Public Information and the Securities Lending Market*

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Abstract

We develop a dynamic model to study how the securities lending market affects the trading and pricing of a stock around the arrival of public information. When investors disagree about the firm's value but agree about how to interpret a public news event given this value, loan fees rise before and fall after the event in proportion to its informativeness. The news reduces the expected stock price after its release when the demand for shorting is high, but has no impact on the pre-announcement price. If little information is expected to arrive, the price can be significantly inflated even when the loan fee is low. When investors disagree more about the news than about firm value, only investors with extreme beliefs take positions before the announcement and the news increases firms' ex-ante valuations by encouraging trade.

JEL: G10, G12, G14, G32

Keywords: securities lending market, loan fees, short-sales constraints, earnings announcement premium, difference of opinions, expected returns

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1 Introduction

The dynamics of the securities lending market around public information events are crucial to understanding how prices aggregate investor beliefs. A large empirical literature shows that short sellers substantially increase their stock trading activity around news announcements.[1] Since short sellers have to borrow shares to establish their positions, such sharp increases in shorting demand should translate to notably higher stock borrow fees and, consequently, higher stock prices. Consistent with this, empirical evidence suggests that the link between short selling and stock returns becomes significantly stronger around news events (e.g., Berkman et al. (2009), Engelberg et al. (2012)).

Despite its importance, the existing theoretical literature is largely unable to analyze the securities lending market around public announcements. Many traditional models focus on static settings, which are limited in their ability to explain dynamics around news events. Moreover, theoretical analyses of dynamics in the lending market (including insightful work by Duffie, Garleanu, and Pedersen (2002) and Atmaz, Basak, and Ruan (2024)) have largely focused on settings in which the flow of information to investors is constant. As such, there remain several open questions. When and how does the securities lending market affect investors' trading behavior around news events? How does a public announcement impact stock borrowing fees and stock returns before and after its release? How does this depend on the extent to which investors disagree about firm value versus the announcement?

To study these questions, we develop a dynamic model in which risk-averse investors trade before and after the release of a public information signal, such as an earnings announcement. They may disagree about the firm's value and/or the interpretation of the public signal. Pessimistic investors may want to short sell the stock, but to do so, they must borrow shares in a competitive lending market from investors who are long the stock.

Consistent with institutional frictions, long investors can only lend out a fraction of their shares, limiting the supply of shares available for shorting. When the aggregate demand for shorting is sufficiently low, short sellers do not need to pay to borrow shares, and the stock price reflects the average valuation of all investors adjusted for a risk premium. However, when the demand for shorting is high relative to the available supply, the stock is "on special" and short investors must pay a positive "loan fee" in equilibrium to borrow shares (as in, e.g., Duffie et al. (2002), Banerjee and Graveline (2014), and Atmaz et al. (2024)). This affects the firm's stock price via two channels: (i) it increases the demand from long investors because

[1] See, e.g., Christophe, Ferri, and Angel (2004), Berkman, Dimitrov, Jain, Koch, and Tice (2009), Berkman and McKenzie (2012), Engelberg, Reed, and Ringgenberg (2012), Alexander, Peterson, and Beardsley (2014), Beneish, Lee, and Nichols (2015), and Clinch and Li (2022).

they earn the fee from lending out a fraction of their shares, and (ii) it limits participation by investors who are not not pessimistic enough to justify paying the short fee.

We characterize investor trading, stock prices, and lending fees around the announcement, and show how they depend on the nature and precision of the public information. We begin by considering a setting where investors may disagree about the firm's value, but agree on the distribution of the public signal given this value i.e., they have "concordant beliefs." In their seminal "no-trade" theorem, Milgrom and Stokey (1982) show that when investors have such beliefs and markets are complete, public news does not lead investors to trade. We show that this result continues to hold in our setting, despite the securities lending constraint.[2] However, the public signal has a potent impact on equilibrium loan fees and stock prices.

We first show that, for a stock on special, loan fees rise before the announcement in proportion to how informative it is. Intuitively, pessimistic investors are willing to pay higher fees to short when they expect an imminent price drop in response to the announcement. The more precise the public signal, the larger the anticipated change in the price, and consequently, the more short sellers are willing to pay to borrow shares. In contrast, loan fees after the announcement decline in proportion to its informativeness. This is because the announcement lowers investor uncertainty about firm value, which reduces pessimistic investors' willingness to pay to short the stock going forward. While this decrease in fees tends to lower the stock price, a more informative signal also reduces the firm's risk premium and, consequently, increases its price. When short-selling demand is high (low) relative to the outstanding supply of shares (i.e., the float), we show that the net effect is a reduction (increase) in the firm's expected stock price.[3]

The fact that loan fees spike prior to a highly-informative public signal might suggest that such an announcement would raise the firm's valuation before its release. However, before the signal's release, investors anticipate and price the magnitude of the fee-driven overvaluation that will remain *after* its release. Since fees are lower following a more informative public signal, this force tends to reduce prices prior to the signal's arrival. In fact, we show that the precision of the public signal has no impact on the firm's ex-ante valuation because the impacts of the announcement on loan fees before and after its release perfectly offset each other. Stated differently, Ross (1989)'s finding that current prices are independent of

[2]Note while markets are not complete in our model, they are effectively complete in that, absent the lending constraint, investors could reach a Pareto efficient outcome via trade. In particular, Brennan and Cao (1996) show that, when investors have CARA utility, payoffs are normal, and investors' beliefs differ only over the mean, the stock alone effectively completes the market.

[3]In particular, this implies that unlike existing work (e.g., Duffie et al. (2002), Atmaz et al. (2024)), higher pre-announcement loan fees do not always predict negative returns on average.

the timing at which information is expected to arrive in the future continues to hold under concordant beliefs, despite the securities-lending constraint.

These findings are pertinent to the empirical work that applies loan fees, short interest/utilization, and/or the supply of shortable shares as proxies for the overvaluation generated by short-sales constraints.[4] Our results indicate that, in addition to these proxies, researchers must also consider impending news arrival when assessing the overvaluation these constraints generate. Notably, our findings imply that a highly-informative announcement can generate a spike in loan fees before its release without affecting the firm's valuation or share utilization.

We next consider the case in which beliefs are not concordant, so that the public signal generates trade. As a natural way to capture this, we assume that the firm's value consists of two distinct components and that investors disagree about only one of them. We show that, when the public signal focuses on one of the two components, it violates concordant beliefs and leads to trade.

Prices and loan fees can exhibit starkly different behavior when beliefs are not concordant. To illustrate this, we consider two cases. First, we consider a public signal that pertains to the component of firm value that investors agree on. For example, investors may differ in their views on a growing firm's long-term prospects but share the belief that its short-term performance, as revealed by an impending earnings announcement, will be poor. In this case, in our model, investors wait until after the announcement to take speculative positions. As a result, loan fees are low before the signal's release and increase afterwards, in proportion to the signal's informativeness.

Second, we consider a public signal that pertains to the component of firm value that investors disagree over. For example, investors may disagree on the firm's core financial outlook, as reflected in earnings, but concur on the likelihood of unpredictable future events like natural disasters or shifts in government policy. In this scenario, we find that, when the stock is not on special, investors take large positions before the announcement and unwind them afterward. When the stock is on special, investors with extreme beliefs follow a similar trading strategy. However, investors with moderate beliefs do the opposite in this case, opening larger positions after the announcement. Moreover, loan fees are high before the signal's release and fall afterwards.

In both scenarios, a more informative announcement raises the overall amount of trade by enabling investors to speculate on the component of value they disagree on while fac-

[4]See, e.g., Nagel (2005), Blocher, Reed, and Van Wesep (2013), Beneish et al. (2015), and Engelberg, Reed, and Ringgenberg (2018).

ing reduced exposure to the component they agree on. When the public signal concerns the component of value investors disagree (agree) over, they achieve this by trading more around (after) the announcement. In either case, a more informative signal leads to higher short-selling demand and higher total loan fees over time, and consequently to higher pre-announcement stock prices. Hence, counter to Ross (1989), investors' expectations of future information arrival alter the equilibrium price.

These findings reinforce the insight from the setting with concordant beliefs: when the signal concerns the value component investors agree on, before its release, the loan fee and share utilization are low and yet the firm can be significantly overvalued. They further indicate that lending fees are not only a function of disagreement and share supply, but also the nature of information arrival over time, which influences investors' ability to efficiently trade on their beliefs.

The rest of the paper is organized as follows. The next section discusses the related literature and our contribution to it. Section 3 presents the model and Section 4 characterizes the equilibrium trading, loan fees, and stock price dynamics around the public announcement. Section 5 derives specific predictions for when investors exhibit concordant beliefs, while Section 6 considers the setting in which investors have non-concordant beliefs and the public announcement generates trading. Finally, Section 7 concludes the paper by outlining several empirical predictions of our model and discussing possible extensions for future work. All proofs are in the Appendix.

2 Related Literature

Several papers study static models of the securities lending market, including Duffie (1996), Blocher et al. (2013), Banerjee and Graveline (2014), and Nezafat and Schroder (2022). Within this literature, Nezafat and Schroder (2022)'s model is closest to ours, as they also allow for a continuum of investors with heterogeneous beliefs and CARA preferences. However, their focus is on showing that, in a noisy rational expectations equilibrium, the loan fee conveys information to investors. They study how fee opacity and private information influence equilibrium fees and prices.

Similar to our analysis, prior work also studies dynamic models of the lending market. Duffie et al. (2002) study securities lending with search frictions, where risk-neutral long and short investors must search for one another and negotiate a fee. In their model, information arrives according to a Poisson arrival rate, so that the expected amount of public information does not vary over time. Atmaz et al. (2024) studies the lending market in a setting where disagreement, and thus lending constraints, vary stochastically over time. They study how stochastic lending constraints can contribute to the risk premium. In their setting, investors disagree about the drift of the firm's dividend process, where dividends have constant volatility; hence, the rate of information arrival is fixed. Finally, in a setting with risk-neutral informed and uninformed investors, Weitzner (2023) shows that the term structure of loan fees reflects expectations about both the degree of over-valuation and the timing of when this overvaluation will be corrected. However, in this setting, the price of the asset remains fixed until the resolution of the uncertainty, and there is no change in demand for shorting over time (since investors are risk-neutral).

We contribute to this work by jointly modeling the cash and lending market of a stock around a public information event. This enables us to study a new set of research questions and introduces novel economic forces. First, the arrival of public news causes the rate of information arrival to vary over time. This leads to our key result that changes in the rate of information arrival can cause the lending fee to vary over time without affecting short utilization or expected stock prices. Second, this enables us to study how different types of disagreement influence stock price and fee dynamics. On the technical front, we further allow for a general distribution of disagreement in most of our analyses.

Our model also relates to the prior work that studies when public information generates trade. Kim and Verrecchia (1991) show that investors trade around announcements when they have different information precisions, though Brennan and Cao (1996) illustrate that this no longer holds upon completing the market via options. More similar to our analysis, in their Section 4.2, He and Wang (1995) study trade in a dynamic setting where investors are initially endowed with private information about one of two components of firm value. They then show that investors open positions prior to subsequent public announcements about this component and close them after. This matches the motive for trade in the "signal disagreement" case we analyze in Section 6 when the stock is not on special.

Finally, our paper contributes to the literature examining how news events influence stock prices by identifying a new set of conditions under which improved information can lead to a reduction in firms' prices, on average. Other studies have identified different mechanisms that could produce similar outcomes. For example, Dutta and Nezlobin (2017) demonstrates that in an overlapping generations model, better public information increases expected prices only for firms with growth rates below a certain threshold. Heinle, Smith, and Verrecchia (2018) shows that public information about a firm's exposure to risk factors has an ambiguous effect on expected prices, and Gollier and Schlee (2011) derives statistical conditions under which better information decreases expected prices in a representative-agent economy.

3 Model

We consider a dynamic model of trade around an information event with a securities lending market. There are four dates indexed by $t \in \{0, 1, 2, 3\}$.

Payoffs and Preferences. There are two securities. The risk-free security is in elastic supply and has a gross rate of return normalized to 1. The risky stock pays terminal cash flows of x at date 3, which are normally distributed with variance σ_x^2. There is a continuum of investors with CARA preferences over their date-3 wealth and risk aversion ρ, indexed by $i \in [0, 1]$. The per-capita supply of shares for the risky security is $Q > 0$.

Timeline. Figure 1 summarizes the timing of the model. Investors trade the risky security at price P_t on $t \in \{0, 1, 2\}$. Between dates 1 and 2, there is a public signal y that is jointly normal with x, with covariance matrix

$$\Sigma_{xy} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}.$$

Note we allow for an additional trading round on date $t = 0$ to study how fees and prices evolve leading up to the information event.

Investor Beliefs. Investors have common beliefs about the covariance matrix Σ_{xy}. However, they may disagree about the means of x and y. Specifically, suppose investor i has beliefs:

$$\mathbb{E}_i[x] = m_i \text{ and } \mathbb{E}_i[y] = \delta_i,$$

where the distribution (density) of $m_i \in [m_L, m_H]$ across investors is $G_m(\cdot)$ ($g_m(\cdot)$, respectively), and the corresponding distribution (density) of $\delta_i \in [\delta_L, \delta_H]$ is $G_\delta(\cdot)$ ($g_\delta(\cdot)$, respectively). Moreover, the distribution of $\{m_i, \delta_i\}$ is fixed and known to investor i at date 0. Disagreement will generate a demand for short selling and lead to an active securities lending market. This is consistent with D'Avolio (2002), who shows empirically that securities lending constraints are linked to disagreement.

Denote investor i's demand for the risky security at date t by D_{it}. We let $\bar{m} \equiv \int m_j dj$ and $\bar{\delta} \equiv \int \delta_j dj$ denote the average investor beliefs. While we do not need to specify which investor's belief corresponds to the truth for most of our results, we must assume true means of x and y when calculating the expected date-2 price and expected returns in Sections 5 and 6; when doing so, we assume that the average investor belief is correct, i.e., $\mathbb{E}[x] = \bar{m}$ and $\mathbb{E}[y] = \bar{\delta}$.

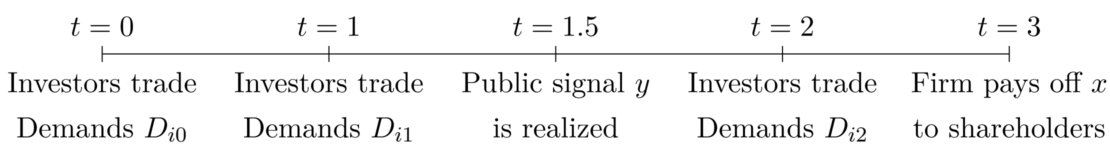
Securities Lending Market. On date t, long investors can lend out at most a fraction $\alpha \in [0, 1)$ of their shares at an endogenously-determined "loan fee" f_t. Hence, investor i's terminal wealth W_{i3} is given by

$$W_{i3} = \sum_{t=0}^{2} D_{it}(x - P_t) + \sum_{t=0}^{2} D_{it} \left[\mathbf{1}(D_{it} > 0)\alpha f_t + \mathbf{1}(D_{it} < 0) f_t \right].$$

The second term reflects that a long investor earns αf_t per share held in loan fees, while a short investor pays f_t per share shorted in loan fees.

Analogous to the stock market, we assume that the securities lending market is perfectly competitive. Hence, as we detail formally below, in equilibrium, one of the following must hold: (i) the loan fee is zero and the demand for shortable shares is lower than the supply, or (ii) the loan fee is positive and the demand for shortable shares equals the supply (as in Banerjee and Graveline (2014)). In the latter case, the stock is said to be "on special." Intuitively, if the fee were positive and the supply of shortable shares exceeded the demand, investors whose shares are not lent out would deviate by lowering the fee they charge to borrow their shares. The assumption that $\alpha < 1$ is important to ensure that a positive loan fee can arise in equilibrium (Duffie (1996)).

Figure 1: Timeline



Equilibrium. An equilibrium consists of demands, prices, and loan fees $\{D_{it}, P_t, f_t\}_{t \in \{0,1,2\}}$ such that:

(i) Investor i chooses her demand D_{it} to maximize her date-t expected utility given her beliefs and information set, i.e.

$$D_{it} = \arg\max \mathbb{E}_{it} \left[-\exp\left\{ -\rho W_{i3} \right\} \right].$$

(ii) The market for the risky security clears on each date, i.e., for each $t \in \{0, 1, 2\}$, $\int D_{jt} dj = Q$.

(iii) For each $t \in \{0, 1, 2\}$, one of the following holds:

a. There is an excess number of shares available to short:

$$\left|\int_0^1 \mathbf{1}\left(D_{jt} < 0\right) D_{jt} dj\right| < \alpha \int_0^1 \mathbf{1}\left(D_{jt} \geq 0\right) D_{jt} dj$$

and the loan fee is zero ($f_t = 0$).

b. The number of shares shorted equals the total supply of shortable shares:

$$\left|\int_0^1 \mathbf{1}\left(D_{jt} < 0\right) D_{jt} dj\right| = \alpha \int_0^1 \mathbf{1}\left(D_{jt} \geq 0\right) D_{jt} dj$$

and the loan fee is strictly positive ($f_t > 0$).

3.1 Discussion of Assumptions

Competitive lending market. Analogous to the stock market, we assume that the lending market is perfectly competitive. This is a simplifying assumption, and implies that either the equilibrium loan fee is positive and all shares are loaned out, or the fee is zero. In practice, there is often sub-100% utilization, and yet fees are non-zero, which reflects imperfect competition. Chen, Kaniel, and Opp (2022) develop a structural model to study this case.

Three trading rounds. Our analysis is readily extended to an arbitrary number of trading rounds. Hence, it also speaks to how lending fees evolve more generally as the rate of information arrival varies over time. However, we focus on three trading periods around a public signal, which spikes the rate of information arrival, because this (i) delivers all of the insights from additional periods, and (ii) helps translate our results into predictions about how loan fees and prices evolve around salient events like earnings announcements.

4 Equilibrium Characterization

We solve for the equilibrium by working backwards.

4.1 Date-2 Equilibrium

Let $\mu_{it} \equiv \mathbb{E}_{it}[P_{t+1}]$ and $\sigma_t^2 \equiv \mathbb{V}_{it}\left[P_{t+1}\right]$ denote investor i's conditional mean and variance of the next date's price on date t, where we define $P_3 = x$. At date 2, these conditional beliefs are given by:

$$\mu_{i2} = \mathbb{E}_i[x|y] = m_i + \beta\left(y - \delta_i\right) \quad \text{and} \quad \sigma_2^2 = \mathbb{V}[x|y] = \sigma_x^2 - \beta^2\sigma_y^2$$

where $\beta \equiv \frac{\sigma_{xy}}{\sigma_y^2}$. Given these beliefs, as we prove formally in the appendix, investor i's optimal demand is given by

$$D_{i2}\left(\mu_{i2}\right) = \begin{cases} \frac{\mu_{i2}-P_{L2}}{\rho\sigma_2^2} & \text{when } \mu_{i2} > P_{L2}; \\ 0 & \text{when } \mu_{i2} \in \left[P_{S2}, P_{L2}\right]; \\ \frac{\mu_{i2}-P_{S2}}{\rho\sigma_2^2} & \text{when } \mu_{i2} < P_{S2}, \end{cases} \tag{1}$$

where $P_{L2} \equiv P_2 - \alpha f_2$ and $P_{S2} \equiv P_2 - f_2$ are the net-of-fees prices for long and short investors, respectively. This reflects the fact that a long investor can lend a fraction α of each share she owns and collect a fee of f_2, so that the net price of buying a share is $P_{L2} = P_2 - \alpha f_2$. In contrast, a short seller pays a borrowing cost of f_2 per share, but receives P_2 from selling it, and so trades at a net price of $P_{S2} = P_2 - f_2$. Importantly, since the net price for longs and shorts is different when $f_2 > 0$, investors whose beliefs μ_{i2} are between P_{S2} and P_{L2} do not participate in the market in this case.

Denote the distribution of μ_{i2} across investors by $G_{\mu 2}\left(\cdot\right)$. For a given net long price P_{L2}, the aggregate demand from long investors is given by

$$Q_L\left(P_{L2}\right) \equiv \int_{P_{L2}}^{\infty} \frac{\mu_{j2} - P_{L2}}{\rho\sigma_2^2} dj = \left(1 - G_{\mu 2}\left(P_{L2}\right)\right) \frac{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > P_{L2}\right] - P_{L2}}{\rho\sigma_2^2}, \tag{2}$$

where we use $\mathbb{E}^{\mathcal{M}}[\cdot]$ to denote the expectation across investor beliefs, as opposed to that over random variables (e.g., $\mathbb{E}^{\mathcal{M}}[\mu_{i2}|\mu_{i2} < t] = G_{\mu 2}(t)^{-1}\int_{j:\mu_{j2}<t}\mu_{j2}dj$). Likewise, for a given net short price P_{S2}, the aggregate short demand is given by

$$Q_S\left(P_{S2}\right) \equiv \int_{-\infty}^{P_{S2}} \frac{\mu_{j2} - P_{S2}}{\rho\sigma_2^2} dj = -G_{\mu 2}\left(P_{S2}\right) \frac{P_{S2} - \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < P_{S2}\right]}{\rho\sigma_2^2}. \tag{3}$$

Now, if there exists a price P_2 that clears the stock market and ensures there are an excess number of shares available to short in equilibrium, $\left|Q_S\left(P_2\right)\right| \le \alpha Q_L\left(P_2\right)$, then this will correspond to an equilibrium with zero loan fees $f_2 = 0$ (i.e., $P_2 = P_{L2} = P_{S2}$). In this case, imposing market clearing implies that the equilibrium price satisfies

$$P_2 = \int \mu_{j2} dj - \rho Q \sigma_2^2 \equiv \bar{P}_2.$$

Applying equations (2) and (3), we can rewrite the condition $|Q_S(P_2)| \leq \alpha Q_L(P_2)$ as

$$\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right] \leq \rho\sigma_2^2 \left(\frac{1}{1 - G_{\mu 2}\left(\bar{P}_2\right)} + \frac{\alpha}{G_{\mu 2}\left(\bar{P}_2\right)}\right)\frac{1}{1-\alpha}Q. \quad (4)$$

Hence, a zero-fee equilibrium exists when the relative beliefs of optimistic and pessimistic investors are not too far apart relative to the asset supply Q, investor risk aversion ρ, and investor uncertainty σ_2^2.

In contrast, if $|Q_S\left(\bar{P}_2\right)| > \alpha Q_L\left(\bar{P}_2\right)$, then the loan fee must be strictly positive (i.e., $f_2 > 0$) for the stock and lending markets to clear. In this case, the market-clearing conditions in the two markets, $Q_L(P_{L2}) + Q_S(P_{S2}) = Q$ and $\alpha Q_L(P_{L2}) = |Q_S(P_{S2})|$ reduce to

$$Q_L\left(P_{L2}\right) = \frac{1}{1-\alpha}Q \quad \text{and} \quad Q_S\left(P_{S2}\right) = -\frac{\alpha}{1-\alpha}Q. \quad (5)$$

To obtain the equilibrium, we can then solve these equations for P_{S2} and P_{L2}. As we show in the next proposition, these equations always have unique solutions, corresponding to the unique equilibrium. Given these solutions, we can apply the definitions of P_{L2} and P_{S2} to obtain the equilibrium price and fee:

$$f_2 = \frac{P_{L2} - P_{S2}}{1-\alpha} \quad \text{and} \quad P_2 = \frac{P_{L2} - \alpha P_{S2}}{1-\alpha}.$$

The following result summarizes the characterization of the date-2 equilibrium.

Proposition 1. *There exists a unique market-clearing price and loan fee on date* 2*. Let* P_{L2} *and* P_{S2} *be the unique solutions to the system of equations* $h_{L2}\left(P_{L2}\right) = 0$ *and* $h_{S2}\left(P_{S2}\right) = 0$*, where*

$$h_{L2}(p) = \left(1 - G_{\mu 2}(p)\right)\frac{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > p\right] - p}{\rho\sigma_2^2} - \frac{1}{1-\alpha}Q \text{ and}$$
$$h_{S2}(p) = G_{\mu 2}(p)\frac{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < p\right] - p}{\rho\sigma_2^2} + \frac{\alpha}{1-\alpha}Q.$$

Then, the equilibrium loan fee is given by

$$f_2 = \frac{1}{1-\alpha}\max\left\{0, P_{L2} - P_{S2}\right\}$$

and the equilibrium price is given by

$$P_2 = \begin{cases} \bar{P}_2 & \text{if } f_2 = 0 \\ \frac{P_{L2} - \alpha P_{S2}}{1-\alpha} & \text{if } f_2 > 0 \end{cases},$$

where $\bar{P}_2 \equiv \int \mu_{j2} dj - \rho Q \sigma_2^2$. *The loan fee is zero if and only if* (4) *holds and strictly positive otherwise.*

To gain intuition for the determinants of the equilibrium price and loan fee, note that when $f_2 > 0$, one can apply (5) to express the net price to long investors and short sellers as

$$P_{L2} = \mu_{L2} - \frac{1}{1-\alpha} \frac{1}{\lambda_{L2}} \rho Q \sigma_2^2 \quad \text{and} \quad P_{S2} = \mu_{S2} + \frac{\alpha}{1-\alpha} \frac{1}{\lambda_{S2}} \rho Q \sigma_2^2,$$

respectively, where $\mu_{L2} \equiv \mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} > P_{L2}]$, $\mu_{S2} \equiv \mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} < P_{S2}]$, $\lambda_{L2} \equiv 1 - G_{\mu 2}(P_{L2})$ and $\lambda_{S2} \equiv G_{\mu 2}(P_{S2})$. This reflects the fact that when $f_2 > 0$ in equilibrium, the mass of long investors λ_{L2} has to bear $\frac{1}{1-\alpha} Q$ shares (per capita) of the risky security in equilibrium, while the mass of short sellers λ_{S2} sells $\frac{\alpha}{1-\alpha} Q$ shares (per capita). Plugging this into the expression for price and rearranging, we can express P_2 as:

$$P_2 = \frac{\lambda_{L2}\mu_{L2} + \lambda_{S2}\mu_{S2} - \rho Q \sigma_2^2}{\lambda_{L2} + \lambda_{S2}} + \frac{\alpha \lambda_{L2} + \lambda_{S2}}{\lambda_{L2} + \lambda_{S2}} f_2,$$

where the loan fee is

$$f_2 = \frac{\mu_{L2} - \mu_{S2}}{1-\alpha} - \frac{1}{(1-\alpha)^2} \left(\frac{1}{\lambda_{L2}} + \frac{\alpha}{\lambda_{S2}} \right) \rho Q \sigma_2^2.$$

On the other hand, recall that when $f_2 = 0$, the price is given by $\bar{P}_2 = \int \mu_{j2} dj - \rho Q \sigma_2^2$.

Together, these results imply that the date-2 price can be expressed as $P_2 = \int \mu_{j2} dj + \Pi_2$, where the premium relative to the average investor belief Π_2 is given by

$$\Pi_2 \equiv -\rho Q \sigma_2^2 + \max\{\eta_2, 0\} \tag{6}$$

where η_2 is the excess valuation generated by lending constraints, is positive when the loan fee is positive, and satisfies

$$\eta_2 = \underbrace{\frac{\lambda_{L2}\mu_{L2} + \lambda_{S2}\mu_{S2}}{\lambda_{L2} + \lambda_{S2}} - \int \mu_{j2} dj + \rho Q \sigma_2^2 \left(1 - \frac{1}{\lambda_{L2} + \lambda_{S2}} \right)}_{\text{investor participation}} + \underbrace{\frac{\alpha \lambda_{L2} + \lambda_{S2}}{\lambda_{L2} + \lambda_{S2}} f_2}_{\text{loan fee}}.$$

When the risky security is not on special (i.e., $f_2 = \eta_2 = 0$), the risk premium reflects the discount that investors require for bearing Q (per-capita) shares of the risky security (i.e., $-\rho\sigma_2^2 Q$). When the risky security is on special (i.e., $f_2, \eta_2 > 0$) however, two other terms also affect the risk premium.

First, the "investor participation" term reflects the difference in prices as a result of limited participation when $f_2 > 0$, which leads to a difference in both the average beliefs of investors who trade in the security (i.e., $\frac{\lambda_{L2}\mu_{L2}+\lambda_{S2}\mu_{S2}}{\lambda_{L2}+\lambda_{S2}} - \int \mu_{j2} dj$) and a difference in the aggregate risk-bearing capacity (i.e., $\rho Q\sigma_2^2(1 - \frac{1}{\lambda_{L2}+\lambda_{S2}})$). Second, the "loan fee" component reflects the fact that the price P_2 is higher because of a non-zero fee. Note that the loan fee increases in the difference between the demands of long and short investors and so increases in disagreement (i.e., $\mu_{L2}-\mu_{S2}$), but decreases in the total supply of the asset Q (which increases the shares available for shorting) and risk aversion ρ. We summarize these observations in the following result.

Corollary 1. *The date-2 price decreases in ρ and Q, and the date-2 loan fee decreases in ρ and Q when it is positive.*

Note that neither component of the premium Π_2 depends on the realization of the public signal y. As a result, the premium, and in particular, whether the security is on special (i.e., whether $\eta_2 > 0$), can be perfectly anticipated by investors prior to the announcement, which simplifies the subsequent analysis.

As a concrete example, suppose that $\mu_{i2} \sim Uniform([\mu_{L2}, \mu_{H2}])$. In the appendix, we show that, in this case,

$$f_2 = \frac{\mu_{H2} - \mu_{L2}}{1-\alpha} - \frac{1+\sqrt{\alpha}}{1-\alpha}\sqrt{\frac{2\rho Q\sigma_2^2\left(\mu_{H2} - \mu_{L2}\right)}{1-\alpha}};$$

$$P_2 = \frac{\mu_{H2} - \alpha\mu_{L2}}{1-\alpha} - \frac{1+\alpha^{\frac{3}{2}}}{1-\alpha}\sqrt{\frac{2\rho Q\sigma_2^2\left(\mu_{H2} - \mu_{L2}\right)}{1-\alpha}}.$$

It is easily seen that f_2 is strictly positive if and only if $\mu_{H2} - \mu_{L2}$ is sufficiently large, and more generally increases in $\mu_{H2} - \mu_{L2}$. Moreover, fixing $\mu_{H2} - \mu_{L2}$, when investors are more uncertain, f_2 is lower because they trade less intensely on their beliefs. Figure 2 illustrates the properties of the equilibrium in this case. In particular, the figure assumes $y = x + \varepsilon$, investors agree about the distribution of ε, and $m_i \sim Uniform([m_L, m_H])$, which leads μ_{i2} to be uniformly distributed.

The figure shows that the fee and price rise with disagreement and fall with the asset's supply. However, α has a non-monotonic impact on the price and fee. Intuitively, a higher α

has two effects. First, it increases the supply of shares that are available for shorting, which relaxes the lending constraint and tends to push down the price. Second, it increases the fee income earned by investors who are long, which tends to push up the price.

Figure 2: Post-Announcement Expected Price and Fee

This figure depicts the equilibrium stock price P_2 and loan fee f_2 in date 2, when $m_i \sim Uniform([m_L, m_H])$ and the public signal y equals $x + \varepsilon$ where investors agree that $\mathbb{E}[\varepsilon] = 0$. The parameters applied are $m_L = 1$, $m_H = 3$, $\sigma_x^2 = \sigma_\varepsilon^2 = 1$, $Q = 0.1$, $\alpha = 0.5$, and $\rho = 0.75$.

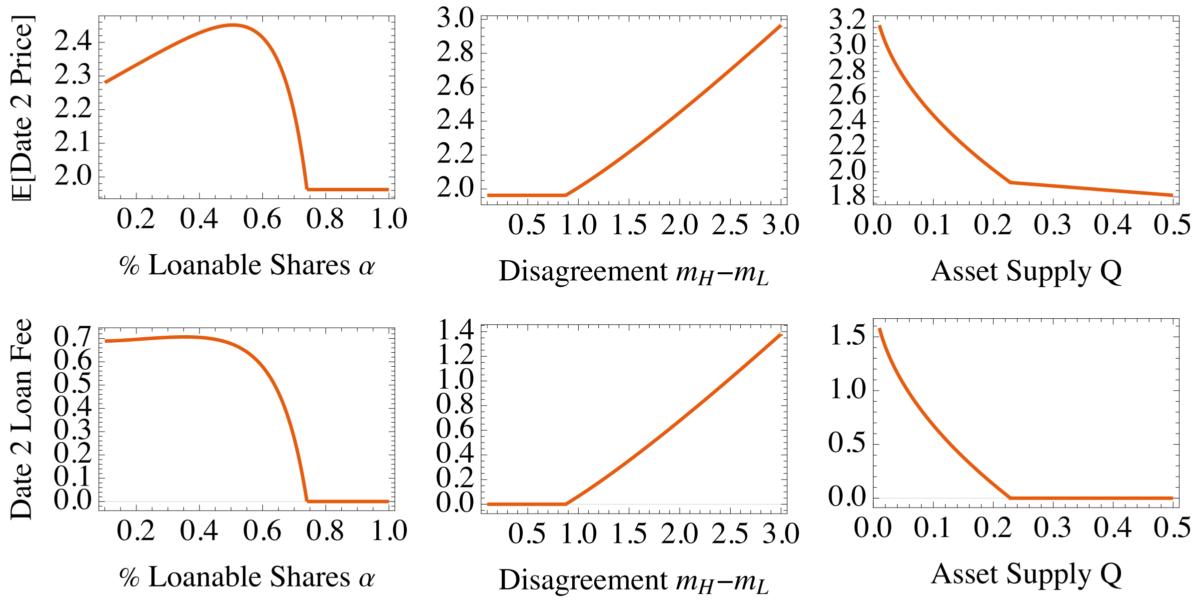


Figure 3 depicts the fraction of investors who long and short in the equilibrium. The figure shows that the dependence of participation on the model parameters often differs depending on whether the lending constraint binds. For instance, while more disagreement leads to more short selling when fees are zero, the opposite holds when fees are positive. In the latter case, an increase in disagreement raises fees, leading only more pessimistic investors to take short positions. Similarly, short selling increases with the asset supply when the fee is positive, as this leads to a lower fee, but declines with asset supply when the fee is zero, as it lowers the stock's price.

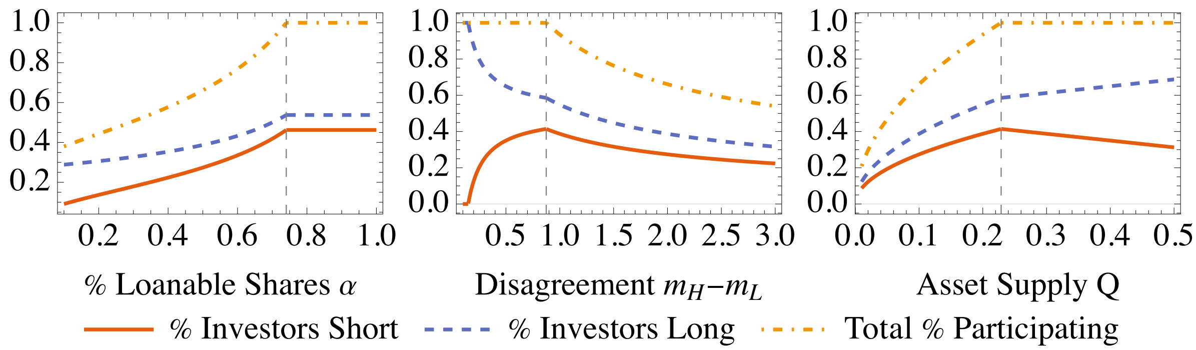
4.2 Date-1 Equilibrium

At date 1, investor i's conditional beliefs about P_2 are

$$\mu_{i1} = \mathbb{E}_{i1}[P_2] = \bar{m} + \beta(\delta_i - \bar{\delta}) + \Pi_2 \quad \text{and} \quad \sigma_1^2 = \mathbb{V}_{i1}[P_2] = \beta^2 \sigma_y^2,$$

Figure 3: Post-Announcement Long and Short Participation

This figure depicts the proportion of investors who take short positions and long positions, as well as the total % taking either a long or short position, in the date-2 equilibrium. In the plots, $m_i \sim Uniform([m_L, m_H])$ and the public signal y equals $x + \varepsilon$, where investors agree that $\mathbb{E}[\varepsilon] = 0$. The dashed vertical lines divide the regions where the loan fee is zero versus positive. The parameters applied are $m_L = 1$, $m_H = 3$, $\sigma_x^2 = \sigma_\varepsilon^2 = 1$, $Q = 0.1$, $\alpha = 0.5$, and $\rho = 0.75$.



where $\bar{m} = \int_j m_j dj$ and $\bar{\delta} = \int_j \delta_j dj$. Analogous to the date-2 equilibrium, we show in the proof of the next proposition that investor i's optimal demand is given by

$$D_1(\mu_{i1}) = \begin{cases} \frac{\mu_{i1} - P_{L1}}{\rho\sigma_1^2} & \text{when } \mu_{i1} > P_{L1}; \\ 0 & \text{when } \mu_{i1} \in [P_{S1}, P_{L1}]; \\ \frac{\mu_{i1} - P_{S1}}{\rho\sigma_1^2} & \text{when } \mu_{i1} < P_{S1}, \end{cases} \tag{7}$$

where $P_{L1} \equiv P_1 - \alpha f_1$ and $P_{S1} \equiv P_1 - f_1$ are the net-of-fees prices for long and short investors, respectively. That is, the investors' optimal demands are myopic in the sense that they speculate based purely on their perceived mean and variance of the next period's price. Given this feature, we can follow similar arguments to those used to derive the date-2 equilibrium to establish the following result.

Proposition 2. *There exists a unique market-clearing price and loan fee on date* 1*. Let* P_{L1} *and* P_{S1} *be the unique solutions to the system of equations* $h_{L1}(P_{L1}) = 0$ *and* $h_{S1}(P_{S1}) = 0$*, where*

$$h_{L1}(p) = (1 - G_{\mu 1}(p)) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i1} | \mu_{i1} > p] - p}{\rho\sigma_1^2} - \frac{1}{1-\alpha} Q \quad \textit{and}$$
$$h_{S1}(p) = G_{\mu 1}(p) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i1} | \mu_{i1} < p] - p}{\rho\sigma_1^2} + \frac{\alpha}{1-\alpha} Q.$$

Then, the equilibrium loan fee is given by

$$f_1 = \frac{1}{1-\alpha} \max\left\{0, P_{L1} - P_{S1}\right\}$$

and the equilibrium price is given by

$$P_1 = \begin{cases} \bar{P}_1 & \text{if } f_1 = 0 \\ \frac{P_{L1} - \alpha P_{S1}}{1-\alpha} & \text{if } f_1 > 0 \end{cases},$$

where $\bar{P}_1 \equiv \int \mu_{j1} dj - \rho \sigma_x^2 Q$*. Finally, the loan fee is strictly positive if and only if*

$$\mathbb{E}^{\mathcal{M}}\left[\mu_{i1} | \mu_{i1} > \bar{P}_1\right] - \mathbb{E}^{\mathcal{M}}\left[\mu_{i1} | \mu_{i1} < \bar{P}_1\right] > \rho \sigma_1^2 \left(\frac{1}{1 - G_{\mu 1}\left(\bar{P}_1\right)} + \frac{\alpha}{G_{\mu 1}\left(\bar{P}_1\right)} \right) \frac{1}{1-\alpha} Q,$$

and zero otherwise.

As we show in the proof of the above proposition, one can express the date-1 price as

$$P_1 = \bar{m} - \rho Q \sigma_x^2 + \max\{\eta_1, 0\} + \max\{\eta_2, 0\}, \tag{8}$$

where $\eta_1 > 0$ if and only if the risky security is on special at date 1 (i.e., $f_1 > 0$). Moreover, since the dispersion in μ_{i1} across investors depends only on the distribution of δ_i, whether or not the security is on special at date 1 depends only on investor disagreement about y (i.e., the dispersion in δ_i across investors). Analogous to Corollary 1, it is straightforward to verify that an increase in ρ or Q lowers the date-1 loan fee when it is positive. Note that any overvaluation generated by the lending constraint at date 2, $\max\{\eta_2, 0\}$, propagates backwards in time, also increasing the price at date 1. Intuitively, investors anticipate the high date-2 price, which raises their willingness to pay for the security in date 1.

Importantly, the above expression highlights that the degree of overvaluation resulting from short-sales constraints depends not only on whether the stock is on special in the current period, but also whether it will be on special in future periods. In Sections 5 and 6, we will characterize how different types of announcements have different implications for loan fee dynamics (i.e., f_1 and f_2) and, consequently, price dynamics (i.e., P_1 and P_2) around the announcement.

4.3 Date-0 Equilibrium

At date 0, investor i's demand for the security only depends on her beliefs about the date-1 price. However, since there is no uncertainty or disagreement about P_1 at date 0, we have the following result.

Proposition 3. *On date zero, the loan fee is always zero, and the price is identical to the date-1 price i.e.,* $P_0 = P_1$.

This result implies that loan fees tend to increase prior to announcements when the lending constraint binds (i.e., $f_1 > 0$), which reflects the increase in short-term information arrival that investors expect on the announcement date. Despite this, there are no abnormal expected stock returns leading up to the announcement.

5 Price and Fee Dynamics Under Concordant Beliefs

We first study how prices and fees evolve around the announcement when investors agree on how to interpret the announcement, in that they have "concordant beliefs" (Milgrom and Stokey (1982)). That is, for all $\{i, j\}$, $f_i(y|x) = f_j(y|x)$. Given the Gaussian information structure, as we show in Lemma A.4 in the appendix, concordant beliefs imply that, after rescaling, y can be expressed as $y = x + \varepsilon$, where all investors i perceive that $\varepsilon \sim_i N(0, \sigma_\varepsilon^2)$ and $\varepsilon \perp x$.[5] The announcement's informativeness is captured by the precision of ε. Despite the fact that short selling is costly, we show that the announcement does not generate trade as in Milgrom and Stokey (1982). As a result, the demand for shortable shares either meets the supply on both dates, leading to positive loan fees, or on neither date.

Proposition 4. *(i) When investors have concordant beliefs, they do not trade after the announcement, i.e.,* $\forall i \in [0, 1]$, $D_{i1} = D_{i2}$.

(ii) The loan fees on dates 1 and 2 satisfy $\frac{f_1}{\sigma_1^2} = \frac{f_2}{\sigma_2^2}$. *Hence, the lending constraint binds before the announcement* ($f_1 > 0$) *if and only if it binds after the announcement* ($f_2 > 0$).

When the lending constraint does not bind, there is no trade because investors' demands in dates 1 and 2 depend on their idiosyncratic beliefs through the ratio $\frac{\mu_{it} - \int \mu_{jt} dj}{\sigma_t^2}$ (which

[5] While a linear projection yields that, after rescaling y by σ_x^2/σ_{xy}, y can always be written in this form, concordant beliefs impose that investors agree about the mean of ε.

follows from substituting the firm's prices into equations (1) and (7)). Under concordant beliefs, this ratio does not vary over time:

$$\frac{\mu_{i1} - \int \mu_{j1} dj}{\sigma_1^2} = \frac{\frac{\sigma_x^2}{\sigma_x^2+\sigma_\varepsilon^2}\left(m_i - \bar{m}\right)}{\frac{\sigma_x^4}{\sigma_x^2+\sigma_\varepsilon^2}} = \frac{\frac{\sigma_\varepsilon^2}{\sigma_x^2+\sigma_\varepsilon^2}\left(m_i - \bar{m}\right)}{\frac{\sigma_x^2\sigma_\varepsilon^2}{\sigma_x^2+\sigma_\varepsilon^2}} = \frac{\mu_{i2} - \int \mu_{j2} dj}{\sigma_2^2}.$$

Intuitively, in this case, a more precise public announcement reduces investor disagreement because investors agree on how to interpret this additional information. However, it also reduces their uncertainty, which makes them speculate more intensely on their idiosyncratic beliefs.

Perhaps surprisingly, even when the lending constraint binds, investors still do not trade after the announcement. This is despite the fact that loan fees typically change after the announcement, as we show below. To understand why, note from equations (1) and (7) that the loan fee affects investor demands only through the ratio $\frac{f_t}{\sigma_t^2}$. Moreover, as we show in the proof of the proposition, the loan fee f_t at each date is determined by the dispersion in investors' idiosyncratic beliefs $\mu_{it} - \int \mu_{jt} dj$, which is proportional across the two dates as discussed above. As a result, $\frac{f_1}{\sigma_1^2} = \frac{f_2}{\sigma_2^2}$ and the component of investor demands driven by the loan fee are identical across the two periods. Given that f_2 clears the lending market in date 2, this market tends to clear at date 1 with a fee of $f_1 = \frac{\sigma_1^2}{\sigma_2^2} f_2$, as this leads investors to take identical positions in the two dates. Note this reasoning also explains part (ii) of the proposition.

5.1 Post-Announcement Price and Loan Fee

The next result, which is illustrated in the upper panels of Figure 4, considers how the announcement impacts fees and prices after its release.

Corollary 2. *Suppose investors have concordant beliefs.*

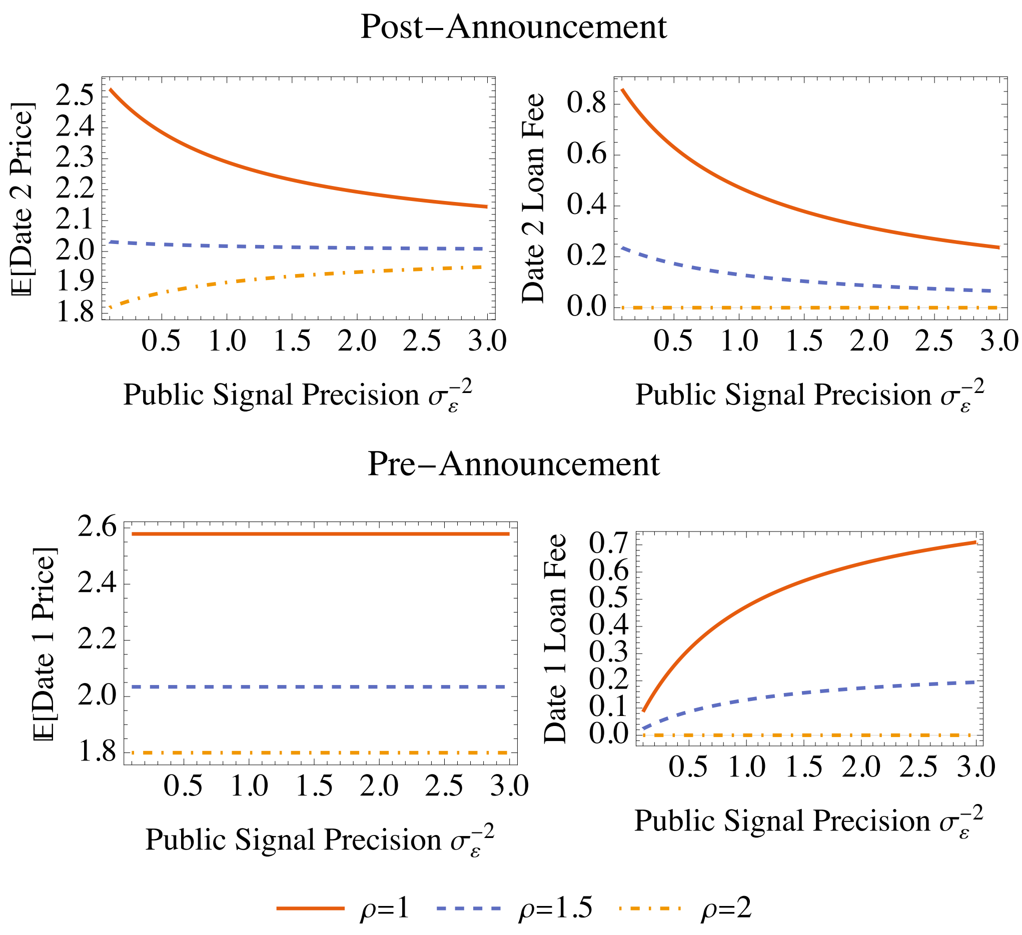
(i) More precise public information decreases the date-2 loan fee when it is positive.

(ii) More precise public information moves the date-2 price towards the average over investors' date-2 expectations of cash flows, i.e., sgn $\left(\frac{\partial P_2}{\partial \sigma_\varepsilon}\right) = sgn(\int \mu_{j2} dj - P_2)$. *As a result, more precise public information decreases (increases) the firm's expected price when* ρQ *is small (large).*

Under concordant beliefs, an increase in the announcement's informativeness reduces the loan fee after its release because it reduces the possibility of disagreement post-announcement.

Figure 4: Expected Prices and Fees Under Concordant Beliefs

This figure depicts the firm's price and loan fee in equilibrium before and after the announcement, when investors have concordant beliefs and $m_i \sim Uniform([m_L, m_H])$. The parameters applied are $m_L = 1$, $m_H = 3$, $\sigma_x^2 = \sigma_\varepsilon^2 = 1$, $Q = 0.1$, and $\alpha = 0.5$.



For example, consider the extreme case in which the announcement resolves almost all information about the firm's value. Then, even the most pessimistic investor has no reason to pay a high fee to short, as they know that the price already closely reflects the firm's value. To see this analytically, recall that the fee enters investors' demands through the ratio $\frac{f_2}{\sigma_2^2}$. All else equal, a decrease in uncertainty σ_2^2 for a fixed f_2 has a larger impact on short investors' demands. However, since market clearing implies short sellers must hold $-\frac{\alpha}{1-\alpha}Q$ shares in equilibrium, the equilibrium loan fee f_2 must decrease for the lending market to clear.

While this pushes down the firm's price, a more informative announcement also reduces the risk premium. When the lending constraint either does not bind or only marginally binds (which, as shown in Corollary 1, occurs when ρQ is high), the risk premium effect dominates and more precise public information increases price. The opposite holds when the lending constraint binds strongly. In fact, we find that more precise public information increases the firm's expected date-2 price if and only if the firm's date-2 price exceeds the average investor's date-2 belief about the firm's expected cash flows, i.e., the overvaluation generated by short constraints dominates the risk premium. Our result is broadly in line with the negative relation between short selling and returns around announcements documented in the empirical literature (e.g., Engelberg et al. (2012)). More specifically, consistent with this prediction, Chang, Hsiao, Ljungqvist, and Tseng (2022) show that firms exhibit smaller declines in stock price after being included in EDGAR (which they interpret as an increase in the availability of public information) when their short-sales constraints are less binding, and this effect is significant around earnings announcements.

5.2 Pre-Announcement Price and Loan Fee

We next consider how public information impacts the firm's date-1 loan fee and expected valuation. The following result, which is illustrated in the lower panels of Figure 4, establishes that, given concordant beliefs, when the announcement is more informative, the pre-announcement loan fee rises. However, the stock price itself does not change.

Proposition 5. *Suppose investors have concordant beliefs. Then,*

(i) When the date-1 loan fee f_1 is positive, it increases in the precision of the public signal σ_ε^{-1}.

(ii) The date-1 stock price P_1 does not depend on the precision of the public signal σ_ε^{-1}.

The reason that a more informative signal raises the pre-announcement fee is essentially the same reason that it lowers the post-announcement fee. When the signal is more informative, pessimistic investors expect larger price movements in the short-term, which raises their willingness to pay to short. In the extreme, if the announcement were completely uninformative, short sellers would expect no price movement in the short term, and thus the fee would be zero.

Despite this, the equilibrium stock price does not increase with signal precision σ_ε^{-1}. The reason is that the excess valuation generated by the lending constraint after the announcement declines as the announcement becomes more informative. Moreover, this reduces the investors' valuations before the announcement. We show that this decline exactly offsets the increase in fee-driven excess valuation in the first period. Formally, recall that the firm's date-1 price can be expressed as

$$P_1 = \bar{m} - \rho Q \sigma_x^2 + \max\{\eta_1, 0\} + \max\{\eta_2, 0\}.$$

The term $P_1 = \bar{m} - \rho Q \sigma_x^2$ is the firm's price when lending constraints do not bind, and is clearly independent of σ_ε. The terms $\max\{\eta_1, 0\}$ and $\max\{\eta_2, 0\}$ capture the excess valuation generated by lending constraints in dates 1 and 2, and while they individually depend on σ_ε when the lending constraint binds, we show that their sum does not.

Ross (1989) shows that current prices do not depend on the timing at which information arrives in the future in a setting without lending constraints.[6] Our result above indicates that this extends to a market with lending constraints under concordant beliefs. In addition, our findings imply that lending fees reflect not only investors' belief dispersion and the supply of shortable shares, but also the expected amount of near-term information arrival.

5.3 Expected Announcement Returns

We next consider expected returns around the announcement. While prior empirical work documents that such returns are related to short-sales constraints (e.g., Berkman et al. (2009)), this relationship has not been analytically explored. Consistent with other models with CARA utility, we focus on expected dollar returns $\mathbb{E}[P_2 - P_1]$.

[6] See also Christensen, de la Rosa, and Feltham (2010) who shows that this result continues to hold in a setting where CARA investors have heterogeneous beliefs but face no short constraints, and Jiang, Yang, Wei, and Zhang (2025), who study the impact of disclosure on the ex-ante cost of capital in a beauty-contest setting.

Notice from equations (6) and (8) that

$$\mathbb{E}\left[P_2 - P_1\right] = \underbrace{\rho Q\left(\sigma_x^2 - \sigma_2^2\right)}_{\text{reduction in risk premium}} - \underbrace{\max\left\{\eta_1, 0\right\}}_{\substack{\text{excess valuation driven by}\\ \text{date 1 lending constraint}}}. \tag{9}$$

An important observation is that the properties of the lending market at date 2, as captured by $\max\{\eta_2, 0\}$, do not enter equation (9). Intuitively, investors anticipate the impact of lending constraints on the price after the announcement and thus it is already reflected in the pre-announcement price. Hence, changes in the lending market after the announcement do not cause the price to increase in our model. Instead, announcement returns are driven by a decline in the risk premium and an unwinding of the excess valuation created by the date-1 lending constraint.

Building on this insight, we next examine how announcement returns vary with key model parameters. First, when either risk aversion or asset supply decreases, the risk premium shrinks and the lending constraint tightens – both effects lead expected returns to decline. Next, when the announcement is more precise, as shown in the prior two sections, the date-1 price does not change, while, for low ρQ, the date-2 price declines. This, in turn, results in lower expected announcement returns. Finally, as disagreement rises, expected returns fall, consistent with the evidence in Berkman et al. (2009) that firms with greater disagreement experience more negative returns around information events. We can show this in the case in which $m_i \sim Uniform([m_L, m_H])$, where we have a straightforward notion of disagreement, $m_H - m_L$.

Corollary 3. *(i) An increase in ρQ lowers the date-1 loan fee when it is positive and raises expected announcement returns.*

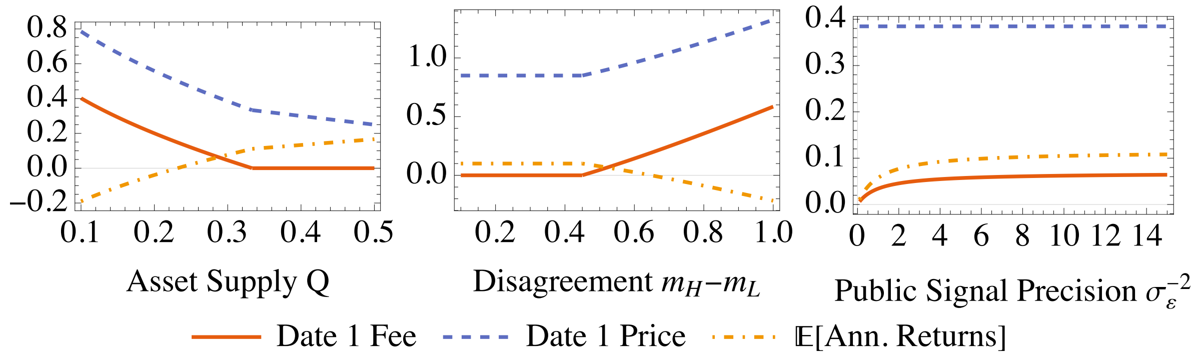
(ii) For ρQ sufficiently low (high), more precise public information lowers (raises) expected announcement returns.

(iii) When $m_i \sim Uniform([m_L, m_H])$ and the date-1 loan fee is positive, an increase in disagreement $m_H - m_L$ raises the date-1 loan fee and lowers expected announcement returns.

This corollary is illustrated in Figure 5. Note parts (i) and (iii) imply that variation in loan fees driven by investor risk aversion, the supply of the firm's shares, or investor beliefs lead to a negative relation between the pre-announcement loan fee and announcement returns. However, part (ii) indicates that, when announcement returns are primarily driven by risk aversion rather than lending constraints, loan fees can be positively associated with expected returns.[7] This distinguishes our setting from existing models of lending markets where loan fees vary only due to disagreement and not time-variation in information arrival, in which loan fees are negatively associated with returns (e.g., Duffie et al. (2002)).

Figure 5: Expected Announcement Returns

This figure depicts expected announcement returns as a function of the asset supply, disagreement, and the public signal's precision, when investor beliefs are concordant and $m_i \sim Uniform([m_L, m_H])$. The parameters applied are $m_L = 0$, $m_H = 1$, $\sigma_x^2 = \sigma_\varepsilon^2 = 1$, $Q = 0.3$, $\alpha = 0.25$, and $\rho = 0.75$.



6 Price and Fee Dynamics When News Generates Trade

We next consider the case in which beliefs are not concordant and thus the announcement generates trade. As a parsimonious and intuitive way to capture the impact of such beliefs, we focus on a particular parametrization. Specifically, we assume now that x can be broken down into two independent (normally-distributed) components $x = x_1 + x_2$ where $x_1 \perp x_2$, $\mathbb{V}_i[x_1] = \sigma_{x1}^2$, and $\mathbb{V}_i[x_2] = \sigma_{x2}^2$. Critically, investors disagree about only one of these components, which we assume without loss of generality is x_1: $\mathbb{E}_i[x_1] = m_i$ and $\mathbb{E}_i[x_2] = 0$.

We consider the following two information structures.

(i) **Signal disagreement.** In this case, the announcement concerns the component of firm value that investors disagree about: $y = x_1 + \varepsilon$, where $\varepsilon \perp \{x_1, x_2\}$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. As an example, investors may disagree about the key aspects of a firm's business such as the demand for its products, and the announcement may provide information on these fundamentals. However, there may be completely unpredictable future events, such as natural disasters, CEO death, government policy changes, etc., that also influence

[7] Recall that Proposition 5 implies that date-1 loan fees increase in the precision of public information.

firm value. Since these events are completely unpredictable, investors neither disagree about them nor find the announcement informative regarding them.

(ii) **Signal agreement.** In this case, the announcement concerns the component of firm value that investors do not disagree about: $y = x_2 + \varepsilon$, where $\varepsilon \perp \{x_1, x_2\}$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. For instance, investors may disagree about the long-term potential of a speculative growth company, but agree that the short-term performance is likely to be poor. Alternatively, there may be a large, forward-looking driver of firm value that investors disagree about, such as the likelihood of a takeover. These events are unlikely to be captured by an earnings announcement, which is, by nature, backwards looking.

The above specification is stylized, but useful to sharply distinguish settings in which investors largely agree on the information revealed by the public announcement from cases in which they do not. In practice, we expect that investors exhibit varying degrees of disagreement about the signal and future information. The next result characterizes how investors trade around the announcement in both cases.

Proposition 6. *(i) In the signal agreement case, investors wait until after the announcement to trade: $D_{j1} = Q$. As a result, the lending constraint never binds before the announcement.*

(ii) In the signal disagreement case,

a. The lending constraint is more likely to bind before than after the announcement, i.e., $f_2 > 0 \Rightarrow f_1 > 0$, but not vice versa.

b. When fees are zero in both periods, investors trade prior to the announcement and scale back their trades post announcement: $D_{j1} - Q = k(D_{j2} - Q)$, for $k \in (0, 1)$.

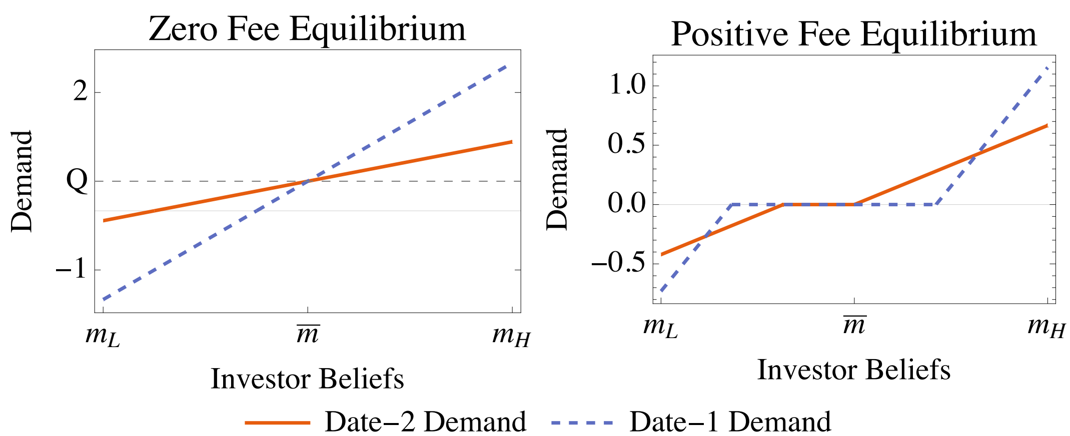
c. When fees are positive in both periods, investors with sufficiently extreme beliefs decrease their absolute positions after the announcement, while those with moderate beliefs weakly increase their positions. Formally, there exist thresholds $m^\dagger, m^\ddagger$ such that, for $m_i < m^\dagger$ or $m_i > m^\ddagger$, $|D_{i2}| < |D_{i1}|$, while, for $m_i \in (m^\dagger, m^\ddagger)$, $|D_{i2}| \geq |D_{i1}|$.

In both cases, investors seek to trade around the announcement, but for different reasons. In the signal agreement case, investors prefer not to be exposed to the announcement risk, as they agree on it. Hence, they wait until after the announcement to trade.[8]

[8] Note further there are no dynamic hedging demands in this setting because the firm's price is driven entirely by information as opposed to, e.g., noise trade. Thus, even if investors anticipate that they will purchase shares in the future, they have no need to hedge against interim price changes (in contrast to models with noise trade). Intuitively, when the announcement leads to a dollar increase in the price, this is perfectly offset by a dollar increase in the stock's expected cash flows, so that interim price movements do not expose the investor to any payoff risk.

Figure 6: Signal Disagreement – Investor Demands

This figure depicts investors' demands in the signal disagreement case, as a function of m_i, when $m_i \sim Uniform([m_L, m_H])$. In the left-hand plot, $Q = 0.5$ and $\alpha = 0.95$, leading to a zero-loan-fee equilibrium. In the right-hand plot, $Q = 0.1$ and $\alpha = 0.4$, leading to a positive-fee equilibrium. In both plots, $m_L = 1$, $m_H = 3$, $\sigma_{x1}^2 = \sigma_{x2}^2 = \sigma_\varepsilon^2 = 1$, and $\rho = 0.5$.



In contrast, in the signal disagreement case, investors are more inclined to trade during the announcement period than after it. Intuitively, by trading during this period, investors obtain exposure to x_1 without also being exposed to the risk of x_2, which they agree about. When fees are zero (e.g., because α or Q is high), this leads them to hold larger absolute positions before the announcement than they do after it (similar to He and Wang (1995)'s Section 4.2). Moreover, this makes the lending constraint more likely to bind before the announcement. As such, this case is generally consistent with the findings in Christophe et al. (2004) and Engelberg et al. (2012) that shorting activity increases before announcements.

However, investors' trading behavior is considerably different when fees are positive both before and after the announcement. In this case, the size of the overall absolute positions taken by investors is fixed by the supply of shortable shares. Only the composition of investors who take these positions can change. As shown in Figure 6, investors with extreme beliefs tend to hold larger positions before the announcement, but those with moderate beliefs hold larger positions after the announcement. Intuitively, highly pessimistic investors take large short positions leading into the announcement because it is an especially favorable time to trade. This tends to raise the loan fee, which discourages moderately pessimistic investors from shorting, who may have shorted absent the announcement. This, in turn, raises the firm's valuation, leading only the most optimistic investors to hold long positions.

In both cases, the overall desire to trade across the two periods (i.e., the amount of trade absent short constraints) rises as a result of the announcement. One way of viewing this result is that the ability to trade before and after the announcement helps to "dynamically complete" the market by allowing investors to trade on different components of firm value by taking positions at different times. We next show that, as a result, loan fees and valuations differ significantly relative to the case in which beliefs are concordant.

6.1 Post-Announcement Price and Loan Fee

In the signal disagreement case, similar to the case of concordant beliefs, the announcement reduces the date-2 loan fee and can reduce the date-2 price by relaxing the lending constraint. However, in the signal agreement case, it raises the loan fee and stock price in proportion to its informativeness.

Corollary 4. *In the signal disagreement case,*

(i) The date-2 loan fee decreases in the precision of the public signal σ_ε^{-1} when this fee is positive.

(ii) The date-2 stock price may either increase or decrease in the precision of the public signal σ_ε^{-1}.

In the signal agreement case,

(i) The date-2 loan fee increases in the precision of the public signal σ_ε^{-1} when this fee is positive.

(ii) The date-2 stock price increases in the precision of the public signal σ_ε^{-1}.

Intuitively, in the signal disagreement case, the stock's attractiveness in speculation declines after the announcement and thus the demand for shorting and loan fees decline. Moreover, these effects are stronger when the announcement is more precise. The effect on prices is more nuanced given that the risk premium also declines post announcement. In contrast, in the signal agreement case, the stock's attractiveness in speculation increases after the announcement, investors take larger absolute positions, and thus fees increase. The risk premium again declines, and thus the price always increases.

6.2 Pre-Announcement Price and Loan Fee

To study the post-announcement fee and price, we focus on the case in which priors are uniformly distributed across investors.[9]

Proposition 7. *Suppose that $m_i \sim Uniform([m_L, m_H])$ and that the lending constraint binds on date 1 and/or date 2.*

(i) In the signal disagreement case, the date-1 price and loan fee increase in the precision of the public signal σ_ε^{-1}.

(ii) In the signal agreement case, the date-1 price increases in the precision of the public signal σ_ε^{-1}. The date-1 loan fee is always zero.

When investors disagree about the signal, a more informative signal increases date-1 short-selling demand since it makes beliefs about the next period's price more extreme. This increases the loan fee at date 1. In contrast, since investors never trade prior to the announcement in the signal agreement case, loan fees are always zero at date 1.

The basic intuition for why the pre-announcement price increases with the precision of the announcement in both cases is that more precise signals drive up the demand for more extreme positions (either before or after the announcement), and thus the overall extent of shorting demand. To see this more precisely, recall that we have

$$P_1 = \bar{m} - \rho Q \sigma_x^2 + \max\{\eta_1, 0\} + \max\{\eta_2, 0\}.$$

In the case of concordant beliefs, recall that $\frac{\partial}{\partial \sigma_\varepsilon} \max\{\eta_1, 0\} = -\frac{\partial}{\partial \sigma_\varepsilon} \max\{\eta_2, 0\}$, i.e., the announcement's impact on lending constraints in dates 1 and 2 perfectly offset, so that $\frac{\partial}{\partial \sigma_\varepsilon} P_1 = 0$. This no longer holds absent concordant beliefs.

In the signal disagreement case, the announcement raises the date-1 loan fee but decreases the date-2 loan fee. However, because it increases the overall demand for shorting, the impact on the date-1 loan fee dominates, i.e., $\frac{\partial}{\partial \sigma_\varepsilon} \max\{\eta_1, 0\} > -\frac{\partial}{\partial \sigma_\varepsilon} \max\{\eta_2, 0\}$. In the signal agreement case, the announcement raises the loan fee after the announcement and has no impact on the fee before the announcement, which is zero. Hence, $\frac{\partial}{\partial \sigma_\varepsilon} P_1 = \frac{\partial}{\partial \sigma_\varepsilon} \max\{\eta_2, 0\} \geq 0$ (with inequality strict when $f_2 > 0$).

[9] While the intuition behind this result appears general, we have not been able to analytically extend it beyond this case.

6.3 Expected Announcement Returns

In the signal agreement case, because investors hold identical positions through the announcement, expected announcement returns are driven entirely by the risk premium. Hence, they are always positive and increasing in the announcement's informativeness. The signal disagreement case resembles the setting with concordant beliefs: announcement returns are again driven by a combination of a reduction in the risk premium and relaxation of the lending constraint. When ρQ is large, so too is the risk premium effect, leading to a positive relation between the signal's precision and expected returns. The opposite holds when ρQ is small.

Corollary 5. *(i) In the signal disagreement case, when $m_i \sim Uniform([m_L, m_H])$, more precise public information lowers (raises) expected announcement returns when ρQ is sufficiently low (high).*

(ii) In the signal agreement case, more precise public information raises expected announcement returns.

7 Implications and Concluding Remarks

Our framework allows us to characterize the impact of public announcements on the dynamics of short-selling fees and stock prices. To reiterate, our analysis focuses on three cases: (i) concordant beliefs, where investors disagree about firm value and the announcement to the same extent, (ii) signal agreement, where investors disagree more about firm value than the announcement, and (iii) signal disagreement, where investors disagree more about the announcement than they do about overall firm value. These cases each generate a different set of predictions on observables that may allow them to be empirically distinguished, which are summarized in Table 1.

Specifically, we show that when investors have concordant beliefs, announcements are likely to be associated with relatively low trading volume but a decrease in loan fees.[10] In this case, expected announcement returns can be positive or negative, but decrease with (ex-ante) disagreement and increase with higher risk considerations (i.e., higher ρQ, corresponding to a larger firm).

In contrast, when investors' beliefs are not concordant, public announcements tend to trigger significant trading activity. In this case, if investors largely agree on the information

[10] In our model, trading volume is literally zero, but we abstract from other motives that may affect trading around announcements such as liquidity needs and mechanical portfolio rebalancing.

revealed by the announcement (signal agreement), our model predicts low loan fees prior to the announcement followed by an increase in fees afterward and positive expected announcement returns. However, if investors disagree more on the information revealed by the announcement than on firm value (signal disagreement), the model implies that loan fees decrease around the announcement, and average returns can either be positive or negative, but again are negatively related to disagreement.

Our model also provides insights into the broader relationship between loan fees and expected returns. When disagreement is high, we find that higher pre-announcement loan fees are negatively associated with announcement returns, consistent with much of the existing empirical literature on the "loan fee" anomaly. However, we also find that the opposite relation can arise when disagreement is relatively low and risk considerations are important (Proposition 5 part (i) and Corollary 3 part (ii)). In practice, this may apply to large firms whose announcements generate systematic risk. As such, the predictive ability of loan fees for future returns may differ around announcements relative to other periods.

Finally, our model also offers predictions on how the precision of public information affects loan fees, prices, and returns, which are summarized in Table 1. These predictions may speak to how changes in information quality impact the lending market (e.g., regulatory shifts that improve the informativeness of earnings announcements). In addition, while we take the nature of the information provided as exogenous, these findings offer some insight into firms' strategic disclosure incentives. For instance, they indicate that by revealing information on components of firm value that investors agree about, firms can raise both pre- and post-announcement prices. In contrast, for firms whose stocks are on special, revealing information about a component of value investors disagree about can reduce prices by attenuating this disagreement (consistent with Chang et al. (2022)). Future work may consider exploring the interaction between strategic disclosure and short-sales constraints further.

Our model may be extended along several dimensions. Our analysis focuses on a setting in which the timing of the public announcement is perfectly anticipated. While this assumption is appropriate for a large class of public news events (e.g., earnings announcements), it would be interesting to consider how our results change if, instead, the timing of the announcement were stochastic.

Similarly, we make the simplifying assumption that investors' prior distributions about fundamentals are fixed and commonly known. This ensures that loan fees around announcements are not stochastic. Atmaz et al. (2024) consider a setting in which disagreement across investors follows a stationary (Ornstein-Uhlenbeck) process, and show that short interest is

Table 1: The impact of public information on observables

The table summarizes how the impacts of the public announcement on the change in fees leading up to its release, the change in fees after its release, expected announcement returns, and investor demands depend on the nature of the information it contains. It further summarizes how the precision of the announcement affects pre- and post-announcement loan fees and prices, announcement returns, and changes in investor demand around the announcement. We focus on settings where the loan fees are positive unless specified otherwise.

Public Announcement, Changes in Loan Fees, Expected Returns, and Trade

Observable	**Concordant beliefs**	**Signal agreement**	**Signal disagreement**
Δ Loan fee before ann.	positive	zero	positive
Δ Loan fee after ann.	negative	positive	negative
Ann. return	negative iff ρQ small	positive	negative iff ρQ small
Δ Inv. demand	no change	more dispersed	more dispersed for moderate beliefs; less dispersed for extreme beliefs

Impact of More Precise Public Announcement on Observables

Observable	**Concordant beliefs**	**Signal agreement**	**Signal disagreement**
Pre-ann. loan fee	higher	zero	higher
Pre-ann. price	no effect	higher	higher
Post-ann. loan fee	lower	higher	lower
Post-ann. price	lower iff ρQ small	higher	lower iff ρQ small
Ann. return	lower iff ρQ small	higher	lower iff ρQ small

positively related to short-selling fees and negatively predicts stock returns. Moreover, they show how higher short-selling risk can lead to lower stock returns and less short-selling activity. It would be interesting to consider how short-selling risk varies across public announcements, and how this interaction affects investor trading and pricing.

A Appendix

A summary of the notation is as follows. We denote investor i's conditional beliefs about next period's payoffs by $\mu_{it} = \mathbb{E}_{it}[P_{t+1}]$ and $\sigma_t^2 = \mathbb{V}_{it}[P_{t+1}]$, where we set $P_3 = x$. We let f_t be the loan fee at time t, and $P_{Lt} = P_t - \alpha f_t$ and $P_{St} = P_t - f_t$ denote the net price to long and short investors, where α is the fraction of their holdings that long investors can lend out.

A.1 Proof of Proposition 1

The proof follows three steps: first, we characterize the investors' demands, then we characterize the equilibrium with zero short fee, and finally we characterize the equilibrium with positive short fee.

Step 1: Optimal date-2 demands

Investor i's optimal demand given her belief μ_{i2}, $D_2(\mu_{i2})$, solves

$$\begin{aligned}
D_2(\mu_{i2}) &\equiv \arg\max_{D_{i2}} \mathbb{E}_i\left[-\exp\left(-\rho D_{i2}\left(x - \mathbf{1}(D_{i2} > 0)P_{L2} - \mathbf{1}(D_{i2} < 0)P_{S2}\right)\right)|y\right] \\
&= \arg\max_{D_{i2}} D_{i2}\left(\mathbb{E}_i[x|y] - \mathbf{1}(D_{i2} > 0)P_{L2} - \mathbf{1}(D_{i2} < 0)P_{S2}\right) - \frac{\rho}{2}D_{i2}^2\mathbb{V}_i[x|y] \\
&= \arg\max_{D_{i2}} D_{i2}\left(\mu_{i2} - \mathbf{1}(D_{i2} > 0)P_{L2} - \mathbf{1}(D_{i2} < 0)P_{S2}\right) - \frac{\rho}{2}D_{i2}^2\sigma_2^2 \\
&\equiv \arg\max_{D_{i2}} \Lambda_2(D_{i2}),
\end{aligned}$$

where the second line applies the MGF of a normal distribution and a monotonic transformation. Notice that, for $D_{i2} \neq 0$, $\Lambda_2''(D_{i2}) < 0$. Moreover, at $D_{i2} = 0$, $\Lambda_2'(D_{i2})$ jumps down from $\mu_{i2} - P_{S2}$ to $\mu_{i2} - P_{L2}$. Hence, $\Lambda_2(\cdot)$ is continuous with decreasing one-sided derivatives and is thus globally concave (e.g., Pollard (2002), p. 311). When

$$\mu_{i2} - P_{L2} \leq 0 \leq \mu_{i2} - P_{S2},$$

we have $\lim_{D_{i2}\to 0^+} \Lambda_2'(D_{i2}) \leq 0 \leq \lim_{D_{i2}\to 0^-} \Lambda_2'(D_{i2})$, so that $D_{i2} = 0$ is the investor's optimal demand. Otherwise, the optimal demand solves $\Lambda_2'(D_{i2}) = 0$. This yields

$$D_2(\mu_{i2}) \equiv \begin{cases} \frac{\mu_{i2}-P_{L2}}{\rho\sigma_2^2} & \text{when } \mu_{i2} > P_{L2}; \\ 0 & \text{when } \mu_{i2} \in [P_{S2}, P_{L2}]; \\ \frac{\mu_{i2}-P_{S2}}{\rho\sigma_2^2} & \text{when } \mu_{i2} < P_{S2}. \end{cases} \tag{10}$$

Step 2: Characterize when there exists a date-2 equilibrium with zero fee

Conjecture the existence of an equilibrium in which $f_2 = 0$. In this case, we have that $P_{L2} = P_{S2} = P_2$, and so

$$D_2(\mu_{i2}) = \frac{\mu_{i2} - P_2}{\rho\sigma_2^2},$$

which, together with market clearing, yields

$$\begin{aligned} \int D_{j2}dj = Q \Leftrightarrow \frac{\int \mu_{j2}dj - P_2}{\rho\sigma_2^2} &= Q \\ \Leftrightarrow P_2 = \int \mu_{j2}dj - \rho Q\sigma_2^2 &\equiv \bar{P}_2. \end{aligned}$$

This is an equilibrium when the resulting short demand is no greater than the supply of shortable shares, so that the securities lending market clears. Letting $G_{\mu 2}(t) \equiv \int \mathbf{1}(\mu_{j2} < t)\, dj$ denote the CDF of investors' beliefs on date 2, this holds when

$$\begin{aligned} &-\int_{L_{\mu 2}}^{\bar{P}_2} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) \leq \alpha \int_{\bar{P}_2}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) \\ \Leftrightarrow\, & G_{\mu 2}\left(\bar{P}_2\right) \frac{\bar{P}_2 - \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right]}{\rho\sigma_2^2} \leq \alpha\left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right) \frac{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \bar{P}_2}{\rho\sigma_2^2} \\ \Leftrightarrow\, & \bar{P}_2 \leq \frac{G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right] + \alpha\left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right]}{G_{\mu 2}\left(\bar{P}_2\right) + \alpha\left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)}. \end{aligned} \tag{11}$$

Applying $\int \mu_{j2}dj = G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right] + \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right]$, we obtain

$$\bar{P}_2 = G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right] + \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \rho Q\sigma_2^2, \tag{12}$$

so that inequality (11) reduces to

$$
\begin{aligned}
-\rho Q\sigma_2^2 \leq & \frac{G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}<\bar{P}_2\right]+\alpha\left(1-G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}>\bar{P}_2\right]}{G_{\mu 2}\left(\bar{P}_2\right)+\alpha\left(1-G_{\mu 2}\left(\bar{P}_2\right)\right)} \\
& -G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}<\bar{P}_2\right]-\left(1-G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}>\bar{P}_2\right] \\
\Leftrightarrow \frac{\rho Q\sigma_2^2}{1-\alpha}\frac{G_{\mu 2}\left(\bar{P}_2\right)+\alpha\left(1-G_{\mu 2}\left(\bar{P}_2\right)\right)}{G_{\mu 2}\left(\bar{P}_2\right)\left(1-G_{\mu 2}\left(\bar{P}_2\right)\right)} & \geq \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}>\bar{P}_2\right]-\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2}<\bar{P}_2\right], \qquad (13)
\end{aligned}
$$

which is the condition stated in the proposition.

Step 3: Characterize when there exists a date-2 equilibrium with positive fee

Next, conjecture an equilibrium in which $f_2 > 0$. In this case, we have $P_{L2} > P_{S2}$. Define $H_{\mu 2} \equiv \sup\{\text{support}(\mu_{i2})\} < \infty$ and $L_{\mu 2} \equiv \inf\{\text{support}(\mu_{i2})\} > -\infty$. The market-clearing condition in the stock market is thus

$$\int_{L_{\mu 2}}^{P_{S2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) + \int_{P_{L2}}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) = Q, \qquad (14)$$

and the market-clearing condition in the securities lending market is

$$-\int_{L_{\mu 2}}^{P_{S2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) = \alpha \int_{P_{L2}}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}). \qquad (15)$$

Substituting (15) into (14), we obtain

$$
\begin{aligned}
(1-\alpha)\int_{P_{L2}}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) &= Q \\
\Leftrightarrow \int_{P_{L2}}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) &= \frac{Q}{1-\alpha}. \qquad (16)
\end{aligned}
$$

Substituting this into equation (14) yields

$$\int_{L_{\mu 2}}^{P_{S2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) = -\frac{\alpha Q}{1-\alpha}. \qquad (17)$$

Equations (16) and (17) enable us to directly solve for P_{L2} and P_{S2}, respectively. Note that, for the left-hand sides of (16) and (17) to be positive and negative, respectively, we must have that $P_{L2} < H_{\mu 2}$ and $P_{S2} > L_{\mu 2}$. So, substituting in investors' demand functions, we can write

$$\int_{P_{L2}}^{H_{\mu 2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) = \int_{P_{L2}}^{H_{\mu 2}} \frac{\mu_{j2} - P_{L2}}{\rho\sigma_2^2} dG_{\mu 2}(\mu_{j2})$$
$$= (1 - G_{\mu 2}(P_{L2})) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} > P_{L2}] - P_{L2}}{\rho\sigma_2^2};$$
$$\int_{L_{\mu 2}}^{P_{S2}} D_2(\mu_{j2})\, dG_{\mu 2}(\mu_{j2}) = \int_{L_{\mu 2}}^{P_{S2}} \frac{\mu_{j2} - P_{S2}}{\rho\sigma_2^2} dG_{\mu 2}(\mu_{j2})$$
$$= G_{\mu 2}(P_{S2}) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} < P_{S2}] - P_{S2}}{\rho\sigma_2^2}.$$

Hence, P_{L2} and P_{S2} must solve the equations

$$h_{L2}(P_{L2}) = (1 - G_{\mu 2}(P_{L2})) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} > P_{L2}] - P_{L2}}{\rho\sigma_2^2} - \frac{Q}{1-\alpha} = 0; \tag{18}$$
$$h_{S2}(P_{S2}) = G_{\mu 2}(P_{S2}) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2} | \mu_{i2} < P_{S2}] - P_{S2}}{\rho\sigma_2^2} + \frac{\alpha Q}{1-\alpha} = 0. \tag{19}$$

We now show that equations (18) and (19) have unique solutions $P_{L2} \in \left(L_{\mu 2} - \frac{\rho Q \sigma_2^2}{1-\alpha}, H_{\mu 2}\right)$ and $P_{S2} \in \left(L_{\mu 2}, H_{\mu 2} + \frac{\alpha\rho Q\sigma_2^2}{1-\alpha}\right)$, respectively. Notice that

$$\lim_{t \to L_{\mu 2} - \frac{\rho Q \sigma_2^2}{1-\alpha}} h_{L2}(t) = \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2}] - L_{\mu 2}}{\rho\sigma_2^2} + \frac{Q}{1-\alpha} - \frac{Q}{1-\alpha} = \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2}] - L_{\mu 2}}{\rho\sigma_2^2} > 0;$$
$$\lim_{t \to H_{\mu 2}} h_{L2}(t) = \frac{\lim_{t \to H_{\mu 2}} \int_t^{H_{\mu 2}} (s - t)\, dG_{\mu 2}(s)}{\rho\sigma_2^2} - \frac{Q}{1-\alpha} = -\frac{Q}{1-\alpha} < 0.$$

Moreover,

$$\frac{\partial}{\partial t} h_{L2}(t) \propto \frac{\partial}{\partial t}\left[\int_t^{H_{\mu 2}} s\, dG_{\mu 2}(s) - t\int_t^{H_{\mu 2}} dG_{\mu 2}(s)\right]$$
$$= -tG'_{\mu 2}(t) - 1 + G_{\mu 2}(t) + tG'_{\mu 2}(t)$$
$$= -(1 - G_{\mu 2}(t)) < 0.$$

Analogously, we have

$$\lim_{t \to H_{\mu 2} + \frac{\alpha\rho Q\sigma_2^2}{1-\alpha}} h_{S2}(t) = \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2}] - H_{\mu 2}}{\rho\sigma_2^2} - \frac{\alpha Q}{1-\alpha} + \frac{\alpha Q}{1-\alpha} = \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i2}] - H_{\mu 2}}{\rho\sigma_2^2} < 0;$$
$$\lim_{t \to L_{\mu 2}} h_{S2}(t) = \frac{\lim_{t \to L_{\mu 2}} \int_{L_{\mu 2}}^{t} (s - t)\, dG_{\mu 2}(s)}{\rho\sigma_2^2} + \frac{\alpha Q}{1-\alpha} = \frac{\alpha Q}{1-\alpha} > 0.$$

Moreover, $\frac{\partial}{\partial t}h_{S2}(t) \propto \left[tG'_{\mu 2}(t) - G_{\mu 2}(t) - tG'_{\mu 2}(t)\right] = -G_{\mu 2}(t) < 0$. The intermediate value theorem now yields the desired results.

We next establish when, consistent with our conjecture, $f_2 > 0$ (which is equivalent to $P_{L2} > P_{S2}$). Note that if $P_{L2} = P_{S2}$, then we can follow the same derivations as in the previous section to obtain $P_{L2} = P_{S2} = \bar{P}_2 = \int \mu_{j2} dj - \rho Q \sigma_2^2$. Moreover, note that, because $h'_{L2} < 0$ and $h'_{S2} < 0$,

$$P_{L2} > \bar{P}_2 \Leftrightarrow h_{L2}\left(\bar{P}_2\right) > 0 \quad \text{and} \quad P_{S2} < \bar{P}_2 \Leftrightarrow h_{S2}\left(\bar{P}_2\right) < 0.$$

Applying equation (12), we obtain:

$$\begin{aligned}
&h_{L2}\left(\bar{P}_2\right) \\
&= -\frac{Q}{1-\alpha} + \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right) \\
&\times \frac{\left\{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \left(G_{\mu 2}\left(\bar{P}_2\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right] + \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \rho Q \sigma_2^2\right)\right\}}{\rho \sigma_2^2} \\
&= \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right) G_{\mu 2}\left(\bar{P}_2\right) \frac{\mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right]}{\rho \sigma_2^2} - Q\left(\frac{1}{1-\alpha} - \left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)\right).
\end{aligned}$$

Now, straightforward manipulations reveal that

$$\begin{aligned}
&h_{L2}\left(\bar{P}_2\right) > 0 \\
\Leftrightarrow\ &\frac{\rho Q \sigma_2^2}{1-\alpha} \frac{G_{\mu 2}\left(\bar{P}_2\right) + \alpha\left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)}{G_{\mu 2}\left(\bar{P}_2\right)\left(1 - G_{\mu 2}\left(\bar{P}_2\right)\right)} < \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} > \bar{P}_2\right] - \mathbb{E}^{\mathcal{M}}\left[\mu_{i2}|\mu_{i2} < \bar{P}_2\right]. \quad (20)
\end{aligned}$$

We can similarly show that $h_{S2}\left(\bar{P}_2\right) < 0$ also is equivalent to condition (20). Hence, condition (20) implies that $P_{L2} > \bar{P}_2$ and $P_{S2} < \bar{P}_2$, and therefore that $f_2 > 0$. In contrast, if condition (20) does not hold, then $P_{L2} - P_{S2} \leq 0$, contradicting the conjecture that $f_2 > 0$ and thus ruling out a positive fee equilibrium. Furthermore, recall from Step 2 that there is a zero fee equilibrium if and only if condition (20) does not hold. Therefore, there is always a unique equilibrium and the fee is positive (zero) in this equilibrium when condition (20) holds (does not hold). □

A.2 Proof of Corollary 1

Recall from the proof of Proposition 1 that $P_2 = \frac{P_{L2} - \alpha P_{S2}}{1-\alpha}$ where P_{L2} and P_{S2} solve $h_{L2}(P_{L2}) = 0$ and $h_{S2}(P_{S2}) = 0$, and where $h'_{L2}(t) < 0$ and $h'_{S2}(t) < 0$. Moreover, it is readily seen that

$\frac{\partial h_{L2}}{\partial Q} = \frac{\partial h_{L2}}{\partial \rho} < 0$ and $\frac{\partial h_{S2}}{\partial Q} = \frac{\partial h_{S2}}{\partial \rho} > 0$. Hence, applying the implicit function theorem, we arrive at $\frac{\partial P_{L2}}{\partial Q} < 0$ and $\frac{\partial P_{S2}}{\partial Q} > 0$, so that P_2 declines in Q and ρ. The same reasoning yields that $f_2 = \frac{P_{L2} - P_{S2}}{1-\alpha}$ declines in Q and ρ. □

A.3 Proof of Proposition 2

To establish this result, we first derive key properties of the date-2 equilibrium. Next, we determine investors' date-1 demands, and then we characterize the equilibria for both positive and zero fees. Finally, we re-express the price in a form that facilitates subsequent analysis.

Step 1: Establish key features of the date-2 equilibrium

Recall that, conditional on date-2 information, investor i's beliefs are given by

$$\mu_{i2} = m_i + \beta (y - \delta_i) \quad \text{and} \quad \sigma_2^2 = \sigma_x^2 - \beta^2 \sigma_y^2$$

where $\beta = \frac{\sigma_{xy}}{\sigma_y^2}$. This implies that

$$\xi_{i2} \equiv \mu_{i2} - \int \mu_{j2} dj = m_i - \bar{m} - \beta \left(\delta_i - \bar{\delta} \right)$$

is constant since $\{m_i, \delta_i\}$ and its distribution is fixed and known to investor i. Using this notation, the next lemma shows that we can rewrite the equations that determine the date-2 equilibrium in a manner that eliminates their dependence on date-2 information.

Lemma A.1. *Let $T_{L2} \equiv P_{L2} - \int \mu_{j2} dj$ and $T_{S2} \equiv P_{S2} - \int \mu_{j2} dj$. Then, the equilibrium conditions (18) and (19) are equivalent to*

$$\tilde{h}_{L2} (T_{L2}) = (1 - G_{\xi 2} (T_{L2})) \frac{\mathbb{E}^{\mathcal{M}} [\xi_{i2} | \xi_{i2} > T_{L2}] - T_{L2}}{\rho \sigma_2^2} - \frac{1}{1-\alpha} Q = 0 \qquad \textit{and} \tag{21}$$

$$\tilde{h}_{S2} (T_{S2}) = G_{\xi 2} (T_{S2}) \frac{\mathbb{E}^{\mathcal{M}} [\xi_{i2} | \xi_{i2} < T_{S2}] - T_{S2}}{\rho \sigma_2^2} + \frac{\alpha}{1-\alpha} Q = 0. \tag{22}$$

Moreover, T_{L2} and T_{S2} are non-random.

Proof of Lemma A.1. By adding and subtracting $\int \mu_{j2} dj$, we can rewrite the equilibrium condition (18) as

$$\left(1 - G_{\xi 2} \left(P_{L2} - \int \mu_{j2} dj \right) \right) \frac{\mathbb{E}^{\mathcal{M}} [\xi_{i2} | \xi_{i2} > P_{L2} - \int \mu_{j2} dj] - (P_{L2} - \int \mu_{j2} dj)}{\rho \sigma_2^2} - \frac{1}{1-\alpha} Q = 0,$$

where $G_{\xi 2}(t) \equiv \int \mathbf{1}(\xi_{j2} < t)dj$. Substituting $T_{L2} = P_{L2} - \int \mu_{j2}dj$, we arrive at equation (21). Equation (22) follows similarly. Since the distribution of ξ_{i2} depends only on $\{m_i, \delta_i\}$, T_{L2} and T_{S2} are constants. □

The next lemma builds on the previous one to show that all randomness in the date-2 price stems from $\int \mu_{j2}dj$ and investors' date-2 demands are non-random.

Lemma A.2. *The date-2 stock price can be expressed as*

$$P_2 = \int \mu_{j2}dj - \rho Q\sigma_2^2 + \max\{\eta_2, 0\} \qquad \textit{where}$$
$$\eta_2 = \frac{T_{L2} - \alpha T_{S2}}{1-\alpha} + \rho Q\sigma_2^2,$$

and η_2 is non-random. Moreover, letting $\Pi_2 \equiv -\rho Q\sigma_2^2 + \max\{\eta_2, 0\}$, investors' date-2 demands satisfy

$$D_2(\mu_{i2}) = \begin{cases} \frac{\xi_{i2} - \Pi_2 + \alpha f_2}{\rho\sigma_2^2} & \textit{when } \xi_{i2} > \Pi_2 - \alpha f_2; \\ 0 & \textit{when } \xi_{i2} \in [\Pi_2 - f_2, \Pi_2 - \alpha f_2]; \\ \frac{\xi_{i2} - \Pi_2 + f_2}{\rho\sigma_2^2} & \textit{when } \xi_{i2} < \Pi_2 - f_2, \end{cases}$$

and are non-random.

Proof of Lemma A.2. Given that either (i) $P_{L2} > \bar{P}_2 > P_{S2}$ and $P_2 = \frac{P_{L2} - \alpha P_{S2}}{1-\alpha} > \bar{P}_2$ or (ii) $P_{L2} \leq \bar{P}_2 \leq P_{S2}$ and $P_2 = \bar{P}_2$, we can write

$$P_2 = \bar{P}_2 + \max\{\eta_2, 0\} = \int \mu_{j2}dj + \Pi_2, \tag{23}$$

where we define $\eta_2 \equiv \frac{P_{L2} - \alpha P_{S2}}{1-\alpha} - \bar{P}_2$. Simplifying, we obtain

$$\begin{aligned} \eta_2 &= \frac{(\int \mu_{j2}dj + T_{L2}) - \alpha(\int \mu_{j2}dj + T_{S2})}{1-\alpha} - \left(\int \mu_{j2}dj - \rho Q\sigma_2^2\right) \\ &= \frac{T_{L2} - \alpha T_{S2}}{1-\alpha} + \rho Q\sigma_2^2 \end{aligned} \tag{24}$$

which is a constant because T_{L2} and T_{S2} are constants. In turn, Π_2 is also constant. Substituting equation (23) into equation (10), we obtain the expression for demand in the lemma. □

Step 2: Optimal date-1 demands

Lemma A.2 implies that investor i's date-1 optimization problem reduces to

$$\begin{aligned}
&\arg\max_{D_{i1}} -\mathbb{E}_{i1}\left[\exp\left\{\begin{array}{c} -\rho D_{i1}\left(P_2 - P_{L1}\mathbf{1}\left(D_{i1}>0\right) - P_{S1}\mathbf{1}\left(D_{i1}<0\right)\right) \\ -\rho D_{i2}\left(x - P_2 + \mathbf{1}\left(D_{i2}<0\right) f_2 + \mathbf{1}\left(D_{i2}>0\right)\alpha f_2\right) \end{array}\right\}\right] \\
&= \arg\max_{D_{i1}} - \exp\left\{\begin{array}{c} \rho D_{i1}\left(P_{L1}\mathbf{1}\left(D_{i1}>0\right) + P_{S1}\mathbf{1}\left(D_{i1}<0\right)\right) \\ -\rho D_{i2}\left(\mathbf{1}\left(D_{i2}<0\right) f_2 + \mathbf{1}\left(D_{i2}>0\right)\alpha f_2\right) \end{array}\right\} \\
&\qquad \times \mathbb{E}_{i1}\left[\exp\left\{-\rho\left(D_{i1}-D_{i2}\right)P_2 - \rho D_{i2} x\right\}\right],
\end{aligned}$$

where we have removed constants stemming from investors' date-0 profits. Note that the expectation is given by:

$$\mathbb{E}_{i1}\left[\exp\left\{-\rho\left(D_{i1}-D_{i2}\right)P_2 - \rho D_{i2}x\right\}\right] = \exp\left\{\begin{array}{c} -\rho\left(D_{i1}-D_{i2}\right)\mathbb{E}_{i1}\left[P_2\right] - \rho D_{i2}\mathbb{E}_{i1}\left[x\right] \\ +\frac{1}{2}\rho^2\left(\begin{array}{c} \left(D_{i1}-D_{i2}\right)^2\mathbb{V}_{i1}\left[P_2\right] + D_{i2}^2\mathbb{V}_{i1}\left[x\right] \\ +2\left(D_{i1}-D_{i2}\right)D_{i2}\mathbb{C}_{i1}\left[P_2,x\right] \end{array}\right) \end{array}\right\}.$$

Hence, investor i's certainty equivalent reduces to

$$\begin{aligned}
\Lambda_1\left(D_{i1}\right) \equiv & \left(D_{i1}-D_{i2}\right)\mathbb{E}_{i1}\left[P_2\right] + D_{i2}m_i \\
& - \frac{\rho}{2}\left(\left(D_{i1}-D_{i2}\right)^2\mathbb{V}_{i1}\left[P_2\right] + 2\mathbb{C}_{i1}\left[P_2,x\right]D_{i2}\times\left(D_{i1}-D_{i2}\right) + D_{i2}^2\mathbb{V}_{i1}\left[x\right]\right) \\
& - D_{i1}\left(P_{L1}\mathbf{1}\left(D_{i1}>0\right) + P_{S1}\mathbf{1}\left(D_{i1}<0\right)\right) + D_{i2}\times\left(\mathbf{1}\left(D_{i2}<0\right)f_2 + \mathbf{1}\left(D_{i2}>0\right)\alpha f_2\right).
\end{aligned}$$

As in the derivation of date-2 demands, when there exists a solution $D_{i1} \neq 0$ to $\Lambda_1'\left(D_{i1}\right) = 0$, this will correspond to the optimal demand; otherwise, the optimal solution is $D_{i1} = 0$. For $D_{i1} \neq 0$, $\Lambda_1'\left(D_{i1}\right) = 0$ reduces to

$$\mathbb{E}_{i1}\left[P_2\right] - P_{L1}\mathbf{1}\left(D_{i1}>0\right) - P_{S1}\mathbf{1}\left(D_{i1}<0\right) - \rho\left(D_{i1}-D_{i2}\right)\mathbb{V}_{i1}\left[P_2\right] - \rho D_{i2}\mathbb{C}_{i1}\left[P_2,x\right] = 0,$$

which implies

$$D_{i1} = \frac{\mathbb{E}_{i1}\left[P_2\right] - P_{L1}\mathbf{1}\left(D_{i1}>0\right) - P_{S1}\mathbf{1}\left(D_{i1}<0\right) + \rho D_{i2}\left(\mathbb{V}_{i1}\left[P_2\right] - \mathbb{C}_{i1}\left[P_2,x\right]\right)}{\rho\mathbb{V}_{i1}\left[P_2\right]}.$$

From (23), we have $P_2 = \bar{m} + \beta\left(y - \bar{\delta}\right) + \Pi_2$, and so

$$\mathbb{V}_{i1}\left[P_2\right] = \beta^2\sigma_y^2$$

and

$$\mathbb{C}_{i1}\left[P_2, x\right] = \beta \mathbb{C}_{i1}\left[y, x\right] = \beta^2 \sigma_y^2,$$

which implies:

$$\Lambda_1'\left(D_{i1}\right) = 0 \Leftrightarrow D_{i1} = \frac{\mathbb{E}_{i1}\left[P_2\right] - P_{L1}\mathbf{1}\left(D_{i1} > 0\right) - P_{S1}\mathbf{1}\left(D_{i1} < 0\right)}{\rho \mathbb{V}_{i1}\left[P_2\right]}.$$

Therefore, investor i's optimal demand given $\mu_{i1} \equiv \mathbb{E}_{i1}\left[P_2\right]$, $D_1(\mu_{i1})$, satisfies

$$D_1(\mu_{i1}) \equiv \begin{cases} \frac{\mu_{i1} - P_{L1}}{\rho \sigma_1^2} & \text{when } \mu_{i1} > P_{L1}; \\ 0 & \text{when } \mu_{i1} \in \left[P_{L1}, P_{S1}\right]; \\ \frac{\mu_{i1} - P_{S1}}{\rho \sigma_1^2} & \text{when } \mu_{i1} < P_{S1}, \end{cases} \tag{25}$$

where $\sigma_1^2 \equiv \mathbb{V}_{i1}\left[P_2\right] = \beta^2 \sigma_y^2$.

Step 3: Characterize when there exists a date-1 equilibrium with zero fee

Note that

$$\mu_{i1} = \mathbb{E}_{i1}\left[\bar{m} + \beta\left(y - \bar{\delta}\right) + \Pi_2\right] = \bar{m} + \beta\left(\delta_i - \bar{\delta}\right) + \max\left\{\eta_2, 0\right\}.$$

When $f_1 = 0$, combining the above equation with equation (25) and the market-clearing condition yields

$$\begin{aligned} P_1 = \bar{P}_1 \equiv & \int \mu_{j1} dj - \rho Q \sigma_1^2 \\ & = \bar{m} - \rho Q(\sigma_1^2 + \sigma_2^2) + \max\left\{\eta_2, 0\right\} \\ & = \bar{m} - \rho Q \sigma_x^2 + \max\left\{\eta_2, 0\right\}, \end{aligned} \tag{26}$$

where the final line applies $\sigma_1^2 + \sigma_2^2 = \mathbb{V}\left[\beta y\right] + \mathbb{V}\left[x|y\right] = \mathbb{V}\left[\mathbb{E}\left[x|y\right]\right] + \mathbb{E}\left[\mathbb{V}\left[x|y\right]\right] = \sigma_x^2$. Following similar steps to the proof of Proposition 1, one can verify that the condition under which the lending constraint does not bind in this equilibrium is the same as the condition for an equilibrium with strictly positive fee to exist. We derive this condition in the next step.

Step 4: Characterize when there exists a date-1 equilibrium with positive fee

When $f_1 > 0$, using arguments as in the proof of Proposition 1, $\{P_{L1}, P_{S1}\}$ solves the following equations:

$$h_{L1}(P_{L1}) \equiv (1 - G_{\mu 1}(P_{L1})) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i1}|\mu_{i1} > P_{L1}] - P_{L1}}{\rho\sigma_1^2} - \frac{Q}{1-\alpha} = 0; \tag{27}$$

$$h_{S1}(P_{S1}) \equiv G_{\mu 1}(P_{S1}) \frac{\mathbb{E}^{\mathcal{M}}[\mu_{i1}|\mu_{i1} < P_{S1}] - P_{S1}}{\rho\sigma_1^2} + \frac{\alpha Q}{1-\alpha} = 0. \tag{28}$$

Following similar arguments to that in the proof of Proposition 1, we can apply the intermediate value theorem to verify that these two equations have unique solutions

$$P_{L1} \in \left(\inf\{\text{support}(\mu_{i1})\} - \sigma_1^2 \frac{\rho Q}{1-\alpha}, \sup\{\text{support}(\mu_{i1})\}\right) \text{ and}$$
$$P_{S1} \in \left(\inf\{\text{support}(\mu_{i1})\}, \sup\{\text{support}(\mu_{i1})\} + \sigma_1^2 \frac{\alpha\rho Q}{1-\alpha}\right).$$

Moreover, similar logic to the proof of Proposition 1 also yields that the lending constraint will bind if and only if $h_{L1}\left(\bar{P}_1\right) > 0$ and $h_{S1}\left(\bar{P}_1\right) < 0$, as, given $h'_{L1}, h'_{S1} < 0$, this implies $P_{L1} > \bar{P}_1 > P_{S1}$. These two inequalities reduce to

$$\frac{\rho Q\sigma_1^2}{1-\alpha} \frac{G_{\mu 1}\left(\bar{P}_1\right) + \alpha\left(1 - G_{\mu 1}\left(\bar{P}_1\right)\right)}{G_{\mu 1}\left(\bar{P}_1\right)\left(1 - G_{\mu 1}\left(\bar{P}_1\right)\right)} < \mathbb{E}^{\mathcal{M}}\left[\mu_{i1}|\mu_{i1} > \bar{P}_1\right] - \mathbb{E}^{\mathcal{M}}\left[\mu_{i1}|\mu_{i1} < \bar{P}_1\right].$$

Step 5: Re-express date-1 price for future proofs

For subsequent results, it is helpful to show that we can express the date-1 price in a similar fashion how we expressed the date-2 price in Lemma A.1. In doing so, we also verify equation (8) in the text. Let $\xi_{i1} \equiv \mu_{i1} - \int \mu_{j1} dj = \beta\left(\delta_i - \bar{\delta}\right)$.

Lemma A.3. *The date-1 price satisfies*

$$P_1 = \bar{m} - \rho Q\sigma_x^2 + \max\{\eta_1, 0\} + \max\{\eta_2, 0\} \qquad \textit{where}$$
$$\eta_1 \equiv \frac{T_{L1} - \alpha T_{S1}}{1-\alpha} + \rho Q\sigma_1^2,$$

and where $T_{L1} \equiv P_{L1} - \int \mu_{j1} dj$ *and* $T_{S1} \equiv P_{S1} - \int \mu_{j1} dj$ *are the solutions to*

$$\tilde{h}_{L1}(T_{L1}) \equiv (1 - G_{\xi 1}(T_{L1})) \frac{\mathbb{E}^{\mathcal{M}}[\xi_{i1}|\xi_{i1} > T_{L1}] - T_{L1}}{\rho\sigma_1^2} - \frac{1}{1-\alpha} Q = 0 \tag{29}$$

and

$$\tilde{h}_{S1}(T_{S1}) \equiv G_{\xi 1}(T_{S1}) \frac{\mathbb{E}^{\mathcal{M}}[\xi_{i1}|\xi_{i1} < T_{S1}] - T_{S1}}{\rho\sigma_1^2} + \frac{\alpha}{1-\alpha} Q = 0. \tag{30}$$

Proof of Lemma A.3. Observe that

$$\begin{aligned} G_{\mu 1}(t) &= \Pr\left(\bar{m} + \beta\left(\delta_i - \bar{\delta}\right) + \Pi_2 < t\right) \\ &= \Pr\left(\beta\left(\delta_i - \bar{\delta}\right) < t - \Pi_2 - \bar{m}\right) \\ &= G_{\xi 1}(t - \Pi_2 - \bar{m}) \end{aligned}$$

and

$$\mathbb{E}^{\mathcal{M}}[\mu_{i1}|\mu_{i1} > t] = \bar{m} + \Pi_2 + \mathbb{E}^{\mathcal{M}}[\xi_{i1}|\xi_{i1} > t - \Pi_2 - \bar{m}].$$

Substituting the above equalities into the equilibrium conditions (31) and (32), we obtain that

$$(1 - G_{\xi 1}(P_{L1} - \Pi_2 - \bar{m})) \frac{\mathbb{E}^{\mathcal{M}}[\xi_{i1}|\xi_{i1} > P_{L1} - \Pi_2 - \bar{m}] - (P_{L1} - \Pi_2 - \bar{m})}{\rho\sigma_1^2} - \frac{Q}{1-\alpha} = 0; \tag{31}$$

$$G_{\mu 1}(P_{S1} - \Pi_2 - \bar{m}) \frac{\mathbb{E}^{\mathcal{M}}[\xi_{i1}|\xi_{i1} < P_{S1} - \Pi_2 - \bar{m}] - (P_{S1} - \Pi_2 - \bar{m})}{\rho\sigma_1^2} + \frac{\alpha Q}{1-\alpha} = 0. \tag{32}$$

Substituting $T_{L1} = P_{L1} - \bar{m} - \Pi_2$ and $T_{S1} = P_{S1} - \bar{m} - \Pi_2$, we obtain equations (29) and (30). Now, note that, when $P_{L1} > \bar{P}_1 > P_{S1}$, we have

$$P_1 = \frac{P_{L1} - \alpha P_{S1}}{1-\alpha} = \bar{m} + \Pi_2 + \frac{T_{L1} - \alpha T_{S1}}{1-\alpha} > \bar{P}_1 = \bar{m} + \Pi_2 - \rho Q \sigma_1^2.$$

In contrast, when $P_{L1} \le \bar{P}_1 \le P_{S1}$, we have $P_1 = \bar{P}_1$. So, we can express the date-1 price more compactly as

$$P_1 = \bar{P}_1 + \max\{\eta_1, 0\} = \bar{m} + \Pi_1 + \Pi_2$$

where we define

$$\begin{aligned} \eta_1 &\equiv \frac{T_{L1} - \alpha T_{S1}}{1-\alpha} + \rho Q \sigma_1^2 \qquad \text{and} \\ \Pi_1 &\equiv -\rho Q \sigma_1^2 + \max\{\eta_1, 0\}. \end{aligned}$$

Substituting $\Pi_2 = -\rho Q \sigma_2^2 + \max\{\eta_1, 0\}$ and applying $\sigma_1^2 + \sigma_2^2 = \sigma_x^2$, which follows from the law of total variance, P_1 can alternatively be written as

$$P_1 = \bar{m} - \rho Q \sigma_x^2 + \max\{\eta_1, 0\} + \max\{\eta_2, 0\},$$

as in equation (8). □

□

A.4 Proof of Proposition 3

At date 0, investor i's expected utility is:

$$\begin{aligned}
&-\mathbb{E}_{i0}\left[\exp\left\{\begin{array}{c} -\rho D_{i0}\left(P_1 - P_0 + \mathbf{1}\left(D_{i0} < 0\right) f_0 + \mathbf{1}\left(D_{i0} > 0\right)\alpha f_0\right) \\ -\rho D_{i1}\left(P_2 - P_1 + \mathbf{1}\left(D_{i1} < 0\right) f_1 + \mathbf{1}\left(D_{i1} > 0\right)\alpha f_1\right) \\ -\rho D_{i2}\left(x - P_2 + \mathbf{1}\left(D_{i2} < 0\right) f_2 + \mathbf{1}\left(D_{i2} > 0\right)\alpha f_2\right) \end{array}\right\}\right] \\
&= -\exp\left\{\begin{array}{c} -\rho D_{i0}\left(P_1 - P_0 + \mathbf{1}\left(D_{i0} < 0\right) f_0 + \mathbf{1}\left(D_{i0} > 0\right)\alpha f_0\right) \\ -\rho D_{i1}\left(-P_1 + \mathbf{1}\left(D_{i1} < 0\right) f_1 + \mathbf{1}\left(D_{i1} > 0\right)\alpha f_1\right) \\ -\rho D_{i2}\left(\mathbf{1}\left(D_{i2} < 0\right) f_2 + \mathbf{1}\left(D_{i2} > 0\right)\alpha f_2\right) \end{array}\right\} \\
&\quad \times \mathbb{E}_{i0}\left[\exp\left\{-\rho \begin{pmatrix} D_{i1} - D_{i2} \\ D_{i2} \end{pmatrix}' \begin{pmatrix} P_2 \\ x \end{pmatrix}\right\}\right] \\
&\propto -\exp\left\{ -\rho D_{i0}\left(P_1 - P_0 + \mathbf{1}\left(D_{i0} < 0\right) f_0 + \mathbf{1}\left(D_{i0} > 0\right)\alpha f_0\right) \right\}.
\end{aligned}$$

This is a monotonic transformation of a piecewise linear function of D_{i0}, where this piecewise linear function has derivative:

$$P_1 - P_0 + \mathbf{1}\left(D_{i0} < 0\right) f_0 + \mathbf{1}\left(D_{i0} > 0\right)\alpha f_0.$$

So, at an optimum, we must have that one of the following holds:

(i) $D_{i0} < 0$ and $P_1 - P_0 + f_0 = 0$;

(ii) $D_{i0} = 0$ and $P_1 - P_0 + f_0 \geq 0$, $P_1 - P_0 + \alpha f_0 \leq 0$;

(iii) $D_{i0} > 0$ and $P_1 - P_0 + \alpha f_0 = 0$.

Note further that these conditions are identical across investors. For the market to clear, we need that, for at least some investors, $D_{i0} > 0$. Thus, we must have:

$$P_1 = P_0 - \alpha f_0. \tag{33}$$

Now, suppose $f_0 > 0$. In this case, equation (33) implies that $P_1 - P_0 + f_0 = (1-\alpha) f_0 > 0$, and so there are no investors for whom $D_{i0} < 0$. This implies that $f_0 = 0$, a contradiction. So, we must have $f_0 = 0$, in which case $P_1 = P_0$, and investors' demands are arbitrary, subject to market clearing $\int D_{j0} dj = Q$. □

A.5 Intermediate Results for Section 5 Proofs

The following lemmas are useful for the subsequent proofs.

Lemma A.4. *When investors have concordant beliefs, a public signal y is informationally equivalent to a statistic*

$$y^* = x + \varepsilon, \tag{34}$$

where $\varepsilon \perp x$ and $\varepsilon \sim_i N(0, \sigma_\varepsilon^2)$.

Proof. Note that $y|x \sim_i N\left(\delta_i + \frac{\sigma_{xy}}{\sigma_x^2}(x - m_i), \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2}\right)$. Concordant beliefs require that, $\forall i_1, i_2 \in [0,1]$, $\mathbb{E}_{i_1}[y|x] = \mathbb{E}_{i_2}[y|x]$, and thus $\mathbb{E}_{i_1}[y|x] = \int_j \mathbb{E}_j[y|x] dj$. This reduces to $\delta_i = \bar{\delta} + \frac{\sigma_{xy}}{\sigma_x^2}(m_i - \bar{m})$. Now, a linear projection under investor i's subjective beliefs yields

$$\begin{aligned} y &=_i \delta_i + \frac{\sigma_{xy}}{\sigma_y^2}(x - m_i) + e \\ &= \bar{\delta} - \frac{\sigma_{xy}}{\sigma_x^2}\bar{m} + \frac{\sigma_{xy}}{\sigma_x^2} x + e, \end{aligned}$$

for $e \perp x; e \sim_i N(0, \sigma_e^2)$. Now, under each investor's subjective measure, y is informationally equivalent to $y^* \equiv \frac{\sigma_x^2}{\sigma_{xy}}(y - \bar{\delta} + \frac{\sigma_{xy}}{\sigma_x^2}\bar{m}) = x + \frac{\sigma_x^2}{\sigma_{xy}} e$, where $\mathbb{E}_i[y^*] = m_i$, which takes the desired form. □

Lemma A.5. *When investors have concordant beliefs, $(1-\beta)T_{L1} = \beta T_{L2}$ and $(1-\beta)T_{S1} = \beta T_{S2}$.*

Proof of Lemma A.5. Recall that, for $\tau \in \{1, 2\}$, $T_{L\tau}$ and $T_{S\tau}$ solve $\tilde{h}_{L\tau}(T_{L\tau}) = 0$ and $\tilde{h}_{S\tau}(T_{S\tau}) = 0$, respectively, where these functions were defined in (21), (22), (29) and (30). Note that that $\xi_{i1} = \mu_{i1} - \int \mu_{j1} dj = \beta(m_i - \bar{m}) \equiv \beta m_i^\Delta$. Hence, $\tilde{h}_{L1}(T_{L1}) = 0$ and $\tilde{h}_{S1}(T_{S1}) = 0$ can be re-expressed as

$$\hat{h}_{L1}(T_{L1}) \equiv \left(1 - G_{m^\Delta}\left(\frac{T_{L1}}{\beta}\right)\right)\left\{\mathbb{E}^{\mathcal{M}}\left[m^\Delta | m^\Delta > \frac{T_{L1}}{\beta}\right] - \frac{T_{L1}}{\beta}\right\} - \frac{\sigma_1^2}{\beta}\frac{\rho Q}{1-\alpha} = 0; \tag{35}$$

$$\hat{h}_{S1}(T_{S1}) \equiv G_{m^\Delta}\left(\frac{T_{S1}}{\beta}\right)\left\{\mathbb{E}^{\mathcal{M}}\left[m^\Delta | m^\Delta < \frac{T_{S1}}{\beta}\right] - \frac{T_{S1}}{\beta}\right\} + \frac{\sigma_1^2}{\beta}\frac{\alpha\rho Q}{1-\alpha} = 0. \tag{36}$$

Similarly, note that $\xi_{i2} = \mu_{i2} - \int \mu_{j2} dj = (1-\beta) m_i^{\Delta}$, which lets us express $\tilde{h}_{L2}(T_{L2}) = 0$ and $\tilde{h}_{S2}(T_{S2}) = 0$ as

$$\hat{h}_{L2}\left(T_{L2}\right) \equiv \left(1 - G_{m^{\Delta}}\left(\frac{T_{L2}}{1-\beta}\right)\right)\left\{\mathbb{E}^{\mathcal{M}}\left[m_i^{\Delta}\middle| m_i^{\Delta} > \frac{T_{L2}}{1-\beta}\right] - \frac{T_{L2}}{1-\beta}\right\} - \frac{\sigma_2^2}{1-\beta}\frac{\rho Q}{1-\alpha} = 0; \tag{37}$$

$$\hat{h}_{S2}\left(T_{S2}\right) \equiv G_{m^{\Delta}}\left(\frac{T_{S2}}{1-\beta}\right)\left\{\mathbb{E}^{\mathcal{M}}\left[m_i^{\Delta}\middle| m_i^{\Delta} < \frac{T_{S2}}{1-\beta}\right] - \frac{T_{S2}}{1-\beta}\right\} + \frac{\sigma_2^2}{1-\beta}\frac{\alpha\rho Q}{1-\alpha} = 0. \tag{38}$$

Notice that

$$\frac{\sigma_2^2}{1-\beta} = \left(\frac{\sigma_\varepsilon^2}{\sigma_x^2+\sigma_\varepsilon^2}\right)^{-1}\frac{\sigma_x^2\sigma_\varepsilon^2}{\sigma_x^2+\sigma_\varepsilon^2} = \sigma_x^2 = \left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_\varepsilon^2}\right)^{-1}\frac{\sigma_x^4}{\sigma_x^2+\sigma_\varepsilon^2} = \frac{\sigma_1^2}{\beta}.$$

Substituting this into expressions (35) and (36), we have that T_{L1} and T_{S1} solve $\hat{h}_{L2}\left(\frac{1-\beta}{\beta}T_{L1}\right) = 0$ and $\hat{h}_{S2}\left(\frac{1-\beta}{\beta}T_{S1}\right) = 0$. Hence, $(1-\beta)T_{L1} = \beta T_{L2}$ and $(1-\beta)T_{S1} = \beta T_{S2}$. □

A.6 Proof of Proposition 4

Part (i) Consider first investors who take long positions in date 2. We have that

$$\begin{aligned}
D_2\left(\mu_{i2}\right) &= \frac{\mu_{i2} - P_2 + \alpha f_2}{\rho\sigma_2^2} \\
&= \frac{(1-\beta)\,m_i + \beta y - ((1-\beta)\bar{m} + \beta y - \rho Q\sigma_2^2 + \max\{\eta_2, 0\}) + \alpha f_2}{\rho\sigma_2^2} \\
&= \frac{(1-\beta)\left(m_i - \bar{m}\right)}{\rho\sigma_2^2} + Q + \frac{\alpha f_2 - \max\left\{\eta_2, 0\right\}}{\rho\sigma_2^2}.
\end{aligned}$$

Next,

$$\begin{aligned}
\mu_{i1} &= \mathbb{E}_i\left[(1-\beta)\,\bar{m} + \beta y - \rho Q\sigma_2^2 + \max\left\{\eta_2, 0\right\}\right] \\
&= (1-\beta)\,\bar{m} + \beta m_i - \rho Q\sigma_2^2 + \max\left\{\eta_2, 0\right\}.
\end{aligned}$$

So,

$$P_1 = (1-\beta)\,\bar{m} + \beta\bar{m} - \rho Q\sigma_1^2 - \rho Q\sigma_2^2 + \max\left\{\eta_1, 0\right\} + \max\left\{\eta_2, 0\right\}.$$

Note the investor's date-1 demand will equal the following, if it is positive:

$$
\begin{aligned}
\frac{\mu_{i1} - P_1 + \alpha f_1}{\rho \sigma_1^2} &= \frac{\beta (m_i - \bar{m}) + \rho Q \sigma_1^2 + \alpha f_1 - \max\{\eta_1, 0\}}{\rho \sigma_1^2} \\
&= \frac{(1-\beta)(m_i - \bar{m})}{\rho \sigma_2^2} + Q + \frac{1-\beta}{\beta} \frac{\alpha \frac{T_{L1} - T_{S1}}{1-\alpha} - \max\left\{\frac{T_{L1} - \alpha T_{S1}}{1-\alpha}, 0\right\}}{\rho \sigma_2^2} \\
&= \frac{(1-\beta)(m_i - \bar{m})}{\rho \sigma_2^2} + Q + \frac{1-\beta}{\beta} \frac{\alpha \frac{\beta}{1-\beta} \frac{T_{L2} - T_{S2}}{1-\alpha} - \frac{\beta}{1-\beta} \max\left\{\frac{T_{L2} - \alpha T_{S2}}{1-\alpha}, 0\right\}}{\rho \sigma_2^2} \\
&= \frac{(1-\beta)(m_i - \bar{m})}{\rho \sigma_2^2} + Q + \frac{\alpha f_2 - \max\{\eta_2, 0\}}{\rho \sigma_2^2} = D_2(\mu_{i2}) > 0,
\end{aligned}
$$

where the second line multiplies by

$$
\frac{\sigma_1^2}{\sigma_2^2} \frac{1-\beta}{\beta} = \frac{\frac{\sigma_x^4}{\sigma_x^2 + \sigma_\varepsilon^2} \frac{\sigma_\varepsilon^2}{\sigma_x^2 + \sigma_\varepsilon^2}}{\frac{\sigma_x^2 \sigma_\varepsilon^2}{\sigma_x^2 + \sigma_\varepsilon^2} \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2}} = 1,
$$

and the third line applies Lemma A.5. Hence, we have that $D_1(\mu_{i1}) = D_2(\mu_{i2})$. Following an analogous series of steps, we can show that investors who take short positions or positions of zero in date 2 have $D_1(\mu_{i1}) = D_2(\mu_{i2})$.

Part (ii) Applying Lemma A.5 and equation (A.6),

$$
\begin{aligned}
f_2 = \max\left\{\frac{T_{L2} - T_{S2}}{1-\alpha}, 0\right\} &= \frac{1-\beta}{\beta} \max\left\{\frac{T_{L1} - T_{S1}}{1-\alpha}, 0\right\} \\
&= \frac{1-\beta}{\beta} f_1 = \frac{\sigma_2^2}{\sigma_1^2} f_1.
\end{aligned}
$$

This immediately verifies that $f_1 > 0 \Leftrightarrow f_2 > 0$.

A.7 Proof of Corollary 2

Totally differentiating equation (37),

$$
\begin{aligned}
&\frac{\partial \hat{h}_{L2}}{\partial \left((1-\beta)^{-1} T_{L2}\right)} \frac{\partial \left(\frac{T_{L2}}{1-\beta}\right)}{\partial \sigma_\varepsilon} + \frac{\partial \hat{h}_{L2}}{\partial \sigma_\varepsilon} = 0 \\
\Leftrightarrow\ &\frac{\partial \hat{h}_{L2}}{\partial \left((1-\beta)^{-1} T_{L2}\right)} \frac{(1-\beta) \frac{\partial T_{L2}}{\partial \sigma_\varepsilon} - T_{L2} \frac{\partial (1-\beta)}{\partial \sigma_\varepsilon}}{(1-\beta)^2} - \frac{\rho Q}{1-\alpha} \frac{\partial}{\partial \sigma_\varepsilon} \left(\frac{\sigma_2^2}{1-\beta}\right) = 0.
\end{aligned}
$$

Similarly, totally differentiating equation (38) yields

$$\frac{\partial \hat{h}_{S2}}{\partial\left(\left(1-\beta\right)^{-1}T_{S2}\right)}\frac{\left(1-\beta\right)\frac{\partial T_{S2}}{\partial\sigma_{\varepsilon}}-T_{S2}\frac{\partial(1-\beta)}{\partial\sigma_{\varepsilon}}}{\left(1-\beta\right)^{2}}+\frac{\alpha\rho Q}{1-\alpha}\frac{\partial}{\partial\sigma_{\varepsilon}}\left(\frac{\sigma_{2}^{2}}{1-\beta}\right)=0.$$

Note in this case that $\frac{\sigma_2^2}{1-\beta}=\sigma_x^2$. So, we obtain

$$\frac{\partial \hat{h}_{L2}}{\partial\left(\left(1-\beta\right)^{-1}T_{L2}\right)}\frac{\left(1-\beta\right)\frac{\partial T_{L2}}{\partial\sigma_{\varepsilon}}-T_{L2}\frac{\partial(1-\beta)}{\partial\sigma_{\varepsilon}}}{\left(1-\beta\right)^{2}}=0$$

$$\frac{\partial \hat{h}_{S2}}{\partial\left(\left(1-\beta\right)^{-1}T_{S2}\right)}\frac{\left(1-\beta\right)\frac{\partial T_{S2}}{\partial\sigma_{\varepsilon}}-T_{S2}\frac{\partial(1-\beta)}{\partial\sigma_{\varepsilon}}}{\left(1-\beta\right)^{2}}=0.$$

It is readily verified that $\frac{\partial \hat{h}_{L2}}{\partial\left(\left(1-\beta\right)^{-1}T_{L2}\right)}<0$ and $\frac{\partial \hat{h}_{S2}}{\partial\left(\left(1-\beta\right)^{-1}T_{S2}\right)}<0$. Hence, the above two equations require that

$$\frac{\partial T_{L2}}{\partial\sigma_{\varepsilon}}=\frac{T_{L2}}{1-\beta}\frac{\partial\left(1-\beta\right)}{\partial\sigma_{\varepsilon}} \tag{39}$$

$$\frac{\partial T_{S2}}{\partial\sigma_{\varepsilon}}=\frac{T_{S2}}{1-\beta}\frac{\partial\left(1-\beta\right)}{\partial\sigma_{\varepsilon}}. \tag{40}$$

Part (i) Subtracting (40) from (39), we obtain

$$\frac{\partial\left(T_{L2}-T_{S2}\right)}{\partial\sigma_{\varepsilon}}=\frac{T_{L2}-T_{S2}}{1-\beta}\frac{\partial\left(1-\beta\right)}{\partial\sigma_{\varepsilon}}.$$

Since $f_2>0\Leftrightarrow T_{L2}>T_{S2}$ and $\frac{\partial(1-\beta)}{\partial\sigma_{\varepsilon}}>0$, this is positive.

Part (ii) Subtracting α times (40) from (39), we obtain

$$\frac{\partial\left(T_{L2}-\alpha T_{S2}\right)}{\partial\sigma_{\varepsilon}}=\frac{T_{L2}-\alpha T_{S2}}{1-\beta}\frac{\partial\left(1-\beta\right)}{\partial\sigma_{\varepsilon}}.$$

Since $\frac{\partial(1-\beta)}{\partial\sigma_{\varepsilon}}>0$,

$$\operatorname{sgn}\left(\frac{\partial\left(T_{L2}-\alpha T_{S2}\right)}{\partial\sigma_{\varepsilon}}\right)=\operatorname{sgn}\left(T_{L2}-\alpha T_{S2}\right)=\operatorname{sgn}\left(\int\mu_{j2}dj-P_{2}\right). \tag{41}$$

Now, let $\bar{Q}$ denote the cutoff level at which the lending constraint no longer binds for $Q>\bar{Q}$, and note from equation (24) that:

$$\lim_{Q\to\bar{Q}^-}(T_{L2}-\alpha T_{S2}) = \lim_{Q\to\bar{Q}^-}(1-\alpha)\eta_2 - (1-\alpha)\rho Q\sigma_2^2 = -(1-\alpha)\rho Q\sigma_2^2 < 0.$$

Moreover, it is readily verified that $\frac{\partial \tilde{h}_{L2}(t)}{\partial t} < 0$, $\frac{\partial \tilde{h}_{S2}(t)}{\partial t} < 0$, $\frac{\partial \tilde{h}_{L2}(t)}{\partial Q} < 0$, and $\frac{\partial \tilde{h}_{S2}(t)}{\partial Q} > 0$. Hence, by the implicit function theorem, $\frac{\partial T_{L2}(t)}{\partial Q} < 0$ and $\frac{\partial T_{S2}(t)}{\partial Q} > 0$, such that $\frac{\partial (T_{L2}-\alpha T_{S2})}{\partial Q} < 0$. This implies that $T_{L2}-\alpha T_{S2}$ is positive if and only if Q is sufficiently small, which, together with equation (41), yields the desired result. □

A.8 Proof of Proposition 5

Given Lemma A.4, we may assume without loss of generality that y takes the form in (34).

Part (i) Observe from equations (35) and (36) that, since $\frac{\sigma_1^2}{\beta} = \sigma_x^2$ does not depend on σ_ε^2, $\frac{T_{L1}}{\beta}$ and $\frac{T_{S1}}{\beta}$ do not depend upon σ_ε^2. Therefore,

$$\begin{aligned}\frac{\partial f_1}{\partial \sigma_\varepsilon} &= \frac{\partial}{\partial \sigma_\varepsilon}\left[\beta \max\left\{0, \frac{T_{L1}-T_{S1}}{\beta(1-\alpha)}\right\}\right] \\ &= \frac{\partial \beta}{\partial \sigma_\varepsilon} \times \max\left\{0, \frac{f_1}{\beta}\right\},\end{aligned}$$

which is strictly negative when $f_1 > 0$.

Part (ii) Applying Lemma A.5, we have $(1-\beta)T_{L1} = \beta T_{L2}$ and $(1-\beta)T_{S1} = \beta T_{S2}$, and thus we can write

$$\begin{aligned}P_1 &= \bar{m} - \rho Q\sigma_x^2 + \beta\max\left\{0, \frac{1}{\beta}\frac{T_{L1}-\alpha T_{S1}}{1-\alpha}\right\} + (1-\beta)\max\left\{0, \frac{1}{1-\beta}\frac{T_{L2}-\alpha T_{S2}}{1-\alpha}\right\} \\ &= \bar{m} - \rho Q\sigma_x^2 + \max\left\{0, \frac{1}{\beta}\frac{T_{L1}-\alpha T_{S1}}{1-\alpha}\right\}.\end{aligned}$$

Now, again, $\frac{T_{L1}}{\beta}$ and $\frac{T_{S1}}{\beta}$ do not depend upon σ_ε^2, and thus P_1 does not depend on σ_ε^2. □

A.9 Equilibrium Under Uniform Expectations

In this section, for future use, we derive the firm's equilibrium price when investors' expectations $\mu_{it} \sim Uniform([\mu_{Lt}, \mu_{Ht}])$ are uniformly distributed. In this case, the equations

$h_{Lt}(P_{Lt}) = 0$ and $h_{St}(P_{St}) = 0$ for $t \in \{1, 2\}$ reduce to

$$h_{Lt}(P_{Lt}) = \max\left\{\frac{\mu_{Ht} - P_{Lt}}{\mu_{Ht} - \mu_{Lt}}, 0\right\}\left(\frac{\mu_{Ht} + P_{Lt}}{2} - P_{Lt}\right) - \sigma_t^2 \frac{\rho Q}{1-\alpha} = 0;$$
$$h_{St}(P_{St}) = \max\left\{\frac{P_{St} - \mu_{Lt}}{\mu_{Ht} - \mu_{Lt}}, 0\right\}\left(\frac{\mu_{Lt} + P_{St}}{2} - P_{St}\right) + \sigma_t^2 \frac{\alpha\rho Q}{1-\alpha} = 0.$$

It is clear that there are no solutions to these equations with $P_{Lt} \geq \mu_{Ht}$ or $P_{St} \leq \mu_{Lt}$. When $P_{Lt} < \mu_{Ht}$ and $P_{St} > \mu_{Lt}$, these equations are quadratic and may be readily solved for P_{Lt} and P_{St}. While each of the equations has two solutions, there is a unique set of solutions $\{P_{St}, P_{Lt}\}$ that in fact satisfy $P_{LT} < \mu_{Ht}$ and $P_{St} > \mu_{Lt}$, which are

$$P_{Lt} = \mu_{Ht} - \sqrt{\frac{2\rho Q\sigma_t^2\left(\mu_{Ht} - \mu_{Lt}\right)}{1-\alpha}} \quad \text{and}$$
$$P_{St} = \mu_{Lt} + \sqrt{\frac{2\alpha\rho Q\sigma_t^2\left(\mu_{Ht} - \mu_{Lt}\right)}{1-\alpha}}.$$

Hence, we have, when $f_t > 0$,

$$f_t = \frac{P_{Lt} - P_{St}}{1-\alpha} = \frac{1}{1-\alpha}\left(\mu_{Ht} - \mu_{Lt} - \left(1 + \sqrt{\alpha}\right)\sqrt{\frac{2\rho Q\sigma_t^2\left(\mu_{Ht} - \mu_{Lt}\right)}{1-\alpha}}\right); \tag{42}$$
$$P_t = \frac{P_{Lt} - \alpha P_{St}}{1-\alpha} = \frac{\mu_{Ht} - \alpha\mu_{Lt}}{1-\alpha} - \frac{1 + \alpha^{\frac{3}{2}}}{1-\alpha}\sqrt{\frac{2\rho Q\sigma_t^2\left(\mu_{Ht} - \mu_{Lt}\right)}{1-\alpha}}. \tag{43}$$

Moreover, the condition for the fee to be strictly positive reduces to

$$\sqrt{\mu_{Ht} - \mu_{Lt}} > \left(1 + \sqrt{\alpha}\right)\sqrt{\frac{2\rho Q\sigma_t^2}{1-\alpha}}.$$

□

A.10 Proof of Corollary 3

Recall that $f_1 = \max\left\{\frac{T_{L1} - T_{S1}}{1-\alpha}, 0\right\}$, where T_{L1} and T_{S1} satisfy equations (29) and (30). Moreover,

$$\mathbb{E}\left[P_2 - P_1\right] = \rho Q\left(\sigma_x^2 - \sigma_2^2\right) - \max\left\{\eta_1, 0\right\}.$$
$$= \rho Q\left(\sigma_x^2 - \sigma_2^2\right) - \max\left\{\frac{T_{L1} - \alpha T_{S1}}{1-\alpha} + \rho Q\sigma_1^2, 0\right\}.$$

Part (i) Since $\tilde{h}'_{L1} < 0$, $\tilde{h}'_{S1} < 0$, $\frac{\partial \tilde{h}_{L1}}{\partial(\rho Q)} < 0$, and $\frac{\partial \tilde{h}_{S1}}{\partial(\rho Q)} > 0$, we have from the implicit function theorem that $\frac{\partial T_{L1}}{\partial(\rho Q)} < 0$ and $\frac{\partial T_{S1}}{\partial(\rho Q)} > 0$. This immediately yields that $\frac{\partial f_1}{\partial(\rho Q)} < 0$ when $f_1 > 0$. Moreover, applying $\sigma_x^2 - \sigma_2^2 = \sigma_1^2$,

$$\begin{aligned}\frac{\partial \mathbb{E}\left[P_2 - P_1\right]}{\partial\left(\rho Q\right)} &= \sigma_x^2 - \sigma_2^2 - \max\left\{\sigma_1^2 + \frac{\partial}{\partial\left(\rho Q\right)}\frac{T_{L1} - \alpha T_{S1}}{1-\alpha}, 0\right\} \\ &= \min\left\{-\frac{\partial}{\partial\left(\rho Q\right)}\frac{T_{L1} - \alpha T_{S1}}{1-\alpha}, \sigma_1^2\right\} > 0.\end{aligned}$$

Part (ii) We have shown that $\mathbb{E}[P_1]$ does not depend upon σ_ε and $\mathbb{E}[P_2]$ increases in σ_ε when ρQ is small and decreases in σ_ε otherwise. The result follows immediately.

Part (iii) Applying $\delta_i = m_i$, we obtain

$$\begin{aligned}\mu_{i1} &= \bar{m} + \beta(\delta_i - \bar{\delta}) + \max\{\eta_2, 0\} \\ &= (1-\beta)\bar{m} + \beta m_i + \max\{\eta_2, 0\} \\ &\sim Uniform([(1-\beta)\bar{m} + \max\{\eta_2, 0\} + \beta m_L, (1-\beta)\bar{m} + \max\{\eta_2, 0\} + \beta m_H]).\end{aligned}$$

Substituting this into equation (42), we obtain that, when $f_1 > 0$,

$$f_1 = \frac{\beta}{1-\alpha}\left(m_H - m_L - \left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q\sigma_1^2\left(m_H - m_L\right)}{\beta\left(1-\alpha\right)}}\right)$$

and

$$\begin{aligned}P_1 &= \frac{P_{L1} - \alpha P_{S1}}{1-\alpha} \\ &= (1-\beta)\bar{m} + \max\{\eta_2, 0\} + \frac{\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_1^2\left(m_H - m_L\right)}{\beta\left(1-\alpha\right)}}\right) \\ &= \bar{m} + \max\{\eta_2, 0\} + \frac{\beta}{1-\alpha}\left(\frac{1+\alpha}{2}(m_H - m_L) - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_1^2\left(m_H - m_L\right)}{\beta\left(1-\alpha\right)}}\right).\end{aligned}$$

Note

$$\begin{aligned}\frac{\partial f_1}{\partial\left(m_H - m_L\right)} &= \frac{\partial}{\partial\left(m_H - m_L\right)}\left[\sqrt{m_H - m_L} \times \frac{\beta}{1-\alpha}\left(\sqrt{m_H - m_L} - \left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q\sigma_1^2}{\beta\left(1-\alpha\right)}}\right)\right] \\ &= \frac{\partial\sqrt{m_H - m_L}}{\partial\left(m_H - m_L\right)}\left(\frac{f_1}{\sqrt{m_H - m_L}} + \frac{\beta}{1-\alpha}\sqrt{m_H - m_L}\right) > 0.\end{aligned}$$

Moreover, when $f_1 > 0$,

$$\frac{\partial \mathbb{E}\left[P_2 - P_1\right]}{\partial (m_H - m_L)} = -\frac{\partial \mathbb{E}\left[P_1\right]}{\partial (m_H - m_L)}$$

$$= -\frac{\partial}{\partial\left(m_H - m_L\right)}\left[\frac{\beta}{1-\alpha}\left(\frac{1+\alpha}{2}\left(m_H - m_L\right) - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_1^2\left(m_H - m_L\right)}{\beta\left(1-\alpha\right)}}\right)\right]$$

$$\propto -\frac{1}{2}\left(1+\alpha\right) + \frac{1}{2}\left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_1^2}{\beta\left(1-\alpha\right)\left(m_H - m_L\right)}}$$

$$= -\frac{1}{2}\frac{1+\alpha}{m_H - m_L}\left(\frac{1-\alpha}{\beta}f_1 + \left(1+\sqrt{\alpha} - \frac{1+\alpha^{\frac{3}{2}}}{1+\alpha}\right)\sqrt{\frac{2\rho Q\sigma_1^2\left(m_H - m_L\right)}{\beta\left(1-\alpha\right)}}\right).$$

As $1+\sqrt{z} - \frac{1+z^{\frac{3}{2}}}{1+z} > 0$ for $z \in (0,1)$, the above expression is negative when $f_1 > 0$.

□

A.11 Proof of Proposition 6

Part (i) In this case, $\mu_{i1} = \mathbb{E}[\bar{m} + \beta y + \Pi_2]$ is identical across investors in this case, and thus their date-1 demands must be identical.

Part (ii.a) Note that, in this case, $\mathbb{E}_i[x] = \mathbb{E}_i[y] = m_i$, and thus

$$\mu_{i1} - \int \mu_{j1} dj = \beta m_i^{\Delta} \qquad \text{and}$$
$$\mu_{i2} - \int \mu_{j2} dj = (1-\beta) m_i^{\Delta}.$$

Therefore, T_{Lt} and T_{St} are again characterized by the equations $\hat{h}_{Lt}\left(T_{Lt}\right) = 0$ and $\hat{h}_{Lt}\left(T_{Lt}\right) = 0$ as defined in (35)–(38). Recall that the date-t loan fee is positive if and only if $T_{Lt} > T_{St}$. Let

$$\chi_L\left(T, z\right) \equiv \left(1 - G_{m^{\Delta}}\left(T\right)\right)\frac{\mathbb{E}^{\mathcal{M}}\left[m_i^{\Delta} | m_i^{\Delta} > T\right] - T}{\rho} - z\frac{1}{1-\alpha}Q;$$
$$\chi_S\left(T, z\right) \equiv G_{m^{\Delta}}(T)\frac{\mathbb{E}^{\mathcal{M}}\left[m_i^{\Delta} | m_i^{\Delta} < T\right] - T}{\rho} + z\frac{\alpha}{1-\alpha}Q.$$

Then, note that the equations for T_{L1} and T_{L2}, $\hat{h}_{L1}\left(T_{L1}\right) = 0$ and $\hat{h}_{L2}\left(T_{L2}\right) = 0$, can be expressed as $\chi_L\left(\frac{T_{L1}}{\beta}, \frac{\sigma_1^2}{\beta}\right) = 0$ and $\chi_L\left(\frac{T_{L2}}{1-\beta}, \frac{\sigma_2^2}{1-\beta}\right) = 0$, respectively. Similarly, the equations for T_{S1} and T_{S2} can be expressed as $\chi_S\left(\frac{T_{S1}}{\beta}, \frac{\sigma_1^2}{\beta}\right) = 0$ and $\chi_S\left(\frac{T_{S2}}{1-\beta}, \frac{\sigma_2^2}{1-\beta}\right) = 0$.

Note that

$$\frac{\partial \chi_L(T,z)}{\partial T} \propto \frac{\partial}{\partial T}\int_T^{\infty}(t-T)\,dG_{m^{\Delta}}(t) = -\int_T^{\infty} dG_{m^{\Delta}}(t) < 0;$$
$$\frac{\partial \chi_S(T,z)}{\partial T} \propto \frac{\partial}{\partial T}\int_{-\infty}^{T}(t-T)\,dG_{m^{\Delta}}(t) = -\int_{-\infty}^{T} dG_{m^{\Delta}}(t) < 0.$$

Moreover, it is immediate that $\frac{\partial}{\partial z}\chi_L(T,z) < 0$ and $\frac{\partial}{\partial z}\chi_S(T,z) > 0$. Hence, the implicit function theorem yields that

$$\chi_L(\mathcal{T}_L(z), z) = 0 \Rightarrow \mathcal{T}_L'(z) = -\left[\frac{\partial \chi_L}{\partial T}\right]^{-1}\frac{\partial \chi_L}{\partial z} < 0;$$
$$\chi_S(\mathcal{T}_S(z), z) = 0 \Rightarrow \mathcal{T}_S'(z) = -\left[\frac{\partial \chi_S}{\partial T}\right]^{-1}\frac{\partial \chi_S}{\partial z} > 0.$$

Now,

$$\frac{\sigma_1^2}{\beta} - \frac{\sigma_2^2}{1-\beta} = \frac{\frac{\sigma_{x1}^4}{\sigma_{x1}^2+\sigma_\varepsilon^2}}{\frac{\sigma_{x1}^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}} - \frac{\frac{\sigma_{x1}^2\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}+\sigma_{x2}^2}{\frac{\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}} = -\frac{\sigma_{x2}^2\sigma_{x1}^2+\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2} < 0.$$

Combining these results, we obtain

$$\frac{T_{L2}}{1-\beta} - \frac{T_{L1}}{\beta} = \int_{\frac{\sigma_1^2}{\beta}}^{\frac{\sigma_2^2}{1-\beta}} \mathcal{T}_L'(z)\,dz < 0;$$
$$\frac{T_{S2}}{1-\beta} - \frac{T_{S1}}{\beta} = \int_{\frac{\sigma_1^2}{\beta}}^{\frac{\sigma_2^2}{1-\beta}} \mathcal{T}_S'(z)\,dz > 0,$$

which gives

$$\frac{T_{L1}-T_{S1}}{\beta} > \frac{T_{L2}-T_{S2}}{1-\beta}.$$

Hence, $T_{L2}-T_{S2} > 0$ implies that $T_{L1}-T_{S1} > 0$.

Part (ii.b) When fees are zero in both periods, we have

$$D_{i2} - Q = \frac{\mu_{i2}-P_2}{\rho\sigma_2^2} - Q = \frac{(1-\beta)(m_i-\bar{m})}{\rho\sigma_2^2};$$
$$D_{i1} - Q = \frac{\mu_{i1}-P_1}{\rho\sigma_1^2} - Q = \frac{\beta(m_i-\bar{m})}{\rho\sigma_1^2}.$$

Now,

$$\frac{D_{i2}-Q}{D_{i1}-Q}=\frac{1-\beta}{\beta}\frac{\sigma_1^2}{\sigma_2^2}=\frac{\frac{\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}}{\frac{\sigma_{x1}^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}}\frac{\frac{\sigma_{x1}^4}{\sigma_{x1}^2+\sigma_\varepsilon^2}}{\frac{\sigma_{x1}^2\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}+\sigma_{x2}^2}=\frac{\frac{\sigma_{x1}^2\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}}{\frac{\sigma_{x1}^2\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}+\sigma_{x2}^2}<1.$$

Part (ii.c) Suppose that fees are positive in both periods. Let $\mathcal{D}_1(m_i)$ and $\mathcal{D}_2(m_i)$ denote investors' date-1 and date-2 demands as a function of their priors m_i, respectively. From the definition of T_{L1}, note that investor i takes a long position on date 1 if and only if

$$\mu_{i1}>P_{L1}\Leftrightarrow\frac{\mu_{i1}-\int\mu_{j1}dj}{\beta}>\frac{T_{L1}}{\beta}$$
$$\Leftrightarrow m_i>\bar{m}+\frac{T_{L1}}{\beta}.$$

That is, $\mathcal{D}_1\left(\bar{m}+\frac{T_{L1}}{\beta}\right)=0$. Similarly, investor i takes a long position in date 2 if and only if $m_i>\bar{m}+\frac{T_{L2}}{1-\beta}$, i.e., $\mathcal{D}_2\left(\bar{m}+\frac{T_{L2}}{1-\beta}\right)=0$. Now, together with the market-clearing condition in the stock, this requires that

$$\int_{\bar{m}+\frac{T_{L1}}{\beta}}^{m_H}\mathcal{D}_1(z)\,dz=\int_{\bar{m}+\frac{T_{L2}}{1-\beta}}^{m_H}\mathcal{D}_2(z)\,dz=\frac{1}{1-\alpha}Q.$$

Recall from Part (ii.a) that $\frac{T_{L1}}{\beta}>\frac{T_{L2}}{1-\beta}$. Therefore, the above equation together with the fact that $\mathcal{D}_2(m_i)>0$ on $(\bar{m}+\frac{T_{L2}}{1-\beta},\bar{m}+\frac{T_{L1}}{\beta})$ yields

$$\int_{\bar{m}+\frac{T_{L1}}{\beta}}^{m_H}\mathcal{D}_1(z)\,dz>\int_{\bar{m}+\frac{T_{L1}}{\beta}}^{m_H}\mathcal{D}_2(z)\,dz.$$

Hence, $\mathcal{D}_1(z)$ must lie above $\mathcal{D}_2(z)$ on some subset of $\left(\frac{T_{L1}}{\beta},m_H\right)$.

Now, among investors who take long positions in both periods (i.e., those with $m_i>\bar{m}+\frac{T_{L1}}{\beta}$), the change in their demands satisfies:

$$\frac{\partial\left[\mathcal{D}_2(m_i)-\mathcal{D}_1(m_i)\right]}{\partial m_i}=\frac{\partial}{\partial m_i}\left[\frac{\mu_{i2}}{\rho\sigma_2^2}-\frac{\mu_{i1}}{\rho\sigma_1^2}\right]=\frac{1}{\rho}\left(\frac{1-\beta}{\sigma_2^2}-\frac{\beta}{\sigma_1^2}\right)<0. \quad (44)$$

Since $\mathcal{D}_1$ and $\mathcal{D}_2$ are continuous, these results imply that there exists a unique point $m^\ddagger\in\left(\bar{m}+\frac{T_{L1}}{\beta},m_H\right)$ such that $\mathcal{D}_1\left(m^\ddagger\right)=\mathcal{D}_2\left(m^\ddagger\right)$, and that $\mathcal{D}_1(z)$ crosses $\mathcal{D}_2(z)$ from below at $m^\ddagger$. An analogous argument can be applied to show that there is a unique point $m^\dagger\in\left(m_L,\bar{m}+\frac{T_{S1}}{\beta}\right)$ where $\mathcal{D}_1(z)$ crosses $\mathcal{D}_2(z)$ from below. Finally, applying $\frac{T_{L1}}{\beta}>\frac{T_{L2}}{1-\beta}$ and $\frac{T_{S1}}{\beta}<\frac{T_{S2}}{1-\beta}$, we have that $\mathcal{D}_2(m_i)>\mathcal{D}_1(m_i)=0$ on $\left(\frac{T_{L2}}{1-\beta},\frac{T_{L1}}{\beta}\right)$, $\mathcal{D}_2(m_i)<\mathcal{D}_1(m_i)=0$

on $\left(\frac{T_{S1}}{\beta}, \frac{T_{S2}}{1-\beta}\right)$, and $\mathcal{D}_2(m_i) = \mathcal{D}_1(m_i) = 0$ on $\left(\frac{T_{S2}}{1-\beta}, \frac{T_{L2}}{1-\beta}\right)$. Therefore, all investors with $m_i \notin (m^\dagger, m^\ddagger)$ and $m_i \in (m^\dagger, m^\ddagger)$ decrease and weakly increase their absolute positions from date 1 to date 2, respectively. $\square$

A.12 Proof of Corollary 4

Signal Disagreement Case

As discussed in the proof of Proposition 6, in this case, we have that $\chi_L\left(\frac{T_{L2}}{1-\beta}, \frac{\sigma_2^2}{1-\beta}\right) = 0$. Now, note that

$$\frac{\partial}{\partial\sigma_\varepsilon}\frac{\sigma_2^2}{1-\beta} = \frac{\partial}{\partial\sigma_\varepsilon}\frac{\frac{\sigma_{x1}^2\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}+\sigma_{x2}^2}{\frac{\sigma_\varepsilon^2}{\sigma_{x1}^2+\sigma_\varepsilon^2}} = -\frac{2\sigma_{x1}^2\sigma_{x2}^2}{\sigma_\varepsilon^3} < 0.$$

Together with the fact that $\frac{\partial\chi_L(T,z)}{\partial T} < 0$, the implicit function theorem yields $\frac{\partial}{\partial\sigma_\varepsilon}\left[\frac{T_{L2}}{1-\beta}\right] > 0$. A similar argument yields $\frac{\partial}{\partial\sigma_\varepsilon}\left[\frac{T_{S2}}{1-\beta}\right] < 0$. Now, when $f_2 > 0$, $T_{L2} > T_{S2}$, and so

$$\begin{aligned}\frac{\partial f_2}{\partial\sigma_\varepsilon} &= \frac{\partial}{\partial\sigma_\varepsilon}(1-\beta)\frac{T_{L2}-T_{S2}}{1-\beta} \\ &= \underbrace{(1-\beta)\frac{\partial}{\partial\sigma_\varepsilon}\left[\frac{T_{L2}-T_{S2}}{1-\beta}\right]}_{>0} + \frac{T_{L2}-T_{S2}}{1-\beta}\underbrace{\frac{\partial(1-\beta)}{\partial\sigma_\varepsilon}}_{>0} > 0.\end{aligned}$$

Signal Agreement Case

In this case, we have $\xi_{i2} = \mu_{i2} - \int \mu_{j2}dj = m_i^\Delta$ and $\sigma_2^2 = \sigma_{x1}^2 + \frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}$. Hence, the equilibrium conditions reduce to

$$\begin{aligned}\tilde{h}_{L2}(T_{L2}) &\propto (1-G_{m^\Delta}(T_{L2}))\frac{\mathbb{E}^{\mathcal{M}}\left[m_i^\Delta|m_i^\Delta > T_{L2}\right]-T_{L2}}{\rho} - \left(\sigma_{x1}^2+\frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}\right)\frac{1}{1-\alpha}Q \\ &= \chi_L\left(T_{L2}, \sigma_{x1}^2+\frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}\right) = 0; \\ \tilde{h}_{S2}(T_{S2}) &\propto G_{m^\Delta}(T_{S2})\frac{\mathbb{E}^{\mathcal{M}}\left[m_i^\Delta|m_i^\Delta < T_{S2}\right]-T_{S2}}{\rho} + \left(\sigma_{x1}^2+\frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}\right)\frac{\alpha}{1-\alpha}Q \\ &= \chi_S\left(T_{S2}, \sigma_{x1}^2+\frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}\right) = 0.\end{aligned}$$

Given that $\sigma_{x1}^2 + \frac{\sigma_{x2}^2\sigma_\varepsilon^2}{\sigma_{x2}^2+\sigma_\varepsilon^2}$ increases with σ_ε, $\frac{\partial \chi_L(T,z)}{\partial T} < 0$, and $\frac{\partial \chi_S(T,z)}{\partial T} < 0$, the implicit function theorem yields $\frac{\partial T_{L2}}{\partial \sigma_\varepsilon} < 0$ and $\frac{\partial T_{S2}}{\partial \sigma_\varepsilon} > 0$. This immediately yields that f_2 decreases in σ_ε. Moreover,

$$\begin{aligned}\frac{\partial}{\partial \sigma_\varepsilon}\mathbb{E}[P_2] &= \frac{\partial}{\partial \sigma_\varepsilon}\left(\mathbb{E}\left[\int \mu_{j2}dj\right] - \rho Q\sigma_2^2 + \max\left\{\frac{T_{L2}-\alpha T_{S2}}{1-\alpha}, 0\right\}\right) \\ &= \underbrace{-\rho Q\frac{\partial \sigma_2^2}{\partial \sigma_\varepsilon}}_{<0} + \max\left\{\underbrace{\frac{\partial}{\partial \sigma_\varepsilon}\frac{T_{L2}-\alpha T_{S2}}{1-\alpha}}_{<0}, 0\right\} < 0.\end{aligned}$$

□

A.13 Proof of Proposition 7

Signal Disagreement Case

We start by deriving the expressions for fees and prices. Starting with the date-2 equilibrium, we have $\mu_{i2} \sim Uniform([(1-\beta)m_L + \beta y, (1-\beta)m_H + \beta y])$. Substituting this into equations (42) and (43), we obtain

$$f_2 = \frac{P_{L2}-P_{S2}}{1-\alpha} = \frac{1-\beta}{1-\alpha}\left(m_H - m_L - \left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q\sigma_2^2\left(m_H-m_L\right)}{\left(1-\beta\right)\left(1-\alpha\right)}}\right)$$

and

$$P_2 = \frac{P_{L2}-\alpha P_{S2}}{1-\alpha} = \beta y + \frac{1-\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_2^2\left(m_H-m_L\right)}{\left(1-\beta\right)\left(1-\alpha\right)}}\right).$$

In contrast, when the lending constraint does not bind, we have

$$P_2 = \beta y + \left(1-\beta\right)\bar{m} - \rho Q\sigma_2^2.$$

We next derive the date-1 equilibrium. Note in this case we have $P_2 = \beta y + \Gamma_2$, where

$$\Gamma_2 \equiv \max\left\{\frac{1-\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_2^2\left(m_H-m_L\right)}{\left(1-\beta\right)\left(1-\alpha\right)}}\right), \left(1-\beta\right)\bar{m} - \rho Q\sigma_2^2\right\}.$$

Therefore, $\mu_{i1} \sim Uniform([\Gamma_2 + \beta m_L, \Gamma_2 + \beta m_H])$. Substituting this into (42) and (43), and applying $\frac{\sigma_1^2}{\beta} = \sigma_{x1}^2$, we obtain

$$f_1 = \frac{P_{L1} - P_{S1}}{1-\alpha} = \beta \left(m_H - m_L - \left(1 + \sqrt{\alpha}\right) \sqrt{\frac{2\rho Q \sigma_{x1}^2 \left(m_H - m_L\right)}{1-\alpha}} \right)$$

and

$$P_1 = \frac{P_{L1} - \alpha P_{S1}}{1-\alpha} = \Gamma_2 + \frac{\beta}{1-\alpha} \left(m_H - \alpha m_L - \left(1 + \alpha^{\frac{3}{2}}\right) \sqrt{\frac{2\rho Q \sigma_{x1}^2 \left(m_H - m_L\right)}{1-\alpha}} \right). \tag{45}$$

Note that

$$f_1 > 0 \Leftrightarrow m_H - m_L > \left(1 + \sqrt{\alpha}\right) \sqrt{\frac{2\rho Q \sigma_{x1}^2 \left(m_H - m_L\right)}{1-\alpha}}; \tag{46}$$

$$f_2 > 0 \Leftrightarrow m_H - m_L > \left(1 + \sqrt{\alpha}\right) \sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{(1-\beta)(1-\alpha)}}.$$

Now, $\frac{\sigma_2^2}{1-\beta} = \sigma_{x1}^2 + \frac{\left(\sigma_{x1}^2 + \sigma_\varepsilon^2\right)\sigma_{x2}^2}{\sigma_\varepsilon^2} > \sigma_{x1}^2$, so $f_2 > 0 \Rightarrow f_1 > 0$. So, we can assume the lending constraint binds in date 1, for otherwise it would not bind in either date. Note in this case we have

$$\frac{\partial f_1}{\partial \sigma_\varepsilon} = \frac{\partial \beta}{\partial \sigma_\varepsilon} \times \frac{f_1}{\beta} < 0.$$

To sign $\frac{\partial P}{\partial \sigma_\varepsilon}$, we next separately consider the cases in which the lending constraint does and does not bind at date 2.

Case 1: $f_2 > 0$. In this case, we obtain that

$$\begin{aligned}
\frac{\partial P_1}{\partial \sigma_\varepsilon} = & -\frac{\partial \beta}{\partial \sigma_\varepsilon} \frac{1}{1-\alpha} \left(m_H - \alpha m_L - \left(1 + \alpha^{\frac{3}{2}}\right) \sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{(1-\beta)(1-\alpha)}} \right) \\
& - \frac{1-\beta}{1-\alpha} \left(1 + \alpha^{\frac{3}{2}}\right) \frac{\partial}{\partial \sigma_\varepsilon} \sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{(1-\beta)(1-\alpha)}} \\
& + \frac{\partial \beta}{\sigma \sigma_\varepsilon} \frac{1}{1-\alpha} \left(m_H - \alpha m_L - \left(1 + \alpha^{\frac{3}{2}}\right) \sqrt{\frac{2\rho Q \sigma_{x1}^2 \left(m_H - m_L\right)}{(1-\alpha)}} \right) \\
= & \frac{1 + \alpha^{\frac{3}{2}}}{1-\alpha} \sqrt{\frac{2\rho Q \left(m_H - m_L\right)}{1-\alpha}} \left\{ \frac{\partial \beta}{\partial \sigma_\varepsilon} \times \left(\sqrt{\frac{\sigma_2^2}{1-\beta}} - \sqrt{\frac{\sigma_1^2}{\beta}} \right) - (1-\beta) \frac{\partial}{\partial \sigma_\varepsilon} \sqrt{\frac{\sigma_2^2}{1-\beta}} \right\}.
\end{aligned}$$

Calculating the expression in brackets and simplifying, we arrive at

$$\frac{\partial P_1}{\partial \sigma_\varepsilon} \propto -\sigma_{x1}^2 \left(2\sigma_\varepsilon^2\sigma_{x1}^2 + \sigma_{x2}^2\left(\sigma_\varepsilon^2 + \sigma_{x1}^2\right) - \sqrt{\left(2\sigma_\varepsilon^2\sigma_{x1}^2 + \sigma_{x2}^2\left(\sigma_\varepsilon^2 + \sigma_{x1}^2\right)\right)^2 - \sigma_{x2}^4\left(\sigma_\varepsilon^2 + \sigma_{x1}^2\right)^2}\right) < 0.$$

Case 2: $f_2 = 0$. In this case, we have

$$\begin{aligned}
\frac{\partial P_1}{\partial \sigma_\varepsilon} &= \frac{\partial}{\partial \sigma_\varepsilon}\left[(1-\beta)\frac{m_L + m_H}{2} - \rho Q \sigma_2^2 + \frac{\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q \sigma_{x1}^2 (m_H - m_L)}{(1-\alpha)}}\right)\right] \\
&= -\frac{\partial \beta}{\partial \sigma_\varepsilon}\frac{m_L + m_H}{2} - \rho Q \frac{\partial \sigma_2^2}{\partial \sigma_\varepsilon} + \frac{1}{1-\alpha}\frac{\partial \beta}{\partial \sigma_\varepsilon}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q \sigma_{x1}^2 (m_H - m_L)}{(1-\alpha)}}\right) \\
&= -\frac{\sigma_\varepsilon \sigma_{x1}^2\left\{-2\sigma_{x1}\left[\left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q (m_H - m_L)}{1-\alpha}} - (1-\alpha)\rho Q \sigma_{x1}\right] + (\alpha+1)(m_H - m_L)\right\}}{(1-\alpha)\left(\sigma_\varepsilon^2 + \sigma_{x1}^2\right)^2} \\
&\propto -(\alpha+1)(m_H - m_L) + 2\sigma_{x1}\left[\left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q (m_H - m_L)}{1-\alpha}} - (1-\alpha)\rho Q \sigma_{x1}\right]. \quad (47)
\end{aligned}$$

We now argue that, when $f_1 > 0$, (47) is negative. Note that

$$\begin{aligned}
\frac{\partial f_1}{\partial [m_H - m_L]} &\propto \frac{\partial}{\partial [m_H - m_L]}\left(m_H - m_L - \left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q \sigma_{x1}^2 (m_H - m_L)}{1-\alpha}}\right) \\
&= 1 - \frac{1}{2}\left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q \sigma_{x1}^2}{1-\alpha}}\frac{1}{\sqrt{m_H - m_L}} \\
&\propto m_H - m_L - \frac{1}{2}\left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q \sigma_{x1}^2 (m_H - m_L)}{1-\alpha}} > f_1 \geq 0.
\end{aligned}$$

This implies that $f_1 > 0$ if and only if $m_H - m_L$ is sufficiently large. Solving, the cutoff level is

$$f_1 > 0 \Leftrightarrow m_H - m_L > \frac{2\left(\sqrt{\alpha}+1\right)^2 \rho Q \sigma_{x1}^2}{1-\alpha}.$$

When $m_H - m_L = \frac{2\left(\sqrt{\alpha}+1\right)^2 \rho Q \sigma_{x1}^2}{1-\alpha}$, (47) equals 0. Moreover, differentiating (47) yields

$$\begin{aligned}
&\frac{\partial}{\partial [m_H - m_L]}\left\{-(\alpha+1)(m_H - m_L) + 2\sigma_{x1}\left[\left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q (m_H - m_L)}{1-\alpha}} - (1-\alpha)\rho Q \sigma_{x1}\right]\right\} \\
&= -(\alpha+1) + \sigma_{x1}\left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q}{1-\alpha}}\frac{1}{m_H - m_L}.
\end{aligned}$$

This is decreasing in $m_H - m_L$ and negative when $m_H - m_L = \frac{2\left(\sqrt{\alpha}+1\right)^2 \rho Q \sigma_{x1}^2}{1-\alpha}$, which implies that (47) is negative whenever $f_1 > 0$, as desired.

Signal Agreement Case

As in the previous case, we start by deriving the expressions for fees and prices. We now have $\mu_{i2} \sim Uniform([m_L + \beta y, m_H + \beta y])$. Substituting this into equations (42) and (43), we obtain

$$f_2 = \frac{P_{L2} - P_{S2}}{1-\alpha} = \frac{1}{1-\alpha}\left(m_H - m_L - \left(1+\sqrt{\alpha}\right)\sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{1-\alpha}}\right)$$

and

$$P_2 = \frac{P_{L2} - \alpha P_{S2}}{1-\alpha} = \beta y + \frac{1}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{1-\alpha}}\right).$$

In contrast, when the lending constraint does not bind, we have

$$P_2 = \bar{m} + \beta y - \rho Q \sigma_2^2.$$

Note that we have $\mu_{i1} = \bar{m} - \rho Q \sigma_2^2 + \max\left\{\eta_2, 0\right\}$, and thus the lending constraint does not bind on date 1. Hence, we can assume the lending constraint binds on date 2 moving forward. In this case, we obtain

$$P_1 = \frac{1}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q \sigma_2^2 \left(m_H - m_L\right)}{1-\alpha}}\right) - \rho Q \frac{\sigma_{x2}^4}{\sigma_{x2}^2 + \sigma_\varepsilon^2},$$

and so

$$\begin{aligned}
\frac{\partial P_1}{\partial \sigma_\varepsilon} &= -\frac{1+\alpha^{\frac{3}{2}}}{1-\alpha}\sqrt{\frac{2\rho Q \left(m_H - m_L\right)}{1-\alpha}}\frac{\partial}{\partial \sigma_\varepsilon}\sqrt{\sigma_{x1}^2 + \frac{\sigma_{x2}^2 \sigma_\varepsilon^2}{\sigma_{x2}^2 + \sigma_\varepsilon^2}} - \rho Q \frac{\partial}{\partial \sigma_\varepsilon}\frac{\sigma_{x2}^4}{\sigma_{x2}^2 + \sigma_\varepsilon^2} \\
&= -\frac{1+\alpha^{\frac{3}{2}}}{1-\alpha}\sqrt{\frac{2\rho Q \left(m_H - m_L\right)}{1-\alpha}}\sqrt{\frac{\sigma_\varepsilon^2 + \sigma_{x2}^2}{\sigma_{x1}^2\left(\sigma_\varepsilon^2 + \sigma_{x2}^2\right) + \sigma_\varepsilon^2 \sigma_{x2}^2}}\frac{\sigma_\varepsilon \sigma_{x2}^4}{\left(\sigma_\varepsilon^2 + \sigma_{x2}^2\right)^2} + \rho Q \frac{\sigma_\varepsilon \sigma_{x2}^2}{\left(\sigma_\varepsilon^2 + \sigma_{x2}^2\right)^{3/2}}.
\end{aligned} \tag{48}$$

Now, following steps similar to the signal disagreement case, one can verify that

$$f_2 > 0 \Leftrightarrow m_H - m_L > \frac{2\left(\sqrt{\alpha}+1\right)^2 \rho Q \left(\sigma_{x1}^2\left(\sigma_\varepsilon^2+\sigma_{x2}^2\right)+\sigma_\varepsilon^2\sigma_{x2}^2\right)}{(1-\alpha)\left(\sigma_\varepsilon^2+\sigma_{x2}^2\right)}.$$

Moreover, for $m_H - m_L = \frac{2\left(\sqrt{\alpha}+1\right)^2 \rho Q \left(\sigma_{x1}^2\left(\sigma_\varepsilon^2+\sigma_{x2}^2\right)+\sigma_\varepsilon^2\sigma_{x2}^2\right)}{(1-\alpha)\left(\sigma_\varepsilon^2+\sigma_{x2}^2\right)}$, expression (48) is negative, and expression (48) clearly decreases in $m_H - m_L$. Hence, for $f_2 > 0$, we have that $\frac{\partial P_1}{\partial \sigma_\varepsilon} < 0$. □

A.14 Proof of Corollary 5

Part (i) We have that

$$\begin{aligned}\mathbb{E}\left[P_2 - P_1\right] &= \max\left\{\rho Q\left(\sigma_x^2 - \sigma_2^2\right) - \eta_1, \rho Q\left(\sigma_x^2 - \sigma_2^2\right)\right\} \\ &= \max\left\{\beta\bar{m} - \frac{\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_{x1}^2\left(m_H - m_L\right)}{1-\alpha}}\right), \rho Q\sigma_1^2\right\},\end{aligned}$$

where the second line applies that in this case, from equation (45),

$$\eta_1 = \rho Q\sigma_1^2 - \beta\bar{m} + \frac{\beta}{1-\alpha}\left(m_H - \alpha m_L - \left(1+\alpha^{\frac{3}{2}}\right)\sqrt{\frac{2\rho Q\sigma_{x1}^2\left(m_H - m_L\right)}{1-\alpha}}\right).$$

Clearly, when $\eta_1 < 0$, $\frac{\partial \mathbb{E}[P_2 - P_1]}{\partial \sigma_\varepsilon} = \frac{\partial\left(\rho Q\sigma_1^2\right)}{\partial \sigma_\varepsilon} < 0$. When $\eta_1 > 0$, since $\frac{\partial \beta}{\partial \sigma_\varepsilon} < 0$, we have

$$\frac{\partial \mathbb{E}\left[P_2 - P_1\right]}{\partial \sigma_\varepsilon} \propto -\bar{m} + \frac{m_H - \alpha m_L}{1-\alpha} - \frac{1+\alpha^{\frac{3}{2}}}{1-\alpha}\sqrt{\frac{2\rho Q\sigma_{x1}^2\left(m_H - m_L\right)}{1-\alpha}}.$$

Setting this equal to 0 and solving for ρQ, this is positive if and only if

$$\rho Q < \frac{\left(1-\sqrt{\alpha}\right)(\alpha+1)^2\left(m_H - m_L\right)}{8\left(\sqrt{\alpha}+1\right)\left(\alpha - \sqrt{\alpha}+1\right)^2\sigma_{x1}^2}. \tag{49}$$

Now, note from (46) that $\eta_1 > 0$ if and only if $\rho Q < \frac{\left(1-\sqrt{\alpha}\right)\left(m_H - m_L\right)}{2\left(\sqrt{\alpha}+1\right)\sigma_{x1}^2}$, which is strictly greater than the expression on the right-hand side of the above equation. Hence, $\frac{\partial \mathbb{E}[P_2 - P_1]}{\partial \sigma_\varepsilon} > 0$ if and only if (49) is satisfied.

Part (ii) In this case, we have that, because the lending constraint never binds in date 1,

$$\mathbb{E}\left[P_2 - P_1\right] = \rho Q\left(\sigma_x^2 - \sigma_2^2\right).$$

Because σ_2^2 increases in σ_ε, expected returns decrease in σ_ε. $\square$

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