Creditor-on-Creditor Violence and Secured Debt Dynamics

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Abstract: Secured lenders have recently demanded a new condition in distressed debt restructurings: competing secured lenders must lose priority. We model the implications of this "creditor-on-creditor violence" trend. In our dynamic model, secured lenders enjoy higher priority in default. However, secured lenders take value-destroying actions to boost their own recovery: they sell assets inefficiently early. We show that this creates an ex-ante tradeoff between secured and unsecured debt that matches recent empirical evidence. Introducing the recent creditor-conflict trend in this model endogenously increases secured credit spreads. Importantly, it also increases ex-ante total surplus: restructurings endogenously introduce efficient state-contingent debt reduction.

Keywords: Continuous-time capital structure models, Liability management, Secured debt, Bankruptcy, Creditor-on-creditor violence, Dropdowns, Uptiers

1 Introduction

Serta Simmons, a leading mattress producer, was struggling financially in 2020. Serta had substantial debt from a leveraged buyout. Its sales were falling due to the pandemic and the growth of online retail. In June 2020, Serta's problems led to a creative debt restructuring. A majority coalition of Serta's secured lenders consented to amend their credit agreement. The amendment allowed Serta to issue new secured debt with a super-priority lien on Serta's assets. The majority coalition exchanged each dollar of their existing secured debt for 74 cents of new "super" secured debt. Crucially, existing secured lenders outside the coalition were not given the exchange opportunity. These excluded lenders, who previously held 30% ($600 million) of the highest priority first-lien secured debt, suddenly had their debt subordinated to more than $1 billion in new debt.[1] This restructuring eliminated $400 million in debt for Serta through the exchange rate paid by coalition lenders.

Practitioners have divided views on these transactions, which have become increasingly common in the last decade (Buccola and Nini, 2022). Firms like Serta euphemistically call these restructurings "liability-management transactions." They argue these restructurings are a beneficial way to lower debt, increase liquidity, and prevent bankruptcies. Lenders and other practitioners call this "creditor-on-creditor violence." They point out that many of these barely legal transactions are quickly followed by bankruptcies. These critics believe this trend is harmful, eroding trust in the corporate borrowing system. To resolve the conflicting views, we build the first theory of how these transactions impact both ex-post and ex-ante firm behavior. We show that both critics and proponents of creditor-on-creditor violence

[1] See https://casetext.com/case/n-star-debt-holdings-lp-v-serta-simmons-bedding-llc and https://www.penews.com/articles/apollo-sues-serta-simmons-and-owner-advent-over-debt-dispute-20200612 and https://bedtimesmagazine.com/2020/06/news-release-serta-simmons-bedding-enters-into-agreement-with-majority-of-lenders-on-deleveraging-and-liquidity-enhancing-transaction/ for details.

are partially right. The trend increases the cost of secured debt due to the anticipation of future lender mistreatment. However, we also vindicate the proponents: surprisingly, the possibility of a Serta-style restructuring increases total surplus when firms borrow ex ante.

Intuitively, firms would like to issue a debt security that: (i) offers tax shields in ex-post good states of the world; and (ii) disappears in ex-post bad states to avoid default costs. Such a security does not exist. However, we show that restructuring-exposed secured debt achieves the same goal. Secured lenders have the highest priority, so they only lose money in a default in very bad states of the world. Accordingly, secured lenders are only willing to exchange face-value haircuts for super seniority in bad states. Because this new type of restructuring only impacts secured lenders, it introduces the state-contingent debt reduction that firms desire ex ante. Thus, the possibility of a Serta-style restructuring actually *benefits* firms when they issue debt. This novel theoretical result from our model provides a positive explanation for an otherwise puzzling fact: firms continue to use debt contracts that are susceptible to "creditor-on-creditor violence" (Buccola and Nini, 2022).

Our main contribution is a theoretical formalization of this intuition. We model a firm that issues risky debt, experiences a shock, and then decides whether to default. We then study how the possibility of an aggressive restructuring changes firm behavior. Since this new legal innovation only impacts secured debt, it changes ex-ante incentives to issue secured debt. To capture this, we model the firm's choice of secured versus unsecured debt. Our second contribution is a new tractable approach to modeling this secured-debt choice based on a realistic tradeoff. We embed these model ingredients (secured debt and aggressive restructurings) in three existing capital-structure models. Across all three models, we confirm the above intuition: the possibility of an aggressive restructuring increases ex-ante firm value. This is surprising because conventional wisdom holds that harming creditors ex post destroys value ex ante through reduced debt capacity. We find the opposite in our novel setting because creditors are only harmed in bad states of the world.

To streamline the exposition, we focus on one baseline model that is realistic and quantitatively matches empirical facts about secured-debt usage. Our baseline model extends the continuous-time capital-structure model of Bolton, Wang, and Yang (2024). A financially constrained firm, facing costly external equity financing, chooses its investment, equity issuance, dividend policy, leverage ratio, secured-debt ratio, and default timing. The firm's capital, a state variable, evolves stochastically according to a jump-diffusion process. The firm adjusts its outstanding short-term debt, the other state variable, to trade off the benefits of debt, e.g., tax shields, with the expected deadweight losses caused by default. As the firm rolls over its debt, it decides what fraction of its new debt to issue as secured debt.

After the arrival of a downward jump shock, equity holders can choose to default. Equity holders default when the cost of the equity injection necessary to repay debt exceeds the continuation value of their future cash flows. In default, secured lenders have first priority on the recovery value of the firm. Crucially, unsecured lenders only get paid after both secured lenders and *priority unsecured claims*, such as employees' unpaid wage claims and unpaid taxes. This creates an incentive for the firm to issue secured debt. The choice of secured debt does not simply reallocate a pie of fixed size between secured and unsecured lenders. Instead, secured debt allows the firm to increase the overall value available to financial lenders in default. Issuing secured debt essentially transfers value from existing priority claim holders, such as employees, to lenders. Departing from Modigliani and Miller (1958), we assume that priority claim holders like employees are too unsophisticated to reprice their claims in response to this transfer. Secured debt thus lowers the firm's total cost of credit: secured-debt usage allows financial lenders as a whole to recover more in default without fairly compensating priority claim holders (employees) ex ante.

While secured debt lowers the firm's cost of credit, it also has a cost: secured lenders have an incentive to push for premature asset sales, even if doing so lowers the firm value, because it ensures they receive full recovery. Without an early sale, it is possible that a severe downward jump shock could leave secured lenders impaired. To capture this incentive, in our model, secured lenders sometimes force an early asset sale. This assumption captures the ability of secured lenders to use covenants or foreclosure threats to manipulate debtors, as we describe in Subsection 2.1. Importantly, these early asset sales destroy value for other claim holders. The firm thus trades off the possibility of an inefficient forced asset sale with the lower cost of credit when deciding how much secured debt to issue ex ante. This tradeoff between secured and unsecured debt is a key contribution of our model.

We introduce creditor-on-creditor violence into this realistic model of corporate policies. We assume that after a negative jump shock, with some exogenous probability, the firm has an opportunity for an aggressive restructuring. If the restructuring offer is accepted by a coalition of lenders, these lenders will then exchange their secured debt for super-secured debt with a lower face value, leaving excluded lenders with a subordinated claim. We thus capture the key features of transactions like Serta's deal. We show that as the probability of an aggressive restructuring opportunity increases, the endogenous cost of secured debt rises. However, the state-contingent nature of these restructurings nonetheless creates value. Specifically, because of equity-issuance costs, the firm's leverage can drift far above target leverage before the firm finds it optimal to reduce leverage with a costly equity issuance. We show that secured lenders optimally accept restructuring offers if and only if leverage has drifted sufficiently far above the target—secured lenders only accept a haircut if the expected default costs, and thus the benefits of super-secured debt, are sufficiently high. This implies that restructurings provide an opportunity to reduce leverage precisely in states of the world where leverage is inefficiently high. Equity holders' moral hazard is limited by the fact that secured lenders will not accept an offer in low-leverage states, where the tax shields are valuable. As a result, ex-ante firm value increases with the ex-post restructuring probability. We also show that the increase in firm value endogenously boosts firm investment.

We validate the central prediction of our model by showing our other model predictions about leverage and secured debt are realistic. The debt policies in our model match the following empirical facts: (i) firms choose a market leverage ratio of 32.8% and a secured-debt ratio of 33.1% in our model, which precisely match the sample averages in Morellec, Nikolov, and Schürhoff (2012) and Benmelech, Kumar, and Rajan (2024); (ii) the difference in credit spreads between a firm's simultaneously issued secured and unsecured debt is 273 basis points, close to the estimate of 222 basis points from Benmelech, Kumar, and Rajan (2022); (iii) "distressed" firms with higher-than-target leverage use more secured debt, consistent with Benmelech, Kumar, and Rajan (2024). Since the choice of secured debt in our model matches this empirical evidence, it is reasonable to think our model accurately captures the effect of the rise in creditor-on-creditor violence, which is difficult to identify empirically. Our model also implies that the ability to issue secured debt can be quite valuable—with our chosen parameters, the legal enforcement of secured-lender rights improves ex-ante firm value by 1.3%. Of course, it is likely that our parameterization only captures particular types of firms.

While our baseline model has many assumptions, our main result is general. We embed aggressive restructurings and a secured-debt choice in two other canonical capital-structure models: the Leland (1994) model and the Modigliani-Miller's static tradeoff model. We show in both settings that aggressive secured-debt restructurings increase firm value ex ante. We likewise show that our results are robust to changing the model to include: (i) long-term debt, (ii) a different restructuring bargaining game, (iii) endogenous restructuring-offer terms, (iv) endogenous secured-lender forced sales, or (v) different subordination assumptions.

Related literature: We make two contributions to the literature. First, we build the first theory of creditor-on-creditor violence. Our main finding, that this novel trend increases ex-ante firm value and investment through state-contingent debt reduction, is thus new to the literature. We build on a long theory literature modeling distressed restructurings, including Gertner and Scharfstein (1991); Bolton and Scharfstein (1996); Fan and Sundaresan (2000); Lambrecht (2001); François and Morellec (2004); Sundaresan and Wang (2007); Brunnermeier and Oehmke (2013); Bolton and Oehmke (2015); Donaldson, Morrison, Piacentino, and Yu (2020); Zhong (2021); Glode and Opp (2023). These models generate many important insights about distressed debt restructurings. Our model differs from these earlier papers by including (i) an ex-ante choice of investment and secured versus unsecured debt; (ii) the tendency for secured lenders to push for asset sales; and (iii) an ex-post restructuring in which only a subset of the most secured lenders exchange their claims, while junior claims are unaffected. As we show, the interactions between these model features are critical for understanding the ex-ante implications of these recent aggressive creditor tactics.

Our second contribution is to show that a novel tradeoff between secured and unsecured debt can produce realistic secured debt choices. Specifically, our model is the first to include a tradeoff in which secured debt can extract value from priority unsecured claims but also leads to premature asset sales. By studying the implications of this novel tradeoff, our model complements the existing theory literature in which different tradeoffs drive the choice between secured and unsecured debt (Bolton and Scharfstein, 1996; Morellec, 2001; Bris and Welch, 2005; Hackbarth, Hennessy, and Leland, 2007; Hackbarth and Mauer, 2012;

Rampini and Viswanathan, 2013; Morellec, Valta, and Zhdanov, 2015; Donaldson, Gromb, and Piacentino, Forthcoming, 2020; Rampini and Viswanathan, 2020; Hu, Varas, and Ying, 2021; Hartman-Glaser, Mayer, and Milbradt, 2023).

Methodologically, we build on earlier continuous-time models of short-term debt such as Bolton, Chen, and Wang (2011); Abel (2018); Geelen (2019); Della Seta, Morellec, and Zucchi (2020); Bolton, Wang, and Yang (2024). We also build on the literature modeling how institutional features of the treatment of debt in default influence ex-ante firm decisions (François and Morellec, 2004; Broadie, Chernov, and Sundaresan, 2007; Antill and Grenadier, 2019). None of these papers study the recent trend of creditor-on-creditor violence.

Finally, we contribute to the recent empirical literature studying creditor-on-creditor violence, including Ivashina and Vallee (2020); Buccola and Nini (2022); Huang, Lewellen, and Wang (2024), by providing the first theory of how this practice impacts firms ex ante.

2 Institutional details

2.1 Secured debt and secured lender control

Unlike unsecured debt, secured debt is explicitly backed by collateral—a specific asset or all of a firm's assets. Outside of bankruptcy, secured lenders have the right to take their collateral if the borrower defaults (e.g., a foreclosure). In contrast, an unsecured lender must first file and win a lawsuit before taking assets from a defaulting borrower.

In bankruptcy, the automatic stay prevents secured lenders from seizing assets. However, secured lenders enjoy the highest priority. A bankruptcy plan can only be confirmed if secured lenders receive full recovery or secured lenders receive the value of their collateral.[2]

[2]Secured debt differs from senior unsecured debt. If a firm has two unsecured lenders, the two debt

We provide further institutional details about secured debt in Internet Appendix I.F.

The high priority of secured lenders incentivizes them to push for a fast sale of their collateral, even if the firm's going concern value is higher than the collateral sale proceeds. This incentive arises when the sale value of a secured lender's collateral is high enough to give the secured lender full recovery, but an uncertain continuation could lead to a future default with lower recovery. Ayotte and Morrison (2009) and Antill (2022) show empirical evidence of inefficient liquidations that benefit secured lenders.

In practice, secured lenders have some ability to push for an asset sale outside of default. For example, secured lenders can use a technical covenant violation to force the appointment of a new sympathetic manager, then promise the manager generous compensation in return for a fast asset sale. Using a discontinuity design to identify the causal effect of a covenant violation, Nini, Smith, and Sufi (2012) show that "the marginal likelihood of observing a forced CEO turnover is 60% higher during the quarter of a covenant violation." Becher, Griffin, and Nini (2022) show that creditors control acquisition activity prior to defaults. Gilson and Vetsuypens (1994) show that "creditors are able to influence corporate policies by... replacing senior management, and influencing the terms of senior executives' compensation" prior to default.

2.2 Priority unsecured claims

As part of its operations, a firm always owes money to employees. This includes, for example, wages or contributions to employee retirement plans that have not yet been paid. Likewise, firms always owe some taxes to the government that have not yet been paid. If a firm files for

contracts can specify that one "senior" lender gets paid before the other "junior" lender in default. Other than this contractual agreement between the two lenders, senior unsecured debt receives no special treatment.

bankruptcy, these wage and tax obligations are priority unsecured claims.[3] These priority unsecured claims must be paid before unsecured lenders. Formally, 11 U.S.C. §507 specifies a certain amount of employee wages, employee benefit contributions, and tax claims that receive priority over general unsecured claims. If unsecured lenders are paid before these priority unsecured claims receive full recovery, the bankruptcy plan cannot be confirmed.[4] Priority unsecured claims can be substantial. For example, in the 2023 bankruptcy of Semrad Law, 38% of the overall liabilities were priority unsecured claims.[5]

While priority unsecured claims must be paid before unsecured claims, secured claims enjoy the highest priority. In our model, this creates a motive to issue secured debt; the firm can obtain cheaper credit by issuing secured debt because secured creditors are paid before the priority unsecured claims that result from the firm's operations. Importantly, we assume that unsophisticated employees do not reprice their wage claims as firms issue secured debt.

2.3 Liability management and creditor-on-creditor violence

When large firms issue secured debt, they typically have a credit agreement that specifies both the terms of the debt and the circumstances under which the terms can be amended. In a recent trend, lenders have begun exploiting loopholes in these credit agreements to protect themselves when firms become distressed. Transactions like the one used by Serta's lenders are called "uptier transactions." In these instances, a coalition of secured lenders and the borrower collude to exploit the amendment terms in a credit agreement. Specifically, credit agreements typically include negative covenants preventing lenders from issuing new liens on assets that would "prime" the existing first liens of secured creditors. However, these

[3] See https://www.law.cornell.edu/uscode/text/11/507.

[4] See https://www.law.cornell.edu/uscode/text/11/1129.

[5] See https://www.inforuptcy.com/browse-filings/delaware-bankruptcy-court/1:23-bk-10512/bankruptcy-case-the-semrad-law-firm-llc.

documents typically allow for a change of these pledges, or a release of liens entirely, if a majority of lenders agree to amend the terms. In uptier transactions, a majority coalition of secured lenders agree to such an amendment in exchange for the ability to receive the new secured debt with the highest priming lien. The excluded lenders are stuck with essentially a second-priority lien on the assets.

Another type of liability-management transaction is called the "dropdown." This was made famous by J. Crew in 2016. In a dropdown, secured lenders have a first lien on a company's assets. The firm exploits loopholes in the credit agreement to transfer these assets to an "unrestricted subsidiary" such that the secured lenders' liens no longer apply. The firm then issues new secured debt backed by the now unencumbered collateral, often to existing lenders. In many instances, prior secured lenders challenge the legality of these transactions, so firms like J. Crew offer a consolation payment to a majority coalition of prior lenders to settle disputes. In this sense, the end outcome of a dropdown is similar to that of an uptier: the firm issues new secured debt that is senior to previously secured debt and only a fraction of prior secured lenders benefit.

Buccola and Nini (2022) provide a detailed description of how these liability-management transactions work. Buccola (2023) includes a list of the many liability-management transactions that have occurred since 2015.

3 Model

We model the partial-equilibrium optimization of a firm in continuous time. A firm chooses its debt level ex ante, then makes decisions ex post to maximize equity value. This section presents our model assumptions, which extend the setup of Bolton, Wang, and Yang (2024) to include the secured debt and aggressive restructurings that are central to our results. In Internet Appendices I.A and I.B, we show that our main results are robust to alternative assumptions.

3.1 Capital and investment

Let K_t denote the firm's capital at time t. Let I_t denote the firm's endogenous investment at t. Capital evolves according to the following stochastic differential equation (SDE):

$$dK_t = K_{t-}\left(\psi\left(\frac{I_{t-}}{K_{t-}}\right) - \delta\right)dt + \sigma K_{t-} d\mathcal{B}_t - (1-Z)K_{t-} d\mathcal{J}_t, \tag{1}$$

where the parameter $\delta \geq 0$ captures capital depreciation and $\sigma > 0$ is the diffusion volatility parameter. We use $K_{t-} = \lim_{s\uparrow t} K_s$ to denote left limits. The process $\mathcal{B}_t$ is a standard Brownian motion.

We assume that jump shocks arrive with an exogenous constant rate λ and the Poisson process $\mathcal{J}_t$ counts these shocks. At each shock, a fraction $1 - Z$ of the firm's capital is destroyed, where $Z \in [0, 1]$ is an independently and identically distributed (i.i.d) random variable, drawn from the following cumulative distribution function (CDF):

$$F(Z) = Z^{\beta}. \tag{2}$$

The parameter $\beta > 0$ determines the distribution of jump shocks. The smaller the level of β the more fat-tailed the distribution of $1/Z$. Jumps play a crucial role in our analysis. Finally, the function $\psi(\,\cdot\,)$ captures the efficacy of investment and is given by:

$$\psi(i) \equiv i - \frac{\xi}{2} i^2, \tag{3}$$

where the parameter ξ captures capital adjustment costs.

3.2 Priority claims and free cash flows

As motivated in Subsection 2.2, we assume in each instant the firm owes ρK_t in priority unsecured claims (e.g., wages) where ρ is an exogenous parameter. This is a simplifying assumption to capture the creation of new claims as the firm pays out previously unpaid wages and taxes. Intuitively, these claims (e.g., wages) increase with firm size K_t. We assume these claims scale with K_t to preserve homogeneity for tractability.

We use the "AK-technology" specification for firm production, which is frequently used in the macroeconomics and corporate finance literatures (e.g., Hayashi, 1982). Under this assumption, the firm's unlevered free cash flow Y_t is given by the following equation:

$$Y_t = \theta K_t - I_t, \tag{4}$$

where the parameter θ captures the firm's productivity, adjusted for tax payments.[6]

3.3 Financing

At time zero, the firm issues X_0 in debt. The firm can costlessly issue new debt at any time before default. It can also pay a cost to issue equity, which allows the firm to reduce its outstanding debt. For tractability, we assume that all debt is short-term and matures immediately.[7] The firm's debt level X_t thus evolves stochastically over time as it (i) issues new short-term debt to cover maturing debt, or pay dividends, or fund investments, and (ii) pays down debt using its free cash flow or equity-issuance proceeds.

In each instant t, out of the total debt X_t issued, a fraction $s_t \in [0, 1]$ is secured and the

[6] Let $A > 0$ denote the firm's capital productivity and $\tau \in (0, 1)$ denote the tax rate on corporate profits. Then, the firm's after-tax free cash flow is given by $Y_t = AK_t - \tau(AK_t - \delta K_t) - I_t = \theta K_t - I_t$, where $\theta \equiv A(1 - \tau) + \tau\delta$.

[7] In Internet Appendix I.B, we show that our main results hold in a model with long-term debt.

remaining fraction $1 - s_t$ is unsecured so that the outstanding secured debt balance is $s_t X_t$ and unsecured debt balance is $(1 - s_t)X_t$.

3.3.1 Credit spreads

The firm's cost of credit depends endogenously on its policies. Let η_t^S denote the endogenous credit spread of secured debt and let η_t^U denote the endogenous credit spread of unsecured debt. Let $Def_{t,t+dt}$ denote the firm's default policy (an indicator function) that equals one if it defaults over the interval of time $[t, t + dt]$ for some small $dt > 0$, and zero otherwise. Let $\mathcal{R}_{t+dt}^{Sec}$ denote the total recovery value received by secured-debt holders in the event of default at time $t + dt$. We describe this recovery in detail in Subsection 3.5. Then the secured credit spread η_t^S is determined by the condition that secured lenders must receive an expected return equal to the exogenous risk-free rate r:

$$\lim_{dt \to 0} s_t X_t (1 + r dt) = \lim_{dt \to 0} \mathbb{E}_t \left[s_t X_t \Big(1 + (r + \eta_t^S) dt \Big) (1 - Def_{t,t+dt}) + \mathcal{R}_{t+dt}^{Sec} Def_{t,t+dt} \right]. \quad (5)$$

Secured lenders could invest the total secured debt $s_t X_t$ at the exogenous risk-free rate r and receive $s_t X_t (1 + r dt)$ at time $t + dt$. We assume that lenders demand a credit spread η_t^S such that they receive the same expected return on the firm's secured debt. This expected return is given by the right side of equation (5). If the firm does not default over the interval $[t, t + dt]$, then lenders get a return equal to sum of the risk-free rate r and the credit spread η_t^S. If the firm defaults over the interval $[t + dt]$, then secured lenders receive the recovery value described in Subsection 3.5. See Appendix A for details.

Unsecured credit spreads are defined by an analogous break-even condition, where the analogous unsecured recovery value $\mathcal{R}^{Unsec}_{t+dt}$ is defined in Subsection 3.5:

$$\begin{aligned}&\lim_{dt\to 0}(1-s_t)X_t(1+rdt)\\=&\lim_{dt\to 0}\mathbb{E}_t\Big[(1-s_t)X_t\Big(1+(r+\eta^U_t)dt\Big)(1-Def_{t,t+dt})+\mathcal{R}^{Unsec}_{t+dt}Def_{t,t+dt}\Big].\end{aligned} \tag{6}$$

3.3.2 Debt coupon payments and the interest tax shields

The firm's debt coupon payment is $C_t dt$ over $[t, t+dt]$ where:

$$C_t \equiv \Bigg(\ r+\eta^S_t s_t+\eta^U_t(1-s_t)\ \Bigg)\ X_t. \tag{7}$$

We assume that the firm receives an interest tax shield $\tau C_t dt$ over the interval $[t, t+dt]$, where τ is the firm's tax rate. This creates an incentive to issue debt. As we discuss in Subsection 3.5, deadweight losses in default create an incentive to avoid excessive debt.

3.3.3 Payouts, equity issuance, and debt dynamics

At any time, equity holders can issue debt and pay themselves the proceeds. That is, equity holders can pay out Δ_U to themselves by increasing the debt level from X_t to $X_t+\Delta_U$. Let U_t denote the cumulative (undiscounted) amount paid out to equity holders by time t.

Likewise, equity holders can raise M_t by issuing external equity at any time to reduce the debt level from X_t to $X_t - M_t$, and incur a total equity issuance cost of $h_0K_t + h_1M_t$. Let N_t denote the cumulative (undiscounted) amount of equity issuance by time t. Let H_t denote the corresponding (undiscounted) cumulative external equity financing costs by time t.

Given our assumptions, in the absence of debt restructurings, the firm's debt balance X_t

evolves according to the following SDE:

$$dX_t = \Big[\underbrace{(1-\tau)C_t}_{\text{Coupon and tax shield}} - \underbrace{Y_t}_{\text{Free cash flow}} \Big] dt + \underbrace{dU_t}_{\text{Payouts}} - \underbrace{dN_t}_{\text{Equity Injections}} . \tag{8}$$

We explain how potential debt restructurings change leverage dynamics in Subsection 3.4.[8]

3.4 Secured lender incentives

Our modeling of secured debt is a key contribution. We include key realistic features of secured debt in our model. The treatment of secured debt in default influences secured lenders' incentives, which in turn impacts default timing and secured lender recovery. Because of this strategic interdependence and equilibrium debt pricing, we build up our characterization of secured lender recovery in steps.

3.4.1 Firm value in default

We assume that firm value in default is πK_t, where $\pi > 0$ is an exogenous parameter. This can be thought of as the recovery value from selling the firm in a liquidation or going-concern sale.[9] The value πK_t is split by all of the firm's claimholders.

3.4.2 Secured debt limit

We assume that secured debt must be fully collateralized. Specifically, the value of a firm's secured debt issued at time t must be less than the total value claimholders would receive if

[8] Purely for technical reasons, we assume equity holders must keep leverage below an exogenous limit via a debt covenant to rule out Ponzi schemes in which the firm constantly issues debt under the self-fulfilling prophecy that it will issue more debt to repay old debt (Auclert and Rognlie, 2016). In our calibration, we set this exogenous debt limit such that equity holders endogenously default before reaching the exogenous leverage limit. This ensures the exogenous limit does not drive our results.

[9] Deadweight losses arise in default regardless of whether the default is resolved through liquidation (Antill, Forthcoming), reorganization (Antill and Hunter, 2023), or going-concern sale (Antill, 2022).

the firm were to default at time t:

$$s_{t-}X_{t-} \leq \pi K_{t-}. \tag{9}$$

This innocuous constraint, which does not bind in our calibration, simply imposes that there is no partially collateralized debt. See Internet Appendix I.F for details on secured debt.

Importantly, equation (9) does not imply that secured debt is risk-free. If a jump shock occurs at time t, then $\pi K_t = Z\pi K_{t-}$ can be insufficient to cover the secured debt $s_{t-}X_{t-}$.

3.4.3 Secured versus unsecured conflict

We assume that secured lenders enjoy the highest priority in default. Combined with equation (9), this gives secured lenders an incentive to force an early default. If secured lenders can force a default before a capital (jump) shock arrives, they have first priority on the firm value πK_t. By equation (9), this implies full recovery for secured debt. However, if a jump shock arrives causing the firm to default before secured creditors force a default, there is a chance that secured lenders will be impaired.

To capture this mechanism, we assume that secured lenders will sometimes push to sell the firm before equity holders would optimally choose to default. That is, over a time increment $[t, t + dt]$, secured lenders take over the firm and sell it with probability

$$\phi\left(\frac{s_{t-}X_{t-}}{K_{t-}}\right)^{\nu} dt, \tag{10}$$

where $\phi, \nu > 0$ are exogenous parameters. If this takeover occurs, the firm shuts down. Secured lenders receive full recovery; the other claimholders split the remaining value (see Subsection 3.5). In Internet Appendix I.A, we show that our results are robust in richer settings where secured lenders endogenously choose whether to force a sale.

3.4.4 Liability management or creditor-on-creditor violence

To our knowledge, our dynamic capital structure model is the first to capture the recent trend of "liability management," also called "creditor-on-creditor violence." We model this as follows. Whenever a jump shock occurs, before lenders learn the corresponding realized value of Z, with probability $\alpha \in [0, 1]$ there is the potential for a liability-management transaction. In this transaction, the firm offers a coalition of secured lenders, owning fraction $\zeta \in [1/2, 1]$ of secured claims, the opportunity to exchange their secured claims worth $\zeta s_{t-}X_{t-}$ for new super secured claims worth $(1-\varepsilon)\zeta s_{t-}X_{t-}$. The haircut rate $\varepsilon \in [0, 1]$, which for now we assume is exogenous for tractability, allows the firm to lower its debt slightly. The coalition accepts if its expected payoff is higher with the exchange than without the exchange. In Internet Appendices I.A and I.B, we show that our results are robust in richer settings where equity holders optimally choose a haircut rate ε.[10] In Internet Appendix I.C, we show that our results also hold in a model extension in which equity holders endogenously choose ζ.

We now provide intuition for how secured lenders decide whether to accept a restructuring offer. Suppose that X_{t-} is very high and a jump shock occurs at t. Because X_{t-} is high, the shock could plausibly lead to a default in which secured lenders are impaired ($\pi Z K_{t-} < s_{t-}X_{t-}$). In this scenario, the secured-lender coalition might prefer to exchange their debt $\zeta s_{t-}X_{t-}$ for $(1-\varepsilon)\zeta s_{t-}X_{t-}$ in super senior debt. The motive for doing this is that the new debt will be less likely to be impaired in a default, leading to a higher expected recovery:

$$\mathbb{E}_{t-}\Big[\min\{\pi Z K_{t-}\,,\,(1-\varepsilon)\zeta s_{t-}X_{t-}\}\,|\,\text{Jump}\Big] > \zeta\mathbb{E}_{t-}\Big[\min\{\pi Z K_{t-}\,,\,s_{t-}X_{t-}\}\,|\,\text{Jump}\Big]. \quad (11)$$

Note that the expectations on both sides of (11) are conditional on the arrival of a jump

[10] Likewise, we show that our results continue to hold if a secured lender's outside option, relative to participating in a coalition, is to be excluded and subordinated in a nonetheless successful restructuring.

shock at t but the realized value of Z is unknown. That is, the expectations in (11) are taken with respect to Z.

Now, suppose that X_{t-} is low. Then it is very unlikely that a jump arrival will lead to a default, let alone one in which secured lenders are impaired. In this case, secured lenders will not accept the offer. They are unwilling to give up a fraction ε of their debt to buy insurance for the unlikely event of impaired recovery. This intuition matches our result: we show that secured lenders optimally accept a restructuring offer if and only if book leverage X_{t-}/K_{t-} exceeds an endogenous threshold. We characterize this threshold in Appendix A.

Importantly, it is possible that secured lenders agree to a restructuring but then later learn that the jump shock is mild enough to avoid a default. When this scenario happens, the firm simply reduces its outstanding debt. This is the equity holders' motive for these *ex-post* restructurings in practice. We will show that this *ex-post restructuring* possibility creates value *ex ante* by effectively expanding the contract space to allow for a state-contingent reduction in debt *ex post* precisely when expected default deadweight losses are large. Mathematically, we can write the debt dynamics with potential restructurings as:

$$dX_t = \Big[\underbrace{(1-\tau)C_t}_{\text{Coupon and tax shield}} - \underbrace{Y_t}_{\text{Free cash flow}} \Big]dt + \underbrace{dU_t}_{\text{Payout}} - \underbrace{dN_t}_{\text{Equity Issue}} - \underbrace{\mathbf{1}_t^R \zeta \varepsilon s_{t-} X_{t-} d\mathcal{J}_t}_{\text{Liability management}}, \tag{12}$$

where $\mathbf{1}_t^R$ is the firm's optimal restructuring policy (an indicator function) that equals one if a restructuring is offered and accepted after the jump shock arrives at time t.

3.5 Recovery by absolute priority rule in default

We can now formalize the treatment of claims in default. Recall that the total firm value in default is πK_t. We further assume that all claimholders split this value according to the absolute priority rule (APR): each claim must receive full recovery before any junior claim receives any recovery. However, the amount of secured claims depends on whether liability management has occurred as we just discussed above.

3.5.1 Recovery with no liability management

If there is no liability-management transaction, then the firm defaults with secured claims worth up to $s_{t-}X_{t-}$ and unsecured claims worth up to $(1-s_{t-})X_{t-}$ subject to APR. Let T_* denote the time of default. Then, secured claims receive first priority on the firm's recovery value so that the recovery value for secured creditors is given by

$$\mathcal{R}_{T_*}^{Sec} \equiv \min\left\{\ s_{T_*}X_{T_*}\ ,\ \pi K_{T_*}\ \right\}. \tag{13}$$

After secured lenders are fully repaid, priority unsecured claimants, e.g., employees, receive second priority. These claims have face value ρK_{T_*} regardless of whether liability management occurs (Subsection 3.2). If there is enough value left over, then unsecured lenders have third priority given by

$$\mathcal{R}_{T_*}^{Unsec} \equiv \min\left\{\ (1-s_{T_*})X_{T_*}\ ,\ \left(\pi K_{T_*} - s_{T_*}X_{T_*} - \rho K_{T_*}\right)^+\ \right\}, \tag{14}$$

where $x^+ \equiv \max\{0, x\}$.

3.5.2 Recovery with liability management

If a liability-management transaction occurs, then the firm has secured claims worth up to $(1-\varepsilon\zeta)s_{T_*}X_{T_*}$ and unsecured claims worth up to $(1-s_{T_*})X_{T_*}$. The total recovery for pre-default secured lenders is then:

$$\mathcal{R}_{T_*}^{Sec} \equiv \min\left\{\ (1-\varepsilon\zeta)s_{T_*}X_{T_*}\ ,\ \pi K_{T_*}\ \right\}, \tag{15}$$

where the participating secured lender coalition is fully repaid before the nonparticipating secured lenders receive anything.

Unsecured lenders have the same seniority position in the capital structure as in the no-liability-management case, but they benefit from the reduced amount of secured debt:

$$\mathcal{R}_{T_*}^{Unsec} \equiv \min\left\{ (1-s_{T_*})X_{T_*} \ , \ \Big(\pi K_{T_*} - (1-\varepsilon\zeta)s_{T_*}X_{T_*} - \rho K_{T_*}\Big)^+ \right\} . \tag{16}$$

Note that the second term in (16) accounts for equity holders' reduced payment $(1-\varepsilon\zeta)s_{T_*}X_{T_*}$ to secured lenders.

3.6 Default timing

Default can occur either because it is forced by secured creditors or chosen by equity holders. Recall that secured creditors can force a default at a rate of $\phi\big(s_tX_t/K_t\big)^\nu$ (Subsection 3.4). In this case, default is captured by the first jump time of a jump process J_t^{sec} with intensity $\phi\big(s_tX_t/K_t\big)^\nu$, defined as $T_S \equiv \inf\{t \geq 0 : J_t^{sec} \neq J_0^{sec}\}$. Alternatively, a default can also occur when equity holders choose to stop paying their debt due to their limited liability protection. We let T_D denote the endogenous time at which equity holders choose to stop paying debt.

In sum, taking both default possibilities into account, we can express the firm's default via an endogenously determined indicator function as follows:

$$Def_{t,t+dt} \equiv \mathbf{1}\Bigg(\ t \leq T_* \leq t+dt \ \Bigg), \tag{17}$$

where T_* is the default time given by the minimum of the two default scenarios: $T_* \equiv T_D \wedge T_S$.

3.7 Firm objective

After issuing debt X_0 at time zero, equity holders choose a payout process U_t, issuance process N_t, investment process I_t, secured debt process s_t, and default time T_D to maximize expected equity payouts. We assume that equity holders have a discount rate γ that is potentially higher than the risk-free rate r. This assumption captures equity holder impatience. Equity holders thus solve the following stochastic optimization problem:

$$P(K_t, X_t) \equiv \sup_{U,N,I,s,T_D} \mathbb{E}_t \Bigg[\int_t^{T_D \wedge T_S} e^{-\gamma(u-t)} \Big(dU_u - dN_u - dH_u \Big) + e^{-\gamma(T_D \wedge T_S - t)} \Big((\pi - \rho) K_{T_D \wedge T_S} - X_{T_D \wedge T_S} \Big)^+ \Bigg]. \tag{18}$$

The first term is an integral that describes the net equity payouts prior to default. The second term corresponds to the possibility that equity holders earn positive recovery in default.[11] Equity holders maximize this objective subject to: the capital stock evolution (1); the credit spreads implied by (5) and (6), given the firm's strategy; debt dynamics (12); and the exogenous limit (9). Note that equity holders account for the impact of their choices on credit spreads and the default likelihood as they are both determined in equilibrium based on the choices of equity holders as described in the previous sections. In effect, equity holders and creditors play a dynamic game and the solution concept that we use here is Markov subgame perfect equilibrium. (Recall that the likelihood that secured creditors force a default depends on the firm's secured-leverage ratio: $s_t X_t / K_t$).

Finally, we note that equity value maximization at $t = 0$ implies that the firm chooses X_0 and the debt composition structure $s_0 \in [0, 1]$ to maximize firm value, the sum of equity

[11] In our baseline parameterization, this final term is always zero: equity holders recover nothing in default.

value and debt proceeds at $t = 0$:

$$\sup_{X_0, s_0} \quad P(K_0, X_0) + X_0 \tag{19}$$

subject to $s_0 X_0 \leq \pi K_0$ and $P(K_0, X_0) > 0$ due to equity holders' limited liability.

4 Model solution

In this section, we present the model solution. Given our functional form assumptions, our model has a homogeneity property. We show that the solution can be restated in terms of a single state variable, $x_t \equiv X_t/K_t$, without loss of generality. Using x_t, which represents book leverage, greatly simplifies analysis. Equity holders' value is $P(K_t, X_t) = p(x_t)K_t$ for a function $p(x_t)$, and firm value is $P(K_t, X_t) + X_t = v(x_t)K_t$ for $v(x_t) = p(x_t) + x_t$. The equity holders' optimal policy at each t is determined by the value of x_t and three endogenous cutoffs: (i) a payout boundary $\underline{x}$, such that the firm issues debt to pay a dividend when $x_t < \underline{x}$; (ii) an equity-issuance boundary $\widehat{x}$, such that the firm issues equity to reduce leverage when $x_t > \widehat{x}$; and (iii) a default boundary $\overline{x}$, such that the firm defaults the first time when $x_t \geq \overline{x}$. Whenever $x_t \in [\underline{x}, \widehat{x}]$, the firm relies on debt financing together with retained earnings. It issues (pays down) debt when the after-tax free cash flow is less than (greater than) interest expenses. The firm optimizes its equity value by choosing its secured-debt policy $s_t = s(x_t)$ and investment $I_t = i(x_t)K_t$ as certain functions of the state variable x_t. Finally, the endogenous credit spreads η_t^S and η_t^U are functions of x_t, taking into account the firm's optimal policies. Lenders optimally accept liability-management offers when x_t exceeds an endogenous cutoff, which we denote by x^R.

Readers less interested in the technical details may skip to Section 5, which uses this characterization of the optimal firm strategy to present the main results of the paper.

4.1 Payout region

The endogenous boundaries, $\underline{x}, \widehat{x}$, and $\overline{x}$ partition the set of all admissible values for x_t into four regions. We now characterize each region, starting with the payout region: $x_t < \underline{x}$.

When x_t is below the endogenous payout boundary $\underline{x}$, the firm makes a lump-sum payment $(\underline{x} - x_t)K_t$ to shareholders. The lump-sum payment is financed with debt, bringing x_t to $\underline{x}$. The equity value function p must then satisfy the following value-continuity condition:

$$p(x) = p(\underline{x}) + \underline{x} - x\,, \quad \text{for} \quad x < \underline{x}\,. \tag{20}$$

Since (20) holds for x close to $\underline{x}$, we obtain the following smooth-pasting condition for $\underline{x}$:

$$p'(\underline{x}) = -1\,, \tag{21}$$

by taking the limit $x \to \underline{x}$. At $x = \underline{x}$, equity holders are indifferent between reducing debt by one dollar and distributing this dollar to shareholders. Since the payout boundary $\underline{x}$ is an optimal choice, we also have the following super-contact condition (see, e.g., Dumas (1991)):

$$p''(\underline{x}) = 0\,. \tag{22}$$

4.2 Equity-issuance region

We next characterize the endogenous equity-issuance region: $\widehat{x} \leq x \leq \overline{x}$. If leverage x_t is in this region, then the firm issues equity by choosing the net issuance proceeds M_t. Define $m_t \equiv M_t/K_t$. As equity value is continuous before and after issuance, the following value-matching condition holds for $x_t \in [\widehat{x}, \overline{x}]$:

$$p(x_t) = \max_{m>0} \Big[p(x_t - m) - [h_0 + (1 + h_1)m] \Big]. \tag{23}$$

This condition implies that the sum of the equity-issuance costs $h_0 + h_1 m_t$ and the dollars injected m_t must equal the value of the equity that old shareholders receive: $p(x_t - m_t) - p(x_t)$.

We define $\widetilde{x} \equiv x_t - m$ and use it to rewrite (23) as:

$$\max_{\widetilde{x}} \quad p(\widetilde{x}) - [h_0 + (1 + h_1)(x_t - \widetilde{x})] \, . \tag{24}$$

Conditional on issuing equity so that $m > 0$, the optimizer for (24) is independent of the value of x_t. Note that for any x_t in the equity-issuance region $[\widehat{x}, \overline{x}]$, the firm chooses the same post-issuance target leverage $\widetilde{x}$. This equity-issuance target leverage is characterized by the argmax of (24) over the region $\widetilde{x} \in [\underline{x}, \widehat{x}]$, since the post-issuance leverage will be below the issuance boundary $\widehat{x}$. Note that the equity-issuance target leverage $\widetilde{x}$ is higher than the payout boundary $\underline{x}$ when each dollar of equity issued has a marginal cost $h_1 > 0$.

Finally, we determine the firm's optimal equity-issuance boundary $\widehat{x}$. Since the target $\widetilde{x}$ does not depend on x, (23) implies that $p(x) = p(\widetilde{x}) - [h_0 + (1 + h_1)(x - \widetilde{x})]$ for any $x \in [\widehat{x}, \overline{x}]$. This holds at the boundary $\widehat{x}$, so $p(\widehat{x}) = p(\widetilde{x}) - [h_0 + (1 + h_1)(\widehat{x} - \widetilde{x})]$. Then,

$$p(x) = p(\widehat{x}) - (1 + h_1)(x - \widehat{x}), \quad [\widehat{x}, \overline{x}]. \tag{25}$$

Since $p(x)$ is continuously differentiable at the endogenous equity-issuance boundary $\widehat{x}$, we can find the equity-issuance boundary $\widehat{x}$ by imposing the following smoothing-pasting condition:

$$\lim_{x \uparrow \widehat{x}} p'(x) = -(1 + h_1). \tag{26}$$

4.3 Default region

Next, we characterize the default region $x > \overline{x}$. Equity holders will not voluntarily default whenever $p(x)$ is strictly positive. Due to equity holders' limited liability, there exists an endogenous threshold $\overline{x}$ above which equity value is zero:

$$p(x) = 0\,, \text{ when } x \geq \overline{x}. \tag{27}$$

Substituting $p(\overline{x}) = 0$ into the equity-valuation equation (25), we obtain $\overline{x} - \widehat{x} = p(\widehat{x})/(1+h_1)$.

Note that since $\overline{x} > \widehat{x}$, a voluntary default cannot occur unless a capital jump shock arrives: a Brownian shock would push the firm into the equity-issuance region before reaching the default region and the firm would issue equity to lower leverage. In the absence of a restructuring, the firm thus defaults if and only if a jump shock arrives with a low Z: $x_t = x_{t-}/Z > \overline{x}$. Rearranging this inequality, we can define a default threshold for the realized jump recovery Z: $Z_*(x) \equiv x/\overline{x}$. Absent restructuring, the firm defaults if and only if $Z < Z_*(x)$. Finally, if a restructuring is accepted, the debt level falls from X_t to $X_t(1 - s_t\zeta\varepsilon)$. This argument implies that the firm then defaults if and only if $Z < Z_*(x(1 - s\zeta\varepsilon)) \equiv Z_*^{res}(x, s)$.

4.4 Earnings retention and debt-financing region

When $\underline{x} < x_t < \widehat{x}$, equity holders do not want to pay out cash ($x_t > \underline{x}$) or issue equity ($x_t < \widehat{x}$). Intuitively, in this region, leverage is too high to justify issuing a debt-financed dividend. However, the costs of leverage deviating from the target level $\underline{x}$ are too small in this region to justify the equity-issuance costs needed to readjust to target leverage. In this region, the firm's leverage thus evolves stochastically. The firm pays down or grows its debt outstanding depending on whether its free cash flow is higher or lower than its interest expense.

Combining equations (1) and (12) and noting there are no payouts and no equity issuance in this region, we can apply Ito's lemma[12] to derive the evolution of $x_t = X_t/K_t$:

$$
\begin{aligned}
dx_t =& \Big(-\theta + i(x_t) + (1-\tau)c_t + x_t\big[-\psi(i(x_t)) + \delta + \sigma^2\big] \Big)\, dt - \sigma x_t dB_t \\
&+ \Big(\frac{x_{t-}(1-\mathbf{1}_t^R \zeta \varepsilon s_{t-})}{Z} - x_{t-}\Big) d\mathcal{J}_t,
\end{aligned} \tag{28}
$$

where $c_t \equiv C_t/K_t = x_t[r + s_t\eta_t^S + (1-s_t)\eta_t^U]$.

In Appendix A.3, we show that the Hamilton-Jacobi-Bellman (HJB) equation for equity value $P(K_t, X_t)$ implies the following HJB for $p(x_t)$ in the region where $x \in (\underline{x}, \widehat{x})$:

$$
\begin{aligned}
(\gamma+\lambda)p(x) = \max_{i,s} & \Big(-\theta + i + (1-\tau)c(x,s)\Big)p'(x) + \frac{1}{2}\sigma^2 x^2 p''(x) \\
&+ \Big(\psi(i) - \delta\Big)\Big(p(x) - xp'(x)\Big) + \phi\big(sx\big)^{\nu}\left[(\pi-\rho-x)^+ - p(x)\right] \\
&+ \lambda\left[(1-\alpha\mathbf{1}^R(x))\int_{Z_*(x)}^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) + \alpha\mathbf{1}^R(x)\int_{Z_*^{res}(x,s)}^1 Zp\Big(x\frac{1-s\zeta\varepsilon}{Z}\Big)dF(Z)\right]
\end{aligned} \tag{29}
$$

subject to $0 \le s \le \min\{1, \frac{\pi}{x}\}$.

We now explain this equation. Recall that $Z^{res}(x,s), Z_*(x)$ are the necessary shock sizes to induce a default with or without a restructuring, respectively. In Appendix A, we derive the equilibrium (scaled) coupon payment function $c(x,s)$ that lenders charge to break even. This incorporates the role of secured debt in determining the credit spread. The first three terms of (29) capture the sensitivity of equity value to continuous stochastic fluctuations in leverage, given the endogenous secured-debt ratio, investment spending, and credit spreads. The fourth term captures the impact of a secured-lender takeover.

The final line of (29) captures the impact of jump shocks. We derive a cutoff x^R and a function $\mathbf{1}^R(x) = \mathbf{1}(x > x^R)$ such that secured lenders optimally accept a restructuring offer if and only if $x_t > x^R$ (i.e., $\mathbf{1}^R(x_t) = 1$). The probability of a restructuring after a jump shock arrival is thus $\alpha\mathbf{1}^R(x)$. The first term on this line captures the scenario where a

[12] See, for example, Lemma 3 of Appendix H of Duffie (2010).

jump shock while lowering K_t does not trigger restructuring. The second term on this line describes the effect of a restructuring triggered by a jump shock arrival.

In this debt-financing region, equity holders choose investment spending $i = i(x_t)$ and secured debt fraction $s_t = s(x_t)$ to maximize the right side of (29). Taking a derivative with respect to i, we can show analytically that the optimal investment level is:

$$i_*(x) = \frac{1}{\xi}\left(1 - \frac{p'(x)}{xp'(x) - p(x)}\right). \tag{30}$$

This condition equates the marginal cost and the marginal benefit of investing for a financially constrained firm facing costly external equity financing.

4.5 Numerical solution

The solution method for our jump-diffusion model is different from pure-diffusion models, which only require local information around x. Moreover, the interdependence between creditor choices (credit spreads and restructuring acceptance) and equity holder choices requires an equilibrium analysis. Our numerical algorithm accounts for this with an iterative approach. We guess a function $p_k(x)$ with associated boundaries $\underline{x}$, $\widehat{x}$, and $\overline{x}$ and then calculate credit spreads and creditors' restructuring acceptance decisions. We then use the HJB equation (29) and other conditions described above to update to a new candidate value function $p_{k+1}(x)$, given equity holders' strategies, creditor behavior and post-jump-shock values derived from $p_k(x)$. We repeat this process until it converges. We provide details in Internet Appendix I.E.

5 Results

This section presents our main results. In Subsection 5.1, we provide intuition for how the firm optimizes the path of its leverage x_t. In Subsection 5.2, we characterize the optimal secured-debt ratio $s_*(x_t)$. In Subsection 5.3, we conduct comparative statics with respect to the parameter α to show our main result: more frequent liability management leads to higher secured credit spreads and lower secured-debt use, but also increases both investment and ex-ante firm value. Finally, Subsection 5.4 shows that our model matches empirical evidence.

5.1 The optimal leverage ratio

First, we develop intuition for our model by studying leverage dynamics implied by our model solution. We solve our model using the parameter values given in Table 1. Recall that whenever $x_t < \underline{x}$, the firm immediately issues debt and pays a dividend to bring leverage up to the optimal leverage target $\underline{x}$. Likewise, whenever $x_t > \widehat{x}$, the firm immediately issues equity to bring leverage down to the recapitalization target $\widetilde{x}$. The firm's leverage thus remains in the range of $[\underline{x}, \widehat{x}]$, almost surely, prior to default (which occurs if a jump shock brings x from $[\underline{x}, \widehat{x}]$ to a value above $\overline{x}$).

[Insert Table 1 here]

Figure 1 displays the model solution in the range of $x_t \in [\underline{x}, \overline{x}]$. As expected, panel A shows that the ex-post firm value declines in x in this range. By definition, $\underline{x}$ is the point at which equity holders are indifferent between keeping leverage fixed or issuing another dollar of debt to pay a dividend. For $x > \underline{x}$, it follows that $p'(x) < -1$ and thus the ex-post firm value $v(x) = p(x) + x$ declines in x. In this sense, the firm's leverage is typically higher than its payout boundary $\underline{x}$. Once leverage rises to $\widehat{x}$, the firm incurs the equity-issuance cost to issue equity and lower leverage. As equity issuance has a marginal cost $h_1 > 0$ per dollar of equity issued, the firm's equity-financed target leverage $\widetilde{x}$ is higher than its debt-financed target $\underline{x}$. For $x > \widehat{x}$, the declining firm value simply reflects the higher equity-issuance costs needed to bring down leverage.

[Insert Figure 1 here]

Interestingly, panel B shows that firm value is concave in x for low x_t and convex in x for high values of x_t. Because of this, panel D shows that investment first falls as leverage rises (debt overhang) for low leverage levels, then increases with leverage (risk-shifting). Panel C shows the obvious result that market leverage $x/v(x)$ increases as book leverage x rises.

5.2 The optimal secured ratio

Next, we illustrate the choice of secured debt in our model. The benefit of secured debt is that it allows firms to lower their cost of credit. This lower cost of credit arises because secured lenders are senior to existing priority unsecured claims, e.g., wages. Issuing secured debt essentially allows the firm to transfer value from priority claim holders, e.g., workers, to secured claim holders. The downside of secured debt is that secured creditors push for early default to ensure full recovery (Subsection 3.4.3). This can lead to an early default that lowers firm value.

We provide intuition for this tradeoff driving the secured-debt choice using comparative statics. In Figure 2, we vary the parameter ρ that captures priority unsecured claims. Panel B shows that as ρ increases, the firm optimally chooses a higher secured-debt ratio: the average secured-debt ratio rises. This is explained by the same intuition described above. As ρ increases, the recovery value available to unsecured creditors declines. Secured debt then becomes more valuable because it lowers the cost of credit by skipping ahead of priority claims. Next, we vary the probability of a forced default: ϕ. As ϕ increases, secured lenders are more likely to push for an early default. While default imposes a deadweight loss, secured lenders do not care about this loss since they still receive full recovery in a forced default. However, equity holders internalize this deadweight loss because it increases the cost of unsecured credit and lowers the expected value of future dividends. This explains why panel A of Figure 2 shows that the average secured-debt ratio falls as ϕ increases.

[**Insert Figure 2 here**]

Figure 1 provides additional insights on secured-debt use. Panel F confirms that unsecured credit spreads are higher than secured credit spreads. Moreover, the gap between secured and unsecured spreads rises with book leverage x as unsecured debt becomes increasingly more risky than secured debt as x increases. This motivates the firm to use more secured debt as x_t rises (panel E).

Finally, to show how the firm's overall financial strategy changes with secured-debt use, we impose an exogenous upper limit $\overline{s}$ on secured debt. We solve our model with an additional $s_t \leq \overline{s}$ constraint. In panel A of Figure 3, we show that ex-ante firm value $v(0)$ increases as the firm is able to use more secured debt (by increasing $\overline{s}$). The firm stops benefiting once $\overline{s}$ rises above the optimal secured-debt ratio, as this constraint no longer binds. Panel B of Figure 3 shows that the increased use of secured debt leads to a higher probability of default due to forced takeovers by secured lenders. Panel F of Figure 3 shows that the firm uses more leverage as its ability to use secured debt rises. As a result of higher leverage, both secured and unsecured credit spreads rise (panels C and D). However, panel E of Figure 3 shows that at a certain point the weighted credit spread η nonetheless falls as $\overline{s}$ rises. This is the benefit of secured debt — it allows the firm to extract value from other priority claimants (e.g., workers) to lower the cost of credit for a given level of leverage.

[Insert Figure 3 here]

5.3 The rise of liability management

In our analysis thus far, we have assumed no liability-management transactions occur ($\alpha = 0$ in Table 1). We now consider the impact of the recent trend toward more frequent liability-management transactions. We set the parameter α to 0.8 and solve our model. Panel A of Figure 4 shows that there exists a cutoff x^R such that secured lenders optimally accept a liability-management transaction if and only if $x > x^R$. Panel B of Figure 4 shows that as the haircut ε falls, secured lenders are more likely to accept an offer (x^R falls).

[Insert Figure 4 here]

Next, we study what happens as liability-management transactions become more common. In panel B of Figure 5, we show that as restructurings become more common (as we increase α), secured credit spreads increase. Secured lenders anticipate having to either get subordinated or pay a haircut in a future restructuring. As a result, panel C of Figure 5 shows that secured-debt use falls as α rises.

[Insert Figure 5 here]

Surprisingly, Figure 5 shows that ex-ante firm value nonetheless increases as α increases (panel A). The intuition is the following. In a restructuring, value is transferred from secured lenders to equity holders. Secured lenders ex-ante price this risk by charging a higher spread. This is the standard pricing mechanism. However, there is also a benefit from a liability-management transaction. This is because lowering debt in bad states of the world (e.g., high leverage states) mitigates debt overhang and thus increases firm value *ex ante*.

In sum, while lenders charge more ex ante, the ex-post flexibility offered by a potential restructuring nonetheless creates value ex ante. This is why ex-ante value increases as liability-management transactions become more common.

Finally, we consider the effect of changing ζ, the fraction of secured lenders that participate in a coalition. Increasing this ζ parameter has two effects. First, increasing ζ lowers the benefit that secured lenders receive from participating in a restructuring. As ζ increases, secured lenders participating in a restructuring must share their super-senior recovery with a larger pool of coalition members, increasing the likelihood that the super-senior class will be impaired. For a given ε, secured lenders participating in a coalition thus want ζ to be low. Because of this, panel B of Figure 6 shows that secured lenders are less likely to accept a restructuring offer as ζ increases: the leverage threshold x^R for acceptance rises as ζ rises. Second, increasing ζ increases the amount of debt forgiveness in a successful restructuring. Conditional on secured lenders accepting a restructuring offer, equity holders want ζ to be as high as possible to maximize the state-contingent debt reduction.

[**Insert Figure 6 here**]

Because of these two conflicting effects, an increase in ζ can cause an increase or a decrease in ex-ante value. Figure 6 shows this. As we increase ζ, we first find that ex-ante firm value increases. This improvement arises because of the positive effect described above (more ex-post debt forgiveness and efficiency gain conditional on a restructuring). However, past a certain level, increasing ζ leads to a reduction in ex-ante firm value. This decline is due to the negative effect of increasing ζ (a lower probability of secured lenders accepting debt restructurings). This suggests that equity holders will want to write debt contracts that allow for amendments with a specific majority (e.g., two-thirds of lenders must approve an amendment). We explore the implications of allowing for an endogenous ζ choice in Internet Appendix I.C.

5.4 Matching empirical evidence

Finally, we show that our model produces realistic firm debt policies. This serves as a validation of the model's prediction regarding the trend toward creditor-on-creditor violence.

Table 2 shows that the model-implied average market leverage (x/v) is 32.8%. The model-implied average secured-debt share is 33.1%. Empirical studies have calculated an average market leverage among Compustat firms equal to 32.5% (e.g., Table 2 of Morellec, Nikolov, and Schürhoff (2012)). Prior studies have also calculated an average secured-debt share equal to 33% (e.g., Table 2 of Benmelech, Kumar, and Rajan (2024)). In this sense, our model perfectly replicates observed market leverage and secured-debt use.

[**Insert Table 2 here**]

Benmelech, Kumar, and Rajan (2022) compare credit spreads on secured and unsecured debt issued by the same firm at the same time. They show that the senior secured credit spread is 222 basis points lower than the junior unsecured credit spread (Table 2 column 4). Our Table 2 replicates the same exercise in our model, showing that secured credit spreads are 273 basis points lower than unsecured credit spreads.

Finally, Benmelech, Kumar, and Rajan (2024) show that firms issue more secured debt in crises and when they are in distress. Panel E of Figure 1 shows that firms in our model use more secured debt as negative shocks drive their leverage above their targets. In this sense, our model replicates this fact.

6 Robustness and discussions

6.1 Robustness

Our model includes many assumptions. However, the intuition behind our main result is quite general. When firms issue debt, they trade off the benefits of debt with the expected deadweight losses in default. Ex post, firms would like to have more debt in good states and less debt in bad states. Serta-style transactions help firms achieve this goal. Equity holders can coerce secured lenders into forgiving debt in exchange for super priority in bad states of the world, reducing expected deadweight losses. However, secured lenders do not agree to these restructurings in good states because of their high priority. Thus, Serta-style restructurings only erase debt in bad states of the world ex post, creating value ex ante. Because this intuition is general, our results are robust to many different modeling assumptions.

Long-term debt: In Internet Appendix I.B, we introduce secured debt and Serta-style restructurings into the standard Leland (1994) model. Our main result continues to hold. This shows that our main result is robust to a setting with long-term debt.

Endogenous restructuring offers (ε, ζ)**:** In Internet Appendices I.A and I.B, we consider models in which equity holders can choose the haircut ε in a restructuring offer. In Internet Appendix I.C, we consider a model in which equity holders can also choose the fraction ζ of secured lenders that participate in a restructuring. Our main result still holds.

Alternative restructuring bargaining protocols: In Internet Appendices I.A and I.B, we consider different restructuring games. Specifically, in our baseline model, if secured lenders reject a restructuring offer, there is no restructuring. In Internet Appendices I.A and I.B, we assume that a continuum of secured lenders simultaneously decide whether they are willing to participate in a coalition. Since each lender is small, each individual lender cannot block a restructuring. A secured lender thus risks being excluded and subordinated in a restructuring if they decline to participate. In these extensions, secured lenders decide whether to accept a restructuring under this less-appealing outside option. Our main result continues to hold.

Endogenous secured-lender takeovers: In Internet Appendix I.A, we assume that secured lenders can choose whether or not to force an early asset sale. This opportunity arises with a fixed probability that depends on the amount of secured debt, similar to our baseline-model assumption. However, when an opportunity arises, secured lenders may choose to force a sale or not. We show that our main result continues to hold.

Subordination in restructurings: In Internet Appendix I.B, we assume that the excluded secured lender in a Serta-style restructuring is subordinated to an unsecured position, rather than a second-lien position. Our main result continues to hold.

6.2 Discussions

6.2.1 Dilution and other motives for secured-debt issuance

In other models, firms issue secured debt to prevent dilution (Demarzo, 2019; Donaldson, Gromb, and Piacentino, Forthcoming, 2020). To keep our model parsimonious, we abstract from this motive for issuing secured debt. However, an informal argument suggests that including dilution protection could strengthen our results. We already find that the ability to issue secured debt can improve firm value by as much as 1.3% (Table 2). Including dilution protection would likely make the benefit of secured debt even greater. This would likely prompt firms to use more secured debt. Since the debt reduction in a restructuring is proportional to the amount of secured debt outstanding, this would likely lead to an even greater positive impact from Serta-style restructurings.

6.2.2 Debtor-in-possession financing

The creditor-on-creditor violence that we model is similar in some ways to the use of debtor-in-possession (DIP) financing in bankruptcy. Firms issue new secured DIP loans in bankruptcy that are senior to existing secured debt. The key difference is that creditor-on-creditor violence occurs before default. As a result, it has the potential to reduce debt before deadweight losses are incurred. As Antill and Hunter (2023) show, deadweight losses due to lost customers can occur the moment a firm's bankruptcy filing becomes publicized. A DIP loan can only be issued once a firm is in bankruptcy. As a result, DIP loans cannot serve the same purpose of creating efficient debt reduction in states of the world in which expected default deadweight losses are high.

6.2.3 Restructuring uncertainty

We assume that there is an exogenous probability $\alpha < 1$ that a restructuring opportunity arises. In principle, equity holders always have the ability to make a restructuring offer. However, there is some uncertainty about whether the courts will view the restructuring as legally consistent with the contracts. Moreover, equity holders cannot be sure that secured lenders will be interested in forming a coalition. Our assumption that $\alpha < 1$ is intended to capture this uncertainty. Nonetheless, we show in Internet Appendix I.A that increasing α all the way to 1 increases ex-ante firm value.

6.2.4 Restructurings in good states of the world

There is a sense in which our result is not robust. If equity holders can force restructurings in good states, where the benefits of debt (e.g., tax shield) are valuable, this will destroy value ex ante. We formalize this in Internet Appendix I.D. This highlights the important role of secured debt in our model. Secured lenders have the highest priority, so they are only willing to accept a debt haircut in exchange for super seniority in very bad states of the world. This is why restructurings create value ex ante.

7 Conclusion

We build a continuous-time capital structure model in which a financially constrained firm facing costly external equity financing chooses its investment, leverage, secured-debt ratio, payout policy, equity issuance, and default timing. We show that the secured-debt share is chosen by a novel tradeoff between a lower cost of credit, due to the ability to extract value from priority unsecured claims like wages, and a higher probability of default, due to secured-lender incentives to push for early asset sales.

Within this model, we introduce a recent phenomenon: secured lenders have used legal loopholes to extract value from other secured lenders when firms become distressed. We show that this recent trend increases the cost of secured debt and endogenously lowers secured-debt use. However, the liability-management transactions create value ex ante by allowing the firm to introduce state-contingent debt reduction. This explains why firms continue to use debt contracts that include these legal loopholes (Buccola and Nini, 2022).

Our model relies on many assumptions. In extensions, we show that our main result is robust to various alternative assumptions. However, to keep the model parsimonious, certain key model objects like the quantity of priority unsecured wage claims are exogenous in all of our extensions. While we conjecture that an ex-ante choice of employment and wages would not change the model forces driving our main result, we leave it to future work to explore how employment choices matter for debt restructurings. Likewise, future research could explore the different roles of different types of priority claims (e.g., wages versus tax claims).

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Table 1: Parameter values

This table shows our baseline parameter values. Whenever applicable, parameter values are annualized.

r	Risk-free rate	0.05
γ	Shareholder discount rate	0.1
σ	Diffusion volatility	0.4
λ	Arrival rate of cashflow jump shocks	1.5
β	Cashflow-jump-shock severity	4.25
θ	Profitability of capital	0.5
π	Recovery value of capital in default	0.7
ρ	Priority claims / capital	0.6
ν	Convexity of secured default risk	4.8
ϕ	Scale of secured default risk	0.7
ξ	Cost of investment	1.1
δ	Depreciation	0
τ	Corporate tax rate	0.21
α	Probability of liability management	0
ζ	Size of secured coalition	0.6
ε	Haircut rate in liability management	0.004
h_0	Equity issue fixed cost	0.05
h_1	Equity issue proportional cost	0.01

Table 2: Simulated moments using model solution

This table displays key simulated moments using the solution of our baseline model, in which the firm can choose any secured-debt share $s \in [0,1]$, and parameter values are given in Table 1. We simulate 100,000 paths for an optimizing firm with a time increment of 0.01 years. Each simulation starts at the 'recapitalization target' $x_0 = \widetilde{x}$ and stops either when (i) the firm is 100 years old or (ii) it defaults at T_*. At each t, we calculate the firm's (i) market leverage; (ii) secured-debt share; (iii) secured credit spread; and (iv) unsecured credit spread. Then, we take the averages of these four objects across all t in all simulated paths and report them in the first four rows, respectively. We also repeat this process for two other cases: an exogenous secured debt limit of $\overline{s} = 1/2$ and no secured debt issuance: $\overline{s} = 0$, but for brevity only report firm value for these two cases. Finally, the last row reports firm value with no debt financing at all.

Moment	
Market leverage	0.328
Secured-debt share	0.331
Secured credit spread	0.0024
Unsecured credit spread	0.0297
Firm value for $\overline{s} = 1/2$	1.706
Firm value for $\overline{s} = 0$	1.684
Firm value with no debt at all	1.479

Figure 1: Features of the model solution

This figure illustrates the model solution. We solve our model numerically using parameter values in Table 1, as described in the main text. In each panel of this figure, we plot how one model object varies with the model state variable: book leverage x. The model objects are defined at the top of each panel. Panel F plots the secured credit spread, the unsecured credit spread, and the weighted credit spread.

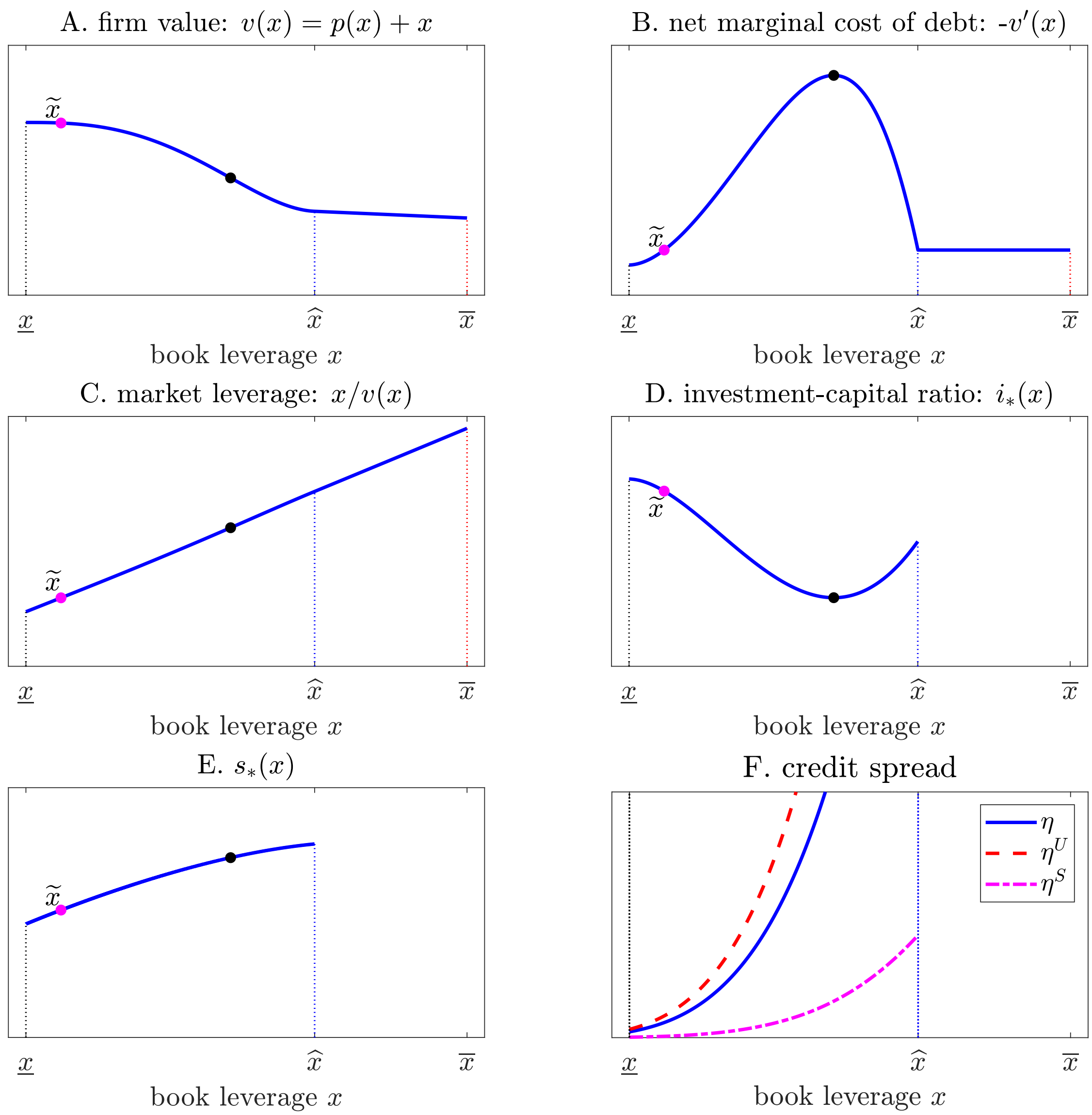


Figure 2: Illustration of the model trade off driving secured debt use

This figure plots how the model-implied secured-debt share varies with the risk of secured lender takeover ϕ and the quantity of priority claims ρ. We solve our model numerically using parameter values in Table 1, as described in the main text. Using our model solution, we simulate 100,000 paths for an optimizing firm with a time increment of 0.01 years. Each simulation starts at the 'recapitalization target' $x_0 = \widetilde{x}$ and stops either when (i) the firm is 100 years old or (ii) it defaults at T_*. We calculate the model-implied expected secured-debt share, by averaging across all instants in all simulated paths. We then repeat this process, varying the parameter ϕ but holding all other parameters fixed. Finally, we repeat this same process varying the parameter ρ but holding all other parameters fixed.

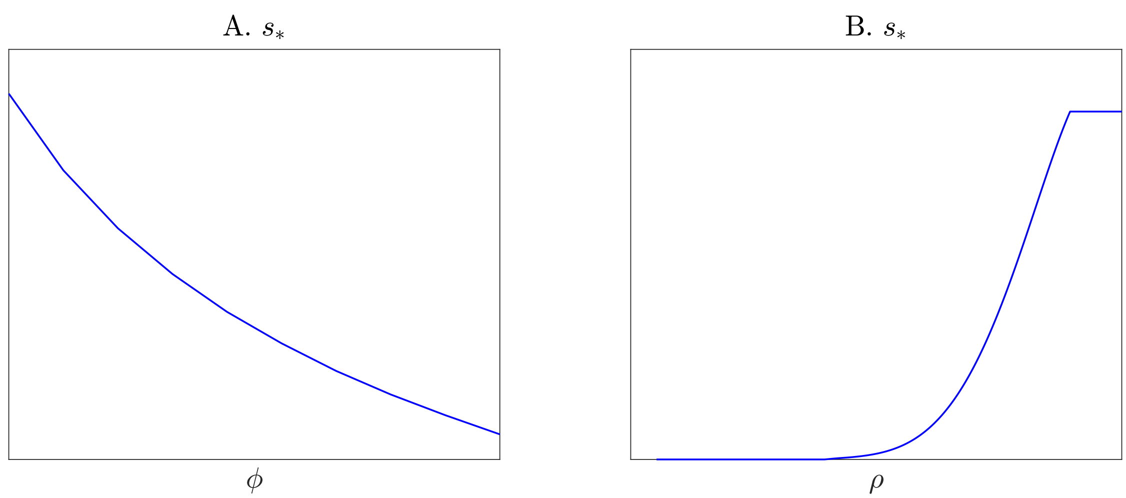


Figure 3: The impact of limiting secured debt use

This figure plots how imposing a cap $\overline{s}$ on secured leverage impacts a firm. We assume the parameter values listed in Table 1. We solve our model numerically, as described in the main text, except that we impose a limit $\overline{s}$ on the secured-debt share. Using our model solution, we simulate 100,000 paths for an optimizing firm with a time increment of 0.01 years. Each simulation starts at the 'recapitalization target' $x_0 = \widetilde{x}$ and stops either when (i) the firm is 100 years old or (ii) it defaults at T_*. We calculate the model objects listed below according to the model solution, averaging across all instants in all simulated paths ($v(0)$ is calculated without simulation). We then repeat this process, varying $\overline{s}$ but holding all other parameters fixed.

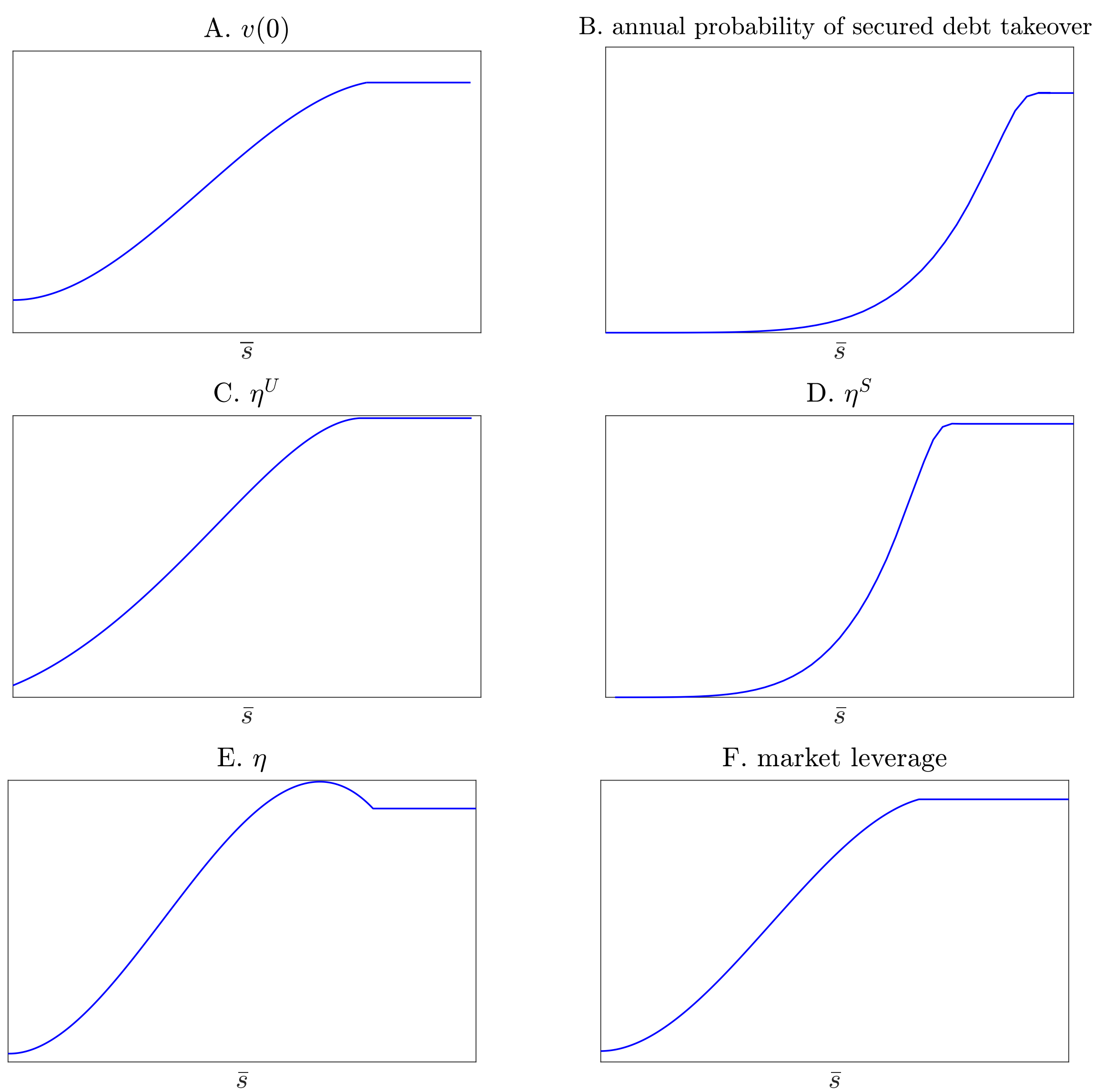


Figure 4: Secured lenders' optimal rule for accepting restructuring offers

This figure illustrates how secured lenders decide whether to accept restructuring offers. We solve our model numerically using $\alpha = 0.8$ (for all other parameter values, see Table 1.) We calculate x^R, the threshold above which secured lenders accept offers and below which they do not. Panel A plots $\alpha \mathbf{1}^R(x)$ as a function of x, where $\mathbf{1}^R(x)$ equals one if and only if $x > x^R$. Panel B shows how x^R varies with ε.

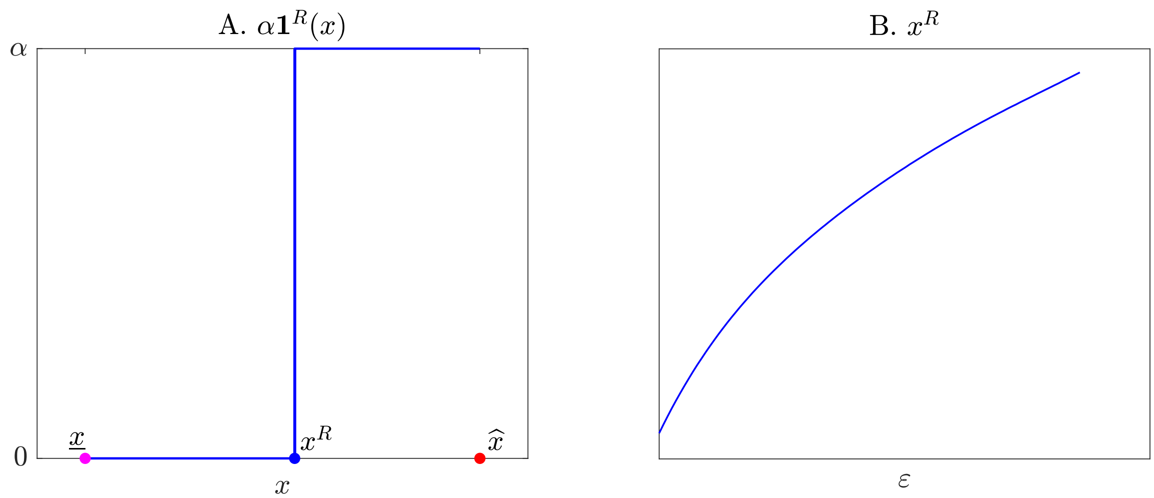


Figure 5: The impact of creditor-on-creditor violence

This figure plots how increasing α, the likelihood of a creditor-on-creditor-violence restructuring, impacts firms. We assume the parameter values listed in Table 1. We solve our model numerically using parameter values in Table 1, as described in the main text. Using our model solution, we simulate 100,000 paths for an optimizing firm with a time increment of 0.01 years. Each simulation starts at the 'recapitalization target' $x_0 = \widetilde{x}$ and stops either when (i) the firm is 100 years old or (ii) it defaults at T_*. We calculate the model objects listed below according to the model solution: (i) the firm value $v(0)$, (ii) the secured credit spread η^S, (iii) the secured-debt share s_*, (iv) the investment policy i_*. We calculate the model-implied expected value of each object, by averaging across all instants in all simulated paths (the firm value $v(0)$ is calculated without simulation). We then repeat this process, varying α but holding all other parameters fixed.

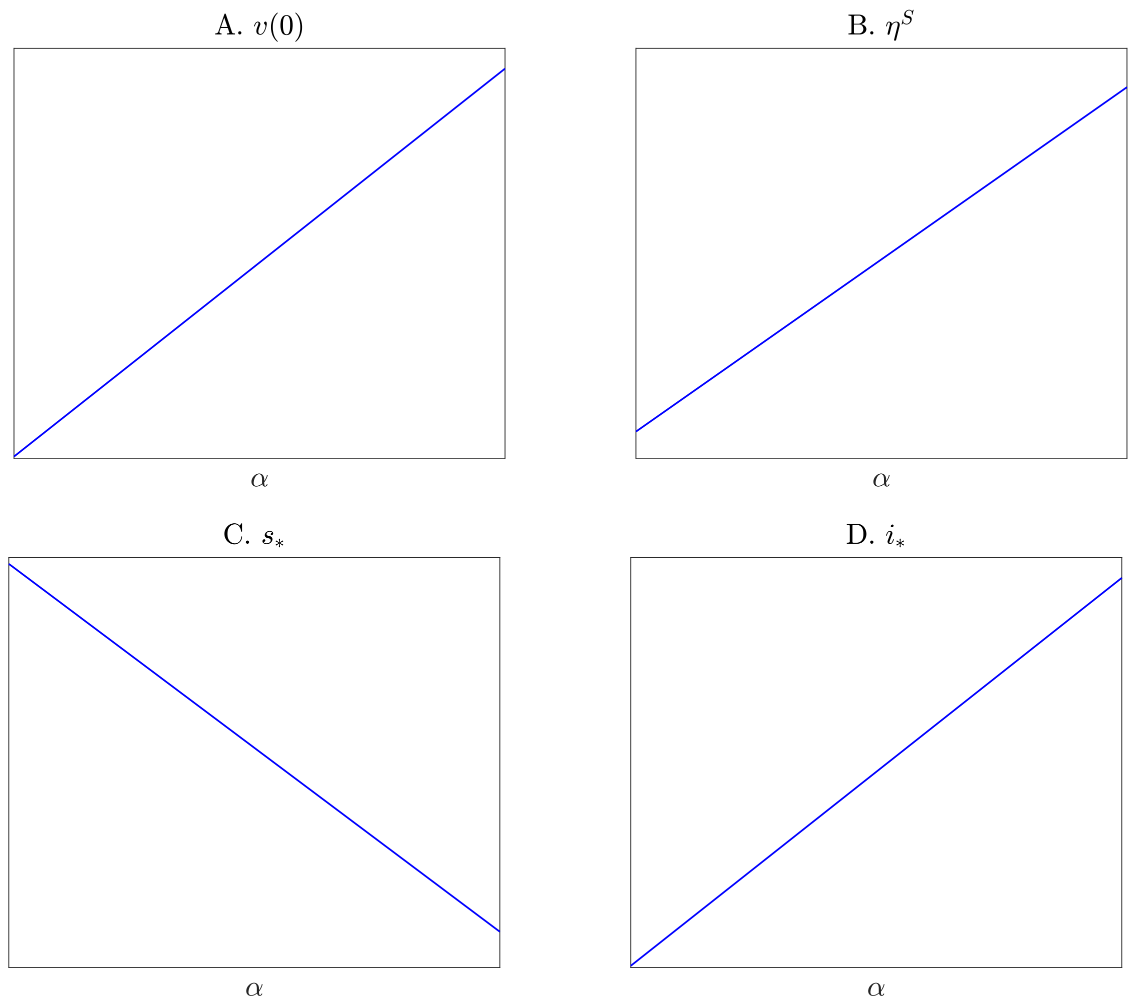
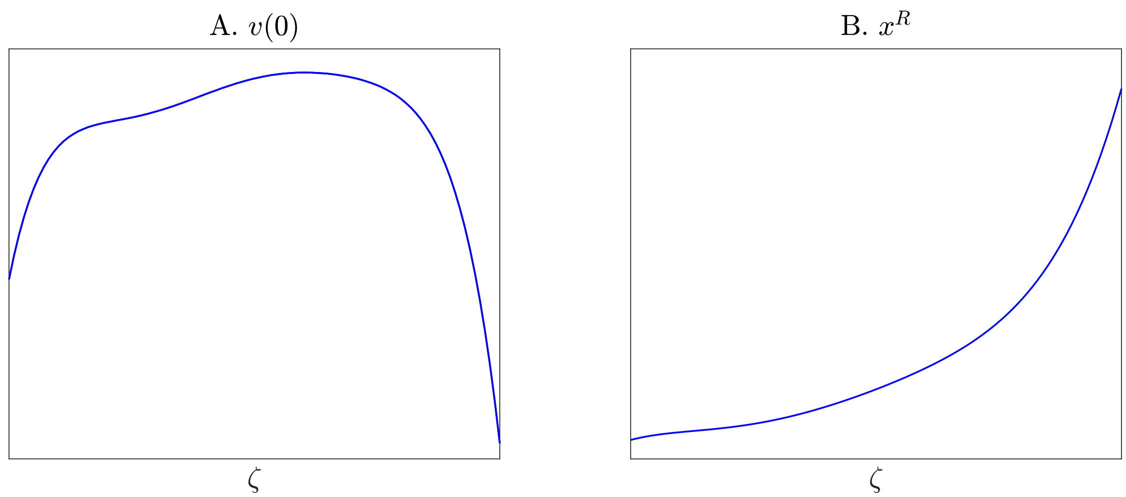


Figure 6: The role of the creditor coalition size

This figure plots how increasing ζ, the size of the creditor coalition in an aggressive restructuring, impacts firms. We solve our model numerically using $\alpha = 0.8$ (for all other parameter values, see Table 1.) Using our model solution, we calculate firm value $v(0)$ and x^R, the threshold above which secured lenders accept offers and below which they do not. Panel A plots how $v(0)$ varies with ζ and Panel B shows how x^R varies with ζ.



Appendix

A Mathematical details

In this appendix, we provide mathematical details. We first derive expressions for the secured credit spread η_t^S and the secured lenders' restructuring-acceptance cutoff x^R. We then calculate the unsecured credit spread η_t^U. Finally, we derive the HJB equation (29).

A.1 Secured credit spread

First, using Ito's lemma, we obtain the process for the (scaled) state variable $x_t = X_t/K_t$ given in (28):

$$
\begin{aligned}
dx_t &= \frac{X_t}{K_t}\left(\frac{dX_t}{X_t} - \frac{dK_t}{K_t} + \left(\frac{dK_t}{K_t}\right)^2\right)\mathbf{1}_{d\mathcal{J}_t=0} + \left(\frac{X_t}{K_t} - \frac{X_{t-}}{K_{t-}}\right)d\mathcal{J}_t \\
&= x_t\left(\frac{-[\theta K_t - I_t - (1-\tau)C_t]dt}{X_t} - \left[\left(\psi\left(\frac{I_{t-}}{K_{t-}}\right) - \delta\right)dt + \sigma d\mathcal{B}_t\right] + \sigma^2 dt\right)\mathbf{1}_{d\mathcal{J}_t=0} \\
&\quad + \left(\frac{X_{t-}(1-\mathbf{1}_t^R\zeta\varepsilon s_{t-})}{K_{t-}Z} - \frac{X_{t-}}{K_{t-}}\right)d\mathcal{J}_t \\
&= \left(-\theta + i(x_t) + (1-\tau)c_t + x_t\left[-\psi(i(x_t)) + \delta + \sigma^2\right]\right)dt - \sigma x_t dB_t \\
&\quad + \left(\frac{x_{t-}(1-\mathbf{1}_t^R\zeta\varepsilon s_{t-})}{Z} - x_{t-}\right)d\mathcal{J}_t.
\end{aligned}
$$

To derive an expression for the secured credit spread η_t^S, we first characterize the potential ways that the firm can default. There are three scenarios to consider, which we discuss below. Recall that in our model, the choice of s_t is Markovian:

$$
s_t = s_*(x_t),
$$

where the function $s_*(\cdot)$ is to be determined.

A.1.1 Scenario 1: forced default

If secured lenders force a default, they will be fully repaid under our assumption given in (9). This scenario occurs with probability $\phi(s_{t-}x_{t-})^{\nu}dt$.

A.1.2 Scenario 2: endogenous default, no restructuring

If a downward jump shock occurs and there is no restructuring, the firm defaults only if $Z < Z_*(x_{t-})$, where

$$Z_*(x) = \max\{Z \in [0,1] : p(x/Z) = 0\}. \tag{A.1}$$

If the firm defaults, secured lenders recover $K_{t-}\min\{s_{t-}x_{t-}, Z\pi\}$. This occurs with probability $\lambda(1-\alpha)F\big(Z_*(x_{t-})\big)dt$. Next, we turn to the restructuring scenario.

A.1.3 Scenario 3: restructuring

We assume the following timing for a restructuring:

1. A capital jump shock occurs.

2. Before anyone sees how bad this shock is (Z), the firm offers an exogenous liability management transaction. With probability α, the firm randomly selects a fraction $\zeta \in [1/2, 1]$ of secured lenders and offers them a haircut rate ε.

3. These chosen lenders decide whether to accept the firm's proposal or not.

   3a. If accepted, the firm's secured debt obligation is reduced by $\zeta\varepsilon s_{t-}X_{t-}$. The value of Z is observed by all parties and the firm decides whether to default.

   3b. If not accepted, there is no restructuring and the firm is in scenario 2 described above.

For the sub-scenario 3b, we simply refer to our analysis for scenario 2.

For the sub-scenario 3a, there are two possibilities. If $Z > Z_*\Big(x_{t-}\big[1 - s_{t-}\zeta\varepsilon\big]\Big)$, the firm does not default and secured lenders receive a total repayment of $K_{t-}s_{t-}x_{t-}(1-\zeta\varepsilon)$ in restructuring.[13] This occurs with probability $\lambda\alpha\Big[1 - F\Big(Z_*\Big(x_{t-}\big[1 - s_{t-}\zeta\varepsilon\big]\Big)\Big)\Big]dt$.

[13] This is also what lenders expect to receive in this scenario before learning whether they are in the

If $Z \leq Z_*\Big(x_{t-}\big[1 - s_{t-}\zeta\varepsilon\big]\Big)$, the firm defaults, a measure $\zeta s_{t-}X_{t-}$ of secured lenders have face value $1-\varepsilon$ while a measure $(1-\zeta)s_{t-}X_{t-}$ have face value 1. It follows that the total recovery to secured lenders is

$$\min\Big\{Z\pi K_{t-}, s_{t-}X_{t-}(1-\zeta\varepsilon)\Big\}. \tag{A.2}$$

This occurs with probability $\lambda\alpha F\ \big(\ Z_*\big(x_{t-}\big[1 - s_{t-}\zeta\varepsilon\big]\big)\ \big)\ dt$.

A.1.4 Combining default scenarios

We now calculate the secured credit spread by combining the default scenarios described above. We drop this time subscript $t-$ and conduct credit spread calculation with the expectation that a jump may arrive at t.

Define

$$Z^S(x,s) = \min\left\{\ Z_*(x), \frac{sx}{\pi}\ \right\}. \tag{A.3}$$

If there is no restructuring, secured lenders receive full recovery if a jump with $Z > Z^S(x,s)$ arrives. This is because either the firm doesn't default ($Z > Z_*(x)$) or there is enough value for full repayment as $Z\pi K > sX$ even if it defaults.

Define

$$x^{res}(x,s) = x(1-s\zeta\varepsilon), \quad Z_*^{res}(x,s) = Z_*(x^{res}\big(x,s)\big) \tag{A.4}$$

and

$$Z^V(x,s) = \min\left\{\ Z_*^{res}(x,s), \frac{sx(1-\zeta\varepsilon)}{\pi}\ \right\}. \tag{A.5}$$

If there is a restructuring, the new face value of debt is $x^{res}(x,s)K$ and secured lenders receive full recovery on their new face value $sX(1-\zeta\varepsilon)$ when $Z > Z^V(x,s)$ by the same logic as above. In the remainder of this subsection, to ease the notation, we will omit the variables x, s, and simply use Z_*, Z_*^{res}, Z^S and Z^V.

Piecing together the above scenarios, the zero-profit condition for secured lenders, which

coalition. A continuum of lenders with measure $s_{t-}X_{t-}$ each have face value 1. They know with probability $1-\zeta$ they will keep face value 1. With probability ζ they will receive face value $1-\varepsilon$. Conditional on no default and restructuring, the expected face value is $1-\zeta\varepsilon$. Multiplying by the mass of the continuum gives the total value above.

requires that the risk-free return must equal the expected return, is:

$$
\begin{aligned}
sX(1+rdt) = & \underbrace{sX(1+(r+\eta^S)dt)\left(\ 1-[\lambda(1-\alpha\mathbf{1}^R(x))F(Z_*)+\lambda\alpha\mathbf{1}^R(x)]dt\ \right)}_{\text{Full recovery unless (shock+no restructure+default) or (shock+restructure)}} \\
& + \underbrace{\lambda\alpha\mathbf{1}^R(x)\big[1-F(Z_*^{res})\big]dt\left[sX(1+(r+\eta^S)dt)(1-\zeta\varepsilon)\right]}_{\text{Full recovery net haircut if (shock+restructure+no default)}} \\
& + \underbrace{\lambda\alpha\mathbf{1}^R(x)dt\left[\Big(F(Z_*^{res})-F(Z^V)\Big)(1-\zeta\varepsilon)sX+\pi K F(Z^V)\mathbb{E}[Z|Z<Z^V]\right]}_{\text{Default recovery if (shock+restructure+default)}} \\
& + \underbrace{\lambda(1-\alpha\mathbf{1}^R(x))dt\left[\Big(F(Z_*)-F(Z^S)\Big)sX+\pi K F(Z^S)\mathbb{E}[Z|Z<Z^S]\right]}_{\text{Default recovery if (shock + no restructure + default)}}. \qquad \text{(A.6)}
\end{aligned}
$$

Note that scenario 1 (forced default) does not appear in (A.6) because secured lenders receive full recovery. Dividing by $sXdt$ and letting dt approach zero, we obtain

$$
\begin{aligned}
&\eta^S - [\lambda(1-\alpha\mathbf{1}^R(x))F(Z_*)+\lambda\alpha\mathbf{1}^R(x)] + \lambda\alpha\mathbf{1}^R(x)\big[1-F(Z_*^{res})\big](1-\zeta\varepsilon) \\
&\quad + \lambda\alpha\mathbf{1}^R(x)\left[\Big(F(Z_*^{res})-F(Z^V)\Big)(1-\zeta\varepsilon)+\frac{\pi}{sx}F(Z^V)\mathbb{E}[Z|Z<Z^V]\right] \\
&\quad + \lambda(1-\alpha\mathbf{1}^R(x))\left[\Big(F(Z_*)-F(Z^S)\Big)+\frac{\pi}{sx}F(Z^S)\mathbb{E}[Z|Z<Z^S]\right] = 0. \qquad \text{(A.7)}
\end{aligned}
$$

Equation (A.7) allows us to write η^S via the following functional form:

$$\eta^S = \boldsymbol{\eta}^S(x,s)\,.$$

Using the assumption $F(Z)=Z^\beta$, which implies that $\mathbb{E}[Z]=\hat{b}\equiv\frac{\beta}{\beta+1}$ and $\int_a^b ZdF(Z)=$

$\frac{\beta}{\beta+1}(b^{\beta+1} - a^{\beta+1})$, we can simplify (A.7) and obtain:

$$\begin{aligned}\boldsymbol{\eta}^S(x,s) =&\lambda(1-\alpha\mathbf{1}^R(x))\left[(Z^S(x,s))^\beta - \frac{\pi\hat{b}}{sx}(Z^S(x,s))^{\beta+1}\right] \\ &- \lambda\alpha\mathbf{1}^R(x)\left[\ \left(\ 1-(Z^V(x,s))^\beta\ \right)\ (1-\zeta\varepsilon) + \frac{\pi\hat{b}}{sx}(Z^V(x,s))^{\beta+1} - 1\right]. \qquad \text{(A.8)}\end{aligned}$$

A.1.5 Condition for secured lenders accepting a restructuring offer

Finally, we consider the condition for secured lenders to accept the transaction. Recall that this is only relevant after a jump shock occurs. Conditional on the arrival of a jump shock, if the coalition of secured lenders reject, their expected recovery per dollar of face value is

$$(1 - F(Z^S(x,s))) + F(Z^S(x,s))\frac{\pi}{sx}\mathbb{E}[Z|Z < Z^S(x,s)]. \qquad \text{(A.9)}$$

The first term captures full recovery if secured lenders are unimpaired and the second term captures fractional recovery when secured lenders are impaired.

If the coalition of secured lenders accept, the expected recovery per dollar of face value is

$$(1 - F(Z_\zeta(x,s)))(1-\varepsilon) + F(Z_\zeta(x,s))\frac{\pi}{sx\zeta}\mathbb{E}[Z|Z < Z_\zeta(x,s)]\,, \qquad \text{(A.10)}$$

where

$$Z_\zeta(x,s) \equiv \min\left\{Z_*^{res}(x,s)\,,\ \frac{sx\zeta(1-\varepsilon)}{\pi}\right\}. \qquad \text{(A.11)}$$

If the new super secured debt is unimpaired, they receive $1-\varepsilon$ dollars per original dollar of face value. This is captured by the first term in (A.10). The second term in (A.10) captures the expected payment if the new super secured debt is impaired.

Taking the expected payments under both scenarios together, secured lenders accept a restructuring offer at x if and only if

$$\begin{aligned}&(1-(Z_\zeta(x,s_*(x)))^\beta)(1-\varepsilon) + (Z_\zeta(x,s_*(x)))^{\beta+1}\frac{\pi\hat{b}}{s_*(x)x\zeta} \\ >&(1-(Z^S(x,s_*(x)))^\beta) + (Z^S(x,s_*(x)))^{\beta+1}\frac{\pi\hat{b}}{s_*(x)x}. \qquad \text{(A.12)}\end{aligned}$$

We use (A.12) to verify that (11) holds and we set $\mathbf{1}^R(x)$ to one if (A.12) holds at x. We define x^R to equal the smallest x such that this holds.

A.2 Unsecured credit spread

We now derive an expression for the unsecured credit spread. To start, we consider the case where no restructuring occurs. Let

$$Z^{U,l,N}(x,s) = \min\left\{Z_*(x), \frac{sx}{\pi-\rho}\right\}. \tag{A.13}$$

Then $Z < Z^{U,l,N}(x_{t-}, s_{t-})$ implies that the firm defaults and $Z(\pi-\rho)K_{t-} < s_{t-}X_{t-}$ so there is nothing left for unsecured creditors in default (assuming no restructuring). Let

$$Z^{U,h,N}(x) = \min\left\{Z_*(x), \frac{x}{\pi-\rho}\right\}. \tag{A.14}$$

Then $Z < Z^{U,h,N}(x_{t-})$ implies $Z(\pi-\rho)K_{t-} < X_{t-}$ so unsecured debt is impaired, while $Z > Z^{U,h,N}(x_{t-})$ implies unsecured debt is unimpaired. It is clear that $Z^{U,h,N} \geq Z^{U,l,N}$. Then, if there is no restructuring, unsecured recovery is 0 for $Z < Z^{U,l,N}(x_{t-}, s_{t-})$, it is $(\pi-\rho)ZK_{t-} - s_{t-}X_{t-}$ for $Z \in [Z^{U,l,N}(x_{t-}, s_{t-}), Z^{U,h,N}(x_{t-})]$, and it is $(1-s_{t-})X_{t-}$ for $Z > Z^{U,h,N}(x_{t-})$.

Next, suppose a restructuring occurs. The analysis is similar, but we must account for the reduced amount of secured debt and a different default threshold:

$$Z^{U,l,V}(x,s) = \min\left\{Z_*^{res}(x,s), \frac{(1-\zeta\varepsilon)sx}{\pi-\rho}\right\}. \tag{A.15}$$

Equity holders default and unsecured lenders receive nothing when $Z < Z^{U,l,V}(x_{t-}, s_{t-})$. Let

$$Z^{U,h,V}(x,s) = \min\left\{Z_*^{res}(x,s), \frac{x\Big[(1-s)+(1-\zeta\varepsilon)s\Big]}{\pi-\rho}\right\}. \tag{A.16}$$

Unsecured lenders receive full recovery if $Z > Z^{U,h,V}(x_{t-}, s_{t-})$ by the same logic described above.

In the following, to ease the notation, we let $f_{sub} = 1 - \zeta\varepsilon$, similarly drop time subscripts $t-$ and use $\mathbf{1}^R$, Z_*, Z_*^{res}, $Z^{U,h,N}$, $Z^{U,l,N}$, $Z^{U,h,V}$ and $Z^{U,l,V}$. Then the breakeven condition for unsecured debt is

$$
\begin{aligned}
&(1-s)X(1+rdt) \\
=&(1-s)X(1+(r+\eta^U)dt)\left(1-\left[\lambda\left(\ (1-\alpha\mathbf{1}^R)F(Z_*)+\alpha\mathbf{1}^R F(Z_*^{res})\ \right)+\phi\Big(\frac{sX}{K}\Big)^\nu\right]dt\right) \\
&+\phi\Big(\frac{sX}{K}\Big)^\nu dt\min\left\{(1-s)X,\big((\pi-\rho)K-sX\big)^+\right\} \\
&+\lambda(1-\alpha\mathbf{1}^R)dt\left(\Big(F(Z_*)-F(Z^{U,h,N})\Big)(1-s)X+\mathbb{E}\Big[((\pi-\rho)ZK-sX)\mathbf{1}_{Z\in(Z^{U,l,N},Z^{U,h,N})}\Big]\right) \\
&+\lambda\alpha\mathbf{1}^R dt\left(\Big(F(Z_*^{res})-F(Z^{U,h,V})\Big)(1-s)X+\mathbb{E}\Big[((\pi-\rho)ZK-f_{sub}sX)\mathbf{1}_{Z\in(Z^{U,l,V},Z^{U,h,V})}\Big]\right).
\end{aligned}
$$

Dividing by $(1-s)Xdt$ and letting dt approach zero, we obtain

$$
\begin{aligned}
\eta^U &- \big[\lambda\big((1-\alpha\mathbf{1}^R)F(Z_*)+\alpha\mathbf{1}^R F(Z_*^{res})\big)+\phi(sx)^\nu\big]+\phi(sx)^\nu\min\left\{1,\frac{(\pi-\rho-sx)^+}{(1-s)x}\right\} \\
&+\lambda(1-\alpha\mathbf{1}^R)\left(F(Z_*)-F(Z^{U,h,N})+\mathbb{E}\left[\frac{(\pi-\rho)Z-sx}{(1-s)x}\mathbf{1}_{Z\in(Z^{U,l,N},Z^{U,h,N})}\right]\right) \\
&+\lambda\alpha\mathbf{1}^R\left(F(Z_*^{res})-F(Z^{U,h,V})+\mathbb{E}\left[\frac{(\pi-\rho)Z-f_{sub}sx}{(1-s)x}\mathbf{1}_{Z\in(Z^{U,l,V},Z^{U,h,V})}\right]\right)=0.
\end{aligned}
$$

Rearranging this expression gives the following expression:

$$\eta^U = \boldsymbol{\eta}^U(x,s),$$

where $\boldsymbol{\eta}^U(\cdot,\cdot)$ is defined as follows:

$$
\begin{aligned}
\boldsymbol{\eta}^U(x,s) =& \phi(sx)^\nu - \phi(sx)^\nu \min\left\{1, \frac{(\pi-\rho-sx)^+}{(1-s)x}\right\} \\
& + \lambda(1-\alpha\mathbf{1}^R(x))(Z^{U,h,N}(x))^\beta + \lambda\alpha\mathbf{1}^R(x)(Z^{U,h,V}(x,s))^\beta \\
& + \lambda\alpha\mathbf{1}^R(x)\frac{f_{sub}s}{(1-s)}\left[(Z^{U,h,V}(x,s))^\beta - (Z^{U,l,V}(x,s))^\beta\right] \\
& - \lambda\alpha\mathbf{1}^R(x)\frac{\hat{b}(\pi-\rho)}{(1-s)x}\left[(Z^{U,h,V}(x,s))^{\beta+1} - (Z^{U,l,V}(x,s))^{\beta+1}\right] \\
& + \lambda(1-\alpha\mathbf{1}^R(x))\left(\frac{s}{1-s}\left[(Z^{U,h,N}(x))^\beta - (Z^{U,l,N}(x,s))^\beta\right]\right) \\
& - \lambda(1-\alpha\mathbf{1}^R(x))\frac{\hat{b}(\pi-\rho)}{(1-s)x}\Big((Z^{U,h,N}(x))^{\beta+1} - (Z^{U,l,N}(x,s))^{\beta+1}\Big).
\end{aligned}
\tag{A.17}
$$

Let $\boldsymbol{\eta}(x,s)$ denote the weighted average credit spread for both secured and unsecured debts: $\boldsymbol{\eta}(x,s) = s\boldsymbol{\eta}^S(x,s) + (1-s)\boldsymbol{\eta}^U(x,s)$. Then, the (scaled) interest payments for both secured and unsecured debts are

$$
c(x,s) \equiv x\Big(r + \boldsymbol{\eta}(x,s)\Big). \tag{A.18}
$$

A.3 HJB equation

Finally, we derive the HJB equation (29). Using (4) and (12), in the debt financing region, we have

$$
dX_t = \Big(-\theta K_t + I_t + (1-\tau)C_t\Big)dt - \mathbf{1}_t^R \zeta\varepsilon s_{t-}X_{t-}d\mathcal{J}_t, \tag{A.19}
$$

where $\mathbf{1}_t^R$ equals one if a restructuring is offered and accepted after a jump arrives at time t and zero otherwise. Here, the debt coupon payment is $C_t = c(x_t, s_t)K_t$, where $c(\cdot,\cdot)$ is given by (A.18).

Note that $\mathbf{1}_t^R = \mathbf{1}^R(x_{t-})$ with probability α and is zero with probability $1-\alpha$, where $\mathbf{1}^R(x)$ is 1 if the secured acceptance condition (A.12) is met and zero otherwise. Combining (1) and (A.19), we can derive the following HJB equation for the equity value function $P(K,X)$

in the interior region:

$$
\begin{aligned}
\gamma P(K,X) \quad = \quad \max_{I,s} & \Bigg(-\theta K + I + (1-\tau)c\Big(\frac{X}{K},s\Big)K\Bigg) P_X(K,X) \\
& + K\Big(\psi\left(\frac{I}{K}\right) - \delta\Big) P_K(K,X) + \frac{1}{2}\sigma^2 K^2 P_{KK}(K,X) \\
& + \lambda\Big[-P(K,X) + (1-\alpha \mathbf{1}^R(X/K)) \int_0^1 P(ZK,X)dF(Z) \\
& + \alpha \mathbf{1}^R(X/K) \int_0^1 P(ZK, X - sX\zeta\varepsilon)dF(Z)\Big] \\
& + \phi\Big(\frac{sX}{K}\Big)^\nu \Big[(\pi K - \rho K - X)^+ - P(K,X) \Big]
\end{aligned} \tag{A.20}
$$

subject to $0 \leq s \leq \min\{1, \frac{\pi}{x}\}$. Using $x = X/K$ and $p(x) = P(K,X)/K$, we have $P_X(K,X) = p'(x)$, $P_K(K,X) = p(x) - xp'(x)$, $KP_{KK}(K,X) = x^2 p''(x)$. Substituting these expression into (A.20) and using $i = I/K$, we derive the following HJB equation for $p(x)$:[14]

$$
\begin{aligned}
(\gamma+\lambda)p(x) = \max_{i,s} & \Big(-\theta + i + (1-\tau)c(x,s)\Big)p'(x) + \frac{1}{2}\sigma^2 x^2 p''(x) \\
& + \Big(\psi(i) - \delta\Big)\Big(p(x) - xp'(x)\Big) + \phi\big(sx\big)^\nu \left[(\pi - \rho - x)^+ - p(x) \right] \\
& + \lambda\Bigg[(1-\alpha\mathbf{1}^R(x)) \int_0^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) + \alpha\mathbf{1}^R(x) \int_0^1 Zp\Big(x\frac{1-s\zeta\varepsilon}{Z}\Big)dF(Z)\Bigg]
\end{aligned} \tag{A.21}
$$

subject to $0 \leq s \leq \min\{1, \frac{\pi}{x}\}$. Using (A.1) and (A.4), we have $p(x/Z) = 0$ for $Z < Z_*(x)$ and $p(x(1-s\zeta\varepsilon)/Z) = 0$ for $Z < Z_*^{res}(x,s)$. Substituting them into (A.21), we obtain (29).

[14] Note that $P(ZK, X - sX\zeta\varepsilon)/(ZK) = p\left((X/K) \times (1 - s\zeta\varepsilon)/Z \right)$.

Internet Appendices

I.A Static model with endogenous debt restructuring

This appendix shows that our results are robust to alternative assumptions. We present a simplified version of our baseline model in a static framework. Because the framework is static, we can tractably model: (i) an endogenous secured-lender choice to sell assets early or not; (ii) an endogenous haircut ε chosen by equity holders when making an aggressive restructuring offer; (iii) a coercive offer in which a secured lender's alternative to participating in a restructuring is being left out of the coalition that restructures their debt.

We show that our main result is robust to these alternative assumptions: we continue to find that increasing the frequency α of aggressive restructurings leads to higher firm value ex ante. Restructuring offers are always accepted: equity holders adjust the haircut ε to make secured lenders indifferent between participating in a coalition or being left out of the coalition. However, in bad states of the world (those with a high probability of a bad shock), secured lenders are willing to accept a higher haircut. In this sense, even with endogenous haircuts, these restructurings lead to state-contingent debt reduction, improving firm value ex ante.

I.A.1 Model assumptions

There are three periods. In the first period, the firm issues debt to trade off tax benefits with deadweight losses in default. It also chooses a secured-debt share, facing the same tradeoff as in our baseline model. In the second period, the firm and its lenders learn whether the probability of a bad shock is high or low. After this, one of three mutually exclusive events occurs: (i) secured lenders have an opportunity to sell assets early, (ii) equity holders have an opportunity to make an aggressive debt restructuring offer, (iii) neither opportunity arises. Specifically, with an exogenous probability, secured lenders have an opportunity to sell assets. Departing from our baseline model, we allow secured lenders to choose whether to do this or not. With a distinct exogenous probability, equity holders have the chance to make an aggressive restructuring offer. Departing from our baseline model, equity holders choose the face-value haircut ε endogenously. Finally, in the third period, everyone learns whether the bad shock has realized. Equity holders decide whether to default and payoffs are realized.

I.A.1.1 Period one

In the first period, the firm chooses its debt face value X and the fraction s to issue as secured debt. Let $F_S \equiv sX$ and $F_U \equiv (1-s)X$ denote the face value of the secured and unsecured debt, respectively.

The firm issues secured debt to a continuum of infinitesimal lenders of measure s. It issues unsecured debt to a continuum of infinitesimal lenders of measure $1-s$. Finally, we assume the firm has an exogenous quantity ρ of priority unsecured claims corresponding to employee wage claims.

There is a competitive lending market. Lenders thus pay equity holders the expected value of their future payoffs in exchange for the debt claims. We derive these expected values below.

I.A.1.2 Period two

In the second period, the firm and its lenders observe the realization of a random variable $\lambda \in \{\lambda_L, \lambda_H\}$. A high realization λ_H of λ corresponds to a high probability of a bad shock. $\lambda = \lambda_H$ is thus a bad state. Let $p_\lambda \equiv \mathbb{P}(\lambda = \lambda_L)$.

With probability ϕF_S^ν, secured lenders have the opportunity to force an asset sale. As we will explain, this choice is endogenous in this model extension. Secured lenders force an asset sale if and only if doing so maximizes their expected payoff. We assume that in a forced asset sale, the firm value is an exogenous parameter π_S. Because secured lenders have the highest priority, they collectively recover $\min\{F_S, \pi_S\}$ in a forced asset sale. Priority claim holders come second, recovering $\min\{\rho, (\pi_S - F_S)^+\}$. Unsecured claims come third, recovering $\min\{F_U, (\pi_S - F_S - \rho)^+\}$. Equity holders come last, recovering $(\pi_S - X - \rho)^+$.

With probability α,[15] equity holders have the opportunity to make a restructuring offer. Equity holders endogenously choose a haircut ε. Each secured lender simultaneously decides whether they are willing to accept the offer. The contract terms require a fraction ζ of secured lenders to accept a restructuring, where $\zeta \in (0,1)$ is exogenous.[16] If a fraction less

[15]For our chosen parameters, $\alpha + \phi F_S^\nu < 1$ at the ex-ante optimal value of F_S.

[16]In Internet Appendix I.C, we show that our results are robust to allowing for an endogenous choice of ζ.

than ζ of the secured lenders are willing to accept, there is no restructuring. If a fraction greater than or equal to ζ are willing to accept, equity holders randomly select a fraction ζ of secured lenders to participate in the coalition. If the restructuring offer is accepted, the coalition lenders continue to enjoy the highest priority on their new debt with total face value $\zeta(1-\varepsilon)F_S$. The excluded lenders, with total face value $(1-\zeta)F_S$, have second priority. We focus on an equilibrium in which all lenders (i) are indifferent between accepting or rejecting the offer and (ii) the offer is accepted. Importantly, off the equilibrium path, if one of the lenders rejects the offer, they are subordinated in the excluded group, since each lender is infinitesimal and cannot impact the likelihood of a successful restructuring. We describe this in greater detail below.

We assume that restructurings and asset sales are mutually exclusive. Thus, with probability $1-\alpha-\phi F_S^{\nu}$, there is no restructuring and no forced asset sale.

I.A.1.3 Period three

In period three, uncertainty is resolved. The probability of a bad shock is λ, where $\lambda \in \{\lambda_L, \lambda_H\}$ is revealed in period two. The firm and its lenders learn in period three whether a bad shock occurs.

Payoffs in the absence of a bad shock: With probability $1-\lambda$, there is no bad shock. We assume parameters such that, in the absence of a bad shock, the firm does not default. This is reasonable, since any other parameters would imply the firm defaults with probability one. Formally, if there is no bad shock, there is no default, and the value of the firm is $\widetilde{V}_H \equiv 1+\tau\widetilde{X}$. The parameter τ is a reduced-form way of capturing the tax benefits of debt. We let $\widetilde{X} = F_U + \widetilde{F}_S$ denote the post-restructuring total value of debt, where $\widetilde{F}_S = F_S$ if there is no restructuring and it equals $F_S(1-\zeta\varepsilon)$ if there is a restructuring. Equity holders repay $\widetilde{X}$ to secured lenders and unsecured lenders, ρ to priority claim holders (employees), and receive a payout of $\widetilde{V}_H - (\widetilde{X}+\rho)$.

If there is a restructuring, the coalition lenders collectively recover $F_S\zeta(1-\varepsilon)$ and the excluded lenders recover $F_S(1-\zeta)$. If there is no restructuring, secured lenders collectively recover F_S. Priority claim holders receive ρ. Unsecured lenders receive their full face value F_U.

Payoffs with a bad shock: With probability λ there is a bad shock. For simplicity, we assume that the tax shield disappears if there is a bad shock. The value of the firm after a bad shock is $Z \sim \text{Unif}[0,1]$. After viewing Z, equity holders decide whether to default. Let $\mathbf{1}_R$ denote an indicator for a restructuring occurring in period two. If equity holders do not default, they receive a payoff

$$P_{eq} \equiv Z - \left[\rho + F_U + F_S - \mathbf{1}_R \varepsilon \zeta F_S\right]. \tag{IA.1}$$

If equity holders do not default, priority claim holders receive ρ, unsecured lenders receive F_U, and secured lenders collectively recover $\widetilde{F}_S$.

Equity holders default if their payoff P_{eq} would be negative, consistent with limited liability. In default, equity holders get nothing and the firm value drops from Z to πZ for a parameter $\pi < 1$. This captures the deadweight losses of default.

In default, if there was no restructuring, secured lenders collectively recover $\min\{F_S, \pi Z\}$. If there was a restructuring, participating secured lenders collectively recover $\min\{\zeta(1-\varepsilon)F_S, \pi Z\}$ while the excluded lenders collectively recover $\min\{(1-\zeta)F_S\,,\,[\pi Z - \zeta(1-\varepsilon)F_S]^+\}$.

In default, unsecured lenders recover $\min\left\{F_U\,,\,(\pi Z - \rho - F_S + \mathbf{1}_R \zeta \varepsilon F_S)^+\right\}$.

I.A.2 Model solution

We solve the model backward. There are no period-three decisions, so we start by solving for the optimal choices in period two.

I.A.2.1 Period two

In period two, everyone knows the realization of λ. Let $\mathbf{1}_D(\varepsilon, \mathbf{1}_R)$ denote an indicator for (IA.1) being negative so that equity holders default. This indicator is a function of Z, but we omit this dependence for notational convenience. Let $\mathbf{1}_D(0) \equiv \mathbf{1}_D(0,0)$ denote the corresponding default indicator if there is no restructuring.

First, consider the collective period-two expected secured lender payoff if there is no restructuring and no asset sale:

$$\text{Sec}_N(\lambda) \equiv (1-\lambda)F_S + \lambda \mathbb{E}\Big[(1-\mathbf{1}_D(0))F_S + \mathbf{1}_D(0)\min\{\pi Z, F_S\}\Big]. \tag{IA.2}$$

Secured lenders endogenously use an asset-sale opportunity if and only if (IA.2) is less than $\min\{F_S, \pi_S\}$. Recall that if an asset-sale opportunity arises, there is no restructuring opportunity, so the secured lenders' choice is simply to sell assets or continue to period three. Let $\mathbf{1}_A(\lambda)$ equal one if this condition for an asset sale is satisfied at λ. This value $\mathbf{1}_A(\lambda)$ is only relevant if an asset-sale opportunity arises.

Next, consider the restructuring choice. If a restructuring opportunity arises, there is no asset-sale opportunity and equity holders choose a haircut ε. If the restructuring succeeds, the collective payoff to excluded secured lenders is

$$\begin{aligned}&\mathrm{Sec}_{Out}(\lambda,\varepsilon) \equiv (1-\lambda)(1-\zeta)F_S \\ &+ \lambda\mathbb{E}\left[(1-\mathbf{1}_D(\varepsilon,1))(1-\zeta)F_S + \mathbf{1}_D(\varepsilon,1)\min\left\{[\pi Z - \zeta(1-\varepsilon)F_S]^+, (1-\zeta)F_S\right\}\right].\end{aligned} \tag{IA.3}$$

A fraction $1-\zeta$ of secured lenders are excluded, so a continuum of lenders of measure $s(1-\zeta)$ are excluded. It follows that the payoff to each identical infinitesimal excluded lender is equal to (IA.3) divided by $s(1-\zeta)$.

In a successful restructuring, the collective payoff to the participating coalition of secured lenders is:[17]

$$\begin{aligned}&\mathrm{Sec}_{In}(\lambda,\varepsilon) \equiv (1-\lambda)(1-\varepsilon)\zeta F_S \\ &+ \lambda\mathbb{E}\big[(1-\mathbf{1}_D(\varepsilon,1))(1-\varepsilon)\zeta F_S + \mathbf{1}_D(\varepsilon,1)\min\{\pi Z, (1-\varepsilon)\zeta F_S\}\big].\end{aligned} \tag{IA.4}$$

A fraction ζ of secured lenders participate in the coalition, so a continuum of measure $s\zeta$ receive the payoff (IA.4). Thus, each identical infinitesimal participating lender receives a payoff equal to (IA.4) divided by $s\zeta$.

Putting this together, a restructuring succeeds in equilibrium if and only if

$$\frac{1}{s\zeta}\mathrm{Sec}_{In}(\lambda,\varepsilon) \geq \frac{1}{s(1-\zeta)}\mathrm{Sec}_{Out}(\lambda,\varepsilon). \tag{IA.5}$$

If this condition holds, there is an equilibrium in which each lender is willing to participate.

[17] Note that in our baseline model, the order of events is: (i) secured lenders learn a jump shock has occurred; (ii) secured lenders decide whether to accept a restructuring offer; (iii) secured lenders learn the size of the jump shock. Analogously, here the order of events is (i) secured lenders learn whether they are in the bad state $\lambda = \lambda_H$; (ii) secured lenders decide whether to accept a restructuring offer; (iii) secured lenders learn whether there is a bad shock, which occurs with probability $\lambda = \lambda_H$ in the bad state or probability $\lambda = \lambda_L$ in the good state.

This is an equilibrium because there are no profitable deviations: (i) each lender is infinitesimal, so rejecting the offer will not make a restructuring fail, and (ii) the payoff to rejecting and getting subordinated (the right side of the above inequality) is less than the payoff from accepting and potentially participating in the coalition (the left side of the above inequality).

Equity holders want as much debt reduction as possible, so they choose ε_* that makes secured lenders indifferent between accepting or rejecting:

$$\frac{1}{s\zeta}\mathrm{Sec}_{In}(\lambda,\varepsilon_*) = \frac{1}{s(1-\zeta)}\mathrm{Sec}_{Out}(\lambda,\varepsilon_*). \tag{IA.6}$$

Again, this entails a different outside option for secured lenders who reject an offer than the corresponding option in our baseline model. In this extension, if a secured lender rejects an offer, then they get subordinated in a successful restructuring.

In summary, if there is a restructuring opportunity, equity holders offer a haircut $\varepsilon_*(\lambda)$ that is endogenously determined by (IA.6). All secured lenders are willing to participate, and a randomly selected fraction ζ of secured lenders get to participate in the coalition. If there is an asset-sale opportunity, secured lenders sell if $\min\{F_S, \pi_S\}$ exceeds (IA.2).

I.A.2.2 Period one

In period one, equity holders choose X, s to maximize the sum of the present expected value of future: (i) equity payoffs; (ii) secured lender payoffs; (iii) unsecured lender payoffs. These expectations account for the subsequent period-two choices. This objective captures a competitive lending market in which the debt proceeds from debt issuance are equal to the expected future payoffs.

The present expected value of future equity is

$$\begin{aligned}\sum_{i=L,H}\mathbb{P}(\lambda=\lambda_i)\times\Bigg(&\phi F_S^{\nu}\mathbf{1}_A(\lambda_i)(\pi_S - X - \rho)^+\\ &+\alpha\Big[(1-\lambda_i)\big(\widetilde{V}_H(\lambda_i) - (\widetilde{F}_S(\lambda_i) + F_U + \rho)\big) + \lambda_i\mathbb{E}\Big[\big(Z - \widetilde{F}_S(\lambda_i) - F_U - \rho\big)^+\Big]\Big]\\ &+(1-\alpha-\phi F_S^{\nu}\mathbf{1}_A(\lambda_i))\Big[(1-\lambda_i)\big(V_H - (F_S + F_U + \rho)\big) + \lambda_i\mathbb{E}\Big[\big(Z - F_S - F_U - \rho\big)^+\Big]\Big]\Bigg),\end{aligned} \tag{IA.7}$$

where $V_H = 1 + \tau X$, $\widetilde{F}_S(\lambda_i) \equiv F_S(1-\zeta\varepsilon_*(\lambda_i))$ and $\widetilde{V}_H(\lambda_i) \equiv 1 + \tau X(1 - s\zeta\varepsilon_*(\lambda_i))$.

The present expected future value of total secured debt payoffs is:

$$\sum_{i=L,H} \mathbb{P}(\lambda = \lambda_i) \times \Bigg(\phi F_S^{\nu} \mathbf{1}_A(\lambda_i) \min\{\pi_S, F_S\}$$
$$+ \alpha \big[\mathrm{Sec}_{Out}(\lambda_i, \varepsilon_*(\lambda_i)) + \mathrm{Sec}_{In}(\lambda_i, \varepsilon_*(\lambda_i)) \big] + \big(1 - \alpha - \phi F_S^{\nu} \mathbf{1}_A(\lambda_i)\big) \mathrm{Sec}_N(\lambda_i) \Bigg) . \tag{IA.8}$$

Finally, the present expected value of unsecured debt is

$$\sum_{i=L,H} \mathbb{P}(\lambda = \lambda_i) \times \Bigg(\phi F_S^{\nu} \mathbf{1}_A(\lambda_i) \min\{F_U, (\pi_S - F_S - \rho)^+\}$$
$$+ \alpha \Bigg[(1 - \lambda_i) F_U + \lambda_i \mathbb{E}\big[(1 - \mathbf{1}_D(\varepsilon_*(\lambda_i), 1)) F_U + \mathbf{1}_D(\varepsilon_*(\lambda_i), 1) \min\{F_U, [\pi Z - \widetilde{F}_S(\lambda_i) - \rho]^+\} \big] \Bigg]$$
$$+ (1 - \alpha - \phi F_S^{\nu} \mathbf{1}_A(\lambda_i)) \times$$
$$\Bigg[(1 - \lambda_i) F_U + \lambda_i \mathbb{E}\big[(1 - \mathbf{1}_D(0)) F_U + \mathbf{1}_D(0) \min\{F_U, (\pi Z - F_S - \rho)^+\} \big] \Bigg] \Bigg), \tag{IA.9}$$

where $x^+ \equiv \max\{0, x\}$ for any $x \in \mathbb{R}$.

I.A.3 Numerical solution

We search numerically over potential values of X, s. For each value, we calculate $\varepsilon_*(\lambda_i)$ and $\mathbf{1}_A(\lambda_i)$. We then calculate and sum the period-one values of (i) equity in (IA.7); (ii) secured debt in (IA.8); and (iii) unsecured debt in (IA.9). We repeat this process until we find values of X, s that maximize this sum.

Our baseline parameter values are the following:

Table IA.1: Parameter values

This table shows our parameter values for our static model extension. Whenever applicable, parameter values are annualized.

p_λ	Probability of good state	0.5
λ_L	Probability of bad shock in good state	0.2
λ_H	Probability of bad shock in bad state	0.8
τ	Reduced-form tax shield	0.03
π	Value in default	0.8
π_S	Value in asset sale	0.57
α	Probability of liability management	0
ϕ	Scale of secured default risk	0.03
ν	Convexity of secured default risk	1
ζ	Fraction of secured lenders in a coalition	0.8
ρ	Priority claims	0.02

I.A.4 Numerical results

The following page shows that our main results continue to hold in this model. We find that ex-ante firm value increases with α: ex-post restructurings create value ex ante. We show that secured credit spreads also rise as α increases, consistent with our baseline model.[18] Moreover, Figure IA.1 includes results from a parameterization in which $\alpha = 1$. This demonstrates that our results are not specific to our baseline assumption that $\alpha < 1$.

We confirm that model tradeoffs play the same role in this extension that they do in our baseline model. As taxes increase, the firm uses more debt. As firm value in default increases (deadweight losses shrink), the firm uses more debt. As the probability of a potential asset sale increases (ϕ rises), the firm uses less secured debt. As the amount of priority unsecured claims rises, the firm uses more secured debt.

In summary, we show that our main results are robust. They hold in a model extension with: (i) endogenous restructuring offers (endogenous ε); (ii) endogenous decisions by secured lenders to sell assets early; and (iii) a distinct restructuring game in which a secured lender's outside option to participating in a coalition is being subordinated in a nonetheless successful restructuring.

[18] We define secured credit spreads as the sum of -1 and the ratio of (i) the total face value F_S of secured debt to (ii) the present value of total secured lender payoffs, which is the expression in (IA.8). In other words, if secured lenders expect to receive 80 cents of future payoffs and the face value of secured debt is one dollar, equity holders issue at a 20% discount to face value and the credit spread is $-1 + (1/.8) = 25\%$.

Figure IA.1: Comparative statics in the static model extension

We assume the parameter values given in Table IA.1. We solve the model of Internet Appendix I.A and calculate: (i) ex-ante firm value, (ii) the ex-ante secured credit spread based on the ratio of the secured-debt face value to the proceeds from issuing secured debt, (iii) total debt, and (iv) the share of secured debt. We then repeat this process, varying α but holding all other parameters fixed. We repeat this process varying τ, π, ϕ, and ρ one at a time. This figure plots how model objects vary with parameters.

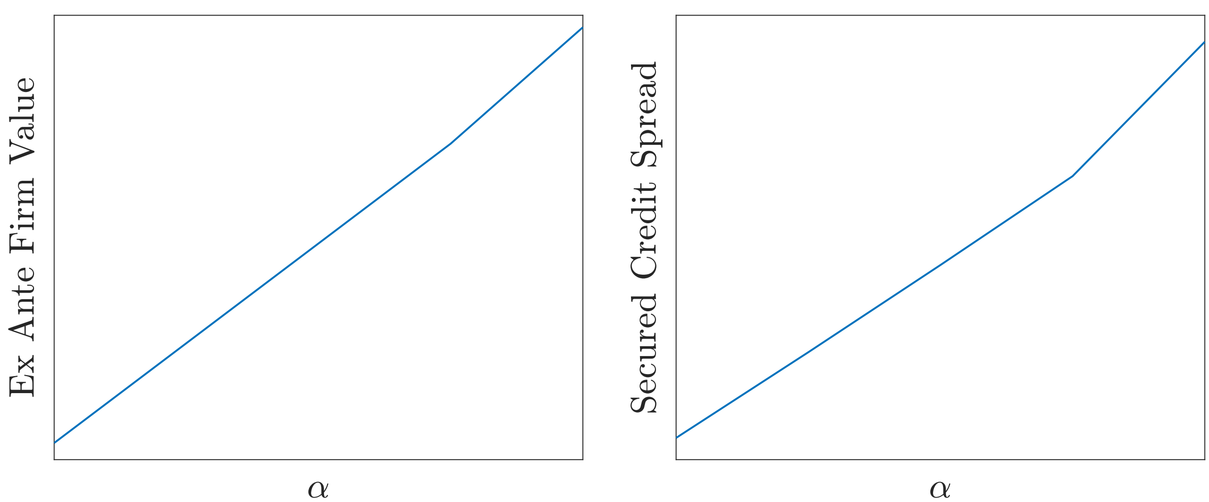
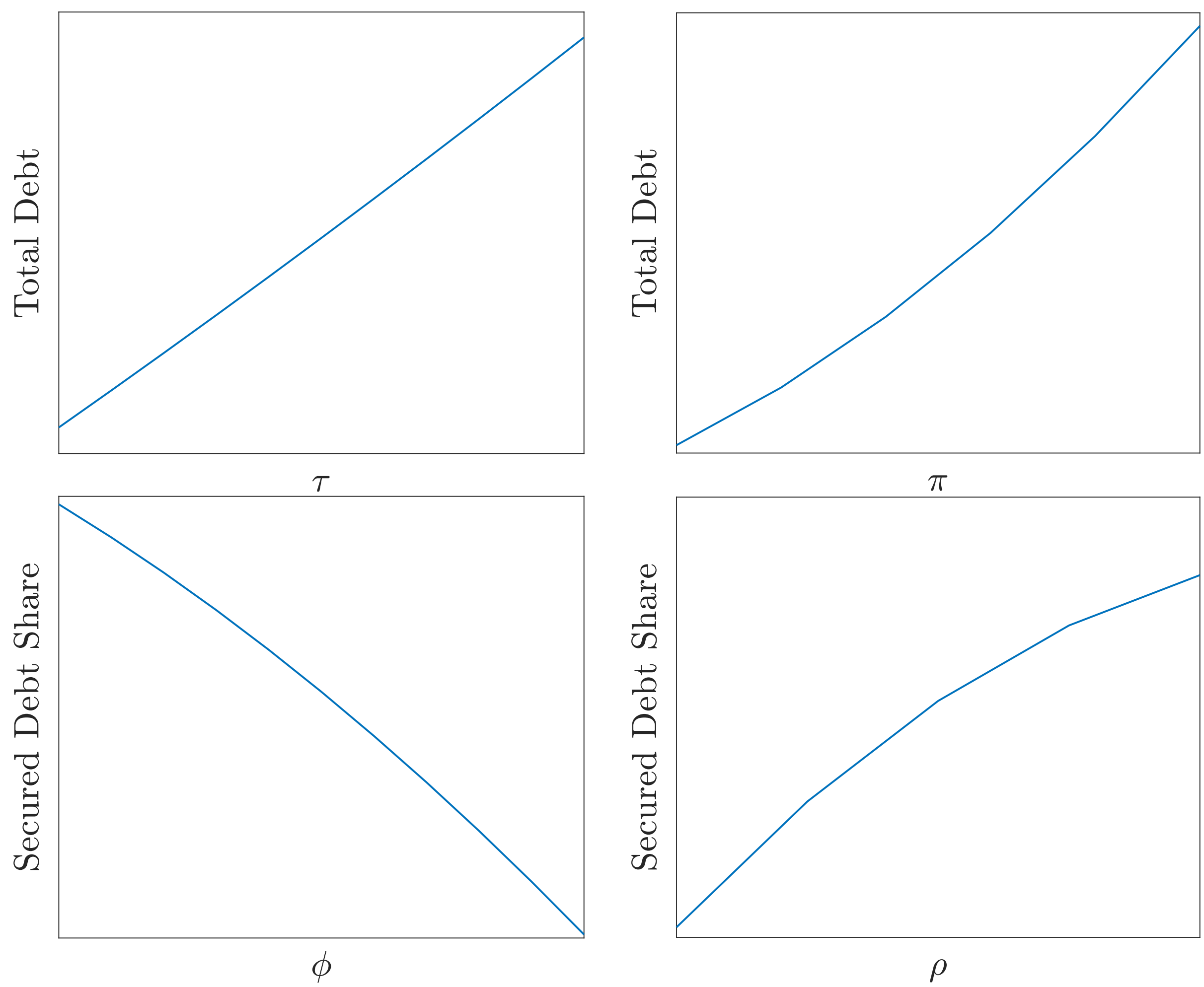


Figure IA.1: Comparative statics in the static model extension, continued



I.B Model extension with long-term debt

In this appendix, we show that our main result holds in the canonical framework of Leland (1994). Importantly, this shows that our main result holds in a setting with long-term debt. Moreover, we show that our main result holds in a setting in which (i) ε is an endogenous choice of equity holders; and (ii) the outside option of a secured lender in a restructuring offer is to be excluded and subordinated in a debt restructuring conducted by another coalition, rather than for the restructuring to not occur. We show that ex-ante firm value and secured credit spreads increase as α, the probability of a restructuring opportunity, increases.

I.B.1 Model assumptions

At time zero, the firm and its lenders play a debt issuance and restructuring game. After time zero, time evolves continuously and equity holders decide when to default in a framework closely resembling Leland (1994).

Time-zero game: We break time zero into four "stages." In the first stage, the firm chooses a coupon C and secured-debt share s and collects debt proceeds equal to the expected present value of future lender cash flows. The debt is priced rationally. The firm's initial EBIT is ω_0. Note that we use ω_t to denote EBIT.

In the second stage, an instant after the first stage, the firm experiences a shock. With probability p_B, the firm learns it is in the "bad state." In the bad state, there is a probability p_Z that the EBIT will subsequently decline from ω_0 to $\omega_0 Z$, where $Z \in (0, 1)$ is an exogenous constant. With probability $1 - p_B$, the firm is in the good state, where the initial EBIT is ω_0 with probability one.

In the third stage, there is a potential restructuring. We assume that restructurings do not occur in the good state. In the bad state, with probability α, equity holders have a restructuring opportunity. They choose an endogenous ε in a restructuring procedure described below. Secured lenders decide endogenously whether to accept. This leads to a potentially new coupon $\widehat{C}$ and secured-debt share $\hat{s}$, as we describe below.

In the fourth stage, the firm learns whether the initial EBIT is ω_0 or $Z\omega_0$. In the good state, it is ω_0. In the bad state, it is ω_0 with probability $1 - p_Z$ and $Z\omega_0$ with probability p_Z.

Ex-post equity holder problem: After these initial four stages, time evolves continuously. For simplicity, we assume that the firm's investment policy is fixed exogenously and the firm's EBIT evolves as

$$d\omega_t = \mu\omega_t dt + \sigma\omega_t d\mathcal{B}_t. \tag{IB.1}$$

In this equation, σ is the volatility and $\mathcal{B}_t$ is a standard Brownian motion. The parameter μ is the drift, which we assume is less than the risk-free rate r.

The firm pays a constant tax rate τ on its EBIT. We assume that the firm earns an interest tax shield on its debt coupon payments. The cash flow to equity holders is thus $(1-\tau)(\omega_t - \widehat{C})dt$ per unit time. Recall that $\widehat{C}, \hat{s}$ are the post-time-zero coupon and secured-debt share, which may differ from C, s if a restructuring occurs at time zero.

We assume that secured lenders force a default with probability $\phi(\hat{s}\widehat{C})^\nu dt$ per unit time. Let T_S denote the first jump time of a Poisson process with constant intensity $\phi(\hat{s}\widehat{C})^\nu$, so secured lenders take over at time T_S.

In default, secured lenders have first priority on the firm value in default $\pi\omega_t$, where π is an exogenous parameter capturing firm value in default. After secured lenders, priority claims worth ρ receive second priority and unsecured claims get third priority. For simplicity, equity receives nothing in default.

Equity holders can also choose to endogenously default in any instant. Given these assumptions, equity holders choose a default time T_D to maximize the expected discounted value of their future cash flows:

$$V_E(\omega; (\widehat{C}, \hat{s})) = \sup_{T_D} \mathbb{E}\left[\int_0^{T_D \wedge T_S} e^{-rt}(1-\tau)(\omega_t - \widehat{C})dt\right]. \tag{IB.2}$$

Debt pricing: Debt is priced under rational expectations. After the time-zero game, the value of secured debt is thus

$$V_S(\omega; (\widehat{C}, \hat{s})) = \mathbb{E}\left[\int_0^{T_D \wedge T_S} e^{-rt}\widehat{C}\hat{s}dt + e^{-r(T_D \wedge T_S)} \min\left\{\pi\omega_{T_D \wedge T_S}, \frac{\hat{s}\widehat{C}}{r}\right\}\right]. \tag{IB.3}$$

In words, secured lenders receive a fraction $\hat{s}$ of the coupon $\widehat{C}$ in each instant until default. If a default occurs, secured lenders have first priority, receiving either their full face value,

$\hat{s}\widehat{C}/r$, or the full firm value $\pi\omega$. The value of unsecured debt is analogously

$$V_U(\omega; (\widehat{C}, \hat{s})) = \mathbb{E}\Bigg[\int_0^{T_D \wedge T_S} e^{-rt}\widehat{C}(1-\hat{s})dt + e^{-r(T_D \wedge T_S)} \min\left\{\left(\pi\omega_{T_D \wedge T_S} - \frac{\hat{s}\widehat{C}}{r} - \rho\right)^+, \frac{(1-\hat{s})\widehat{C}}{r}\right\}\Bigg]. \tag{IB.4}$$

In words, unsecured lenders receive a fraction $1-\hat{s}$ of the coupon $\widehat{C}$ in each instant until default. If a default occurs, unsecured lenders have third priority after the secured face value $\hat{s}\widehat{C}/r$ and priority claims ρ. They thus receive either their full face value, $(1-\hat{s})\widehat{C}/r$, or the residual firm value $\pi\omega - \hat{s}\widehat{C}/r - \rho$.

Restructuring: We can now describe the time-zero restructuring game. Recall that if the bad state is realized in the second stage, then there is a restructuring opportunity in the third stage with probability α. We assume that equity holders choose an endogenous haircut ε. Each secured lender simultaneously decides whether they will participate in the restructuring if they are invited to join the coalition. The credit agreement requires a fraction ζ of secured lenders to amend the credit agreement. If a fraction less than ζ of secured lenders are willing to participate, there is no restructuring. If a fraction equal to or greater than ζ are willing to participate, then equity holders randomly select a fraction ζ of secured lenders from those willing to form the coalition that participates in the restructuring. The remaining fraction $1-\zeta$ of secured lenders are subordinated: a lender's outside option to participating might thus be getting subordinated in a restructuring. For tractability, we assume in this extension that the excluded lenders' claims are subordinated all the way to unsecured claims. This makes the subsequent value function calculation far simpler. Formally, if a restructuring offer is accepted, the total coupon falls from C to

$$\widehat{C} \equiv C\Big[1 - s\zeta\varepsilon\Big]. \tag{IB.5}$$

As a result, the total secured-debt share falls from s to

$$\hat{s} \equiv \frac{s\zeta(1-\varepsilon)}{1-s\zeta\varepsilon}, \tag{IB.6}$$

because the excluded fraction $1-\zeta$ of secured lenders become unsecured.

Restructuring payoffs: We assume there is originally a continuum of infinitesimal secured lenders with measure s. In a successful restructuring, a measure ζs of this continuum participates. Given the restructuring, the total coupon is $\widehat{C}$ and a fraction $\hat{s}$ of all debt is secured. Thus, in the instant before the fourth stage, the total expected value of post-restructuring secured debt is $(1-p_Z)V_S\big(\omega;(\widehat{C},\hat{s})\big)+p_Z V_S\big(Z\omega;(\widehat{C},\hat{s})\big)$, which takes into account the potential for EBIT to fall from ω_0 to $Z\omega_0$ in the fourth stage. Given this is the total value of secured debt and there is a measure ζs of identical participating coalition lenders, the value of each infinitesimal coalition lender's claim is:

$$\frac{1}{\zeta s}\Big[(1-p_Z)V_S\big(\omega;(\widehat{C},\hat{s})\big)+p_Z V_S\big(Z\omega;(\widehat{C},\hat{s})\big)\Big]. \tag{IB.7}$$

We conjecture an equilibrium in which a measure ζs of secured lenders participate in the restructuring on the equilibrium path. In such an equilibrium, if a secured lender declines the restructuring offer, they get subordinated: each lender is infinitesimal, so even if one lender deviates from being willing to participate, the measure of participating lenders is unchanged so the restructuring goes through. For tractability, we assume in this extension that any subordinated lender becomes unsecured. We assume there is originally a continuum of infinitesimal unsecured lenders of measure $1-s$. The subordinated secured lenders have measure $s(1-\zeta)$, implying that the post-restructuring measure of unsecured debt is $1-s+s(1-\zeta)=1-s\zeta$. As described above, the total expected value of post-restructuring unsecured debt is $(1-p_Z)V_U\big(\omega;(\widehat{C},\hat{s})\big)+p_Z V_U\big(Z\omega;(\widehat{C},\hat{s})\big)$. The value of each infinitesimal non-coalition lender's claim is thus:

$$\frac{1}{1-s\zeta}\Big[(1-p_Z)V_U\big(\omega;(\widehat{C},\hat{s})\big)+p_Z V_U\big(Z\omega;(\widehat{C},\hat{s})\big)\Big]. \tag{IB.8}$$

Restructuring equilibrium: Given these payoffs, an equilibrium with a successful restructuring exists if (IB.7) is weakly greater than (IB.8). In that case, each individual secured lender is willing to participate in a coalition: they receive (IB.7) if they are chosen to participate in the coalition, and they are infinitesimal so if they reject the offer the restructuring nonetheless passes and they get (IB.8).

Given these lender strategies, equity holders choose an offer ε_* that makes secured lenders indifferent between accepting and rejecting. Formally, we calculate ε_* to equate (IB.7) and

(IB.8) given the subsequent ex-post equity default timing and given the dependence of $\widehat{C}, \hat{s}$ on ε.

Moving backward in time, at time zero, equity holders maximize ex-ante firm value. Define

$$\mathcal{V}\big(\omega; (\widehat{C}, \hat{s})\big) \equiv V_E\big(\omega; (\widehat{C}, \hat{s})\big) + V_U\big(\omega; (\widehat{C}, \hat{s})\big) + V_S\big(\omega; (\widehat{C}, \hat{s})\big). \tag{IB.9}$$

Then equity holders choose s, C to maximize total firm value:

$$\begin{aligned}(1 - p_B)\mathcal{V}\big(\omega; (C, s)\big) + p_B(1 - \alpha)\Big[p_Z \mathcal{V}\big(Z\omega; (C, s)\big) + (1 - p_Z)\mathcal{V}\big(\omega; (C, s)\big)\Big] \\ + p_B\alpha\Big[p_Z \mathcal{V}\big(Z\omega; (\widehat{C}(\varepsilon_*), \hat{s}(\varepsilon_*))\big) + (1 - p_Z)\mathcal{V}\big(\omega; (\widehat{C}(\varepsilon_*), \hat{s}(\varepsilon_*))\big)\Big].\end{aligned} \tag{IB.10}$$

I.B.2 Model solution

We solve the model backwards. The equity holder problem is standard (Leland, 1994) with one small change. Due to the possibility of secured lenders forcing an asset sale, the effective interest rate is

$$\hat{r} = r + \phi(\hat{s}\widehat{C})^{\nu}, \tag{IB.11}$$

which takes into account the exogenous chance of default. The equity holder value function V_E solves the following standard Hamilton-Jacobi Bellman (HJB) equation:

$$\hat{r}V_E(\omega) = (\omega - \widehat{C})(1 - \tau) + V_E'(\omega)\mu\omega + V_E''(\omega)\frac{\sigma^2\omega^2}{2}. \tag{IB.12}$$

Following standard arguments, the relevant general solution is

$$V_E(\omega) = A_1\omega^{\psi} + (1 - \tau)\Big[\frac{\omega}{\hat{r} - \mu} - \frac{\widehat{C}}{\hat{r}}\Big], \tag{IB.13}$$

where ψ is the negative root of

$$\frac{\sigma^2}{2}z(z - 1) + \mu z - \hat{r} = 0. \tag{IB.14}$$

The function V_E must be smooth, so value matching and smooth pasting at the default boundary ω_D imply that

$$0 = V_E(\omega_D) = A_1\omega_D^{\psi} + (1-\tau)\Big[\frac{\omega_D}{\hat{r}-\mu} - \frac{\widehat{C}}{\hat{r}}\Big] \tag{IB.15}$$

$$0 = V_E'(\omega_D) = \psi A_1\omega_D^{\psi-1} + (1-\tau)\frac{1}{\hat{r}-\mu}. \tag{IB.16}$$

Then we can solve

$$\omega_D = \widehat{C}\frac{\hat{r}-\mu}{\hat{r}}\frac{\psi}{\psi-1}, \tag{IB.17}$$

$$A_1 = \omega_D^{-\psi}(1-\tau)\Big[\frac{\widehat{C}}{\hat{r}} - \frac{\omega_D}{\hat{r}-\mu}\Big]. \tag{IB.18}$$

Next, we turn to secured debt. The HJB for secured lenders is

$$\hat{r}V_S(\omega) = \hat{s}\widehat{C} + V_S'(\omega)\mu\omega + V_S''(\omega)\frac{\sigma^2\omega^2}{2} + \phi(\widehat{C}\hat{s})^{\nu}\min\Big\{\pi\omega, \frac{\hat{s}\widehat{C}}{r}\Big\}.$$

The final term introduces some complications in the solution: if $\pi\omega_D < \frac{\hat{s}\widehat{C}}{\hat{r}}$, the general solution features two regions: in one region, secured lenders anticipate full recovery in a takeover, while in the other region they expect partial recovery in a takeover. Applying value matching and smooth pasting across these regions delivers a closed-form solution.

Next, we turn to unsecured debt. The HJB for unsecured lenders is

$$\hat{r}V_U(\omega) = (1-\hat{s})\widehat{C} + V_U'(\omega)\mu\omega + V_U''(\omega)\frac{\sigma^2\omega^2}{2} + \phi(\widehat{C}\hat{s})^{\nu}\min\left\{\Big(\pi\omega - \frac{\hat{s}\widehat{C}}{r} - \rho\Big)^+, \frac{(1-\hat{s})\widehat{C}}{r}\right\}.$$

Again, the final term introduces several regions, and we calculate the value function by imposing value matching and smooth pasting across the regions.

Following these steps, for any $\widehat{C}, \hat{s}$, we can calculate $V_E(\omega; (\widehat{C}, \hat{s}))$, $V_S(\omega; (\widehat{C}, \hat{s}))$ and $V_U(\omega; (\widehat{C}, \hat{s}))$. Going backward in time, for any C, s, we can calculate $\varepsilon_*, \widehat{C}, \hat{s}$ by equating (IB.7) and (IB.8). We move back to the first stage, searching numerically for C, s values that maximize the firm value (IB.10).

I.B.3 Model results

Following a common assumption (Leland, 1994), we assume that a fraction $1 - \chi$ of firm value is destroyed in default: we assume $\chi = 0.6$. We assume the value in default is then the perpetuity value of the remaining cashflows: $\pi = (1-\tau)(1-\chi)/(r-\mu) = 7.9$. The following table lists the rest of our parameter assumptions.

Table IB.1: Parameter values

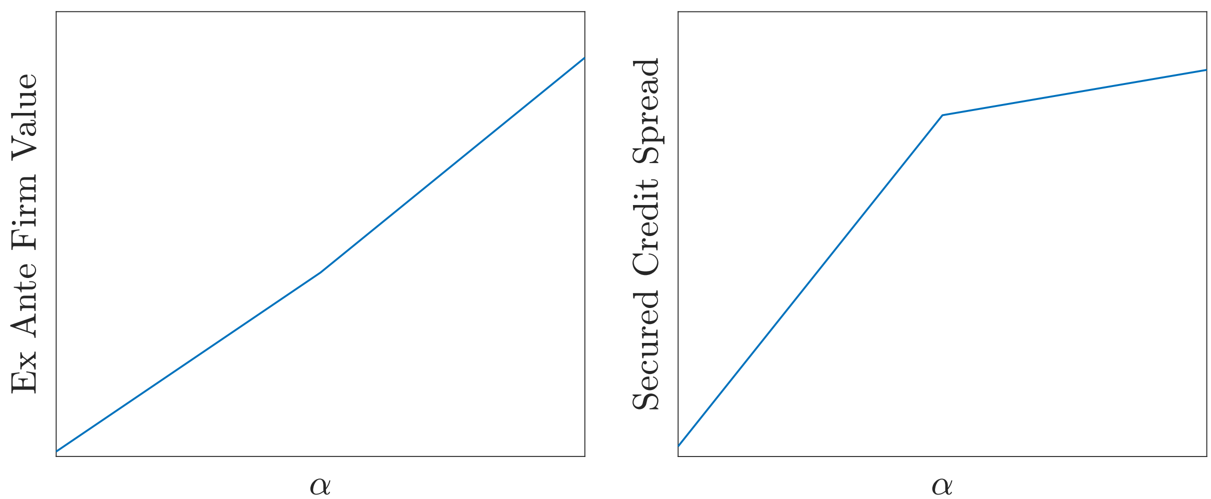
This table shows our parameter values for our model extension based on Leland (1994). Whenever applicable, parameter values are annualized.

μ	Drift	0.01
r	Risk-free rate	0.05
σ	Volatility	0.45
τ	Tax rate	0.21
π	Value in default	7.9
ρ	Priority claims	0.5
ϕ	Scale of secured default risk	0.0001
ν	Convexity of secured default risk	2
Z	Value after shock in bad state	0.3
p_B	Probability of bad state	0.5
p_Z	Probability of shock in bad state	0.5
ζ	Fraction of participating secured lenders	0.8
α	Probability of a restructuring opportunity	0

Solving the model with these parameters, we confirm the main results from our baseline model: as we increase α, ex-ante firm value and secured credit spreads[19] both increase. Figure IB.1 shows the result.

Figure IB.1: Comparative statics in the model extension based on Leland (1994)

We assume the parameter values given in Table IB.1. We solve the model of Internet Appendix I.B and calculate: (i) ex-ante firm value, and (ii) the ex-ante secured credit spread based on the ratio of the secured-debt cash flows to the proceeds from issuing secured debt. We then repeat this process, varying α but holding all other parameters fixed. This figure plots how model objects vary with α.



[19] As in Internet Appendix I.A, we define the secured credit spread as the sum of $-r$ and the ratio of (i) the flow payment Cs to secured lenders to (ii) the amount that secured lenders pay for the debt. Thus, if a firm facing a risk-free rate of 5% issues secured debt with flow payment $Cs = 0.25$ and raises one dollar, then the secured credit spread is $-0.05 + (.25/1) = 20\%$.

I.C Model extension with endogenous coalition size

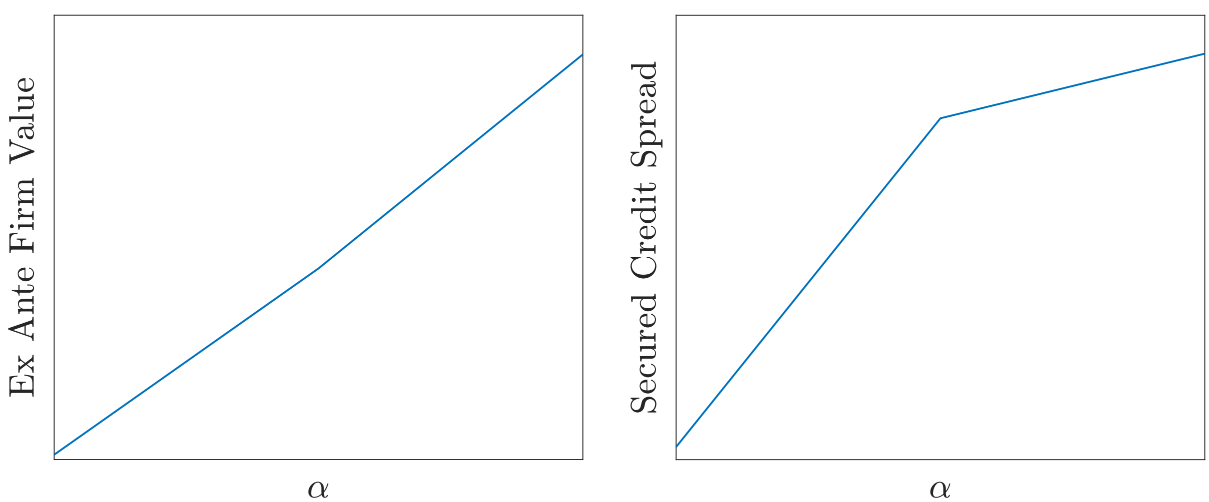
In this appendix, we show that our results are robust to allowing for an endogenous choice of ζ, the fraction of secured lenders that participate in a coalition in a restructuring. For simplicity, we use the setup of Internet Appendix I.B. We make one change: in addition to choosing the debt coupon C and secured-debt share s at time zero, equity holders choose ζ as well. This choice occurs at the first "stage" of the time-zero game and equity holders maximize the same objective: total firm value including debt-issuance proceeds. This choice represents the choice of credit-agreement terms regarding the fraction of lenders required to amend the agreement. Debt is priced rationally, taking into account how ζ impacts a potential future restructuring.

In the third stage of the time-zero game, the restructuring game is the same as described in Internet Appendix I.B. Equity holders endogenously choose a haircut ε. Secured lenders simultaneously decide whether they are willing to accept the proposal. As in Internet Appendix I.B, we solve for the value ε_* that supports an equilibrium in which (i) all secured lenders are indifferent between accepting or rejecting (we equate (IB.7) and (IB.8)) and (ii) all secured lenders accept in equilibrium, implying the restructuring succeeds. By making secured lenders indifferent between accepting and rejecting, equity holders extract the highest possible reduction in debt.

Other than allowing ζ to be an endogenous choice, we assume the same parameter values described in Table IB.1. Figure IC.1 shows that our main result continues to hold. As we increase α, the probability of a restructuring opportunity, the ex-ante firm value and secured credit spread both increase.

Figure IC.1: Comparative statics in the model extension with endogenous coalition size

We assume the parameter values given in Table IB.1, except that ζ is an endogenous time-zero choice. We solve the model of Internet Appendix I.B, except that equity holders choose ζ in the first stage of the time-zero game. We calculate: (i) ex-ante firm value, and (ii) the ex-ante secured credit spread based on the ratio of the secured-debt cash flows to the proceeds from issuing secured debt. We then repeat this process, varying α but holding all other parameters fixed. This figure plots how model objects vary with α.



I.D Illustrative model

This section presents a simple model to illustrate the intuition behind our results. We show how the potential for a liability-management transaction impacts a firm with one class of debt. We then explain how this impact will depend on a firm's use of secured debt, motivating our main model.

The illustrative model has three dates $t = 0, 1, 2$. At $t = 0$, a firm chooses how much debt to issue. At $t = 1$, the firm and its lender observe a signal about the future operations. There is then the potential for a liability-management transaction like Serta's transaction. We refer to this as a restructuring. At $t = 2$, the firm and lender observe the firm's value. The firm defaults or pays back debt, giving any residual value to shareholders.

Specifically, at $t = 0$, the firm issues fairly priced debt. Both the firm and its lender are risk neutral and have a discount rate of zero. The firm chooses the date-two repayment X that it will owe to the lender at $t = 2$. At $t = 0$, the lender gives the firm the expected value of the firm's future repayment, which takes into account the possibility of a restructuring at $t = 1$ or default at $t = 2$. In this sense, debt is issued at a competitively priced discount to face value, determined by rational expectations. We provide details below.

At $t = 1$, there is the potential for a restructuring. We let $\mathbf{1}_R$ denote an indicator equal to one if a restructuring offer is accepted. If the restructuring is accepted, a fraction ζ of lenders exchange each dollar of their old debt for $1 - \varepsilon$ dollars of new senior debt, where $\zeta \in [0.5, 1]$ and $\varepsilon \in [0, 1]$ are exogenous parameters. In other words, if a restructuring is accepted, the total debt owed at $t = 2$ is reduced from X to $X\left(1 - \zeta\varepsilon\right)$. We define

$$\tilde{X} = X\left(1 - \mathbf{1}_R \zeta \varepsilon\right) \tag{ID.1}$$

as the debt owed at $t = 2$, taking into account the possibility of a restructuring at $t = 1$.

Additionally, at $t = 1$, the firm and lender learn whether the firm's operations are healthy. With probability $1-\lambda$, the firm and the lender observe that the firm's operations are healthy. In this case, everyone knows the firm's value is certain to equal $1 + \tau\tilde{X}$. We normalize the unlevered after-tax firm value to one for simplicity. The parameter $\tau > 0$ captures the tax benefits of debt per dollar of debt. This can be thought of as a reduced-form approach to modeling both the tax rate and the coupon rate.

With probability λ, the firm experiences a negative shock. In this case, the tax shield is not realized. The firm value is $Z \sim \text{Uniform}(0,1)$, where we assume the uniform distribution for simplicity. Let $\mathbf{1}_Z$ denote an indicator for a negative shock at $t=1$.

Finally, if the firm experiences a negative shock, it defaults if the firm value Z is less than the debt owed $\tilde{X}$. In default, the firm is only worth πZ for an exogenous parameter $\pi < 1$ that captures default costs.

Given this, the ex-ante firm value is

$$\max_{X} \; \mathbb{E}\Big[\left(1 - \mathbf{1}_Z \right) (1 + \tau \tilde{X}) + \mathbf{1}_Z Z \left(1 - (1-\pi)\mathbf{1}(Z < \tilde{X}) \right) \Big]. \tag{ID.2}$$

Because debt is fairly priced and equity holders receive the debt proceeds at issuance, equity holders simply choose X at $t=0$ to maximize (ID.2).

The following proposition characterizes the impact of restructurings on ex-ante firm value.

Proposition 1. *Suppose that a restructuring occurs with probability α after a negative shock ($\mathbf{1}_Z = 1$) and with probability zero for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (ID.2) *increases with the probability of restructuring α.*

Proposition 2. *Suppose that a restructuring occurs with probability zero after a negative shock ($\mathbf{1}_Z = 1$) and with probability α for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (ID.2) *decreases with the probability of restructuring α.*

Intuitively, equity holders would like to realize the tax benefits of debt without risking the deadweight loss of default. Ideally, equity holders would issue a state-contingent debt contract that is cancelled in bad states of the world before default. In practice, many frictions make such a security infeasible (e.g., difficulty in verifying bad states, moral hazard, etc). However, restructurings introduce state-contingent repayment. If restructurings occur in the states where the tax-shield is valuable, this destroys value. If restructurings occur in states where default is likely, they create value ex ante.

I.D.1 Secured and unsecured debt

The above results show that if restructurings are accepted in relatively good states of the world, they will harm firms ex ante. If restructurings are only accepted in bad states of the world, they will benefit firms ex ante.

When deciding whether to accept a restructuring, lenders trade off a lower face value with higher seniority. Higher seniority is particularly beneficial if seniority is lower to start with. For this reason, a restructuring offer aimed at unsecured creditors is more likely to succeed than a restructuring offer aimed at secured creditors. In other words, secured lenders are likely to only accept when the firm is very likely to default (bad states), while unsecured lenders are likely to accept when default is less likely (better states). Because of the legal constraints discussed in the main text, this new form of liability-management transaction targets secured lenders. We thus expect these transactions to only be accepted by secured lenders in bad states of the world where default is close. Because of this, the above results suggest restructurings will improve ex-ante firm value. In the main text, we show this in our realistic dynamic model.

I.D.2 Proofs for illustrative model

Recall Proposition 1 states the following:

Proposition 1: *Suppose that a restructuring occurs with probability α after a negative shock ($\boldsymbol{1}_Z = 1$) and with probability zero for healthy firms ($\boldsymbol{1}_Z = 0$). Then ex-ante firm value* (ID.2) *increases with the probability of restructuring α.*

Proof: Under the stated assumption, for any fixed X, firm value is

$$(1-\lambda)(1+\tau X) + \lambda(1-\alpha)\Big(\int_X^1 Z dZ + \int_0^X \pi Z dZ\Big) + \lambda\alpha\Big(\int_{\widehat{X}}^1 Z dZ + \int_0^{\widehat{X}} \pi Z dZ\Big), \quad \text{(ID.3)}$$

where $\widehat{X} \equiv X(1-\zeta\varepsilon)$. Evaluating integrals in (ID.3), we obtain

$$(1-\lambda)(1+\tau X) + \lambda(1-\alpha)\Big(\frac{1-X^2}{2} + \frac{\pi X^2}{2}\Big) + \lambda\alpha\Big(\frac{1-\widehat{X}^2}{2} + \frac{\pi\widehat{X}^2}{2}\Big). \quad \text{(ID.4)}$$

Rearranging,

$$(1-\lambda)(1+\tau X) + \lambda\Big(\frac{1-(1-\pi)X^2}{2}\Big) + \lambda\alpha(1-\pi)\Big(\frac{X^2-\widehat{X}^2}{2}\Big). \quad \text{(ID.5)}$$

The last term in (ID.5) is positive, so increasing α increases firm value for any chosen X, so firm value increases with α.

Recall Proposition 2 states the following:

Proposition 2: *Suppose that a restructuring occurs with probability zero after a negative shock ($\mathbf{1}_Z = 1$) and with probability α for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (ID.2) *decreases with the probability of restructuring α.*

Proof: Under the stated assumption, for any fixed X, we can apply the same steps to show that firm value is

$$(1-\lambda)(1+\tau(\alpha\widehat{X}+(1-\alpha)X))+\lambda\Big(\int_X^1 ZdZ+\int_0^X \pi ZdZ\Big), \tag{ID.6}$$

where $\widehat{X} \equiv X(1-\zeta\varepsilon)$. Rearranging (ID.6), we obtain

$$(1-\lambda)(1+\tau X)+\lambda\Big(\int_X^1 ZdZ+\int_0^X \pi ZdZ\Big)+(1-\lambda)\tau\alpha(\widehat{X}-X). \tag{ID.7}$$

The last term in (ID.7) is negative, so increasing α lowers firm value for any chosen X, so firm value decreases with α.

I.E Algorithm for numerical solution

This appendix describes the algorithm for the numerical solution in the baseline model. First, it is helpful to define a few operators. For any guess of the equity value function $p(x)$, investment choice i, and secured-debt choice s, define:

$$\begin{aligned}\mathcal{A}p^{i,s}(x) \equiv& \frac{1}{2}\sigma^2 x^2 p''(x) + \Big(-\theta + i + (1-\tau)c(x,s) - x\Big(\psi(i)-\delta\Big)\Big)p'(x) \\ & - \Big(\gamma+\lambda+\delta-\psi(i)\Big)p(x) + \phi\big(sx\big)^{\nu}\left[(\pi-\rho-x)^{+} - p(x)\right].\end{aligned}$$

Define

$$\mathcal{A}p(x) \equiv \max_{i\in\mathbb{R}, 0\le s\le \min\{1,\frac{\pi}{x}\}} \mathcal{A}^{i,s}p(x), \tag{IE.1}$$

which corresponds to the optimal choice of i, s in the HJB equation (29). Finally, for any guess of $p(x), s(x)$ and $\mathbf{1}^R(x)$ (the secured-lender restructuring acceptance rule), define the operator:

$$\mathcal{B}p(x) \equiv (1-\alpha\mathbf{1}^R(x))\int_0^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) + \alpha\mathbf{1}^R(x)\int_0^1 Zp\Big(x\frac{1-s(x)\zeta\varepsilon}{Z}\Big)dF(Z). \tag{IE.2}$$

This operator corresponds to the impact of jump shocks in the HJB equation (29). Recall that $\widehat{x}$ is the equity-issuance boundary and $\underline{x}$ is the boundary for issuing debt to pay a dividend. Our model solution (20)-(29) implies that the equity value function $p(x)$ should satisfy the following variational inequality[20]:

$$\max\{\mathcal{A}p(x) + \lambda\mathcal{B}p(x), 1+p'(x)\} = 0, \quad x\in(x_{min},\widehat{x}) \tag{IE.3}$$

with boundary conditions:

$$p'(x_{min}) = -1, \quad p'(\widehat{x}) = -(1+h_1), \tag{IE.4}$$

where $x_{min}\in(0,\underline{x})$ is any sufficiently small number and the equity-issuance boundary $\widehat{x}$ is

[20]Note that $\mathcal{B}p(x) = (1-\alpha\mathbf{1}^R(x))\int_{Z_*(x)}^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) + \alpha\mathbf{1}^R(x)\int_{Z_*^{res}(x,s(x))}^1 Zp\Big(x\frac{1-s(x)\zeta\varepsilon}{Z}\Big)dF(Z)$.

determined by (23), which implies

$$\mathcal{M}p(\widehat{x}) = p(\widehat{x}).$$

Here, $\mathcal{M}p(x)$ denotes the equity value after equity financing:

$$\mathcal{M}p(x) \equiv \max_{m>0} \Big[p(x-m) - h_0 - (1+h_1)m\Big]. \tag{IE.5}$$

We numerically solve for a solution to this variational inequality by completing the following steps:

1. We fix a guess $\widehat{x}$ of the equity-issuance boundary. We fix an arbitrarily small positive number x_{min}: we choose this number such that any firm will optimally set $\underline{x} > x_{min}$. We discretize the interval $[x_{min}, \widehat{x}]$. We use a uniform grid with $N+1$ points: $x_n = x_{min} + (n-1)\Delta x$, $n = 1, 2, \ldots, N+1$, where $\Delta x = \frac{\widehat{x} - x_{min}}{N}$.

2. We fix an initial guess of the lender acceptance rule $\{\mathbf{1}_0^R(x_n)\}_{n=1,2,\ldots,N+1}$. We fix an initial guess of the equity value function $\{p_0(x_n)\}_{n=1,2,\ldots,N+1}$, investment policy $\{i_0(x_n)\}_{n=1,2,\ldots,N+1}$ and secured-debt policy $\{s_0(x_n)\}_{n=1,2,\ldots,N+1}$. Given these guesses, we calculate $\{Z_*(x_n), c(x_n, s_0(x_n))\}_{n=1,2,\ldots,N+1}$ following the formulas derived in Appendix A.

3. We fix a penalty parameter $\Upsilon > 0$. Given guesses $\{\mathbf{1}_k^R(x_n), p_k(x_n), s_k(x_n), i_k(x_n), Z_*(x_n), c(x_n, s_k(x_n))\}_{n=1,2,\ldots,N+1}$, we use the variational inequality (IE.3) to update to a new guess $\{p_{k+1}(x_n)\}_{n=1,2,\ldots,N+1}$:

$$\mathcal{A}^{i_k(x), s_k(x)} p_{k+1}(x) + \lambda \mathcal{B} p_k(x) + \Upsilon \Big(1 + p'_{k+1}(x)\Big) \mathbf{1}_{1+p'_k(x) \geq 0} = 0, \tag{IE.6}$$

$$p'_{k+1}(x_{min}) = -1, \quad p'_{k+1}(\widehat{x}) = -(1+h_1). \tag{IE.7}$$

We evaluate the derivatives using an upwind finite-difference scheme. We define $\mathcal{B}$ by inputting $s_k, \mathbf{1}_k^R$ into (IE.2). In the region $x \geq \widehat{x}$, we derive from (25) and (27) that

$$p_{k+1}(x) = \max\{0, p_{k+1}(\widehat{x}) - (1+h_1)(x - \widehat{x})\}, \quad x \geq \widehat{x}. \tag{IE.8}$$

Once we calculate the updated equity value function p_{k+1}, we calculate updated policies i_{k+1}, s_{k+1} by solving the maximization (IE.1) with p_{k+1}. Given $\{p_{k+1}, i_{k+1}, s_{k+1}, \mathbf{1}_k^R\}$, we calculate $\{Z_*(x_n), c(x_n, s_{k+1}(x_n))\}_{n=1,2,\ldots,N+1}$ following the formulas derived in Appendix A. Finally, given these other updated values, we calculate the updated secured-lender acceptance rule $\{\mathbf{1}_{k+1}^R(x_n)\}_{n=1,2,\ldots,N+1}$ to equal one when (A.12) is satisfied and zero otherwise.

4. We repeat step 3, iteratively constructing a new guess $k+1$ from each prior guess k, until the equity value function converges: we repeat until $\|p_k - p_{k+1}\|$ is sufficiently small. Once this converges for some step k_*, this delivers the other model objects from step 3: $i_{k_*}, s_{k_*}, \mathbf{1}_{k_*}^R$, etc.

5. Given the equity value function $\{p_{k_*}(x_n)\}_{n=1,2,\ldots,N+1}$, we calculate $\mathcal{M}p_{k_*}(\widehat{x})$ by equation (IE.5). If $|\mathcal{M}p_{k_*}(\widehat{x}) - p_{k_*}(\widehat{x})|$ is sufficiently small, (23) is satisfied and we are done. If it is not small, we return to step 1 with a new guess for $\widehat{x}$ and repeat until $|\mathcal{M}p_{k_*}(\widehat{x}) - p_{k_*}(\widehat{x})|$ is sufficiently small. Specifically, fixing a small step size $\widehat{\delta}$, we update

$$\widehat{x}_{new} = \widehat{x} - \widehat{\delta}\mathbf{1}\left(\mathcal{M}p_{k_*}(\widehat{x}) > p_{k_*}(\widehat{x}) \right) + \widehat{\delta}\mathbf{1}\left(\mathcal{M}p_{k_*}(\widehat{x}) < p_{k_*}(\widehat{x}) \right). \tag{IE.9}$$

Once we have finished these steps, we can use the equity value function p to define the endogenous payout boundary:

$$\underline{x} = \inf\{x \in [x_{min}, \widehat{x}] : p'(x) < -1\},$$

and the endogenous default boundary:

$$\overline{x} := \inf\{x > 0 : p(x) = 0\}.$$

I.F Secured debt details

Unlike unsecured debt, secured debt is explicitly backed by collateral. Article 9 of the Uniform Commercial Code (UCC) outlines the legal treatment of secured debts outside of bankruptcy.[21] A secured debt contract gives the lender a "security interest" in a specified asset of the debtor, which is a voluntary lien on the asset. The lender registers this security interest in a public database ("perfecting the security interest") so that other potential lenders are aware which of the debtor's assets already have liens on them.[22] Once registered, the secured lender has the right to seize the specified collateral if the debtor defaults on the loan. Unlike unsecured lenders, the secured lender can take collateral from a defaulting debtor without asking for court permission as long as they do not "breach the peace" by doing so.[23] When physically seizing collateral is impractical, a foreclosure is a straightforward way to transfer ownership of the asset to the secured lender. Importantly, these secured-lender rights only apply if the assets are held by the borrowing company.

A given piece of collateral can have multiple liens on it. A lender can file a security interest in a piece of collateral even after an existing lender has filed a security interest. However, the second lender gets a "second lien" and only receives value in a foreclosure after the first lien-holder is paid back in full.[24]

If a debtor files for bankruptcy, the automatic stay prevents secured lenders from seizing collateral. In exchange, the bankruptcy code gives secured lenders special treatment. Essentially, a bankruptcy plan can only be confirmed if secured lenders receive full recovery or secured lenders receive the value of their collateral.[25] Specifically, 11 U.S.C. §1129(a)8 requires all classes of claims to approve a bankruptcy plan. If this condition is not met, a plan can only be confirmed under the conditions of 11 U.S.C. §1129(b). Section 1129(b)2 requires

[21] See https://www.law.cornell.edu/ucc/9.

[22] See https://www.nolo.com/legal-encyclopedia/how-attach-perfect-security-interest-under-the-ucc.html.

[23] See https://www.nolo.com/legal-encyclopedia/what-secured-debt.html.

[24] See https://www.forbes.com/advisor/business-loans/what-is-a-ucc-filing.

[25] If a creditor holds a secured claim with a face value that exceeds the value of the loan collateral, the bankruptcy court gives the creditor a secured claim with a face value equal to collateral value and an unsecured "deficiency" claim equal to the difference between the original face value and the collateral value. See https://content.next.westlaw.com/practical-law/document/I68760bc7169611e598db8b09b4f043e0/Deficiency-Claim?viewType=FullText&transitionType=Default&contextData=(sc.Default)#:~:text=In%20bankruptcy%2C%20a%20general%20unsecured,is%20not%20secured%20by%20collateral.

that secured lenders (i) get to retain their liens on secured assets (or the sale proceeds from selling those assets in bankruptcy) and (ii) receive deferred cash payments with a present value equal to the claim or asset value.[26] In other words, a firm cannot exit bankruptcy unless secured lenders exit bankruptcy in the same position that they entered the bankruptcy (or better).

[26]See https://www.law.cornell.edu/uscode/text/11/1129.