Payout-Based Asset Pricing

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Abstract

We propose a payout-based approach for the evaluation of the asset pricing implications of production models. Our approach recovers the implicit return process of an optimizing firm from its observed payout processes without the need to measure firm investment or specify investor preferences, by answering the following question: given the firm's production and financing technology, what are the equity and debt rates of return that induce the firm to optimally provide the observed equity and debt payouts? We simulate the canonical representative firm model and use our approach to explore whether the model-implied U.S. aggregate returns match the properties of their empirical counterparts. We find that the canonical model gives rise to three important asset pricing puzzles regarding aggregate equity and debt returns, indicating the need for additional features that generate more realistic asset pricing properties.

JEL Classification: E10; E13; G10; G11; G12; G35.

Keywords: Payout-Based Asset Pricing; Production-Based Asset Pricing; Investment-Based Asset Pricing; Market Returns; Return Predictability.

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1 Introduction

We introduce a new approach within the production-based asset pricing paradigm, called payoutbased asset pricing, that backs out the implicit return process of an optimizing firm from its observed payout process. Our approach hinges on the insight of Cochrane (1988) that producer optimality conditions, which describe relationships between equilibrium firm-related quantities (such as investment and payout) and asset prices and returns, can be exploited in a number of ways, including the following: "model a statistical process for quantities, derive the process for prices, and compare those prices or the corresponding returns to data". By evaluating the realism of the asset pricing implications of particular firm specifications and identifying asset pricing "puzzles", our approach provides guidance towards building better production-based asset pricing models.¹

One of the main benefits of our payout-based approach is that it sidesteps the need to measure investment accurately. Much of the production-based asset pricing literature focuses on evaluating the asset pricing properties of production models by testing model-implied relationships between firm investment and asset returns. However, firm investment is conceptually ambiguous because of capital heterogeneity (Gonçalves, Xue and Zhang (2020) and Belo, Gala, Salomao and Vitorino (2022)) and hard to measure and align with asset prices (Liu, Whited and Zhang (2009), Bai, Li, Xue and Zhang (2024), and Belo, Deng and Salomao (2023)). Instead, our approach relies on firm payout, which is much easier to define and measure.

While our approach is quite general and can be applied to any firm, in this paper we focus on a representative firm, and, in particular, on a stand-in firm for the aggregate U.S. market. For our baseline exercise, we specify the technology of an equity-financed representative firm, derive its optimal payout policy, and propose a statistical model for the firm's observed payout process. Then, we back out the representative firm's cost of capital (i.e., the firm's expected return) by answering the following question: given the firm's production technology, what is the cost of capital so that the firm *optimally* generates the observed payout process? We also extend our approach by considering a representative firm that chooses its equity and debt payout policy by jointly optimizing its capital budgeting and capital structure decisions. Specifying statistical processes for the firm's equity and debt payout allows us to jointly recover the levered firm's cost of equity and debt (i.e., the expected return of aggregate equity and debt claims) and, hence, evaluate how closely their properties match the properties of the corresponding observed aggregate returns. In short, we take the properties of firm payout as given and retrieve the model-implied returns of firm-issued assets under the assumption that the firm optimizes. Importantly, the asset pricing implications of particular firm

¹As pointed out in Cochrane (1988), the aforementioned use of the producer's first order conditions essentially mirrors the use of household optimality conditions in consumption-based asset pricing (Lucas (1978)). Hence, identifying production-based asset pricing "puzzles", which guide towards adopting more realistic models of firm behavior (for example, by refining the assumptions on firm production technology or firm-side frictions), is a direct analog of identifying consumption-based "puzzles" (e.g., the "equity premium puzzle" of Mehra and Prescott (1985)), which guide towards building more realistic models of household behavior (for example, by considering different types of household preferences or investor-side frictions).

specifications can be evaluated in the absence of any assumptions about investor preferences, so progress on the modeling of the production side of the economy is not held back by the need to finesse the modeling of the household sector.

In both our baseline exercise and its extension, our aim is to evaluate the asset pricing implications of our payout-based approach within a canonical model of the firm. To that end, we consider a representative firm with a constant-returns-to-scale production function, which optimizes its payout decision in the presence of quadratic capital adjustment costs and faces no frictions when raising new funding. Since our aim is to back out aggregate U.S. returns, we calibrate the representative firm's productivity and payout processes to match the properties of the corresponding aggregate U.S. series. Although our approach can accommodate more complex models of the firm, our focus on the canonical model makes our findings relevant to the widest possible swath of the productionbased asset pricing literature.

Our model simulations give rise to three asset pricing puzzles. The main one, the "return sensitivity puzzle", is that the model-implied firm cost of capital is too sensitive to changes in the firm payout ratio (defined as firm payout over output), leading to counterfactually strong predictability of future firm returns. When we consider a levered representative firm, we show that this puzzle is effectively an "equity return sensitivity puzzle", as it is due to the fact that the cost of equity is too sensitive to changes in the equity payout ratio, which induces counterfactually high equity return volatility and strong predictability of future equity returns. We also identify two debt return-related puzzles: we find that model-implied debt returns are too low on average and display almost no time variation (the "debt return level puzzle" and " debt return volatility puzzle", respectively). The three asset pricing puzzles provide useful guidance on the directions towards the canonical model of the firm needs to be adjusted in order to generate realistic asset pricing implications.

In particular, the return sensitivity puzzle suggests that, in the model, the payout policy of the representative firm is counterfactually insensitive to equity discount rate fluctuations: the optimal equity payout ratio varies too little when the equity discount rate changes, so the only way to match the properties of the observed U.S. aggregate equity payout ratio is by having a counterfactually volatile cost of equity. Hence, in order to achieve more realistic expected equity returns while matching the properties of the equity payout ratio process, we need models that include features which increase the sensitivity of the firm's equity payout policy to equity discount rate variation. For example, one such feature would be the inclusion of misvaluation shocks, in the spirit of Warusawitharana and Whited (2016).

The two debt return puzzles largely reflect the limitations of the trade-off theory of capital structure. In our model, the firm optimizes its capital structure by trading off the tax benefits of debt against financial distress costs, so optimal leverage is increasing in the cost of debt, due to the interest tax shield effect. It follows that, under the payout-based approach, the low average leverage (and, hence, low average debt payout) observed in the United States can be reconciled with firm optimization only if the average cost of debt is counterfactually low. In other words, taking leverage levels (and payouts) as given, the implied cost of debt is counterfactually low. Notably, the "debt return level puzzle" we identify is the flipside of the well-known "capital structure puzzle" in the corporate finance literature: taking the cost of debt as given, the observed leverage levels are too low to be consistent with standard trade-off models under reasonable parameter calibrations. Relatedly, the "debt return volatility puzzle" we identify is due to the tight connection between the firm's optimal leverage policy and its cost of debt in the model: since the aggregate leverage of U.S. firms is very persistent and displays little volatility over short horizons, the model endogenously generates similar properties for the implied cost of debt. Hence, jointly addressing the two debt puzzles is likely to require modeling firms' capital structure decisions as involving elements beyond taxes and financial distress costs. The capital structure literature is a fertile ground for thinking along those lines. For example, the introduction of financial flexibility considerations, in the spirit of DeAngelo, DeAngelo and Whited (2011), would significantly weaken the connection between firms' cost of debt and their level of leverage, potentially helping generate more realistic debt return moments.

Our proposed payout-based asset pricing framework contributes to the production-based asset pricing literature, which connects the production side of the economy with asset prices (see Kogan and Papanikolaou (2012) and Zhang (2017) for literature reviews). We do not introduce a new model within the production-based paradigm – rather, we identify important empirical puzzles associated with the canonical firm specification, highlighting its limitations and helping guide the literature towards production models with more realistic asset pricing moments. Our methodological contribution is crucial for that purpose: our payout-based approach allows us to separately identify equity and debt return puzzles that are fundamental and pertinent to a large cross-section of the production-based asset pricing literature, as opposed to arising from either mismeasured variables or unconventional theoretical assumptions. Below, we elaborate on those issues further, in the context of discussing the differences between our payout-based approach and other approaches adopted in the production-based asset pricing literature.

Our approach is mainly related to (but also differs in crucial aspects from) the investment-based asset pricing strand of the production-based literature, which empirically examines the relationship between realized firm returns and realized investment returns, without either deriving the economy's SDF or backing out equilibrium expected returns (e.g., Cochrane (1991)). Furthermore, it shares some elements with the strand of the production-based literature that derives the economy's stochastic discount factor (SDF) from the representative firm's optimality conditions (e.g., Cochrane (1988), Jermann (2010), Belo (2010), Cochrane (2021)).

The investment-based asset pricing literature, which builds on the q-theory of investment, typically focuses on the canonical model of the firm that we consider in our paper.² However, the way that

²That literature relies on the insight that, under linear homogeneity, firm returns are equal to investment returns (Cochrane (1991) and Restoy and Rockinger (1994)). Notable contributions include, among others, Liu et al. (2009), Belo, Xue and Zhang (2013), Lin and Zhang (2013), Liu and Zhang (2014), Gonçalves et al. (2020), Belo et al. (2022),

it evaluates the asset pricing implications of production models differs from ours. In particular, the papers in that literature take both firm quantities (such as investment and output) and asset prices (such as debt and equity returns) as given and test restrictions associated with the model-implied equality between firm and investment returns.³ In short, the main aim of the investment-based literature is to empirically evaluate whether firms' investment policies are aligned with their cost of capital.⁴ In contrast, our approach takes the firms' payout realizations as given and backs out model-implied asset prices under the assumption that firms optimize, with the aim of comparing model-implied and observed asset prices and returns (both realized and expected).

It follows that the accurate measurement of investment returns and, hence, firm investment is critical for the standard investment-based approach. However, firm investment is subject to important measurement issues due to both differences between the historical and current cost of physical capital (Bai et al. (2024)) and the existence of multiple capital inputs, such as working and intangible capital (see, e.g., Gonçalves et al. (2020) and Belo et al. (2022)). As a result, measured investment returns may deviate from observed firm returns not just due to model misspecification, but also due to measurement errors, confounding the ability of the standard investment-based approach to correctly evaluate the asset pricing properties of production models.

In contrast, our approach derives model-implied returns from easy-to-measure payout data by assuming that a firm's observed payout process is optimal. As we document in the main body of our paper, focusing on firms' payout (rather than investment) processes yields important empirical benefits, consistent with the conjecture that payout is better measured than investment. In particular, we show that payout ratios forecast aggregate firm returns much better than investment rates. Furthermore, for the same model, when we calculate model-implied firm returns derived from payout data and model-implied firm returns derived from investment data, we find that the former are better aligned with observed firm returns than the latter. Apart from yielding empirical advantages related to measurement issues, shifting the focus from investment to payout also has theoretical benefits. In particular, although modeling investment and payout are mathematically equivalent in the context of a firm financed by the issuance of one type of claim (typically common stock), that is no longer true in a model that features a firm that finances itself by issuing multiple types of securities: in that case, separately modeling the payout process of each security allows for the recovery of each security's return process, something that would be impossible to do by modeling the firm's investment process. In our model extension, we exploit that feature in order to

Li, Ma, Wang and Yu (2023), and Belo et al. (2023).

³The closest paper to ours in that literature is Cochrane (1991), which, like ours, focuses on aggregate returns (other papers in this literature focus on the cross-section of returns). In particular, Cochrane (1991) postulates a firm production technology and uses aggregate U.S. investment data in order to retrieve the time-series of U.S. realized aggregate investment returns, statistically testing their similarity with the empirically observed realized aggregate equity returns.

⁴Hence, investment-based asset pricing exploits the insight of Cochrane (1988) that one of the ways to use household and firm optimality conditions is as follows: "we can model the joint stochastic process for prices and quantities and test whether the restrictions implied by the first order condition hold". As Cochrane (1988) points out, that is the approach adopted by Hansen and Singleton (1983) for testing household optimality conditions.

separately retrieve the model-implied equity and debt returns of a levered representative firm and evaluate their properties, underscoring the breadth and flexibility of our approach.

The literature that focuses on retrieving the economy's SDF using firms' optimality conditions can be broadly classified in two categories. Some of those papers (such as Belo (2010) and Cochrane (2021)) introduce models with non-standard firm production functions and, in particular, production functions that allow firms to shift output across states of nature. The main benefit of those complex firm specifications is that they generate firm optimality conditions that allow for the economy's SDF to be fully recovered. The second category includes papers such as Cochrane (1988), Jermann (2010) and Jermann (2013): in those papers, firms have standard production functions (i.e., production technologies which do not allow firms to shift resources across states of nature), so state prices are recovered via the "complete technologies" assumption, i.e., the assumption of an equal number of types of capital (and production processes) and states of nature. In particular, those papers consider an equity-financed representative firm with "complete technologies" (two types of capital, each an input in a corresponding production technology, and two states of nature), propose a statistical model for its investment process, and recover the implicit state prices from the firm's optimality conditions.

Our framework is closer to the second category of papers, but differs from them in two key ways. First, we aim to recover only the firm's returns, rather than the economy's SDF, which allows us to sidestep the restrictive "complete technologies" assumption: our model accommodates a continuum of states of nature, while relying on a standard neoclassical firm specification, which makes the asset pricing findings documented in our paper relevant to a wide class of models featured in the existing literature. Of course, our approach has the obvious limitation that, by not recovering the economy's SDF, it can only be used to price claims on a particular payout process. However, that limitation is not as restrictive as it first may appear: as we show when we consider a levered representative firm, we can use our payout-based approach in order to price multiple claims on the same firm provided we know the firm's production and financing technology and have a statistical model for the corresponding payout processes. Second, we statistically model the firm's payout – rather than investment – process, which, as discussed above, entails significant empirical and theoretical benefits.

Finally, a number of production-based asset pricing papers propose partial equilibrium models that include an exogenous SDF (or exogenous risk neutral dynamics). In these models, firms' expected returns arise from their corporate policies, which determine the covariance of firms' returns with the SDF.⁵ We propose a different approach: instead of specifying an exogenous SDF, we consider a

⁵That literature builds on the models developed in Berk, Green and Naik (1999) and Zhang (2005) and includes, inter alia, the models in Kogan and Papanikolaou (2010), Belo and Lin (2012), Belo and Yu (2013), Jones and Tuzel (2013), Belo, Lin and Bazdresch (2014), Belo, Li, Lin and Zhao (2017), Li (2018), Belo, Lin and Yang (2018), Kogan, Li and Zhang (2023), Grigoris and Segal (2023), Grigoris, Hu and Segal (2023), and Belo and Li (2023). Among the papers in that literature, Belo and Li (2023) is the closest to ours: they use an exogenous SDF, but rely on firms' optimality decisions to write the SDF in closed form as a function of variables related to firm investment and

statistical model of the firm's payout process and assume that the firm optimizes its payout decision. The main benefit of our approach is that firm payout is observable: the payout processes can be calibrated to corporate payout data, ensuring that the model calibration is based on quantities, as opposed to prices. Thus, our approach sidesteps the problem of calibrating SDF parameters, which is often done by matching asset pricing moments (i.e., a subset of the moments the model is meant to explain).

The rest of this paper is organized as follows. Section 2 introduces our payout-based asset pricing framework and describes our baseline model. Section 3 reports the quantitative output of our baseline model. Section 4 discusses an extension of our model that features a levered representative firm, which allows us to study the properties of equity and debt returns separately. Finally, Section 5 concludes. The Internet Appendix includes model derivations, as well as details on data sources and empirical measures, that are omitted from the main text.

2 Payout-Based Asset Pricing: Baseline Model

In this section, we introduce our payout-based asset pricing approach in the context of the canonical firm model with an unlevered representative firm (which we call the baseline model). First, we outline the baseline model and discuss its properties. Then, we discuss the payout-based approach to retrieve model-implied asset prices and returns in the baseline model. Finally, we contrast our approach with an approach that models the firm's observed investment process and illustrate the benefits of our payout-based approach.

2.1 Baseline model of the firm

We consider an optimizing equity-financed representative firm with an infinite horizon. All derivations for the results in this section can be found in Internet Appendix A.

2.1.1 Setting

The operating profit of the representative firm is given by

$$\Pi(K,Z) = \alpha \cdot Z \cdot K = \alpha \cdot Y,\tag{1}$$

where K is the firm's capital stock, Z is an exogenous productivity process, $Y = Z \cdot K$ is the firm's output, and $\alpha \in (0, 1)$ is the firm's operating profit margin.⁶ The exogenous productivity process

profitability.

⁶Our specification is consistent with a constant returns to scale production function that includes additional inputs (such as energy, purchased services, and costlessly adjustable labor). Footnote 4 in Gonçalves et al. (2020)

Z satisfies $Z_t = e^{z_t}$, where z is a stationary process that has law of motion

$$z_{t+1} = \mu_z + \phi_z (z_t - \mu_z) + \sigma_z \epsilon_{t+1}^z,$$
(2)

with $\epsilon_{t+1}^z \sim N(0,1)$, $\phi_z \in (0,1)$, and $\sigma_z > 0$. Capital depreciates at a constant rate $\delta \in [0,1]$ per period, so capital accumulation satisfies

$$K_{t+1} = I_t + (1 - \delta)K_t,$$
(3)

where I is the firm's investment. Finally, we assume that the firm faces capital adjustment costs, with the adjustment cost function being

$$\Phi(K,I) = \frac{a}{2} \cdot (I/K)^2 \cdot K.$$
(4)

As standard, we assume that the capital adjustment costs are tax-deductible. It follows that the firm's flow payout is given by

$$D_t = (1 - \tau) \left(\Pi(K_t, Z_t) - \Phi(K_t, I_t) \right) - I_t + \tau \delta K_t,$$
(5)

where $\tau \in (0, 1)$ is the corporate tax rate.

The firm's manager chooses investment I and payout D in order to maximize the cum-payout firm value V_t ,

$$V_t = \max_{\{I_{t+h}, D_{t+h}\}_{h=0}^{\infty}} \{D_t + \sum_{h=1}^{\infty} \mathbb{E}_t[M_{t,t+h}D_{t+h}]\},\tag{6}$$

where $\{M_{t,t+h}\}_{h=1}^{\infty}$ is the set of stochastic discount factors, the properties of which the firm takes as given when optimizing.

2.1.2 The firm's optimal investment and payout policy

The firm's problem can be rewritten recursively as

$$V(K_t, Z_t; \{f_t(M_{t,t+h})\}_{h=1}^{\infty}) = \max_{\{I_t\}} \{D_t + \mathbb{E}_t[M_{t,t+1}V(K_{t+1}, Z_{t+1}; \{f_{t+1}(M_{t+1,t+1+h})\}_{h=1}^{\infty})]\}, \quad (7)$$

where $f_t(\cdot)$ denotes the probability distribution conditional on information available at time t. This specification is consistent with the fact that the firm picks the optimal investment-payout policy taking the conditional distribution of the current and future SDFs as given.

provides a detailed discussion.

The firm's first order condition is

$$\underbrace{\mathbb{E}_{t}[M_{t+1}\partial_{K}V(K_{t+1}, Z_{t+1}; \{f_{t+1}(M_{t+1,t+1+h})\}_{h=1}^{\infty})]}_{\equiv q_{t}} = 1 + (1 - \tau)\partial_{I}\Phi(K_{t}, I_{t}),$$
(8)

where q is Tobin's marginal q. That condition yields the familiar investment policy

$$I_t = I(K_t; q_t) = \frac{q_t - 1}{a(1 - \tau)} K_t,$$
(9)

which specifies that the firm's optimal investment is proportional to its capital stock and increasing in the marginal q.⁷

Plugging the firm's optimal investment policy into Equation 5, we get the firm's optimal payout policy,

$$D_t = D(K_t, Z_t; q_t) = \left[(1 - \tau)\alpha Z_t - \frac{q_t^2 - 1}{2a(1 - \tau)} + \tau \delta \right] K_t.$$
(10)

The firm's optimal payout at period t is proportional to its capital stock K_t , increasing in productivity Z_t , and decreasing in q_t . The negative relationship between firm payout and marginal q is intuitive: higher marginal q implies higher investment, which reduces the firm resources available to be paid out. Notably, the firm's marginal q is a sufficient statistic for investor preferences as regards characterizing the firm's optimal payout behavior – no other information regarding conditional SDF distributions is needed. In other words, any configuration of conditional SDF distributions that yields the same q process leads the firm to optimally choose the same investment and payout processes.

Using the envelope condition, we obtain

$$q_t = \mathbb{E}_t \left[M_{t+1} \left((1-\tau) (\partial_K \Pi(K_{t+1}, Z_{t+1}) - \partial_K \Phi(K_{t+1}, I_{t+1}) \right) + \tau \delta + (1-\delta) q_{t+1} \right) \right].$$
(11)

Notably, we can rewrite Equation 11 as

$$\mathbb{E}_t \left[M_{t+1} \cdot R_{t+1}^I \right] = 1, \tag{12}$$

where $R_{t+1}^I = \frac{(1-\tau)(\partial_K \Pi(K_{t+1}, Z_{t+1}) - \partial_K \Phi(K_{t+1}, I_{t+1})) + \tau \delta + (1-\delta)q_{t+1}}{q_t}$ is the one-period investment return, as in Cochrane (1991). Intuitively, taking the SDF as given, the firm should adjust its investment (and, therefore, payout) until the investment return is such that the firm is adequately compensated for the risk that it takes and, thus, no arbitrage opportunities remain.

⁷Since the firm's capital stock has to be non-negative, investment needs to satisfy $I_t \ge -(1-\delta)K_t$ for all t. We assume that capital adjustment costs are sufficiently large so that the non-negativity constraint never binds and, hence, the firm always optimally picks an interior solution for investment. In particular, we assume that the adjustment cost parameter a satisfies the condition $a > \frac{1}{(1-\tau)(1-\delta)}$. That condition ensures the feasibility of the interior solution, given the non-negativity of the firm's marginal q. Indeed, for $0 \le q_t < 1$, we have $I_t = \frac{q_t-1}{a(1-\tau)}K_t > (q_t-1)(1-\delta)K_t \ge -(1-\delta)K_t$. For $q_t \ge 1$, the interior optimality condition yields $I_t \ge 0 \ge -(1-\delta)K_t$.

2.1.3 Asset prices and returns

In Internet Appendix A, we show the firm's optimality conditions imply that the (cum-payout) firm value V satisfies

$$V_t = D_t + q_t K_{t+1},$$
 (13)

so the ex-payout firm value P satisfies

$$P_t = q_t K_{t+1}.\tag{14}$$

As a result, the firm's average Tobin's q, denoted by Q and defined as the ex-payout value of the firm per unit of capital (i.e., $Q_t \equiv \frac{P_t}{K_{t+1}} = \frac{V_t - D_t}{K_{t+1}}$) is identical to the firm's marginal q:

$$Q_t = q_t. (15)$$

It follows that the firm return (i.e., the return on the firm's equity), given by $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{V_{t+1}}{V_t-P_t}$, is identical to the investment return (i.e., the return on resources invested in the firm's production process):

$$R_{t+1} = R_{t+1}^I.$$
 (16)

Much of the investment-based asset pricing literature directly calculates firm returns R (using financial market data) and investment returns R^{I} (using firm accounting data) and tests moment conditions implied by Equation 16. While that approach has been fruitful, it is not ideal for the purpose of evaluating the asset pricing implications of production models, as mismatches between R and R^{I} can be a consequence of measurement errors, especially given the difficulty in aligning the timing of financial market and accounting data. We take a different approach: we retrieve the model-implied firm returns R^* by specifying a statistical model for the firm's payout process D and compare the properties of model-implied returns R^* with those of observed returns R, calculated using financial market data.

2.2 The Payout-Based Approach

For any given set of conditional SDF distributions, Equation 11 implies a particular q process, which potentially depends on both the current state of the economy and the properties of future SDFs. Plugging that q into the optimal investment and payout policies, I(K;q) and D(K,Z;q), respectively, yields the firm's investment and payout series. This is the approach taken by the subset of the production-based asset pricing literature that specifies exogenous SDFs (e.g., Zhang (2005)). As we discuss next, we take a different approach: instead of specifying an exogenous SDF, we specify a statistical model for the firm's payout process, which allows us to recover the model-implied q process and, hence, back out the firm's model-implied price and return.

2.2.1 Statistical model of the firm's payout process

To operationalize our approach, we consider a statistical model of the firm's payout process, D. In particular, we assume that D satisfies

$$D = d \cdot Z \cdot K = d \cdot Y,\tag{17}$$

where the stochastic process d (firm payout per unit of output) has law of motion

$$d_{t+1} = \mu_d + \phi_d \cdot (d_t - \mu_d) + \sigma_{d,t} \cdot \epsilon^d_{t+1}, \tag{18}$$

where $\phi_d \in (0,1)$, $\epsilon_{t+1}^d \sim N(0,1)$ and $corr(\epsilon_{t+1}^z, \epsilon_{t+1}^d) = \rho_{z,d}$. The conditional volatility process is

$$\sigma_{d,t} = \sigma_d \cdot \sqrt{d_t^{max} - d_t},\tag{19}$$

where $\sigma_d > 0$ and d_t^{max} is the conditional upper bound of d, given by

$$d_t^{max} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2} \cdot \varphi \right) \varphi + \tau \delta \right],$$
(20)

where $\varphi = \min\{1/(a(1-\tau)), 1-\delta\}.$

The statistical model for the firm's payout process is specified so that the payout is admissible, i.e., consistent with the firm's optimizing behavior. In particular, the payout process is specified to be linear in the firm's capital stock K: given its technology, the firm's optimal payout is always (i.e., for any SDF specification) proportional to its capital stock, so a postulated D process that violates that restriction cannot be consistent with any SDF. Furthermore, as shown in Internet Appendix A, the assumption that the realizations of d have the conditional upper bound given by Equation 20 ensures that our payout specification implies feasible firm payout and investment processes: in the absence of that bound, the postulated payout process could have realizations too large to be satisfied by any feasible firm investment policy. Inter alia, as will be made clear when we derive the implications of our model, that upper bound ensures that the implicit marginal q of the firm is always real-valued and non-negative.

2.2.2 Model-implied marginal q

We now show that imposing the statistical model for the firm's payout D allows us to back out the firm's q and, hence, all the information regarding investor preferences that is relevant to the firm's decisions.

Indeed, imposing the condition that the firm's optimal payout equals the postulated payout yields

$$D(K_t, Z_t; q_t) = d_t Z_t K_t.$$
(21)

Using Equation 10, we can rewrite the expression above as

$$\left[(1-\tau)\alpha Z_t - \frac{q_t^2 - 1}{2a(1-\tau)} + \tau \delta \right] K_t = d_t Z_t K_t,$$
(22)

so we can solve for the model-implied q, denoted by q^* , as a function of z_t and d_t :

$$q_t^* = q^*(z_t, d_t) = \sqrt{1 + 2a(1 - \tau)\left[((1 - \tau)\alpha - d_t)e^{z_t} + \tau\delta\right]}.$$
(23)

As discussed previously, our d specification (which imposes the restriction that $d_t \leq d_t^{max}$ for all t) ensures that q^* is always real-valued and non-negative. The model-implied q is a function of z and d, but (due to the constant returns to scale production function) does not depend on the level of the firm's capital stock, K. In particular, q^* is increasing in (log) productivity z: intuitively, higher productivity implies that the firm has more resources available to allocate to either investment or payout so, for a given level of payout ratio d, higher productivity implies a higher investment rate, which implies a higher marginal q. On the other hand, q^* is decreasing in d: for a given level of z, a higher payout ratio d implies a lower investment rate and, hence, a lower model-implied q.

2.2.3 Model-implied asset prices and returns

We can now turn to the asset pricing implications of the model. Given the equivalence between average and marginal q in the model, the firm's model-implied average q is given by

$$Q_t^* = Q^*(z_t, d_t) = \sqrt{1 + 2a(1 - \tau)\left[((1 - \tau)\alpha - d_t)e^{z_t} + \tau\delta\right]},$$
(24)

and inherits all the properties of q^* .

The model-implied realized return, denoted by R^* , is given by

$$R_{t+1}^* = \frac{d_{t+1}e^{z_{t+1}} + Q^*(z_{t+1}, d_{t+1})\left(1 - \delta + \frac{Q^*(z_{t+1}, d_{t+1}) - 1}{a(1-\tau)}\right)}{Q^*(z_t, d_t)},\tag{25}$$

so the model-implied expected return of the firm is

$$\mathcal{R}^*(z_t, d_t) = \mathbb{E}\left[R_{t+1}^* \mid z_t, d_t\right].$$
(26)

We evaluate the functions $Q^*(z, d)$ and $\mathcal{R}^*(z, d)$ using Equations 24 and 26 and the laws of motion for z and d (Equations 2 and 18, respectively). The integral needed for the evaluation of the $\mathcal{R}^*(z,d)$ function is computed using Gauss-Hermite quadrature, with 31 grid points per shock.

Figure 1 displays the model-implied average q function, $Q^*(z, d)$, and the model-implied expected return function, $\mathcal{R}^*(z,d)$, under the calibration described in Section 3.1. Panels A and B show the value of Q^* for different values of z and d, respectively, keeping the other variable constant. Consistent with our previous discussion, Q^* is increasing in z and decreasing in d. Panels C and D show the value of \mathcal{R}^* for different values of z and d, respectively, everything else constant. \mathcal{R}^* is almost completely flat in z and strongly increasing in d. As regards z, there are two opposing forces operating on the model-implied expected return. On the one hand, higher current productivity increases the firm's current operating profit and, hence, tends to increase the firm's optimal payout, so, keeping the payout ratio d fixed, the model implies a lower expected return. On the other hand, due to the persistence of process z, higher current productivity implies higher future productivity, which entices the firm to increase current investment and lower current payout. Hence, for d to be kept unchanged, the model-implied expected return needs to be higher. Under our parametrization, these two forces offset each other, producing a model-implied expected return that is effectively insensitive to z. As regards the payout ratio process, an increase in d, other things equal, raises the postulated payout of the firm, without affecting the firm's operating profit. As a result, the firm's model-implied cost of capital rises in order to rationalize a lower investment and a higher payout. Crucially, while the firm's model-implied expected return exhibits very moderate variation across different values of z, it is very sensitive with respect to d, underscoring the importance of payout ratio fluctuations as a driver of model-implied expected returns. For the same reason, the firm's Q^* is much more sensitive to changes in d than to changes in z.

Figure 1 suggests that the main source of variation in firm value and expected returns in our model is variation in the payout ratio d, whereas productivity fluctuations have a muted effect. To illustrate that point, we consider a single 100-year simulation path of the model and report the paths of variables z and d, as well as the paths of the firm's model-implied Q and expected return, in Figure 2. As seen in Panels A and B, both z and d exhibit substantial variation across time. Nevertheless, Panels C and D show that Q and expected returns vary mainly due to variation in d. Specifically, Panel C plots the simulated path of the firm's Q, as well as two counterfactual paths, each allowing for time variation in only one state variable. Similarly, Panel D plots the simulated path, and the two counterfactual paths, of the firm's expected return. As seen in both panels, almost all of the variation in the firm's Q and its expected return is due to variation in d.

2.3 Payout-based vs. investment-based approach

Our approach recovers model-implied asset prices by imposing a statistical model for the firm's payout process. As pointed out by Cochrane (1988), as long as the statistical model for the firm's payout process matches the observed (i.e., equilibrium) payout process of the firm, our approach

recovers the firm's true return process.⁸ Alternatively, we could back out asset prices by specifying a statistical model for the firm's investment process.

In the case of an unlevered firm, the two approaches are essentially identical from a theoretical point of view, as proposing a statistical model for the firm's investment process implies a statistical model for the firm's payout process, and vice versa. To see this, note that firm optimization implies that we can write the firm's investment rate i = I/K as a function of z and d:

$$i_t = \frac{\sqrt{1 + 2a(1-\tau)\left[((1-\tau)\alpha - d_t)e^{z_t} + \tau\delta\right]} - 1}{a(1-\tau)}.$$
(27)

Hence, specifying a statistical model for any two of those processes implies a statistical model for the third. Our approach relies on recovering q (and, thus, firm returns) by modeling processes zand d, so the model-implied i is given by Equation 27. We could, instead, model z and i, recover the implicit d process (from Equation 27), and then proceed as we do. In addition, as we show below, modeling z and i allows us to directly recover the firm's return process by imposing firm optimization, without the need to consider the firm's payout process at all. Thus, in the case of the unlevered firm, shifting the focus from investment to payout is not important for theoretical reasons, but because it shifts the focus from investment to payout data. ⁹

Why do we want to shift the focus from investment to payout data? As discussed in the introduction of our paper, properly defining and measuring investment, which is essential for calibrating a statistical investment process, poses significant challenges. In contrast, firm payout can be unambiguously defined and measured, allowing for a more precise evaluation of production-based asset pricing models. To illustrate that point, we compare the asset pricing implications of our payout-based approach against those of the investment-based approach (i.e., the approach in which investment, rather than payout, is the statistically modeled process) for exactly the same technological specification of the representative firm (i.e., for exactly the same model of the representative firm) and, in particular, for our baseline model specification. As discussed above, since the firm in our baseline model is equity-financed, the two approaches should yield *identical* model-implied returns if firm investment and payout are perfectly measured. As we show next, that is not the case empirically: the payout-based approach generates much more realistic asset pricing implications than the investment-based approach, suggesting that the investment-based approach suffers from more severe measurement issues than the payout-based one.

 $^{^{8}\}mathrm{We}$ formalize that point through a simple two-period general equilibrium model, presented in Internet Appendix B.

⁹It is important to stress that, whereas modeling d and modeling i are essentially equivalent in the context of a firm financed by the issuance of one type of claim (typically common stock), that is no longer the case when we consider a firm financed by issuing multiple types of claims. As we show in Section 4, which features a model with a representative firm that issues both equity and debt, modeling d^b and d^e (the debt and equity payout process, respectively) allows us to back out the firm's cost of debt and cost of equity separately, something that cannot be achieved by modeling i. Hence, in that case, focusing on payout, rather than investment, provides clear theoretical advantages.

2.3.1 Implementation

As pointed out in Cochrane (1991), any production-based model in which investment return (R^{I}) realizations and firm return (R) realizations have to coincide every period can be trivially rejected. For that reason, the level of success of those models, including ours, needs to be evaluated on a more realistic standard: the degree of similarity between model-implied firm returns R^* (which are identical to investment returns R^{I} in the model) and actual firm returns R, calculated from financial market data. In particular, we compare the two approaches in two dimensions, in the spirit of Cochrane (1991). First, are actual aggregate returns forecastable better by investment-related or payout-related variables? Second, are realized aggregate returns more similar to the realized returns backed out by the investment-based approach or the payout-based realized approach?

To implement the investment-based approach, we replace the statistical model for the firm's payout ratio d with a statistical model for the firm's investment rate process i (investment per unit of capital, i.e., i = I/K), retaining all the other aspects of our model. In particular, we assume that i has law of motion

$$i_{t+1} = \mu^i(i_t) + \sigma^i(i_t) \cdot \epsilon^i_{t+1},$$
(28)

where $\epsilon_{t+1}^i \sim N(0,1)$, $corr(\epsilon_{t+1}^z, \epsilon_{t+1}^i) = \rho_{z,i}$, and functions μ^i and σ^i are such that process *i* is both stationary and admissible.¹⁰ Then, we retrieve an expression for the model-implied average and marginal *q* of the firm as a function of *i*:

$$Q_t^* = Q^*(z_t, i_t) = q_t^* = 1 + a(1 - \tau)i_t.$$
(29)

It follows that the model-implied firm return is

$$R_{t+1}^* = \frac{(1-\tau)(\alpha e^{z_{t+1}} + \frac{a}{2}i_{t+1}^2) + \tau\delta + (1-\delta)Q^*(z_{t+1}, i_{t+1})}{Q^*(z_t, i_t)}.$$
(30)

so the model-implied expected return of the firm is given by

$$\mathcal{R}^*(z_t, i_t) = \mathbb{E}\left[R_{t+1}^* \mid z_t, i_t\right].$$
(31)

Note that, for our purposes, we do not need to specify functions μ^i and σ^i in any more detail. A full specification is necessary only if we need to back out the time series of model-implied expected returns, which is not necessary for our comparison exercise.

For the same reason, in our comparison exercise, we implement the payout-based approach without

¹⁰For example, both those requirements are satisfied by

 $i_{t+1} = \mu_i + \phi_i \cdot (i_t - \mu_i) + \sigma_i \cdot \sqrt{i_t + \varphi} \cdot \epsilon_{t+1}^i,$

where $\phi_i \in (0, 1)$: that process is stationary and has a lower bound at $-\varphi$, ensuring its admissibility.

the need to adopt the specification of Equations 17 and 18 for d; instead, we assume that d satisfies

$$d_{t+1} = \mu^d(d_t) + \sigma^d(d_t) \cdot \epsilon^d_{t+1}$$

where functions μ^d and σ^d are such that process d is stationary and admissible. Obviously, the specification of Equations 17 and 18 is a special case of that generic law of motion. Hence, the results of our comparison exercise are applicable to a very broad array of implementations of the payout-based and the investment-based approach.

2.3.2 Return predictability regressions

The first part of our comparison exercise focuses on return predictability. The payout-based and the investment-based approach differ on the identity of the signal to use in order to forecast aggregate returns. As seen in Equation 26, the payout-based approach implies that aggregate returns should be forecastable by lagged aggregate payout ratios d = D/Y with a positive sign. On the other hand, the investment-based approach suggests that aggregate returns should be forecastable by lagged investment rates i = I/K (Equation 31) with a negative sign. To compare the two approaches, we regress observed U.S. aggregate firm returns on lagged empirical proxies of U.S. aggregate d and aggregate i and report our findings in Table 1.

Panel A of Table 1, which presents our benchmark results, uses annual U.S. aggregate firm returns over the 1974–2017 sample period as the dependent variable. As detailed in Internet Appendix D, those returns are obtained from the Davydiuk, Richard, Shaliastovich and Yaron (2023) dataset. Column two, titled "d", reports the output of regressions of aggregate returns on lagged payout ratios d, following the payout-based approach. Columns three to ten report the output of regressions of aggregate returns on lagged investment rates i, as per the investment-based approach, using eight different empirical measures of i, the details for the construction of which are discussed in Internet Appendix E.

We find that aggregate firm returns are strongly forecastable by lagged payout ratios: the slope coefficient is positive (1.36) and statistically significant at the 1% level, with the regression adjusted R^2 being 8.62%. In contrast, we find no evidence of forecasting ability for any of the eight investment rate measures: seven out of the eight slope coefficients have a negative sign, consistent with the prediction of the investment-based approach, but none of those is significant at any conventional level. Furthermore, the adjusted R^2 is quite small for all eight regressions, ranging from -2.44% to 3.79%.

In the interest of robustness and following Cochrane (1991), in Panel B we report the output of regressions in which the dependent variable is annual U.S. aggregate equity returns from the Davydiuk et al. (2023) dataset, keeping the same sample period as before (1974–2017). We find

that equity returns are even more strongly forecastable by payout ratios than firm returns: the slope coefficient is 1.87 and significant at the 1% level, with the regression adjusted R^2 increasing to 12.30%. On the other hand, the forecasting power of investment rates continues to be negligible: none of the eight slope coefficients is statistically significant and the regression adjusted R^2 s continue to be low (3.49% at the highest).¹¹

Finally, in Panel C of Table 1 we continue to focus on annual equity returns, but now extend the sample period to 1950–2017. Since the Davydiuk et al. (2023) dataset starts in 1974, we consider CRSP value-weighted equity returns. Notably, including the earlier years does not materially affect the forecasting power of payout ratios: the slope coefficient is 1.69, significant at the 1% level, and the regression adjusted R^2 is 9.04%. On the other hand, the inclusion of the early part of the sample period substantially enhances the predictive ability of investment rates. In particular, all eight investment rate coefficients are negative, with half of them being statistically significant. The best performing measure is i_{WG} : the slope coefficient is -3.86, significant at the 5% level, and the regression adjusted R^2 is 5.86%. Still, the payout ratio continues to be a better predictor of aggregate equity returns than the investment rate.

The improvement of the return forecasting ability of investment rates when we include the early part of the sample period is not surprising: Cochrane (1991), focusing on the 1947–1987 period, provides strong evidence that aggregate equity returns are forecastable by investment rates. However, as evidenced by the findings of Panel B, the forecast power of investment rates diminishes in the latter part of the sample period. A potential explanation for the deterioration in the forecast performance of investment rates in recent years is the increased importance of intangible capital (see, for example, Corrado, Haskel, Jona-Lasinio and Iommi (2022) and Crouzet, Eberly, Eisfeldt and Papanikolaou (2022)), which increases the difficulty of accurately measuring firm capital and investment.

Finally, we explore the joint forecasting power of the payout ratio d and the investment rate i for aggregate returns by considering return predictability regressions that use the two signals together as regressors, for eight different empirical measures of i. We report our findings in Table 2. Panel A considers the forecastability of annual aggregate firm returns in the 1974–2017 sample period. Across all eight specifications, the payout ratio coefficient is positive and strongly significant. On the other hand, the investment rate coefficient is non-significant across all specifications and, in addition, has the theory-implied (i.e., negative) sign only for i measures calculated using COMPU-STAT data. Our findings remain essentially unchanged when we replace firm returns with equity returns (Panel B). Finally, in Panel C, we continue to focus on equity returns, but extend the sample period to 1950–2017. We find that the forecast coefficient of the payout ratio d is still positive and statistically significant across specifications, but (with one exception) investment rate

¹¹Our findings are almost identical when we consider CRSP value-weighted equity returns instead of the Davydiuk et al. (2023) equity returns, as the correlation between the two time series is 0.997.

coefficients are non-significant. The exception is the coefficient on on i_{WG} , which is significant at the 10% level – in fact, the specification that includes both d and i_{WG} as forecast signals is the best-performing one, as it has an adjusted R^2 of 11.16%.

In summary, U.S. aggregate returns are strongly predictable by the aggregate payout ratio d, consistent with the predictions of the payout-based approach. On the other hand, investment rates exhibit return forecast power only when we include the early (1950–1973) part of the sample, but not when we focus on the post-1973 period, casting severe doubt on whether using the investment-based approach is appropriate for testing the asset pricing implications of production-based models.

2.3.3 Payout-based vs. investment-based returns

The second part of our comparison exercise consists of comparing the model-implied realized firm returns backed out by the payout-based and investment-based approaches with the realized aggregate returns observed in the United States. To do so, we set the realizations of processes z, d, and i equal to the corresponding observed aggregate values (calculated as discussed in the calibration section), and set all parameter values to the calibrated values of Table 4. Given the time series for z, d and i, the model-implied realized annual returns are calculated using Equation 25 for the payout-based approach and Equation 30 for the investment-based approach.

Our findings are reported in Table 3. Panel A focuses on the 1974–2017 sample period and benchmarks annual model-implied returns (from both approaches), denoted by R^* , against annual U.S. aggregate firm returns obtained from the Davydiuk et al. (2023) dataset, denoted by R. In particular, column two reports the properties of payout-based implicit returns, whereas columns three to ten report the properties of model-implied returns backed out by eight different implementations of the investment-based approach, each of which corresponds to a different empirical proxy for the aggregate investment rate i, as discussed before.

As seen in Panel A, U.S. aggregate firm returns have a mean of 7.86% and a standard deviation of 14.88%. The payout-based approach retrieves model-implied returns that have unconditional moments that are quite close to the corresponding moments of actual returns: their mean is 5.64% and their standard deviation is 14.83%. Crucially, the model-implied returns are highly positively correlated with observed returns: the unconditional correlation coefficient is 0.57. Following the logic of Cochrane (1991) on the timing of investment expenditures, we also calculate the unconditional correlation between our model-implied returns and actual returns shifted by six months, denoted by $R^{shifted}$ – for example, the shifted return for the year 2000 is the return from July of 1999 to June of 2000.¹² However, shifting the timing of actual returns lowers their association with their model-implied counterparts, as the correlation coefficient drops to 0.22, suggesting that the

 $^{^{12}}$ To calculate the shifted annual aggregate firm returns, we use the quarterly aggregate firm returns of the Davydiuk et al. (2023) dataset.

payout-based approach matches firm returns better when the standard timing convention is used.

We now turn to model-implied returns retrieved using the investment-based approach – our findings are reported in the last eight columns of Panel A. None of the eight implementations of the investment-based approach are able to retrieve model-implied returns that match both the mean and the standard deviation of actual returns. In particular, implementations that match the observed return mean (such as those associated with i_{WG} and i_{FAA}^p) generate counterfactually smooth returns, whereas implementations that that match the observed return volatility (such as those associated with *i* measures derived using COMPUSTAT data) generate returns with counterfactually low means. Importantly, all eight implementations generate model-implied returns that are uncorrelated with actual returns when we use the standard timing convention: the correlation coefficients range from -0.07 to 0.14. When we shift the timing of actual returns, the correlation coefficient between model-implied and actual returns increases for half the implementations and declines for the rest, with the best performing implementation being the one associated with i_{WG} , which achieves a correlation coefficient of 0.36. In short, adopting the timing shift proposed by Cochrane (1991) for the purpose of matching the timing of investment spending, generally improves the performance of the investment-based approach, as intended. Still, even focusing on just the return correlation metric, the payout-based approach performs better: the return correlation coefficient associated with the payout-based approach (using the standard timing) is more than 1.5 times the return correlation coefficient of the best-performing implementation of the investment-based approach (using the shifted timing).

Panel B of Table 3 contrasts model-implied realized returns with observed realized equity returns in the 1974–2017 period. As we see, the main findings of Panel A still apply.¹³ Finally, Panel C continues to focus on observed equity returns, but extends the sample period to 1950–2017. The payout-based approach generates model-implied returns that match the first two moments of observed returns: model-implied returns have a mean of 7.77% and a standard deviation of 16.17%, both of which are quite close to their empirical counterparts (8.84% and 17.40%, respectively). The correlation between model-implied and actual returns is 0.32 using the standard timing and 0.07using the shifted timing, underscoring the fact that payout does not suffer from the timing issues of investment regarding firm return alignment. Turning to the implementations of the investmentbased approach, we note that, as before, no implementation is able to jointly match the first two moments of observed returns. Furthermore, the correlation of model-implied and actual returns is either negative or very close to zero under the standard timing. Shifting the timing increases the correlation in all eight implementations: the correlation coefficients range from 0.02 to 0.24, with the best performing implementation being the one associated with i_{WG} . In short, the payout-based approach continues to dominate the investment-based approach even when we include the early post-war years.

¹³As before, we get very similar results when we use CRSP value-weighted equity returns instead of the Davydiuk et al. (2023) equity returns.

Finally, we consider two robustness exercises. First, to illustrate the fact that our findings do not hinge on our assumptions regarding the firm productivity process, we calculate model-implied returns assuming that aggregate productivity Z is constant, as in Cochrane (1991), and show that our results do not materially change. Second, to check the robustness of our results to alternative values of the model parameters, we calculate model-implied returns by using implementation-specific parameters (estimated using a Non-Linear Least Squares approach), which provide each implementation with its best chance to match firm returns, and confirm the superior performance of out payout-based approach. The details are provided in Internet Appendix E.

In short, we find that the payout-based approach generates model-implied firm returns that are much more connected to observed firm returns than any implementation of the investment-based approach. Thus, shifting the focus from investment to payout data, as our framework does, yields more realistic asset pricing implications even in models of firms that finance themselves with one type of security.

3 Baseline Model: Quantitative Results

This section provides the quantitative results of our baseline model. We start by describing our calibration process. Then, we retrieve the model-implied U.S. aggregate expected returns in the 1974–2017 sample period and document that they give rise to an "excess sensitivity" puzzle. Finally, we simulate our model and discuss its asset pricing implications.

3.1 Model calibration

We report our model calibration in Table 4. Tax and technological parameters are calibrated following the extant literature: we set $\tau = 0.35$ and $\delta = 0.15$, following DeAngelo et al. (2011). Since the capital adjustment cost specification in DeAngelo et al. (2011) is not comparable to ours, we set the adjustment cost parameter such that the adjustment cost annual expense is 10% of the firm's capital in the steady state (which corresponds to less than 5% of the firm's annual output in the steady state).¹⁴ The resulting value of a = 9.947 is squarely within the range of values used in the prior quantitative literature, as documented by Li et al. (2023). Finally, we set the profit margin parameter to a = 0.15, which is the estimated value in Li et al. (2023) and also very close to the estimate obtained in Gonçalves et al. (2020).

The rest of the parameters are calibrated so as to match empirical moments. The data sample used to calculate those moments comprises annual observations of aggregate output Y and payout

¹⁴Mathematically, we set $\frac{a}{2}(1-\tau)i_{ss}^2 = 0.1$, which implies $a = 0.2/((1-\tau)i_{ss}^2)$. We, then, set the steady state investment-to-capital ratio to $i_{ss} = e^g - 1 + \delta$, as dictated by the capital accumulation equation, where g = 0.025 in order to match the average output growth in our dataset.

D from 1974 to 2017. We construct those measures using CRSP and COMPUSTAT data, as well as the dataset in Davydiuk et al. (2023), with D representing total payout of U.S. public firms to equity and debt investors (which includes dividends, interest payments, equity repurchases and issuances, and debt paydowns and issuances). The sample period is restricted by the Davydiuk et al. (2023) dataset, which is important for our analysis since it provides information on debt payouts as well as the market value of corporate debt. Internet Appendix D provides details on the data sources and empirical measurement for Y and D. It also discusses the methodology we use to generate the productivity (Z) time series, which relies on combining the Y and D data with the budget constraint and capital accumulation equation, in a fashion analogous to how the aggregate investment-to-capital ratio is calculated in Cochrane (1991).

The time series for the U.S. aggregate firm payout ratio (i.e., firm payout divided by firm output) from 1974 to 2017 is plotted in Figure 3. The figure also plots its two components, the aggregate equity payout ratio and the aggregate debt payout ratio. As seen in the figure, the firm payout ratio exhibits considerable time variation, taking both positive and negative values over the sample period. Notably, the payout ratio turns sharply negative in the late 1990s and spikes up during the global financial crisis.

We set $\mu_z = 0.983$, $\phi_z = 0.745$, and $\sigma_z = 0.061$ to match the average log productivity level, the autocorrelation of the log productivity process, and the volatility of the log productivity autoregressive shocks, respectively. The payout process parameters are set to $\mu_d = 0.015$, $\phi_d = 0.595$, and $\sigma_d = 0.073$ in order to match, respectively, the mean and the autocorrelation of the empirical payout-to-output ratio d, as well as the volatility of the payout-to-output ratio autoregressive shocks normalized by lagged $\sqrt{d_t^{max} - d_t}$, in line with Equation 18.¹⁵ Finally, we set $\rho_{d,z} = -0.134$ so as to match the unconditional correlation between the d and z autoregressive shocks.

3.2 Model-implied expected U.S. returns in the 1974–2017 period

One of the main benefits of specifying a statistical process for the firm's payout is that it allows us to retrieve the time series of the model-implied *expected* returns. For this exercise, we set all parameter values to the calibrated values of Table 4. To retrieve the time series of the expected annual returns, we use the laws of motion for z and d (Equations 2 and 18, respectively), evaluating the resulting integrals of Equation 26 using Gauss-Hermite quadrature with 31 grid points per shock, and set the realizations of the z and d shocks to the values needed so that our model generates exactly the U.S. aggregate productivity and payout realizations observed in the 1974-2017 period (calculated as discussed in the calibration section).

¹⁵Note that we calibrate *d* to match the *observed* firm payout data. This approach is consistent with the discussion in Cochrane (1988). Furthermore, it is consistent with the implications of our two-period model in Internet Appendix B: our payout-based approach yields the same asset pricing implications as a fully specified general equilibrium model if and only if the statistical model for the payout process reflects the properties of the equilibrium (i.e., observed) payout process in the general equilibrium economy.

Panel A of Figure 4 plots the 1974–2017 time series of our model-implied expected returns. As seen in the graph, expected returns exhibit substantial time variation – their standard deviation is 7.01%, accounting for about half of the volatility of realized returns (14.83%). Furthermore, expected returns are countercyclical: their unconditional correlation with the aggregate D/P ratio is 0.76. As implied by theory, model-implied expected returns exhibit strong positive comovement with the payout ratio D/Y – the unconditional correlation between the two series is 0.92. For example, we document a big spike in expected returns during the recent financial crisis, consistent with the spike in the aggregate payout ratio shown in Figure 3: in the context of an optimizing firm, a big increase in the payout ratio and, hence, a cut in investment spending relative to resources is rationalized only by an increase in the firm's cost of capital.

As seen in Equation 26, model-implied expected returns vary over time due to variation in either the payout ratio d or log productivity z. To calculate the sensitivity of model-implied returns to those two signals, we regress model-implied realized returns on lagged d and lagged z. Furthermore, to evaluate how well our model matches the properties of time variation in actual aggregate returns, we run the same predictability regressions for actual returns and compare the forecast coefficients between model-implied and actual returns. Our findings are reported in Table 5.

Panel A considers model-implied returns. When we regress model-implied returns on lagged d, the slope coefficient is 2.59, significant at the 1% level, and the adjusted R^2 of the regression is 30.06%. On the other hand, lagged log productivity z has no forecasting ability for model-implied returns: the slope coefficient is 0.05 (and not statistically significant) and the regression adjusted R^2 is negative. We get similar results when we include both signals as regressors: the coefficient on d is 2.62 (and strongly significant), whereas the coefficient on z is -0.08 (and non-significant). Panel B reports the output of the same predictability regressions for actual aggregate returns. Payout ratios do forecast actual returns: the slope coefficient is positive and significant at the 1% level. However, its magnitude (1.36) is about half of the magnitude of the corresponding forecast coefficient for model-implied returns and the regression adjusted R^2 is only 8.62%, suggesting that actual returns are much less sensitive to fluctuations in the payout ratio than model-implied returns and that, consistent with that, variation in the payout ratio accounts for a much smaller share of the overall variation in actual returns than in model-implied returns. Similarly to modelimplied returns, actual returns cannot be predicted by lagged z: both the slope coefficient and the regression adjusted R^2 are negligible. Finally, when both signals are included, only lagged d enters significantly and, again, its magnitude is much lower than the magnitude of the corresponding forecast coefficient for model-implied returns.

In short, we document that model-implied expected returns exhibit counterfactually large sensitivity to changes in the payout ratio, giving rise to a "return sensitivity puzzle". To illustrate that point, we run return forecast regressions on lagged d and plot the fitted values of model-implied and actual returns (green solid line and red dashed line, respectively) in Panel B of Figure 4. As seen in the figure, the fitted values are significantly more volatile for model-implied returns than for actual returns. In the next section, we simulate our model and show that the return sensitivity puzzle is a fundamental property of the canonical production-based model, rather than an artifact of the aggregate U.S. productivity and payout realizations in the 1974–2017 sample period.

3.3 Model simulation

To evaluate the asset pricing implications of the canonical model, we run 10,000 model simulations, each of which consists of 44 annual observations (after a burn-in period of 1,000 years) in order to match the size of our sample period. In our simulations, we update z and d according to their law of motion (with no state space discretization). Table 6 provides key asset pricing statistics in the data and in model simulations. Importantly, none of those statistics was used as a target moment for calibrating the model. For each simulation statistic, we report the median value across the 10,000 simulations, as well as the corresponding 5th and 95th percentiles.

Panel A of Table 6 presents unconditional moments of the output growth, the payout yield, and the return of the representative firm. As we see, the model generates realistic output growth properties: the output growth of the simulated firm closely matches the first and second moments of the U.S. aggregate output growth. In addition, in line with the properties of actual U.S. aggregate returns, simulated firm returns are uncorrelated with the firm's output growth rates, weakly positively correlated with the firm's productivity shocks, and strongly negatively correlated with the firm's payout shocks. However, the correlation between firm returns and payout shocks is much stronger in the model than in data (-0.85 and -0.54, respectively), suggesting that model-implied realized returns are counterfactually sensitive to payout fluctuations. That feature of the model is consistent with the return sensitivity puzzle discussed in the previous section: positive payout shocks are associated with counterfactually high increases in the firm's expected return, which generate large drops in realized returns through the present value identity.

The return sensitivity puzzle is also reflected in the properties of the firm's payout yield: while the model is able to match the payout yield mean and autocorrelation reasonably well, payout yield volatility is counterfactually high, indicating that model-implied firm prices are too volatile compared to the data. In particular, the payout yield has an unconditional mean of 1.59%, an unconditional volatility of 2.47% and unconditional autocorrelation of 0.44 in the data, with the median model values being 2.59%, 4.47% and 0.53, respectively. Notably, the 90% model range for payout yield volatility does not include the empirical value, underscoring the inability of the canonical model to generate plausible asset price volatility.

On the other hand, the model is able to generate unconditional return moments broadly consistent with data: the model-implied median values for the firm return mean, volatility, and autocorrelation (5.47%, 12.81%, and -0.20, respectively) are quite close to the corresponding empirical values

(7.86%, 14.88%, and -0.17, respectively). Despite that success, the model is not perfect: the actual U.S. firm return mean is above the 95th percentile of its simulated moment values and, hence, unlikely to be generated in the model. Consistent with our previous results, expected returns are quite volatile in the model: the median unconditional volatility of $\mathbb{E}[R]$ is 5.94%, accounting for a large share of realized return volatility.

Panel B of Table 6 reports the output of regressions of annual returns, R_{t+1} , on lagged payout yields, D_t/P_t . Those forecasting regressions allow us to explore the properties of time variation in expected returns in a setting analogous to the one typically used in the evaluation of consumptionbased asset pricing models. In the data, high payout yields forecast high future returns: the predictive coefficient is 1.81 (and significant at the 1% level) and the regression adjusted R^2 is 9.20%. The model yields a median forecast coefficient of 1.43 and a median regression adjusted R^2 of 23.68%, with the corresponding empirical values being well within the 90% range of simulated outcomes.

In our model, return predictability is strongly associated with fluctuations in the payout ratio d = D/Y: when the payout ratio is relatively high, the model-implied expected return is high in order to justify the implicit low investment rate, which suggests a positive relationship between the firm's payout ratio, D/Y, and its future return. Notably, there is no mechanical relationship between the payout ratio and future returns, as the payout ratio (unlike the payout yield) is not scaled by firm value. Panel C of Table 6 reports the output of regressions of R_{t+1} on D_t/Y_t . Both in the data and in our model, the predictive coefficient is positive. However, our simulated expected results exhibit excess sensitivity to payout ratios, consistent with our findings in the previous section. In particular, the median model-implied forecast coefficient on D/Y is much higher than its empirical counterpart: 1.94 in the model, compared with 1.36 in the data. Furthermore, the median adjusted R^2 is 23.68% of the return forecasting regression, more than double the corresponding empirical value of 8.62%, indicating that the payout ratio accounts for a counterfactually high share of return variation in the model. Despite the fact that the empirical value of the forecast coefficient is within the 90% simulation range, the disparity between the empirical value and the model median value is substantial and (as seen in Panel B of Figure 4, which refers to the 1974-2017 sample period) can be economically meaningful. In addition, the value of the adjusted R^2 in the associated return forecasting regression is outside the 90% simulation range, indicating that the model consistently overemphasizes the ability of payout ratio fluctuations to account for return fluctuations.

In summary, the canonical production-based model matches unconditional return moments reasonably well, but performs less well with respect to conditional return moments, as it generates a return sensitivity puzzle. Our findings suggest that the canonical model features a representative firm that is *counterfactually insensitive* to discount rate fluctuations: taking firm productivity as given, the model suggests that the firm payout ratio varies too little when discount rates change, so the only way to match the properties of the observed U.S. aggregate firm payout is by having very sensitive (in fact, counterfactually sensitive) discount rates. It follows that, to better match the conditional properties of aggregate returns, the model needs additional features that increase the sensitivity of the representative firm's optimal payout policy to cost of capital variation.

3.4 Sensitivity analysis

To further evaluate the asset pricing implications of the model, we consider how key moments change when we change the value of particular technological parameters: the capital adjustment cost parameter a, the depreciation rate δ , and the operating profit margin α . Table 7 reports the simulation results of those sensitivity analysis exercises: for each exercise, we re-run the model simulation as before, changing the value of one parameter and keeping the values of all other parameters the same as in the benchmark calibration.

We start with changes in the capital adjustment cost parameter a. Its benchmark value is a = 9.947, and we now consider the values a = 5.947 and a = 13.947. As we see in Table 7, changes in a have a large impact on the average level of firm returns: high values of a generate lower firm returns on average compared to the benchmark calibration. This is because, other things equal, an increase in the capital adjustment cost parameter lowers the firm's desired average investment level and, hence, increases its average desired payout level. For the firm to optimally provide the observed average payout level, the average cost of capital needs to be lower, so that the firm invests more and pays out less on average. Furthermore, expected and realized return volatility, as well as the return forecast coefficient on the payout ratio, are increasing in a. The intuition is straightforward: other things equal, higher values of a lower the responsiveness of investment and payout (as shares of output) to cost of capital fluctuations, so the firm's payout ratio is less sensitive to discount rate changes. It follows that the implied cost of capital needs to fluctuate more for the firm to optimally match the observed payout ratio process, increasing both the volatility of expected and realized returns and the return responsiveness to d.

Can the return sensitivity puzzle be solved by a lower value of the capital adjustment cost parameter a? The answer is no: changing the value of a entails a trade-off between matching the average level of firm returns and matching the return forecast coefficient. As we see in Table 7, setting a = 5.947 reduces the forecast coefficient to 1.71 (which is still higher than the empirical value of 1.36), but increases the return mean to 8.13%, above the average return level observed empirically (7.86%). A further reduction in the value of a would bring the model forecast coefficient closer to its empirical value, but would generate counterfactually high return levels.

Next, we turn to the effect of changes in the capital depreciation rate δ , which has a benchmark value of $\delta = 0.15$. Here, we consider simulations with $\delta = 0.05$ and $\delta = 0.25$. As shown in Table 7, average model-implied returns are decreasing in the depreciation rate. The reason is that,

other things equal, a higher capital depreciation rate lowers the desire of firm to invest resources internally and, therefore, can increase the firm's desired payout above its observed value. It follows that, in order for the firm to optimally provide the observed payout, its implicit cost of capital needs to be lower, so that the firm raises its investment and cuts its payout to the observed level. Furthermore, an increase in δ is associated with an increase in the sensitivity of investment to cost of capital fluctuations, which also increases the sensitivity of the payout ratio d. As a result, matching the observed volatility of d entails a less volatile implied cost of capital. Indeed, the discount rate volatility is decreasing in δ : it ranges from 6.88% for $\delta = 0.05$ to 5.14% for $\delta = 0.25$. Not surprisingly, the return sensitivity to the payout ratio is also decreasing in δ , although the effect is muted: for $\delta = 0.25$, the return forecast coefficient is 1.60, higher than its empirical value of 1.36.

Finally, we consider the impact of different values of the operating profit margin α : we consider the values $\alpha = 0.05$ and $\alpha = 0.25$. Increasing the value of α increases the average level of returns. Intuitively, higher profitability per unit of output increases the desire of the firm to invest and, hence, lower its payout. Hence, for the firm to optimally provide the observed level of payout, its cost of capital needs to be higher, which discourages investment and encourages payout. The return responsiveness to the payout ratio is sharply decreasing in α : the return forecast coefficient is 3.89 for $\alpha = 0.05$ and 1.34 for $\alpha = 0.25$. However, since the average level of returns is also very sensitive to the value of α , matching the return forecast coefficient would imply a counterfactually high level of average returns.

4 Adding Firm Leverage

In this section, we retain all our previous assumptions about the representative firm, with the exception that we now allow for leverage. The assumption that the firm can finance itself using both equity and debt not only makes the model more realistic, but allows us to separately recover the implicit returns on the firm's debt and equity. In particular, proposing a statistical model for the firm's debt and equity payout processes allows us to back out the representative firm's model-implied equity and debt returns and study their properties.

4.1 Model

We consider a representative firm that determines its optimal debt and equity payouts by jointly optimizing its investment and capital structure decisions. In the interest of simplicity, the only type of debt we consider is one-period risk-free debt. In particular, following Hennessy and Whited (2005), we assume that the firm is subject to a collateral constraint which ensures that all firm-issued debt is riskless. The firm optimally chooses its capital structure each period by considering a

tradeoff between debt costs and benefits. As a result of the tax deductability of interest payments, debt is beneficial to the firm as it generates an interest tax shield. On the other hand, debt entails financial distress costs, which we model in reduced form as convex leverage costs.¹⁶

4.1.1 Setting

The firm's capital structure decision has the following features. Each period, the firm issues oneperiod risk-free bonds: in particular, at t, the firm issues F_{t+1} one-period safe bonds, each with price p_t and face value 1. Therefore, the firm raises $B_{t+1} = p_t \cdot F_{t+1}$ at t and agrees to repay F_{t+1} at t+1. It follows that, at t, the one-period bond (net) yield is $y_t = 1/p_t - 1$, the one-period gross borrowing rate is $R_{t+1}^b = 1 + y_t = 1/p_t$, and the market value of debt is $P_{t+1}^b = p_t \cdot F_{t+1} = B_{t+1}$. The firm can expense its interest payments, so the period t+1 interest tax shield is $\tau(R_{t+1}^b - 1)B_{t+1}$ and the after-tax gross debt return is $R_{t+1}^{b,a} = R_{t+1}^b - \tau(R_{t+1}^b - 1)$. Debt issuance has to satisfy a collateral constraint, discussed below, that puts an upper bound to the firm's leverage.¹⁷

It follows that the firm's period t debt payout, denoted by D_t^b , is the difference between the repayment of existing debt and the funds raised by issuing new debt:

$$D_t^b = F_t - B_{t+1} = R_t^b B_t - B_{t+1}.$$
(32)

Since interest payments are tax deductible, debt yields a benefit to the firm in the form of a tax shield. In the absence of any countervailing leverage cost, the firm would choose to borrow up to its collateral constraint. Instead, we assume that leverage entails costs to the firm (such as potential costs of financial distress), which we model in reduced form by assuming that the firm pays a (non-deductible) cost $G_t = G(B_t, K_t)$ at period t.¹⁸ In particular, we assume that

$$G(B_t, K_t) = \frac{\kappa}{2} \left(\frac{B_t}{K_t}\right)^2 K_t, \tag{33}$$

¹⁷Since the firm issues safe debt, its pre-tax cost of debt R_{t+1}^b is the (pre-tax) risk-free rate in the economy: $1 = \mathbb{E}_t[M_{t+1}R_{t+1}^b] = \mathbb{E}_t[M_{t+1}]R_{t+1}^b = \frac{R_{t+1}^b}{R_{t+1}^f} \implies R_{t+1}^b = R_{t+1}^f = \frac{1}{\mathbb{E}_t[M_{t+1}]}$. It follows that the firm's after-tax cost of debt, $R_{t+1}^{b,a} = R_{t+1}^b - \tau(R_{t+1}^b - 1)$, also depends solely on the SDF and, thus, is taken as given by the firm. Our derivation implicitly assumes that the investor tax rate, denoted by τ^i , is zero. If, instead, $\tau^i > 0$, then R_{t+1}^b is still the pre-tax gross risk-free rate, but the investor Euler equation is $1 = \mathbb{E}_t[M_{t+1}(R_{t+1}^b - \tau^i(R_{t+1}^b - 1))]$. In our paper, we assume $\tau > \tau^i = 0$. That assumption ensures that the firm optimally chooses positive debt (i.e., that $B_{t+1} \ge 0$) when the bond yield is positive (i.e., when $y_t \ge 0$).

¹⁶Note that financial distress costs arise despite the fact that debt is riskless from the perspective of outside investors. Those costs refer to the operational and financial costs that the firm incurs to ensure that it always pays back its debt. For example, Hennessy and Whited (2005) propose a model in which financial distress costs arise from the fact that the firm may have to engage in costly fire sales of capital in order to raise resources to pay back the firm's (safe) debt in full. In our model, we do not take a stand on the particular nature of financial distress costs.

¹⁸The non-deductibility of the leverage cost does not impact our results. To see that, consider a model with tax-deductible leverage cost and leverage cost parameter κ' . We can easily show that, under the parametrization $\kappa = (1 - \tau)\kappa'$, that model is identical to our model.

for $\kappa > 0$, i.e., that the leverage cost per unit of capital is increasing and convex in the firm's debt per unit of capital $\frac{B_t}{K_t}$.¹⁹

Firm borrowing has to satisfy a collateral constraint that ensures that the firm issues safe debt. In particular, the amount promised to the debtholders cannot exceed the minimum resources available to them, i.e.,

$$F_{t+1} \le (1-\delta)K_{t+1} + (1-\tau)\alpha Z_{t+1}^{\min}K_{t+1} + \tau\delta K_{t+1} + \tau (R_{t+1}^b - 1)B_{t+1} - G(B_{t+1}, K_{t+1}), \quad (34)$$

where Z_{t+1}^{min} is the minimum value that Z_{t+1} can attain conditional on the information available at period t. The right-hand side collects the minimum resources available to the firm's creditors. Those resources consist of the value of the firm's undepreciated capital, plus the combined value of the firm's operating profit, depreciation tax shield, and interest tax shield, minus the leverage cost. Using the definition of the after-tax bond return, we can rewrite the collateral constraint as

$$R_{t+1}^{b,a}b_{t+1} \le (1-\delta) + (1-\tau)\alpha Z_{t+1}^{min} + \tau\delta - \frac{\kappa}{2}b_{t+1}^2,$$
(35)

where $b_{t+1} \equiv \frac{B_{t+1}}{K_{t+1}} = \frac{P_t^b}{K_{t+1}}$ is the firm's (market) leverage ratio, i.e., the market value of firm debt per unit of capital.

At each period t, the firm's manager chooses investment I_t , equity payout D_t^e , and borrowing B_{t+1} in order to maximize the cum-payout value of firm equity V_t^e :

$$V_t^e = \max_{\{I_{t+h}, D_{t+h}^e, B_{t+1+h}\}_{h=0}^{\infty}} \{D_t^e + \sum_{h=1}^{\infty} \mathbb{E}_t[M_{t,t+h}D_{t+h}^e]\},\tag{36}$$

where $\{M_{t,t+h}\}_{h=1}^{\infty}$ is the set of stochastic discount factors, taken as given by the firm, and D_t^e is the period t equity payout of the firm, given by

$$D_t^e = (1 - \tau)(\Pi(K_t, Z_t) - \Phi(I_t, K_t)) + \tau \delta K_t - I_t - R_t^{b,a} B_t + B_{t+1} - G(B_t, K_t).$$
(37)

4.1.2 The firm's optimal policies

The firm's problem can be rewritten recursively as

$$V^{e}(K_{t}, F_{t}, Z_{t}; \{f_{t}(M_{t,t+h})\}_{h=1}^{\infty}) = \max_{\{I_{t}, B_{t+1}\}} \{D_{t}^{e} + \mathbb{E}_{t}[M_{t,t+1}V^{e}(K_{t+1}, F_{t+1}, Z_{t+1}; \{f_{t+1}(M_{t+1,t+1+h})\}_{h=1}^{\infty})]\}$$
(38)

¹⁹In principle, B can be negative, in which case the firm holds cash and y represents the (pre-tax) net interest rate that the firm receives on its cash position. In that case, G is assumed to reflect the pecuniary impact of the agency cost of holding cash. In our model, B is negative if and only if y is negative. In our simulation, that is a relatively rare event, as it only happens in less than 0.5% of the years.

where, as before, $f_t(\cdot)$ denotes the distribution conditional on information available at time t. The firm's optimality conditions for investment and debt jointly determine the firm's equity and debt payout policies, taking the SDF properties as given.

We start with the firm's capital structure choice. The firm's interior optimality condition for B_{t+1} yields

$$1 + \mathbb{E}_t \left[M_{t,t+1} \left(\partial_F V^e(K_{t+1}, F_{t+1}, Z_{t+1}; \{ f_t(M_{t+1,t+1+h}) \}_{h=1}^\infty \right) \right) R_{t+1}^b \right] = 0,$$
(39)

and the envelope condition with respect to F_t is

$$\partial_F V^e(K_t, F_t, Z_t; \{f_t(M_{t,t+h})\}_{h=1}^{\infty}) = -\frac{R_t^{b,a}}{R_t^b} - \frac{\partial_B G_t}{R_t^b}.$$
(40)

Together, Equations 39 and 40 yield the Euler equation

$$\mathbb{E}_t \left[M_{t,t+1} \cdot (R_{t+1}^{b,a} + \partial_B G_{t+1}) \right] = 1.$$

$$\tag{41}$$

The left-hand side of the equation is the present value of the firm's effective cost of one additional unit of debt raised at period t: at period t + 1, the firm pays both the after-tax return $R_{t+1}^{b,a}$ and the marginal leverage cost $\partial_B G_{t+1}$. Conversely, the right-hand side of the equation is the marginal benefit to the firm of one unit of additional debt raised at t, which is always equal to 1. Intuitively, for a given SDF, the firm's optimal capital structure is the one that eliminates any arbitrage opportunities for the firm. Thus, we get a simple trade-off condition: the optimal (interior) capital structure of the firm is the one that equates the firm's discounted marginal cost and marginal benefit of debt.

Since $R_{t+1}^{b,a}$, K_{t+1} , and B_{t+1} are known at period t, and using the fact that $\mathbb{E}_t \left[M_{t,t+1} \cdot R_{t+1}^b \right] = 1$, we can solve for the firm's optimal leverage ratio b_{t+1} :

$$b_{t+1} = \frac{\tau}{\kappa} (R_{t+1}^b - 1) = \frac{\tau}{\kappa} y_t.$$
(42)

The firm's optimal leverage ratio fluctuates over time only due to fluctuations of the firm's cost of debt and, in particular, is increasing in the debt yield, as a higher yield is associated with a more valuable interest tax shield. As regards comparative statics, the optimal leverage ratio is increasing in the corporate tax rate τ (as a higher tax rate implies a larger tax shield) and decreasing in the leverage cost parameter κ (as higher financial distress costs discourage debt issuance).

We now turn to the firm's investment policy. The firm's interior optimality condition for investment I_t is

$$\underbrace{\mathbb{E}_{t}[M_{t+1}\partial_{K}V^{e}(K_{t+1}, B_{t+1}, Z_{t+1}; \{f_{t}(M_{t+1,t+1+h})\}_{h=1}^{\infty})]}_{\equiv q_{t}} = 1 + (1 - \tau)\partial_{I}\Phi(K_{t}, I_{t}), \qquad (43)$$

which yields the firm's investment function:

$$I_t = \frac{q_t - 1}{a(1 - \tau)} K_t.$$
 (44)

As in the case of the unlevered firm, the firm's optimal investment is increasing in its marginal q and is proportional to its capital stock.²⁰

After solving for the firm's optimal investment and capital structure policies, we are ready to characterize the firm's optimal debt and equity payout policies. Substituting the firm's optimal investment policy (Equation 44) and optimal debt policy (Equation 42) into Equation 32, we get the firm's optimal debt payout policy,

$$D_t^b = \left[R_t^b b_t - \frac{\tau}{\kappa} y_t \left((1 - \delta) + \frac{q_t - 1}{a(1 - \tau)} \right) \right] K_t.$$

$$\tag{46}$$

Similarly, substituting the firm's optimal investment and debt policy into Equation 37, we derive the firm's optimal equity payout policy,

$$D_t^e = \left[\alpha(1-\tau)Z_t + \tau\delta - \frac{q_t^2 - 1}{2a(1-\tau)} - R_t^{b,a}b_t + \frac{\tau}{\kappa}y_t\left((1-\delta) + \frac{q_t - 1}{a(1-\tau)}\right) - \frac{\kappa}{2}b_t^2\right]K_t.$$
 (47)

In summary, the firm's optimality conditions determine the firm's optimal debt and equity payout policies (given by Equations 46 and 47, respectively). Notably, the firm's period t optimal payouts are functions of one contemporaneous exogenous variable (productivity Z_t), two contemporaneous endogenous variables (the firm's marginal q_t and the bond yield y_t), and one pre-determined endogenous variable (R_t^b – note that b_t is a function of R_t^b through Equation 42). Hence, taking the exogenous productivity process Z as given, the firm's marginal Tobin's q and the bond yield processes (q and y, respectively) encode exactly the same information as the firm's debt and equity payout processes (D^b and D^e , respectively). In what follows, we exploit that insight as follows: we propose a statistical model for the firm's debt and equity payout processes and then use that statistical model to recover the model-implied q and y processes, and, hence, the firm's model-implied debt and equity return processes.

$$q_{t} = \mathbb{E}_{t} \left[M_{t,t+1} \left((1-\tau) \left(\partial_{K} \Pi(K_{t+1}, Z_{t+1}) - \partial_{K} \Phi(I_{t+1}, K_{t+1}) \right) + \tau \delta - \partial_{K} G(B_{t+1}, K_{t+1}) + (1-\delta) q_{t+1} \right) \right].$$
(45)

This condition is analogous to Equation 11 for the unlevered firm.

 $^{^{20}}$ Using the envelope condition with respect to K_t , we can show that the firm's optimal investment decision satisfies the condition

4.1.3 Statistical model of the firm's debt and equity payout processes

We consider a statistical model for the firm's equity payout process D^e and debt payout process D^b . In particular, we assume that the equity payout process satisfies

$$D^e(K, Z, d) = d^e \cdot Z \cdot K \cdot = d^e \cdot Y, \tag{48}$$

and the debt payout process $D^{b,d}$ satisfies

$$D^{b}(K, Z, d^{b}) = d^{b} \cdot Z \cdot K = d^{b} \cdot Y,$$

$$\tag{49}$$

where d^e and d^b are the firm's equity payout and debt payout, respectively, per unit of output. Hence, the statistical model for the firm's total payout process is $D = D^e + D^b = d \cdot Y$, where $d \equiv d^e + d^b$ is the firm's total payout per unit of output.

The law of motion for d is given by Equation 18, except that the conditional upper bound of d is given by

$$d_t^{max} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2} \cdot \varphi \right) \varphi + \tau \delta + \frac{\kappa}{2} b_t^2 \right].$$
(50)

As in the case of the unvelered firm, this upper bound for d ensures that the firm's model-implied investment and payout processes are feasible and that the firm's model-implied marginal q is realvalued and non-negative (see Internet Appendix C).

The debt payout ratio process, d^b , is stationary, with law of motion

$$d_{t+1}^b = \mu_b + \phi_b \cdot (d_t^b - \mu_b) + \sigma_b \cdot \epsilon_{t+1}^b, \tag{51}$$

where $\phi_b \in (0,1), \sigma_b > 0, \epsilon_{t+1}^b \sim N(0,1), \operatorname{corr}(\epsilon_{t+1}^z, \epsilon_{t+1}^b) = \rho_{z,b}$, and $\operatorname{corr}(\epsilon_{t+1}^d, \epsilon_{t+1}^b) = \rho_{d,b}$.

Finally, the equity payout ratio process, d^e , is implicitly determined by the relationship $d^e = d - d^b$.

4.1.4 Model-implied q and y

We can now impose the condition that the firm's optimal payout policies need to match the statistical model of the firm's payout processes. We start with the firm's debt payout, which yields

$$D_t^b = d_t^b Z_t K_t. ag{52}$$

Substituting for the firm's debt payout policy (Equation 46) and using Equation 42 in order to write R_t^b as a function of b_t , we get the following expression for the model-implied bond yield:

$$y_t^* = y^*(z_t, d_t, d_t^b, b_t^*) = \frac{\kappa}{\tau} \frac{\left(\frac{\kappa}{\tau} b_t^* + 1\right) b_t^* - d_t^b e^{z_t}}{(1 - \delta) + \frac{q_t^* - 1}{a(1 - \tau)}},$$
(53)

where q_t^* and b_t^* are the firm's model-implied marginal q and leverage ratio, respectively.

We now turn to the firm's equity payout. The condition is

$$D_t^e = d_t^e Z_t K_t, (54)$$

so substituting for the firm's equity payout policy (Equation 47), imposing the expression for the model-implied bond yield (Equation 53) and rearranging, we get an expression for the firm's model-implied marginal q:

$$q_t^* = q^*(z_t, d_t, b_t^*) = \sqrt{1 + 2a(1 - \tau) \left[(\alpha(1 - \tau) - d_t) e^{z_t} + \tau \delta + \frac{\kappa}{2} (b_t^*)^2 \right]}.$$
(55)

In summary, imposing consistency between the firm's optimal policies and the statistical model for the firm's payout allows us to recover q_t^* and y_t^* as functions of z_t , d_t , and d_t^b , as well as the predetermined model-implied leverage ratio (b_t^*) . Moreover, Equation 32 and the firm's debt optimality condition (Equation 42) yield the following expression for the evolution of the firm's model-implied leverage ratio:

$$b_{t+1}^* = \frac{\left(\frac{\kappa}{\tau}b_t^* + 1\right)b_t^* - d_t^b c^{z_t}}{(1-\delta) + \frac{q_t^* - 1}{a(1-\tau)}}.$$
(56)

Hence, Equations 53, 55, and 56 jointly allow us to back out the model-implied q and y processes from the statistical processes z, d^b , and d.

4.1.5 Model-implied asset prices and returns

In Internet Appendix C, we show that the firm's optimality conditions imply that the ex-payout equity value is given by

$$P_t^e = q_t K_{t+1} - B_{t+1} = (q_t - b_{t+1}) K_{t+1}.$$
(57)

As a result, the ex-payout firm value is

$$P_t = P_t^e + B_{t+1} = q_t K_{t+1}, (58)$$

which implies that the firm's average Tobin's q, denoted by Q, is equal to its marginal Tobin's q:

$$Q_t = q_t. (59)$$

Therefore, the model-implied Q is given by

$$Q_t^* = Q^*(z_t, d_t, b_t^*) = \sqrt{1 + 2a(1 - \tau) \left[\left(\alpha(1 - \tau) - d_t \right) e^{z_t} + \tau \delta + \frac{\kappa}{2} (b_t^*)^2 \right]},$$
(60)

so the model-implied firm return is

$$R_{t+1}^* = \frac{d_{t+1}e^{z_{t+1}} + Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) \left(1 - \delta + \frac{Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) - 1}{a(1-\tau)}\right)}{Q^*(z_t, d_t, b_t^*)}.$$
(61)

and the firm's model-implied expected return (i.e., its model-implied cost of capital) is

$$\mathcal{R}^*(z_t, d_t, d_t^b, b_t^*) = \mathbb{E}\left[R_{t+1}^* \mid z_t, d_t, d_t^b, b_t^*\right].$$
(62)

Importantly, we can also characterize the firm's model-implied equity and debt returns separately. As regards the equity return, $R_{t+1}^e = (P_{t+1}^e + D_{t+1}^e)/P_t^e$, Equations 57 and 59 imply the model-implied equity return is

$$R_{t+1}^{e,*} = \frac{(d_{t+1} - d_{t+1}^b)e^{z_{t+1}} + \left(Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) - \frac{\tau}{\kappa}y^*(z_{t+1}, d_{t+1}, d_{t+1}^b, b_{t+1}^*)\right)\left(1 - \delta + \frac{Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) - 1}{a(1-\tau)}\right)}{Q^*(z_t, d_t, b_t^*) - \frac{\tau}{\kappa}y^*(z_t, d_t, d_t^b, b_t^*)}$$

$$(63)$$

which can be rewritten as

$$R_{t+1}^{e,*} = \frac{d_{t+1}e^{z_{t+1}} + Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) \left(1 - \delta + \frac{Q^*(z_{t+1}, d_{t+1}, b_{t+1}^*) - 1}{a(1-\tau)}\right) - \left(\frac{\kappa}{\tau} b_{t+1}^* + 1\right) b_{t+1}^*}{Q^*(z_t, d_t, b_t^*) - b_{t+1}^*}, \quad (64)$$

so the model-implied expected equity return (i.e., the firm's cost of equity) is

$$\mathcal{R}^{e,*}(z_t, d_t, d_t^b, b_t^*) = \mathbb{E}\left[R_{t+1}^{e,*} \mid z_t, d_t, d_t^b, b_t^*\right].$$
(65)

Finally, the model-implied debt return satisfies

$$R_{t+1}^{b,*} = \frac{\kappa}{\tau} b_{t+1}^* + 1.$$
(66)

The firm's model-implied cost of debt is recovered from the firm's debt optimality condition (Equation 42), and is increasing in the firm's model-implied leverage ratio. Intuitively, the implicit cost of debt is the rate that makes the firm *optimally* issue the amount of debt consistent with the statistical model of the firm's payout processes. Recall that the firm's optimal leverage ratio is increasing in the firm's cost of debt. Hence, when statistical processes z, d and d^b jointly imply that the firm's leverage ratio is high, the model-implied cost of debt needs to be high in order to justify the firm's leverage choice. Substituting Equation 56 into Equation 66, and using Equation 59, we get the following expression for the firm's model-implied expected debt return,

$$\mathcal{R}^{b,*}(z_t, d_t, d_t^b, b_t^*) = \frac{\kappa}{\tau} \frac{\left(\frac{\kappa}{\tau} b_t^* + 1\right) b_t^* - d_t^b e^{z_t}}{(1 - \delta) + \frac{Q^*(z_t, d_t, b_t^*) - 1}{a(1 - \tau)}} + 1 = y^*(z_t, d_t, d_t^b, b_t^*) + 1, \tag{67}$$

which equals the model-implied realized debt return, $R_{t+1}^{b,*}$, as the firm issues one-period riskless debt.

To solve for the model-implied expected firm, equity, and debt returns, we use Equations 62, 65, and 67, as well as the expression for the firm's model-implied Q (Equation 60), the laws of motion of processes z, d, and d^b , and the law of motion for the model-implied leverage ratio, b^* (Equation 56).

Figures 5, 6, and 7 display the model-implied expected firm return \mathcal{R}^* , expected equity return $\mathcal{R}^{e,*}$, and expected debt return $\mathcal{R}^{b,*}$, respectively, as functions of the four state variables: productivity z, firm payout ratio d, debt payout ratio d^b , and (pre-determined) leverage ratio b.²¹ For our calculations, we use the calibrated parameter values discussed in the next section and Gauss-Hermite quadrature (with 31 grid points per shock) to compute the integrals needed for the evaluation of the \mathcal{R}^* and $\mathcal{R}^{e,*}$ functions.

As seen in Figure 5, the firm's expected return is sharply increasing in the total payout ratio d, whereas it is essentially flat with respect to productivity z, the debt payout ratio d^b , and the leverage ratio b. Thus, as in the case of the unlevered firm, most of the variation in the firm's cost of capital arises from fluctuations in the total payout ratio d. As seen in Panel C (in which d is kept fixed, whereas the relative magnitudes of d^e and d^b change), it is the size of the overall payout d that matters for the magnitude of the firm's expected return, but not its composition.

The composition of the firm's total payout ratio becomes relevant when we focus on the firm's expected equity and debt returns separately. As seen in Panel C of Figures 6 and 7, both the firm's model-implied expected equity return and its debt return are decreasing in the debt payout ratio d^b , holding everything else (including the total payout ratio d) fixed. For a given value of d, changes in d^b imply opposite changes in d^e , so they reflect payout composition effects. An increase in the debt payout ratio d^b , everything else equal (including existing debt ratio b), suggests that the firm issues less new debt. Hence, taking into account the positive relationship between the firm's optimal debt issuance and the cost of debt in our model, the firm's cost of debt needs to be low in order to justify the firm's leverage choice. As regards the cost of equity, a higher value of d^b implies a lower equity payout ratio d^e , reducing the expected return on equity. Note that, as discussed above, despite the fact that a higher d^b implies both a lower cost of debt and a lower cost

²¹Recall the notation and timing of leverage choice in our model: b_t is a choice variable at period t - 1, so b_t is predetermined at t.

of equity, the firm's overall implicit cost of capital is virtually unchanged: that is due to the fact that the firm's implied leverage ratio is also lower (due to the issuance of less new debt), so the firm substitutes relatively cheap debt with relatively expensive equity.

On the other hand, both the firm's cost of debt and its cost of equity are increasing in the firm's (pre-determined) leverage ratio b (Panel D of Figures 6 and 7). This is because a higher b, keeping everything else (and, in particular d^b) the same, implies more issuance of new debt by the firm: simply put, if the stock of the firm's existing debt is higher, the only way for the firm to have the same debt payout is to issue more new debt. Since higher debt issuance is optimal only if the firm's cost of debt is higher, the model-implied cost of debt is increasing in b. Furthermore, higher issuance of new debt implies that that the firm has more resources available for either equity payout or investment so, keeping d and d^b (and, hence, d^e) constant, a higher b implies more investment and, hence, a higher model-implied Q, increasing the model-implied expected equity return.

In order to quantify the contribution of each state variable to the overall volatility of key asset pricing measures, we run a single 100-year simulation of the model and report the paths of the firm's model-implied Q, expected firm return, expected equity return, and expected debt return in Figure 8. As seen in Panels A, B, and C, fluctuations in the firm payout ratio d account for almost all of the variation in the firm's Q, firm expected returns, and equity expected returns. On the other hand, the volatility of the firm's debt returns is mainly due to changes in the (pre-determined) leverage ratio b.

4.2 Quantitative results

We investigate the quantitative properties of the model with a levered representative firm by considering a simulation exercise. The calibrated parameter values for our model are reported in the last column of Table 4. All parameters common to both the levered and the unlevered firm model are calibrated using the methodology discussed in the previous section.²² The new parameters are the leverage cost parameter κ and the debt payout ratio parameters (μ_b , ϕ_b , σ_b , $\rho_{z,b}$, and $\rho_{d,b}$). The leverage cost parameter κ is calibrated to match the average U.S. aggregate leverage ratio,²³ whereas the debt payout parameters are calibrated to match the corresponding moments of the U.S. aggregate debt payout ratio. As before, we run 10,000 model simulations, each of which consists of 44 annual observations (after an initial period of 1,000 years, to reduce the dependence on initial conditions). We report empirical and simulated moments for firm returns, equity returns, and debt

 $^{^{22}}$ All common parameter values are identical across the two models, with the exception of the values of the productivity parameters, which change slightly because the implied productivity process changes due to the different budget constraints in the two models.

²³We back out the time series for the aggregate leverage ratio (market value of debt per unit of capital) by multiplying aggregate market value of debt per unit of output by productivity, which is equal to output per unit of capital.

returns in Tables 8, 9, and 10, respectively.²⁴

We first consider the properties of firm returns, presented in Table 8. As seen in the table, the simulated moments for the levered firm are almost identical to the simulated moments of the unlevered firm discussed in the previous section (Table 6). In particular, the model is able to adequately match the unconditional moments of U.S. aggregate firm returns (Panel A), but returns exhibit excess sensitivity to payout ratios, with the median value of the forecast coefficient across simulations being 1.92 (Panel C). The only new component of Table 8 is Panel D, which reports the output of return forecasting regressions on equity and debt payout ratios. In both the data and the model, both ratios have a positive forecast coefficient, underscoring the ability of the model to accurately capture important return predictability attributes. Interestingly, we find that the excess sensitivity of the firm's return to its payout ratio is due to its excess sensitivity to the firm's equity payout ratio, whereas the sensitivity of firm returns to the firm's debt payout ratio matches the data. In particular, the coefficient on the equity payout ratio has a median value of 1.93, more than double the value of its empirical counterpart - in fact, the empirical value (0.87) is outside the 90% simulation range, underscoring the difficulty of the model to match the empirical forecast coefficient. We delve deeper into that puzzle by considering the firm's equity and debt returns separately.

We start with equity returns. As seen in Panel A of Table 9, the model generates realistic correlations of equity returns with output growth rates, productivity shocks, equity payout shocks, and debt payout shocks. However, simulated equity payout yields are much higher on average, and more volatile, than their empirical counterparts. On the other hand, the model is able to generate realistic equity return unconditional moments: the average equity return is 10.42% and the unconditional equity return volatility is 24.79%, both being close to their empirical values (8.96% and 17.76%, respectively).

The model is able to produce equity returns that qualitatively match the predictability properties of U.S. aggregate equity returns, as evidenced by the fact that the sign of the slope coefficients in equity return predictability regressions of simulated data matches the sign of the corresponding slope coefficients in predictability regressions of aggregate U.S. returns (Panels B–D of Table 9). However, as seen in Panel C, model-implied equity returns are too sensitive to fluctuations in the total payout ratio D/Y: the median forecasting coefficient in simulated data is 3.99, more than twice the magnitude of the forecasting coefficient for U.S. aggregate equity returns (1.91). Relatedly, variation in the firm's payout ratio explains a much larger share of equity return variation in the

²⁴Under the law of motion for z shown in Equation 2, $Z_t^{min} = 0$ for all t, since normally distributed shocks have infinite support. As discussed in Internet Appendix C, a lower bound of $Z_t^{min} = 0$ sometimes leads to a binding collateral constraint. To ensure that the collateral constraint binds very infrequently (and, hence, can be ignored when solving the model), we introduce the following slight modification: we set the value of Z_{t+1}^{min} to be equal to the value that Z_{t+1} would take if the realization of the productivity shock at t+1 were equal to four standard deviations below zero. Since realizations below four standard deviations from zero are extremely rare for normally distributed shocks, our modification helps ensure that the collateral constraint almost never binds without substantially violating our distributional assumptions for z.
model than in the data: the median adjusted R^2 of the forecasting regression is 24.30% in the model, but only 12.22% in the data. When we consider debt and equity payout ratios separately (Panel D), we find that model-implied equity returns are counterfactually sensitive to equity payout ratios, but not to debt payout ratios: in the model, the median value of the equity return forecast coefficient on the equity payout ratio is 4.01, much higher than the empirical value of 1.25, whereas the corresponding values for the coefficient on the debt payout are 3.97 and 3.14, respectively. Therefore, the "return sensitivity puzzle" documented in the previous section is really an "equity return sensitivity puzzle": in the model, equity returns are counterfactually sensitive to equity payout ratios.

Next, we turn to debt returns (Table 10). We find that the model is able to generate realistic unconditional moments for debt payout yields, but not for debt returns: model-implied debt returns are quite low on average (unconditional mean of 1.01%) and almost constant over time (unconditional volatility of 0.14%), whereas actual U.S. aggregate debt returns have an unconditional mean of 4.84% and an unconditional volatility of 7.47%. Since the model generates essentially constant debt returns, it is unable to match the empirical debt return predictability properties: as seen in Panels B–D, all simulated forecasting coefficient estimates are extremely close to zero. In short, our model generates two puzzles regarding debt returns: model-implied debt returns are both too low on average and counterfactually smooth.

Overall, our model yields an "equity return sensitivity puzzle" and two puzzles for debt returns, a "debt return level puzzle" and a "debt return volatility puzzle". The first puzzle suggests that the equity payout of the representative firm is too insensitive to fluctuations in the equity discount rate, so matching the properties of the observed U.S. aggregate equity payout requires a counterfactually sensitive cost of equity. Our findings suggest that, in order for a model of the firm to jointly match the properties of equity returns and equity payouts, it would need to include features that make the firm's equity payout policy more responsive to equity discount rate variation. One such potential feature would be misvaluation shocks, following Warusawitharana and Whited (2016).

The two puzzles on debt returns reflect the limitations of the trade-off theory of capital structure. In the model, optimal leverage is increasing in the cost of debt (as a higher cost of debt yields a higher interest tax shield), so low average leverage (and, hence, low average debt payouts) are consistent with firm optimization only if the average cost of debt is low. Hence, the "debt return level puzzle" is related to the well-documented "capital structure puzzle" in the corporate finance literature, which states that the level of firm leverage that we observe is, on average, too low to be consistent with standard trade-off theory models. In other words, corporate finance models take asset returns as given and generate predictions about firm quantities: in particular, they state that, given the observed average cost of debt, the model-implied observed average leverage level is too high. In contrast, our approach takes firm quantities as given and generates predictions about asset returns: it states that, given the observed average leverage leverage (and, hence, the observed average debt payout), the empirically observed cost of debt is too high. The tight model-implied association between the firm's optimal leverage and its cost of debt is also the cause of the second debt return puzzle: the aggregate leverage of U.S. firms exhibits high persistence and low volatility over short horizons, so model needs to generate similar properties for the implied cost of debt. Our findings suggest that, in order to generate more realistic debt return properties, we need to model firms' capital structure decisions as involving elements beyond taxes and financial distress costs. For example, allowing firms to consider financial flexibility, in the spirit of DeAngelo et al. (2011), would weaken the connection between leverage and cost of debt, potentially generating more realistic debt return moments.

5 Conclusion

We propose a payout-based approach within the production-based asset pricing paradigm. Our approach allows us to elicit the return properties of optimizing firms by proposing a statistical model for the firms' payout processes, without the need to either recover the economy's SDF or to impose any assumptions about investor preferences.

Whereas our approach is applicable to any firm, in this paper we consider an optimizing representative firm and explore to what extent our model can match the properties of aggregate U.S. returns. Our model is intentionally simple: our goal is to highlight the asset pricing implications of our payout-based approach for the canonical representative firm model, so that our asset pricing findings are applicable to a wide class of models proposed in the production-based asset pricing literature. We find that our model-implied returns match some of the salient properties of aggregate U.S. returns, but also give rise to important asset pricing puzzles, suggesting the need for models of the firm that feature more sophisticated production and financing technology specifications.

Given the broadness of our framework, several new potential research paths are possible. For instance, future work could extend our payout-based asset pricing approach to the individual firms or firm portfolios, helping to better understand the implications of different firm technology specifications for the cross-section of firm returns without the need to measure firm-level investment.

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Fig. 1: Model-implied Q and expected firm return as functions of the state variables

This figure presents the model-implied average Tobin's q, denoted by Q, and the model-implied expected return of the representative firm in the quantitative model with an unlevered firm. Panels A and B of this figure plot the firm's Q as a function of the state variable z and d, respectively, keeping the other state variable constant. Panels C and D of this figure plot the equilibrium expected firm return as a function of the state variable z and d, respectively, keeping the other state variable constant.



Fig. 2: Simulated paths: model with unlevered firm

This figure reports the output of a 100-year simulation of the model with an unlevered representative firm. Panels A and B plot the simulated paths of the state variables z and d, respectively. Panels C and D plot the simulated path of the firm's average Tobin's q, denoted by Q, and the firm's expected return (in black solid line), respectively, as well as the Q path and expected return path when z and d varies (in red and blue dotted line, respectively) and the other state variable is kept constant.



Fig. 3: U.S. aggregate payout ratio

This figure plots the annual time series of the U.S. aggregate firm payout ratio (firm payout divided by firm revenue) from 1974 to 2017 (solid green line). The figure also plots the annual time series of the U.S. aggregate equity and debt payout ratio (dashed red line and blue dotted line, respectively) for the same time period.



Panel A: Model-Implied Expected Returns

Fig. 4: Properties of model-implied expected returns

Panel A of this figure plots the time series of model-implied expected returns. Panel B of this figure plots the time series of the fitted values from regressions of annual realized returns, in the model and the data, on lagged payout ratios (solid green line and dashed red line, respectively). Shaded areas represent NBER recessions.



Fig. 5: Expected return of the levered firm as a function of the state variables

This figure presents the model-implied firm expected return in the model with a levered representative firm. In particular, Panels A, B, C, and D of this figure plot the firm's expected return as a function of the state variable z, d, d^b , and b, respectively, keeping the other state variables constant.



Fig. 6: Equity expected return of the levered firm as a function of the state variables

This figure presents the model-implied equity expected return in the model with a levered representative firm. In particular, Panels A, B, C, and D of this figure plot the firm's equity expected return as a function of the state variable z, d, d^b , and b, respectively, keeping the other state variables constant.



Fig. 7: Debt expected return of the levered firm as a function of the state variables

This figure presents the model-implied debt expected return in the model with a levered representative firm. In particular, Panels A, B, C, and D of this figure plot the firm's debt expected return as a function of the state variable z, d, d^b , and b, respectively, keeping the other state variables constant.



Fig. 8: Simulated paths: model with levered firm

This figure reports the output of a 100-year simulation of the model with a levered representative firm. In particular, Panels A, B, C, and D plot the simulated path of the firm's average Tobin's q (denoted by Q), the firm's expected return, the firm's expected equity return, and the firm's expected debt return, respectively (in solid black line). Furthermore, each plot reports the corresponding variable path when each of z, d, d^b or b varies (in blue, red, green, and turquoise dotted line, respectively) and the other state variables are kept constant.

Table 1: Return forecasting regressions

This table reports the output of U.S. aggregate return forecasting regressions for nine different forecasting variables: the payout ratio d (defined as aggregate payout over aggregate output) and eight different proxies of the investment rate i (defined as aggregate investment over aggregate capital). In particular, Panel A reports the output of forecasting regressions of U.S. aggregate firm returns for the 1974–2017 sample period, Panel B reports the output of forecasting regressions of U.S. aggregate equity returns for the 1974–2017 sample period, and Panel C reports the output of forecasting regressions of U.S. aggregate equity returns for the 1950–2017 sample period. We report Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

$\mathbf{P}_{2} = \mathbf{P}_{2} $									
Forecasting variable	d		i _{NIPA}		(1514) $i_{FAA}^{p,i}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i^{p,w}_{COMP}$	$i_{COMP}^{p,i,w}$
Coefficient	1.36	-1.76	-0.25	0.01	-1.07	-0.76	-0.93	-0.46	-0.54
s.e.	[0.40]	[1.68]	[0.86]	[1.35]	[2.20]	[0.47]	[0.58]	[0.47]	[0.56]
Adj. R^2	8.62%	0.05%	-2.30%	-2.44%	-1.82%	3.79%	3.50%	-0.20%	-0.11%
Panel B: Equity Returns (1974 – 2017)									
Forecasting variable	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$
Coefficient	1.91	-2.72	-0.61	-0.73	-2.05	-0.91	-1.12	-0.59	-0.71
s.e.	[0.47]	[1.94]	[0.95]	[1.64]	[2.52]	[0.58]	[0.72]	[0.57]	[0.68]
Adj. R^2	12.22%	1.60%	-1.84%	-2.16%	-0.90%	3.48%	3.36%	0.08%	0.22%
	I	Panel C	: Equity	Returns	s (1950 -	- 2017)			
Forecasting variable	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$
Coefficient	1.69	-3.86	-1.44	-2.37	-3.33	-0.52	-0.37	-0.86	-0.92
s.e.	[0.34]	[1.59]	[0.96]	[1.42]	[1.58]	[0.50]	[0.59]	[0.47]	[0.53]
Adj. R^2	9.04%	5.86%	1.18%	0.94%	2.66%	-0.18%	-1.02%	1.99%	1.60%

Table 2: Return forecasting regressions

This table reports the output of forecasting regressions of U.S. aggregate returns on the aggregate payout ratio d (defined as aggregate payout over aggregate output) and the aggregate investment rate i (defined as aggregate investment over aggregate capital), for eight different empirical measures of the investment rate. In particular, Panel A reports the output of forecasting regressions of U.S. aggregate firm returns for the 1974–2017 sample period, Panel B reports the output of forecasting regressions of U.S. aggregate equity returns for the 1974–2017 sample period, and Panel C reports the output of forecasting regressions of U.S. aggregate equity returns for the 1974–2017 sample period. We report Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

Panel A: Firm Returns (1974 – 2017)										
Forecasting variable	i_{WG}	i_{NIPA}	i^p_{FAA}	$i_{FAA}^{p,i}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$		
d coefficient	1.37	1.52	1.55	1.50	1.16	1.16	1.31	1.32		
s.e.	[0.46]	[0.53]	[0.51]	[0.48]	[0.44]	[0.43]	[0.52]	[0.51]		
i coefficient	0.02	0.64	1.50	1.04	-0.50	-0.56	-0.10	-0.10		
s.e.	[1.41]	[0.74]	[1.14]	[1.54]	[0.43]	[0.54]	[0.43]	[0.50]		
Adj. R^2	6.34%	7.21%	7.91%	6.83%	8.76%	8.28%	6.44%	6.41%		
	Panel B: Equity Returns (1974 – 2017)									
Forecasting variable	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$		
d coefficient	1.86	2.05	2.07	2.01	1.70	1.70	1.87	1.88		
s.e.	[0.56]	[0.63]	[0.62]	[0.58]	[0.53]	[0.53]	[0.62]	[0.61]		
i coefficient	-0.31	0.59	1.26	0.78	-0.51	-0.57	-0.09	-0.08		
s.e.	[1.66]	[0.83]	[1.33]	[1.84]	[0.51]	[0.65]	[0.50]	[0.59]		
Adj. R^2	10.07%	10.53%	10.78%	10.21%	11.78%	11.41%	10.07%	10.05%		
	Pan	el C: Eq	uity Ret	urns (19	50 - 2017	·)				
Forecasting variable	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$		
d coefficient	1.40	1.58	1.59	1.54	1.64	1.67	1.55	1.56		
s.e.	[0.38]	[0.48]	[0.41]	[0.36]	[0.39]	[0.39]	[0.41]	[0.39]		
i coefficient	-2.77	-0.70	-1.14	-2.34	-0.31	-0.12	-0.54	-0.56		
s.e.	[1.64]	[0.94]	[1.32]	[1.60]	[0.48]	[0.57]	[0.45]	[0.50]		
Adj. R^2	11.16%	8.22%	8.16%	9.63%	8.10%	7.68%	8.94%	8.72%		

Table 3: Payout-based vs. investment-based implicit returns

This table reports the properties of model-implied realized firm returns for nine different implementations, under the assumption that firm productivity is constant. The nine implementations are, in order, the payout-based approach (column two) and eight implementations of the investment-based approach (columns three to ten). Each panel reports the unconditional moments of model-implied firm returns and corresponding observed aggregate U.S. returns, with the panels differing in the observed return measure employed. In particular, Panel A considers U.S. aggregate firm returns for the 1974–2017 sample period, Panel B considers U.S. aggregate equity returns for the 1974– 2017 sample period, and Panel C considers U.S. aggregate equity returns for the 1950–2017 sample period.

Panel A: Firm Returns (1974 – 2017)									
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$
Average R	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%
Volatility of R	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%
Average R^*	5.64%	5.09%	1.20%	7.42%	3.04%	1.54%	-2.85%	1.24%	-1.93%
Volatility of R^*	14.83%	3.87%	5.60%	6.01%	4.79%	13.23%	8.89%	16.25%	12.32%
$\operatorname{Corr}(R^*, R)$	0.57	0.08	-0.07	-0.03	-0.03	0.14	0.13	0.11	0.11
$\operatorname{Corr}(R^*, R^{shifted})$	0.22	0.36	0.21	0.11	0.13	0.07	0.07	0.07	0.08
Panel B: Equity Returns (1974 – 2017)									
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$
Average R	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%
Volatility of R	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%
Average R^*	5.64%	5.09%	1.20%	7.42%	3.04%	1.54%	-2.85%	1.24%	-1.93%
Volatility of R^*	14.83%	3.87%	5.60%	6.01%	4.79%	13.23%	8.89%	16.25%	12.32%
$\operatorname{Corr}(R^*, R)$	0.61	0.10	-0.05	-0.03	-0.03	0.14	0.13	0.12	0.12
$\operatorname{Corr}(R^*, R^{shifted})$	0.21	0.38	0.24	0.13	0.15	0.05	0.05	0.05	0.06
		Panel	C: Equit	ty Retur	ns (1950	– 2017)			
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$
Average R	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%
Volatility of R	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%
Average R^*	7.77%	4.81%	0.81%	8.72%	5.35%	2.75%	-1.19%	2.68%	-0.05%
Volatility of R^*	16.17%	4.25%	5.81%	5.76%	4.75%	11.55%	8.01%	14.72%	11.60%
$\operatorname{Corr}(R^*, R)$	0.32	-0.03	-0.08	-0.22	-0.22	0.04	0.01	0.03	0.02
$\operatorname{Corr}(R^*, R^{shifted})$	0.07	0.24	0.26	0.02	0.05	0.08	0.08	0.14	0.15

Table 4: Model calibration

This table reports the calibrated parameters in our quantitative model. For each parameter, the first column provides its description, the second column shows its symbol, and the third and fourth columns report its calibrated value in the model with an unlevered and a levered firm, respectively.

		Calibrated Value			
Parameter Description	\mathbf{Symbol}	Unlevered Firm	Levered Firm		
Adjustment Cost Parameter	a	9.947	9.947		
Depreciation Rate	δ	0.150	0.150		
Corporate Tax Rate	au	0.350	0.350		
Profit Margin	α	0.150	0.150		
Leverage cost parameter	κ	_	0.004		
Average z	μ_z	0.983	0.975		
Autocorrelation of z	ϕ_z	0.745	0.741		
Volatility Parameter of z	σ_z	0.061	0.060		
Average d	μ_d	0.015	0.015		
Autocorrelation of d	ϕ_d	0.595	0.595		
Volatility Parameter of d	σ_d	0.073	0.073		
Average d^b	μ_b	_	-0.006		
Autocorrelation of d^b	ϕ_b	_	0.168		
Volatility Parameter of d^b	σ_b	_	0.021		
$\operatorname{Correlation}(z \ , d)$	$ ho_{z,d}$	-0.134	-0.134		
Correlation(z , d^b)	$ ho_{z,b}$	_	-0.065		
Correlation (d, d^b)	$ ho_{d,b}$	_	0.568		

Table 5: Model-implied vs. actual return predictability regressions

This table reports the output of return predictability regressions. In Panel A, the dependent variable is annual model-implied returns, whereas in Panel B the dependent variable is annual actual returns. The sample period is 1974–2017.

Panel A: Model-implied returns								
d coefficient	2.59		2.62					
s.e.	[0.83]		[0.73]					
z coefficient		0.05	-0.08					
s.e.		[0.11]	[0.24]					
Adj. R^2	30.06%	-2.34%	28.56%					
Panel	B: Actu	al retur	ns					
d coefficient	1.36		1.97					
	1.00		1.57					
s.e.	[0.40]		[0.49]					
s.e. z coefficient	[0.40]	0.05	1.37 [0.49] -0.03					
s.e. z coefficient s.e.	[0.40]	0.05 $[0.17]$	1.37 [0.49] -0.03 [0.16]					

Table 6: Empirical and simulated moments: unlevered firm

This table reports empirical and simulated asset pricing moments. For each moment, it reports its description, its notation, its empirical value, and its median and 5th and 95th percentile values across 10,000 simulations of the model with an unlevered representative firm. Panel A reports unconditional moments. Panel B reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged payout yields. Panel C reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged payout ratios. For regression coefficients estimated in the data, we also provide statistical significance information, with *, **, and *** reflecting statistical significance at the 10% level, 5% level, and 1% level, respectively. We calculate Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

Description	Notation	Data	Median	$\mathbf{Q}(5\%)$	$\mathbf{Q}(95\%)$					
Panel A: Unconditional moments										
Average Output Growth	$\mathbb{E}[\Delta Y/Y]$	2.76%	2.41%	0.70%	4.21%					
Volatility of Output Growth	$\sigma[\Delta Y/Y]$	5.89%	7.47%	6.38%	8.63%					
Corr. of Return with Output Growth	$corr(R,\Delta Y/Y)$	0.03	0.01	-0.18	0.20					
Corr. of Return with Prod. Shock	$corr(R, z - \mathbb{E}[z])$	0.25	0.32	0.12	0.49					
Corr. of Return with Payout Shock	$corr(R, d - \mathbb{E}[d])$	-0.54	-0.85	-0.90	-0.78					
Average Payout Yield	$\mathbb{E}[D/P]$	1.59%	2.59%	0.86%	4.40%					
Volatility of Payout Yield	$\sigma[D/P]$	2.47%	4.47%	3.52%	5.70%					
Autocorrelation of Payout Yield	$\mathbb{AC}[D/P]$	0.44	0.53	0.34	0.68					
Average Return	$\mathbb{E}[R]$	7.86%	5.47%	4.66%	6.34%					
Volatility of Return	$\sigma[R]$	14.88%	12.81%	10.78%	15.24%					
Reward-to-Risk	$\mathbb{E}[R]/\sigma[R]$	0.53	0.43	0.36	0.50					
Autocorrelation of Return	$\mathbb{AC}[R]$	-0.17	-0.20	-0.37	-0.02					
Volatility of $\mathbb{E}[R]$	$\sigma[\mathbb{E}[R]]$	—	5.94%	4.47%	8.03%					
Panel B:	Regression of R_t .	$+1$ on D_t	$/P_t$							
Forecast Coefficient	b	1.81***	1.43	0.97	2.00					
Adjusted R^2	R_{adj}^2	9.20%	23.68%	14.11%	35.25%					
Panel C:	Regression of R_t .	$+1$ on D_t	$/Y_t$							
Forecast Coefficient	b	1.36***	1.94	1.20	2.89					
Adjusted R^2	R_{adj}^2	8.62%	21.59%	12.33%	32.51%					

Table 7: Sensitivity analysis

This table reports empirical and simulated asset pricing moments. For each moment, it shows its notation, its empirical value, and its median value across 10,000 simulations of the model with an unlevered representative firm for different parameter configurations. Panel A reports unconditional moments. Panel B reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged payout yields. Panel C reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged payout ratios.

			a		δ		(α		
Moment	Data	Benchmark	5.947	13.947	0.05	0.25	0.05	0.25		
Panel A: Unconditional moments										
$\mathbb{E}[R]$	7.86%	5.47%	8.13%	3.66%	14.09%	-3.17%	-2.40%	12.21%		
$\sigma[R]$	14.88%	12.81%	12.18%	13.06%	14.71%	11.16%	16.77%	11.47%		
$\mathbb{E}[R]/\sigma[R]$	0.53	0.43	0.67	0.28	0.96	-0.28	-0.14	1.06		
$\mathbb{AC}[R]$	-0.17	-0.20	-0.20	-0.20	-0.20	-0.20	-0.22	-0.19		
$\sigma[\mathbb{E}[R]]$	_	5.94%	5.60%	6.08%	6.88%	5.14%	8.97%	4.93%		
		Panel B	: Regressi	ion of R_{t}	$_{\pm 1}$ on D_t/L	P_t				
b	1.81	1.43	1.07	1.70	1.77	1.14	1.71	1.33		
R^2_{adj}	9.20%	23.68%	23.43%	23.77%	24.01%	23.43%	31.09%	20.36%		
		Panel C	: Regressi	ion of R_{t}	$+1$ on $D_t/2$	Y_t				
b	1.36	1.94	1.71	2.04	2.35	1.60	3.89	1.34		
R^2_{adj}	8.62%	21.59%	21.52%	21.60%	21.68%	21.50%	24.91%	19.16%		

Table 8: Empirical and simulated moments: returns of levered firm

This table reports empirical and simulated asset pricing moments. For each moment, it shows its description, its notation, its empirical value, and its median and 5th and 95th percentile values across 10,000 simulations of the model with a levered representative firm. Panel A reports unconditional moments of firm returns. Panel B reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged firm payout yields. Panel C reports the slope coefficient and the adjusted R^2 of regressions of firm returns on lagged firm payout ratios. Panel E reports the slope coefficients and the adjusted R^2 of regressions of firm returns on lagged firm payout ratios. Panel E reports the slope coefficients and the adjusted R^2 of regressions of firm returns on lagged firm payout ratios. For regression coefficients estimated in the data, we also provide statistical significance information, with *, **, and *** reflecting statistical significance at the 10% level, 5% level, and 1% level, respectively. We calculate Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

				Model						
Description	Notation	Data	Median	$\mathbf{Q}(5\%)$	$\mathbf{Q}(\mathbf{95\%})$					
Panel A: Unconditional moments										
Average Output Growth	$\mathbb{E}[\Delta Y/Y]$	2.76%	2.38%	0.71%	4.18%					
Volatility of Output Growth	$\sigma[\Delta Y/Y]$	5.89%	7.43%	6.38%	8.58%					
Corr. of Firm Return with Output Growth	$corr(R,\Delta Y/Y)$	0.03	0.02	-0.18	0.21					
Corr. of Firm Return with Prod. Shock	$corr(R, z - \mathbb{E}[z])$	0.24	0.32	0.12	0.49					
Corr. of Firm Return with Payout Shock	$corr(R, d - \mathbb{E}[d])$	-0.54	-0.85	-0.90	-0.78					
Average Firm Payout Yield	$\mathbb{E}[D/P]$	1.59%	2.58%	0.87%	4.32%					
Volatility of Firm Payout Yield	$\sigma[D/P]$	2.47%	4.44%	3.50%	5.64%					
Autocorrelation of Firm Payout Yield	$\mathbb{AC}[D/P]$	0.44	0.53	0.34	0.68					
Average Firm Return	$\mathbb{E}[R]$	7.86%	5.43%	4.65%	6.28%					
Volatility of Firm Return	$\sigma[R]$	14.88%	12.71%	10.65%	15.06%					
Reward-to-Risk	$\mathbb{E}[R]/\sigma[R]$	0.53	0.43	0.36	0.51					
Autocorrelation of Firm Return	$\mathbb{AC}[R]$	-0.17	-0.20	-0.37	-0.02					
Volatility of $\mathbb{E}[R]$	$\sigma[\mathbb{E}[R]]$	_	5.89%	4.45%	7.96%					
Panel B: Re	gressions of R_{t+1}	on D_t/I	P_t							
Forecast Coefficient	b	1.81***	1.43	0.97	1.98					
Adjusted R^2	R_{adj}^2	9.20%	23.51%	14.21%	34.89%					
Panel C: Re	Panel C: Regressions of R_{t+1} on D_t/Y_t									
Forecast Coefficient	b	1.36***	1.92	1.20	2.86					
Adjusted R^2	R_{adj}^2	8.62%	21.45%	12.44%	32.32%					
Panel D: Regressie	ons of R_{t+1} on D	t^e/Y_t and	D_t^b/Y_t							
D_t^e/Y_t Forecast Coefficient	b_e	0.87*	1.93	1.10	2.99					
D_t^b/Y_t Forecast Coefficient	b_b	2.29**	1.95	0.82	3.18					
Adjusted R^2	R_{adj}^2	10.10%	21.48%	11.98%	32.70%					

Table 9: Empirical and simulated moments: equity returns of levered firm

This table reports empirical and simulated asset pricing moments. For each moment, it shows its description, its notation, its empirical value, and its median and 5th and 95th percentile values across 10,000 simulations of the model with a levered representative firm. Panel A reports unconditional moments of equity returns. Panel B reports the slope coefficient and the adjusted R^2 of regressions of equity returns on lagged equity payout yields. Panel C reports the slope coefficient and the adjusted R^2 of regressions of equity returns on lagged firm payout ratios. Panel D reports the slope coefficients and the adjusted R^2 of regressions of equity returns on lagged firm payout ratios. Panel D reports the slope coefficients and the adjusted R^2 of regressions of equity returns on lagged firm payout ratios. Panel D reports the slope coefficients and the adjusted R^2 of regressions coefficients estimated in the data, we also provide statistical significance information, with *, **, and *** reflecting statistical significance at the 10% level, 5% level, and 1% level, respectively. We calculate Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

				Model	
Description	Notation	Data	Median	Q(5%)	$\mathbf{Q}(95\%)$
Panel A: U	nts				
Corr. of Equity Return with Output Growth	$corr(R^e, \Delta Y/Y)$	0.03	-0.02	-0.23	0.18
Corr. of Equity Return with Prod. Shock	$corr(R^e, z - \mathbb{E}[z])$	0.28	0.29	0.08	0.47
Corr. of Equity Return with Equity Payout Shock	$corr(R^e, d^e - \mathbb{E}[d^e])$	-0.27	-0.49	-0.63	-0.29
Corr. of Equity Return with Debt Payout Shock	$corr(R^e, d^b - \mathbb{E}[d^b])$	-0.26	-0.44	-0.59	-0.24
Average Equity Payout Yield	$\mathbb{E}[D^e/P^e]$	2.49%	6.42%	2.69%	11.60%
Volatility of Equity Payout Yield	$\sigma[D^e/P^e]$	2.54%	8.42%	5.02%	16.34%
Autocorrelation of Equity Payout Yield	$\mathbb{AC}[D^e/P^e]$	0.54	0.49	0.26	0.66
Average Equity Return	$\mathbb{E}[R^e]$	8.96%	10.42%	7.00%	15.16%
Volatility of Equity Return	$\sigma[R^e]$	17.76%	24.79%	16.41%	41.85%
Reward-to-Risk	$\mathbb{E}[R^e]/\sigma[R^e]$	0.50	0.42	0.33	0.49
Autocorrelation of Equity Return	$\mathbb{AC}[R^e]$	-0.17	-0.18	-0.36	0.01
Volatility of $\mathbb{E}[R^e]$	$\sigma[\mathbb{E}[R^e]]$	_	13.14%	7.27%	31.68%
Panel B: Regr	essions of R^e_{t+1} on I	D_t^e/P_t^e			
Forecast Coefficient	b	1.64^{*}	1.47	0.91	2.26
Adjusted R^2	R_{adj}^2	4.40%	23.71%	9.68%	54.58%
Panel C: Regr	essions of R^e_{t+1} on .	D_t/Y_t			
Forecast Coefficient	b	1.91***	3.99	2.08	7.53
Adjusted R^2	R_{adj}^2	12.22%	24.30%	14.37%	35.80%
Panel D: Regression	s of R^e_{t+1} on D^e_t/Y_t	and D_t^b	Y_t		
D_t^e/Y_t Forecast Coefficient	b_e	1.25***	4.01	1.99	7.67
D_t^b/Y_t Forecast Coefficient	b_b	3.14***	3.97	1.50	8.17
Adjusted R^2	R_{adj}^2	14.50%	24.35%	13.85%	36.15%

Table 10: Empirical and simulated moments: debt returns of levered firm

This table reports empirical and simulated asset pricing moments. For each moment, it shows its description, its notation, its empirical value, and its median and 5th and 95th percentile values across 10,000 simulations of the model with a levered representative firm. Panel A reports unconditional moments of debt returns. Panel B reports the slope coefficient and the adjusted R^2 of regressions of debt returns on lagged debt payout yields. Panel C reports the slope coefficient and the adjusted the adjusted R^2 of regressions of debt returns on lagged firm payout ratios. Panel D reports the slope coefficients and the adjusted R^2 of regressions of debt returns on lagged firm payout ratios. Panel D reports the slope coefficients and the adjusted R^2 of regressions of debt returns on lagged firm payout ratios and lagged productivity. For regression coefficients estimated in the data, we also provide statistical significance information, with *, **, and *** reflecting statistical significance at the 10% level, 5% level, and 1% level, respectively. We calculate Newey and West (1987) standard errors, with lag selection as in Newey and West (1994).

				Model			
Description	Notation	Data	Median	Q(5%)	$\mathbf{Q}(95\%)$		
Panel A: Unconditional moments							
Corr. of Debt Return with Output Growth	$corr(R^b, \Delta Y/Y)$	-0.02	0.03	-0.21	0.27		
Corr. of Debt Return with Prod. Shock	$corr(R^b, z - \mathbb{E}[z])$	-0.13	0.03	-0.17	0.23		
Corr. of Debt Return with Equity Payout Shock	$corr(R^b, d^e - \mathbb{E}[d^e])$	-0.21	-0.11	-0.29	0.08		
Corr. of Debt Return with Debt Payout Shock	$corr(R^b, d^b - \mathbb{E}[d^b])$	-0.15	0.06	-0.14	0.25		
Average Debt Payout Yield	$\mathbb{E}[D^b/B]$	-1.69%	-1.52%	-3.00%	0.16%		
Volatility of Debt Payout Yield	$\sigma[D^b/B]$	7.13%	6.33%	4.85%	12.55%		
Autocorrelation of Debt Payout Yield	$\mathbb{AC}[D^b/B]$	0.14	0.14	-0.08	0.34		
Average Debt Return	$\mathbb{E}[R^b]$	4.84%	1.01%	0.57%	1.17%		
Volatility of Debt Return	$\sigma[R^b]$	7.47%	0.14%	0.06%	0.27%		
Reward-to-Risk	$\mathbb{E}[R]/\sigma[R^b]$	0.65	6.71	2.59	17.99		
Autocorrelation of Debt Return	$\mathbb{AC}[\mathbb{R}^b]$	-0.04	0.94	0.78	0.98		
Volatility of $\mathbb{E}[R^b]$	$\sigma[\mathbb{E}[R^b]]$	_	0.15%	0.08%	0.27%		
Panel B: Regr	ressions of R^b_{t+1} on .	D_t^b/P_t^b					
Forecast Coefficient	b	-0.01	0.00	-0.01	0.00		
Adjusted R^2	R_{adj}^2	-2.44%	3.67%	-1.41%	19.48%		
Panel C: Reg	ressions of R^b_{t+1} on	D_t/Y_t					
Forecast Coefficient	b	-0.23	0.00	-0.02	0.02		
Adjusted R^2	R_{adj}^2	-1.27%	1.03%	-1.49%	17.34%		
Panel D: Regression	ns of R^b_{t+1} on D^e_t/Y_t	t_t and D_t^{b}	P/Y_t				
D_t^e/Y_t Forecast Coefficient	b_e	-0.46*	0.01	-0.01	0.03		
D_t^b/Y_t Forecast Coefficient	b_b	0.19	-0.01	-0.04	0.00		
Adjusted R^2	R_{adj}^2	-0.93%	10.71%	0.26%	29.99%		

Internet Appendix

This Internet Appendix is organized as follows. Section A reports the derivations for the baseline payout-based asset pricing model, i.e., the model that features an unlevered representative firm. Section B presents our two-period general equilibrium model, which is used to illustrate that, if the statistical model for the firm's payout process matches the true equilibrium payout process, then the payout-based approach retrieves the true model-implied returns. Section C provides the derivations for the payout-based asset pricing model with an levered representative firm. Section D describes our data sources and discusses the construction of the empirical measures that we use in our quantitative analysis. Finally, Section E provides the details for our implementation of the investment-based asset pricing approach.

A Derivations for the Baseline Payout-Based Model

This section provides the derivations of the results for our baseline payout-based asset pricing model, which features an unlevered representative firm.

A.1 The firm's optimization problem

In what follows, in the interests of notational convenience, we drop the dependence on the conditional distribution of current and future SDFs (which the firm takes as given) from the firm's value function and, thus, instead of writing $V(K_t, Z_t; \{f_t(M_{t,t+h})\}_{h=1}^{\infty})$, we write $V(K_t, Z_t)$.

The firm's first order condition is

$$\mathbb{E}_{t}[M_{t+1}\partial_{K}V(K_{t+1}, Z_{t+1})] = 1 + (1 - \tau)\partial_{I}\Phi(K_{t}, I_{t}).$$
(IA.1)

We define $q_t \equiv \mathbb{E}_t[M_{t+1}\partial_K V(K_{t+1}, Z_{t+1})]$, so we can write

$$q_t = 1 + (1 - \tau)\partial_I \Phi(K_t, I_t).$$
(IA.2)

That condition yields the firm's investment function $I_t = I(K_t, q_t)$.

The envelope condition (with respect to K_t) is

$$\partial_{K}V(K_{t}, Z_{t}) = (1 - \tau)(\partial_{K}\Pi(K_{t}, Z_{t}) - \partial_{K}\Phi(K_{t}, I_{t})) + \tau\delta + (1 - \delta)\mathbb{E}_{t}[M_{t+1}\partial_{K}V(K_{t+1}, Z_{t+1})],$$
(IA.3)

so, using the definition for q_t , we can write

$$\partial_K V(K_t, Z_t) = (1 - \tau)(\partial_K \Pi(K_t, Z_t) - \partial_K \Phi(K_t, I_t)) + \tau \delta + (1 - \delta)q_t.$$
(IA.4)

Finally, the investment return is

$$R_{t+1}^{I} = \frac{(1-\tau)(\partial_{K}\Pi(K_{t+1}, Z_{t+1}) - \partial_{K}\Phi(K_{t+1}, I_{t+1})) + \tau\delta + (1-\delta)q_{t+1}}{q_{t}}, \qquad (IA.5)$$

so, using Equation IA.4, the equilibrium investment return satisfies

$$R_{t+1}^{I} = \frac{\partial_{K} V(K_{t+1}, Z_{t+1})}{q_{t}}.$$
(IA.6)

Plugging in Equation IA.6 into IA.1, we get

$$\mathbb{E}_t[M_{t+1}R_{t+1}^I q_t] = q_t, \tag{IA.7}$$

which yields Equation 11.

A.2 Pricing the firm

Following Liu et al. (2009), we start by noting that functions $\Pi(Z, K) = \alpha Z K$ and $\Phi(K, I) = \frac{a}{2} \left(\frac{I}{K}\right)^2 K$ have the following properties:

$$\Pi(Z,K) = K \cdot \partial_K \Pi(Z,K), \qquad (IA.8)$$

and

$$\Phi(K,I) = K \cdot \partial_K \Phi(K,I) + I \cdot \partial_I \Phi(K,I), \qquad (IA.9)$$

respectively.

Using Equation IA.9, the firm's investment optimality condition can be written as follows:

$$q_t = 1 + \partial_I \Phi(K_t, I_t) = 1 + (1 - \tau) (\Phi(K_t, I_t) - K_t \partial_K \Phi(K_t, I_t)) / I_t.$$
(IA.10)

Recall that the firm payout is given by

$$D_t = (1 - \tau)(\Pi(Z_t, K_t) - \Phi(I_t, K_t)) - I_t + \tau \delta K_t,$$
(IA.11)

so, using Equations IA.8 and IA.10, the firm's optimal payout satisfies

$$D_t = (1 - \tau) \left(\partial_K \Pi(Z_t, K_t) - \partial_K \Phi(K_t, I_t) \right) \cdot K_t - q_t I_t + \tau \delta K_t = \partial_K D_t \cdot K_t - q_t I_t.$$
(IA.12)

Equation IA.12 implies that

$$\mathbb{E}_t[M_{t+1}D_{t+1}] = \mathbb{E}_t[M_{t+1}(\partial_K D_{t+1} \cdot K_{t+1} - q_{t+1}I_{t+1})].$$
(IA.13)

We can now use the optimality condition $q_t = \mathbb{E}_t \left[M_{t+1} \left(\partial_K D_{t+1} + (1-\delta)q_{t+1} \right) \right]$ to rewrite Equation IA.13 as follows:

$$\mathbb{E}_{t}[M_{t+1}D_{t+1}] = (q_{t} - \mathbb{E}_{t}[M_{t+1}(1-\delta)q_{t+1}])K_{t+1} - \mathbb{E}_{t}[M_{t+1}q_{t+1}I_{t+1}] = q_{t}K_{t+1} - \mathbb{E}_{t}[M_{t+1}q_{t+1}K_{t+2}]$$
(IA.14)

Iterating and applying the law of iterated expectations, we get

$$\mathbb{E}_t[M_{t+1}D_{t+1}] = q_t K_{t+1} - \mathbb{E}_t[M_{t+2}(D_{t+2} + q_{t+2}K_{t+3})], \qquad (IA.15)$$

which yields

$$\mathbb{E}_t[M_{t+1}D_{t+1}] + \mathbb{E}_t[M_{t+2}D_{t+2}] = q_t K_{t+1} - \mathbb{E}_t[M_{t+2}q_{t+2}K_{t+3}].$$
 (IA.16)

Finally, iterating forward and imposing the transversality condition $\lim_{n\to\infty} \mathbb{E}_t[M_{t+n}q_{t+n}K_{t+n+1}] = 0$, we get

$$q_t K_{t+1} = \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t+s} D_{t+s} \right] = V_t - D_t,$$
 (IA.17)

or, equivalently,

$$V_t = D_t + q_t K_{t+1}.$$
 (IA.18)

It is important to note that we only use the firm's optimality conditions for the above derivation. In other words, Equation IA.18 holds for any conditional SDF distributions that the firm takes as given. Of course, different assumptions about the conditional SDF distributions lead to a different q process (and, hence, a different firm value process), but the message of Equation IA.18 is simple: due to the linear homogeneity conditions (Equations IA.8 and IA.9), the only information regarding conditional SDF distributions that is needed to price the firm is q. Thus, the statistical model for the firm's payout process is useful only for one thing: it allows us to back out the firm's equilibrium q process and, hence, to pin the down the equilibrium price of the firm. We can now turn to firm returns. The firm's return is

$$R_{t+1} = \frac{V_{t+1}}{V_t - D_t} = \frac{D_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}},$$
(IA.19)

which yields

$$R_{t+1} = \frac{\frac{D_{t+1}}{K_{t+1}} + q_{t+1}((1-\delta) + \frac{q_{t+1}-1}{a(1-\tau)})}{q_t}.$$
 (IA.20)

The investment return is

$$R_{t+1}^{I} = \frac{(1-\tau)\left(\partial_{K}\Pi(K_{t+1}, Z_{t+1}) - \partial_{K}\Phi(K_{t+1}, I_{t+1})\right) + \tau\delta + (1-\delta)q_{t+1}}{q_{t}},$$
 (IA.21)

which yields

$$R_{t+1}^{I} = \frac{\alpha(1-\tau)Z_{t+1} + \frac{(q_{t+1}-1)^2}{2a(1-\tau)} + \tau\delta + (1-\delta)q_{t+1}}{q_t}.$$
 (IA.22)

We can rewrite the investment return using Equation 5, which implies that

$$(1-\tau)\alpha Z_{t+1} = \frac{D_{t+1}}{K_{t+1}} + \frac{(q_{t+1})^2 - 1}{2a(1-\tau)} - \tau\delta.$$
 (IA.23)

Plugging the expression of IA.23 in Equation IA.22, we get, after some algebra,

$$R_{t+1}^{I} = \frac{\frac{D_{t+1}}{K_{t+1}} + (1-\delta)q_{t+1} + \frac{(q_{t+1}-1)q_{t+1}}{a(1-\tau)}}{q_t}.$$
 (IA.24)

From Equations IA.20 and IA.24, it is obvious that, in equilibrium, the firm's return and the investment return are equal state-by-state:

$$R_{t+1} = R_{t+1}^I. (IA.25)$$

A.3 Payout ratio upper bound

We show that the upper bound specification for the firm's payout ratio d, given by Equation 20, leads to feasible investment and payout processes for the firm.

At each period t, the firm needs to choose investment and payout policies that satisfy its budget constraint, given Z_t (which is exogenous) and K_t (which is predetermined). The firm has finite resources available for payout (consisting of the sum of its after-tax operating profit and its proceeds from sales of undepreciated installed capital, taking into account the firm's capital adjustment costs), so there is a maximal amount of payout that the firm is able to provide. Furthermore, since the firm's capital stock has to be always non-negative, the capital accumulation equation $K_{t+1} = (1-\delta)K_t + I_t$ implies that firm investment needs to satisfy $I_t \ge -(1-\delta)K_t$ for all t, which further constrains the maximal firm payout.^{IA.2}

In particular, the maximum payout that the firm is able to provide at period t, denoted by D_t^{max} , is given by the solution of the following static problem:

$$D_t^{max} = \max_{\{I_t\}} \left\{ (1-\tau) \left(\alpha Z_t K_t - \frac{a}{2} \left(I_t / K_t \right)^2 K_t \right) - I_t + \tau \delta K_t \right\},$$
(IA.26)

such that $I_t \ge -(1 - \delta)K_t$. It can be easily shown that the investment level that maximizes resources is $I_t = -\varphi K_t$, where $\varphi \equiv \min\{1/(a(1 - \tau)), 1 - \delta\}$, which yields a maximum payout level of

$$D_t^{max} = (1 - \tau)\alpha Z_t K_t - (1 - \tau)\frac{a}{2}\varphi^2 K_t + \varphi K_t + \tau \delta K_t.$$
 (IA.27)

It follows that the maximum payout per unit of output is

$$\frac{D_t^{max}}{Y_t} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2} \varphi \right) \varphi + \tau \delta \right],$$
(IA.28)

which is identical to the expression for the conditional upper bound of the payout ratio, d_t^{max} , in our model (Equation 20).

Next, we show that our specification for d ensures that the capital non-negativity constraint never binds and, hence, the model-implied firm marginal q is such that the firm's optimal investment always satisfies the interior optimality condition $I_t = \frac{q_t - 1}{a(1-\tau)}K_t$.

Fix t and assume that $K_t \ge 0$. The interior optimal investment satisfies the capital non-negativity constraint if (and only if) $\frac{q_t-1}{a(1-\tau)}K_t \ge -(1-\delta)K_t$. This condition is trivially satisfied for $K_t = 0$. For $K_t > 0$, the expression is equivalent to $q_t \ge 1-a(1-\tau)(1-\delta)$. Since economic logic suggests that the firm's marginal q is always non-negative, ensuring that the firm's interior optimality condition satisfies the capital non-negativity constraint implies ensuring the following lower bound for the firm's marginal q: $q_t \ge max\{0, 1 - a(1 - \tau)(1 - \delta)\}$. If that lower bound is satisfied, then the firm's investment policy at t is given by its interior optimality condition and K_{t+1} is non-negative. Iterating from $K_0 > 0$, it follows that we need to ensure that the model specification is such that the firm's model-implied marginal q satisfies $q_t \ge max\{0, 1 - a(1 - \tau)(1 - \delta)\}$ for all t.

All left to do is to confirm that our specification for the payout ratio d, which imposes a conditional upper bound on that process, leads to a model-implied q process that satisfies the condition above. First, consider the case that $a \ge \frac{1}{(1-\tau)(1-\delta)}$. Then, we need to show that our d process leads to a model-implied q process that satisfies $q_t \ge 0$ for all t. Indeed, for any $d_t \le d_t^{max} = (1 - \tau)\alpha + e^{-z_t} \left[\frac{1}{2a(1-\tau)} + \tau \delta\right]$, Equation 23 yields a real-valued (and non-negative) q_t^* . Now, consider

^{IA.2}By assumption, the firm has initial capital stock $K_0 > 0$, so the capital non-negativity constraint is trivially satisfied at the initial period.

the case that $a < \frac{1}{(1-\tau)(1-\delta)}$. We need to show that our d process leads to a model-implied q process that satisfies $q_t \ge 1 - a(1-\tau)(1-\delta)$ for all t. Indeed, for any $d_t \le d_t^{max} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2} \cdot (1-\delta) \right) (1-\delta) + \tau \delta \right]$, Equation 23 yields a real-valued q_t^* that satisfies $q_t^* \ge 1 - a(1-\tau)(1-\delta) > 0$.

B The Two-Period Model

This section contains the details and derivations for our two-period model.

B.1 The general equilibrium model

The economy has two periods (denoted by t and t + 1) and consists of a representative equityfinanced firm and a representative household. There is a single good (the price of which is normalized to one) that can be either consumed or used as a capital input in the firm's production, and all quantities are expressed in units of that good. The firm optimizes its investment-payout decision, whereas the household optimizes its consumption-saving decision.

B.2 The firm's problem

The representative firm is endowed with initial capital stock $K_t > 0$ and faces an exogenous stochastic productivity process Z, to be specified below, with realizations Z_t and Z_{t+1} . The only factor of production is capital, and the firm's output Y (which is equal to its operating profit Π) is given by function $\Pi(K, Z)$. The firm faces capital adjustment costs, with the adjustment cost function being $\Phi(K, I)$. At period t, Z_t is realized and then the firm decides how much of the profit will be distributed to the shareholders and how much will be invested in new capital. At period t + 1, Z_{t+1} is realized and then the firm is liquidated, so the entirety of the firm's profit, as well as the value of the remaining capital, is distributed to the shareholders as a payout.

The only asset in the economy is a claim on the firm's payout (i.e., the firm's equity), which is normalized to one share. We denote the period t cum-payout value of the firm by V_t and the ex-payout value of the firm by P_t , i.e., $P_t = V_t - D_t$. Thus, the gross return from investing in the equity of the firm from t to t + 1 is given by

$$R_{t+1} = \frac{D_{t+1}}{P_t} = \frac{D_{t+1}}{V_t - D_t}.$$
 (IA.29)

The firm chooses payout D_t and investment I_t to maximize the cum-payout value of the firm, V_t :

$$V_t = \max_{\{D_t, I_t\}} (D_t + \mathbb{E}_t [M_{t+1}D_{t+1}]), \qquad (IA.30)$$

where M_{t+1} is the stochastic discount factor (SDF) in the economy, subject to the capital accumulation process $K_{t+1} = (1 - \delta) \cdot K_t + I_t$, where δ is the one-period capital depreciation rate, and the one-period budget constraints,

$$D_t = \Pi(K_t, Z_t) - I_t - \Phi(K_t, I_t), \quad D_{t+1} = \Pi(K_{t+1}, Z_{t+1}) + (1 - \delta) \cdot K_{t+1}.$$

Note that, although the SDF is endogenous in our economy, it is taken as given by the firm when optimizing.

Imposing the expression for capital accumulation and the budget constraints, the firm's problem simplifies to

$$\max_{\{I_t\}} (D_t + \mathbb{E}_t \left[M_{t+1} \left(\Pi((1-\delta)K_t + I_t, Z_{t+1}) + (1-\delta)((1-\delta)K_t + I_t) \right) \right] \right),$$
(IA.31)

so that the only choice variable for the firm is investment I_t (as the firm's payout D_t is retrieved from the period t budget constraint). Assuming an interior solution, the firm's payout optimality condition is

$$-\partial_I D_t = \mathbb{E}_t \left[M_{t+1} \cdot (\partial_K \Pi((1-\delta)K_t + I_t, Z_{t+1}) + (1-\delta)) \right],$$
(IA.32)

so optimality is achieved when the marginal cost of an extra unit of investment (due to the reduction in the current payout) is equated with the present discounted value of the marginal gain in future payout due to the increased current investment. The firm's period t budget constraint yields $\partial_I D_t = -(1 + \partial_I \Phi(K_t, I_t))$, so we can rewrite the firm's optimality condition as

$$1 = \mathbb{E}_t \left[M_{t+1} \cdot \frac{\partial_K \Pi((1-\delta)K_t + I_t, Z_{t+1}) + (1-\delta)}{1 + \partial_I \Phi(K_t, I_t)} \right].$$
 (IA.33)

The intuition is simple: taking M_{t+1} as given, the firm adjusts its investment I_t (and, hence, its payout D_t) until the point that its optimality condition is satisfied. As a result, the firm's optimality condition yields the firm's payout policy: conditional on state variables K_t and Z_t , for any given M_{t+1} , the firm chooses the particular investment (and, thus, payout) level that is consistent with its optimization objective.^{IA.3}

In what follows, we assume that the firm's operating profit function is $\Pi(K, Z) = Z \cdot K$, that the capital adjustment cost function is $\Phi(K, I) = \frac{a}{2} \cdot (I/K)^2 \cdot K$, that the productivity process satisfies $Z_{t+1} = \bar{Z}^{1-\phi_z} \cdot Z_t^{\phi_z} \cdot e^{\epsilon_{z,t+1}}$, where $\bar{Z} > 0$, $\phi_z \in [0, 1)$ and $\epsilon_{z,t+1} \sim N(-\sigma_z^2/2, \sigma_z^2)$, and that capital

$$R_{t+1}^{I} = \frac{\partial_{K} D_{t+1}}{1 + \partial_{I} \Phi(K_{t}, I_{t})} = \frac{\partial_{K} \Pi((1-\delta)K_{t} + I_{t}, Z_{t+1}) + (1-\delta)}{1 + \partial_{I} \Phi(K_{t}, I_{t})}$$

so the firm's optimality condition reduces to $1 = \mathbb{E}_t [M_{t+1} \cdot R_{t+1}^I]$, as in Cochrane (1991).

^{IA.3}It is worth noting that we can express the firm's optimality condition in more familiar terms by considering the investment return, R^{I} , defined as the gross return of an extra unit of firm capital

fully depreciates within one period (i.e., $\delta = 1$).

The firm's marginal Tobin's q is defined as $q_t \equiv \mathbb{E}_t [M_{t+1}\partial_K \Pi(K_{t+1}, Z_{t+1})] = \mathbb{E}_t [M_{t+1}Z_{t+1}]$. It follows that the firm's optimal optimal investment policy is given by

$$I_t = I(K_t, Z_t; q_t) = \frac{q_t - 1}{a} K_t.$$
 (IA.34)

Substituting the firm's optimal investment policy into its period t budget constraint yields the firm's optimal payout policy:

$$D_t = D(K_t, Z_t; q_t) = \left[Z_t - \frac{q_t^2 - 1}{2a} \right] K_t.$$
 (IA.35)

Note that optimal firm investment is increasing in, and optimal firm payout is decreasing in, q_t : for any $\{K_t, Z_t\}$, the higher the firm's marginal q, the more the firm invests and the less it pays out at time t.

It is easy to see that $P_t = \mathbb{E}_t [M_{t+1}D_{t+1}] = \mathbb{E}_t [M_{t+1}Z_{t+1}] K_{t+1} = q_t K_{t+1} = q_t I_t$, so the firm's investment and payout policies implicitly depends on asset prices (and, in particular, the ex-dividend firm price P_t).

B.3 The household's problem

We now turn to the representative household. It is endowed with the entirety of the firm's outstanding equity, so initial wealth is $W_t = V_t$. The household consumes C_t and purchases ω_t shares of the representative firm in order to maximize its utility

$$\max_{\{C_t,\omega_t\}} \left(U(C_t,\theta_t) + \beta \cdot \mathbb{E}_t \left[U(C_{t+1},\theta_{t+1}) \right] \right)$$
(IA.36)

where β is the subjective discount factor, and $U(C, \theta)$ is the household's utility function, which has as its arguments household consumption C and the taste shifter θ , an exogenous stochastic process to be specified later. The household is able to shift resources across time by investing in the firm's equity, so the household optimizes subject to the following one-period budget constraints:

$$C_t = W_t - \omega_t P_t, \quad C_{t+1} = \omega_t D_{t+1}.$$

We can combine the household's two one-period budget constraints into the intertemporal budget constraint

$$C_{t+1} = (C_t - W_t) \cdot R_{t+1}.$$
 (IA.37)

Imposing the intertemporal budget constraint simplifies the household's problem to

$$\max_{\{C_t\}} \left(U(C_t) + \beta \cdot \mathbb{E}_t \left[U((W_t - C_t) \cdot R_{t+1}) \right] \right).$$
(IA.38)

Therefore, the only choice variable for the household is consumption C_t , with optimal ω_t being pinned down by the period t budget constraint. The household's optimality condition is the familiar Euler equation,

$$1 = \mathbb{E}_t \left[\beta \frac{\partial_C U((W_t - C_t) \cdot R_{t+1}, \theta_{t+1})}{\partial_C U(C_t, \theta_t)} \cdot R_{t+1} \right].$$
(IA.39)

Again, the intuition is straightforward: taking the properties of the firm return R_{t+1} as given, the household chooses consumption C_t so that its optimality condition is satisfied.

In what follows, we assume that the household's utility function is $U(C,\theta) = \theta \cdot \log(C)$ and that process θ has law of motion $\theta_{t+1} = \overline{\theta}^{1-\phi_{\theta}} \cdot \theta_t^{\phi_{\theta}} \cdot e^{\epsilon_{\theta,t+1}}$, where $\overline{\theta} > 0$, $\phi_{\theta} \in [0,1)$ and $\epsilon_{\theta,t+1} \sim N(-\sigma_{\theta}^2/2, \sigma_{\theta}^2)$. Furthermore, we assume that shocks $\epsilon_{\theta,t+1}$ and $\epsilon_{z,t+1}$ are independent of each other.

The household's optimal consumption policy is ^{IA.4}

$$C_t = C(\theta_t; W_t) = \frac{W_t}{1 + \beta(\theta_t/\bar{\theta})^{\phi_{\theta} - 1}}.$$
 (IA.40)

Recall that $W_t = V_t = P_t + D_t$, so the household's optimal consumption policy implicitly depends on asset prices (and, in particular, the ex-dividend firm price P_t).

B.4 Equilibrium

In equilibrium, both the goods market and the asset market clear. At period t, the goods market clears when the firm's output equals the sum of consumption demand from the household, investment demand from the firm, and capital adjustment costs: $\Pi(Z_t, K_t) = C_t + I_t + \Phi(K_t, I_t)$. At period t + 1, the only demand for the good is consumption demand, so the market clearing condition is $\Pi(Z_{t+1}, K_{t+1}) = C_{t+1}$. Using the firm's and household's budget constraints, it can easily be shown that the two goods market clearing conditions above reduce to a single goods market clearing condition: $C_t = D_t$. For the asset market to clear, the period t asset supply (normalized to one share) needs to equate the period t asset demand ω_t , so the asset market clearing condition is $\omega_t = 1$.

$$1 = \mathbb{E}_t \left[\beta \frac{\theta_{t+1}}{\theta_t} \left(\frac{C_t}{(W_t - C_t)R_{t+1}} \right) R_{t+1} \right] = \beta \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \right] \left(\frac{C_t}{W_t - C_t} \right),$$

which, solving for C_t , yields Equation IA.40

^{IA.4}The household's optimality condition is

It is easy to show that the two market clearing conditions, one for the payout market and one for the asset market, can be substituted by one – if one of the two markets clears, the other also clears. We choose to focus on the goods market clearing condition. We have

$$C(\theta_t; W_t) = D(K_t, Z_t; q_t). \tag{IA.41}$$

The condition above yields the following a quadratic equation for q_t :

$$\left(\frac{1}{2}\beta(\theta_t/\bar{\theta})^{\phi_{\theta}-1}+1\right)q_t^2 - q_t - \frac{1}{2}\beta(\theta_t/\bar{\theta})^{\phi_{\theta}-1}\left(2aZ_t+1\right) = 0.$$
 (IA.42)

That quadratic equation has two solutions, one positive and one negative. Since a negative q is economically inadmissible, the equilibrium q is

$$q_t = q(Z_t, \theta_t) = \frac{1 + \sqrt{1 + (\beta(\theta_t/\bar{\theta})^{\phi_\theta - 1} + 2)\beta(\theta_t/\bar{\theta})^{\phi_\theta - 1}(2aZ_t + 1)}}{\beta(\theta_t/\bar{\theta})^{\phi_\theta - 1} + 2}.$$
 (IA.43)

Thus, the firm's equilibrium q is a function of the two exogenous state variables, firm productivity Z and household taste θ . It follows that q fluctuations reflect two types of shocks, productivity (i.e., supply) shocks and taste (i.e., demand) shocks.

It follows that the firm's equilibrium price is

$$P_t = q_t I_t = \frac{1}{a} q(Z_t, \theta_t) (q(Z_t, \theta_t) - 1) K_t,$$
 (IA.44)

and the firm's equilibrium return is

$$R_{t+1} = \frac{D_{t+1}}{P_t} = \frac{Z_{t+1}}{q(Z_t, \theta_t)}.$$
 (IA.45)

Finally, the household's equilibrium consumption realizations are

$$C_t = \left[Z_t - \frac{q(Z_t, \theta_t)^2 - 1}{2a} \right] K_t, \qquad C_{t+1} = Z_{t+1} \frac{q(Z_t, \theta_t) - 1}{a}, \tag{IA.46}$$

and the firm's equilibrium payout-to-output process realizations are

$$\frac{D_t}{Y_t} = 1 - \frac{q^2(Z_t, \theta_t) - 1}{2aZ_t}, \qquad \frac{D_{t+1}}{Y_{t+1}} = 1.$$
 (IA.47)

In what follows, we show that, if we impose a statistical model of the representative firm's equilibrium (i.e., observed) payout process, our payout-based asset pricing approach retrieves the true equilibrium firm returns from the firm's optimal payout policy. We also show the equivalent result for consumption-based asset pricing: if we impose a statistical model of the representative household's equilibrium (i.e., observed) consumption process and of the representative firm's payout process, the consumption-based asset pricing approach backs out the true equilibrium firm returns from the representative household's optimal consumption policy.

B.5 Payout-based asset pricing

We assume that the representative firm faces the same problem (and, hence, has the same optimal payout policy) as in the true economy, and we postulate a statistical model for the firm's equilibrium (i.e., observed) payout process D. Importantly, for the payout-based approach to exactly back out the asset pricing implications of the true general equilibrium economy, the statistical model for process D needs to be chosen carefully so that it has the same properties as the firm's equilibrium payout process in the true economy.

In particular, consider the following statistical model for the firm's payout process:

$$D_t = d_t \cdot Y_t,$$

where Y is the observed firm output and d is a postulated statistical process (the "payout ratio" process).

The firm's optimal payout policy solves the firm's optimization problem and is given by Equation IA.35. To retrieve the asset pricing implications of our payout-based approach, we impose the condition that the firm's optimal payout coincides with the postulated statistical payout process, i.e., the condition

$$D(K_t, Z_t; q_t) = d_t \cdot Y_t.$$

Solving for the firm's model-implied marginal q, we get:

$$q_t^* = q^*(Z_t, d_t) = \sqrt{1 + 2a(1 - d_t)Z_t}.$$
(IA.48)

Thus, in the payout-based framework the model-implied marginal q is a function of the firm's productivity Z and the payout ratio d. Notably, the model-implied q does not explicitly reflect the household's taste process θ . It follows that the model-implied firm return is

$$R_{t+1}^* = \frac{Z_{t+1}}{q_t^*} = \frac{Z_{t+1}}{\sqrt{1 + 2a(1 - d_t)Z_t}}.$$
 (IA.49)

The payout-based approach yields the same asset pricing results as the true economy as long as the postulated d process coincides with the firm's equilibrium D/Y process in the true economy. In particular, we need

$$d_t = \frac{D_t}{Y_t} = 1 - \frac{q^2(Z_t, \theta_t) - 1}{2aZ_t}, \qquad d_{t+1} = \frac{D_{t+1}}{Y_{t+1}} = 1.$$
 (IA.50)

Indeed, it is easy to see that plugging the above value for the realization of d_t in Equations IA.48 and IA.49 yields $q_t^* = q_t$ and $R_{t+1}^* = R_{t+1}$, respectively. Essentially, the information in process d = D/Y allows us to recover all the information about process θ that we need in order to price the firm, even if our approach does not explicitly consider the household's optimization problem.

B.6 Consumption-based asset pricing

We assume that the representative household faces the same problem (and, hence, has the same optimal consumption policy) as in the true economy, and we postulate a statistical model for the household's equilibrium consumption process C and the firm's equilibrium payout process D. Since C = D in the true economy, all we need is to specify a statistical model for equilibrium consumption C. For the consumption-based approach to exactly back out the asset pricing implications of the true general equilibrium economy, the statistical model for process C needs to be chosen carefully so that it matches the household's equilibrium consumption process in the true economy.

The household's optimal consumption policy solves the household's optimization problem and is given by Equation IA.40. To retrieve the asset pricing implications of the consumption-based approach, we impose the condition that the household's optimal consumption coincides with the postulated statistical consumption (also known as "endowment") process e, i.e., the condition

$$C(\theta_t; W_t) = e_t$$

Solving for the firm's model-implied price P, we get:

$$P_t^* = P(\theta_t, e_t) = \beta(\theta_t/\bar{\theta})^{\phi_\theta - 1} e_t.$$
(IA.51)

Thus, in the consumption-based framework the model-implied price P is a function of the household taste θ and the endowment e. Notably, the model-implied P does not explicitly reflect firm-related variables (capital K and productivity Z). Hence, the model-implied firm return is

$$R_{t+1}^* = \frac{e_{t+1}}{P_t^*} = \frac{(\theta_t/\bar{\theta})^{1-\phi_\theta}}{\beta} \frac{e_{t+1}}{e_t}.$$
 (IA.52)

The consumption-based approach yields the same asset pricing results as the true economy if the postulated e process coincides with the firm's equilibrium C process in the true economy. Hence, we need

$$e_t = C_t = \left[Z_t - \frac{q(Z_t, \theta_t)^2 - 1}{2a} \right] K_t, \qquad e_{t+1} = C_{t+1} = Z_{t+1} \frac{q(Z_t, \theta_t) - 1}{a}.$$
 (IA.53)

Plugging the above values for the realization of e_t and e_{t+1} in Equations IA.51 and IA.52 yields

 $P_t^* = P_t$ and $R_{t+1}^* = R_{t+1}$, respectively. Thus, the information in process *e* allows us to back out all the information about processes *K* and *Z* that we need for pricing the firm, even though we do not explicitly consider the firm's optimization problem.

C Derivations for the Payout-Based Model with Firm Leverage

This section provides the derivations of the results for the extension of our payout-based asset pricing model, which features a levered representative firm.

C.1 Pricing the firm

As in case of the unlevered firm, we price the firm by following the approach outlined in Liu et al. (2009).

Since the operating profit function and the capital adjustment cost function are the same as in the case of the unlevered firm, Equations IA.8 and IA.9 are still satisfied. Furthermore, the leverage cost function $G(B, K) = \frac{\kappa}{2} \left(\frac{B}{K}\right)^2 K$ satisfies

$$G(B,K) = K \cdot \partial_K G(B,K) + B \cdot \partial_B G(B,K).$$
(IA.54)

Using Equations IA.8, IA.9, and IA.54 we can write the firm's optimal equity payout as

$$D_t^e = K_t \cdot \partial_K D_t - q_t I_t - (R_t^{b,a} + \partial_B G_t) B_t + B_{t+1}, \qquad (IA.55)$$

 \mathbf{SO}

$$\mathbb{E}_{t}[M_{t,t+1}D_{t+1}^{e}] = \mathbb{E}_{t}[M_{t,t+1}(K_{t+1} \cdot \partial_{K}D_{t+1} - q_{t+1}I_{t+1} - (R_{t+1}^{b,a} + \partial_{B}G_{t+1})B_{t+1} + B_{t+2})].$$
(IA.56)

We use the firm's investment and debt optimality conditions, $q_t = \mathbb{E}_t \left[M_{t,t+1} \left(\partial_K D_{t+1} + (1-\delta) q_{t+1} \right) \right]$ and $1 = \mathbb{E}_t \left[M_{t,t+1} \left(R_{t+1}^{b,a} + \partial_B G_{t+1} \right) \right]$, respectively, to rewrite Equation IA.56 as follows:

$$\mathbb{E}_{t}[M_{t,t+1}D_{t+1}^{e}] = (q_{t} - \mathbb{E}_{t}[M_{t,t+1}(1-\delta)q_{t+1}])K_{t+1} - \mathbb{E}_{t}[M_{t,t+1}q_{t+1}I_{t+1}] - B_{t+1} + \mathbb{E}_{t}[M_{t,t+1}B_{t+2}],$$
(IA.57)

which yields

$$\mathbb{E}_t[M_{t,t+1}D_{t+1}^e] = (q_t K_{t+1} - B_{t+1}) - \mathbb{E}_t[M_{t,t+1}(q_{t+1}K_{t+2} - B_{t+2})].$$
(IA.58)

Iterating, using the fact that $M_{t,t+2} = M_{t,t+1}M_{t+1,t+2}$, and applying the law of iterated expecta-
tions, we get

$$\mathbb{E}_t[M_{t,t+1}D_{t+1}^e] = (q_t K_{t+1} - B_{t+1}) - \mathbb{E}_t[M_{t,t+2}(D_{t+2}^e + q_{t+2}K_{t+3} - B_{t+3})], \quad (IA.59)$$

which yields

$$\mathbb{E}_t[M_{t,t+1}D_{t+1}^e] + \mathbb{E}_t[M_{t,t+2}D_{t+2}^e] = (q_tK_{t+1} - B_{t+1}) - \mathbb{E}_t[M_{t,t+2}(q_{t+2}K_{t+3} - B_{t+3})].$$
(IA.60)

Finally, iterating forward and imposing the transversality condition

$$\lim_{n \to \infty} \mathbb{E}_t [M_{t,t+n}(q_{t+n}K_{t+n+1} - B_{t+n+1})] = 0,$$

we get

$$q_t K_{t+1} - B_{t+1} = \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t,t+s} D_{t+s}^e \right] = P_t^e,$$
(IA.61)

so the market value of (ex-payout) equity is given by

$$P_t^e = q_t K_{t+1} - B_{t+1} = (q_t - b_{t+1}) K_{t+1}.$$
 (IA.62)

C.2 Equity returns

The firm's equity return is

$$R_{t+1}^e = \frac{D_{t+1}^e + P_{t+1}^e}{P_t^e} = \frac{d_{t+1}^e Z_{t+1} K_{t+1} + (q_{t+1} - b_{t+2}) K_{t+2}}{(q_t - b_{t+1}) K_{t+1}},$$
(IA.63)

which yields

$$R_{t+1}^{e} = \frac{d_{t+1}^{e} Z_{t+1} + (q_{t+1} - b_{t+2}) \left((1 - \delta) + \frac{q_{t+1} - 1}{a(1 - \tau)} \right)}{q_t - b_{t+1}}.$$
 (IA.64)

Note that Equation 32 implies that, in equilibrium,

$$b_{t+2} = \frac{R_{t+1}^b b_{t+1} - d_{t+1}^b Z_{t+1}}{(1-\delta) + \frac{q_{t+1}-1}{a(1-\tau)}},$$
(IA.65)

so we can rewrite the expression above as

$$R_{t+1}^{e} = \frac{d_{t+1}Z_{t+1} + q_{t+1}\left((1-\delta) + \frac{q_{t+1}-1}{a(1-\tau)}\right) - R_{t+1}^{b}b_{t+1}}{q_t - b_{t+1}}.$$
 (IA.66)

Finally, taking into account the firm's optimal capital structure policy (Equation 42), the expression above can be rewritten as

$$R_{t+1}^{e} = \frac{d_{t+1}Z_{t+1} + q_{t+1}\left(\left(1-\delta\right) + \frac{q_{t+1}-1}{a(1-\tau)}\right) - \left(\frac{\kappa}{\tau}b_{t+1} + 1\right)b_{t+1}}{q_t - b_{t+1}}.$$
 (IA.67)

C.3 Payout ratio upper bound

We show that the specification for the upper bound of the firm's payout ratio d, given by Equation 50, generates feasible model-implied investment and payout processes for the levered firm. The logic of our derivation follows the logic of the corresponding derivation for the unlevered firm.

At each period t, the firm needs to choose policies that satisfy its budget constraint, given Z_t (which is exogenous) and K_t and F_t (which are predetermined). The choice of B_{t+1} does not affect the total resources that the firm has available to pay out to all claimholders, D_t , as any choice of B_{t+1} leads to fully offsetting changes in the firm's debt and equity payout (see Equations 32 and 37, respectively). Hence, the maximum total payout that the firm is able to provide at period t, denoted by D_t^{max} , is given by the solution of the following static problem:

$$D_t^{max} = \max_{\{I_t\}} \left\{ (1-\tau) \left(\alpha Z_t K_t - \frac{a}{2} \left(I_t / K_t \right)^2 K_t \right) - I_t + \tau \delta K_t + \tau (R_t^b - 1) B_t - \frac{\kappa}{2} \left(B_t / K_t \right)^2 K_t \right\},$$
(IA.68)

such that $I_t \ge -(1-\delta)K_t$. As in the case of the unlevered firm, the investment level that maximizes resources is $I_t = -\varphi K_t$, where $\varphi \equiv \min\{1/(a(1-\tau)), 1-\delta\}$, which yields a maximum total payout level of

$$D_t^{max} = (1-\tau)\alpha Z_t K_t - (1-\tau)\frac{a}{2}\varphi^2 K_t + \varphi K_t + \tau \delta K_t + \tau (R_t^b - 1)B_t - \frac{\kappa}{2} (B_t/K_t)^2 K_t.$$
(IA.69)

It follows that the firm's maximum total payout per unit of output is

$$\frac{D_t^{max}}{Y_t} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2}\varphi \right)\varphi + \tau\delta + \tau (R_t^b - 1)b_t - \frac{\kappa}{2}b_t^2 \right],$$
(IA.70)

and, since optimal leverage satisfies $b_t = \frac{\tau}{\kappa} (R_t^b - 1)$ (see Equation 42), the expression above simplifies to

$$\frac{D_t^{max}}{Y_t} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2}\varphi \right)\varphi + \tau\delta + \frac{\kappa}{2}b_t^2 \right],$$
(IA.71)

which is identical to the expression for the conditional upper bound of the total payout ratio, d_t^{max} (Equation 50).

We now proceed to demonstrate that our specification for d ensures that the capital non-negativity constraint never binds and, hence, the firm always optimally picks an interior solution for investment. The interior investment condition of the levered firm, given by Equation 44, is identical to the interior investment condition of the unlevered firm. Thus, following the same steps as for the unlevered firm, we can show that we need to ensure that the model specification is such that the firm's marginal q satisfies $q_t \ge max\{0, 1 - a(1 - \tau)(1 - \delta)\}$ for all t.

We conclude by showing that our specification for the payout ratio d leads to a model-implied q process that satisfies the condition above. First, assume that $a \ge \frac{1}{(1-\tau)(1-\delta)}$. In that case, we need to show that our d process leads to a model-implied q process that satisfies $q_t \ge 0$ for all t. We can easily see that, for any $d_t \le d_t^{max} = (1-\tau)\alpha + e^{-z_t} \left[\frac{1}{2a(1-\tau)} + \tau\delta + \frac{\kappa}{2}b_t^2\right]$, Equation 55 yields a real-valued (and non-negative) q_t^* . Now, assume that $a < \frac{1}{(1-\tau)(1-\delta)}$. In that case, we need to show that our d process generates a model-implied q process that satisfies $q_t \ge 1 - a(1-\tau)(1-\delta)$ for all t. Indeed, for any $d_t \le d_t^{max} = (1-\tau)\alpha + e^{-z_t} \left[\left(1 - \frac{(1-\tau)a}{2} \cdot (1-\delta)\right)(1-\delta) + \tau\delta + \frac{\kappa}{2}b_t^2 \right]$, Equation 55 yields a real-valued q_t^* that satisfies $q_t^* \ge 1 - a(1-\tau)(1-\delta)$.

C.4 Collateral constraint

We need to ensure that the firm's optimal interior leverage ratio, given by Equation 42, always satisfies the firm's collateral constraint (Equation 35) and, hence, the collateral constraint never binds.

We start by rewriting the collateral constraint as a quadratic inequality:

$$\frac{\kappa}{2}b_{t+1}^2 + R_{t+1}^{b,a}b_{t+1} - \left((1-\delta) + (1-\tau)\alpha Z_{t+1}^{min} + \tau\delta\right) \le 0.$$
(IA.72)

Due to its nature, the firm's collateral constraint applies only when the firm borrows, i.e., when $b_{t+1} \ge 0$, whereas there is no constraint when the firm holds cash, i.e., when $b_{t+1} < 0$. It follows that the firm's collateral constraint is satisfied if and only if

$$b_{t+1} \le \frac{\sqrt{(R_{t+1}^{b,a})^2 + 2\kappa[(1-\delta) + (1-\tau)\alpha Z_{t+1}^{min} + \tau\delta] - R_{t+1}^{b,a}}}{\kappa}.$$
 (IA.73)

Consider the firm's optimal interior leverage ratio, given by $b_{t+1} = \frac{\tau}{\kappa}(R_{t+1}^b - 1)$. The interior optimum satisfies the collateral constraint if

$$\frac{\tau}{\kappa}(R_{t+1}^b - 1) \le \frac{\sqrt{(R_{t+1}^{b,a})^2 + 2\kappa[(1-\delta) + (1-\tau)\alpha Z_{t+1}^{min} + \tau\delta] - R_{t+1}^{b,a}}}{\kappa}.$$
 (IA.74)

We can show that the condition above is satisfied for all $R_{t+1}^b \leq \bar{R}_{t+1}^b$, where \bar{R}_{t+1}^b is a time-varying upper bound that depends on the value of Z_{t+1}^{min} . For our derivations, we assume that Z^{min} is always high enough so that R^b is always below the time-varying upper bound and, hence, the firm's collateral constraint never binds.

D Data Sources and Empirical Measures

We map output Y_t , and payout D_t to the corresponding measures for the aggregate public corporate sector in the United States. To do so, we rely on annual data from CRSP and COMPUSTAT, obtained from WRDS, as well as the dataset in Davydiuk et al. (2023) – henceforth, the DRSY dataset – obtained directly from the article's Journal of Finance webpage. The sample period for most of our analysis is determined by the DRSY dataset, which contains annual data from 1974 to 2017. Internet Appendix E discusses how we extend the sample period to 1950 for the analysis in Section

We measure annual output as $Y_t = P_t^e \cdot (Y_t/P_t^e)$, where P_t^e is the aggregate market value of equity reported in the DRSY dataset and Y_t/P_t^e is calculated using COMPUSTAT data. Specifically, in the annual COMPUSTAT dataset, we aggregate firm-level Y_t (measured as REVT) and P_t^e (measured as CSHO·PRCC_F when PRCC_F is available and CSHO·PRCC_C when PRCC_F is not available) for all firms for which Y_t and P_t^e are available and which have a fiscal year ending in December. For firms in our annual COMPUSTAT dataset which don't have a fiscal year ending in December, we use data from the quarterly COMPUSTAT dataset: we aggregate firm-level Y_t (measured as REVTQ) and P_t^e (measured as CSHOQ·PRCCQ) for all firms for which Y_t and P_t^e are available as of December of each year. Finally, we measure Y_t/P_t^e as the sum of firm-level Y_t divided by the sum of firm-level P_t^e .

We measure aggregate corporate payout as $D_t = Y_t \cdot d_t$, where the payout ratio d is given by

$$d_t = \frac{P_t^e}{Y_t} \cdot \left(\frac{D_t^e}{P_t^e} + \frac{D_t^b}{P_t^e}\right). \tag{IA.75}$$

In the expression above, D_t^e and D_t^b is the aggregate debt and equity payout of U.S. firms, respectively. The P_t^e/Y_t term is the inverse of the Y_t/P_t^e term obtained from COMPUSTAT data, as described above. The D_t^b/P_t^e term is directly obtained from the DRSY dataset. Finally, we calculate the D_t^e/P_t^e term using data from CRSP, which is the original data source for D_t^e/P_t^e in the DRSY dataset, as follows.^{IA.5} First, we retrieve the subset of the CRSP dataset which includes

$$\hat{D}_t^e = \gamma_{t-1} \cdot D_t^e - \Delta \gamma_t \cdot P_t^e$$

^{IA.5}We do not use the D_t^e/P_t^e values from the DRSY dataset for two reasons. First, the average DRSY D_t^e/P_t^e ratio is 1.7%, which implies a very high cash flow duration for the equity market. In contrast, our average D_t^e/P_t^e ratio is 2.5%. Second, to account for equity cross-holdings, DRSY assume that the return that corporations get on their equity portfolio is the same as the return that other investors get on their equity portfolio. While this assumption is reasonable, it has the effect that their D_t^e measure partially reflects the market value of firms, mixing cash flows with asset prices. Specifically, let D_t^e be the equity payout measured directly from CRSP, \hat{D}_t^e the payout from the portfolio that accounts for equity cross-holdings, and γ_t the fraction of the equity market held by public firms. DRSY assume $(D_t^e + P_t^e)/P_{t-1}^e = (\hat{D}_t^e + \gamma_t \cdot P_t^e)/(\gamma_{t-1} \cdot P_{t-1}^e)$, which allows them to measure their equity payout as

so P_t^e affects the DRSY D_t^e . Instead, our assumption is that the payout yield that public corporations get on their equity portfolio is the same as the payout yield that other investors get on their equity portfolio. Using the notation above, our assumption implies $\hat{D}_t^e = (\gamma_t \cdot P_t^e) \cdot (D_t^e/P_t^e) = \gamma_t \cdot D_t^e$, which does not include any asset pricing effect. Nonetheless, the correlation between the DRSY D/P measure and our measure is above 0.90, so the two measures

public firms incorporated in the United States (SHRCD = 10 or 11) trading on NYSE, Amex, or Nasdaq (EXCHCD = 1,2, or 3). Then, we measure the market value of equity monthly for each PERMNO (as |PRCC|·SHROUT) and carry it forward when there are missing observations. We measure net payout at the PERMNO level as $D_t^e = P_{t-1}^e \cdot (1+R_t^e) - P_t^e$ (where R_t^e is based on the RET variable in CRSP) – recall that P_t^e refers to the market value of equity (rather than price per share), so D_t^e retrieves the entirety of the firm's net equity payout (dividends plus equity repurchases, minus equity issuances), rather than just dividends. We assume that the first month of non-missing market equity is the firm's entry month in the public market portfolio so that $P_{t-1}^e = 0$ and $D_t^e = -P_t^e$ for the firm at that month. Moreover, in the delisting month we set $P_t^e = 0$ and $D_t^e = P_{t-1}^e \cdot (1 + R_t^e)$, where R_t^e is measured from the actual return or the delisting return depending on availability (when the return and delisting return are not available on the delisting month, we set $R_t^e = -1$ so that $D_t^e = 0$ over that month). After measuring P_t^e and D_t^e monthly at the PERMNO level, we aggregate over time (from January to December) to obtain annual D_t^e for each PERMNO and then aggregate across PERMNOs to obtain aggregate annual D_t^e values. Similarly, we aggregate P_t^e across PERMNOs at the end of each December to obtain the aggregate P_t^e . Finally, we compute the aggregate D_t^e/P_t^e and use it in Equation IA.75.

Finally, we measure productivity $Z_t = Y_t/K_t$ in a way that allows us to not take a stand on how to measure investment or capital, which is advantageous given that measuring physical capital is prone to non-trivial measurement errors (see, e.g., Bai et al. (2024)) and that firms can have different sources of capital beyond physical capital (see for example Gonçalves et al. (2020) and Belo et al. (2022)). Specifically, we start by taking our calibrated δ , τ , a, and α values as given, together with the Y_t and D_t series (and thus the d_t series) described above. We, then, set the initial value for Z_t in 1974 (the first year in our sample) to its steady-state value and update the Z_t series as follows (consistent with our model)^{IA.6}:

$$q_t = \sqrt{1 + 2a(1-\tau)(\tau\delta + [\alpha(1-\tau) - d_t]Z_t)},$$
 (IA.76)

$$i_t = \frac{q_t - 1}{a(1 - \tau)},$$
 (IA.77)

$$K_{t+1} = (1 - \delta + i_t) \cdot K_t, \qquad (IA.78)$$

$$Z_{t+1} = Y_{t+1} / K_{t+1}. (IA.79)$$

have very similar dynamics, with the main difference being that our measure has a higher mean.

^{IA.6} It should be stressed that our methodology for backing out aggregate productivity Z does not impose our asset pricing model into the data, but follows directly from the firm's budget constraint and the capital accumulation equation, similar in spirit to the methodology used in Cochrane (1991). To see that, note that Equations IA.76 and IA.77 can be combined into one equation that reflects the firm's budget constraint.

In the expressions above, *i* denotes the investment-to-capital ratio (i.e., i = I/K). We follow an analogous procedure for the model with firm leverage, except that we also use the expression for the evolution of *b* (Equation 56), since *q* also depends on *b*.

We can now turn to returns. The firm return is given

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t},\tag{IA.80}$$

where P_t is measured as described previously and D_t is measured as in Equation IA.75. The returns in the DRSY dataset differ from ours because we do not use their D_t^e measure (as discussed in Footnote IA.5). However, the differences are not large: the correlation between the two firm return measures is 0.995. Moreover, our return measure makes it somewhat harder for the model to match the data, as the DRSY measure implies higher average and more volatile aggregate returns. In particular, the DRSY measure implies $\mathbb{E}[R] = 7.0\%$ and $\sigma[R] = 14.2\%$, whereas our measure implies $\mathbb{E}[R] = 7.9\%$ and $\sigma[R] = 14.9\%$. Similarly, the equity and debt returns are given by

$$R_{t+1}^e = \frac{P_{t+1}^e + D_{t+1}^e}{P_t^e},$$
 (IA.81)

and

$$R_{t+1}^b = \frac{P_{t+1}^b + D_{t+1}^b}{P_t^b},$$
(IA.82)

respectively.

E Investment-Based Approach Implementation

This section provides the details for the measurement of aggregate investment, as well as additional details on the empirical performance of different implementations of the investment-based approach compared to the payout-based approach.

E.1 Investment measurement

We consider eight empirical measures for the aggregate investment rate i. The first measure, denoted by " i_{WG} ", is the I/K measure from Welch and Goyal (2008). We obtain the time series of that measure, which is expressed in quarterly terms, directly from Amit Goyal's webpage and annualize it by multiplying it by 4. The second measure (" i_{NIPA} ") is calculated by proxying aggregate investment by the "Real Gross Private Domestic Investment" series from National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis. In particular, we sum the amounts of the quarterly investment series within each year to obtain the annual series. Then, we recover the annual i = I/K series by applying a method analogous to that used in Cochrane (1991), adjusting for the fact that our capital accumulation equation differs from the one used in Cochrane (1991).^{IA.7}

Our next two measures are calculated using data from the Fixed Assets Account (FAA) of the Bureau of Economic Analysis. The first measure (" i_{FAA}^p ") is a physical investment rate, calculated as in Bai et al. (2024): aggregate investment I is proxied by the sum of investment in nonresidential equipment and structures (FAA Table 2.7) and aggregate capital K is proxied by the sum of the current-cost net stock of nonresidential equipment and structures (FAA Table 2.7). The second measure (" $i_{FAA}^{p,i}$ ") is a physical and intangible investment rate. To calculate it, for each year we add the amount of intangible investment and capital to the amount of physical investment and capital, respectively. Intangible investment and capital is proxied by investment and current-cost net stock of nonresidential intellectual property products (FAA Tables 2.7 and 2.1, respectively).

The last four investment rate measures $("i^p_{COMP}", "i^{p,i}_{COMP}", "i^{p,w}_{COMP}", and "i^{p,i,w}_{COMP}")$ are calculated using firm-level COMPUSTAT data: i_{COMP}^p is the physical investment rate, $i_{COMP}^{p,i}$ is the physical and intangible investment rate, $i_{COMP}^{p,w}$ is the physical and working capital investment rate, and $i^{p,w,i}$ is the combined investment rate from all three capital types. For each type of investment rate, we calculate the corresponding types of aggregate capital stock and aggregate investment flow by aggregating firm-level data, as follows. To calculate firm-level physical investment, we follow Gonçalves et al. (2020): we use the law of motion $I_t = K_{t+1} - K_t + Depr_t$, and define firm-level depreciation as variable DP and firm-level physical capital as variable PPENT (net property, plant, and equipment). The stock of working capital and the associated investment flow is calculated following Gonçalves et al. (2020): we proxy for working capital by ACT (current assets) and we calculate its investment flow using the working capital law of motion (under the assumption of zero depreciation): $I_t^w = K_{t+1}^w - K_t^w$. Finally, we calculate firm-level intangible investment by following the methodology of Peters and Taylor (2017). In particular, we set the missing values of XRD and XSGA (which reflect R&D and SG&A) to zero when total assets is available. We also set to zero all missing values of RDIP, which reflects the portion of R&D that does not enter the SG&A variable in COMPUSTAT. Then, we calculate Pure SG&A as XSGA - (XRD-RDIP) and replace these values with XSGA if either XRD > XSGA or if XRD < COGS. Intangible investment is, then, equal to 0.3*Pure SG&A + XRD. To be consistent with the timing in the model, the aggregate investment rate in year t is defined as $i_t = I_t/K_t$, where I_t is the aggregate investment during year t and K_t is the aggregate capital stock at the end of year t-1.^{IA.8} When computing the aggregate investment rates, we are careful to account for the fact that different firms may have different types of missing information. In particular, analogously to our approach in Section D of the Internet Appendix,

^{IA.7}In our model, capital accumulation satisfies $i_{t+1} = (I_{t+1}/I_t) \cdot (i_t/(1-\delta+i_t))$ and the steady state of the investment rate, used to initialize the *i* series, is given by $i_{ss} = \mathbb{E}[I_{t+1}/I_t] - 1 + \delta$.

^{IA.8}Since the COMPUSTAT dataset starts in 1950, we are unable to calculate I_{1950} and K_{1950} directly from COM-PUSTAT data. Instead, we estimate those values as follows: we deflate the corresponding I_{1951} and K_{1951} values either by the growth rates of I_{FAA}^{p} and K_{FAA}^{p} (for the calculation of i_{COMP}^{p} and $i_{COMP}^{p,w}$) or by the growth rates of $I_{FAA}^{p,i}$ and $K_{FAA}^{p,i}$ (for the calculation of $i_{COMP}^{p,w,i}$).

each aggregate variable is divided by the fraction of aggregate market equity that corresponds to the firms for which the variable of interest is not missing. Finally, in order to align the timing of accounting data across firms, we calculate all firm measures for a given year using December COMPUSTAT data. For the firms with fiscal years not ending in December (for which we cannot use the annual COMPUSTAT database), we use quarterly COMPUSTAT data whenever available (in which case flow variables reflect the sum of the corresponding quarterly variables within the year).

To obtain the model-implied returns R^* for each implementation of the investment-based approach, we also need the time series of aggregate productivity Z. For each implementation, we retrieve a distinct Z series that is consistent with the calculation of aggregate capital K, as follows: aggregate productivity satisfies Z = Y/K in the model, so we divide the series of aggregate output with the implementation-specific series of aggregate capital. For the implementation associated with i_{WG} , we do not directly observe K, so the aggregate capital series is retrieved using the identity K = I/i, where i is the implementation-specific investment rate and I is calculated as follows: we recover the series of quarterly investment growth by inverting Equation IA.1 in Cochrane (1991) and, then, calculate the quarterly I series by normalizing the initial investment to one. To recover the quarterly investment growth series, we use a = 13.22 and $\delta = 0.025$.^{IA.9}

Finally, for the calculation of R^* , we use different capital depreciation rates δ across investmentbased implementations in order to be consistent with the depreciation rate used for the calculation of the corresponding *i*. In particular, we use $\delta = 0.10$ and $\delta = 0.15$ for the implementations associated with i_{WG} and i_{NIPA} , respectively. For the rest of the implementations, we set $\delta = \mathbb{E}[i] - (e^g - 1)$, where *g* is the average log growth rate of aggregate output *Y*. Adjusting the depreciation rates in that fashion ensures that the investment-based approach implies a steady-state growth rate that is in line with the data. It should be stressed that the depreciation rate adjustment is not crucial for our findings: when we use $\delta = 0.15$ across implementations of the investment-based approach, we get very similar results, with the only difference being that model-implied returns match with the average level of actual returns less well.

E.2 Model-implied returns with constant productivity

We repeat the exercise of Table 3 with one change: we calculate model-implied returns under the assumption that firm productivity Z is constant (and equal to its steady-state value), as in Cochrane (1991), and report our findings in Table IA.1.

^{IA.9} From implementations associated with i measures calculated using aggregate capital series obtained from sources other than COMPUSTAT, aggregate output Y and aggregate capital K do not correspond to the same sample of firms. To address that issue, we multiply the implementation-specific measure of aggregate capital by the average of the ratio of the capital measure calculated using COMPUSTAT over the capital measure calculated using the non-COMPUSTAT source.

As seen in Panels A and B, which consider the period from 1974 to 2017, our findings remain almost identical to those in the corresponding panels of Table 3: the payout-based approach generates model-implied returns that are substantially more similar to actual returns than the model-implied returns retrieved using the investment-based approach. When the start the sample period in 1950 (Panel C), the payout-based approach continues to dominate, but the relative performance of the investment-based approach improves. In particular, none of the eight implementations of the investment-based approach is able to jointly match the mean and the volatility of actual returns, in contrast to the payout-based approach, which successfully matches both moments. On the other hand, the investment-based approach generates model-implied returns that exhibit modest positive correlation with actual (time-shifted) returns – the correlation coefficients range from 0.07to 0.47, comparable in magnitude to the correlation between the model-implied returns retrieved with the payout-based approach and the (standard-time) actual returns, which is 0.42. In sum, the assumption of constant productivity does not materially alter the results documented in Table 3. with the only exception being a slight improvement in the correlation between model-implied returns (for both the payout-based approach and the investment-based approach) and actual returns when we consider the long (1950–2017) sample, and, hence, attributable to return observations associated with the early years of the long sample.

E.3 Model-implied returns with estimated parameters

To check the robustness of our findings in Table 3 to alternative values of the firm's technological parameters, we repeat the exercise by considering implementation-specific estimated parameters, which provide each implementation with its best chance to match firm returns. In particular, in Table IA.2, the values of the capital adjustment cost parameter a and the operating profit margin parameter α are estimated by a Non-Linear Least Squares (NLS) regression of actual realized returns on model-implied realized returns. The rest of the model parameters are fixed at the calibrated values reported in Table 4.

Panel A focuses on firm returns in the 1974–2017 sample period. Both the payout-based approach and most of the implementations of the investment-based approach are able to match the level of average returns. However, both the payout-based approach and the investment-based approach generate firm returns that are too smooth compared to their empirical counterparts. Notably, the payout-based approach dominates all implementations of the investment-based approach: the model-implied returns retrieved using the payout-based approach exhibit a standard deviation of 8.09%, whereas the unconditional volatility of returns backed out using the investment-based approach ranges from 1.81% to 5.61%. The payout-based approach also dominates as regards the correlation of model-implied returns with actual returns: the payout-based approach generates an unconditional correlation of 0.58 (under the standard timing convention for actual returns), whereas the corresponding correlation for the different implementation of the investment-based approach ranges from -0.05 to 0.37 (under the more favorable shifted timing for actual returns). It is worth noting that the best-performing implementation of the investment-based approach is the one associated with i_{WG} , which is also the best-performing investment-based implementation in Table 3.^{IA.10}

Panel B focuses on CRSP value-weighted equity returns in the 1950–2017 sample period. Again, both the payout-based approach and most of the implementations of the investment-based approach generate returns that exhibit a realistic mean, but counterfactually low volatility. Furthermore, the correlation between model-implied and actual returns generated by the payout-based approach (0.30) is about the same as the highest correlation generated across the implementations of the investment-based approach (0.31). Overall, the performance of the payout-based approach is comparable to that of the best-performing implementation of the investment-based approach (which, as before, is the implementation associated with i_{WG}) in the long sample, suggesting that including the early (i.e., 1950–1973) period is favorable to the investment-based approach, consistent with our findings in Table 3.

^{IA.10}Our findings are very similar when we replace U.S. aggregate firm returns with U.S. aggregate equity returns for the 1974–2017 period. In the interest of space, we leave those results untabulated.

Table IA.1: Payout-based vs. investment-based returns (constant productivity)

This table reports the properties of model-implied realized firm returns for nine different implementations, under the assumption that firm productivity is constant. The nine implementations are, in order, the payout-based approach (column two) and eight implementations of the investment-based approach (columns three to ten). Each panel reports the unconditional moments of model-implied firm returns and corresponding observed aggregate U.S. returns, with the panels differing in the observed return measure employed. In particular, Panel A considers U.S. aggregate firm returns for the 1974–2017 sample period, Panel B considers U.S. aggregate equity returns for the 1974– 2017 sample period, and Panel C considers U.S. aggregate equity returns for the 1950–2017 sample period.

Panel A: Firm Returns $(1974 - 2017)$												
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Average R	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%			
Volatility of R	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%			
Average R^*	5.56%	5.19%	1.26%	7.99%	3.37%	1.63%	-2.80%	1.29%	-1.89%			
Volatility of R^*	14.03%	3.11%	5.14%	3.17%	2.92%	13.07%	8.78%	16.23%	12.31%			
$\operatorname{Corr}(R^*,R)$	0.57	0.08	-0.09	-0.13	-0.11	0.16	0.14	0.12	0.12			
$\operatorname{Corr}(R^*, R^{shifted})$	0.20	0.39	0.19	0.12	0.14	0.08	0.08	0.08	0.09			
Panel B: Equity Returns (1974 – 2017)												
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Average R	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%	8.96%			
Volatility of R	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%	17.76%			
Average R^*	5.56%	5.19%	1.26%	7.99%	3.37%	1.63%	-2.80%	1.29%	-1.89%			
Volatility of R^*	14.03%	3.11%	5.14%	3.17%	2.92%	13.07%	8.78%	16.23%	12.31%			
$\operatorname{Corr}(R^*,R)$	0.60	0.12	-0.06	-0.10	-0.09	0.16	0.15	0.13	0.13			
$\operatorname{Corr}(R^*, R^{shifted})$	0.17	0.42	0.23	0.17	0.19	0.05	0.06	0.05	0.07			
Panel C: Equity Returns (1950 – 2017)												
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Average R	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%			
Volatility of R	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%			
Average R^*	7.43%	4.78%	0.75%	8.90%	5.41%	2.88%	-1.06%	2.72%	0.00%			
Volatility of R^*	14.96%	3.07%	5.08%	3.06%	2.85%	11.30%	7.86%	14.66%	11.57%			
$\operatorname{Corr}(R^*,R)$	0.42	0.11	-0.03	-0.19	-0.18	0.04	0.00	0.04	0.02			
$\operatorname{Corr}(R^*, R^{shifted})$	0.07	0.47	0.35	0.19	0.21	0.07	0.07	0.14	0.15			

Table IA.2: Payout-based vs. investment-based returns (estimated parameters)

This table reports the properties of model-implied realized firm returns for nine different implementations, using implementation-specific parameter estimates. The nine implementations are, in order, the payout-based approach (column two) and eight implementations of the investment-based approach (columns three to ten). Panel A reports the parameter values for each implementation. Panel B panel reports the unconditional moments of model-implied firm returns and corresponding observed aggregate U.S. firm returns for the 1974–2017 sample period.

		Panel A	: Firm I	Returns	(1974 - 2)	2017)						
Part I: Parameter values												
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Adj. Cost Parameter a	1.87	31.21	2.59	9.47	13.47	0.61	1.22	0.41	0.67			
Profit Margin α	0.11	0.36	0.17	0.07	0.13	0.14	0.23	0.19	0.27			
Part II: Unconditional moments												
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Average R	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%	7.86%			
Volatility of R	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%	14.88%			
Average R^*	6.01%	7.91%	7.57%	1.84%	1.16%	7.21%	7.00%	6.91%	6.82%			
Volatility of R^*	8.09%	5.61%	3.84%	4.01%	4.40%	2.74%	2.95%	1.81%	2.17%			
$\operatorname{Corr}(R^*, R)$	0.58	0.08	-0.02	-0.07	-0.06	-0.07	-0.03	-0.03	-0.02			
$\operatorname{Corr}(R^*, R^{shifted})$	0.28	0.37	0.20	0.13	0.14	-0.05	-0.02	0.03	0.04			
		Panel B	: Equity	Returns	(1950 –	2017)						
	Part I: Parameter values											
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Adj. Cost Parameter a	0.59	38.78	17.25	30.91	67.20	0.93	1.58	0.94	1.27			
Profit Margin α	0.08	0.40	0.35	0.14	0.31	0.14	0.21	0.20	0.27			
		Part	II: Unco	onditiona	l momer	nts						
Model	d	i_{WG}	i_{NIPA}	i^p_{FAA}	$i^{p,i}_{FAA}$	i^p_{COMP}	$i_{COMP}^{p,i}$	$i_{COMP}^{p,w}$	$i_{COMP}^{p,i,w}$			
Average R	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%	8.84%			
Volatility of R	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%	17.40%			
Average R^*	8.39%	7.07%	6.68%	4.24%	3.85%	7.72%	7.06%	7.80%	7.48%			
Volatility of R^*	6.95%	6.00%	7.92%	5.84%	6.16%	3.44%	3.71%	2.98%	3.18%			
$\operatorname{Corr}(R^*, R)$	0.30	0.00	-0.08	-0.24	-0.22	0.01	0.01	-0.02	-0.01			
$\operatorname{Corr}(R^*, R^{shifted})$	0.12	0.31	0.23	0.13	0.15	0.09	0.13	0.13	0.16			