \varsigma) \;<\; \theta^i_{\text{cand,up}}(x, \chi^b, \sigma, \varsigma),$$

i.e., bank i is strictly riskier than bank b at parameter x. Since $\mathcal{D}_{\text{cand},b}(x) \subsetneq \mathcal{D}_{\text{cand},i}(x)$, no depositor of type x moves from bank b to bank i (moving to a strictly riskier bank is dominated by staying or holding cash). Because this holds for almost every realization of $\widetilde{\chi}^i_{w'}$, the aggregate inflow to bank i from bank b depositors is zero.

Without inflows, bank i's default boundary reverts to its isolated-economy level and bank b may only receive inflows (from bank i depositors fleeing to b when $\chi^b > 0$), so by the argument of Internet Appendix 1.7.1:

$$\theta^i_{\text{cand,up}}(\underline{\chi}, \chi^b, \sigma, \varsigma) \;\geq\; \theta^i_B \;>\; \theta^b_B \;\geq\; \theta^b_{\text{cand,down}}(\underline{\chi}, \chi^b, \sigma, \varsigma).$$

This establishes $\theta^b_{\text{cand,down}}(\underline{\chi}, \ldots) < \theta^i_{\text{cand,up}}(\underline{\chi}, \ldots)$. By definition of the infimum, for any $\epsilon > 0$ small enough, $\underline{\chi} + \epsilon\varsigma$ belongs to the set $\{x : \theta^b_{\text{cand,down}}(x, \chi^b, \sigma, \varsigma) \geq \theta^i_{\text{cand,up}}(x, \chi^b, \sigma, \varsigma)\}$. However, at parameter $\underline{\chi} + \epsilon\varsigma$, depositor w believes other depositors' parameters lie in $[\underline{\chi} + \epsilon\varsigma - \varsigma,\; \underline{\chi} + \epsilon\varsigma]$. Of these, all but a fraction ϵ have parameters below $\underline{\chi}$ and therefore do not move to bank i. The aggregate inflow to bank i is thus bounded by $\mathcal{O}(\epsilon)$, and by continuity of the solvency conditions in the inflow,

$$\theta^i_{\text{cand,up}}(\underline{\chi} + \epsilon\varsigma, \chi^b, \sigma, \varsigma) \;\geq\; \theta^i_B - \mathcal{O}(\epsilon) \;>\; \theta^b_B + \mathcal{O}(\epsilon) \;\geq\; \theta^b_{\text{cand,down}}(\underline{\chi} + \epsilon\varsigma, \chi^b, \sigma, \varsigma)$$

contradicting $\underline{\chi} + \epsilon$ being in the set.

Since the contradiction holds for every $\sigma, \varsigma > 0$, no equilibrium of the ς-elaboration game exhibits the reverse-flight ordering at any χ^i. In particular, no sequence of elaboration equilibria converges to the reverse-flight equilibrium as $\varsigma \downarrow 0$ and $\sigma \downarrow 0$. The reverse-flight equilibrium is not robust. □

Part (ii): The baseline equilibrium is robust. No depositor at bank b moves to bank i in the baseline equilibrium, so the equilibrium cutoffs do not depend on χ^i; perturbations of χ^i are therefore irrelevant.

For perturbations of χ^b: the IEODS construction of Internet Appendix 1.7.2 applies to the elaboration game. The Step-0 bounds and the ordering $\theta_B^i > \theta_B^b$ that initiate the iteration are independent of χ^b. By the law of large numbers, the aggregate deposit flows at each IEODS round are deterministic functions of the distribution $F_{\widetilde{\chi}|\chi^b,\varsigma}$ and the round-ℓ signal sets. The monotonicity properties of Claim 17—in particular, the strict separation $\mathfrak{D}_b(\ell) \subsetneqq \mathfrak{D}_i(\ell)$ and the emptiness of the transfer set $\mathfrak{M}^i(w,\ell,\sigma)$—carry through at every round, because they rely on the no-non-run-reallocation assumption $R^b + \xi_w^b > 1 + \chi^i \xi_w^i$ (which does not involve χ^b) and the ordering of default sets (which is preserved by the IEODS monotonicity).

Let $\mathfrak{O}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,\varsigma)$ and $\mathfrak{I}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,\varsigma)$ denote the aggregate flows at the IEODS limits. Under condition (b) ($\widetilde{\chi}^b_{w'}(\varsigma) \in [0,1]$ a.s.) and the convergence in probability $\widetilde{\chi}^b_{w'}(\varsigma) \to \chi^b$, dominated convergence ensures

$$\mathfrak{O}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,\varsigma) \xrightarrow{\varsigma\downarrow 0} \mathfrak{O}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,0), \qquad \mathfrak{I}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,\varsigma) \xrightarrow{\varsigma\downarrow 0} \mathfrak{I}^j_{\text{cand}}(\theta,\chi^i,\chi^b,\sigma,0).$$

Taking $\sigma \downarrow 0$ and applying the limiting characterization of Theorem 2 establishes that the elaboration equilibria converge to the baseline equilibrium, confirming robustness. □