Bonds vs. Equities: Information for Investment*

Huifeng Chang[†], Adrien d'Avernas[‡], Andrea L. Eisfeldt[§]

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Abstract

Why do credit spreads explain firm investment better than equity volatility does? In a standard corporate finance setting, this can be explained as a consequence of credit spreads and *asset* volatility having unambiguous relationships with investment, while *equity* volatility sends a mixed signal: Elevated volatility raises the option value of equity and increases investment for financially sound firms, but it exacerbates debt overhang and decreases investment for firms close to default. Overall, our study clarifies the structural and empirical relationships between investment, leverage, credit spreads, volatility, and Tobin's q.

Keywords: Credit Spreads, Uncertainty, Investment, Equity Volatility, Leverage, Debt Overhang

JEL Classifications: E22, E32, G31

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[†]Fudan University, School of Management, email: huifengchang@fudan.edu.cn

[‡]Stockholm School of Economics, email: adrien.davernas@hhs.se

[§]UCLA Anderson School of Management and NBER, email: andrea.eisfeldt@anderson.ucla.edu

1 Introduction

Economists and practicioners alike have long argued that the macroeconomy has a tighter connection with bond markets than with equity markets. Bond market measures—such as credit spreads—have substantial predictive power for aggretate investment, employment and output.[1] And while a large literature emphasizes the link between equity market measures—such as equity volatility—and investment,[2] credit spreads outperform these equity market measures in predicting economic activity.[3]

So why do bond market measures predict economic activity better than equity market measures? Using straightforward empirics and a parsimonious model, we argue that while credit spreads have an unambiguous (negative) relationship with firm-level investment, equity volatility is a mixed signal of positive option value and negative debt overhang. However, once we control for the debt overhang problem with credit spreads, *asset* volatility is an unambiguously *positive* signal for investment.

We establish four main empirical facts. First, as documented by Gilchrist, Sim, and Zakrajšek (2014), credit spreads drive out equity volatility in an empirical model of the sensitivity of firm-level investment to equity volatility and credit spreads. However, we show that this result is due to systematic heterogeneity in the elasticity of investment to equity volatility in the cross-section of firms. The elasticity of investment to equity volatility is positive for firms far enough away from default, and negative otherwise. These systematically different signs in the cross section wash out in pooled data and confound aggregate inference. By contrast, the elasticity of investment to credit spreads is always negative. Our model is consistent with the interpretation in Gilchrist, Sim, and Zakrajšek (2014) that financial frictions are important for understanding the equity and bond market information. Our model and empirical evidence support misalignment of debt and equityholders' interests as the key friction that drives our results.

[1] Friedman and Kuttner (1992) show that the spread between commercial paper and Treasury bills forecasts recessions. Gilchrist and Zakrajšek (2012) use firm-level data to construct a credit spread measure with substantial predictive power for aggregate investment, employment and output. See also the important contributions of Friedman and Kuttner (1998); Stock and Watson (1989); Bernanke (1990), Gertler and Lown (1999); Gilchrist, Yankov, and Zakrajšek (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2014); Krishnamurthy and Muir (2017).

[2] Bloom (2009) uses implied equity volatility to proxy for uncertainty and shows that shocks to uncertainty forecast lower investment. See Panousi and Papanikolaou (2012) for related evidence that the negative relation between idiosyncratic equity volatility and investment is stronger when managerial ownership is higher.

[3] Gilchrist, Sim, and Zakrajšek (2014) show that controlling for credit spreads substantially reduces the predictive power of equity volatility for investment. Philippon (2009) shows that at the aggregate level credit spreads outperform Tobin's q in predicting investment. See also Christiano, Motto, and Rostagno (2014) and Arellano, Bai, and Kehoe (2019).

Second, we provide empirical evidence against the hypothesis that bond markets predict investment better because they have more smart money. To do this, we repeat the above analysis using credit spreads constructed using equity market data, leverage ratios, and historical default rates as inputs into a structural model.[4] These fair-value credit spreads are constructed without any bond market data and thus cannot be driven by bond market investors. Empirical results using this equity-market measure of bond spreads are virtually identical to those using bond-market spreads.

Third, both equity volatility and credit spreads are in large part driven by asset volatility and leverage, as predicted by structural models of credit risk.[5] More of the variation in credit spreads is explained by leverage while more of the variation in equity volatility is explained by asset volatility, especially for more financially sound firms. And, credit spreads have higher loadings on leverage, while equity volatility loads more on asset volatility. This is intuitive, given the priority of debt versus equity in firms' capital structures and, together with our next fact, the result whereby equity volatility mainly reflects asset volatility is helpful for understanding why equity volatility might positively impact investment decisions for more financially sound firms.

The fourth main result is that the sensitivity of investment to *asset* volatility is positive for all firms. This may seem surprising, given the large literature on the negative effects of uncertainty on investment.[6] Our analysis differs in two ways from that literature. First, we emphasize the importance of considering unlevered asset volatility vs. equity volatility. Second, we examine the relation between investment rates and the level of asset volatility; in contrast, the uncertainty literature focuses on changes in volatility. While we show that the level of *asset* volatility is an unambiguously *positive* signal for investment, our results are not necessarily a challenge to the wait-and-see mechanism of Bloom (2009) or Alfaro, Bloom, and Lin (2018). Those papers study the effect of a change in volatility and typically use equity volatility. In the data, it appears that firms' sensitivity to the level of asset volatility is positive, while changes in volatility can have a temporary negative effect. Our study shows

[4]See Arora, Bohn, and Zhu (2005) and Nazeran and Dwyer (2015).

[5]Building on the seminal work of Merton (1974) and Leland (1994), Atkeson, Eisfeldt, and Weill (2017) show theoretically that under very minimal assumptions, the inverse of equity volatility is bounded above by distance to insolvency and below by distance to default. Empirically, they document a tight log-linear relationship between the inverse of equity volatility and credit spreads. See also Campbell and Taksler (2003), who show that idiosyncratic equity volatility explains as much of the cross-sectional variation in bond yields as credit spreads do. Our empirical work also addresses the role of the fundamental part of credit spreads in driving our results, as opposed to the non-fundamental part emphasized by Collin-Dufresn et al. (2001) and Gilchrist and Zakrajšek (2012).

[6]Our result that asset volatility has a positive relation to investment is consistent with the "Oi-Hartman-Abel" effect (Oi, 1961; Hartman, 1972; Abel, 1983) in which firms can expand to take advantage of positive shocks and shrink to avoid negative ones, making them risk-loving. However, our results for the relationship between equity volatility and investment suggest that leverage is at least one key driver of option value.

why it is crucial to distinguish between equity and asset volatility, and between levels and changes in studies of the relation between measures of uncertainty and investment.

At least two interpretations of the novel result whereby asset volatility is robustly positively related to investment are possible. First, as in our model, asset volatility can boost the option value of equity, alleviate the debt overhang effect, and incentivize equityholders to invest more (a causal channel). Alternatively, the uncertainty from future investment could feed back into the volatility of current asset values (an endogeneity channel). We show using lags and leads of asset volatility, subsamples of R&D-intensive and high-investment-rate firms, and instrumental variables, that the first explanation is more likely.

We build a simple model of investment to provide intuition for our empirical results. The model features a firm with a given level of asset volatility and capital structure (the level of debt) in place.[7] At date zero, equityholders choose the level of investment. At date one, equityholders observe productivity and output and choose whether or not to default. We assume a separation of debt and equity holders. Equityholders only maximize the value of their claim when making decisions. The first-order conditions for investment and the threshold for productivity below which equityholders choose to default, along with the given asset volatility and debt level, pin down credit spreads, leverage, equity volatility, and Tobin's q.

To understand our firm-level empirical results, we perform comparative statics holding different observable firm-level variables constant in the model. The main results are as follows: Only credit spreads and asset volatility are clean signals for investment. Credit spreads capture debt overhang, while asset volatility measures option value. As a result, holding asset volatility constant, the elasticity of investment with respect to *credit spreads* is always *negative* (due to debt overhang). Holding credit spreads constant, the elasticity of investment with respect to *asset volatility* is always *positive* (due to option value). Equity volatility is an ambiguous signal, because it is a compound signal of the negative effects of debt overhang and the positive effects of option value. We use a numerical example to show that our model can generate the change in sign observed empirically across firms with different levels of financial soundness. With respect to leverage, we show that holding asset volatility constant, the elasticity of investment is always negative. However, controlling for leverage rather than credit spreads, the elasticity of investment with respect to asset volatility has an ambiguous sign. The reason is that if asset volatility increases while holding leverage constant, there are two effects. First, option value increases. But second, debt overhang also increases because when holding leverage constant at the higher volatility, the default

[7]We provide a fully dynamic version with endogenous debt and equity based on DeMarzo and He (2020) in the Online Appendix.

threshold for productivity increases and distance to default shrinks. Distance to default could shrink faster or more slowly than option value increases, and these two effects compete for the overall effect of a change in volatility holding leverage constant.

We also extend the intuition in Philippon (2009) that links structural measures of credit risk to Tobin's q. We show that credit spreads do indeed capture the loss in equityholders' marginal return from investment from debt overhang.[8] However, due to bondholders' position in the capital structure, another important driver of Tobin's q is missing from credit spreads. When debt and equityholders' are not perfectly aligned, credit spreads do not capture the option value of higher asset volatility, while Tobin's q does. The baseline model in Philippon (2009) cannot be used to understand our findings that the sensitivity of investment to equity volatility changes sign in the cross-section because in that model leverage does not affect firm value or investment (i.e. the Modigliani and Miller's (1958) theorem holds).[9] In contrast to the aggregate results in Philippon (2009), in firm-level data, although asset volatility and credit spreads add additional information for investment, they do not drive out traditional Tobin's q.

Our study yields some important suggestions for future empirical work on firm-level investment and some open avenues for research on aggregate investment. First, researchers should use asset volatility rather than equity volatility to measure the effects of option values and/or uncertainty.[10] The effect of the level of asset volatility on investment is unambiguously positive, while the effect of a change in volatility can be negative. Second, controlling for leverage is not as clean as controlling for credit spreads. Only credit spreads hold distance to default and the effect of financial frictions such as debt overhang constant.

[Figure 1 and Figure 2 here.]

Although the focus of our study is at the firm level, our findings suggest fruitful directions for future work on the relation between equity volatility (a common measure of uncertainty), credit spreads, and aggregate economic activity. In Figure 1, we plot the time series and cross-section of the estimated firm-level elasticities of investment with respect to equity volatility. Firms with lower credit spreads that are further from default display a positive elasticity of investment, while firms with higher credit spreads display a negative elasticity. Aggregate

[8]See also Proposition 2 in Philippon (2009), which expresses q as approximately equal to $\frac{\psi}{\delta(1+r)}\frac{1+r_t}{1+y_t}$, where r is the risk-free rate, y is the corporate bond yield, ψ is leverage, and δ is the risk-neutral default rate. Figure I presents numerical results for the full model.

[9]The appendix of that paper relaxes the assumption of no bankruptcy costs, but does not allow for incentive misalignment between debt and equityholders.

[10]Choi and Richardson (2016) and Choi, Richardson, and Whitelaw (2022) also emphasize the difference between equity and asset volatility. Jurado, Ludvigson, and Ng (2015) provides evidence that standard measures of uncertainty based on conditional volatilities are imperfect uncertainty measures.

effects are driven by the movement of the entire cross-section of firms away from and closer to their respective default boundaries. Thus, a positive shock to equity volatility has a more strongly negative impact on investment when the entire cross section of firms is closer to default. In contrast, Figure 2 shows that the elasticity of investment to credit spreads is negative for all firm-quarters. We also confirm that our micro-results aggregate with a recursive vector autoregression (VAR) model of the aggregate time series of investment, asset volatility, and credit spreads. These results confirm that the aggregate investment response to a positive shock to asset volatility is positive, while the response to a positive shock to credit spreads is negative.

The remainder of the paper is organized as follows. Section 2 presents our firm-level empirical results. In Section 3, we show that our results hold at the aggregate level. Section 4 presents our model to build economic intuition, and Section 5 concludes.

2 Empirical Results: Firm Level

In this section, we establish our four main stylized facts. First, we show that credit spreads drive out equity volatility in a horse race to predict firm-level investment rates. Second, we show that this result is not due to information in bond markets, but instead to the nonlinear transformation of asset volatility and leverage that credit spreads and equity volatility represent. To do this, we use credit spreads constructed from equity market data and a structural model. Third, we show that leverage is the main driver of credit spreads, while asset volatility drives more of the variation in equity volatility. Finally, we show that the sensitivity of investment to asset volatility is positive for all firms.

We also provide several robustness checks, including using different measures of asset and equity volatility. We examine the relationship between investment and levels vs. changes in asset volatility, as well as the relation between investment, credit spreads, asset volatility, and Tobin's q at the firm level. We provide empirical evidence that, in the data as well as in our model, controlling for credit spreads is superior to controlling for leverage when trying to hold firms' financial soundness constant. We study the information in credit spreads that is driven by fundamental default risk vs. other factors using decompositions based on fair value spreads and the excess bond premium of Gilchrist and Zakrajšek (2012). To address whether the evidence supports the option value of higher volatility leading to higher investment rates—as opposed to higher investment rates leading to higher volatility—we show that investment has a stronger relationship with lags of volatility, that investment rates are not more positively correlated with volatility for high-R&D and high-investment-rate firms, and that our results hold using an instrumental variables approach. Finally, we conduct tests to document evidence in support of the debt overhang channel described in our model using results from firms with no bond spreads and zero leverage, as well as firms with more and less binding covenant restrictions, following Kermani and Ma (2020).

Our quarterly dataset, which describes firms' credit spreads, asset and equity volatilities, and investment rates (as well as controls), covers the period from 1984 to 2018. We use S&P's Compustat quarterly database for firm-level accounting variables. To compute equity volatility, we use daily returns from the Center for Research in Security Prices (CRSP) database. In robustness checks, we also use implied volatility from Option Metrics for the shorter available sample (1996-2018). Credit spreads are collected from the Lehman/Warga (1984-2005) and ICE databases (1997-2018). Appendix Section A details sample construction and precise definitions for each variable we study. Our main sample contains 1,407 unique firms and 48,672 firm-quarter observations. Table 1 presents notation, short variable descriptions, sample coverage, and summary statistics.

[Table 1 here.]

To establish our four key stylized facts, we start by presenting a set of firm-level panel regressions of investment rates on lagged measures of volatility and credit spreads:

$$\log[I/K]_{i,t} = \beta_1 \log X^{\sigma}_{i,t-1} + \beta_2 \log X^{cs}_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}, \tag{1}$$

where $\log[I/K]_{i,t}$ is the log of the investment rate of firm i in quarter t; $X^{\sigma}_{i,t-1}$ denotes measures of volatility (such as idiosyncratic equity volatility $\sigma^{e}_{i,t-1}$ or idiosyncratic asset volatility $\sigma_{i,t-1}$); and $X^{cs}_{i,t-1}$ denotes measures of credit risk (such as credit spreads $cs_{i,t}$, fair value spreads $\hat{cs}_{i,t-1}$, or market leverage $[MA/ME]_{i,t-1}$), all lagged by one quarter. Our main analysis uses idiosyncratic equity volatility computed using realized returns and removing factor exposures. Robustness checks include using total equity volatility and implied total equity volatility. For asset volatility, our baseline analysis uses idiosyncratic asset volatility computed using delevered idiosyncractic equity returns. Robustness checks include using total asset volatility, total implied asset volatility, idiosyncratic asset volatility derived from Merton's model and removing factor exposures, and residual asset volatility using the residual of a panel regression of realized idiosyncratic equity volatility on leverage. We control for firm and time fixed effects (η_i and λ_t). Our control variables $\mathbf{X}_{i,t-1}$ include the lag of firm i's return on equity, log tangibility, log sale ratio, log income ratio, and log Tobin's q.

2.1 Main Firm-level Results

This section presents our four key empirical results and related extensions and robustness checks.

2.1.1 Credit Spreads vs. Equity Volatility

We begin by documenting the fact that credit spreads drive out equity volatility in explaining firm-level investment. Table 2 presents the estimation results of Equation (1) using idiosyncratic equity volatility and credit spreads. Columns 1 and 2 show that the individual relationships between investment rates and both equity volatility and credit spreads are negative. Column 3 shows that when credit spreads and equity volatility are included together, the magnitude of the coefficient on equity volatility is cut by about one-third while the coefficient on credit spreads is essentially unchanged. This is the main result in Gilchrist, Sim, and Zakrajšek (2014).

How should we interpret the fact that credit spreads drive out equity volatility? We show that credit spreads drive out equity volatility in explaining firm-level investment because the sign of the relationship between equity volatility and investment changes sign systematically in the cross-section of firms. Firms far from their default boundary display a positive elasticity of investment to equity volatility, whereas this elasticity is negative for less financially sound firms. The changing sign washes out the effect of equity volatility in pooled data, and confounds aggregate inference.

[Table 2 here.]

To see this, consider Columns 4-6 of Table 2, in which we sort firms into terciles based on their credit spreads each quarter.[11] The coefficient on equity volatility is positive for firms with low credit spreads, but negative for firms with medium credit spreads and even more negative for firms with high credit spreads. Columns 7 and 8 show that, controlling for the negative effect on investment rates from the interaction between credit spreads and equity volatility, the effect of equity volatility is positive.

[Table 3 here.]

In our model, it is total volatility that matters for investment decisions. The literature on uncertainty studies idiosyncratic volatility, so we begin with that volatility measure. Table 3 shows that the sign change in the cross-section of firms for the relation between equity

[11] This method of splitting uses quarter-specific cutoffs. Using fixed cutoffs to sort all firm-quarter observations leads to similar results.

volatility and investment is preserved using lagged total realized equity volatility. This is not surprising, given that the majority of firm-level volatility is idiosyncratic, as shown in Table 1.

Another concern regarding the comparability between the information in credit spreads and equity volatility computed using realized returns is that while credit spreads are forward looking, realized volatility is backward looking. To address this issue, Table 4 replicates the results from Table 2 using implied volatility of at-the-money options with 30 days to expiration for the shorter sample starting in 1996.

[Table 4 here.]

Next, we turn to the question of why credit spreads tend to have a more robust relationship with firm-level investment. One set of answers relies on segmented markets. Perhaps there is smarter (and more institutional) money in bond markets, or maybe equity markets are more prone to bubbles and mispricing. There are also more fundamental explanations, such as the fact that bonds capture downside risk better while equity values include growth options. Like Philippon (2009), our results suggest that the reason is likely fundamental. To show this, we repeat the analysis in Table 2 but replace credit spreads with fair value spreads. We construct fair value spreads based on Moody's method described in Nazeran and Dwyer (2015). Moody's constructs a mapping between firms' distance to default based on equity market data, leverage, and empirical default frequencies (Moody's EDF). Fair value spreads are then computed using the cumulative EDF's and constant assumptions for the loss given default, the market equity Sharpe ratio, and the correlation between asset returns and market equity returns.

[Table 5 here.]

The results are presented in Table 5 and are qualitatively identical to Table 2. The coefficient on equity volatility goes from significantly positive to significantly negative as firms' credit spreads increase, while the coefficient on the fair value spread remains significantly negative across terciles of fair value spreads.[12] Since fair value spreads are constructed with only equity market information, the results in Table 5 cannot be driven by differences in equity vs. bond markets' investor bases or susceptibility to mispricing or bubbles.

[Table 6 here.]

[12] Online Appendix Table A1 presents these results using implied equity volatility.

The results in Table 5 are not driven by bond market information. However, it could still be that the residuals of credit spreads after controlling for fundamentals have bond-market-specific information that drives out equity volatility. Relatedly, perhaps fair value spreads don't explain much of the variation in credit spreads. The latter is not the case. The R^2 from a univariate panel regression of credit spreads on fair value spreads is 50%. Table 6 shows that the former is also not the case. This table confirms that it is the information in fair value spreads that drives out the information in equity volatility in explaining investment, by examining the effects of fair value spreads vs. the residual of credit spreads regressed on fair value spreads. Comparing the R^2 of Columns 1 and 2 shows that fair value spreads explain more of the variation in firm-level investment rates than the residuals of credit spreads after controlling for fair value spreads, and the coefficient on fair value spreads is economically and statistically more significant. Comparing Columns 4 and 5 shows that the residual bond market information after controlling for fair value spreads and equity volatility (leaving only bond-market-specific information) cannot drive out equity volatility. The coefficient on equity volatility in Column 5 is nearly the same as in Column 3 in both magnitude and significance.

2.1.2 Credit Spreads vs. Asset Volatility

While the previous results documented systematic variation in the elasticity of investment to equity volatility, we now document the robust *positive* elasticity of investment with respect to *asset* volatility. Because equity volatility is asset volatility scaled up by the firm's market-value leverage, it is not a clean measure of asset volatility. We construct our baseline measure of idiosyncratic asset volatility by first delevering equity returns and then computing the idiosyncratic volatility of these delevered returns. Our results are robust to several other measures of asset volatility. We present results using total asset volatility and implied asset volatility computed by delevering implied equity volatility in the main text.[13]

[Table 7 here.]

In Table 7, we study the relationship between volatility and investment, replacing idiosyncratic *equity* volatility $\sigma^e_{i,t}$ with idiosyncratic *asset* volatility $\sigma_{i,t}$. Asset volatility has a robustly positive relationship with investment. The coefficient that describes the relationship between firm-level investment rates and asset volatility is positive and statistically significant in the full sample and for all levels of credit spreads.

[13] We present results using idiosyncratic asset volatility implied by Merton's model and the residual of idiosyncratic equity volatility regressed on firm leverage in Online Appendix Tables A3 and A4, respectively.

[Table 8 here.]

In Table 8, we show that the relationship between firm-level investment rates and total asset volatility is also robustly positive. The coefficient on asset volatility is positive overall and for all subsamples of firm credit spreads. These coefficients are larger in magnitude and slightly more significant that the coefficients in Table 7.

[Table 9 here.]

In Table 9, we replicate the same exercise but with implied asset volatility. In our model, equityholders make investment decisions as a function of debt overhang and the distribution of future productivity realizations. Implied volatility may capture forward-looking risk better than our baseline measure using realized volatility. Table 9 shows that the results using implied asset volatility are economically stronger than those using realized total asset volatility. Also, we show in Online Appendix Table A2 that the coefficient on asset volatility is half the size and less statistically significant using realized total asset volatility for the same (smaller) sample of firm-year observations for which implied volatility is available than in Table 9 using implied volatility. This result lends support to the idea that it is the expectation of future asset volatility, not past realizations, that drives changes in investment.[14]

The fact that asset volatility is robustly positively related to investment may seem surprising, given the emphasis on a negative relationship between volatility and investment in the literature on uncertainty and investment (see Bloom (2009) and the large literature following that important paper.) Our results are not necessarily inconsistent with that literature. The underlying theory and timing in those models differs from ours. We emphasize the misalignment of debt and equityholders' returns to investment and the fact that the level of equity volatility reflects both debt overhang and investment option value. The uncertainty literature emphasizes the "wait-and-see" effect of an increase in volatility and focuses on the relationship between investment and *changes* in volatility. In wait-and-see models of investment with fixed adjustment costs, firms reduce investment in the short run when they expect volatility to increase. Even in those models, however, investment increases in the long run once higher volatility is realized and firms are pushed outside of their inaction regions more often.

[Table 10 here.]

[14] Lettau and Ludvigson (2002) emphasize the importance of information about future investment returns contained in asset prices.

Table 10 shows the empirical relationship between investment and both the level of and changes in asset volatility, controlling for firm-level credit spreads.[15] In the full regressions in Columns 2 and 5, the coefficient on the level of asset volatility is positive while the coefficient on the change in asset volatility is negative. The positive coefficient on the level of asset volatility is two to three times as large as the coefficient on the changes.

[Table 11 here.]

There is a caveat, however, for the result for changes in asset volatility, controlling for the level of asset volatility. Table 11 shows that the coefficient on lags of volatility is positive. Thus, it is possible that the negative coefficient on the change in volatility is significant because this lagged value enters the change in volatility with a negative sign. Possibly consistent with this explanation, Column 4 in Table 10 shows that when controlling for credit spreads only, there is no significant relationship between the change in asset volatility and investment.

2.1.3 Drivers of Equity Volatility and Credit Spreads

Structural models of credit risk show that both equity volatility and credit spreads are functions of leverage and asset volatility. However, there is no sign change in the relation between credit spreads and investment in the cross-section of firms with different levels of financial soundness. We argue that the change of sign in the relation between equity volatility and investment arises because whereas equity volatility mainly captures the upside option value of investment, it also measures the downside pressure from leverage and debt overhang when a firm is close to default. On the other hand, although credit spreads are also a function of asset volatility, they mainly capture the debt overhang effects on investment from leverage. We show that, consistent with this explanation, variation in equity volatility is mainly driven by variation in asset volatility (for both levels and changes), while variation in credit spreads is mainly driven by variation in leverage (for both levels and changes).

To show this, we consider the loadings of credit spreads and equity volatility on asset volatility and leverage as estimated by the following equation:

$$\log y_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

where $y_{i,t}$ is either equity volatility ($\sigma^e_{i,t}$) or the credit spreads ($cs_{i,t}$), and $[MA/ME]_{i,t}$ is firm-level leverage. We estimate the equation both in levels and in first differences. For asset

[15] Table A5 in the Online Appendix presents these results using implied equity volatility.

volatility, we use asset volatility derived from Merton's model $\widetilde{\sigma}_{i,t}$ for this exercise, instead of delevered equity volatility, so that the empirical decomposition in levels is not mechanical.

[Table 12 here.]

Table 12 summarizes the results.[16] Panel A reports coefficients for the loadings of the levels of equity volatility and credit spreads on the levels of asset volatility and leverage. Coefficients for equity volatility on asset volatility are about double those on leverage. The bottom panel of Table 12 reports the partial R^2, and shows that asset volatility explains 58% of the variation in equity volatility after controlling for time and firm fixed effects, while leverage explains 11%. By contrast, the loadings for credit spreads on leverage are more than three times as large as the loadings on asset volatility. Asset volatility explains little to none of the variation in credit spreads after controlling for firm and time fixed effects, while leverage explains abut 21%. The results in Panel B for the loadings of changes in equity volatility and credit spreads on changes in leverage and asset volatility display patterns similar to the level results in terms of magnitudes and significance. The bottom panel shows that changes in asset volatility explain a substantial amount of variation in changes in equity volatility, controlling for firm and time fixed effects (74%). For credit spreads, changes in leverage explain more variation than changes in asset volatility, but the magnitudes are small.[17]

The results in Table 12 show why an increase in equity volatility could either signal an increase in asset volatility (positive for investment) or leverage (negative for investment). Although credit spreads are also a combination of asset volatility and leverage, the loading of credit spreads on asset volatility is not large enough to ever drive a positive relation between credit spreads and investment.

[Table 13 and Table 14 here.]

In Table 2 we emphasized the change in sign in the elasticity of investment with respect to equity volatlity for firms with high and low credit spreads. In Table 13 we provide evidence consistent with the idea that this may be because equity volatility is driven more by asset volatility for firms with low credit spreads and more by leverage for firms with high credit spreads. We report loadings on and partial R^2 for equity volatility on asset volatility and leverage by credit spread tercile. The loadings on asset volatility are monotonically

[16] To address endogeneity concerns, in Online Appendix Table A7, we use industry-level regressors—constructed as a simple average of all firms in the same industry, excluding the firm itself—instead of using the firm's asset volatility and leverage directly. This exercise shows similar patterns, whereby equity volatility loads more on asset volatility and credit spreads loads more on leverage.

[17] See Collin-Dufresn et al. (2001) for a related result.

decreasing in credit spreads while the loadings on leverage are monotonically increasing. The same pattern holds for partial R^2. Table 14 presents the analogous results for credit spreads. The loadings of credit spreads on both asset volatility and leverage are larger for firms with the highest vs. lowest credit spreads (though the pattern for leverage is not monotonic for the level of spreads) and the partial R^2 for leverage are much higher for high credit spread firms.

2.1.4 Credit Spreads vs. Leverage

[Table 15 here.]

Given the decomposition of equity volatility as levered asset volatility, a natural question is whether controlling for leverage is sufficient, or better than, controlling for credit spreads. The answer is no. Table 15 shows that, using leverage instead of credit spreads to control for firms' financial soundness, the coefficient that describes the relationship between asset volatility and investment, while always positive, is not significant for medium- and high-credit-spread firms. We show in our model that one reason for this may be that it is not enough to hold leverage constant to isolate the option value effect of asset volatility. This is because even with constant leverage, distance to default can still vary. Only credit spreads hold distance to default (and thus the driving force of debt overhang) constant.[18]

2.1.5 Tobin's q

The q-theory of investment predicts a strong relationship between investment and the ratio of firms' market values of capital to the replacement values. Philippon (2009) was the first to point out the structural relationship between Tobin's q and credit spreads in structural models of credit risk, such as those of Merton (1974) and Leland (1994). Atkeson, Eisfeldt, and Weill (2017) emphasize the structural relationship between inverse equity volatility and credit spreads. Together, these papers describe a structural relationship between q, credit spreads, asset volatility, and equity volatility. Taking these structural relationships into account should prove helpful when when developing econometric specifications for empirical work on investment.

In Section 4, we confirm the theoretical links between q, credit spreads, and asset volatility. We also show that when debt and equityholders are not perfectly aligned, Tobin's q cannot be fully captured by credit spreads. Our result is intuitive. When debt holders do not capture the upside of investment returns due to their subordination in the capital structure, credit spreads will not be able to capture the full marginal returns to investment.

[18]Table A8 replicates the result in Table 15 using implied volatility.

The option value of investment is best captured by asset volatility. Thus, credit spreads and asset volatility both contain important and distinct information for investment. In our model, however, as in most models of investment returns, Tobin's q is still the theoretically best predictor of investment.

[Table 16 here.]

Philippon (2009) shows that an aggregate measure of credit spreads empirically outperforms an aggregate equity-market-based measure of Tobin's q in data from 1953 to 2007. We show that this is not the case in firm-level data from 1984 to 2018. Table 16 presents the results of comparing the ability of Tobin's q in predicting firm-level investment rates with the ability of credit spreads and asset volatility. At the firm level, Tobin's q is a strong predictor of investment rates and is not subsumed by credit spreads. Columns 1 to 3 of Table 16 review the relationship between investment rates, asset volatility, and credit spreads from Table 7 for comparison. Panel A presents results without additional firm-level controls and Panel B includes these controls. Column 4 shows that the coefficient on Tobin's q is positive and highly significant, and the R^2 of that univariate regression with time and firm fixed effects is higher than for either credit spreads or asset volatility. Column 5 shows that including credit spreads does not drive out Tobin's q. Comparing Columns 2, 4, and 5 of Panel A shows that without additional controls the economic significance of credit spreads declines more than that of q when both are used together to explain investment rates. However, the decline in economic significance is similar in both variables when additional firm-level controls are included in Panel B. Finally, Column 6 shows that when all three key variables for investment—q, asset volatility, and credit spreads—are included, each one remains strongly significant. Note that our study is still consistent with the large literature documenting that Tobin's q works better in theory than empirically, since Tobin's q does not drive out credit spreads or asset volatility, as it theoretically should in our model.[19]

There are two key differences in our empirical analysis compared with that of Philippon (2009). First, his study is at the aggregate level, while ours is at the firm level. Second, more than 80% of our observations come from between 1995 and 2018, while the sample of Philippon's (2009) sample ends in 2007. Our firm-level results are consistent with the aggregate results of Andrei, Mann, and Moyen (2019), who demonstrate that the relation between aggregate investment and aggregate Tobin's q became remarkably tight after 1995.

[19]Examples of earlier work that has shown that a simple regression of investment on Tobin's q performs quite poorly include Fazzari, Hubbard, and Petersen (1988); Kaplan and Zingales (1997); Gilchrist and Himmelberg (1995); Erickson and Whited (2000); Gomes (2001); Cooper and Ejarque (2003); Moyen (2004); and Abel and Eberly (2011), among others.

2.1.6 Excess Bond Premium

Our analysis of bond-market credit spreads vs. fair value spreads from equity market data in Tables 5 and 6, as well as the results for the drivers of variation in credit spreads in Table 12, document the relationship between fundamental default risk, credit spreads, and investment rates. We emphasize this relationship in our model of debt overhang and equityholders' investment incentives. Thus, our discussion emphasizes that credit spreads are in large part driven by asset volatility and leverage, while a long-standing literature finds that a nontrivial fraction of credit spreads cannot be explained by credit risk. Prior work has questioned the role of fundamental default risk in explaining changes in credit spreads (Collin-Dufresn, Goldstein, and Martin, 2001) and in explaining macroeconomic aggregates such as employment, output, and inventories (Gilchrist and Zakrajšek, 2012). Gilchrist and Zakrajšek (2012) decompose aggregate credit spreads into two components: a component that captures the movements in default risk based on fundamentals (the predicted component) and a residual component (the excess bond premium). They show that in the aggregate, the excess bond premium has substantial predictive content for future economic activity and outperforms the component of credit spreads predicted by fundamentals.

To further address the role of fundamentals vs. additional bond market information in the context of our study, we construct firm-level excess bond spreads following Gilchrist and Zakrajšek (2012) and show that both the part of credit spreads explained by default risk fundamentals and the excess bond premium contain important information for investment. Thus, we do not question whether the excess bond premium (Gilchrist and Zakrajšek, 2012) or the residual of credit spread changes on fundamentals (Collin-Dufresn, Goldstein, and Martin, 2001) contain interesting information for asset pricing or macroeconomic studies. Instead, we provide evidence that the fundamental part of credit spreads explains firm-level investment rates and provide evidence that our main results are driven by the structural relationship between credit spreads and asset and equity volatility.

First, we estimate the following panel regression:

$$\log cs_{i,m}[k] = \boldsymbol{\gamma}'\mathbf{X}_{i,m}[k] + \epsilon_{i,m}[k],$$

where the log of credit spreads on bond k issued by firm i in month m is regressed on a vector of bond-specific characteristics $\mathbf{X}_{i,m}[k]$ for bond k issued by firm i.[20] We then build

[20]The bond characteristics $\mathbf{X}_{i,m}[k]$ include the firm's distance-to-default; the bond's amount outstanding, duration, and coupon rate; and an indicator variable for callable bonds. It also includes the interactions of callability with these bond characteristics; firm's distance-to-default; the level, slope, and curvature of the Treasury yield curve; and the realized monthly volatility of the daily 10-year Treasury yield, which reflects the value of the call option embedded in callable bonds. Industry fixed effects and credit rating fixed effects

firm-level quarterly excess bond premia as the quarterly average of the residuals for all bonds issued by the firm during that quarter: $\log(ebp_{i,q}) = \frac{1}{3}\sum_{m=q_1}^{q_3} \frac{1}{N_{i,m}^k} \sum_{k=1}^{N_{i,m}^k} \epsilon_{i,m}[k]$, where q_n is the nth month of quarter q and $N_{i,m}^k$ is the number of bonds of firm i in month m.

[Table 17 and Table 18 here.]

Table 17 replicates our main results from Table 2 and shows that the fundamental part of credit spreads greatly reduces the economic significance of equity volatility in explaining investment. The excess bond premium, on the other hand, does not change (without additional controls) or slightly improves (with additional controls) the economic significance of equity volatility in explaining investment. The firm-level excess bond premium is, however, in itself a strong predictor of investment. This is consistent with the notion that an increase in the firm-level excess bond premium reflects an increase in the cost of the firm's capital and, as a result, a contraction in future investments, as emphasized by Gilchrist and Zakrajšek (2012).

Table 18 replicates our main results from Table 7 showing that asset volatility is robustly positively related to firm-level investment rates for controlling for either of the two components of credit spreads, the excess bond premium or the credit spreads minus the excess bond premium.

2.2 Additional Firm-level Results

In this section, we provide additional results to test for endogeneity, verify if our results still hold for firms with zero reported financial leverage, and use a measure of covenant tightness as an additional measure of financial soundness.

2.2.1 Endogeneity

We hypothesize that the positive correlation between investment and asset volatility is most likely driven by one of the following two causal mechanisms. Either (i) an increase in business risk renders the value of assets in place more volatile and incentivizes firms to invest more, or (ii) due to higher investments, the value of the firm's assets become more uncertain. We argue that the former is the more likely explanation for our results.

[Table 19 and Table 20 here.]

are included as well.

The first evidence in support of the direction of causality running from volatility to investment rather than the other way around is in Table 11.[21] If asset volatility increases because of uncertainty driven by higher investment rates, we would expect an increase in investment to cause an increase in asset volatility during the investment period. On the other hand, if, as we argue, higher asset volatility increases the option value of investment, then we would expect a stronger relationship between investment and lags of asset volatility. Table 11 shows that the coefficients on lagged asset volatility are statistically and economically larger than those on leads of asset volatility. Note that our main analyses focus on the effect of the first lag of volatility.

The second set of evidence in support of the idea that higher risky investment is not causing higher equity or asset volatility is that the relationship between investment and either equity volatility or asset volatility is generally weaker for both high R&D firms and high investment firms. Table 19 presents the results for equity volatility, replicating the results in Table 2 for firms with high and low R&D and firms with high and low investment. With and without additional controls, and for both R&D and investment, firms that invest more display a weaker link between volatility and investment. Thus, it seems unlikely that high investment is causing higher equity volatility. Table 20 presents the results for asset volatility. For R&D, both with and without additional controls, the relationship between asset volatility and investment is stronger for low R&D firms. The relationship between investment and asset volatility is marginally stronger for high investment firms, but the differences are not statistically significant. The p-values for the difference between the coefficients on asset volatility for low and high investment firms with and without additional firm-level controls are 0.66 and 0.55, respectively.

Instrumental Variables Next, we address endogeneity in estimating the impact of equity and asset volatility on investment by instrumenting for firm-level volatility with industry-level exposures to volatility shocks.

We follow the instrumental variables strategy of Alfaro, Bloom, and Lin (2018). First, we estimate sensitivities to energy, currencies, treasuries, and policy at the industry level as the factor loadings of a regression of a firm's daily stock return on the price growth of energy and 7 currencies, return on treasury bonds, and changes in daily policy uncertainty from Baker, Bloom, and Davis (2016). That is, for firm i in industry j, the sensitivity β_j^c is estimated as follows:

$$r_{i,t} = \alpha_j + \sum_c \beta_j^c \cdot r_t^c + \varepsilon_{i,t},$$

[21]See Table A6 for leads and lags of implied asset volatility.

where $r_{i,t}$ is the daily risk-adjusted return on firm i, r_t^c is the change in the price of commodity c, and α_j is industry j's intercept.

The risk-adjusted returns $r_{i,t}$ are the residuals from running firm-level time-series regressions of daily CRSP stock returns on the Carhart (1997) four-factor asset pricing model. We estimate the risk-adjusted returns and the sensitivity β_j^c yearly and using the same 10-year window.

Next, for these 10 aggregate market price shocks (oil, 7 currencies, treasuries, and policy), we multiply the absolute value of their time-varying sensitivities $|\beta_j^c|$ by their implied volatilities σ_t^c. This provides 10 instruments for lagged equity volatility, as follows:

$$z_{i,t-1}^c = |\beta_j^c| \cdot \sigma_{t-1}^c.$$

The key difference between our study and theirs is that we estimate the impact of the level of volatility on the level of investment and not the impact of shocks to volatility. Thus, our instruments are for the level of volatility while they construct instruments for shocks to volatility as $|\beta_j^c| \cdot \Delta\sigma_{t-1}^c$.

The second difference concerns the measure of asset volatility. Our analysis directly constructs instrumented asset volatility as opposed to delevering instrumented equity volatility. We argue this is a more precise way to instrument asset volatility as it removes the effect of leverage in the very first step so that leverage is less likely to drive the results. We first generate risk-adjusted asset returns $r_{i,t}^a$ as the residuals of regressing firms' *unlevered* equity returns on the Carhart factors. We construct asset volatility as the standard deviation of $r_{i,t}^a$, and estimate the sensitivities of asset returns to the 10 aggregate market price shocks by estimating the following equation:

$$r_{i,t}^a = \alpha_j + \sum_c \beta_j^{c,a} \cdot r_t^c + \varepsilon_{i,t}.$$

Then we construct instruments for lagged asset volatility as:

$$z_{i,t-1}^{c,a} = |\beta_j^{c,a}| \cdot \sigma_{t-1}^c.$$

By contrast, Alfaro, Bloom, and Lin (2018) generate their instrument for asset volatility by deleveraging instrumented equity volatility using firm-level leverage directly, and they use the same set of instruments for asset volatility and equity volatility. We refer the reader to Alfaro, Bloom, and Lin (2018) for further details on the construction of the instrumental variables.

[Table 21 here.]

In Table 21, we show that our main results hold in the instrumental variable regression: Asset volatility has an unconditional positive impact on investment.[22]

2.2.2 Zero-leverage Firms

[Table 22 here.]

Our analysis so far has focused on the subset of firms with observable credit spreads. In Table 22 we demonstrate that our main results are preserved for firms without observable bond spreads. Columns 1 and 2 (without additional firm-level controls) and Columns 4 and 5 (adding controls) use distance to default to proxy for firms' financial soundness for firms that have financial leverage but not bond spreads. Columns 1 and 4 show that the sign of the relationship between equity volatility and investment changes sign in the cross-section of firms, as measured by their distance to default. Distance to default is larger for more financially sound firms, so the positive interaction term between distance to default and equity volatility indicates that the relationship between equity volatility and investment is positive for more financially sound firms and negative for firms closer to their default boundary. The coefficient on distance to default is positive, consistent with more financially sound firms having higher investment rates. The coefficient on equity volatility is negative. This coefficient corresponds to the relationship between investment and equity volatility when distance to default is zero. The sign is consistent with the result in Table 2 in which the positive coefficient on equity volatility in Columns 7 and 8 corresponds to the relationship between investment and equity volatility when log credit spreads are equal to zero.

Columns 2 and 5 show that when controlling for distance to default, the relationship between *asset* volatility and investment is positive—as in Table 7 and as predicted by our model in which higher volatility indicates a greater option value of investment.

Columns 3 and 6 in Table 22 report results for firms with zero reported financial leverage. These firms may still have operating leverage, but for these firms equity volatility and asset volatility, measured by delevering equity volatility using financial leverage, are equal. For firms with zero financial leverage, the impact of equity volatility is either negative or insignificant, consistent with our results for levered firms' being driven by the presence of debt overhang.

[22]Note that the low Kleibergen-Paap F-statistic indicates that the excluded instruments are correlated with the endogenous regressors, but only weakly.

2.2.3 Covenant Tightness

[Table 23 here.]

In our model, the main force that drives investment lower for high credit spread firms is debt overhang. Thus, in the data, we expect the interaction between credit spreads and equity volatility to be stronger for firms with potentially tighter covenants, and for the effect of credit spreads to be more negative for these firms. Table 23 confirms that this is the case. First, we split the sample into two groups according to covenant tightness following the measurement developed by Kermani and Ma (2020). The "Tight Covenant" subsample contains firms for which the overall measure of the distance between actual financial ratios and covenant thresholds are below the median. The remaining firms are assigned to the "Slack Covenant" group. We estimate Equation (1) with the interaction term for each covenant-tightness subsample. Table 23 shows that all of the coefficients are larger in absolute value and are statistically more significant for the tight covenant subsample, in support of our model with debt overhang as a key investment distortion.[23]

3 Empirical Results: Aggregate Level

Although our main focus is at the firm level, we provide evidence indicating that our results may be extended to aggregate effects and leave a full aggregate study for future work.[24]

Time Series To understand the implications of our findings for the aggregate time series, we first review the plots of the elasticity of investment rates with respect to equity volatility and credit spreads across time and across firms. In Figure 1, we compute the overall coefficient on equity volatility at each credit spread level using estimates on equity volatility ($\log \sigma^e_{i,t}$) and the interaction term ($\log \sigma^e_{i,t} \times \log cs_{i,t}$) reported in the Column 7 of Table 2. Each line represents the elasticity of investment to equity volatility for a particular percentile of the credit spread distribution. More financially sound firms, with lower credit spreads, are represented by the top blue line, while less sound firms, with higher credit spreads, are represented by the bottom red line. As can be seen in the figure, the entire distribution of

[23]The literature on risk shifting has proposed a different explanation for the effect of equity volatility on investment. In particular, Eisdorfer (2008) finds evidence of risk shifting in the investment decisions of financially distressed firms using an aggregate volatility measure. In the Internet Appendix Section B, we describe our replication and show that we are unable to replicate the main empirical result that financially distressed firms increase their investment rate when aggregate equity volatility increases.

[24]See Lee (2016) for a macroeconomic model emphasizing the positive role of volatility for aggregate outcomes.

these elasticities shifts over time together with the distribution of credit spreads. In particular, the coefficient is negative for the whole cross-section of firms during the Great Recession, while it is mainly positive in the late 1980s. The other important takeaway from this figure is that the change in sign of the elasticity of investment with respect to credit spreads is made evident by the fact that the lower-percentile lines tend to lie above the zero line, while the higher credit spread percentiles lie below it.

Figure 2 plots the elasticity of investment with respect to credit spreads in the cross-section of firms with higher and lower equity volatilities. Firms with lower equity volatility have less negative elasticities of investment with respect to credit spreads as implied by the negative coefficient on the interaction term in Table 2 Column 7. However, the entire distribution of these elasticities is always negative.

VAR Analysis We use VAR analysis to show that our key micro-level result, whereby the level of asset volatility has a positive impact on investment, holds at the macro-level. We aggregate the variables in our sample and estimate a simple VAR consisting of the three endogeneous variables: the log of idiosyncratic asset volatility ($\log \sigma_t$), the log of credit spread ($\log cs_t$), and the log of investment rate ($\log[I/K]_t$).[25] We employ a standard recursive ordering technique and consider two identification schemes, one in which credit spreads have an immediate impact on asset volatility and one in which asset volatility has an immediate impact on credit spreads.

[Figure 3 here.]

Figure 3 reports the impulse responses of investment rates to credit spreads and asset volatility using the two specifications. As can be clearly seen in the figure by comparing the left and right panels, credit spreads have a negative impact on investment while asset volatility has a positive impact. Comparing panels (a) and (c) of Figure 3, the positive impact of asset volatility on investment is somewhat economically and statistically larger in the first specification, though both are strongly and significantly positive. The slightly stronger result in panel (a) highlights the importance of controlling for credit spreads in order to uncover the option value effect of asset volatility as a strong positive signal for investment in the aggregate.

[25] We use the value-weighted average of $\sigma_{i,t}$, $cs_{i,t}$, and $[I/K]_{i,t}$ to generate the corresponding aggregate time series. We seasonally adjust the investment rate time series by subtracting a seasonal average computed over the previous 5 years. All variables are detrended using the HP filter with weight $1,600$.

4 A Model of Debt Overhang and Option Value

In this section, we develop a simple but general credit risk model to analyze the investment choices of a firm facing productivity risk which has outstanding debt already in place. Two forces drive the investment decision: debt overhang and the option value of equity. The key violation of the Modigliani and Miller (1958) theorem in our model is that the incentives of equity and debtholders are not aligned. Equity holders choose investment and face a tradeoff for investment between the option value of investment against losses from debt overhang. Conditional on a firm's credit spread, asset volatility is a clean measure of the positive effect of the option value of investment. And, conditional on a firm's underlying asset volatility, credit spreads are a clean measure of the negative impact of debt overhang on investment. Thus, credit spreads and asset volatility are jointly unambiguous signals for investment. By contrast, the signals provided by credit spreads and equity volatility are ambiguous and can change in the cross-section.

For the analysis in the main text we assume that the firm's liquidation value is zero, or that bankruptcy costs are 100%. However, bankruptcy costs are not necessary for our results and we show in the Internet Appendix Section C that our results hold for all values of bankruptcy costs between zero and 100%. All proofs are relegated to the Internet Appendix Section C. For ease of notation, we sometimes write $f_x(x) \equiv \frac{\partial f(x)}{\partial x}$.

Consider a firm in a two-date economy that has funded itself partly with debt, that is, it has leverage in place at date zero. Given this level of debt in place and the underlying distribution of productivity shocks at date one, shareholders choose how much to invest in the firm subject to a convex total cost of investment. At date one, a random productivity shock is realized, and after observing output, shareholders decide whether to default or not. To streamline the analysis, we make the following assumptions regarding the firm and its investments.

First, we assume a simple linear production function and convex total investment cost. The model requires some concavity for an interior optimum. Either a convex investment cost or a concave production function is sufficient. For simplicity, we use a linear productive function and a convex investment adjustment cost.

Assumption 1 (Investments)**.** *The firm has the option to invest in capital which will produce output at date one equal to iz, where i is investment and z is a random productivity shock realized at date one. The convex function $\phi(i)$ captures the total cost of investment including resource costs and any adjustment costs.*

Second, we assume that debt is in place at date zero, that there is a separation between debt and equity holders, and that the value of the firm in default is zero. Again, we show that our results are robust to a relaxation of Assumption 2 that features complete or partial recovery of the firms' assets upon bankruptcy in the Internet Appendix Section C. The key assumption is the separation of debt and equityholders.

Assumption 2 (Debt and Equity)**.** *The firm is funded by debt and equity with misaligned interests. The debt claim has a given face value b that is due at date one after output is realized. After output is realized at date one, shareholders decide whether or not to default. Upon default, the entirety of the firm's value is lost. Furthermore, shareholders cannot liquidate the firm ($i \geq 0$).*

Next, we normalize the interest rate to zero, normalize the mean productivity shock to one, and assume risk neutral asset pricing in Walrasian markets.

Assumption 3 (Pricing)**.** *All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities equal to their expected payoff with respect to a risk-neutral distribution $F(z;\sigma)$ of firm's asset productivity z with full support on $[0,\infty)$. We normalize the size of the productivity shock by assuming that $\mathbb{E}[z] = 1$.*

Given our assumptions about payouts and pricing, it follows that the value of equity e and debt d are given by

$$\begin{aligned} e(b,\sigma) &= \max_{i,\underline{z}} \int_{\underline{z}}^{\infty} (iz - b)dF(z;\sigma) - \phi(i), \\ d(b,\sigma) &= (1 - F(\underline{z}(b,\sigma);\sigma))b, \end{aligned}$$

where $\underline{z}$ is the threshold productivity level below which equityholders choose to default. The value of equity is the value of output less the face value of debt for realizations of productivity above the default threshold, less the cost of investment. The value of debt is the face value times the cumulative probability of productivity realizations above the default threshold.

The first-order conditions for investment i and the default threshold $\underline{z}$ imply that, at an optimum, i and $\underline{z}$ satisfy

$$\int_{\underline{z}}^{\infty} z dF(z;\sigma) = \phi_i(i), \tag{2}$$

$$i\underline{z} = b. \tag{3}$$

The first equation states that the marginal benefit of investment equals the marginal cost. The left-hand side of the second equation is the output lost at the default threshold and thus this equation states that at the default threshold the output lost equals the face value of debt. In other words, the left-hand side $i\underline{z}$ represents the lowest level of production such that the value of equity is not negative after repaying the debt.

The credit spread of the firm is defined as:[26]

$$cs(\underline{z}, \sigma) \equiv F(\underline{z}; \sigma)/(1 - F(\underline{z}; \sigma)). \tag{4}$$

We define book leverage as b.[27] To streamline our analysis, we also make assumptions on the distribution of productivity shocks, $F(z; \sigma)$, which are satisfied by most risk distributions used in financial theory (including the Black–Scholes–Merton model and most standard models).

Assumption 4 (Vega)**.** *The distribution of the productivity shock $F(z; \sigma)$ is such that vega is always positive:*

$$\nu(\underline{z}, \sigma) = \frac{\partial}{\partial \sigma} \mathbb{E}\left[(z - \underline{z})^{+}\right] > 0$$

for $\underline{z} > 0$. Furthermore, the standard deviation σ of z is a finite moment of the distribution $F(z; \sigma)$.

The model has two free parameters, leverage b and asset volatility σ, and two endogenous decision variables, investment i and the default threshold $\underline{z}$. We use this simple model to study the behavior of investment following changes in the key observable variables from our empirical analysis: asset volatility σ, leverage b, credit spreads cs, equity volatility σ^e, and Tobin's q. Without measurement error, in our model simply observing two non-perfectly correlated functions of the parameters and endogenous variables is sufficient to identify these parameters. We describe several comparative statics results. Rather than doing comparative statics with respect to the model's parameters, we perform comparative statics for the key observable variables from our empirical work holding other key observable variables constant. In doing so, we directly provide theoretical insights from our model for the empirical tests in Section 2.

In Proposition 1, we provide the elasticities of investment when observing asset volatility and credit spreads. Given Assumptions 1-4, the signs of these partial derivatives match our empirical results. When credit spreads increase, holding asset volatility constant, the debt overhang problem intensifies and equityholders have lower incentives to invest. As asset

[26]The credit spread is the difference between the yield of the corporate bond y and the risk-free rate. As the risk-free rate is assumed to be 0 in this simple model, and the yield is given by $y = b/d - 1$, we get $cs = F/(1 - F)$.

[27]We already normalized the size of the firm by assuming that there is no capital in place in the first time period and that $\mathbb{E}[z] = 1$.

volatility increases, holding credit spreads constant, the option value of equity alleviates the debt overhang problem and induces equityholders to invest more. When there is no debt ($b = 0$) and therefore no credit risk ($\underline{z} = 0$), these partial derivatives are equal to 0 and investment is undistorted.

Proposition 1 (Credit Spreads and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to credit spreads is given by*

$$\frac{\partial i}{\partial cs} = -\frac{\underline{z}(1 - F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \leq 0. \tag{5}$$

Holding credit spreads constant, the partial derivative of investment with respect to asset volatility is given by

$$\frac{\partial i}{\partial \sigma} = \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)} \geq 0. \tag{6}$$

The first part of Proposition 1 shows the negative impact of credit spreads on investment. The numerator of Equation (5) $\underline{z} \times (1 - F(\underline{z};\sigma))^2$ represents the marginal product lost in default $\underline{z}$ times a term that arises due to the nonlinearity of credit spreads with respect to the default probability. If credit spreads were instead approximated with F, that term would be equal to 1. In the denominator, the role of the convexity of the adjustment cost function appears. If the cost of adjusting the stock of capital is more convex in investment, the impact of a higher credit spread is attenuated, since firms do not have to adjust the stock of capital that much to reduce the marginal cost of investment. Our results also hold with linear investment costs but a concave production function. In that case, when the production function is more concave the effect of higher credit spreads is smaller because equityholders do not have to reduce investment by as much to increase the marginal product of investment. With either convex adjustment costs or a concave production function the denominator of Equation (5) will be negative and all other terms are positive.

The second part of Proposition 1 shows that investment reacts positively to an increase in volatility because the payout to shareholders is nonlinear with limited downside and unlimited upside; that is, vega $\nu(\underline{z},\sigma)$ is positive. Thus, in this simple model with fairly general and standard assumptions, the signs of the effects of credit spreads and asset volatility on investment are unambiguous. Increases in credit spreads cs signal increases in the negative effect of the debt-overhang burden, and increases in asset volatility σ signal increases in the positive effect of the option value of equity.

In the Internet Appendix Section D, we show that our results hold in a setting with leverage dynamics with endogenous debt issuance. We extend the framework of DeMarzo and He (2020) to include an investment function and show that Proposition 1 still holds.

[Figure 4 here.]

In Figure 4, we illustrate the optimal investment function with a log-normal distribution of risk. The comparative statics in Proposition 1 are clearly illustrated for this standard risk distribution.

We now compare the straightforward roles of credit spreads and asset volatility in determining investment with the more intricate relation between *leverage* and asset volatility in investment decisions. This analysis exemplifies why credit spreads and asset volatility are clean empirical measures of the effects of debt overhang and option value on investment decisions. It also shows why controlling for credit spreads is superior to controlling for leverage in empirical studies of investment.

Proposition 2 (Leverage and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to leverage is given by*

$$\frac{\partial i}{\partial b} = -\frac{\underline{z} f(\underline{z};\sigma)}{\varphi(i,\underline{z},\sigma)} \leq 0, \tag{7}$$

where

$$\varphi(i,\underline{z},\sigma) \equiv \phi_{ii}(i)i - \underline{z}^2 f(\underline{z};\sigma) > 0.$$

Holding leverage constant, the partial derivative of investment with respect to volatility is given by

$$\frac{\partial i}{\partial \sigma} = \frac{i}{\varphi(i,\underline{z},\sigma)}\big(\nu(\underline{z},\sigma) - \underline{z} F_\sigma(\underline{z};\sigma)\big). \tag{8}$$

Proposition 2 shows that if, instead of controlling for credit spreads cs, we instead control for leverage b, the elasticities of investment become more intricate. In Equation (7), the numerator still represents the marginal product lost to default. In the denominator, the term φ captures the feedback loop between investment and default decisions. Following a decrease in investment, shareholders default more often as output and incentives to pay back the debt decrease. That additional force was not present in Proposition 1, since changing credit spreads $cs(\underline{z};\sigma)$ controls for the default decision $\underline{z}$ directly. Holding leverage constant instead controls for $b = i\underline{z}$ (see the first-order condition for $\underline{z}$ in Equation (3)), which is a function of both i and $\underline{z}$. This term φ is always positive due to the second-order conditions for a maximum, and the sign of the effect of leverage on investment, holding asset volatility constant, is always negative.

However, turning to the effect of asset volatility on investment, holding leverage constant, the sign now becomes ambiguous. Intuitively, there are two effects of increasing asset volatility while holding leverage constant. The first is that the option value of investment increases. The second is that the debt-overhang problem also increases. To hold leverage $b = i\underline{z}$ constant as asset volatility increases, the default threshold $\underline{z}$ must change and the distance to default could shrink faster than the increase in the option value. The term $\nu(\underline{z}, \sigma) - \underline{z}F_\sigma(\underline{z}; \sigma)$ captures this horse race between option value and what is lost in default as asset volatility increases. If the option value effect is strong, this term will be positive. If the increase in asset volatility moves a large probability mass into the default region ($\underline{z}F_\sigma(\underline{z}; \sigma) > 0$), this term can be negative. In other words, when the marginal increase in investment returns lost to default $zF_\sigma(\underline{z}; \sigma)$ dominates the marginal increase in the option value $\nu(\underline{z}, \sigma)$, shareholders reduce investment following an increase in volatility.

[Figure 5 here.]

Which effect dominates is highly dependent on the shape of the distribution $F(z; \sigma)$. In Figure 5, we plot the optimal investment decision as a function of asset volatility σ when holding leverage b constant and assuming a log-normal distribution for z. The monotonic relation between leverage and investment, holding asset volatility constant, is clear. However, the relation between investment and asset volatility, holding leverage constant, is nonmonotonic. When leverage is high, the option-value effect dominates, while the debt-overhang effect dominates when leverage is low.

Next, we consider the changes in investment when observing credit spreads and equity volatility, and illustrate the intuition our model suggests for the empirical finding whereby the sign of the elasticity of investment with respect to equity volatility changes sign in the cross-section of more and less distressed firms. First, we define equity volatility as

$$\sigma^e(\underline{z}, \sigma) \equiv \frac{\sigma}{\mathbb{E}\left[(z - \underline{z})^+\right]}.$$

Thus, equity is simply levered asset volatility,[28] where the denominator represents the impact

[28] Given our model, equity volatility could include the impact of investment and the truncation of equity volatility above the default threshold and be given by

$$\frac{\sqrt{\mathbb{V}ar\left[i(z - \underline{z})^+ - \phi(i)\right]}}{\mathbb{E}\left[i(z - \underline{z})^+ - \phi(i)\right]}.$$

In this case, our key insight—equity volatility is an ambiguous signal for investment—still holds, but the elasticities become undecipherable.

of leverage on equity volatility. If the debt burden from leverage b increases, then the default threshold $\underline{z}$ increases as well and equity's expected payoff per unit of capital $\mathbb{E}\left[(z-\underline{z})^{+}\right]$ decreases. Conversely, if the firm is funded entirely by equity ($b=0$), then $\underline{z}$ is equal to zero—the lower bound of the support. In that case, equity volatility is equal to asset volatility ($\sigma^{e}(\underline{z},\sigma)=\sigma$), since $\mathbb{E}[z]=1$.

Proposition 3 (Credit Spreads and Equity Volatility)**.** *Holding equity volatility constant, the partial derivative of investment with respect to credit spreads is given by*

$$\frac{\partial i}{\partial cs}=-\frac{\underline{z}(1-F(\underline{z};\sigma))^{2}}{\phi_{ii}(i)}\xi_{cs}(\underline{z},\sigma), \tag{9}$$

where

$$\xi_{cs}(\underline{z},\sigma)\equiv\frac{\int_{\underline{z}}^{\infty}z/\underline{z}dF_{\sigma}(z;\sigma)\sigma^{e}_{\underline{z}}(\underline{z},\sigma)+f(\underline{z};\sigma)\sigma^{e}_{\sigma}(\underline{z},\sigma)}{f(\underline{z};\sigma)\sigma^{e}_{\sigma}(\underline{z},\sigma)-F_{\sigma}(\underline{z};\sigma)\sigma^{e}_{\underline{z}}(\underline{z},\sigma)}.$$

Holding credit spreads constant, the partial derivative of investment with respect to equity volatility is given by

$$\frac{\partial i}{\partial\sigma^{e}}=\frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)}\xi_{\sigma^{e}}(\underline{z},\sigma), \tag{10}$$

where

$$\xi_{\sigma^{e}}(\underline{z},\sigma)\equiv\frac{f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^{e}_{\sigma}(\underline{z},\sigma)-F_{\sigma}(\underline{z};\sigma)\sigma^{e}_{\underline{z}}(\underline{z},\sigma)}.$$

We define the wedges ξ_{cs} and $\xi_{\sigma^{e}}$ to clarify the distinction between Propositions 3 and 1. It is easiest to start with the relation between investment and equity volatility, holding credit spreads constant. To understand the additional complication that arises when equity as opposed to asset volatility is used as a signal of uncertainty, it is useful to look at the partial derivative of equity volatility with respect to asset volatility σ and the default threshold $\underline{z}$:

$$\sigma^{e}_{\sigma}(\underline{z},\sigma)=\frac{1}{\mathbb{E}\left[(z-\underline{z})^{+}\right]}-\frac{\sigma\nu(\underline{z},\sigma)}{\mathbb{E}\left[(z-\underline{z})^{+}\right]^{2}}\qquad\text{and}\qquad\sigma^{e}_{\underline{z}}(\underline{z},\sigma)=\frac{\sigma\left(1-F(\underline{z};\sigma)\right)}{\mathbb{E}\left[(z-\underline{z})^{+}\right]^{2}}\geq 0.$$

As shown in these equations, when the option value impact of asset volatility $\nu(\underline{z},\sigma)$ is large, equity volatility decreases following a positive shock to asset volatility. Indeed, the increase in the payoff to equityholders (denominator of σ^{e}) gets larger than the relative increase in asset volatility (numerator of σ^{e}). Add to that effect that the partial derivative of σ^{e} with respect to the default threshold is positive and to keep the credit spread cs constant, the default threshold $\underline{z}$ needs to decrease, and it is not surprising anymore that following a positive asset volatility shock, equity volatility might decrease. Corollary 1 makes that argument explicit.

Corollary 1 (Equity Volatility and Asset Volatility)**.** *If the total derivative of the default threshold with respect to asset volatility is such that*

$$\frac{d\underline{z}}{d\sigma} < \frac{\sigma\nu(\underline{z},\sigma) - \mathbb{E}\left[(z-\underline{z})^{+}\right]}{\sigma(1-F(\underline{z};\sigma))},$$

then the total derivative of equity volatility with respect to asset volatility is negative:

$$\frac{d\sigma^{e}(\underline{z},\sigma)}{d\sigma} < 0.$$

These additional forces are captured by the wedges ξ_{cs} and ξ_{σ^e}. The forces driving these wedges lead the signs of the elasticities of Proposition 3 to be highly dependent on the shape of the risk distribution F and the level of leverage and volatility of the firm, in contrast to the robustly positive signs of the elasticity for asset volatility in Proposition 1.

These nonmonotonicities also complicate the mapping of investment decisions in the (cs, σ^e)-space. Lemma 1 formally states this complication.

Lemma 1 (Existence of Credit Spread and Equity Volatility Pair)**.** *Given* $(cs, \sigma^e) \in [0,1] \times \mathbb{R}^{+}$*, there does not always exist a solution* $(\underline{z}, \sigma) \in \mathbb{R}^{+} \times \mathbb{R}^{+}$ *to the following system of two equations:*

$$cs = \frac{F(\underline{z};\sigma)}{1-F(\underline{z};\sigma)}, \qquad \sigma^{e} = \frac{\sigma}{\mathbb{E}\left[(z-\underline{z})^{+}\right]}.$$

Furthermore, the solution might not be unique.

Given the result in Lemma 1, to illustrate the results for equity volatility, instead of directly plotting investment as a function of cs and σ^e, we show the sign of the wedges in the (cs, σ)-space for two distributions: a log-normal distribution and a log-normal mixture distribution. Figure 6 presents the results. In the case of the log-normal distribution, the wedges are either: (a) both positive (white area), implying that the signs of the elasticities are identical to those in Proposition 1; (b) both negative (light gray area), implying that the signs of the elasticities are opposite to those in Proposition 1; (c) or the wedge for credit spreads is negative and the wedge for equity volatility is positive (dark grey area).

[Figure 6 here.]

The mixture distribution is a mixture of two log-normal distributions (see the caption of Figure 6 for details) and is therefore bimodal. This risk distribution could correspond to a technology in which the productivity shock is drawn from either a bad (low mean) or a good (high mean) distribution. In this case, an increase in uncertainty could have a large effect on the option value without substantially impacting default risk—the dark gray area, where the elasticities with respect to credit spreads and equity volatility are both negative. In the example in Figure 6, fixing asset volatility at 0.3, the elasticity of investment with respect to equity volatility is positive for a low credit spread levels ($cs \leq 0.15$) and negative for a high levels of credit spreads ($0.30 \leq cs \leq 0.8$). Thus, in this example, we observe the same change of sign for equity volatility in the cross-section of firms with high and low credit spreads as we documented in our empirical setting. Also consistent with our empirical results, the elasticity with respect to credit spreads is negative in both of these intervals.

Our model can also speak to whether credit spreads can effectively summarize the information in Tobin's q. When debt and equityholders are not aligned, the answer is no. In our model, as in most standard models, q fully summarizes the marginal benefit of investment. Our model also shows that while credit spreads can effectively capture the disincentive to invest when some output is lost below the default threshold due to debt in the capital structure, the information in asset volatility summarizing the option value to equity holders above the default threshold is missing from credit spreads. This makes sense, since bondholders receive a constant payoff above the default threshold.

We illustrate our simple model's prediction for the relationship between Tobin's q and investment. As in Philippon (2009), we define Tobin's q by the market value of the firm scaled by its end-of-period assets:

$$q = \int_{\underline{z}}^{\infty} z dF(z;\sigma) = \phi_i(i).$$

As is the case in most models of investment, Tobin's q equals the marginal cost of investment, $\phi_i(i)$, as implied by the first-order condition for investment in Equation (2). Thus, observing q directly pins down the investment level i, and credit spreads and asset volatility have no additional predictive information for investment.[29] Of course, in the presence of measurement error, other signals for investment incentives not perfectly correlated with q

[29] This result also holds if debt holders can recover a fraction α of the firm's capital after default. Indeed, in that case Tobin's q becomes

$$q = \int_{\underline{z}}^{\infty} z dF(z;\sigma) + \alpha \int_{0}^{\underline{z}} z dF(z;\sigma) = (1-\alpha)\phi_i(i) + \alpha,$$

since $\int_0^\infty z dF(z;\sigma) = 1$.

can have additional predictive content, as in our empirical analysis.

We next explain why credit spreads are an incomplete signal for q because, while they can capture debt overhang, they don't capture option value. To see this, suppose F is a normal distribution. In this case we have:

$$q = (1 - F(\underline{z}; \sigma)) + \sigma^2 f(\underline{z}; \sigma). \tag{11}$$

The first term in Equation (11) captures the fact that returns to investment are lost below the default threshold. This can be captured by credit spreads as $cs = F/(1 - F)$. However, the second term depends on asset volatility and this option-value effect is not captured by credit spreads.

5 Conclusion

We provide evidence and a simple model supporting the idea that bond market measures predict economic activity better than equity market measures because of the precise nonlinear transformation of leverage and asset volatility that credit spreads and equity volatility—the most commonly used measures of bond and equity markets used in macroeconomic forecasting—represent. Equity volatility is asset volatility levered up by the size of the firm's equity cushion while credit spreads measure the distance to default (the effective size of the firm's equity cushion). Thus, both credit spreads and equity volatility are driven by leverage and asset volatility. However, while higher credit spreads predict lower investment for all firms, equity volatility is an ambiguous signal for investment. We show that for healthy firms, higher equity volatility signals greater option value and better investment opportunities. But for more distressed firms, greater equity volatility exacerbates the debt-overhang problem. Importantly, both in our empirical analysis and model, *asset* volatility is an unambiguous signal for investment: a *higher* level of asset volatility predicts a *higher* rate of investment.

Our theoretical and empirical explanation for these facts build on one violation of Modigliani and Miller (1958), namely a separation of debt and equity holders and a resulting misalignment of investment incentives. While credit spreads are a clean signal of the negative effect of debt overhang on investment, and asset volatility is a clean signal of the positive effect of option value on investment, the information in equity volatility and leverage is mixed and ambiguous. Our study generally sheds light on the theoretical and empirical structural relationships between credit spreads, asset volatility, equity volatility (and changes in volatility), Tobin's q, and the fundamental and non-fundamental components of credit spreads.

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Figures and Tables

Figure 1: This figure presents the elasticity of investment with respect to equity volatility across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log credit spread: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in Column 7 of Table 2 on $\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$, the elasticity at each cutoff point is computed as $\beta_1 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

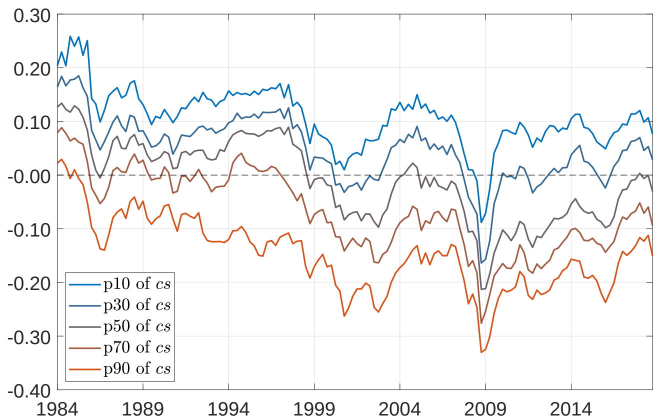


Figure 2: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log equity volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in Column 7 of Table 2 on $\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$, the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

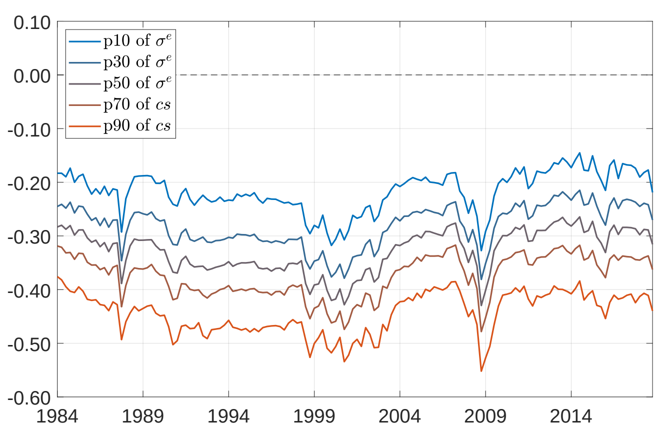
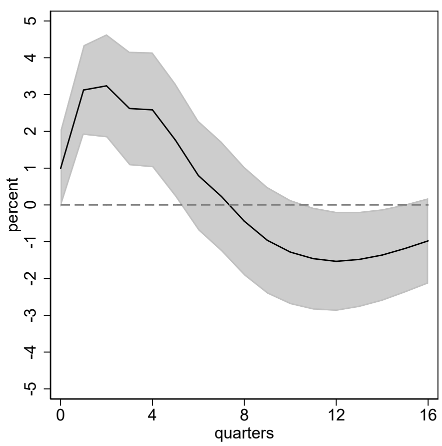
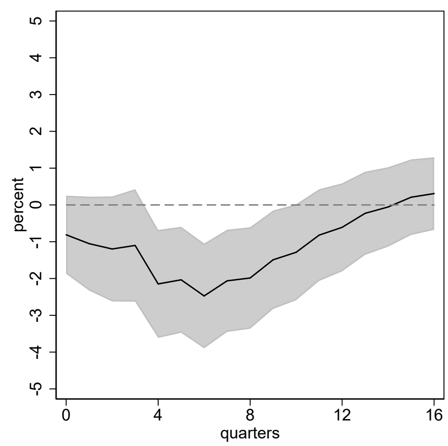


Figure 3: This figure plots the impulse responses of investment to an orthogonalized 1 standard deviation shock to asset volatility and credit spread. The VAR is estimated using four lags of each endogenous variable. Subfigures (a) and (b) correspond to the recursive ordering: (cs, σ, I/K). Subfigures (c) and (d) correspond to the recursive ordering: (σ, cs, I/K). The shaded bands represent the 95% confidence interval.

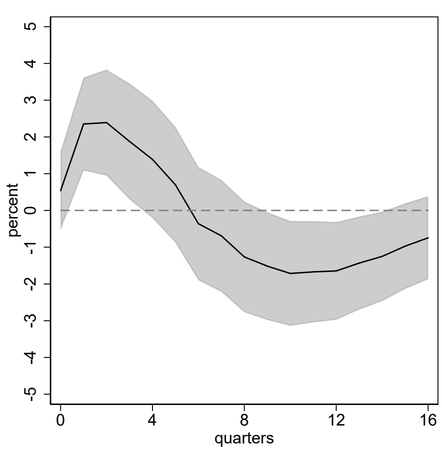
(a) Asset volatility



(b) Credit spread



(c) Asset volatility



(d) Credit spread

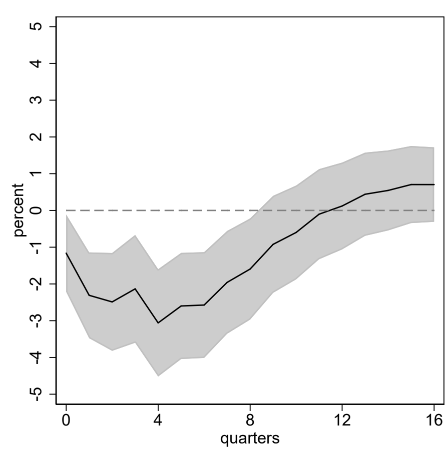


Figure 4: Optimal investment with log-normal distribution. The left figure shows the level of investment i as a function of credit spreads cs for different levels of asset volatility σ, while the right figure shows the level of investment i as a function of asset volatility σ for different levels of credit spreads cs. The adjustment cost function is given by: $\phi(i) = i^{\gamma}$ with $\gamma = 2$.

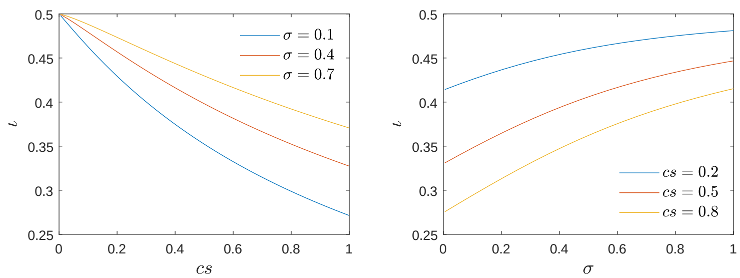


Figure 5: Optimal investment with log-normal distribution. The left figure shows the level of investment i as a function of leverage b for different levels of asset volatility σ, while the right figure shows the level of investment i as a function of asset volatility σ for different levels of leverage b. The adjustment cost function is given by: $\phi(i) = i^{\gamma}$ with $\gamma = 2$.

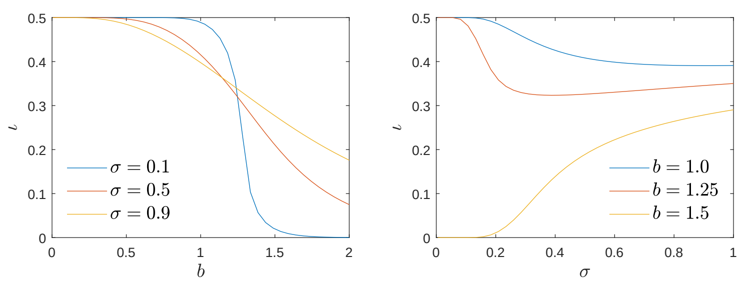


Figure 6: Sign of Wedges for Log-Normal and Log-Normal Mixture. These figures show the sign of the wedges of Proposition 3 in the (cs,σ)-space for the log-normal distribution (left) and a log-normal mixture distribution (right). The mixture distribution is a mixture of two log-normal distributions drawn with 50% probability with parameters $(\mu_1, \hat{\sigma})$ and $(\mu_2, \hat{\sigma})$ such that the unconditional mean of z is 1 and the standard deviation of z is σ. We set $\hat{\sigma} = 0.2$ in this example.

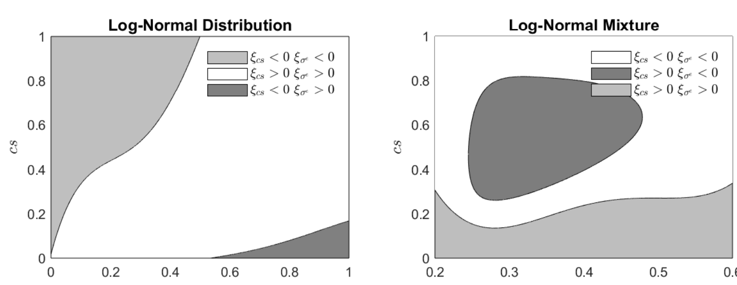


Table 1: This table documents the summary statistics of our firm-quarter variables. See Appendix A for a detailed variable description.

	Method	Coverage	N	Mean	SD	Min	Max
Credit spreads cs	GZ spread as in Gilchrist and Zakrajšek (2012)	1984-2018	48672	296.35	240.01	8.73	1912.25
Fair value spreads $\widehat{cs}$	Computed using Moody's EDF measure	1984-2018	39167	160.09	251.53	9.15	1821.42
Idiosyncratic equity volatility σ^e	Computed using realized idiosyncratic equity returns	1984-2018	48672	0.28	0.16	0.06	2.12
Implied equity volatility $\widehat{\sigma}^e$	Implied by at-the-money 30-day forward put options	1996-2018	21262	0.34	0.15	0.12	1.58
Total equity volatility σ^{eT}	Computed using realized equity returns	1984-2018	48472	0.35	0.19	0.07	2.26
Idiosyncratic asset volatility σ	Deleveraged by firm-level leverage	1984-2018	48029	0.14	0.07	0.01	0.77
Implied asset volatility $\widehat{\sigma}$	Deleveraged by firm-level leverage	1996-2018	21163	0.19	0.08	0.03	0.78
Total asset volatility σ^T	Deleveraged by firm-level leverage	1984-2018	48040	0.18	0.09	0.02	1.06
Merton's idiosyncratic asset volatility $\widetilde{\sigma}$	Deleveraged using a Merton approach	1984-2018	45886	0.18	0.10	0.03	1.53
Distance to default DD	Merton DD as in Bharath and Shumway (2008)	1984-2018	46348	5.85	3.24	-1.90	23.67
Leverage $[MA/ME]$	Ratio of market value of assets to market value of equity	1984-2018	48086	2.23	1.72	1.04	50.47
Return on equity	Cumulative return realized over the quarter	1984-2018	47919	0.15	0.50	-0.98	16.73
Tangibility ratio	Capital stock divided by total assets	1984-2018	38423	0.69	0.40	0.02	3.98
Sales ratio	Sales divided by lagged capital	1984-2018	47944	1.52	2.81	0.03	49.88
Income ratio	Operating income divided by lagged capital	1984-2018	45313	0.20	0.32	-3.33	6.34
Tobin's q	Market value divided by replacement value of capital	1984-2018	36431	2.49	3.79	-1.37	63.86

Table 2: This table documents the relationship between investment, equity volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.147***		-0.051***	0.053***	-0.036*	-0.109***	0.819***	0.619***
	(-8.74)		(-3.53)	(2.97)	(-1.76)	(-4.35)	(9.99)	(7.26)
$\log cs_{i,t-1}$		-0.273***	-0.259***	-0.134***	-0.283***	-0.426***	-0.465***	-0.313***
		(-13.43)	(-13.06)	(-3.73)	(-6.26)	(-10.19)	(-17.28)	(-11.00)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$							-0.156***	-0.111***
							(-10.55)	(-7.15)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	48642	48642	48642	15546	15489	14749	48642	31714
R-squared	0.107	0.132	0.133	0.145	0.135	0.123	0.141	0.214

Table 3: This table documents the relationship between investment, total equity volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^{eT}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^{eT}_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma^{eT}_{i,t-1}$	-0.147***		-0.039**	0.110***	-0.031	-0.118***	0.854***	0.616***
	(-7.62)		(-2.31)	(5.33)	(-1.20)	(-4.16)	(10.67)	(7.35)
$\log cs_{i,t-1}$		-0.273***	-0.263***	-0.140***	-0.286***	-0.425***	-0.439***	-0.287***
		(-13.43)	(-13.20)	(-3.92)	(-6.29)	(-10.05)	(-17.96)	(-11.00)
$\log \sigma^{eT}_{i,t-1} \times \log cs_{i,t-1}$							-0.160***	-0.109***
							(-11.05)	(-7.09)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	48469	48642	48469	15463	15463	14705	48469	31599
R-squared	0.106	0.132	0.132	0.147	0.135	0.122	0.141	0.213

Table 4: This table documents the relationship between investment, implied equity volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \hat{\sigma}^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \widehat{\sigma}^e_{i,t-1}$	-0.294***		-0.086**	0.124**	-0.053	-0.267***	1.219***	1.056***
	(-7.15)		(-2.01)	(2.47)	(-0.78)	(-3.71)	(7.74)	(6.37)
$\log cs_{i,t-1}$		-0.273***	-0.301***	-0.195***	-0.334***	-0.491***	-0.545***	-0.372***
		(-13.43)	(-9.19)	(-3.93)	(-4.71)	(-5.82)	(-13.44)	(-8.44)
$\log \widehat{\sigma}^e_{i,t-1} \times \log cs_{i,t-1}$							-0.231***	-0.190***
							(-8.49)	(-6.47)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	21549	48642	21549	8580	7172	4840	21549	14747
R-squared	0.118	0.132	0.144	0.145	0.133	0.153	0.157	0.238

Table 5: This table documents the relationship between investment, equity volatility, and fair value spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log \hat{cs}_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log \hat{cs}_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.139***		0.021	0.087***	0.032	-0.043*	0.365***	0.308***
	(-7.57)		(1.42)	(4.77)	(1.47)	(-1.70)	(8.28)	(6.80)
$\log \widehat{cs}_{i,t-1}$		-0.170***	-0.174***	-0.116***	-0.107***	-0.240***	-0.270***	-0.169***
		(-16.63)	(-17.21)	(-4.68)	(-4.67)	(-12.93)	(-17.25)	(-9.90)
$\log \sigma^e_{i,t-1} \times \log \widehat{cs}_{i,t-1}$							-0.078***	-0.063***
							(-8.15)	(-6.24)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	39162	39162	39162	12465	12461	12095	39162	25481
R-squared	0.107	0.152	0.152	0.148	0.133	0.151	0.159	0.218

Table 6: This table documents the relationship between fair value spreads, the residual in credit spreads after removing the effect of fair value spread, equity volatility and investment at the firm-quarter level from 1984 to 2018. The residual $e_{i,t}$ in column (2) is obtained from the regression $\log cs_{i,t} = \beta \log \widehat{cs}_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$, with R^2 of 58% (the R^2 is 50% without any fixed effects) and the residual $e_{i,t}$ in column (5) is obtained from the regression $\log cs_{i,t} = \beta_1 \log \widehat{cs}_{i,t} + \beta_2 \log \sigma^e_{i,t} + \eta_i + \lambda_t \epsilon_{i,t}$, with R^2 of 60% (the R^2 is 52% without any fixed effects). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log \widehat{cs}_{i,t-1} + \beta_3 e_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$
$\log \sigma^e_{i,t-1}$			-0.139***	0.021	-0.138***
			(-7.57)	(1.42)	(-7.66)
$\log \widehat{cs}_{i,t-1}$	-0.170***			-0.174***	
	(-16.63)			(-17.21)	
$e_{i,t-1}$		-0.147***			-0.153***
		(-6.20)			(-6.53)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Observations	39162	39162	39162	39162	39162
R-squared	0.152	0.107	0.107	0.152	0.114

Table 7: This table documents the relationship between investment, asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma_{i,t-1}$	0.233***		0.208***	0.175***	0.163***	0.201***	0.172***
	(15.52)		(13.92)	(9.60)	(7.82)	(8.12)	(11.07)
$\log cs_{i,t-1}$		-0.271***	-0.252***	-0.120***	-0.280***	-0.420***	-0.156***
		(-13.34)	(-12.65)	(-3.36)	(-6.16)	(-10.29)	(-7.70)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	48061	48061	48061	15475	15320	14433	31445
R-squared	0.121	0.132	0.148	0.157	0.144	0.132	0.221

Table 8: This table documents the relationship between investment, total asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^T_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^T_{i,t-1}$	0.268***		0.235***	0.230***	0.193***	0.215***	0.194***
	(16.51)		(14.36)	(10.59)	(8.07)	(7.92)	(10.90)
$\log cs_{i,t-1}$		-0.271***	-0.245***	-0.117***	-0.277***	-0.410***	-0.154***
		(-13.38)	(-12.33)	(-3.22)	(-6.13)	(-10.00)	(-7.60)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	48065	48065	48065	15474	15331	14429	31418
R-squared	0.126	0.132	0.151	0.164	0.147	0.133	0.221

Table 9: This table documents the relationship between investment, implied asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Implied asset volatility is deleveraged equity volatility implied from options. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \widehat{\sigma}_{i,t-1}$	0.306***		0.289***	0.291***	0.280***	0.185***	0.222***
	(9.59)		(9.21)	(6.13)	(5.16)	(3.29)	(6.03)
$\log cs_{i,t-1}$		-0.333***	-0.324***	-0.174***	-0.349***	-0.628***	-0.178***
		(-10.77)	(-10.54)	(-3.65)	(-5.20)	(-7.33)	(-5.87)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	21450	21450	21450	8595	7111	4774	14759
R-squared	0.123	0.143	0.161	0.161	0.145	0.165	0.239

Table 10: This table documents the relationship between investment, level of asset volatility, shock to asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \Delta \log \sigma_{i,t-1} + \beta_3 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$	0.294***	0.263***	0.208***		0.214***
	(15.11)	(13.56)	(13.92)		(10.38)
$\Delta \log \sigma_{i,t-1}$	-0.132***	-0.119***		0.010	-0.085***
	(-11.13)	(-10.16)		(1.61)	(-6.55)
$\log cs_{i,t-1}$		-0.249***	-0.252***	-0.271***	-0.157***
		(-12.53)	(-12.65)	(-13.40)	(-7.85)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Controls					✓
Observations	47568	47568	48061	47568	31260
R-squared	0.126	0.152	0.148	0.132	0.223

Table 11: This table documents the relationship between investment, asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018 for different lags and leads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \sum_{\tau=-4}^{4} \beta_\tau \log \sigma_{i,t+\tau} + \beta_5 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.245***	-0.160***
	(-11.85)	(-8.11)
$\log \sigma_{i,t-4}$	0.071***	0.063***
	(6.31)	(5.25)
$\log \sigma_{i,t-3}$	0.046***	0.033***
	(4.99)	(3.07)
$\log \sigma_{i,t-2}$	0.058***	0.045***
	(6.68)	(4.17)
$\log \sigma_{i,t-1}$	0.097***	0.098***
	(11.17)	(9.46)
$\log \sigma_{i,t}$	0.034***	0.014
	(3.85)	(1.21)
$\log \sigma_{i,t+1}$	0.022***	0.011
	(2.69)	(1.09)
$\log \sigma_{i,t+2}$	0.016*	0.012
	(1.95)	(1.22)
$\log \sigma_{i,t+3}$	0.008	0.009
	(0.91)	(0.85)
$\log \sigma_{i,t+4}$	-0.009	-0.012
	(-0.89)	(-1.07)
Firm FE	✓	✓
Time FE	✓	✓
Controls		✓
Observations	41051	27675
R-squared	0.161	0.230

Table 12: This table presents the loadings of equity volatility and credit spreads on asset volatility (derived from Merton's model) and leverage at the firm-quarter level from 1984 to 2018. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. All variables are standardized to have mean zero and unit variance. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level. The panels below present the partial R^2 given time and firm fixed effects, that is, the percentage reduction in the residual sum of squares (RSS) by adding each variable in addition to time and firm fixed effects.

$$\log y_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	$\log \sigma^e_{i,t}$	$\log cs_{i,t}$	Panel B: Changes	$\Delta \log \sigma^e_{i,t}$	$\Delta \log cs_{i,t}$
$\log \widetilde{\sigma}_{i,t}$	0.817***	0.131***	$\Delta \log \widetilde{\sigma}_{i,t}$	0.707***	0.020***
	(78.99)	(13.51)		(82.16)	(3.50)
$\log[MA/ME]_{i,t}$	0.419***	0.438***	$\log[MA/ME]_{i,t}$	0.089***	0.181***
	(46.56)	(30.02)		(20.56)	(23.87)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	45401	45401	Observations	44491	44491
R-squared	0.869	0.575	R-squared	0.787	0.310

Partial R^2

Panel C: Levels	$\log \sigma^e_{i,t}$	$\log cs_{i,t}$	Panel D: Changes	$\Delta \log \sigma^e_{i,t}$	$\Delta \log cs_{i,t}$
$\log \widetilde{\sigma}_{i,t}$	0.574	0.003	$\Delta \log \widetilde{\sigma}_{i,t}$	0.736	0.000
$\log[MA/ME]_{i,t}$	0.110	0.210	$\Delta \log[MA/ME]_{i,t}$	0.010	0.022

Table 13: This table presents the loadings of equity volatility on asset volatility (derived from Merton's model) and leverage at the firm-quarter level from 1984 to 2018 across subsamples sorted by terciles every quarter on credit spreads. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. All variables are standardized to have mean zero and unit variance. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level. The panels below present the partial R^2 given time and firm fixed effects, that is, the percentage reduction in the residual sum of squares (RSS) by adding each variable in addition to time and firm fixed effects.

$$\log \sigma^e_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	low cs	mid cs	high cs	Panel B: Changes	low cs	mid cs	high cs
$\log \widetilde{\sigma}_{i,t}$	0.882***	0.850***	0.715***	$\Delta \log \widetilde{\sigma}_{i,t}$	0.756***	0.700***	0.673***
	(63.34)	(66.94)	(40.51)		(52.75)	(49.77)	(52.72)
$\log[MA/ME]_{i,t}$	0.388***	0.398***	0.402***	$\Delta \log[MA/ME]_{i,t}$	0.078***	0.088***	0.091***
	(21.50)	(24.35)	(40.15)		(5.35)	(10.94)	(17.55)
Firm FE	✓	✓	✓	Firm FE	✓	✓	✓
Time FE	✓	✓	✓	Time FE	✓	✓	✓
Observations	15383	15194	14824	Observations	15155	14922	14414
R-squared	0.919	0.874	0.813	R-squared	0.839	0.782	0.755

Partial R^2

Panel C: Levels	low cs	mid cs	high cs	Panel D: Changes	low cs	mid cs	high cs
$\log \widetilde{\sigma}_{i,t}$	0.766	0.680	0.445	$\Delta \log \widetilde{\sigma}_{i,t}$	0.800	0.733	0.688
$\log[MA/ME]_{i,t}$	0.003	0.016	0.167	$\Delta \log[MA/ME]_{i,t}$	0.004	0.005	0.018

Table 14: This table presents the loadings of credit spreads on asset volatility (derived from Merton's model) and leverage at the firm-quarter level from 1984 to 2018 across subsamples sorted by terciles every quarter on credit spreads. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. All variables are standardized to have mean zero and unit variance. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level. The panels below present the partial R^2 given time and firm fixed effects, that is, the percentage reduction in the residual sum of squares (RSS) by adding each variable in addition to time and firm fixed effects.

$$\log cs_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	low cs	mid cs	high cs	Panel B: Changes	low cs	mid cs	high cs
$\log \widetilde{\sigma}_{i,t}$	0.039***	0.042***	0.067***	$\Delta \log \widetilde{\sigma}_{i,t}$	0.013	0.017**	0.031***
	(4.72)	(6.18)	(7.52)		(1.26)	(2.11)	(3.90)
$\log[MA/ME]_{i,t}$	0.204***	0.149***	0.265***	$\Delta \log[MA/ME]_{i,t}$	0.081***	0.133***	0.198***
	(10.21)	(11.80)	(26.76)		(3.40)	(8.71)	(22.28)
Firm FE	✓	✓	✓	Firm FE	✓	✓	✓
Time FE	✓	✓	✓	Time FE	✓	✓	✓
Observations	15383	15194	14824	Observations	15155	14922	14414
R-squared	0.919	0.874	0.813	R-squared	0.308	0.359	0.428

Partial R^2

Panel C: Levels	low cs	mid cs	high cs	Panel D: Changes	low cs	mid cs	high cs
$\log \widetilde{\sigma}_{i,t}$	0.000	0.001	0.000	$\Delta \log \widetilde{\sigma}_{i,t}$	0.000	0.000	0.001
$\log[MA/ME]_{i,t}$	0.039	0.048	0.245	$\Delta \log[MA/ME]_{i,t}$	0.001	0.009	0.064

Table 15: This table documents the relationship between investment, asset volatility , leverage, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log[MA/ME]_{i,t-1} + \beta_3 \log cs_{i,t-1} + \beta_4 \log \sigma_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma_{i,t-1}$	0.233***		0.031*	0.069***	0.008	0.015	0.604***	0.441***
	(15.52)		(1.89)	(3.96)	(0.36)	(0.57)	(6.33)	(4.58)
$\log[MA/ME]_{i,t-1}$		-0.522***	-0.504***	-0.526***	-0.547***	-0.467***	-0.444***	-0.387***
		(-20.86)	(-18.38)	(-8.27)	(-13.03)	(-13.37)	(-16.79)	(-10.39)
$\log cs_{i,t-1}$							-0.334***	-0.223***
							(-8.23)	(-5.19)
$\log \sigma_{i,t-1} \times \log cs_{i,t-1}$							-0.103***	-0.068***
							(-5.99)	(-3.87)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	48061	47650	47650	15233	15267	14342	47650	31212
R-squared	0.121	0.168	0.168	0.176	0.171	0.149	0.176	0.234

Table 16: This table documents the relationship between investment, asset volatility, credit spreads, and Tobin's q at the firm-quarter level from 1984 to 2018. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log q_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Without controls

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$	0.233***		0.208***			0.174***
	(15.52)		(13.92)			(10.55)
$\log cs_{i,t-1}$		-0.273***	-0.252***		-0.178***	-0.173***
		(-13.43)	(-12.65)		(-7.82)	(-7.82)
$\log q_{i,t-1}$				0.211***	0.178***	0.156***
				(15.37)	(12.79)	(11.09)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Observations	48061	48642	48061	35879	35879	35563
R-squared	0.121	0.132	0.148	0.157	0.170	0.180

Panel B: With controls

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$	0.217***		0.203***			0.172***
	(13.69)		(12.93)			(11.07)
$\log cs_{i,t-1}$		-0.195***	-0.182***		-0.156***	-0.156***
		(-9.50)	(-9.12)		(-7.40)	(-7.70)
$\log q_{i,t-1}$				0.155***	0.125***	0.097***
				(11.25)	(9.21)	(7.24)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls	✓	✓	✓	✓	✓	✓
Observations	33655	33977	33655	31714	31714	31445
R-squared	0.198	0.197	0.212	0.200	0.209	0.221

Table 17: This table documents the relationship between equity volatility, the predictable component of credit spreads, the excess bond premium, and investment at the firm-quarter level from 1984 to 2018. Following Gilchrist and Zakrajšek (2012), the excess bond premium $ebp_{i,t}$ is the quarterly average of the residual $\epsilon_{i,t}$ of a panel regression for credit spreads: $\log cs_{i,t} = \gamma' \mathbf{X}_{i,t} + \epsilon_{i,t}$. The vector of bond-specific characteristics $\mathbf{X}_{i,t}$ include the firm's distance-to-default; bond's amount outstanding, duration, coupon rate, industry fixed effects, and credit rating fixed effects; an indicator variable for callable bonds; the interactions of callability with these bond characteristics; the level, slope, and curvature of the Treasury yield curve; and the realized monthly volatility of the daily 10-year Treasury yield. Control variables for the regression of investment include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2[\log cs_{i,t-1} - \log ebp_{i,t-1}] + \beta_3 \log ebp_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$
$\log \sigma^e_{i,t-1}$	-0.147***	-0.149***	-0.044***	-0.038**	-0.042**	0.028
	(-8.74)	(-8.82)	(-2.64)	(-2.25)	(-2.45)	(1.52)
$\log ebp_{i,t-1}$		-0.219***			-0.097***	
		(-10.45)			(-4.23)	
$\log cs_{i,t-1} - \log ebp_{i,t-1}$			-0.225***			-0.162***
			(-8.06)			(-5.41)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls				✓	✓	✓
Observations	48642	46653	46421	31714	30500	30352
R-squared	0.107	0.117	0.117	0.200	0.203	0.206

Table 18: This table documents the relationship between asset volatility, the predictable component of credit spreads, the excess bond premium, and investment at the firm-quarter level from 1984 to 2018. Following Gilchrist and Zakrajšek (2012), the excess bond premium $ebp_{i,t}$ is the quarterly average of the residual $\epsilon_{i,t}$ of a panel regression for credit spreads: $\log cs_{i,t} = \gamma' \mathbf{X}_{i,t} + \epsilon_{i,t}$. The vector of bond-specific characteristics $\mathbf{X}_{i,t}$ include the firm's distance-to-default; bond's amount outstanding, duration, coupon rate, industry fixed effects, and credit rating fixed effects; an indicator variable for callable bonds; the interactions of callability with these bond characteristics; the level, slope, and curvature of the Treasury yield curve; and the realized monthly volatility of the daily 10-year Treasury yield. Control variables for the regression of investment include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 [\log cs_{i,t-1} - \log ebp_{i,t-1}] + \beta_3 \log ebp_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$		0.213***	0.244***		0.167***	0.198***
		(13.36)	(15.75)		(9.92)	(11.85)
$\log ebp_{i,t-1}$	-0.211***	-0.168***		-0.093***	-0.065***	
	(-10.02)	(-7.90)		(-4.08)	(-2.85)	
$\log cs_{i,t-1} - \log ebp_{i,t-1}$			-0.260***			-0.186***
			(-10.05)			(-6.80)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls				✓	✓	✓
Observations	45879	45879	45879	30103	30103	30103
R-squared	0.110	0.127	0.139	0.204	0.213	0.221

Table 19: This table documents the relationship between investment, equity volatility, and credit spreads at the firm-quarter level from 1984 to 2018 for different levels of research and development and investment ratios. The low (high) R&D category corresponds to firms sorted below (above) the quarterly median of R&D. The low (high) investment category corresponds to firms sorted every quarter in the first (third) tercile of the investment ratio $[I/K]_{i,t}$. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) low R&D	(2) high R&D	(3) low R&D	(4) high R&D	(5) low inv.	(6) high inv.	(7) low inv.	(8) high inv.
$\log \sigma^e_{i,t-1}$	1.170***	0.834***	0.873***	0.493**	0.648***	0.619***	0.536***	0.500***
	(6.38)	(4.76)	(4.91)	(2.57)	(6.50)	(6.49)	(4.90)	(5.06)
$\log cs_{i,t-1}$	-0.563***	-0.466***	-0.385***	-0.235***	-0.398***	-0.261***	-0.321***	-0.148***
	(-8.41)	(-8.45)	(-5.09)	(-3.63)	(-12.69)	(-8.47)	(-9.12)	(-4.58)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.215***	-0.163***	-0.157***	-0.087**	-0.127***	-0.115***	-0.106***	-0.085***
	(-6.60)	(-4.91)	(-4.91)	(-2.37)	(-7.15)	(-6.41)	(-5.46)	(-4.54)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls			✓	✓			✓	✓
Observations	7930	7903	5451	5988	15271	15144	10218	9608
R-squared	0.187	0.189	0.246	0.249	0.190	0.108	0.231	0.166

Table 20: This table documents the relationship between investment, asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018 for different levels of research and development and investment ratios. The low (high) R&D category corresponds to firms sorted below (above) the quarterly median of R&D. The low (high) investment category corresponds to firms sorted every quarter in the first (third) tercile of the investment ratio $[I/K]_{i,t}$. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) low R&D	(2) high R&D	(3) low R&D	(4) high R&D	(5) low inv.	(6) high inv.	(7) low inv.	(8) high inv.
$\log \sigma_{i,t-1}$	0.206***	0.156***	0.154***	0.112***	0.131***	0.141***	0.106***	0.121***
	(6.72)	(5.71)	(4.32)	(4.11)	(7.25)	(9.41)	(5.15)	(7.15)
$\log cs_{i,t-1}$	-0.284***	-0.243***	-0.175***	-0.106***	-0.245***	-0.095***	-0.190***	-0.021
	(-5.37)	(-5.89)	(-3.22)	(-2.77)	(-12.51)	(-4.46)	(-8.58)	(-0.90)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls			✓	✓			✓	✓
Observations	7868	7788	5422	5931	15090	14895	10138	9491
R-squared	0.188	0.189	0.247	0.253	0.189	0.112	0.231	0.168

Table 21: This table presents instrumental variable results for equity and asset volatility at the firm-year level from 1990 to 2018. The IV approach follows that of Alfaro, Bloom, and Lin (2018). Realized annual volatility measures are instrumented with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks: the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker, Bloom, and Davis (2016). Annual realized equity volatility σ^e is the 12-month standard deviation of daily stock returns from CRSP. Annual realized asset volatility σ is the 12-month standard deviation of daily stock returns from CRSP unlevered using the daily market-to-book ratio of equity. Control variables include yearly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one year). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the 3-digit SIC industry.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$
$\log \sigma^e_{i,t-1}$	-0.649		-0.770*	
	(-1.27)		(-1.89)	
$\log \sigma_{i,t-1}$		0.598***		0.816***
		(3.15)		(3.05)
$\log cs_{i,t-1}$	-0.101	-0.335***	0.082	-0.237***
	(-0.55)	(-8.15)	(0.67)	(-4.97)
First Moments	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	4543	4649	3865	3993
Kleibergen-Paap F	2.864	6.289	2.190	4.343
Sargan-Hansen p-val	0.243	0.150	0.454	0.145

Table 22: This table documents the relationship between investment, equity volatility, and asset volatility at the firm-quarter level from 1984 to 2018 for firms without observable credit spreads. Columns 1, 2, 4, and 5 use the subsample of firms without observable credit spreads but positive leverage. Columns 4 and 6 use the subsample of firms with zero leverage: The Compustat variables dlcq and dlttq are both equal to 0. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 DD_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times DD_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) no bonds	(2) no bonds	(3) zero lev.	(4) no bonds	(5) no bonds	(6) zero lev.
$\log \sigma^e_{i,t-1}$	-0.184***		-0.153***	-0.143***		0.003
	(-7.26)		(-7.76)	(-9.57)		(0.11)
$\log \sigma^e_{i,t-1} \times DD_{i,t-1}$	0.011***			0.040***		
	(3.69)			(14.54)		
$\log \sigma_{i,t-1}$		0.327***			0.213***	
		(22.23)			(16.31)	
$DD_{i,t-1}$	0.053***	0.072***		0.079***	0.045***	
	(5.56)	(9.60)		(13.84)	(16.71)	
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls				✓	✓	✓
Observations	234804	230532	56206	103332	102101	20085
R-squared	0.066	0.076	0.053	0.159	0.161	0.167

Table 23: This table documents the relationship between equity volatility, credit spreads, and investment at the firm-year level from 1984 to 2018 for firms with different covenant tightness. The tight (slack) covenant sample includes firms with the covenant distance to threshold below (above) the median. Each observation is a firm-year. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) slack	(2) tight	(3) slack	(4) tight
$\log \sigma^e_{i,t-1}$	0.458***	0.556***		
	(3.13)	(3.52)		
$\log \sigma_{i,t-1}$			0.110***	0.139***
			(5.09)	(5.51)
$\log cs_{i,t-1}$	-0.251***	-0.408***	-0.131***	-0.296***
	(-5.01)	(-8.02)	(-3.98)	(-7.78)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.081***	-0.096***		
	(-2.95)	(-3.60)		
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls	✓	✓	✓	✓
Observations	10128	8664	10065	8553
R-squared	0.222	0.218	0.225	0.222

Appendices

A Data and Definitions

This section discusses the data sources used for the empirical analysis and the construction of variables.

Data Collection We use S&P's Compustat quarterly database from 1984:Q1 to 2018:Q4. We exclude firms in the financial sector (SIC code 6000 to 6999) and utility sector (SIC code 4900 to 4949); firms not in the panel for at least 3 years; and observations with missing investment rate or equity volatility and with negative sales. We use daily returns from the Center for Research in Security Prices (CRSP) database. Implied volatilities are from OptionMetrics data starting in 1996. Bond prices come from the Lehman/Warga (1984-2005) and ICE databases (1997-2018). These selection criteria yields 1,407 unique firms with 48,672 firm-quarter observations. To ensure that our results are not driven by extreme values, we trim every regression variable at the 1st and 99th percentiles. We provide summary statistics in Table 1 and describe how we construct our key variables below.

Investment We define investment rate as capital expenditures in quarter t scaled by net property, plant, and equipment in quarter $t-1$.

Equity Volatility Idiosyncratic equity volatility is constructed in two steps. For each firm i and fiscal quarter t, we extract daily idiosyncratic equity returns using the Carhart (1997) four-factor model:

$$r_{i,t_d} - r^f_{t_d} = \alpha_i + \boldsymbol{\beta}'_i \mathbf{f}_{t_d} + u_{i,t_d}, \tag{12}$$

where $t_d = 1, ..., D_t$ denotes trading days in the quarter. In equation (12), r_{i,t_d} is the daily equity return, $r^f_{t_d}$ is the risk-free rate, and $\boldsymbol{f}_{t_d}$ are the factors. We obtain the OLS residuals $\hat{u}_{i,t_d}$ by running the regression in equation (12) and define idiosyncratic equity volatility as the standard deviation of these residuals. The idiosyncratic equity volatility of firm i in quarter t is given by

$$\sigma^e_{i,t} = \sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t}\left(\hat{u}_{i,t_d} - \frac{1}{D_t}\sum_{d=1}^{D_t}\hat{u}_{i,t_d}\right)^2}. \tag{13}$$

Total equity volatility σ^{eT} is defined as the standard deviation of equity returns and is given by

$$\sigma^{eT}_{i,t} = \sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t}\left(r_{i,t_d} - \frac{1}{D_t}\sum_{d=1}^{D_t} r_{i,t_d}\right)^2}. \tag{14}$$

We only keep observations for quarters with more than 30 trading days ($D_t > 30$).

In addition to realized equity volatility measures, we use an implied equity volatility measure implied by at-the-money 30-day-forward put options equity volatility from OptionMetrics, denoted by $\hat{\sigma}^e$.

Credit Spreads We follow Gilchrist and Zakrajšek (2012) to compute bond-level credit spreads. First, we construct a theoretical risk-free bond that exactly replicates the promised cash flows. Suppose at time t a bond i of firm k promises cash flows $\{C(s), s = 1, 2, ..., S\}$, which are paid in time $\{t_s, s = 1, 2, ...S\}$ from today. We can calculate the price of the corresponding risk-free bond by discounting the promised cash flows as follows:

$$p_{it}^f[k] = \sum_{s=1}^{S} C(s) \exp\left(-y_t^T[t_s]t_s\right), \tag{15}$$

where $y_t^T[t_s]$ is the continuously compounded zero-coupon Treasury yield for time horizon t_s at time t from Gürkaynak, Sack, and Wright (2007).

Then we convert bond prices to yields [30] and define the credit spread of an individual bond as the difference between the yield of the actual bond and the yield of the corresponding risk-free bond: $cs_{i,t}[k] = y_{i,t}[k] - y_{i,t}^f[k]$. We then compute the credit spread of a firm i in quarter t as the quarterly average of the credit spreads of all bonds issued by that firm: $cs_{i,t} = \frac{1}{3}\sum_{m=t_1}^{t_3} \frac{1}{N_{i,m}^k} \sum_{k=1}^{N_{i,m}^k} cs_{i,m}[k]$, where t_n is the nth month of quarter t and $N_{i,m}^k$ is the number of bonds of firm i in month m.

Firm-level Leverage Firm-level leverage is defined as the ratio of the market value of assets to the market value of equity: $[MA/ME]_{i,t} = \frac{MA_{i,t}}{ME_{i,t}}$. Market value of equity ($ME_{i,t}$) is the product of share price and number of shares outstanding. Market value of assets ($MA_{i,t}$) is built as book value of assets plus market value of equity minu book value of equity. Following Davies, Fama, and French (2000), book value of equity is defined as book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit, minus book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) for book value of preferred stock. If this procedure generates missing values, we measure stockholders' equity as book value of common equity plus par value of preferred stock, or book value of assets minus total liabilities.

Return on equity, Tangibility, Sales, Income, and Tobin's q Return on equity is the cumulative equity return realized over a quarter. Tangibility is property, plant, and equipment divided by total assets. Sales and income ratios are given by sales and operating income before depreciation divided by lagged property, plant, and equipment. Following Erickson and Whited (2012), we construct the numerator of Tobin's q as book debt plus market value of equity minus book assets and the denominator is capital stock.

[30] From bond price p, we first compute yield-to-maturity as $YTM = \frac{CP+(FV-p)/M}{(FV+p)/2}$, where CP denotes annual coupon, FV denotes face value, and M denotes the maturity of the bond. Then we define yield y as the effective annual yield $y = \left(1 + \frac{YTM}{2}\right)^2 - 1$.

Asset Volatility and Distance to Default For our main measure of idiosyncratic asset volatility, we first delever equity returns with the firm's leverage to obtain asset returns according to $r^a_{i,t} = \frac{r_{i,t}}{[MA/ME]_{i,t-1}}$. Note that we generate leverage $[MA/ME]_{i,t}$ at daily frequency by using daily equity prices. Then we follow the same procedures used to generate equity volatility—that is, we first obtain idiosyncratic asset returns using the classic Carhart (1997) four-factor model:

$$r^a_{i,t_d} - r^f_{t_d} = \alpha_i + \boldsymbol{\beta}_i' \mathbf{f}_{t_d} + u^a_{i,t_d}, \tag{16}$$

and then construct idiosyncratic asset volatility as the standard deviation of the idiosyncratic asset returns:

$$\sigma_{i,t} = \sqrt{\frac{1}{D_t} \sum_{d=1}^{D_t} \left(\hat{u}^a_{i,t_d} - \frac{1}{D_t} \sum_{d=1}^{D_t} \hat{u}^a_{i,t_d} \right)^2}. \tag{17}$$

Similarly, total asset volatility σ^T is defined as the standard deviation of equity returns and is given by

$$\sigma^T_{i,t} = \sqrt{\frac{1}{D_t} \sum_{d=1}^{D_t} \left(r^a_{i,t_d} - \frac{1}{D_t} \sum_{d=1}^{D_t} r^a_{i,t_d} \right)^2}. \tag{18}$$

We also construct the measure of firm-level idiosyncratic asset volatility based on Merton's (1974) model, denoted by $\widetilde{\sigma}$. Asset value V and (total) asset volatility σ_V can be obtained from a two-equation system as follows:

$$E = VN(d_1) - e^{-rT} BN(d_2) \tag{19}$$

$$\sigma_E = \left(\frac{V}{E} \right) N(d_1) \sigma_V \tag{20}$$

where

$$d_1 = \frac{\ln(V/B) + (r + 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}}, \quad d_2 = d_1 - \sigma_V \sqrt{T}.$$

The inputs for the two-equation system are (i) market value of equity E, measured by the product of stock price and the number of shares outstanding; (ii) equity volatility σ_E, measured by the annualized realized volatility of daily stock returns in each month; (iii) face value of debt B, measured as the sum of the firm's current liabilities and one-half of its long-term liabilities; (iv) debt maturity (forecasting horizon) $T = 1$; and (v) risk-free rate r, measured by the annualized monthly return on 90-day Treasury bills.

Instead of solving this two-equation system directly, we implement the iterative procedure proposed by Bharath and Shumway (2008).[31] We linearly interpolate the quarterly value of debt to a daily frequency and estimate asset value at a daily frequency. To construct idiosyncratic asset volatility, we use the daily asset values to generate times series of daily asset returns. With time series of daily asset returns, we calculate the idiosyncratic asset volatility using the same methodology used for idiosyncratic equity volatility. In addition to this realized asset volatility measure, we also use an implied asset volatility measure. Implied

[31] Gilchrist and Zakrajšek (2012) also adopt this iterative procedure.

asset volatility is constructed as delevered implied equity volatility—that is, implied equity volatility times the market value of equity divided by the market value of assets.

Also, after we obtain the asset value V and total asset volatility σ_V, the distance to default (DD) can easily be computed according to the following equation:

$$DD = \frac{\ln(V/B) + (\mu - 0.5\sigma_V^2)T}{\sigma_V \sqrt{T}}.$$

We also construct the measure of firm-level idiosyncratic asset volatility using a reduced-form regression of the log of idiosyncratic equity volatility on the log of firm level leverage:

$$\log \sigma^e_{i,t} = \beta \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}. \tag{21}$$

We use the residuals obtained from this regression as the log of the residual asset volatility, denoted by $\log \mathring{\sigma}_{i,t}$.

Fair Value Spreads We use a proprietary data set from Moody's on its public firm Expected Default Frequency (EDF) Metric, which is an equity-based measure of a firm's probability of default. The core model used to generate the EDF metric belongs to the class of option-pricing based, structural credit risk models pioneered by Black and Scholes (1973) and Merton (1974). The Vasicek-Kealhofer (VK) model summarizes information on asset volatility, the market value of assets, and the default point in one metric, the distance to default (DD), and then maps the DD to obtain the EDF metric. The DD-to-EDF mapping step uses the empirical distribution of DD and frequency of realized defaults. Nazeran and Dwyer (2015) provide a detailed description of their methodology. Most important for our purpose, the EDF credit risk measure relies only on equity market inputs and does not contain bond market information.

Using the EDF credit risk measure, we construct a cumulative EDF (CEDF) over T years by assuming a flat term structure—that is, $CEDF_T = 1 - (1 - EDF)^T$. Then we convert our physical measure of default probabilities (CEDF) to risk-neutral default probabilities (CQDF) using the following equation:

$$CQDF_T = N\left[N^{-1}\left(CEDF_T\right) + \lambda\rho\sqrt{T}\right],$$

where N is the cumulative distribution function for the standard normal distribution, λ is the market Sharpe ratio, and ρ is the correlation between the underlying asset returns and market returns. Given this risk-neutral default probability measure, the spread of a zero-coupon bond with duration T can be computed as

$$\hat{cs} = -\frac{1}{T}\log(1 - CQDF_T \cdot LGD),$$

where LGD stands for the risk-neutral expected loss given default. We follow Moody's convention and set $T = 5$, $LGD = 60\%$, $\lambda = 0.546$, and $\rho = \sqrt{0.3}$ to build our "fair value spread" measure $\hat{cs}$. We successfully match $39,925$ fair value spreads with our firm-quarter observations.

Covenant Tightness To provide empirical support for the debt-overhang channel, we use a covenant tightness measure based on a firm's outstanding loans. Data on covenant specifications and thresholds for loans are from DealScan. There are 18 types of covenants in the data. We first compute the distance between the actual financial ratio and the covenant threshold for each type of covenant, normalized by the firm-specific standard deviation of the actual financial ratios. We then use the minimum of the normalized distances to measure the overall covenant tightness for the firm in each quarter. See Kermani and Ma (2020) for more details on the covenant tightness measure.[32]

[32]We thank Yueran Ma and Amir Kermani for sharing their data with us.

Internet Appendix

A Additional Robustness Checks

In this appendix, we provide several robustness checks for the results discussed above and show that they yield similar results. Table 1 in the main text presents summary statistics for all variables used in this appendix. Table A1 is a robustness check for Table 5. Tables A2, A3, and A4 are related to Table 7. Table A5 is robustness check for Table 10. Table A6 checks the results in Table 11. Table A7 is a robustness check for Table 12, and Table A8 is for Table 15.

In our main text, we mainly use realized volatilities because it covers a longer sample period. However, implied volatility might be a better measure in terms of capturing forward-looking risk. Here we present a series of robustness checks using implied volatility instead of realized volatility. In Table A1, we find the same qualitative results as in Table 5 by using implied equity volatility instead of realized idiosyncratic equity volatility. In Table A2, we replicate Table 8 and regress investment rate on total asset volatility and credit spreads, but using a restricted sample of firms with an observable implied asset volatility. The idea is to use the same sample and compare the estimation results of using implied asset volatility vesus realized total asset volatility. By comparing the results in Table A2 with those in Table 9, We show that the coefficient on asset volatility increases by 50% by using implied asset volatility instead of realized asset volatility. This confirms that it's the expectations of future volatility that impact the investment rate. Table A5, Table A6, and Table A8 replicate the exercise in Table 10, Table 11, and Table 15 by replacing realized idiosyncratic asset volatility with implied asset volatility. We find that all of our results hold with implied asset volatility and are even stronger than those with realized asset volatility.

An additional alternative measure of asset volatility is given by the asset volatility derived from Merton's model. Table A3 replicates Table 7, confirming again that our results are robust to using different measures of asset volatility.

In light of the loadings of equity volatility on asset volatility and leverage, we consider another reduced-form measure of asset volatility: the residual of a panel regression of the log of idiosyncratic equity volatility on the log of firm-level leverage with time and firm fixed effects, denoted $\mathring{\sigma}_{i,t}$. Thus, this measure captures the changes in equity volatility that are orthogonal to changes in leverage. Table A4 confirms again that once we control for changes in leverage, an increase in volatility is associated with an increase in future investments.

In Table A7, for each firm, we compute the average asset volatility and leverage by averaging cross the industry it belongs to and excluding itself. Then we estimate the loadings of firm's credit spreads and equity volatility on the industry-average asset volatility and leverage and confirms our findings presented in Table 12.

Table A1: This table documents the relationship between investment, implied equity volatility, and fair value spread at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \hat{\sigma}^e_{i,t-1} + \beta_2 \log \hat{cs}_{i,t-1} + \beta_3 \log \hat{\sigma}^e_{i,t-1} \times \log \hat{cs}_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all	(8) all
$\log \widehat{\sigma}^e_{i,t-1}$	-0.254***		0.070*	0.183***	0.096	-0.040	0.426***	0.375***
	(-5.94)		(1.83)	(3.77)	(1.54)	(-0.68)	(5.31)	(4.44)
$\log \widehat{cs}_{i,t-1}$		-0.170***	-0.199***	-0.185***	-0.125***	-0.222***	-0.278***	-0.177***
		(-16.63)	(-13.73)	(-6.26)	(-3.96)	(-8.02)	(-12.76)	(-6.74)
$\log \widehat{\sigma}^e_{i,t-1} \times \log \widehat{cs}_{i,t-1}$							-0.084***	-0.070***
							(-5.18)	(-3.81)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	20056	39162	20056	7565	6850	4763	20056	13602
R-squared	0.116	0.152	0.163	0.136	0.142	0.175	0.168	0.233

Table A2: This table documents the relationship between investment, total asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. We restrict the sample of firms to firms with observable implied asset volatility. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^T_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma^T_{i,t-1}$	0.206***		0.191***	0.185***	0.174***	0.131***	0.144***
	(8.77)		(8.47)	(6.17)	(4.68)	(2.84)	(5.89)
$\log cs_{i,t-1}$		-0.335***	-0.327***	-0.168***	-0.344***	-0.655***	-0.171***
		(-10.78)	(-10.61)	(-3.47)	(-5.10)	(-7.55)	(-5.64)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	21315	21315	21315	8571	7067	4712	14668
R-squared	0.117	0.143	0.155	0.154	0.141	0.163	0.237

Table A3: This table documents the relationship between investment, asset volatility derived from Merton's model, and credit spreads at the firm-quarter level from 1984 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \widetilde{\sigma}_{i,t-1}$	0.093***		0.110***	0.118***	0.093***	0.112***	0.086***
	(6.41)		(7.93)	(6.67)	(4.73)	(4.83)	(5.96)
$\log cs_{i,t-1}$		-0.262***	-0.268***	-0.123***	-0.283***	-0.465***	-0.157***
		(-12.79)	(-13.22)	(-3.37)	(-6.39)	(-10.65)	(-7.34)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	45817	45817	45817	14970	14649	13767	30176
R-squared	0.104	0.130	0.135	0.149	0.133	0.123	0.214

Table A4: This table documents the relationship between investment, residual asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Residual asset volatility is the residual of realized idiosyncratic equity volatility on leverage. For each column, we obtain $\log \mathring{\sigma}_{i,t}$ from the regression of the log of idiosyncratic equity volatility on the log of firm-level leverage, controlling for the same set of control variables that are used in the regression model of that column. Columns 4-6 use subsamples sorted by terciles every quarter on credit spreads. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \mathring{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \mathring{\sigma}_{i,t-1}$	0.002		0.030**	0.097***	0.019	-0.022	0.058***
	(0.14)		(2.00)	(5.12)	(0.87)	(-0.83)	(3.58)
$\log cs_{i,t-1}$		-0.271***	-0.271***	-0.120***	-0.294***	-0.461***	-0.154***
		(-13.45)	(-13.39)	(-3.23)	(-6.40)	(-10.89)	(-7.34)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	47435	47435	47433	15151	15165	14291	30990
R-squared	0.100	0.132	0.132	0.147	0.136	0.117	0.210

Table A5: This table documents the relationship between investment, level of implied asset volatility, shock to implied asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018. Implied asset volatility is deleveraged equity volatility implied from options. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1 \log \widehat{\sigma}_{i,t-1} + \beta_2 \Delta \log \widehat{\sigma}_{i,t-1} + \beta_3 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$
$\log \widehat{\sigma}_{i,t-1}$	0.358***	0.333***	0.289***		0.248***
	(9.81)	(9.28)	(9.21)		(6.06)
$\Delta \log \widehat{\sigma}_{i,t-1}$	-0.197***	-0.167***		-0.004	-0.105***
	(-7.33)	(-6.36)		(-0.20)	(-3.83)
$\log cs_{i,t-1}$		-0.318***	-0.324***	-0.331***	-0.176***
		(-10.26)	(-10.54)	(-10.69)	(-5.82)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Controls					✓
Observations	21076	21076	21450	21076	14534
R-squared	0.125	0.162	0.161	0.142	0.241

Table A6: This table documents the relationship between investment, implied asset volatility, and credit spreads at the firm-quarter level from 1984 to 2018 for different lags and leads. Implied asset volatility is deleveraged equity volatility implied from options. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \sum_{\tau=-4}^{4} \beta_\tau \log \hat{\sigma}_{i,t+\tau} + \beta_5 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.318***	-0.186***
	(-9.77)	(-5.75)
$\log \hat{\sigma}_{i,t-4}$	0.082***	0.069**
	(2.90)	(2.16)
$\log \hat{\sigma}_{i,t-3}$	0.084***	0.063*
	(3.37)	(1.95)
$\log \hat{\sigma}_{i,t-2}$	0.070***	0.026
	(2.61)	(0.92)
$\log \hat{\sigma}_{i,t-1}$	0.154***	0.128***
	(6.00)	(3.68)
$\log \hat{\sigma}_{i,t}$	0.010	-0.006
	(0.40)	(-0.19)
$\log \hat{\sigma}_{i,t+1}$	0.045**	0.034
	(2.03)	(1.23)
$\log \hat{\sigma}_{i,t+2}$	-0.063***	-0.054*
	(-2.62)	(-1.87)
$\log \hat{\sigma}_{i,t+3}$	0.065***	0.071**
	(2.60)	(2.30)
$\log \hat{\sigma}_{i,t+4}$	-0.038	-0.054*
	(-1.30)	(-1.68)
Firm FE	✓	✓
Time FE	✓	✓
Controls		✓
Observations	17345	12078
R-squared	0.173	0.243

Table A7: This table presents the loadings of credit spreads on the industry average of asset volatility and leverage at the firm-quarter level from 1984 to 2018. For a firm i in industry k at time t, we compute the industry average of log asset volatility excluding itself as $\frac{1}{N_k-1}\sum_{j\neq i}\log\widetilde{\sigma}_{j,t}$ and the industry average of firm-level leverage as $\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. All variables are standardized to have mean zero and unit variance. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level. The panels below present the partial R^2 given time and firm fixed effects, that is, the percentage reduction in the residual sum of squares (RSS) by adding each variable in addition to time and firm fixed effects.

$$\log y_{i,t} = \beta_1 \log\widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	$\log\sigma^e_{i,t}$	$\log cs_{i,t}$	Panel B: Changes	$\Delta\log\sigma^e_{i,t}$	$\Delta\log cs_{i,t}$
$\frac{1}{N_k-1}\sum_{j\neq i}\log\widetilde{\sigma}_{j,t}$	0.242***	0.054	$\Delta\frac{1}{N_k-1}\sum_{j\neq i}\log\widetilde{\sigma}_{j,t}$	0.125***	0.022**
	(9.64)	(1.49)		(13.41)	(2.02)
$\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.077***	0.139***	$\Delta\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.031***	0.095***
	(5.30)	(7.23)		(6.19)	(12.31)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	45401	45401	Observations	44491	44491
R-squared	0.403	0.460	R-squared	0.173	0.298

Partial R^2

Panel C: Levels	$\log\sigma^e_{i,t}$	$\log cs_{i,t}$	Panel D: Changes	$\Delta\log\sigma^e_{i,t}$	$\Delta\log cs_{i,t}$
$\frac{1}{N_k-1}\sum_{j\neq i}\log\widetilde{\sigma}_{j,t}$	0.017	0.001	$\Delta\frac{1}{N_k-1}\sum_{j\neq i}\log\widetilde{\sigma}_{j,t}$	0.006	0.000
$\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.007	0.016	$\Delta\frac{1}{N_k-1}\sum_{j\neq i}\log[MA/ME]_{j,t}$	0.001	0.005

Table A8: This table documents the relationship between investment, implied asset volatility, leverage, and credit spreads at the firm-quarter level from 1984 to 2018. Implied asset volatility is deleveraged equity volatility implied from options. Columns 4-6 use subsamples sorted by terciles on credit spreads. Control variables include return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). See Table 1 and Section A for detailed variable definitions. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

$$\log[I/K]_{i,t} = \beta_1\log\hat{\sigma}_{i,t-1} + \beta_2\log[MA/ME]_{i,t-1} + \beta_3\log cs_{i,t-1} + \beta_4\log\hat{\sigma}_{i,t-1}\times\log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log\widehat{\sigma}_{i,t-1}$	0.306***		-0.057	0.074	-0.012	-0.249***	1.194***	1.120***
	(9.59)		(-1.45)	(1.52)	(-0.19)	(-3.33)	(6.06)	(5.16)
$\log[MA/ME]_{i,t-1}$		-0.622***	-0.657***	-0.560***	-0.634***	-0.704***	-0.585***	-0.518***
		(-14.71)	(-12.88)	(-5.91)	(-7.54)	(-9.04)	(-10.86)	(-7.06)
$\log cs_{i,t-1}$							-0.525***	-0.425***
							(-7.75)	(-5.73)
$\log\widehat{\sigma}_{i,t-1}\times\log cs_{i,t-1}$							-0.219***	-0.199***
							(-6.44)	(-5.25)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	21450	21269	21269	8460	7088	4763	21269	14664
R-squared	0.123	0.175	0.175	0.169	0.158	0.187	0.191	0.256

B Risk Shifting

We attempted to reproduce the empirical findings documented in Panel A of Table II in Eisdorfer (2008), where they regress investment intensity on expected volatility and find that the coefficient on expected volatility is negative for health firms while positive for distressed firms. We tried to follow Eisdorfer (2008) as close as possible in all the steps but failed to replicate his result. Below we describe the details about the data and the regressions.

Variables The dependent variable, investment intensity, is measured by the ratio of capital expenditures to PP&E at the beginning of the year. The key independent variable, expected volatility is obtained by applying a GARCH $(1, 1)$ model to monthly returns of NYSE market index from 1927-2002. For each calendar year, the expected volatility is measured by the 12-month-ahead forecasted volatility conditional on information available in the last month of the year before. This expected annual volatility is a linear function of the expected volatility for the next month, so it is sufficient to regress investment on expected volatility for the first month of the year. The variable measuring financial distress is Altman's Z-score[33]. Firms with Z-scores below 1.81 at the beginning of the year are classified as distressed.

The control variables include (i) firm size, which is estimated by the log of the market value of the firm's total assets; (ii) market-to-book ratio, estimated by equity market value divided by equity book value; (iii) leverage, estimated by the ratio of the book value of total debt to book value of total assets; (iv) cash flow, estimated by the ratio of operating cash flow to PP&E at the beginning of the year; (v) the NBER recession dummy variable; (vi) default spread, estimated by the Moody's BAA-AAA corporate bond yield spread; (vii) interest rate, measured by the nominal return on 1-month Treasury bills.

The construction of the market value of assets in Eisdorfer (2008) is different from the procedure we document in our main text. Instead of using an iterative procedure, asset value is computed by directly solving the two-equation system as in Equation (19). The definition of some input variables are also slightly different: the equity volatility σ_E is measured by the realized monthly stock return volatility in the subsequent year; the face value of debt B is measured by the total liability of firms; the debt maturity T is measured as $T = \frac{1}{TD}(0.5STD + 5LTD)$; the risk-free rate r is measured by the 1-year Treasury bill yield.

Sample The data are obtained from CRSP and COMPUSTAT. As in Eisdorfer (2008), we only include firms that are traded on the NYSE, AMEX, and Nasdaq, and have non-

[33]The formula to compute Z-score is Z-score = 1.2 x (Working capital/Total assets)+1.4 x (Retained earnings/Total assets)+3.3 x (Earnings before interest and taxes/Total assets)+ 0.6 x (Market value of equity/Book value of total liabilities) + 0.999 x (Sales/Total assets)

missing observations for asset value, investment intensity, and Z-score. The sample period is 1963 to 2002. According to Eisdorfer (2008), their final sample contains 52,112 firm-year observations. The sample we generated using the filters described above contains 55,462 observations (Sample I), and if we further trim every regression variables at the 1 and 99 percentiles, the regression sample contains 51,266 observations (Sample II).

Table 9 reports the results from OLS regressions of investment intensity on expected volatility for financially healthy firms and distressed firms, controlling for firm size, market-to-book ratio, leverage, cash flow, the recession dummy, the default spread, and the interest rate. Panel A presents the regression results using Sample I while Panel B presents the results using Sample II. In Panel A, the coefficient on expected volatility for distressed firms is negative and insignificant, which is supposed to be positive and marginally significant in Eisdorfer (2008). As documented above, we followed Eisdorfer (2008) as close as possible but failed to generate a final sample that is exactly the same, so a possible reason for the discrepancy might be that they are some sample filters not documented. In Panel B where we use the trimmed data, we can see that the coefficient on expected volatility is more negative and significant for distressed firms, which is actually consistent with our main results. Meanwhile, the sign and significance of other coefficients in Panel B are roughly consistent with those in Eisdorfer (2008).

Table 9: This table presents the results from OLS regressions of investment intensity on expected volatility for financially healthy firms and distressed firms, controlling for firm size, market-to-book ratio, leverage, cash flow,the recession dummy, the default spread, and the interest rate. Distressed firms are firms with Z-scores below 1.81 at the beginning of the year. The sample is at firm-year level from 1963 to 2002. Panel A uses Sample I which is obtained by applying filters documented in the text of Eisdorfer (2008). Panel B uses Sample II which is generated by further trimming investment intensity, size, market-to-book ratio, leverage and cash flow at the 1 and 99 percentiles. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels

	Panel A: Sample I		Panel B: Sample II	
	(1)	(2)	(3)	(4)
	Healthy Firms	Distressed Firms	Healthy Firms	Distressed Firms
Exp. volatility	-1.884**	-2.431	-1.264**	-3.684***
	(-2.21)	(-0.92)	(-2.32)	(-3.50)
Log_size	0.002***	-0.005***	-0.003***	-0.003***
	(4.17)	(-3.57)	(-7.56)	(-4.86)
Market-to-book	0.000***	0.000	0.017***	0.005***
	(3.12)	(0.07)	(46.18)	(8.15)
Leverage	-0.119***	0.010	-0.063***	0.015*
	(-20.27)	(1.04)	(-15.59)	(1.76)
Lagged cash flow	0.019***	0.002	0.086***	0.012***
	(21.91)	(1.44)	(47.36)	(4.28)
Recession	-0.016***	-0.010	-0.009***	-0.002
	(-6.48)	(-1.24)	(-6.15)	(-0.60)
Default spread	-0.853**	-1.003	-0.410*	0.770*
	(-2.54)	(-0.88)	(-1.91)	(1.66)
Interest rate	0.776***	0.481***	0.763***	0.509***
	(16.57)	(3.13)	(25.48)	(8.19)
Constant	0.185***	0.141***	0.118***	0.087***
	(37.14)	(8.69)	(35.10)	(10.58)
Observations	46179	9283	43309	7957
R-squared	0.026	0.003	0.123	0.027

C Proofs

Shareholders maximize their expected cash flow and decide when to default. Thus, the value of equity is given by

$$e = \max_{i,\underline{z}} \left\{ \mathbb{E}\Big[(iz - b)\mathbb{1}\{z \geq \underline{z}\}\Big] - \phi(i) \right\}.$$

The first-order conditions for investment i and the default boundary $\underline{z}$ are given by

$$\begin{aligned} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) = 0, \\ -f(\underline{z};\sigma)(i\underline{z} - b) = 0. \end{aligned}$$

The second-order conditions for investment i and the default boundary $\underline{z}$ are given by

$$\begin{aligned} -\phi_{ii}(i) < 0, \\ -f(\underline{z};\sigma)i < 0, \\ \phi_{ii}(i)f(\underline{z};\sigma)i + f(\underline{z};\sigma)^2\underline{z}^2 > 0. \end{aligned} \tag{22}$$

Thus, $\phi_{ii}(i)i + f(\underline{z};\sigma)\underline{z}^2 > 0$.

In the following sections, we derive the partial derivatives of equity with respect to (i) credit spreads and asset volatility, (ii) leverage and asset volatility, (iii) credit spreads and equity volatility, (iv) Tobin's q and asset volatility, and (v) Tobin's q and credit spreads to rationalize our empirical results.

Assume we observe $\boldsymbol{\theta}$ and we want to derive the partial derivatives of $\mathbf{x}$ with respect to $\boldsymbol{\theta}$. Since $\mathbf{x}$ is the solution to a system of nonlinear equations $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$, we need to use the multivariate implicit function theorem:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_k}\right].$$

Proof of Proposition 1

If we observe cs and σ, we get

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ F(\underline{z};\sigma)/(1 - F(\underline{z};\sigma)) - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z}f(\underline{z};\sigma) \\ 0 & f(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 \end{bmatrix}$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) \\ F_\sigma(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we get

$$\frac{\partial i}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = -\frac{\underline{z}(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} < 0$$

and

$$\begin{aligned}
\frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \frac{F_\sigma(\underline{z};\sigma)}{(1-F(\underline{z};\sigma))^2} \\
&= \frac{\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma)}{\phi_{ii}(i)} + \frac{\underline{z}(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{F_\sigma(\underline{z};\sigma)}{(1-F(\underline{z};\sigma))^2} \\
&= \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)} > 0.
\end{aligned}$$

The sign of both partial derivatives comes directly from our assumptions.

Proof of Proposition 2

If we observe b and σ, we get

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_k}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ i\underline{z} - b \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} b & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z} f(\underline{z}; \sigma) \\ \underline{z} & i \end{bmatrix}$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial b}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF_\sigma(z; \sigma) dz \\ 0 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive

$$\frac{\partial i}{\partial b} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = -\frac{\underline{z} f(\underline{z}; \sigma)}{\phi_{ii}(i) i - \underline{z}^2 f(\underline{z}; \sigma)} < 0,$$

$$\begin{aligned} \frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z; \sigma) dz \\ &= \frac{i \int_{\underline{z}}^{\infty} z dF_\sigma(z; \sigma) dz}{\phi_{ii}(i) i - \underline{z}^2 f(\underline{z}; \sigma)} \\ &= \frac{i(\nu(\underline{z}, \sigma) - \underline{z} F_\sigma(\underline{z}; \sigma))}{\phi_{ii}(i) i - \underline{z}^2 f(\underline{z}; \sigma)}. \end{aligned}$$

Proof of Proposition 2

If we observe cs and σ^e, we get

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_k}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z; \sigma) - \phi_i(i) \\ F(\underline{z}; \sigma)/(1 - F(\underline{z}; \sigma)) - cs \\ \frac{\sigma}{\mathbb{E}[(z - \underline{z})^+]} - \sigma^e \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \\ \sigma \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma^e \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z}f(\underline{z};\sigma) & \int_{\underline{z}}^{\infty} z dF_{\sigma}(z;\sigma) \\ 0 & f(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 & F_{\sigma}(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 \\ 0 & \sigma^e_{\underline{z}} & \sigma^e_{\sigma} \end{bmatrix}$$

where

$$\begin{aligned}
\sigma^e_{\underline{z}} &= -\frac{\sigma \bar{\mu}_{\underline{z}}(\underline{z},\sigma)}{\bar{\mu}(\underline{z},\sigma)^2} = \frac{\sigma\left(1-F(\underline{z};\sigma)\right)}{\bar{\mu}(\underline{z},\sigma)^2}, \\
\sigma^e_{\sigma} &= \frac{\bar{\mu}(\underline{z},\sigma) - \sigma\nu(\underline{z},\sigma)}{\bar{\mu}(\underline{z},\sigma)^2} \\
\bar{\mu}(\underline{z},\sigma) &= \mathbb{E}\left[(z-\underline{z})^{+}\right],
\end{aligned}$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma^e}\right] = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive

$$\begin{aligned}
\frac{\partial i}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} &= \frac{(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z dF_{\sigma}(z;\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) + \underline{z}f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}{F_{\sigma}(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) - f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}, \\
&= -\frac{\underline{z}(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z/\underline{z} dF_{\sigma}(z;\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) + f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}{f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma) - F_{\sigma}(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial i}{\partial \sigma^e} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{13} &= -\frac{1}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z dF_{\sigma}(z;\sigma) f(\underline{z};\sigma) + \underline{z}f(\underline{z};\sigma)F_{\sigma}(\underline{z};\sigma)}{F_{\sigma}(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) - f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)} \\
&= \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)} \frac{f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma) - F_{\sigma}(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}.
\end{aligned}$$

Positive Liquidation Value Given that the price of debt with positive liquidation value α is given by

$$D = (1-F(\underline{z};\sigma))B + i\alpha \int_0^{\underline{z}} zK dF(z;\sigma),$$

we define the credit spreads with positive liquidation value as

$$\widetilde{cs} = \frac{F(\underline{z};\sigma) - \alpha/bi\int_0^{\underline{z}} zdF(z;\sigma)}{1 - F(\underline{z};\sigma) + \alpha/bi\int_0^{\underline{z}} zdF(z;\sigma)}.$$

where $1-\alpha$ represents bankruptcy costs. For readability, we define

$$\widetilde{F}(i,\underline{z},\sigma) = F(\underline{z};\sigma) - \alpha/bi\int_0^{\underline{z}} zdF(z;\sigma).$$

Thus, we can write

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} zdF(z;\sigma) - \phi_i(i) \\ \frac{\widetilde{F}(i,\underline{z},\sigma)}{1-\widetilde{F}(i,\underline{z},\sigma)} - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as

$$\left[\frac{\partial\mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z}f(\underline{z};\sigma) \\ -\alpha/b\int_0^{\underline{z}} zdF(z;\sigma)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 & f(\underline{z};\sigma)(1-\alpha)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 \end{bmatrix}$$

and the partial derivatives as

$$\left[\frac{\partial\boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$\left[\frac{\partial\boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} zdF_\sigma(z;\sigma) \\ \widetilde{F}_\sigma(i,\underline{z},\sigma)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial\mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we get

$$\frac{\partial i}{\partial cs} = \left[\frac{\partial\mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = -\frac{\underline{z}(1-\widetilde{F}(i,\underline{z},\sigma))^2}{\alpha/i\int_0^{\underline{z}} zdF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \leq 0$$

and

$$
\begin{aligned}
\frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \frac{\widetilde{F}_\sigma(i,\underline{z},\sigma)}{(1-\widetilde{F}(i,\underline{z},\sigma))^2} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma)+\phi_{ii}(i)(1-\alpha)} + \frac{\underline{z}(1-\widetilde{F}(i,\underline{z},\sigma))^2}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma)+\phi_{ii}(i)(1-\alpha)} \frac{\widetilde{F}_\sigma(i,\underline{z},\sigma)}{(1-\widetilde{F}(i,\underline{z},\sigma))^2} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) + \underline{z}\widetilde{F}_\sigma(i,\underline{z},\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma)+\phi_{ii}(i)(1-\alpha)} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) + \underline{z}F_\sigma(\underline{z};\sigma) - \alpha\int_0^{\underline{z}} z dF_\sigma(z;\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma)+\phi_{ii}(i)(1-\alpha)} \\
&= \frac{\nu(\underline{z},\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma)+\phi_{ii}(i)(1-\alpha)} \geq 0.
\end{aligned}
$$

D Endogenous Leverage Dynamics

In this appendix, we extend the framework of DeMarzo and He (2020) to include an investment function. We solve numerically the Markov perfect equilibrium and confirm that our results hold in Figure 1. We refer to DeMarzo and He (2020) for proofs of the existence and uniqueness of the Markov perfect equilibrium.

We assume that agents are risk neutral with an exogenous discount rate of $r > 0$. The firm's assets-in-place generate operating cash flow at the rate of Y_t, which evolves according to a geometric Brownian motion:

$$dY_t/Y_t = \mu_t dt + \sigma dZ_t,$$

where Z_t is a standard Brownian motion. A firm has at its disposal an investment technology with adjustment costs, such that $\iota_t Y_t$ spent allows the firm to grow its capital stock by $\mu(\iota_t)Y_t dt$, where $\mu(\cdot)$ is increasing and concave. Denote by B the aggregate face value of outstanding debt that pays a constant coupon rate of $c > 0$. The firm pays corporate taxes equal to $\pi(Y_t - cF_t)$. We assume that debt takes the form of exponentially maturing coupon bonds with a constant amortization rate ξ. Equity holders control the outstanding debt B_t through an endogenous issuance/repurchase policy $d\Gamma_t$ but cannot commit to a policy. Thus, the evolution of the outstanding face value of debt follows

$$dB_t = d\Gamma_t - \xi B_t dt.$$

In the unique Markov equilibrium, given the debt price $p(Y, B)$, the firm's issuance policy $d\Gamma_t = G_t dt$, and default time τ, maximize the market value of equity:

$$E(Y, B) = \max_{\tau, \iota_t, G_t} \mathbb{E}_t \Bigg[\int_t^\tau e^{-r(s-t)} \big[(1 - \iota_s)Y_s - \pi(Y_s - cB_s) - (c + \xi)B_s \\ + G_s p_s\big] ds \Bigg| Y_t = Y, B_t = B \Bigg].$$

Similarly, the equilibrium market price of debt must satisfy

$$p(Y, B) = \mathbb{E}_t \left[\int_t^\tau e^{-(r+\xi)} (c + \xi) ds \middle| Y_t = Y, B_t = B \right].$$

The Hamilton-Jacobi-Bellman (HJB) equation for equity holders is

$$rE(Y,B) = \max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s \qquad (23)$$
$$+ Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B) + \frac{1}{2}\sigma^2Y^2E_{YY}(Y,B)\Big].$$

Thus, in equilibrium it must be that

$$p(Y,B) = -E_B(Y,B).$$

The first-order condition for the investment rate is given by

$$1 = \mu_\iota(\iota)E_Y(Y,B).$$

In the following, we define $\{\iota(Y,B), G(Y,B)\}$ as

$$\{\iota(Y,B), G(Y,B)\} = \arg\max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s$$
$$+ Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B)$$
$$+ \frac{1}{2}\sigma^2Y^2E_{YY}(Y,B)\Big].$$

In this setting with scale-invariance, the relevant measure of leverage is given by

$$y_t \equiv Y_t/B_t,$$

and the equity value function $E(Y,B)$ and debt price $p(Y,B)$ satisfy

$$E(Y,B) = E\left(Y/B,1\right) \equiv e(y)B \quad \text{and} \quad p(Y,B) = p(Y/B,1) \equiv p(y).$$

We also define the following:

$$\iota(Y,B) \equiv \iota(y) \quad \text{and} \quad G(Y,B) \equiv g(y)B.$$

Thus, we can rewrite (23) as follows

$$(r+\xi)e(y) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)ye'(y) + \frac{1}{2}\sigma^2y^2e''(y)\Big]. \qquad (24)$$

The optimal default boundary is such that

$$e'(y_b) = 0.$$

The higher bound is such that

$$e'(y) = \phi y - \rho,$$

which corresponds to the value of equity without a default option. We can solve for ϕ and ρ with

$$(r+\xi)(\phi y - \rho) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)\phi y\Big].$$

Thus,

$$\begin{aligned}\rho &= \frac{(1-\tau)c+\xi}{r+\xi},\\ \phi &= \frac{1-\iota^\star-\pi}{r-\mu(\iota^\star)},\\ 1 &= \mu'(\iota^\star)\phi.\end{aligned}$$

The HJB for $p(Y,B)$ is given by

$$rp(Y,B) = c + \xi(1-p(Y,B)) + (G-\xi B)p_B(Y,B) + \mu(Y,B)Yp_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 p_{YY}(Y,B),$$

where we define $\mu(Y,B) \equiv \mu(\iota(Y,B)) \equiv \mu(y)$.

Thus, we can write the HJB for $p(y)$ as

$$rp(y) = c + \xi(1-p(y)) - (g(y)-\xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y), \tag{25}$$

where $g(y) = G(Y,B)/B$. We need $g(y)$ to be such that $p(y) = e'(y)y - e(y)$. From (24), we get

$$\begin{aligned}(r+\xi)e'(y)y = {} & (1-\iota(y))y - \pi y - \iota'(y)y^2 + (\mu(y)+\xi)y^2 e''(y) + (\mu(y)+\xi)ye'(y) + \mu'(y)y^2 e'(y)\\ & + \frac{1}{2}\sigma^2 y^3 e'''(y)) + \sigma^2 y^2 e''(y).\end{aligned}$$

Thus,

$$(r+\xi)(e'(y)y-e(y)) = (1-\pi)c+\xi-\iota'(y)y^2+(\mu(y)+\xi)ye''(y)+\mu'(y)y^2e'(y) + \frac{1}{2}\sigma^2y^2e'''(y)+\frac{1}{2}\sigma^2y^2e''(y).$$

Thus, $g(y)$ is such that

$$\begin{aligned} & c+\xi-(g(y)-\xi)p'(y)y+\mu(y)yp'(y)+\frac{1}{2}\sigma^2y^2p''(y) \\ & =(1-\pi)c+\xi-\iota'(y)y^2+(\mu(y)+\xi)y^2e''(y)+\mu'(y)y^2e'(y) \\ & +\frac{1}{2}\sigma^2y^3e'''(y))+\frac{1}{2}\sigma^2y^2e''(y). \end{aligned}$$

With further algebra, we get

$$-gp'(y)y = -\pi c-\iota'(y)y^2+\mu'(y)y^2e'(y).$$

Since $\mu'(\iota)e'(y)=1$ and $\mu'(y)=\mu'(\iota)\iota'(y)$, we get

$$g(y)=\frac{\pi c}{p'(y)y}.$$

Plugging the solution for $g(y)$ in (25) yields

$$(r+\xi)p(y)=(1-\pi)c+\xi+(\mu(y)+\xi)yp'(y)+\frac{1}{2}\sigma^2y^2p''(y).$$

We solve numerically for the solution using ODE45 in Matlab. We use the following pseudo-algorithm.

1. Start with $y_L=0$ and $y_H=H$, where H is a sufficiently large number.

2. Given $y_b=1/2(y_L+y_H)$, $e(y_b)=0$, and $e'(y_b)=0$, we solve for $e(y)$ on $[y_b,y_B]$ where Y_B is a large number.

3. Check if $|e(Y_B)-(\phi y_B-\rho)|\le\varepsilon$, where $\varepsilon>0$ is a small number. If $e(Y_B)-(\phi y_B-\rho)>\varepsilon$, set $y_L=y_b$ and repeat 2-3. If $e(y_B)-(\phi y_B-\rho)<-\varepsilon$, set $y_H=y_b$ and repeat 2-3. Otherwise, move to 4.

4. Start with $pp_L=0$ and $pp_H=H$, where H is a sufficiently large number.

5. Given $pp_b=1/2(pp_L+pp_H)$, $p(y_b)=0$, $p'(y_b)=pp_b$ we solve for $p(y)$ on $[y_b,y_B]$.

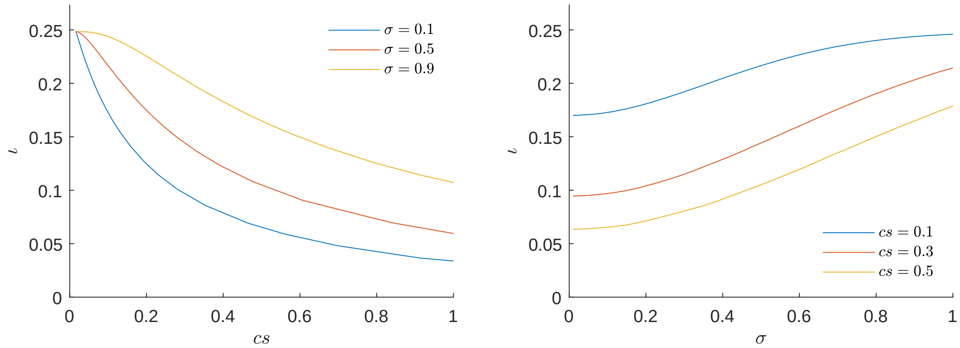


Figure 1: Optimal investment in dynamic setting with $\mu(\iota) = \frac{\log(1+\kappa\iota)}{\kappa}$, $\kappa = 100$, $r = 0.05$, $\xi = 1/8$, $c = 0.05$, $\pi = 0.3$.

6. Check if $|p(y_B) - \rho| \leq \varepsilon$ If $p(Y_B) - \rho > \varepsilon$, set $p_H = p_b$ and repeat 2-3. If $p(y_B) - \rho < -\varepsilon$, set $pp_L = pp_b$ and repeat 4-5. Otherwise, move to 7.

7. Check if $|p'(y_b) - e''(y_b)y_b| \leq \varepsilon$. If not, increase the precision of the ODE45 solver and restart from 1.