Index Investing and Sentiment Spillover

Abstract

We develop a dynamic model of index investing that can reconcile key cross-sectional differences between index and non-index stocks. In our model, investors with extrapolative expectations create sentiment, and index investing spills the sentiment on an index stock to all other index stocks. Primarily due to this spillover mechanism, we find that when index investors are mostly extrapolators, all consistent with empirical evidence, index stocks have higher and more volatile prices, comove more with other index stocks, exhibit stronger negative price autocorrelation, and have higher trading volume than comparable non-index stocks. Our model also reconciles the recently observed "disappearing index effect" and delivers novel implications on the flow-performance relation for index funds, the response of investor portfolios to their subjective beliefs, and the welfare costs of index investing.

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1 Introduction

The investment management industry has experienced a substantial shift from active management to passive index investing in recent decades. For instance, according to the 2024 Investment Company Fact Book, the total net assets of index funds in the U.S. grew from \$1.88 trillion in 2010 to \$13.3 trillion in 2023. Regarding the relative share of index funds in the investment industry, these numbers correspond to 19% and 48%, respectively.[1] This trend is often attributed to the poor after-fee performance of active funds relative to their benchmark index, leading to the practical advice for retail traders to opt for more cost-effective index funds. Corresponding to this shift, numerous empirical studies (discussed below) examine the effects of index investing and the cross-sectional differences between index and non-index stocks. For instance, the research shows that index stocks have higher and more volatile prices, comove more with other index stocks, exhibit stronger negative autocorrelation, and have higher trading volume than comparable non-index stocks. Moreover, empirical works also show that the "index effect" (stocks added to an index experiencing higher abnormal returns) has diminished recently, and there is a positive flow-performance relation for index funds, akin to active funds.

In this paper, we develop a dynamic equilibrium model of index investing that can simultaneously reconcile all the empirical regularities discussed above. Our model also delivers novel implications on the response of investor portfolios to their subjective beliefs and the welfare costs of index investing. The key feature of our model is the presence of index investors with extrapolative expectations. Indeed, index investing has become a popular investment strategy, particularly among retail investors, who are shown to have such expectations. For instance, growing survey evidence shows that many individual investors' stock return expectations are extrapolative, i.e., they expect higher (lower) future stock returns following a series of high (low) returns (e.g., Vissing-Jorgensen (2003), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Cassella and Gulen (2018), Da, Huang, and Jin (2021)). To our knowledge, ours is the first theory to study index investing under extrapolative expectations, enabling it to simultaneously explain all the empirical evidence discussed above.

Specifically, we consider an economy with multiple risky stocks that are claims to uncertain dividends (cash flows). An index fund, which passively tracks an index based on a subset of stocks, is also available for trading. We refer to a stock as an *index stock* if it belongs to this index and a *non-index stock* if not. In this economy, risk-averse investors differ in their beliefs and investment profiles. In terms of beliefs, investors either have rational or extrapolative expectations. In terms of investment profiles, investors either trade individual stocks or the index fund. In particular, stock extrapolators trade individual stocks but not the index fund. In line with survey evidence, these investors' expectations about future stock prices are driven by past stock prices. Index extrapolators, on the other hand, trade the index

[1] See, https://www.icifactbook.org/. In a recent study, Chinco and Sammon (2024) argue that the size of passive ownership could be double of the reported values of index investing if one takes into account of other passive investors such as internal and closet indexers.

fund but not individual stocks, and their expectations about future index performance are driven by past index performance. Rational stock (index) investors have correct expectations and invest only in individual stocks (the index fund). We follow the literature (e.g., Barberis, Greenwood, Jin, and Shleifer (2015)) and refer to extrapolators' expectations as "sentiment" since a higher (lower) value of it implies that the average expectation across investors is relatively optimistic (pessimistic) on that asset. The prices of index and non-index stocks and the index level are determined endogenously in equilibrium.

We first determine the equilibrium and show that the presence of index investors, i.e., index investing, generates a novel *sentiment spillover* such that the sentiment of an index stock affects not only the price of that stock but also the prices of all other stocks in the index. In particular, when index investors are mostly extrapolators, the price of an index stock positively relates to the sentiment of all other index stocks. This positive spillover effect arises because a good cash flow shock to an index stock increases the price of that stock and the index level (and more so in the presence of stock extrapolators whose additional sentiment-driven demand amplifies the cash flow shocks). Consequently, index extrapolators expect the index to do well in the future and demand more index fund shares. Since the fund allocates investors' demand across index stocks to track the index, all index stocks experience a higher demand, generating a positive relation between the sentiment of an index stock and the prices of all index stocks. This mechanism not only amplifies the initial cash flow shock for that stock but also generates a positive relation among sentiments in all index stocks, thereby further boosting the index sentiment.

The presence of both index and non-index stocks in our model enables us to study cross-sectional differences between them within the same economy. We find that when index investors are mostly extrapolators, all consistent with empirical evidence, index stocks have higher prices (Harris and Gurel (1986), Shleifer (1986), Greenwood and Sammon (2024)), are more volatile (Sullivan and Xiong (2012), Ben-David, Franzoni, and Moussawi (2018), Coles, Heath, and Ringgenberg (2022)), comove more with other index stocks (Greenwood and Sosner (2007), Wurgler (2010), Boyer (2011), Coles, Heath, and Ringgenberg (2022)), exhibit stronger negative autocorrelations (Ben-David, Franzoni, and Moussawi (2018), Baltussen, van Bekkum, and Da (2019), Höfler, Schlag, and Schmeling (2023)), and have higher trading volume (Vijh (1994), Coles, Heath, and Ringgenberg (2022)) than otherwise identical non-index stocks in equilibrium. Therefore, our one key contribution is to argue that the documented cross-sectional differences between index and non-index stocks could very well be due to the marginal index investor having extrapolative expectations. We further show that these results are robust and emerge not only when index investors are new to the market but also when existing stock investors switch to index investing. We elaborate more on the mechanisms driving these results below.

The presence of index investors generates an "index effect" by leading to relatively higher prices for index stocks than comparable non-index stocks, primarily due to higher demand for index stocks when index investors are new to the market. More notably, we find that an increase in extrapolative indexers leads to a weaker index effect. This result arises because, unlike a rational indexer, an extrapolative indexer makes the index stocks and the index relatively more volatile due to the sentiment spillover and amplification mechanisms discussed above. Thus, risk-averse investors' demands for index stocks and the index fund are relatively smaller under more extrapolative indexers, leading to a limited increase in index stock prices. The empirical evidence shows that the index effect was particularly strong in the 1980s and 1990s but has diminished significantly recently. For instance, Greenwood and Sammon (2024) show that the abnormal price increase associated with stock added to the S&P 500 was 7.4% in the 1990s but less than 1% in the past decade. Greenwood and Sammon (2024) offer several plausible explanations for this trend. Our finding here offers yet another possible explanation. Namely, the recent rise in the index trading activity by retail investors, who tend to have extrapolative expectations, may have also played a role for the disappearing index effect.

When index investors are mostly extrapolators, the positive sentiment spillover effect naturally leads to stronger comovement among index stocks. We also find that an increase in extrapolative indexers leads to relatively stronger price reversals on average, i.e., negative autocorrelation, for index stocks than non-index stocks. Again, this result occurs because a good cash flow shock to an index stock increases the overall index level, which in turn induces index extrapolators to expect a good index performance in the future, leading them to increase their index fund demand, resulting in an even higher index level. In subsequent periods, index extrapolators' expectations become less bullish since the initial rise in the index is assigned less weight in their expectations, leading to reduced demands and lower prices on average. When there are more index extrapolators, this mechanism gets amplified, leading to a stronger price reversal and more negative price autocorrelation.

Furthermore, we show that index stocks experience higher trading volume than non-index stocks, and an increase in extrapolative indexers leads to a greater trading volume difference than an increase in rational indexers. In our model, stock trading activity occurs due to the sentiment-driven disagreement among rational and extrapolative investors. Due to the spillover effect, sentiment in index stocks is more volatile than that in non-index stocks. Therefore, rational and extrapolative investors disagree more strongly and frequently with each other, resulting in more intense trading activity in index stocks. This mechanism also leads to trades on index stocks even though they do not encounter cash flow shocks. For example, following a positive cash flow shock to any index stock, other index stock prices react positively. Rational stock investors know these index stocks' fundamentals are unchanged and thus are willing to trade with index investors.

In addition to addressing observed cross-sectional differences between index and non-index stocks, our model also has several other novel implications. We find that when index investors are mostly extrapolators, our model generates a positive flow-performance relation for the index fund, consistent with empirical evidence (Goetzmann and Massa (2003), Dannhauser and Pontiff (2024), Anadu et al. (2020), Broman (2022)). Intuitively, the good performance of index fund induces index extrapolators to be more optimistic about its future performance, leading to a net fund inflow.

We further show that when more extrapolators invest in index funds, existing stock investors' portfolio allocations become less sensitive to their subjective beliefs. This result arises because an increase in extrapolative indexers leads to a higher volatility for index stocks, making stock investors more reluctant to act on their beliefs. Moreover, due to positive sentiment spillover, stock extrapolators may even hold less of an index stock so as to reduce aggregate risk exposure given that index stocks are positively correlated. Therefore, our result here may help understand the so-called "attenuation puzzle," that portfolio responses to beliefs being much smaller in the data than what standard theories predict (e.g., Amromin and Sharpe (2013), Ameriks et al. (2020), Giglio et al. (2021), Dahlquist and Ibert (2024)).

We also examine how index investing affects investors' welfare and find that welfare loss of switching to index investing for rational stock investors is higher when there are more extrapolative stock investors. This finding is intuitive. The rational stock investors expect to make larger profits and would be unwilling to switch to index investing when there are more stock extrapolators who, compared to index extrapolators, generate more profit opportunities in individual stocks for them.

Our paper adds to the extensive literature on subjective expectations in financial markets. More specifically, motivated by the growing survey evidence showing that many investors' stock return expectations are extrapolative, several theories are developed to study asset pricing implications of such expectations (e.g., Cutler, Poterba, and Summers (1991), De Long et al. (1990), Hong and Stein (1999), Barberis et al. (2015, 2018), Jin and Sui (2022), Atmaz (2022), Li and Liu (2023), Atmaz et al. (2024)). Among these works, the paper with the closest framework to ours is Barberis et al. (2015). They consider a single-stock economy and show that the presence of extrapolative investors can help reconcile various features of stock market returns while also being consistent with survey evidence on investor expectations. Differently from them, our analysis is based on a multiple-stock framework and additionally incorporates an index fund and index investors. These differences enable us to complement this literature by generating several novel implications. For instance, our implications on the cross-sectional differences between index and non-index stocks, the flow-performance relation for index funds, and the welfare costs of index investing cannot be obtained in these works.

Our paper also directly contributes to the growing theoretical literature on index investing. In this literature, several theories examine the asset pricing implications of index investing within a dynamic framework like ours (Grégoire (2020), Chabakauri and Rytchkov (2021), Jiang, Vayanos, and Zheng (2022)). Grégoire (2020) shows that index investing leads to a greater comovement among index stocks. Chabakauri and Rytchkov (2021) find that lockstep trading due to indexing increases market volatility and stock return comovements, whereas reduction in risk sharing diminishes these effects, leading to an overall decrease in volatility and an ambiguous effect on comovement. Jiang, Vayanos, and Zheng (2022) demonstrate that index investing disproportionately reduces the financing costs of the largest firms and leads to increased industry concentrations. Several others study index investing within a static asymmetric information setting (Liu and Wang (2023), Baruch and Zhang (2022), Bond and Garcia

(2022), Gârleanu and Pedersen (2022), Buss and Sundaresan (2023)). Even though the main focus of these studies is how index investing affects information production, price informativeness, and market efficiency, some of them have asset pricing implications like us. For example, Baruch and Zhang (2022) show that increased index investing results in higher stock comovement, Buss and Sundaresan (2023) demonstrate that it leads to higher and more volatile asset prices. Bond and Garcia (2022) show that more index investing leads to more pronounced return reversals but lower individual stock trading.

Our paper differs from all the above works on index investing regarding numerous aspects related to methodology, mechanisms, and predictions. In particular, none of them has our sentiment spillover mechanism nor simultaneously explains the observed cross-sectional differences between the index and non-index stocks as well as the positive flow-performance relation for index funds as we do. Furthermore, to the best of our knowledge, our analysis is also the first one to reconcile the empirical fact that index investing leads to higher trading volume for index stocks. In fact, the only other study we are aware of that examines this quantity is Bond and Garcia (2022), which obtains the opposite prediction that more index investing leads to less stock trading activity.

Finally, our paper is also related to the literature on benchmarking concerns since several theories show that active fund managers tilting their portfolios towards stocks that compose their benchmark index can generate different implications for index and non-index stocks. For instance, Basak and Pavlova (2013) find that benchmarking concerns can generate index effect, higher volatility, and comovement for stocks in the benchmark index. Buffa and Hodor (2023) show that heterogeneity in benchmark incentives can create spillovers leading to a negative return comovement among stocks belonging to the same benchmark. Pavlova and Sikorskaya (2022) show that if a stock's benchmarking intensity increases, its price increases. Our mechanism differs from these works significantly as it is based on the presence of extrapolative investors and an index fund, thereby leading to several other novel implications. Determining whether the index investing or the alternative mechanisms based on active fund managers' benchmarking concerns drive the documented regularities ultimately requires a careful empirical analysis, a task that is beyond the scope of this paper. In reality, both considerations are likely to play significant roles. That said, due to the recent trend from active to passive investing, the explanatory power of the index investing channel is likely to grow in the future.

The remainder of the paper is organized as follows. Section 2 introduces our model and Section 3 determines the equilibrium. Section 4 studies cross-sectional implications of our model and Section 5 explores the flow-performance relation, portfolio response to beliefs, and welfare effects. Section 6 concludes. Appendix A contains all the proofs, Appendix B discusses the parameter values.

2 Model

2.1 Securities Market

We consider an economy with an infinite horizon evolving in continuous time. The uncertainty in the economy is generated by an N-dimensional Brownian motion $\boldsymbol{\omega}_t = [\omega_{1t}, \omega_{2t}, \ldots, \omega_{Nt}]^\intercal$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with its associated filtration denoted by $\{\mathcal{F}_t\}$. The securities available for trading are a riskless asset, N risky stocks, and an index fund. The riskless asset is in perfectly elastic supply and pays a constant interest rate r.

Each stock n, $n = 1, \ldots, N$, is in fixed positive supply of Q_n units. We denote the $N \times 1$ stock supply vector by $\boldsymbol{Q} \equiv [Q_1, Q_2, \ldots, Q_N]^\intercal$. Each stock n is a claim to the dividend (cash flow) D_{nt}, with the dividend vector $\boldsymbol{D}_t \equiv [D_{1t}, D_{2t}, \ldots, D_{Nt}]^\intercal$ following

$$d\boldsymbol{D}_t = \boldsymbol{\mu}_D dt + \boldsymbol{\sigma}_D d\boldsymbol{\omega}_t, \tag{1}$$

where the $N \times 1$ vector of constants $\boldsymbol{\mu}_D$ and $N \times N$ matrix of constants $\boldsymbol{\sigma}_D$ capturing the (conditional) mean and volatility of the dividend changes, respectively. To illustrate the effects of index investing clearly, we assume uncorrelated dividends, i.e., $\boldsymbol{\sigma}_D$ is a diagonal matrix. We denote the variance of dividend changes by $\boldsymbol{\Sigma}_D \equiv \mathrm{Var}_t [d\boldsymbol{D}_t] / dt = \boldsymbol{\sigma}_D \boldsymbol{\sigma}_D^\intercal$. The price of each stock n, S_{nt}, is determined endogenously in equilibrium with the stock price vector $\boldsymbol{S}_t \equiv [S_{1t}, S_{2t}, \ldots, S_{Nt}]^\intercal$ is posited to follow

$$d\boldsymbol{S}_t = \boldsymbol{\mu}_{St} dt + \boldsymbol{\sigma}_{St} d\boldsymbol{\omega}_t, \tag{2}$$

where the (possibly stochastic) $N \times 1$ vector $\boldsymbol{\mu}_{St}$ and $N \times N$ matrix $\boldsymbol{\sigma}_{St}$ are the (conditional) mean and volatility of the stock price changes, respectively. We also denote the variance of stock price changes by $\boldsymbol{\Sigma}_{St} \equiv \mathrm{Var}_t [d\boldsymbol{S}_t] / dt = \boldsymbol{\sigma}_{St} \boldsymbol{\sigma}_{St}^\intercal$.

In this economy, there is a capitalization-weighted index that consists of the first M stocks, $1 \leq M \leq N$. The index level at time $t \geq 0$ is given by

$$I_t = \boldsymbol{q}^\intercal \boldsymbol{S}_t, \tag{3}$$

where the $N \times 1$ vector of constants $\boldsymbol{q}$ is proportional to the index stocks' supply and is given by

$$\boldsymbol{q} = \frac{1}{\sum_{m=1}^M Q_m} [Q_1, Q_2, \ldots, Q_M, 0 \ldots, 0]^\intercal. \tag{4}$$

Under this specification, the index is equivalent to holding $q_j = Q_j / \sum_{m=1}^M Q_m$ shares in an index stock j, $j = 1, \ldots, M$, and no shares in a non-index stock n, $n = M+1, \ldots, N$.[2] Investors can trade the

[2]The normalization by $\sum_{m=1}^M Q_m$ in (4) does not play an economic role in our results. We consider this scaling to have

index through a passive index fund, e.g., an exchange-traded fund (ETF), whose each share replicates the index without any tracking error, and thus, is a claim to q_j shares in each index stock j. Hence, at time t, each fund share costs I_t and yields a dollar return of $dI_t + D_{It}dt$ over the next instant dt, where $dI_t = \boldsymbol{q}^\intercal d\boldsymbol{S}_t$ is the change in the index level driven by the capital gains/losses in index stocks and $D_{It} = \boldsymbol{q}^\intercal \boldsymbol{D}_t$ is the total dividend paid out by the index fund. Accordingly, the index dividend and its level follow

$$dD_{It} = \mu_{DI}dt + \sigma_{DI}d\omega_{It}, \tag{5}$$

$$dI_t = \mu_{It}dt + \sigma_{It}d\omega_{It}, \tag{6}$$

where the constants $\mu_{DI} \equiv \boldsymbol{q}^\intercal \boldsymbol{\mu}_D$ and $\sigma_{DI} \equiv \sqrt{\boldsymbol{q}^\intercal \boldsymbol{\Sigma}_D \boldsymbol{q}}$ capture the mean and the volatility of the index dividend changes, and the scalars $\mu_{It} \equiv \boldsymbol{q}^\intercal \boldsymbol{\mu}_{St}$ and $\sigma_{It} = \sqrt{\boldsymbol{q}^\intercal \boldsymbol{\Sigma}_{St} \boldsymbol{q}}$ represent the mean and the volatility of the index changes, respectively, and $\omega_{It} \equiv (1/\sigma_{DI})\boldsymbol{q}^\intercal \boldsymbol{\sigma}_D \boldsymbol{\omega}_t$ is a standard one-dimensional Brownian motion with its associated filtration denoted by $\left\{\mathcal{F}_t^I\right\}$.[3] We note that ω_{It} contains less information than $\boldsymbol{\omega}_t$, i.e., $\mathcal{F}_t^I \subseteq \mathcal{F}_t$, which is intuitive since observing only the index yields less information than observing each stock price individually.

2.2 Investors' Beliefs and Investment Profiles

The economy is populated by a continuum of investors who differ in their beliefs and investment profiles. In terms of beliefs, an investor can either be *rational* or *extrapolative.* Regarding her investment profile, an investor can either be a *stock* or *index* investor.

Rational stock investors, denoted by r, have a population mass of π_r and can invest in the riskless asset and N individual stocks. They do not invest in the index fund since it would be redundant given their stock investments. These investors observe individual stock prices $\boldsymbol{S}_t$ (and dividends $\boldsymbol{D}_t$) and have correct expectations about their means and volatilities. Hence, from their point of view, stock dividends and prices follow (1) and (2), respectively. Similarly, *rational index investors,* denoted by $\mathcal{R}$, have a population mass of $\pi_{\mathcal{R}}$ and can invest in the riskless asset and the index fund, but not in individual stocks. Accordingly, we assume that these investors observe the index level I_t (and its dividend D_{It}) and have correct expectations about the mean and volatility of index level changes. Thus, from their point of view, index dividend and level follow (5) and (6), respectively.

Stock extrapolators, denoted by e, have a population mass of π_e, and akin to r-type investors, can invest in the riskless asset and N individual stocks but not the index fund. These investors observe individual stock prices $\boldsymbol{S}_t$ (and dividends $\boldsymbol{D}_t$) and agree on their volatilities but misperceive their means.

a weighted-average construction for $\boldsymbol{q}$ so that the choice of number of stocks M in the index does not affect the magnitude of index returns.

[3] As we demonstrate in the Proof of Proposition 1 in Appendix A, due to consistency, we have $d\omega_{It} = (1/\sigma_{DI})\boldsymbol{q}^\intercal \boldsymbol{\sigma}_D d\boldsymbol{\omega}_t = (1/\sigma_{It})\boldsymbol{q}^\intercal \boldsymbol{\sigma}_{St} d\boldsymbol{\omega}_t$ in equilibrium.

We follow the tractable formulation in Barberis et al. (2015) and model extrapolators' expectation of stock price changes as an exponentially decaying weighted average of past stock price changes:

$$\mathrm{E}_t^e\left[d\boldsymbol{S}_t\right]/dt = \boldsymbol{X}_t \qquad \text{where} \qquad \boldsymbol{X}_t = \int_{-\infty}^{t} \kappa e^{-\kappa(t-s)} d\boldsymbol{S}_{s-dt}. \tag{7}$$

The n^{th} entry of the $N \times 1$ vector $\boldsymbol{X}_t$ gives the stock extrapolators' conditional expectation of the n^{th} stock, X_{nt}. We follow the literature and refer to the process $\boldsymbol{X}_t$ as *"stock sentiment"* and the parameter κ as *"degree of extrapolation."*[4] A higher degree of extrapolation κ implies that stock extrapolators assign more weights to the most recent stock performance relative to distant ones while forming their expectations.[5] Thus, from stock extrapolators' point of view, stock prices follow

$$d\boldsymbol{S}_t = \boldsymbol{X}_t dt + \boldsymbol{\sigma}_{St} d\boldsymbol{\omega}_t^e, \tag{8}$$

where $\boldsymbol{\omega}_t^e$ is an N-dimensional Brownian motion under their subjective probability measure $\mathbb{P}^e$. Accordingly, they also view the dividend dynamics

$$d\boldsymbol{D}_t = \boldsymbol{\mu}_{Dt}^e dt + \boldsymbol{\sigma}_D d\boldsymbol{\omega}_t^e, \tag{9}$$

where $\boldsymbol{\mu}_{Dt}^e = \boldsymbol{\mu}_D + \boldsymbol{\sigma}_D \boldsymbol{\sigma}_{St}^{-1}\left(\boldsymbol{X}_t - \boldsymbol{\mu}_{St}\right)$ is their subjective mean of the dividends changes.

Index extrapolators, denoted by $\mathcal{E}$, have a population mass of $\pi_{\mathcal{E}}$, and akin to rational indexers, can invest in the riskless asset and the index fund, but not in individual stocks. These investors observe only the index level I_t but not the prices of individual stocks. Differently from rational indexers, they agree on the index volatility σ_{It} but misperceive its mean in a way that their expectation of index changes is an exponentially decaying weighted average of past index changes

$$\mathrm{E}_t^{\mathcal{E}}\left[dI_t\right]/dt = X_{It} \qquad \text{where} \qquad X_{It} = \int_{-\infty}^{t} \kappa e^{-\kappa(t-s)} dI_{s-dt}. \tag{10}$$

Therefore, index extrapolators expect higher (lower) index fund returns following a good (bad) index performance. Again, a higher value of κ implies that these extrapolators assign more weights to the most recent index performance relative to distant ones while forming their expectations. We note that the *"index sentiment"* X_{It} satisfies $X_{It} = \boldsymbol{q}^\intercal \boldsymbol{X}_t$. Therefore, even without index extrapolators, $\pi_{\mathcal{E}} = 0$,

[4] We note that referring to the process $\boldsymbol{X}_t$ as the sentiment is also consistent with the widespread usage of the term in the literature. For instance, Baker and Wurgler (2007) broadly define the sentiment as the belief about future cash flows and investment risks that are not justified by the facts at hand, and Brown and Cliff (2004) show that the past stock returns are important determinants of commonly employed sentiment measures in empirical studies.

[5] We view the extrapolative belief formation as an investor-specific behavior rather than stock-specific. Therefore, the parameter κ is common to all stocks. That said, our model remains tractable under the alternative formulation of stock-specific degree of extrapolation κ_n, $n = 1, \ldots, N$. Our model can also accommodate a more general affine functional form for extrapolators' expectation: $\mathrm{E}_t^e\left[d\boldsymbol{S}_t\right]/dt = \overline{\boldsymbol{\lambda}} + \mathrm{diag}\left(\boldsymbol{\lambda}\right)\boldsymbol{X}_t$ for constant vectors $\overline{\boldsymbol{\lambda}}$ and $\boldsymbol{\lambda}$. Given the evidence in Barberis et al. (2015), we focus on the case $\overline{\boldsymbol{\lambda}} = \boldsymbol{0}$ and $\boldsymbol{\lambda} = \boldsymbol{1}$.

there is a non-trivial sentiment on the index as long as some stock extrapolators exist in the economy, $\pi_e > 0$. From index extrapolators' point of view, the index evolves according to

$$dI_t = X_{It}dt + \sigma_{It}d\omega_{It}^{\mathcal{E}}, \tag{11}$$

where $\omega_{It}^{\mathcal{E}}$ is a one-dimensional Brownian motion under their subjective probability measure $\mathbb{P}^{\mathcal{E}}$. They also view the index dividend dynamics as

$$dD_{It} = \mu_{DI}^{\mathcal{E}}dt + \sigma_{DI}d\omega_{It}^{\mathcal{E}}, \tag{12}$$

where $\mu_{DI}^{\mathcal{E}} = \mu_{DI} + \sigma_{DI}\sigma_{It}^{-1}\left(X_{It} - \mu_{It}\right)$ is their subjective mean of the index dividend changes.

Extrapolators' beliefs in our model are motivated by the survey evidence, which shows that individual investors' stock return expectations are extrapolative, i.e., they expect higher (lower) future stock returns following a series of high (low) returns. The survey evidence for extrapolative expectations is present both at the aggregate level (e.g., Vissing-Jorgensen (2003), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Cassella and Gulen (2018)) and at the individual stock level (Da, Huang, and Jin (2021)).[6] In contrast, the evidence for extrapolative expectations of institutional investors is weak or mixed (e.g., Da, Huang, and Jin (2021), Dahlquist and Ibert (2024), Nagel and Xu (2023)). Therefore, we can interpret extrapolators in our model as a group of unsophisticated retail investors who rely on simple extrapolative expectation formation to estimate future returns of assets they trade, consistent with the survey evidence. Rational investors can be thought of as a group of sophisticated institutional investors and professional traders who can estimate the stock and index return dynamics sufficiently accurately due to their technological advantages and expertise.

Remark 1 (Further discussion on index investors). In our analysis, we do not specify why some investors trade the index fund over individual stocks. The literature offers various possible economic reasons for index investing, which include trading costs, information costs, cognitive and attention costs, and management fees. In our analysis, we abstract from such costs to focus on the equilibrium implications of extrapolative index investors in a clear setting without committing to one particular cost over others. That said, a simple way to incorporate costs in our framework would be to introduce a holding cost of ϵdt over the next instant dt for each risky asset investors trade. The presence of such costs would provide an incentive for investors to trade the index fund with a cost ϵdt over trading all individual stocks with a total cost of $N\epsilon dt$.[7]

[6]See also Egan, MacKay, and Yang (2022) and Cassella et al. (2024) for non-survey based evidence and Afrouzi et al. (2023) for experimental evidence for extrapolative expectations. Moreover, in our specification, all investors' unconditional expectations are the same and equal to the true one, consistent with Adam, Matveev, and Nagel (2021) who show that survey expectations of stock returns are unconditionally approximately unbiased.

[7]Other works on index investing considering a frictionless economy like ours include Chabakauri and Rytchkov (2021) and Jiang, Vayanos, and Zheng (2022). For an equilibrium in the presence of index participation costs, see Bond and Garcia (2022).

We also note that index investors do not trade individual stocks in our model, so there is no reason for them to pay attention and observe the individual stock prices. Towards that, we ensure that index investors' consumption and portfolio strategies are adapted to the filtration $\left\{\mathcal{F}_t^I\right\}$ generated by ω_{It}. Alternatively, one could assume that indexers have full attention, observe all individual stock prices, and work with the filtration $\{\mathcal{F}_t\}$ generated by $\boldsymbol{\omega}_t$ but are restricted to invest only in the index fund. We find that both approaches yield similar results in our frictionless framework.

2.3 Investors' Preferences and Optimization

Each i-type investor, $i = r, e, \mathcal{R}, \mathcal{E}$, is endowed with identical initial wealth W_0 and a constant absolute risk aversion (CARA) preferences with identical absolute risk aversion coefficient $\gamma > 0$ and time discount rate $\rho > 0$.[8] Each investor optimally chooses her intertemporal consumption c_{it} and an admissible portfolio strategy (adapted to the respective filtration) to maximize her subjective expected utility from a life-time consumption

$$\mathrm{E}^i\left[\int_0^\infty e^{-\rho t}\frac{e^{-\gamma c_{it}}}{-\gamma}dt\right],$$

subject to her dynamic budget constraint

$$dW_{it} = \begin{cases} rW_{it}dt + \boldsymbol{\psi}_{it}^{\intercal}\left(d\boldsymbol{S}_t + \boldsymbol{D}_t dt - r\boldsymbol{S}_t dt\right) - c_{it}dt & \text{for } i = r, e, \\ rW_{it}dt + \psi_{it}\left(dI_t + D_{It}dt - rI_t dt\right) - c_{it}dt & \text{for } i = \mathcal{R}, \mathcal{E}, \end{cases} \tag{13}$$

where E^i denotes the unconditional expectation under i-type investors' subjective beliefs $\mathbb{P}^i$, the $N \times 1$ vector $\boldsymbol{\psi}_{it}$ denotes the portfolio of the i-type stock investors, $i = r, e$, as the number of shares in individual stocks, and the scalar ψ_{it} denotes the portfolio of the i-type index investors, $i = \mathcal{R}, \mathcal{E}$, as the number of shares in the index fund.

3 Equilibrium

In this section, we determine the equilibrium in our index investing economy with extrapolative expectations. We find that index investing generates a novel sentiment spillover effect such that the sentiment of an index stock affects not only the price of that stock but also the prices of all other index stocks.

The equilibrium in our economy is defined in a standard way. The economy is said to be in equilibrium if stock prices $\boldsymbol{S}_t$, the index level I_t, and consumption-portfolio strategies of stock investors

[8] Our model could be extended to incorporate heterogeneous risk aversion among investor types, such as index investors to be more risk-averse than stock investors, as in Chabakauri and Rytchkov (2021).

$(c_{it}, \boldsymbol{\psi}_{it})_{i=r,e}$ and index investors $(c_{it}, \psi_{it})_{i=\mathcal{R},\mathcal{E}}$ are such that all investors optimally choose their strategies given prices and beliefs, and the stock market clears for all t,

$$\pi_r \boldsymbol{\psi}_{rt} + \pi_e \boldsymbol{\psi}_{et} + \left(\pi_{\mathcal{R}} \psi_{\mathcal{R}t} + \pi_{\mathcal{E}} \psi_{\mathcal{E}t}\right) \boldsymbol{q} = \boldsymbol{Q}. \tag{14}$$

We employ the standard stochastic dynamic programming method (e.g., Merton (1971)) to solve for each investor's optimal consumption and portfolio strategies and apply the stock market clearing condition (14) to obtain the equilibrium.[9] Proposition 1 characterizes the equilibrium in our index investing economy with extrapolators by presenting the prices of individual stocks, the index level, and the investors' consumption and portfolio strategies in equilibrium.

Proposition 1 (**Equilibrium**). *In the index investing economy with extrapolators, the equilibrium prices of individual stocks are given by*

$$\boldsymbol{S}_t = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{X}_t + \frac{1}{r}\boldsymbol{D}_t, \tag{15}$$

where the $N \times 1$ vector of constants $\boldsymbol{A}$ and the $N \times N$ matrix of constants $\boldsymbol{B}$ solve systems of non-linear equations provided in Appendix A, and the equilibrium stock sentiment follows

$$d\boldsymbol{X}_t = \kappa \boldsymbol{\Lambda} \left(\bar{\boldsymbol{X}} - \boldsymbol{X}_t\right) dt + \frac{\kappa}{r} \boldsymbol{\Lambda} \boldsymbol{\sigma}_D d\boldsymbol{\omega}_t, \tag{16}$$

where $\boldsymbol{\Lambda} = \left(\boldsymbol{I}_N - \kappa \boldsymbol{B}\right)^{-1}$ and $\bar{\boldsymbol{X}} = \boldsymbol{\mu}_D / r$, with $\boldsymbol{I}_N$ denoting the $N \times N$ identity matrix.

The equilibrium index level is given by

$$I_t = A_I + B_I X_{It} + \frac{1}{r} D_{It}, \tag{17}$$

where $A_I = \boldsymbol{q}^\intercal \boldsymbol{A}$ and B_I satisfies $\boldsymbol{B}^\intercal \boldsymbol{q} = B_I \boldsymbol{q}$, and the equilibrium index sentiment follows

$$dX_{It} = \kappa \Lambda_I \left(\bar{X}_I - X_{It}\right) dt + \frac{\kappa}{r} \Lambda_I \sigma_{DI} d\omega_{It}, \tag{18}$$

where $\Lambda_I = \left(1 - \kappa B_I\right)^{-1}$ and $\bar{X}_I = \mu_{DI} / r$.

The equilibrium consumption and portfolio strategies of i-type stock investor, $i = r, e$, are given by

$$c_{it} = rW_{it} - \frac{1}{\gamma} \ln(\gamma r) - \frac{1}{\gamma}\left(F_i + \boldsymbol{G}_i^\intercal \boldsymbol{X}_t - \tfrac{1}{2} \boldsymbol{X}_t^\intercal \boldsymbol{H}_i \boldsymbol{X}_t\right) \qquad \text{and} \qquad \boldsymbol{\psi}_{it} = \boldsymbol{K}_i + \boldsymbol{L}_i \boldsymbol{X}_t, \tag{19}$$

[9] We note that the bracket term in (14) is the total number of index fund shares held by index investors at time t. Since each index fund share is a claim to $\boldsymbol{q}$ shares of individual stocks, the last term $\left(\pi_{\mathcal{R}} \psi_{\mathcal{R}t} + \pi_{\mathcal{E}} \psi_{\mathcal{E}t}\right) \boldsymbol{q}$ captures the total individual stock demand coming from the index fund.

and those of i-type index investor, $i = \mathcal{R}, \mathcal{E}$, are given by

$$c_{it} = rW_{it} - \frac{1}{\gamma}\ln(\gamma r) - \frac{1}{\gamma}\left(F_i + G_i X_{It} - \tfrac{1}{2}H_i X_{It}^2\right) \qquad and \qquad \psi_{it} = K_i + L_i X_{It}, \tag{20}$$

where the scalars F_i, G_i, H_i, the $N \times 1$ vector of constants $\boldsymbol{G}_i$, and the $N \times N$ symmetric matrix of constants $\boldsymbol{H}_i$ solve systems of non-linear equations provided in Appendix A, and the scalars K_i and L_i are given by (A.27) *and* (A.28)*, the $N \times 1$ vector of constant $\boldsymbol{K}_i$ and the $N \times N$ matrix of constants $\boldsymbol{L}_i$ are given by* (A.10) *and* (A.11).

Proposition 1 shows that, in the presence of extrapolators, individual stock prices take simple linear forms and are driven not only by their cash flows (dividends) $\boldsymbol{D}_t$ but also by the sentiment $\boldsymbol{X}_t$. In the absence of index investors, the coefficient $\boldsymbol{B}$, which captures the sensitivity of prices to sentiment, becomes a diagonal matrix. Consequently, each stock's sentiment affects only its own price without affecting the prices of other stocks. For instance, in an economy with five stocks, $N = 5$, if there are no index investors, the coefficient matrix $\boldsymbol{B}$ takes the form

$$\boldsymbol{B} = \begin{bmatrix} + & 0 & 0 & 0 & 0 \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & + & 0 \\ 0 & 0 & 0 & 0 & + \end{bmatrix}. \tag{21}$$

In this case, each stock price is positively associated with its sentiment (past performance). This finding is well-established in the extrapolative expectations literature (e.g., Barberis, Greenwood, Jin, and Shleifer (2015)) and arises because, following a series of positive stock returns, stock extrapolators expect the stock prices to increase further in the future and increase their stock demand, leading to a positive relation between the past performance of a stock and its price.

With index investors present, stock prices become more involved due to the *sentiment spillover*: each index stock's sentiment affects not only its own stock price but also the prices of all other index stocks. In other words, off-diagonal entries of the coefficient matrix $\boldsymbol{B}$ corresponding to index stocks become non-zero in equilibrium. For instance, when the index consists of the first three stocks, $M = 3$, if the index investors are mostly extrapolators, the coefficient matrix in equilibrium becomes

$$\boldsymbol{B} = \begin{bmatrix} + & + & + & 0 & 0 \\ + & + & + & 0 & 0 \\ + & + & + & 0 & 0 \\ 0 & 0 & 0 & + & 0 \\ 0 & 0 & 0 & 0 & + \end{bmatrix}. \tag{22}$$

In this case, the price of an index stock is also positively related to the sentiment of all other index stocks. This positive spillover effect arises because following a positive cash flow shock to an index stock increases the sentiment and the price of that stock, thus, the index level. Consequently, index extrapolators expect the index to rise further and demand more index fund shares. Since the fund allocates investors' demand across index stocks in a fixed proportion to replicate the index for each share it issues, all index stocks experience a higher demand, generating a positive relation between the price of an index stock and the sentiment of all other index stocks.[10] As we demonstrate in Section 4, this spillover effect plays a crucial role in explaining the strong comovement among index stocks, and other empirically documented cross-sectional differences between index and non-index stocks.

Turning to the stock sentiment process (16), we see that it follows an N-dimensional mean-reverting Ornstein-Uhlenbeck process under the objective measure. Thus, it generates predictable variations in individual stock prices for rational investors such that a high sentiment $\boldsymbol{X}_t$ signals inflated current prices. We refer to the key quantity $\boldsymbol{\Lambda}$ in (16) as the "*stock amplification term*" since it captures the extent to which the changes in the stock sentiment, and thus the stock prices, are amplified following cash flow shocks. We see that when the coefficient matrix $\boldsymbol{B}$ has non-zero off-diagonal terms due to sentiment spillover, so does the amplification matrix $\boldsymbol{\Lambda} = (\boldsymbol{I}_N - \kappa \boldsymbol{B})^{-1}$. This implies that, in the presence of index investors, the rational expectation of an index stock's future sentiment depends on the current sentiments of all other index stocks. In particular, when index investors are mostly extrapolators, a higher sentiment in an index stock leads to higher expected sentiment for all other index stocks. We also note that the persistence of the sentiment process is given by $\kappa\boldsymbol{\Lambda}$ and the (conditional) variance of changes in sentiment by

$$\boldsymbol{\Sigma}_X \equiv \mathrm{Var}_t\left[d\boldsymbol{X}_t\right]/dt = \frac{\kappa^2}{r^2}\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D\boldsymbol{\Lambda}^\intercal. \tag{23}$$

Using the well-known properties of multi-dimensional Ornstein-Uhlenbeck processes, we know that in the long run, the process $\boldsymbol{X}_t$ has a stationary Gaussian distribution when all the eigenvalues of $\kappa\boldsymbol{\Lambda}$ have positive real parts. In this case, the ergodic distribution of $\boldsymbol{X}_t$ is characterized by its long-run mean $\bar{\boldsymbol{X}} = \boldsymbol{\mu}_D/r$ and variance

$$\mathrm{Var}\left[\boldsymbol{X}_\infty\right] \equiv \lim_{t\to\infty}\mathrm{Var}\left[\boldsymbol{X}_t\right] = \mathrm{vec}^{-1}\left[\left(\kappa\boldsymbol{\Lambda}\oplus\kappa\boldsymbol{\Lambda}\right)^{-1}\mathrm{vec}\left[\boldsymbol{\Sigma}_X\right]\right], \tag{24}$$

where $\oplus$ is the Kronecker sum and vec is the stacking operator that stacks the columns of a matrix into a vector and vec^{-1} is the operator that reshapes a vector back into a matrix.[11]

[10] In contrast, in the presence of extrapolators, if index investors are predominantly rational, the prices of index stocks positively relate to their own sentiment but they negatively relate to the sentiment of other index stocks. The latter negative relation arises because following a high sentiment in one of the index constituents, index level increases and rational index investors correctly expect the index to fall in the future from its currently inflated level. Thus, they reduce their index fund demand, leading all other index stock prices to decrease, resulting in a negative relation between the price of an index stock and the sentiment of all other index stocks.

[11] Our comprehensive numerical analysis demonstrates that for a wide range of reasonable parameter values, including our baseline calibration, stationary equilibria in which all the eigenvalues of $\kappa\boldsymbol{\Lambda}$ have positive real parts exist in our model.

Proposition 1 also reveals that the equilibrium index level (17) takes a similar linear form and is driven by its cash flow D_{It} and its sentiment X_{It}. As a key feature, we find that the index level increases in its sentiment (past performance), i.e., $B_I > 0$.[12] This positive relation arises as long as some extrapolators, be it stock or index extrapolators, are present in the economy. For instance, with only stock extrapolators present, positive cash flow shocks to some index stocks increase the index level and the sentiment on those stocks. Extrapolative stock investors would push those index stock prices further up and thus the index level, generating a positive coefficient B_I. With only index extrapolators present, a rise in the index level also raises the sentiment of index investors, who would demand more index fund shares, pushing the index further up, again generating a positive coefficient B_I. We also see that the index sentiment process (18) follows a one-dimensional Ornstein-Uhlenbeck process driven by a single Brownian motion ω_{It} generating the filtration $\left\{\mathcal{F}_t^I\right\}$. We refer to the term $\Lambda_I = (1 - \kappa B_I)^{-1}$ as the *index amplification term* since it plays a similar role to that of stocks and determines the extent to which index sentiment, and thus the index level, is amplified following cash flow shocks.

Looking at equilibrium consumption and portfolio strategies in (19)–(20), we see that stock investors' strategies are driven by $\boldsymbol{X}_t$, and thus are adapted to the filtration $\{\mathcal{F}_t\}$. Whereas, strategies of index investors are driven by X_{It} and are adapted to the filtration $\left\{\mathcal{F}_t^I\right\}$, with the relation between two filtrations satisfying $\mathcal{F}_t^I \subseteq \mathcal{F}_t$ for all t. Since stock investors observe the prices (and dividends) of each individual stock, their consumption and portfolio decisions are based on a finer information set than those of index investors, who observe only the index level (and index dividend). We also see a convex relation between investors' consumption and sentiments. Typically, the matrix $\boldsymbol{H}_i$ is positive definite for stock investors, and the scalar H_i is positive for index investors, implying that investors consume more in extreme (high or low) sentiment states. In these extreme states, all types of investors expect to make large gains from their respective portfolios and thus increase their consumption due to the income effect. We further show that investors' equilibrium portfolios are linearly related to sentiment. In particular, in equilibrium, we obtain index extrapolators' fund demand to be positively related to index sentiment, i.e., $L_{\mathcal{E}} > 0$, while that of rational index investors' to be negatively related, i.e., $L_{\mathcal{R}} < 0$. A rising index level makes index extrapolators more bullish about future index level; thus, they demand more of the index fund. Conversely, rational index investors correctly anticipate that the currently inflated index level will revert to a lower level in the future and thus demand less of the index fund.

4 Cross-Sectional Effects of Index Investing

Having determined the equilibrium, in this section, we examine the implications of our model for cross-sectional differences between index and non-index stocks. We show that when index investors are

[12] Since the constant B_I satisfies a representation $\boldsymbol{B}^\intercal \boldsymbol{q} = B_I \boldsymbol{q}$, one could think of B_I as the eigenvalue corresponding to the eigenvector $\boldsymbol{q}$ of the matrix $\boldsymbol{B}^\intercal$. However, we refrain from this interpretation as there is no guarantee that the exogenous vector $\boldsymbol{q}$ arises as an eigenvector of the endogenous matrix $\boldsymbol{B}^\intercal$.

predominantly extrapolators, all consistent with empirical evidence, index stocks have higher and more volatile prices, comove more with other index stocks, exhibit stronger negative price autocorrelations, and have higher trading volume than otherwise identical non-index stocks. We further show that the index effect is much smaller when index investors are predominantly extrapolators. We also show that these empirically consistent patterns are robust and emerge not only when index investors are new to the market but also when existing investors switch from trading individual stocks to index.

Before we get to our results, we briefly discuss how we present them in our tables.[13] Since our model has a rich investor space, the effects of index investors can differ depending on whether some stock investors are extrapolative or not and which type of index investor is more dominant in the economy. Therefore, we find it necessary to consider several economies with different investor compositions to understand the effects of index investing. Towards that, in columns I, II, and III of our tables, we consider economies without any stock extrapolators, $\pi_e = 0$. In particular, column I provides the basic benchmark economy in which all individual stock investors are rational without any index investors, $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}}) = (1, 0, 0, 0)$. In columns II and III, respectively, we demonstrate the marginal effects of rational and extrapolative index investors by introducing them to this basic benchmark economy. Paralleling column I, column IV provides a benchmark economy without index investors but with stock extrapolators present, $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}}) = (0.5, 0.5, 0, 0)$, which can be thought of as the multi-stock generalization of Barberis et al. (2015). Again, we illustrate the effects of rational and extrapolative index investors by introducing them to this benchmark economy in columns V and VI, respectively. Given their relevance, we will pay particular attention to columns III and VI, which capture the effects when the marginal index investor is extrapolative.

4.1 Stock Price and Index Effect

Starting with Harris and Gurel (1986) and Shleifer (1986), numerous empirical works document that stocks experience higher (lower) prices following inclusion into (removal from) the S&P 500 and other major indices. This index effect was particularly strong in the 1980s and 1990s but has diminished significantly recently. For instance, Greenwood and Sammon (2024) show that the abnormal price increase associated with stock added to the S&P 500 was 7.4% in the 1990s but less than 1% in the past decade. We here argue that the recent rise in the index trading by retail investors, who tend to have extrapolative expectations, could be partly behind the disappearing index effect. To that end, using the equilibrium prices in Proposition 1, we present the average prices of index stocks and (otherwise identical) non-index stocks in equilibrium under different investor compositions in Table 1.

[13] We discuss the parameter values employed in our numerical analysis in Appendix B. In particular, we set the baseline value of our key parameter, the degree of extrapolation, to $\kappa = 0.5$, consistent with its empirical estimate in Barberis et al. (2015) and Cassella and Gulen (2018). Moreover, to demonstrate the effects of higher and lower degrees of extrapolation, we also consider the values of κ that are one standard deviation higher (0.70) and lower (0.30) than its mean, using the empirical estimate of its standard deviation of 0.2 in Cassella and Gulen (2018).

Table 1. Stock price and index effect. This table reports the average stock price among index stocks (Ind) and non-index stocks (Non) as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ when the stock sentiment is at its long-run average $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$. All other parameter values are as in Table B1.

		No Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
		$(1,0,0,0)$	$(1,0,.5,0)$	$(1,0,0,.5)$	$(.5,.5,0,0)$	$(.5,.5,.5,0)$	$(.5,.5,0,.5)$
κ	Stock	I	II	III	IV	V	VI
	Ind	430.00	446.67	430.74	382.37	430.74	383.44
0.3	Non	430.00	430.00	430.00	382.37	382.37	382.37
	Diff	**0**	**16.67**	**0.74**	**0**	**48.37**	**1.07**
	Ind	430.00	446.67	430.45	381.44	430.45	382.08
0.5	Non	430.00	430.00	430.00	381.44	381.44	381.44
	Diff	**0**	**16.67**	**0.45**	**0**	**49.01**	**0.64**
	Ind	430.00	446.67	430.32	381.03	430.32	381.49
0.7	Non	430.00	430.00	430.00	381.03	381.03	381.03
	Diff	**0**	**16.67**	**0.32**	**0**	**49.29**	**0.46**

Table 1, column I, presents the prices of a typical index and a non-index stock under the basic benchmark economy in which all investors are rational and invest in individual stocks. Since there are no index investors, all stocks have identical prices in this benchmark economy. As columns II and III illustrate, introducing index investors into this basic economy generates an index effect by increasing the prices of index stocks while keeping the prices of non-index stocks the same. More notably, we see that the price difference between index and non-index stocks is much smaller when index investors are extrapolators. For instance, under the baseline value of $\kappa = 0.50$, the price difference is 16.67 when index investors are rational (column II) but is only 0.45 when index investors are extrapolators (column III). As columns V and VI show, a similar conclusion also holds under the presence of stock extrapolators.

The index effect arises when index investors are introduced because their demand for the index fund pushes up the prices of index stocks in equilibrium. However, less obviously and more interestingly, the index effect becomes weaker in equilibrium when index investors are predominantly extrapolators. The intuition for this result can easily be understood through the shifts in the supply and demand curves of existing stock investors. Introduction of index investors reduces the residual supply of index stocks available to stock investors, with the magnitude of the shift depending on the indexers' type. When index investors are extrapolators, they make the index more volatile due to the fluctuations in their sentiment (see Section 4.2). More volatile index leads to a smaller demand for the index fund and thus, to a smaller reduction in the supply curve faced by existing stock investors. Moreover, due to more volatile index stocks, the aggregate demand curve of stock investors shifts downward, overall generating small index effects as illustrated in columns III and VI of Table 1.

On the other hand, introduction of rational indexers when there are no stock extrapolators does not affect volatilities of index stocks. This leads to a moderate upward shift in the supply curve faced by existing stock investors without changing their demand curve, generating a relatively large index effect as observed in column II. Column V also shows that with stock extrapolators present, entry of rational indexers leads to an even larger index effect. This is because in addition to the larger upward shift in the supply curve, the aggregate demand curve of existing stock investors also shifts upward due to reduced volatility of index stocks, which makes them attractive for investors.

As discussed earlier, the index effect has diminished in the past decade. Greenwood and Sammon (2024) offer several plausible explanations for this behavior. Our finding here offers yet another possible explanation that has yet to be considered in the literature. Namely, the rise in the trading activity of retail investors, whose expectations are typically extrapolative, could be partly behind this phenomenon. For instance, it is reported that during the first six months of 2020, retail investors accounted for roughly 20% of the shares traded in the U.S. stock market, roughly doubling its level from 2010.[14] The rise of retail trading activity in the past decade is typically attributed to the recent zero-commission trading rules and popularity of simple trading applications such as Robinhood. It is argued that these developments incentivized existing retail investors to trade more and encouraged many new retail investors to enter the market. We would also like to highlight that a weaker index effect also emerges in our analysis if index investors are not new investors but existing investors who switch from trading individual stocks to index investing, as demonstrated in Section 4.6.

Table 1 also shows that when index investors are predominantly extrapolators, a higher degree of extrapolation κ reduces the price gap between the index and non-index stocks. A higher κ means that extrapolators assign more weight to the most recent prices than distant ones while forming their expectations. Therefore, investors' beliefs and sentiments become less persistent and more volatile, leading to more volatile stock prices. In equilibrium, risk-averse investors are willing to hold these more volatile stocks only if their prices are lower. When index investors are predominantly extrapolators, the prices of index stocks are affected more and become lower, leading to reduced index effect. We highlight that this may be another channel contributing to the reduction in the index effect. Cassella and Gulen (2018) document that as more young investors enter the market, the average degree of extrapolation increases. Given that most retail traders who enter the market recently are shown to be young and financially unsophisticated investors, it may be the case that the average degree of extrapolation has also increased in the last decade, diminishing the index effect.[15]

[14] See, `https://www.wsj.com/articles/individual-investor-boom-reshapes-u-s-stock-market-11598866200`.

[15] For instance, a recent survey by Financial Industry Regulatory Authority finds that 66% of the investors who opened a brokerage account for the first time in 2020 were under 45 years old (Lush et al. (2021)). Moreover, the median age of the trading platform Robinhood is reported to be 31, with one million of new accounts that opened in 2020 belonging to younger investors with an average age of 19 (see, `https://www.reuters.com/article/business/factbox-the-us-retail-trading-frenzy-in-numbers-idUSKBN29Y2PW/`). The young and inexperienced investors ex-

4.2 Stock Volatility

Another key empirical regularity in index investing literature is that more index investing makes index stocks more volatile (e.g., Sullivan and Xiong (2012), Ben-David et al. (2018), Coles, Heath, and Ringgenberg (2022)). To address this finding, we present the equilibrium volatility of individual stock price changes in Proposition 2.[16]

Proposition 2 (Equilibrium stock volatility). *In the index investing economy with extrapolators, the equilibrium price change volatility of stock n, $n = 1, \ldots, N$, is given by the square root of the n^{th} row n^{th} column entry of the variance-covariance matrix of stock price changes*

$$\mathbf{\Sigma}_S = \frac{1}{r^2}\mathbf{\Lambda}\mathbf{\Sigma}_D\mathbf{\Lambda}^\intercal. \tag{25}$$

Proposition 2 shows that, in equilibrium, the price change volatility of an individual stock is constant and impacted by the amplification term $\mathbf{\Lambda}$ that arises in the presence of extrapolative investors. Previous research (e.g., Barberis et al. (2015)) has already identified investors' extrapolative expectations amplifying stock prices. Intuitively, positive cash flow shocks lead to higher prices, increasing extrapolators' expectations about future stock prices. Thus, they increase their security demand, which pushes the prices further up. An opposite mechanism amplifies the price decrease following negative cash flow shocks. However, the extent to which index and non-index stocks are affected by this amplification mechanism has not yet been studied. This is not a trivial issue because as discussed in Section 3, the presence of index investors creates a spillover effect: the price of an index stock depends not only on its own sentiment but also on all other index stocks' sentiments. Thus, in the presence of index investors, the volatility of an individual stock depends not only on the extent to which extrapolators amplify that stock but also on the extent to which extrapolators amplify other index stocks. To understand how index investing with extrapolators impacts the volatilities of the index and non-index stocks, we present the average volatility among index and non-index stocks in Table 2.

Table 2 reveals that index stocks have higher volatilities than non-index stocks when the index investors are predominantly extrapolative (columns III and VI). This finding is intuitive given the sentiment spillover and amplification mechanisms discussed in Section 3. Namely, the entrance of extrapolative indexers effectively amplifies the responses of index stock prices to their own cash flow shocks as extrapolative indexers' demand for index fund positively reacts to index stocks' sentiments. In addition, given the spillover effect, one index stock's price also responds to other index stocks' cash flow shocks and thus sentiments, generating an additional volatility. In contrast, when most index investors are rational, index stocks have the same or lower volatility than non-index stocks, as depicted

trapolating past trends is also documented in Vissing-Jorgensen (2003) and Greenwood and Nagel (2009).

[16] As is well known, in a framework with CARA preferences with normally distributed dividends like ours, it is more natural to look at the additive quantities such as price changes rather than actual returns which may not be well-defined for some values given that prices are normally distributed.

Table 2. Stock volatility. This table reports the average price change volatility among index stocks (Ind) and non-index stocks (Non) as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

		No Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
		$(1,0,0,0)$	$(1,0,.5,0)$	$(1,0,0,.5)$	$(.5,.5,0,0)$	$(.5,.5,.5,0)$	$(.5,.5,0,.5)$
κ	Stock	I	II	III	IV	V	VI
	Ind	10.00	10.00	10.77	13.97	13.40	15.06
0.3	Non	10.00	10.00	10.00	13.97	13.97	13.97
	Diff	**0**	**0**	**0.77**	**0**	**-0.57**	**1.09**
	Ind	10.00	10.00	10.78	14.04	13.45	15.14
0.5	Non	10.00	10.00	10.00	14.04	14.04	14.04
	Diff	**0**	**0**	**0.78**	**0**	**-0.59**	**1.10**
	Ind	10.00	10.00	10.79	14.07	13.48	15.18
0.7	Non	10.00	10.00	10.00	14.07	14.07	14.07
	Diff	**0**	**0**	**0.79**	**0**	**-0.59**	**1.11**

in columns II and V, respectively. Particularly, in the economy represented in column II, there are no extrapolators and hence no sentiment. Therefore, the fluctuations in individual stock prices are only due to the fluctuations in their cash flows, which are identically distributed, leading to the same volatility for all stocks. With stock extrapolators present, the sentiment spillover still leads to an additional volatility for index stocks since the price of an index stock still responds (negatively) to cash flow shocks of other index stocks. However, the entrance of rational indexers effectively increases the relative population size of rational investors who can now better absorb extrapolators' sentiment-driven demand, thus dampening price fluctuations and leading to less volatile prices for index stocks (column V).

Table 2 also shows that in the presence of extrapolators, a higher degree of extrapolation κ leads to a higher stock price volatility. As discussed earlier, a higher κ means that extrapolators assign more weight to the most recent prices than distant ones while forming their expectations. Thus, cash flow shocks lead to larger movements in their beliefs and consequently prices. That said, in line with Barberis et al. (2015), we find that this effect's economic magnitude is small due to countering demand from rational investors.

4.3 Stock Comovement

Empirical evidence robustly shows that index stocks have significantly higher correlations than non-index stocks, and the stocks added to an index begin to comove more with other index stocks and less with non-index stocks (e.g., Greenwood and Sosner (2007), Wurgler (2010), Boyer (2011), Coles, Heath, and Ringgenberg (2022)).[17] Due to the key sentiment spillover mechanism discussed in Section 3, our model generates a rich correlation structure among stocks that can help explain these findings. To that end, Proposition 3 presents the equilibrium price change correlation between any two stocks m and n.

Proposition 3 (Equilibrium stock comovement). *In the index investing economy with extrapolators, the equilibrium price change correlation between stocks m and n for $m, n = 1, \ldots, N$ is given by*

$$\rho_{mn} \equiv \mathrm{Corr}_t\left[dS_{mt}, dS_{nt}\right] = \frac{\boldsymbol{e}_m^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_n}{\sqrt{\left(\boldsymbol{e}_m^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_m\right)\left(\boldsymbol{e}_n^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_n\right)}}, \tag{26}$$

where $\boldsymbol{e}_n$ is an $N \times 1$ elementary vector with its n^{th} element being 1 and others being 0.

Proposition 3 shows that the equilibrium price change correlation between any two stocks is constant and depends on the stock variance-covariance matrix $\boldsymbol{\Sigma}_S$, and thus, the amplification term $\boldsymbol{\Lambda}$ in the presence of extrapolators. To understand the behavior of the stock comovement, we use the pairwise correlations in (26) and obtain the average correlation among index stocks and non-index stocks as

$$\rho_{Ind} = \frac{2}{M(M-1)} \sum_{m=1}^{M-1} \sum_{n=m+1}^{M} \rho_{mn}, \qquad \rho_{Non} = \frac{2}{(N-M)(N-M-1)} \sum_{m=M+1}^{N-1} \sum_{n=m+1}^{N} \rho_{mn},$$

respectively, and report them in Table 3.[18]

The key finding of Table 3 is that index stocks have positive and higher pairwise correlations than non-index stocks as in the data when index investors are predominantly extrapolators. In contrast, as columns II and V illustrate, when index investors are predominantly rational, the correlations among index stocks are either zeros or negative. Thus, they are either the same or lower than those of non-index stocks. These results arise because, as discussed in Section 3, in the presence of index investing extrapolators, the price of an index stock depends not only on its sentiment but also on all other index stocks' sentiments. Whether the sentiments of other index stocks affect an individual stock price positively or negatively depends on which type of index investors are relatively more dominant. When index extrapolators are more dominant, all else being equal, a good (bad) past performance of an index stock induces them to expect high (low) index performance in the future, leading them to increase (decrease) their demand for the index fund. Since the index fund allocates investor demand across index stocks in a fixed proportion to replicate the index, other stocks in the index also experience a high (low) demand, generating a positive comovement among index stocks. In contrast, when rational index

[17] Relatedly, Vijh (1994) and Barberis, Shleifer, and Wurgler (2005) show that after addition to the S&P 500 index, a stock's beta with S&P 500 index increases.

[18] In Table 3, for generality, we report the cross-sectional average correlations, but they are also equal to pairwise correlations between any two index and non-index stocks since the cash flows of all stocks are identical and mutually independent under our baseline calibration. Therefore, the correlation between an index and a non-index stock is zero, which is not reported in Table 3 for brevity.

Table 3. Stock comovement. This table reports the average pairwise price change correlation among index stocks (Ind) and non-index stocks (Non) as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

		No Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
		$(1,0,0,0)$	$(1,0,.5,0)$	$(1,0,0,.5)$	$(.5,.5,0,0)$	$(.5,.5,.5,0)$	$(.5,.5,0,.5)$
κ	Stock	I	II	III	IV	V	VI
	Ind	0	0	13.74	0	-8.82	13.88
0.3	Non	0	0	0	0	0	0
	Diff	**0**	**0**	**13.74**	**0**	**-8.82**	**13.88**
	Ind	0	0	13.95	0	-8.93	14.05
0.5	Non	0	0	0	0	0	0
	Diff	**0**	**0**	**13.95**	**0**	**-8.93**	**14.05**
	Ind	0	0	14.05	0	-8.97	14.11
0.7	Non	0	0	0	0	0	0
	Diff	**0**	**0**	**14.05**	**0**	**-8.97**	**14.11**

investors are more dominant, a good (bad) past performance of an index stock induces them to decrease (increase) their demand for the index fund, since they believe the index fund is relatively overvalued (undervalued). Therefore, other index stocks experience low (high) demand following a good (bad) past performance of an index stock, generating a negative comovement among index stocks.

Table 3 also reveals that in the presence of extrapolators, a higher degree of extrapolation κ leads to a stronger comovement among index stocks. This result arises because a higher κ leads to more volatile beliefs for extrapolators. Therefore, their index fund demand changes more following cash flow shocks, leading to a stronger positive comovement among index stocks.

4.4 Stock Autocorrelation

Empirical research also documents that indexing leads to stronger negative autocorrelation in stock prices and index levels (Ben-David et al. (2018), Baltussen, van Bekkum, and Da (2019), Höfler, Schlag, and Schmeling (2023)). In particular, Baltussen, van Bekkum, and Da (2019) show that index return autocorrelation has turned significantly negative across 20 major market indexes recently, coinciding with the growth of index investing. Moreover, Ben-David et al. (2018) and Höfler, Schlag, and Schmeling (2023) find that, in the cross-section, stocks with a high passive ETF ownership have much stronger return reversal (negative autocorrelation) than those with a low passive ETF ownership. To examine whether our model can capture these facts, we present the autocorrelation of stock price changes in equilibrium in Proposition 4.

Proposition 4 (Equilibrium stock autocorrelation). *In the index investing economy with extrapolators, the equilibrium price change autocorrelation of stock n, $n = 1, \ldots, N$, over the periods (t_0, t_1) and (t_2, t_3) for any $t_0 \le t_1 \le t_2 \le t_3$ is given by*

$$\rho_n(t_0, t_1, t_2, t_3) = \mathrm{Corr}\left[S_{nt_1} - S_{nt_0}, S_{nt_3} - S_{nt_2}\right] = \frac{\mathrm{Cov}\left[S_{nt_1} - S_{nt_0}, S_{nt_3} - S_{nt_2}\right]}{\sqrt{\mathrm{Var}\left[S_{nt_1} - S_{nt_0}\right]\mathrm{Var}\left[S_{nt_3} - S_{nt_2}\right]}}, \tag{27}$$

where $\mathrm{Cov}\left[S_{nt_1} - S_{nt_0}, S_{nt_3} - S_{nt_2}\right]$ *is given by the* n^{th}*row* n^{th}*column entry of the covariance matrix*

$$\begin{aligned}\mathrm{Cov}[\boldsymbol{S}_{t_1} - \boldsymbol{S}_{t_0}, \boldsymbol{S}_{t_3} - \boldsymbol{S}_{t_2}] = \boldsymbol{B}\Big[\Big(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\Big) - \Big(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_0)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_0)}\Big)\Big]\mathrm{Var}\left[\boldsymbol{X}_\infty\right]\boldsymbol{B}^\intercal \\ + \frac{1}{r^2}\left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\right)\boldsymbol{\Lambda}^{-1}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}(t_1-t_0)}\right)\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D\boldsymbol{B}^\intercal,\end{aligned} \tag{28}$$

and $\mathrm{Var}\left[S_{nt_{k+1}} - S_{nt_k}\right]$ *is given by the* n^{th}*row* n^{th}*column entry of the variance matrix*

$$\mathrm{Var}\left[\boldsymbol{S}_{t_{k+1}} - \boldsymbol{S}_{t_k}\right] = \frac{1}{r^2}\boldsymbol{\Sigma}_D\tau + 2\boldsymbol{B}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}\tau}\right)\mathrm{Var}\left[\boldsymbol{X}_\infty\right]\boldsymbol{B}^\intercal + \frac{2}{r^2}\boldsymbol{B}\boldsymbol{\Lambda}^{-1}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}\tau}\right)\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D, \tag{29}$$

where $\tau = t_{k+1} - t_k$ *and* $\mathrm{Var}\left[\boldsymbol{X}_\infty\right]$ *is as in* (24)*.*

As Proposition 4 illustrates, the autocorrelations of stock price changes are constant in equilibrium but take a complex form in our model. To better understand how the stock serial dependence behaves in our model, in Table 4, we present the autocorrelation between stock price changes in the previous quarter and the changes in next quarter.[19]

Table 4, columns I and II, reveal that individual stock price changes are serially uncorrelated when there are no extrapolators in the economy. This finding is intuitive since in these economies in which all investors rational stock prices do not depend on sentiment $\boldsymbol{X}_t$, which is the source of the predictability. As column III shows, the presence of extrapolative index investors makes index stocks have price reversals on average, i.e., negative autocorrelation. This occurs because a good cash flow shock to an index stock increases the overall index level, which in turn induces index extrapolators to expect a good index performance in the future, leading them to increase their index fund demand, resulting in an even higher index level. In subsequent periods, index extrapolators' expectations become less bullish since the initial rise of index level is assigned a progressively less weight over time, leading to diminishing demands and lower prices on average, and thus a negative autocorrelation. For these reasons, we also

[19]For brevity, we only present the autocorrelation between stock price changes in the previous quarter and the changes in next quarter, even though the autocorrelation $\rho_n(t_0, t_1, t_2, t_3)$ we present in Proposition 4 is more general and holds for any price changes between periods (t_0, t_1) and (t_2, t_3). This choice is motivated by our numerical investigations, which show that the main message of Table 4 on the effects of increase in index investors remains unchanged if we examine autocorrelation at different horizons. Moreover, we also find the effects of degree of extrapolation κ remain the same if we consider longer future horizons, typically up to four quarters. However, for horizons longer than that the effects of κ may differ due to the complex autocorrelation dynamics in our model. Since the relevant empirical studies typically employ horizons shorter than a quarter in their analysis (e.g., Ben-David et al. (2018), Baltussen, van Bekkum, and Da (2019), Höfler, Schlag, and Schmeling (2023)), we base our analysis on relatively shorter quarter-on-quarter autocorrelations.

Table 4. Stock autocorrelation. This table reports the average price change autocorrelation between the previous quarter and the next quarter among index stocks (Ind) and non-index stocks (Non) as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_\mathcal{R}, \pi_\mathcal{E})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

		No Stock Extrapolators $(\pi_r, \pi_e, \pi_\mathcal{R}, \pi_\mathcal{E})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_\mathcal{R}, \pi_\mathcal{E})$		
		$(1, 0, 0, 0)$	$(1, 0, .5, 0)$	$(1, 0, 0, .5)$	$(.5, .5, 0, 0)$	$(.5, .5, .5, 0)$	$(.5, .5, 0, .5)$
κ	Stock	I	II	III	IV	V	VI
	Ind	0	0	-0.58	-2.36	-2.09	-2.98
0.3	Non	0	0	0	-2.36	-2.36	-2.36
	Diff	**0**	**0**	**-0.58**	**0**	**0.27**	**-0.62**
	Ind	0	0	-0.92	-3.79	-3.35	-4.77
0.5	Non	0	0	0	-3.79	-3.79	-3.79
	Diff	**0**	**0**	**-0.92**	**0**	**0.44**	**-0.98**
	Ind	0	0	-1.23	-5.07	-4.48	-6.36
0.7	Non	0	0	0	-5.07	-5.07	-5.07
	Diff	**0**	**0**	**-1.23**	**0**	**0.59**	**-1.29**

obtain negatively autocorrelated stock prices in the presence of stock extrapolators even without any index investors (column IV). Introducing extrapolative index investors into this economy makes index stocks to have even stronger price reversals than non-index stocks, consistent with empirical evidence.[20] In contrast, as column V shows, an increase in rational index investors leads to a weaker negative autocorrelation for index stocks. This latter result occurs because following a good cash flow shock to an index constituent, stock extrapolators push up that stock's price, consequently raising the overall index level. Rational index investors expect the index to decrease from its current inflated level. Thus, they reduce their index fund demand, leading to a less price appreciation and negative price autocorrelations.

We also show that in the presence of extrapolators, a higher degree of extrapolation κ leads to a stronger negative autocorrelation for one-quarter price changes. Under a higher κ, extrapolators' expectations are more sensitive to price changes. Hence, cash flow shocks lead to stronger effects on current prices and subsequent price reversals.

4.5 Stock Trading Volume

We next explore the stock trading activity in our economy to see whether our model can capture the empirical fact that the index stocks experience higher trading volumes and turnovers than non-index stocks (e.g., Vijh (1994), Coles, Heath, and Ringgenberg (2022)). Towards that, we first denote each

[20]Naturally, the presence of extrapolative index investors also leads to a negative autocorrelation for the overall index level, consistent with findings of Baltussen, van Bekkum, and Da (2019).

i-type stock investor's portfolio changes by $d\boldsymbol{\psi}_{it} = \boldsymbol{\mu}_{\psi it}dt + \boldsymbol{\sigma}_{\psi it}d\boldsymbol{\omega}_t$ where the m^{th} row n^{th} column entry of the diffusion term $\boldsymbol{\sigma}_{\psi it}$ capturing that investor's (unpredictable) trade in the stock m following a cash flow (dividend) shock ω_{nt}. Similarly, we denote i-type index investor's portfolio changes by $d\psi_{it} = \mu_{\psi it}dt + \sigma_{\psi it}d\omega_{It}$, with $\sigma_{\psi it}$ capturing her unpredictable trade in the index fund following an index shock ω_{It}, which implies $\sigma_{\psi it}\boldsymbol{q}$ as the corresponding trade in individual stocks. We then consider a measure of stock trading volume that is commonly employed in continuous-time settings (e.g., Xiong and Yan (2010), Longstaff and Wang (2012)), which sums over the (population weighted) absolute value of these unpredictable trades

$$\boldsymbol{V}_t \equiv \frac{1}{2}\sum_{i=r,e}\pi_i\left|\boldsymbol{\sigma}_{\psi it}\right|\mathbf{1}_N + \frac{1}{2}\sum_{i=\mathcal{R},\mathcal{E}}\pi_i\left|\sigma_{\psi it}\right|\boldsymbol{q}, \tag{30}$$

where the adjustment $1/2$ is to prevent double counting of the shares traded across investors, $\mathbf{1}_N$ denotes the $N \times 1$ vector with all entries equal to one, and $|\boldsymbol{\sigma}_{\psi it}|$ denotes the (entry-by-entry) absolute value of the diffusion matrix $\boldsymbol{\sigma}_{\psi it}$. Proposition 5 reports the equilibrium trading volume in individual stocks in our model.

Proposition 5 (Equilibrium stock trading volume). *In the index investing economy with extrapolators, the equilibrium trading volume of stock n, $n = 1, \ldots, N$, is given by the n^{th} entry of the vector*

$$\boldsymbol{V}_t = \frac{1}{2}\frac{\kappa}{r}\sum_{i=r,e}\pi_i\left|\boldsymbol{L}_i\boldsymbol{\Lambda}\boldsymbol{\sigma}_D\right|\mathbf{1}_N + \frac{1}{2}\frac{\kappa}{r}\sum_{i=\mathcal{R},\mathcal{E}}\pi_i\left|L_i\Lambda_I\sigma_{DI}\right|\boldsymbol{q}. \tag{31}$$

In our model, as long as a stock is held by investors with different beliefs, there is a non-trivial trading activity on that stock that is captured by our trading volume measure in Proposition 5. As (31) shows, the degree of extrapolation κ and the amplification terms $\boldsymbol{\Lambda}$ and Λ_I affect each individual stock's trading volume directly. In particular, due to the sentiment spillover effect, trades in an index stock occurs not only due to its own cash flow shocks but also due to shocks to the cash flows of all other index stocks. For instance, following a positive cash flow shock on any index stock, other index stock prices positively react to the shock. Rational stock investors, who have a finer information set, would know that these index stocks' cash flows are unchanged and thus are willing to trade with index investors. To better understand its behavior, we present the average trading volume among index and non-index stocks in Table 5.

Table 5 reveals that in the presence of index investors and extrapolators, consistent with empirical evidence, index stocks experience higher trading volumes than non-index stocks. In particular, column III shows that index extrapolators' fund demand itself can generate a trading activity in index stocks even when all stock investors are rational. In this case, a positive cash flow shock to any index stock increases the index extrapolators' demand for the fund due to their more bullish expectations. To satisfy this demand, the index fund issues more shares by buying appropriate amounts of all index stocks from

Table 5. Stock trading volume. This table reports the average trading volume among index stocks (Ind) and non-index stocks (Non) stocks as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

		No Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
		$(1, 0, 0, 0)$	$(1, 0, .5, 0)$	$(1, 0, 0, .5)$	$(.5, .5, 0, 0)$	$(.5, .5, .5, 0)$	$(.5, .5, 0, .5)$
κ	Stock	I	II	III	IV	V	VI
	Ind	0	0	2.00	2.18	2.87	3.44
0.3	Non	0	0	0	2.18	2.18	2.18
	Diff	**0**	**0**	**2.00**	**0**	**0.69**	**1.26**
	Ind	0	0	3.28	3.60	4.73	5.68
0.5	Non	0	0	0	3.60	3.60	3.60
	Diff	**0**	**0**	**3.28**	**0**	**1.13**	**2.08**
	Ind	0	0	4.57	5.01	6.59	7.92
0.7	Non	0	0	0	5.01	5.01	5.01
	Diff	**0**	**0**	**4.57**	**0**	**1.58**	**2.91**

rational stock investors, thereby generating a trade in each index stock. Table 5, Column IV, shows that significant trading activity can emerge even in the absence of index investors, solely due to the belief disagreement among stock investors. In the absence of sentiment spillover, a cash flow shock to an index stock would lead to trades only on that stock. As we can see in columns V and VI, index investing increases the trading volume of index stocks even further. This is again due to the sentiment spillover. Moreover, comparing the column V to column VI shows that the trading volume of index stocks is higher when index investors are extrapolators. This difference arises because, as discussed in Section 4.2, index stocks are more volatile when index investors are predominantly extrapolators. This implies that index stock sentiments, which are based on index stock prices, are also more volatile. Thus, investors have stronger and more frequent disagreements with each other, resulting in more intensive trading activity on index stocks.

Finally, we find that in the presence of extrapolators, a higher degree of extrapolation κ leads to higher trading volumes in stocks. This finding is intuitive since a higher κ means extrapolators are more sensitive to price changes, and thus, they trade more aggressively following cash flow shocks. Similarly, under a higher degree of extrapolation, rational investors also trade more aggressively against extrapolators since they know that prices will reverse more quickly.

4.6 Effects of Switching from Stock to Index Investing

So far, we have demonstrated our results by comparing an economy with only stock investors to the same economy but with new index investors incorporated. These comparisons were sufficient to demonstrate our main point that the documented cross-sectional differences between the index and non-index stocks can be reconciled when the marginal index investor is an extrapolator. However, in these comparisons, the population size of an economy with index investors is necessarily larger than that of the economy without index investors. In this section, we show that our main points remain valid even if index investors are not new entrants but existing stock investors who switch from trading individual stocks to an index fund, without altering the population size of the economy. To that end, in Table 6, we present our main economic quantities under different switching investor compositions. To capture switching for all investor types, differently from our earlier Tables, we only consider economies in which rational and extrapolative stock investors already exist so they can switch. For brevity, we also present our results when the degree of extrapolation is fixed at its baseline value of $\kappa = 0.5$.

Comparing the first and the last columns of Table 6 shows that when a stock extrapolator switches to index trading, index stocks have higher and more volatile prices, comove more with other index stocks, exhibit stronger negative price autocorrelations, and have higher trading volume than otherwise identical non-index stocks. On the other hand, when a rational stock investor switches to index investing, as the middle column shows, the prices of index stocks become less volatile, negatively correlate with prices of other index stocks, and have weaker negative autocorrelation than non-index stocks. These results are all in line with our earlier conclusions in Sections 4.1–4.5. They are not surprising since our results primarily depend on what type of index investor is dominant, the marginal investor, irrespective of whether they are switchers or new to the market. However, since switching changes the relative size of extrapolators and rational investors effectively trading the index and non-index stocks, the mechanisms behind these results slightly differ from previous ones. We discuss these results for each economic quantity below.

Stock price: We see that, when the the stock sentiment is at its average level, $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$, switching does not affect index stock prices, and the price difference between index and non-index stocks comes from lower non-index stock prices. In this case, stock investors and their corresponding index investors effectively have the same demand for each index stock. As a result, the switching to index investing does not influence the prices of index stocks.[21] On the other hand, when a stock investor switches to index trading, she cease trading non-index stocks, reducing the total demand, and thus the prices, of such stocks. The magnitude of the price reduction in non-index stocks depends on the relative

[21]We note that the only quantity in Table 6 that depends on the sentiment level is the stock price, and for brevity, we only report the average stock price by fixing the stock sentiment at its long-run average, $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$. However, our main message of Table 6 that the index stocks have higher prices than non-index stocks and the price difference is lower when switchers are extrapolators remains valid for other values of sentiment in which index stock prices are different than their benchmark economy counterparts.

Table 6. Effects of switching from stock to index investing. This table reports the average quantities among index stocks (Ind) and non-index stocks (Non) as well as their difference (Diff) in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ when the degree of extrapolation is $\kappa = 0.5$. The stock price is evaluated when the stock sentiment is at its long-run average $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$. All other parameter values are as in Table B1.

		$(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
	Stock	$(.5, .5, 0, 0)$	$(.25, .5, .25, 0)$	$(.5, .25, 0, .25)$
	Ind	381.44	381.44	381.44
Stock price	Non	381.44	284.17	380.90
	Diff	**0**	**97.27**	**0.54**
	Ind	14.04	16.17	12.84
Volatility	Non	14.04	17.14	12.19
	Diff	**0**	**-0.97**	**0.65**
	Ind	0	-12.32	9.80
Comovement	Non	0	0	0
	Diff	**0**	**-12.32**	**9.80**
	Ind	-3.79	-5.53	-2.83
Autocorrelation	Non	-3.79	-6.13	-2.20
	Diff	**0**	**0.60**	**-0.63**
	Ind	3.60	3.97	3.41
Trading volume	Non	3.60	2.93	2.08
	Diff	**0**	**1.04**	**1.33**

population size of remaining investors trading such stocks. Switching by extrapolators increases the relative size of rational stock investors trading non-index stocks, who now can absorb the remaining stock extrapolators' sentiment-driven demand without affecting prices much. Therefore, non-index stocks become less volatile (see also below) and command relatively higher prices in equilibrium. In contrast, rational investors' switching leads to much lower prices for non-index stocks. This occurs because the remaining rational stock investors have limited capacity to offset the price pressure of stock extrapolators, who are relatively more dominant now, which leads to more volatile and thus even lower prices for non-index stocks in equilibrium.[22]

Volatility: Switching by extrapolators leads to lower volatility for index and non-index stocks, with a more pronounced decrease for non-index stocks. In contrast, when switchers are rational, the volatility of both types of stocks increases with a more pronounced increase for non-index stocks. When stock extrapolators switch to index investing, their expectation, which now depends on the index, becomes less

[22]As in Section 4.1, one could also understand this result in terms of shifts in the supply and demand curves. Namely, switching increases the residual supply of non-index stocks available to remaining stock investors. Moreover, the aggregate demand curve of such investors shifts upward (downward) due to lower (higher) volatility of non-index stocks under switching by extrapolators (rational investors).

volatile since the index itself is less volatile than individual stock prices, due to the usual diversification effect. Thus, by switching, extrapolators' expectations become less sensitive to cash flow shocks and induce lower volatility for index stocks. Moreover, under switching by extrapolators, rational stock investors become more dominant for the non-index stocks. Therefore, they can better absorb the remaining extrapolators' sentiment-driven demand, dampening the amplification effect, leading to much lower volatility for non-index stocks. In contrast, when rational stock investors switch to index investing, index stocks become more volatile because after switching they can no longer tailor their portfolios for each index stock independently, which reduces their capacity to offset stock extrapolators' volatile beliefs. Moreover, since the remaining rational investors' ability to counter the stock extrapolators' sentiment-driven demand for non-index stocks reduces, we obtain an even higher volatility for non-index stocks, as the middle column of Table 6 illustrates.

Comovement: We observe that switching by extrapolators (rational investors) leads to a positive (negative) correlation among index stocks without affecting the correlation structure of non-index stocks. As in Section 4.3, this result is also due to the positive (negative) sentiment spillover arising when the marginal index investor is extrapolative (rational).

Autocorrelation: When a stock extrapolator switches from trading stocks to an index fund, price reversals of all stocks get weaker with the effects being more pronounced for non-index stocks. In contrast, under rational switchers, the price reversal gets stronger for all stocks with non-index stocks having stronger reversals. As we have discussed, when extrapolators switch, their expectations become less sensitive to cash flow shocks of index stocks. Therefore, cash flow shocks lead to smaller effects on current prices and weaker subsequent price reversals. These effects are more pronounced for non-index stocks simply because rational stock investors can absorb the remaining stock extrapolators' sentiment-driven demand more, leading to lower effects on current and subsequent prices. In contrast, by switching, rational investors reduce their capacity to offset stock extrapolators' volatile beliefs, resulting in stronger effects on current and subsequent prices, particularity for non-index stocks, since they cease trading such stocks.

Trading volume: Index stocks experience higher trading volume than non-index stocks, and the difference is greater when switchers are extrapolators rather than rational. These results arise due to two channels in our model. First, when a stock investor switches to index investing, she ceases trading non-index stocks. All else being equal, this channel naturally reduces the trading activity in non-index stocks. Second, by switching investors alter the risk characteristics and effective disagreement in all stocks in equilibrium. Under extrapolative switchers, all stocks become less volatile with the effect being more pronounced for non-index stocks (discussed above for volatility). Thus, extrapolators' beliefs, and thus demand, become relatively less sensitive to cash flow shocks resulting in relatively fewer trades in them, with the effect being more pronounced for non-index stocks. For non-index stocks, both channels reinforce each other leading to much lower trading activity in them. In contrast, under rational switchers, all stock prices and sentiments become more volatile, resulting in more aggressive trading activity in stocks, with the effect being more pronounced for non-index stocks. Therefore, index stocks which are subject to only the second channel experience a higher trading volume than non-index stocks, which are additionally subject to first channel which leads to a lower trading volume.

Taken together, our analysis in this section confirms the main conclusions of Sections 4.1–4.5: the documented cross-sectional differences between the index and non-index stocks could very well be due to the marginal index investor having an extrapolative expectation. While our earlier analysis is better suited to capture the rise of new retail investors entering the market, the current analysis on switching investors can also be thought of as capturing the recent trend of existing investors moving away from active management to passive index investing (e.g. Anadu et al. (2020)). Both trends, as we observe, are likely to exert a significant influence on asset prices. The fact that both cases yield similar cross-sectional implications in our model is reassuring that our main message remains valid, regardless of the relative importance of each channel.

As we discuss in Introduction, existing theories on index investing obtains some but not all our implications of this section. Moreover, some of these implications can also arise under alternative mechanisms that are not based on index investing. For instance, as also discussed in introduction, benchmarking concerns can generate index effect, higher volatility, and positive comovement for stocks in the benchmark index. The return comovement is also shown to arise in theories based on style investing (Barberis and Shleifer (2003)), limited attention and category-based learning (Peng and Xiong (2006)), and time-varying costs of holding an active fund (Vayanos and Woolley (2013)). That said, to the best of our knowledge, ours is the first theory to simultaneously reconcile all the observed cross-sectional differences we discuss in this section. Moreover, as we illustrate in the next section, our model also generates novel implications for index fund flow-performance relation, how investors' portfolios respond to their beliefs, and welfare effects of index investing.

5 Further Implications of Index Investing

In the preceding section, we examined the implications of our model for the documented cross-sectional differences between index and non-index stocks. Due to its richness, our model has other noteworthy equilibrium implications, which we study in this section. We first show that when index investors are predominantly extrapolators, consistent with empirical evidence, the relation between index fund flows and index performance becomes positive. We then demonstrate that an increase in extrapolative indexers makes index stock portfolios less responsive to investors' subjective expectations. Finally, we examine how index investing affects investors' welfare and find that as more stock extrapolators switch to index investing, the welfare loss of switching to index investing gets lower (higher) for rational (extrapolative) investors.

5.1 Index Fund Flow and Performance

One of the most extensively studied topics in finance is how fund flows relate to performance. The vast majority of the works in this literature focus on the flow-performance relation for active mutual funds and find a positive relation between fund flows and past performance (e.g., Chevalier and Ellison (1997), Sirri and Tufano (1998), Coval and Stafford (2007)). This "return chasing" behavior is typically attributed to the investors using past performance as a signal to learn about stock selection skill of the fund manager or investors' extrapolative expectations. With the rise of index investing, several empirical works also look at the flow-performance relation for index funds and find a similar positive relation between index fund flows and current index performance (Goetzmann and Massa (2003)) or past index performance (Dannhauser and Pontiff (2024), Anadu et al. (2020), Broman (2022)).[23] In this section, we examine whether our model can capture this documented behavior for index funds.

In our model, the total market value of the assets the index fund manages at a given point in time, the index fund size, is given by

$$P_{It} = I_t Q_{It}, \tag{32}$$

where the index level I_t is as in Proposition 1 and Q_{It} is the total number of index fund shares held by index investors at time t, and is given by

$$Q_{It} = (\pi_{\mathcal{R}} K_{\mathcal{R}} + \pi_{\mathcal{E}} K_{\mathcal{E}}) + (\pi_{\mathcal{R}} L_{\mathcal{R}} + \pi_{\mathcal{E}} L_{\mathcal{E}})\, X_{It}. \tag{33}$$

We see from (32) that the index fund size varies either due to the changes in the index level I_t (capital gains/losses) or changes in the aggregate fund demand Q_{It} (fund flows). We define the index fund flow over the next instant, dF_t, as the dollar change in the fund size arising only from the changes in aggregate fund demand:

$$dF_t \equiv I_t dQ_{It}. \tag{34}$$

To examine the flow-performance relation in our model, we first look at how index fund flows over the next instant dF_t covary with index changes dI_t in equilibrium in Proposition 6.

Proposition 6 (**Equilibrium flow-performance relation**). *In the index investing economy with extrapolators, the equilibrium covariance between index fund flows and index changes is given by*

$$\mathrm{Cov}_t\left[dF_t, dI_t\right]/dt = (\pi_{\mathcal{R}} L_{\mathcal{R}} + \pi_{\mathcal{E}} L_{\mathcal{E}})\, \kappa \big(\tfrac{1}{r}\Lambda_I \sigma_{DI}\big)^2 I_t. \tag{35}$$

Proposition 6 shows that whether the fund flow-performance relation is positive or negative in our model is determined by the sign of the constant $\pi_{\mathcal{R}} L_{\mathcal{R}} + \pi_{\mathcal{E}} L_{\mathcal{E}}$. From (33), we see that this term captures

[23] Clifford, Fulkerson, and Jordan (2014) find a similar return-chasing behavior for ETFs, which are mostly passively managed and track an index.

Table 7. Index fund flow-performance relation. This table reports the flow sensitivity term $\pi_{\mathcal{R}}L_{\mathcal{R}} + \pi_{\mathcal{E}}L_{\mathcal{E}}$ in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

	No Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			With Stock Extrapolators $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
	$(1, 0, 0, 0)$	$(1, 0, .5, 0)$	$(1, 0, 0, .5)$	$(.5, .5, 0, 0)$	$(.5, .5, .5, 0)$	$(.5, .5, 0, .5)$
κ	I	II	III	IV	V	VI
0.3	0	0	**2.08**	0	-1.04	**1.04**
0.5	0	0	**2.05**	0	-1.02	**1.02**
0.7	0	0	**2.04**	0	-1.02	**1.02**

the sensitivity of flows to past performance (sentiment) of the index, since $dQ_{It}/dX_{It} = \pi_{\mathcal{R}}L_{\mathcal{R}} + \pi_{\mathcal{E}}L_{\mathcal{E}}$. Thus, when this term is positive, a higher past performance leads to a higher current demand for the index, generating a positive flow-performance relation in our model. To investigate its sign, we quantify and present the flow sensitivity term $\pi_{\mathcal{R}}L_{\mathcal{R}} + \pi_{\mathcal{E}}L_{\mathcal{E}}$ in Table 7.

Table 7 shows that when index investors are predominantly extrapolators, we obtain a positive flow-performance relation for the index fund as in the data. This is not surprising, since positive cash flow shocks to index stocks would increase the index level and during these times extrapolative indexers increase their demand for the index fund, creating a positive fund net flow while also pushing further up the prices of index stocks. In contrast, when the marginal index investor is rational, there is either no flow-performance relation (column II) or a negative one (column V). Intuitively, when there are no extrapolators in the stock market (column II), there is no stock sentiment, and thus, rational indexers fund demand does not depend on past fund performance. In the presence of stock extrapolators (column V), positive cash flow shocks to index stocks would raise the index level above and beyond the level justified by its fundamentals from the viewpoint of rational index investors. Therefore, they reduce their demand for the index fund, incurring a negative flow-performance relation.[24]

5.2 Portfolio Response to Beliefs

Recently, there is a growing interest in not only inferring investors' beliefs using survey data but also whether investors act on those beliefs when forming their portfolios. Several studies show that investors indeed adjust their portfolios in response to the changes in their subjective return expectations (e.g., Amromin and Sharpe (2013), Ameriks et al. (2020), Giglio et al. (2021), Dahlquist and Ibert (2024)). However, they show that portfolio responses are much smaller than what standard portfolio theories

[24]We note that the main message of Table 7 remains valid if index investors are not new entrants but switchers. We also highlight that the necessary condition, which enables our model to reconcile the documented flow-performance relation (i.e., the marginal index investor having an extrapolative expectations), is also the same condition that helps it explain the documented cross-sectional differences between index and non-index stocks in Section 4.

predict, thus often call this phenomenon "attenuation puzzle." In this section, we examine how stock investors' portfolios respond to their subjective beliefs to see whether our model can shed light on these attenuation effects. To that end, we consider the following (contemporaneous) regression

$$\boldsymbol{\psi}_{it}^{n} = \alpha_i^n + \beta_i^n \boldsymbol{\Pi}_{it}^n + \varepsilon_{it}^n, \tag{36}$$

where $\boldsymbol{\psi}_{it}^{n}$ is the portfolio holdings of the i-type stock investor, $i = r, e$, in terms of the number of shares in the stock n, $n = 1, 2, \cdots, N$, and $\boldsymbol{\Pi}_{it}^n$ is that investor's subjective risk premium on stock n, which is given by the n^{th} element of the vector $\mathrm{E}_t^i\left[d\boldsymbol{S}_t + \boldsymbol{D}_t dt - r\boldsymbol{S}_t dt\right]/dt$. Proposition 7 reports the portfolio response coefficient β_i^n in the regression (36) for each stock investor type.

Proposition 7 (**Equilibrium portfolio response to beliefs**). *In the index investing economy with extrapolators, the equilibrium portfolio response coefficient in the regression* (36) *for rational and extrapolative stock investors are given by*

$$\beta_r^n = -\frac{\boldsymbol{e}_n^\intercal \boldsymbol{L}_r \mathrm{Var}\left[\boldsymbol{X}_\infty\right]\left(r\boldsymbol{B} + \kappa \boldsymbol{\Lambda} \boldsymbol{B}\right)^\intercal \boldsymbol{e}_n}{\boldsymbol{e}_n^\intercal \left(r\boldsymbol{B} + \kappa \boldsymbol{\Lambda} \boldsymbol{B}\right) \mathrm{Var}\left[\boldsymbol{X}_\infty\right]\left(r\boldsymbol{B} + \kappa \boldsymbol{\Lambda} \boldsymbol{B}\right)^\intercal \boldsymbol{e}_n}, \tag{37}$$

$$\beta_e^n = \frac{\boldsymbol{e}_n^\intercal \boldsymbol{L}_e \mathrm{Var}\left[\boldsymbol{X}_\infty\right]\left(\boldsymbol{I}_N - r\boldsymbol{B}\right)^\intercal \boldsymbol{e}_n}{\boldsymbol{e}_n^\intercal \left(\boldsymbol{I}_N - r\boldsymbol{B}\right) \mathrm{Var}\left[\boldsymbol{X}_\infty\right]\left(\boldsymbol{I}_N - r\boldsymbol{B}\right)^\intercal \boldsymbol{e}_n}, \tag{38}$$

where $\boldsymbol{e}_n$ is an $N \times 1$ elementary vector with its n^{th} element being 1 *and others being* 0 *and* $\mathrm{Var}\left[\boldsymbol{X}_\infty\right]$ *is as in* (24).

Proposition 7 reveals that in the presence of extrapolators, portfolio response coefficients take rich forms and depend on sentiment-related quantities, such as the portfolio sensitivity to sentiment $\boldsymbol{L}_i$, price sensitivity to sentiment $\boldsymbol{B}$, the stock amplification term $\boldsymbol{\Lambda}$, and the sentiment uncertainty $\mathrm{Var}\left[\boldsymbol{X}_\infty\right]$. Since these terms differ for index and non-index stocks, we present the average portfolio response coefficients both for index and non-index stocks in Table 8.

Table 8 shows that an increase in extrapolative indexers makes index stock portfolios less sensitive to subjective expectations. Moreover, this weaker portfolio response is present for both rational and extrapolative stock investors, as comparing column III to column I and column VI to column IV shows. The intuition for this results is as follows. An increase in extrapolative indexers leads to a higher volatility for index stocks (Section 4.2). Thus, stock investors' demand for such stocks becomes less responsive to changes in their subjective risk premium when these changes are driven by own cash flow shocks. Moreover, due to the positive sentiment spillover, stock extrapolators' subjective risk premium on an index stock also increases due to positive cash flow shocks to other index stocks. Consequently, stock extrapolators may hold less of this index stock to reduce aggregate risk exposure given that index stocks are positively correlated, further attenuating their portfolio response to subjective beliefs. In contrast, an increase in rational indexers leads to a stronger portfolio response to subjective expectations for index stocks as columns II and V show, due to lower risk and negative spillover effects. Table 8

Table 8. Portfolio response to beliefs. This table reports the average portfolio response coefficient among index stocks (Ind) and non-index stocks (Non) for rational and extrapolative stock investors in equilibrium under different population shares $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$ and degree of extrapolation κ. All other parameter values are as in Table B1.

		Rational Stock Investors $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$			Extrapolative Stock Investors $(\pi_r, \pi_e, \pi_{\mathcal{R}}, \pi_{\mathcal{E}})$		
		$(.5, .5, 0, 0)$	$(.5, .5, .5, 0)$	$(.5, .5, 0, .5)$	$(.5, .5, 0, 0)$	$(.5, .5, .5, 0)$	$(.5, .5, 0, .5)$
κ	Stock	I	II	III	IV	V	VI
0.3	Ind	2.47	2.53	**2.09**	1.07	1.17	**0.94**
	Non	2.47	2.47	**2.47**	1.07	1.07	**1.07**
0.5	Ind	2.45	2.51	**2.06**	1.04	1.14	**0.91**
	Non	2.45	2.45	**2.45**	1.04	1.04	**1.04**
0.7	Ind	2.44	2.50	**2.05**	1.03	1.13	**0.90**
	Non	2.44	2.44	**2.44**	1.03	1.03	**1.03**

also reveals that stock investors' portfolio responses to their subjective expectations decrease in the degree of extrapolation κ. This result also occurs due to the higher risk channel since as the degree of extrapolation increases each stock becomes more volatile.

The extant literature offers some possible explanations for the observed attenuation effect. For example, Giglio et al. (2021) argue that it might be due to retail investors' infrequent trading and low confidence about their beliefs. Dahlquist and Ibert (2024) point out that the investment mandates could limit the responsiveness of asset managers' portfolio allocation to their beliefs. Our contribution in this section is to demonstrate that the presence of extrapolative indexers can also make stock investors portfolios appear less responsive to their beliefs due to higher volatility and sentiment spillover.

5.3 Index Investing and Welfare Loss

In this section we examine how index investing affects investors' welfare. Since index investors cannot tailor their portfolios for each index stock independently, absent any costs, they typically have lower indirect utility than stock investors. To study the welfare loss to becoming an index investor in our frictionless economy, we follow Chabakauri and Rytchkov (2021) and consider the certainty equivalent loss (CEL). In our setting, equilibrium CEL for an investor type is defined as the dollar amount a stock investor would be willing to give up in any state to become indifferent to being an index investor in that state. That is, CEL for rational and extrapolative investors, denoted by η_{rt} and η_{et}, respectively, solve

$$J^r\left(W_t - \eta_{rt}, \boldsymbol{X}_t, t\right) = J^{\mathcal{R}}\left(W_t, X_{It}, t\right), \tag{39}$$

$$J^e\left(W_t - \eta_{et}, \boldsymbol{X}_t, t\right) = J^{\mathcal{E}}\left(W_t, X_{It}, t\right), \tag{40}$$

where J^i is the i-type investor's indirect utility function defined at time t as

$$J^i\left(W_{it}, \boldsymbol{X}_t, t\right) = \max_{(c_i, \boldsymbol{\psi}_i)} \mathrm{E}_t^i\left[\int_t^\infty e^{-\rho u}\frac{e^{-\gamma c_{iu}}}{-\gamma}du\right], \qquad \text{for} \qquad i = r, e,$$

$$J^i\left(W_{it}, X_{It}, t\right) = \max_{(c_i, \psi_i)} \mathrm{E}_t^i\left[\int_t^\infty e^{-\rho u}\frac{e^{-\gamma c_{iu}}}{-\gamma}du\right], \qquad \text{for} \qquad i = \mathcal{R}, \mathcal{E},$$

with $X_{It} = \boldsymbol{q}^\intercal \boldsymbol{X}_t$. Proposition 8 presents the equilibrium CEL for rational and extrapolative investors.

Proposition 8 (**Equilibrium certainty equivalent loss**). *In the index investing economy with extrapolators, the equilibrium certainty equivalent loss for rational and extrapolative investors are given by*

$$\eta_{rt} = \frac{1}{\gamma r}\big[\big(F_{\mathcal{R}} + G_{\mathcal{R}}X_{It} - \tfrac{1}{2}H_{\mathcal{R}}X_{It}^2\big) - \big(F_r + \boldsymbol{G}_r^\intercal \boldsymbol{X}_t - \tfrac{1}{2}\boldsymbol{X}_t^\intercal \boldsymbol{H}_r \boldsymbol{X}_t\big)\big], \tag{41}$$

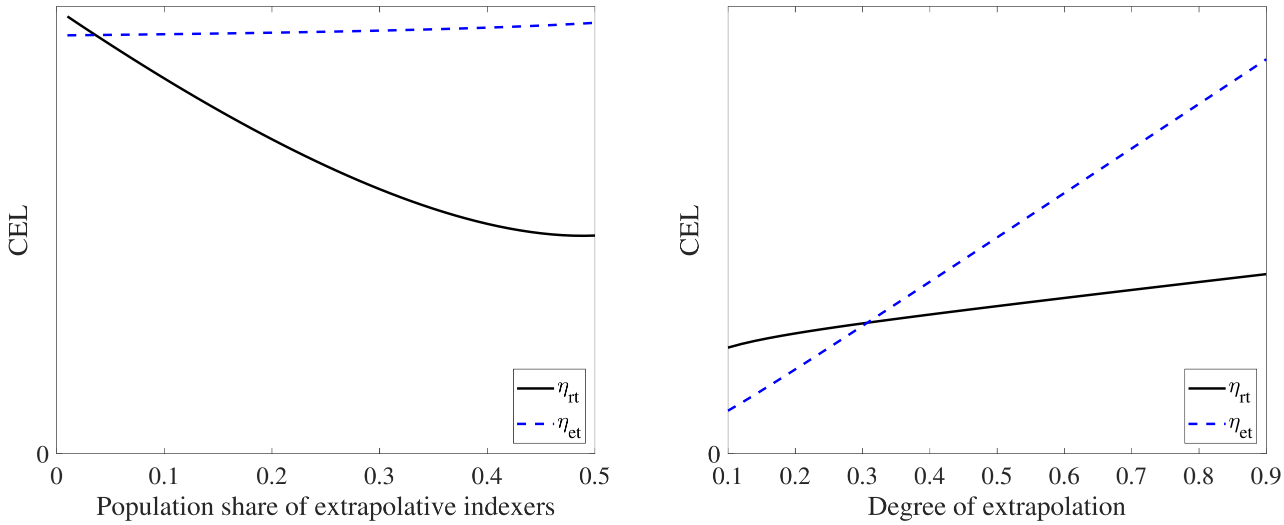
$$\eta_{et} = \frac{1}{\gamma r}\big[\big(F_{\mathcal{E}} + G_{\mathcal{E}}X_{It} - \tfrac{1}{2}H_{\mathcal{E}}X_{It}^2\big) - \big(F_e + \boldsymbol{G}_e^\intercal \boldsymbol{X}_t - \tfrac{1}{2}\boldsymbol{X}_t^\intercal \boldsymbol{H}_e \boldsymbol{X}_t\big)\big]. \tag{42}$$

Proposition 8 reveals that CEL for rational and extrapolative investors are state dependent and takes a quadratic form in stock and index sentiments. To understand its average behavior, we focus on the state when the stock sentiment is at its long-run average $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$ and illustrate the effects of switching by extrapolators and the degree of extrapolation on CEL in Figure 1.[25]

Figure 1 shows that CEL is positive, which indicates that in the absence of any costs, given choice, both rational and extrapolative investors would prefer to be stock investors rather than index investors.[26] As highlighted above, index investors cannot tailor their portfolios for each index stock independently, thus they have a lower indirect utility than stock investors. Panel A also illustrates that as more stock extrapolators switch to index investing, CEL for rational investors decreases indicating that welfare loss of switching to index investing for them is higher when there are more extrapolative stock investors. This finding is intuitive. Rational stock investors expect to make larger profits and would be unwilling to switch to index investing when there are more stock extrapolators who, compared to index extrapolators,

[25]In Figure 1, we are only interested in the sign and the directional effects on CEL. Therefore, we choose not to provide the magnitudes on the y-axis so as to prevent comparisons between CEL for rational and extrapolative investors. As highlighted Brunnermeier, Simsek, and Xiong (2014), comparing indirect utilities of investors with different beliefs is not straightforward and may not be economically meaningful. Moreover, we note that the patterns depicted in Figure 1 remain the same when the index covers all the stocks in the economy, i.e., $M = N$.

[26]That said, as discussed in Remark 1 of Section 2, introducing relatively higher costs for stock trading would provide incentives for index investing.



Panel A. Effects of $\pi_\mathcal{E}$

Panel B. Effects of κ

Figure 1. Certainty equivalent loss. These panels plot the equilibrium certainty equivalent loss (CEL) for rational (η_{rt}) and extrapolative (η_{et}) investors against the population share of extrapolative indexers $\pi_\mathcal{E}$ (Panel A) and the degree of extrapolation κ (Panel B) when the stock sentiment is at its long-run average $\boldsymbol{X}_t = \bar{\boldsymbol{X}}$. The population composition $(\pi_r, \pi_e, \pi_\mathcal{R}, \pi_\mathcal{E})$ is $(0.25, 0.5 - \pi_\mathcal{E}, 0.25, \pi_\mathcal{E})$ in Panel A and $(0.25, 0.25, 0.25, 0.25)$ in Panel C. All other parameter values are as in Table B1.

generate more profit opportunities in individual stocks for them. In contrast, CEL for extrapolative investors rises (slightly) as more of them become indexers. As discussed in Section 4.6, when a stock extrapolator switches to index investing, price amplification effect is dampened. This means for a given positive (negative) sentiment change, remaining stock extrapolators can buy (sell) stocks at a lower (higher) price, increasing their perceived profits from such trades. Therefore, these extrapolators would be less willing to forgo these perceived profits and switch to index investing.[27]

On the other hand, Figure 1, Panel B, reveals that when the degree of extrapolation κ increases, both rational and extrapolative stock investors become less willing to switch to index investing. As discussed in Section 4.5 for trading volume, a higher κ means both rational and extrapolative stock investors trade more aggressively on their beliefs. Therefore, they would be more reluctant to become index investors since doing so would mean forgoing larger expected profits from their respective stock investments.

[27]For brevity, we do not provide the plot for the case of switching rational investors, but for similar reasons, we also find that as more rational stock investors switch to index investing, CEL for both rational and extrapolative investors increase.

6 Conclusion

In this paper, we develop a dynamic equilibrium model of index investing in the presence of investors with extrapolative expectations. Our model generates rich implications that support the extensive empirical evidence on the cross-sectional differences between index and non-index stocks regarding their prices, volatilities, comovements, auto-correlations, and trading volume. We also provide an analysis on the flow-performance relation for index funds, the response of investor portfolios to their subjective beliefs, and the welfare costs of index investing.

Our main results are as follows. When index investors are mostly extrapolators, all consistent with empirical evidence, index stocks have higher and more volatile prices, comove more with other index stocks, exhibit stronger negative price autocorrelation, have higher trading volume than comparable non-index stocks, and there is a positive relation between index fund flows and index performance. An increase in extrapolative indexers leads to a smaller price difference between index and non-index stocks and makes index stock portfolios less responsive to investors' subjective expectations. Finally, by examining the welfare consequences of index investing, we find that as more stock extrapolators switch to index investing, the welfare loss of switching to index investing gets lower (higher) for rational (extrapolative) investors.

To demonstrate the equilibrium implications of extrapolative index investors in a clear setting, in this paper, we abstract away from any costs and other institutional features. However, our framework should be able to accommodate (per-period) index fund management fees similar to those considered in static settings (e.g., Bond and Garcia (2022), Gârleanu and Pedersen (2022)) in a tractable manner and study the effects of such costs. Our framework may also be extended to incorporate active funds in addition to the index fund to study the joint determination of asset prices and portfolio allocation across active and passive funds as in Gârleanu and Pedersen (2022). We leave these considerations, and many other relevant ones, for future research.

Appendix A: Proofs

Proof of Proposition 1. To determine the equilibrium in the index investing economy with extrapolators, we first solve each investor's optimization problem. We begin with stock investors. The dynamic budget constraint (13) of each i-type stock investors, $i = r, e$, can be rewritten as

$$dW_{it} = rW_{it}dt + \boldsymbol{\psi}_{it}^{\intercal}\boldsymbol{\Pi}_{it}dt + \boldsymbol{\psi}_{it}^{\intercal}\boldsymbol{\sigma}_{St}d\boldsymbol{\omega}_t^i - c_{it}dt. \tag{A.1}$$

where $\boldsymbol{\Pi}_{it}$ is the $N \times 1$ vector of subjective stock risk premia perceived by them and is given by $\boldsymbol{\Pi}_{it} = \boldsymbol{\mu}_{St} + \boldsymbol{D}_t - r\boldsymbol{S}_t$ for $i = r$ and $\boldsymbol{\Pi}_{it} = \boldsymbol{X}_t + \boldsymbol{D}_t - r\boldsymbol{S}_t$ for $i = e$. Moreover, the definition of stock sentiment in (7) implies its dynamics as $d\boldsymbol{X}_t = -\kappa\boldsymbol{X}_t dt + \kappa d\boldsymbol{S}_t$, which is perceived by investors as

$$d\boldsymbol{X}_t = \boldsymbol{\mu}_{\boldsymbol{X}t}^i dt + \kappa\boldsymbol{\sigma}_{St}d\boldsymbol{\omega}_t^i, \tag{A.2}$$

where $\boldsymbol{\mu}_{\boldsymbol{X}t}^i = \kappa\left(\boldsymbol{\mu}_{St} - \boldsymbol{X}_t\right)$ for $i = r$ and $\boldsymbol{\mu}_{\boldsymbol{X}t}^i = 0$ for $i = e$, which follows from the fact that stock extrapolators' subjective Brownian motion is related to the objective one as

$$d\boldsymbol{\omega}_t^e = d\boldsymbol{\omega}_t + \boldsymbol{\sigma}_{St}^{-1}\left(\boldsymbol{\mu}_{St} - \boldsymbol{X}_t\right)dt. \tag{A.3}$$

From the theory of stochastic control, the optimal consumption and portfolio strategies of i-type stock investors', $i = r, e$, satisfy the Hamilton–Jacobi–Bellman (HJB) equation

$$\begin{aligned} 0 = \max_{(c_i,\boldsymbol{\psi}_i)} \frac{e^{-\rho t}e^{-\gamma c_{it}}}{-\gamma} + J_t^i + J_W^i\left[rW_{it} + \boldsymbol{\psi}_{it}^{\intercal}\boldsymbol{\Pi}_{it} - c_{it}\right] + \frac{1}{2}J_{WW}^i\boldsymbol{\psi}_{it}^{\intercal}\boldsymbol{\Sigma}_{St}\boldsymbol{\psi}_{it} \\ + J_{\boldsymbol{X}}^{i\intercal}\boldsymbol{\mu}_{\boldsymbol{X}t}^i + \frac{1}{2}\kappa^2\mathrm{tr}\left[J_{\boldsymbol{XX}}^i\boldsymbol{\Sigma}_{St}\right] + \kappa J_{W\boldsymbol{X}}^{i\intercal}\boldsymbol{\Sigma}_{St}\boldsymbol{\psi}_{it}, \end{aligned} \tag{A.4}$$

where $J^i\left(W_{it}, \boldsymbol{X}_t, t\right) = \max_{(c_i,\boldsymbol{\psi}_i)} \mathrm{E}_t^i\left[\int_t^\infty e^{-\rho u}\frac{e^{-\gamma c_{iu}}}{-\gamma}du\right]$ is i-type stock investor's indirect utility function with its partial derivative with respect to x is denoted by J_x^i and $\mathrm{tr}\left[\boldsymbol{M}\right]$ is the trace of a square matrix $\boldsymbol{M}$, denoting the sum of elements on the main diagonal of $\boldsymbol{M}$.

We proceed by conjecturing a linear equilibrium in which prices of individual stocks take the form (15) and i-type stock investor's indirect utility taking the form

$$J^i\left(W_{it}, \boldsymbol{X}_t, t\right) = -e^{-\rho t}e^{-\gamma rW_{it}}e^{F_i + \boldsymbol{G}_i^{\intercal}\boldsymbol{X}_t - \frac{1}{2}\boldsymbol{X}_t^{\intercal}\boldsymbol{H}_i\boldsymbol{X}_t}, \tag{A.5}$$

for some scalar F_i, $N \times 1$ vector of constants $\boldsymbol{A}$ and $\boldsymbol{G}_i$, and $N \times N$ matrix of constants $\boldsymbol{B}$ and $\boldsymbol{H}_i$. The stock price conjecture implies its dynamics as

$$d\boldsymbol{S}_t = \boldsymbol{\Lambda}\left(\frac{1}{r}\boldsymbol{\mu}_D - \kappa\boldsymbol{B}\boldsymbol{X}_t\right)dt + \frac{1}{r}\boldsymbol{\Lambda}\boldsymbol{\sigma}_D d\boldsymbol{\omega}_t, \tag{A.6}$$

and (16), where the amplification term $\boldsymbol{\Lambda}$ is as in the proposition. From the above dynamics, we immediately have the volatility and variance-covariance matrices of individual stocks as

$$\boldsymbol{\sigma}_S = \frac{1}{r}\boldsymbol{\Lambda}\boldsymbol{\sigma}_D \qquad \text{and} \qquad \boldsymbol{\Sigma}_S = \frac{1}{r^2}\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D\boldsymbol{\Lambda}^\intercal, \tag{A.7}$$

along with the subjective stock risk premia as

$$\boldsymbol{\Pi}_{it} = \begin{cases} \left(\frac{1}{r}\boldsymbol{\Lambda}\boldsymbol{\mu}_D - r\boldsymbol{A}\right) - \left(r\boldsymbol{B} + \kappa\boldsymbol{\Lambda}\boldsymbol{B}\right)\boldsymbol{X}_t & \text{for } i = r, \\ -r\boldsymbol{A} - \left(r\boldsymbol{B} - \boldsymbol{I}_N\right)\boldsymbol{X}_t & \text{for } i = e, \end{cases} \tag{A.8}$$

and the expected change in the sentiment as

$$\boldsymbol{\mu}^i_{\boldsymbol{X}t} = \begin{cases} \kappa\boldsymbol{\Lambda}\left(\frac{1}{r}\boldsymbol{\mu}_D - \boldsymbol{X}_t\right) & \text{for } i = r, \\ 0 & \text{for } i = e. \end{cases} \tag{A.9}$$

Taking the first-order conditions of (A.4) with respect to c_i and $\boldsymbol{\psi}_i$ after substituting the partial derivatives $J^i_t = -\rho J^i$, $J^i_W = -\gamma r J^i$, $J^i_{WW} = \gamma^2 r^2 J^i$, $J^i_{\boldsymbol{X}} = \left(\boldsymbol{G}_i - \boldsymbol{H}_i\boldsymbol{X}_t\right)J^i$, $J^i_{\boldsymbol{XX}} = [-\boldsymbol{H}_i + \left(\boldsymbol{G}_i - \boldsymbol{H}_i\boldsymbol{X}_t\right)\left(\boldsymbol{G}_i - \boldsymbol{H}_i\boldsymbol{X}_t\right)^\intercal]J^i$, and $J^i_{W\boldsymbol{X}} = -\gamma r\left(\boldsymbol{G}_i - \boldsymbol{H}_i\boldsymbol{X}_t\right)J^i$, along with (A.8) and (A.9) gives the optimal consumption and portfolio strategy as in (19) where the portfolio terms are

$$\boldsymbol{K}_i = \begin{cases} \frac{1}{\gamma r}\left[\kappa\boldsymbol{G}_i + \boldsymbol{\Sigma}_S^{-1}\left(\frac{1}{r}\boldsymbol{\Lambda}\boldsymbol{\mu}_D - r\boldsymbol{A}\right)\right] & \text{for } i = r, \\ \frac{1}{\gamma r}\left[\kappa\boldsymbol{G}_i - \boldsymbol{\Sigma}_S^{-1} r\boldsymbol{A}\right] & \text{for } i = e, \end{cases} \tag{A.10}$$

$$\boldsymbol{L}_i = \begin{cases} -\frac{1}{\gamma r}\left[\boldsymbol{\Sigma}_S^{-1}\left(r\boldsymbol{B} + \kappa\boldsymbol{\Lambda}\boldsymbol{B}\right) + \kappa\boldsymbol{H}_i\right] & \text{for } i = r, \\ -\frac{1}{\gamma r}\left[\boldsymbol{\Sigma}_S^{-1}\left(r\boldsymbol{B} - \boldsymbol{I}_N\right) + \kappa\boldsymbol{H}_i\right] & \text{for } i = e, \end{cases} \tag{A.11}$$

Substituting the optimal consumption and portfolio strategy into the HJB equation (A.4) and rearranging gives

$$\begin{aligned} 0 = {} & -rF_i + r - \rho - r\ln(\gamma r) - \frac{\gamma^2 r^2}{2}\boldsymbol{K}_i^\intercal\boldsymbol{\Sigma}_S\boldsymbol{K}_i + \frac{\kappa^2}{2}\operatorname{tr}\left[\left(\boldsymbol{G}_i\boldsymbol{G}_i^\intercal - \boldsymbol{H}_i\right)\boldsymbol{\Sigma}_S\right] + \frac{\kappa}{r}\boldsymbol{G}_i^\intercal\boldsymbol{\Lambda}\boldsymbol{\mu}_D\boldsymbol{1}_{i=r} \\ & + \left[-r\boldsymbol{G}_i^\intercal - \gamma^2 r^2\boldsymbol{K}_i^\intercal\boldsymbol{\Sigma}_S\boldsymbol{L_i} - \kappa^2\boldsymbol{G}_i^\intercal\boldsymbol{\Sigma}_S\boldsymbol{H}_i - \kappa\left(\frac{1}{r}\boldsymbol{\mu}_D^\intercal\boldsymbol{\Lambda}^\intercal\boldsymbol{H}_i + \boldsymbol{G}_i^\intercal\boldsymbol{\Lambda}\right)\boldsymbol{1}_{i=r}\right]\boldsymbol{X}_t \\ & - \frac{1}{2}\boldsymbol{X}_t^\intercal\left[-r\boldsymbol{H}_i + \gamma^2 r^2\boldsymbol{L}_i^\intercal\boldsymbol{\Sigma}_S\boldsymbol{L}_i - \kappa^2\boldsymbol{H}_i^\intercal\boldsymbol{\Sigma}_S\boldsymbol{H}_i - 2\kappa\boldsymbol{H}_i^\intercal\boldsymbol{\Lambda}\boldsymbol{1}_{i=r}\right]\boldsymbol{X}_t, \end{aligned} \tag{A.12}$$

where the indicator function $\boldsymbol{1}_{i=r}$ takes the value 1 if $i = r$ and 0 if $i = e$. Thus, by the method of undetermined coefficients, for $i = r, e$, we must have

$$F_i = \frac{1}{r}\left[r - \rho - r\ln(\gamma r) - \frac{\gamma^2 r^2}{2}\boldsymbol{K}_i^\intercal \boldsymbol{\Sigma}_S \boldsymbol{K}_i + \frac{\kappa^2}{2}\mathrm{tr}\left[\left(\boldsymbol{G}_i\boldsymbol{G}_i^\intercal - \boldsymbol{H}_i\right)\boldsymbol{\Sigma}_S\right] + \frac{\kappa}{r}\boldsymbol{G}_i^\intercal \boldsymbol{\Lambda}\boldsymbol{\mu}_D \mathbf{1}_{i=r}\right], \tag{A.13}$$

and

$$\mathbf{0}_{N\times 1} = -r\boldsymbol{G}_i - \gamma^2 r^2 \boldsymbol{L}_i^\intercal \boldsymbol{\Sigma}_S \boldsymbol{K}_i - \kappa^2 \boldsymbol{H}_i^\intercal \boldsymbol{\Sigma}_S \boldsymbol{G}_i - \kappa\left(\frac{1}{r}\boldsymbol{H}_i^\intercal \boldsymbol{\Lambda}\boldsymbol{\mu}_D + \boldsymbol{\Lambda}^\intercal \boldsymbol{G}_i\right)\mathbf{1}_{i=r}, \tag{A.14}$$

$$\mathbf{0}_{N\times N} = -r\boldsymbol{H}_i + \gamma^2 r^2 \boldsymbol{L}_i^\intercal \boldsymbol{\Sigma}_S \boldsymbol{L}_i - \kappa^2 \boldsymbol{H}_i^\intercal \boldsymbol{\Sigma}_S \boldsymbol{H}_i - 2\kappa \boldsymbol{H}_i^\intercal \boldsymbol{\Lambda}\mathbf{1}_{i=r}, \tag{A.15}$$

Next, we solve the index investors' problem following similar steps to those for stock investors. The dynamic budget constraint (13) of each i-type index investors', $i = \mathcal{R}, \mathcal{E}$, can be rewritten as

$$dW_{it} = rW_{it}dt + \psi_{it}\Pi_{it}dt + \psi_{it}\sigma_{It}d\omega_{It}^i - c_{it}dt. \tag{A.16}$$

where the scalar Π_{it} is the subjective index risk premia perceived by them and is given by $\Pi_{it} = \mu_{It} + D_{It} - rI_t$ for $i = \mathcal{R}$ and $\Pi_{it} = X_{It} + D_{It} - rI_t$ for $i = \mathcal{E}$. Moreover, the definition of index sentiment in (10) implies its dynamics as $dX_{It} = -\kappa X_{It}dt + \kappa dI_t$, which is perceived by investors as

$$dX_{It} = \mu_{X_I t}^i dt + \kappa\sigma_{It}d\omega_{It}^i, \tag{A.17}$$

where $\mu_{X_I t}^i = \kappa(\mu_{It} - X_{It})$ for $i = \mathcal{R}$ and $\mu_{X_I t}^i = 0$ for $i = \mathcal{E}$, which follows from the fact that index extrapolators' subjective Brownian motion is related to the objective one as

$$d\omega_{It}^{\mathcal{E}} = d\omega_{It} + \sigma_{It}^{-1}(\mu_{It} - X_{It})\,dt. \tag{A.18}$$

From the theory of stochastic control, the optimal consumption and portfolio strategies of i-type index investors, $i = \mathcal{R}, \mathcal{E}$, satisfy the Hamilton–Jacobi–Bellman (HJB) equation

$$\begin{aligned} 0 = \max_{(c_i,\psi_i)} \frac{e^{-\rho t}e^{-\gamma c_{it}}}{-\gamma} + J_t^i + J_W^i\left[rW_{it} + \psi_{it}\Pi_{it} - c_{it}\right] + \frac{1}{2}J_{WW}^i\psi_{it}^2\sigma_{It}^2 \\ + J_{X_I}^i\mu_{X_I t}^i + \frac{1}{2}J_{X_I X_I}^i\kappa^2\sigma_{It}^2 + \kappa J_{WX_I}^i\sigma_{It}^2\psi_{it}, \end{aligned} \tag{A.19}$$

where $J^i(W_{it}, X_{It}, t) = \max_{(c_i,\psi_i)} \mathrm{E}_t^i\left[\int_t^\infty e^{-\rho u}\frac{e^{-\gamma c_{iu}}}{-\gamma}du\right]$ is i-type index investor's indirect utility function.

Given the stock price form (15), the index level becomes

$$I_t = \boldsymbol{q}^\intercal \boldsymbol{A} + \boldsymbol{q}^\intercal \boldsymbol{B}\boldsymbol{X}_t + \frac{1}{r}D_{It}. \tag{A.20}$$

We define $A_I = \boldsymbol{q}^\intercal \boldsymbol{A}$ and posit that there exists a scalar B_I satisfying

$$B_I \boldsymbol{q}^\intercal = \boldsymbol{q}^\intercal \boldsymbol{B}, \tag{A.21}$$

which along with $X_{It} = \boldsymbol{q}^\intercal \boldsymbol{X}_t$ allows us to rewrite (A.20) as in (17). Taking the dynamics of the index (17) yields

$$dI_t = \Lambda_I \left(\frac{1}{r}\mu_{DI} - \kappa B_I X_{It} \right) dt + \frac{1}{r}\Lambda_I \sigma_{DI} d\omega_{It}, \tag{A.22}$$

$$dX_{It} = \kappa \Lambda_I \left(\frac{1}{r}\mu_{DI} - X_{It} \right) dt + \frac{\kappa}{r}\Lambda_I \sigma_{DI} d\omega_{It}, \tag{A.23}$$

where $\Lambda_I = (1-\kappa B_I)^{-1}$ is the index amplification term, $\sigma_{DI} = \sqrt{\boldsymbol{q}^\intercal \boldsymbol{\Sigma}_D \boldsymbol{q}}$ is the index dividend volatility, and ω_{It} is the standard Brownian motion under the objective measure defined as

$$d\omega_{It} = \frac{1}{\sigma_{DI}} \boldsymbol{q}^\intercal \boldsymbol{\sigma}_D d\boldsymbol{\omega}_t.$$

From the above dynamics, we immediately have the volatility and variance of the index as $\sigma_I = \Lambda_I \sigma_{DI}/r$ and $\Sigma_I \equiv \sigma_I^2$, the subjective index risk premia as

$$\Pi_{it} = \begin{cases} \left(\frac{1}{r}\Lambda_I \mu_{DI} - r A_I \right) - \left(r B_I + \kappa \Lambda_I B_I \right) X_{It} & \text{for } i = \mathcal{R}, \\ -r A_I - \left(r B_I - 1 \right) X_{It} & \text{for } i = \mathcal{E}, \end{cases} \tag{A.24}$$

and the subjective expected change in the index sentiment as

$$\mu^i_{X_I t} = \begin{cases} \kappa \Lambda_I \left(\frac{1}{r}\mu_{DI} - X_{It} \right) & \text{for } i = \mathcal{R}, \\ 0 & \text{for } i = \mathcal{E}. \end{cases} \tag{A.25}$$

We note that since $I_t = \boldsymbol{q}^\intercal \boldsymbol{S}_t$, for consistency, we also have the index variance $\Sigma_I = \Lambda_I^2 \sigma_{DI}^2 / r^2 = \boldsymbol{q}^\intercal \boldsymbol{\Sigma}_S \boldsymbol{q}$. This can be seen from the fact that $\boldsymbol{q}^\intercal \Lambda_I = \boldsymbol{q}^\intercal \boldsymbol{\Lambda}$, which also implies that $d\omega_{It} = (1/\sigma_I) \boldsymbol{q}^\intercal \boldsymbol{\sigma}_S d\boldsymbol{\omega}_t$.

We take the i-type index investor's indirect utility as

$$J^i \left(W_{it}, X_{It}, t \right) = -e^{-\rho t} e^{-\gamma r W_{it}} e^{F_i + G_i X_{It} - \frac{1}{2} H_i X_{It}^2}, \tag{A.26}$$

for some scalars F_i, G_i, and H_i and obtain the first-order conditions of (A.19) with respect to c_i and ψ_i after substituting the partial derivatives $J^i_t = -\rho J^i$, $J^i_W = -\gamma r J^i$, $J^i_{WW} = \gamma^2 r^2 J^i$, $J^i_{X_I} = \left(G_i - H_i X_{It} \right) J^i$, $J^i_{X_I X_I} = \left[-H_i + \left(G_i - H_i X_{It} \right)^2 \right] J^i$, and $J^i_{W X_I} = -\gamma r \left(G_i - H_i X_{It} \right) J^i$, along with (A.24) and (A.25) gives the optimal consumption and portfolio strategy as in (20) where the portfolio terms are

$$K_i = \begin{cases} \frac{1}{\gamma r}\left[\kappa G_i + \mathit{\Sigma}_I^{-1}\left(\frac{1}{r}\Lambda_I \mu_{DI} - rA_I\right)\right] & \text{for } i = \mathcal{R}, \\ \frac{1}{\gamma r}\left[\kappa G_i - \mathit{\Sigma}_I^{-1} r A_I\right] & \text{for } i = \mathcal{E}, \end{cases} \tag{A.27}$$

$$L_i = \begin{cases} -\frac{1}{\gamma r}\left[\mathit{\Sigma}_I^{-1}\left(rB_I + \kappa\Lambda_I B_I\right) + \kappa H_i\right] & \text{for } i = \mathcal{R}, \\ -\frac{1}{\gamma r}\left[\mathit{\Sigma}_I^{-1}\left(rB_I - 1\right) + \kappa H_i\right] & \text{for } i = \mathcal{E}, \end{cases} \tag{A.28}$$

Substituting the optimal consumption and portfolio strategy into the HJB equation (A.19) and rearranging gives

$$\begin{aligned} 0 = & -rF_i + r - \rho - r\ln(\gamma r) - \frac{\gamma^2 r^2}{2}\mathit{\Sigma}_I K_i^2 + \frac{\kappa^2}{2}\left(G_i^2 - H_i\right)\mathit{\Sigma}_I + \frac{\kappa}{r} G_i \Lambda_I \mu_{DI} \mathbf{1}_{i=\mathcal{R}} \\ & + \left[-rG_i - \gamma^2 r^2 L_i \mathit{\Sigma}_I K_i - \kappa^2 H_i \mathit{\Sigma}_I G_i - \kappa\left(\frac{1}{r} H_i \Lambda_I \mu_{DI} + \Lambda_I G_i\right)\mathbf{1}_{i=\mathcal{R}}\right] X_{It} \\ & - \frac{1}{2}\left[-rH_i + \gamma^2 r^2 \mathit{\Sigma}_I L_i^2 - \kappa^2 \mathit{\Sigma}_I H_i^2 - 2\kappa H_i \Lambda_I \mathbf{1}_{i=\mathcal{R}}\right] X_{It}^2, \end{aligned} \tag{A.29}$$

where the indicator function $\mathbf{1}_{i=\mathcal{R}}$ takes the value 1 if $i = \mathcal{R}$ and 0 if $i = \mathcal{E}$. By the method of undetermined coefficients, for $i = \mathcal{R}, \mathcal{E}$, we must have

$$F_i = \frac{1}{r}\left[r - \rho - r\ln(\gamma r) - \frac{\gamma^2 r^2}{2}\mathit{\Sigma}_I K_i^2 + \frac{\kappa^2}{2}\left(G_i^2 - H_i\right)\mathit{\Sigma}_I + \frac{\kappa}{r} G_i \Lambda_I \mu_{DI} \mathbf{1}_{i=\mathcal{R}}\right], \tag{A.30}$$

and

$$0 = -rG_i - \gamma^2 r^2 L_i \mathit{\Sigma}_I K_i - \kappa^2 H_i \mathit{\Sigma}_I G_i - \kappa\left(\frac{1}{r} H_i \Lambda_I \mu_{DI} + \Lambda_I G_i\right)\mathbf{1}_{i=\mathcal{R}}, \tag{A.31}$$

$$0 = -rH_i + \gamma^2 r^2 \mathit{\Sigma}_I L_i^2 - \kappa^2 \mathit{\Sigma}_I H_i^2 - 2\kappa H_i \Lambda_I \mathbf{1}_{i=\mathcal{R}}. \tag{A.32}$$

To determine the constants in prices and indirect value functions, and hence verify our conjecture, we next impose the stock market clearing condition (14). Using investors portfolios in (19)– (20) and the fact $X_{It} = \boldsymbol{q}^\intercal \boldsymbol{X}_t$, we obtain the following system by the method of undetermined coefficients

$$\left(\pi_r \boldsymbol{K}_r + \pi_e \boldsymbol{K}_e\right) + \left(\pi_\mathcal{R} K_\mathcal{R} + \pi_\mathcal{E} K_\mathcal{E}\right)\boldsymbol{q} = \boldsymbol{Q}, \tag{A.33}$$

$$\left(\pi_r \boldsymbol{L}_r + \pi_e \boldsymbol{L}_e\right) + \left(\pi_\mathcal{R} L_\mathcal{R} + \pi_\mathcal{E} L_\mathcal{E}\right)\boldsymbol{q}\boldsymbol{q}^\intercal = \boldsymbol{0}_{N\times N}. \tag{A.34}$$

We jointly solve for three $N \times N$ matrices $\boldsymbol{B}$, $\boldsymbol{H}_r$, $\boldsymbol{H}_e$ and three scalars B_I, $H_\mathcal{R}$, $H_\mathcal{E}$ using six equations: (A.15) for $i = r, e$, (A.32) for $i = \mathcal{R}, \mathcal{E}$, (A.21), and (A.34). We next determine the three $N \times 1$ vectors $\boldsymbol{A}$, $\boldsymbol{G}_r$, $\boldsymbol{G}_e$ and two scalars $G_\mathcal{R}$ and $G_\mathcal{E}$ using five equations: (A.14) for $i = r, e$, (A.31) for $i = \mathcal{R}, \mathcal{E}$, and (A.33). Substituting these into (A.13) and (A.30) yields the scalars F_i for $i = r, e, \mathcal{R}, \mathcal{E}$. □

Proof of Proposition 2. The price change volatility of stock n is readily given by the square root of the n^{th} row n^{th} column entry of the variance-covariance matrix of stock price changes, which using the stock price dynamics in (A.6), is given by $\boldsymbol{\Sigma}_S = (1/r^2)\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D\boldsymbol{\Lambda}^\intercal$. □

Proof of Proposition 3. The price change correlation between stocks m and n is immediately given by its definition

$$\rho_{mn} = \text{Corr}_t\left[dS_{mt}, dS_{nt}\right] = \frac{\text{Cov}_t\left[\boldsymbol{e}_m^\intercal d\boldsymbol{S}_t, \boldsymbol{e}_n^\intercal d\boldsymbol{S}_t\right]}{\sqrt{\text{Var}_t\left[\boldsymbol{e}_m^\intercal d\boldsymbol{S}_t\right]\text{Var}_t\left[\boldsymbol{e}_n^\intercal d\boldsymbol{S}_t\right]}} = \frac{\boldsymbol{e}_m^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_n}{\sqrt{\left(\boldsymbol{e}_m^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_m\right)\left(\boldsymbol{e}_n^\intercal \boldsymbol{\Sigma}_S \boldsymbol{e}_n\right)}},$$

where $\boldsymbol{e}_n$ is an $N \times 1$ elementary vector with its n^{th} element being 1 and others being 0. □

Proof of Proposition 4. The price change autocorrelation of stock n, over the periods (t_0, t_1) and (t_2, t_3) for any $t_0 \leq t_1 \leq t_2 \leq t_3$ is by definition given by (27). The numerator in (27) is the n^{th} row n^{th} column entry of the covariance matrix $\text{Cov}\left[\boldsymbol{S}_{t_1} - \boldsymbol{S}_{t_0}, \boldsymbol{S}_{t_3} - \boldsymbol{S}_{t_2}\right]$, which after stock prices in (15) substituted in becomes

$$\text{Cov}[\boldsymbol{S}_{t_1} - \boldsymbol{S}_{t_0}, \boldsymbol{S}_{t_3} - \boldsymbol{S}_{t_2}] = \boldsymbol{B}\text{Cov}[\boldsymbol{X}_{t_1} - \boldsymbol{X}_{t_0}, \boldsymbol{X}_{t_3} - \boldsymbol{X}_{t_2}]\boldsymbol{B}^\intercal + \frac{1}{r}\text{Cov}[\boldsymbol{D}_{t_1} - \boldsymbol{D}_{t_0}, \boldsymbol{X}_{t_3} - \boldsymbol{X}_{t_2}]\boldsymbol{B}^\intercal. \tag{A.35}$$

To derive the covariances in (A.35), we employ the fact that $\boldsymbol{X}_t$ is a multi-dimensional Ornstein-Uhlenbeck process, which has a stationary Gaussian distribution when all the eigenvalues of $\boldsymbol{\Lambda}$ have positive real parts. Under the stationary of $\boldsymbol{X}_t$, we have its steady-state unconditional autocovariance for any $\tau \geq 0$ as

$$\text{Cov}\left[\boldsymbol{X}_t, \boldsymbol{X}_{t+\tau}\right] = e^{-\kappa\boldsymbol{\Lambda}\tau}\text{Var}\left[\boldsymbol{X}_\infty\right], \tag{A.36}$$

where $\text{Var}\left[\boldsymbol{X}_\infty\right]$ is the long-run variance of $\boldsymbol{X}_t$ and is given by (24). Using (A.36), we obtain the first covariance in (A.35) readily as

$$\text{Cov}\left[\boldsymbol{X}_{t_1} - \boldsymbol{X}_{t_0}, \boldsymbol{X}_{t_3} - \boldsymbol{X}_{t_2}\right] = \left[\left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\right) - \left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_0)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_0)}\right)\right]\text{Var}\left[\boldsymbol{X}_\infty\right],$$

and the second covariance in (A.35) as

$$\begin{aligned}
\text{Cov}\left[\boldsymbol{D}_{t_1} - \boldsymbol{D}_{t_0}, \boldsymbol{X}_{t_3} - \boldsymbol{X}_{t_2}\right] &= \boldsymbol{\sigma}_D\text{Cov}\left[\int_{t_0}^{t_1} d\boldsymbol{\omega}_u, \boldsymbol{X}_{t_1}\right]\left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\right)^\intercal \\
&= \frac{1}{r}\boldsymbol{\sigma}_D\left[\boldsymbol{\Lambda}^{-1}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}(t_1-t_0)}\right)\boldsymbol{\Lambda}\boldsymbol{\sigma}_D\right]^\intercal\left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\right)^\intercal \\
&= \frac{1}{r}\left(e^{-\kappa\boldsymbol{\Lambda}(t_3-t_1)} - e^{-\kappa\boldsymbol{\Lambda}(t_2-t_1)}\right)\boldsymbol{\Lambda}^{-1}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}(t_1-t_0)}\right)\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D.
\end{aligned}$$

The denominator terms in (27) is obtained from the variance matrix $\text{Var}\left[\boldsymbol{S}_{t_{k+1}} - \boldsymbol{S}_{t_k}\right]$, which after

(15) substituted in becomes

$$\mathrm{Var}[\boldsymbol{S}_{t_{k+1}} - \boldsymbol{S}_{t_k}] = \frac{1}{r^2}\mathrm{Var}[\boldsymbol{D}_{t_{k+1}} - \boldsymbol{D}_t] + \boldsymbol{B}\mathrm{Var}[\boldsymbol{X}_{t_{k+1}} - \boldsymbol{X}_t]\boldsymbol{B}^\intercal + \frac{2}{r}\boldsymbol{B}\mathrm{Cov}[\boldsymbol{X}_{t_{k+1}} - \boldsymbol{X}_t, \boldsymbol{D}_{t_{k+1}} - \boldsymbol{D}_t]\,. \quad \text{(A.37)}$$

Letting $\tau = t_{k+1} - t$, we obtain the first variance in (A.37) immediately as $\mathrm{Var}\left[\boldsymbol{D}_{t_{k+1}} - \boldsymbol{D}_t\right] = \boldsymbol{\Sigma}_D\tau$, and the second variance term as

$$\mathrm{Var}\left[\boldsymbol{X}_{t_{k+1}} - \boldsymbol{X}_t\right] = \mathrm{Var}\left[\boldsymbol{X}_{t+\tau}\right] + \mathrm{Var}\left[\boldsymbol{X}_t\right] - 2\mathrm{Cov}\left[\boldsymbol{X}_{t+\tau}, \boldsymbol{X}_t\right] = 2\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}\tau}\right)\mathrm{Var}\left[\boldsymbol{X}_\infty\right],$$

after employing (A.36). Finally, the covariance term in (A.37) becomes

$$\mathrm{Cov}\left[\boldsymbol{X}_{t_{k+1}} - \boldsymbol{X}_t, \boldsymbol{D}_{t_{k+1}} - \boldsymbol{D}_t\right] = \frac{1}{r}\boldsymbol{\Lambda}^{-1}\left(\boldsymbol{I}_N - e^{-\kappa\boldsymbol{\Lambda}\tau}\right)\boldsymbol{\Lambda}\boldsymbol{\Sigma}_D,$$

which along with earlier terms substituted in (A.37) yields (29). □

Proof of Proposition 5. To determine the trading volume of individual stocks, we first use investors' portfolio strategies in (19) and (20) and obtain the changes in their portfolios as $d\boldsymbol{\psi}_{it} = \cdots dt + (\kappa/r)\boldsymbol{L}_i\boldsymbol{\Lambda}\boldsymbol{\sigma}_D d\boldsymbol{\omega}_t$, for $i = r, e$, and $d\psi_{it} = \cdots dt + (\kappa/r)L_i\Lambda_I\sigma_{DI}d\omega_{It}$, for $i = \mathcal{R}, \mathcal{E}$. Substituting the diffusion terms into the trading volume measure (30), we immediately obtain the trading volume as in (31). □

Proof of Proposition 6. The equilibrium covariance between index fund flows and index changes is determined using (A.22)–(A.23) and (33)–(34) which implies $dF_t = I_t\left(\pi_\mathcal{R}L_\mathcal{R} + \pi_\mathcal{E}L_\mathcal{E}\right)dX_{It}$, yielding (35) immediately. □

Proof of Proposition 7. Investors' portfolio response coefficient in the regression (36) is given by definition

$$\beta_i^n = \frac{\mathrm{Cov}\left[\boldsymbol{\psi}_{it}^n, \boldsymbol{\Pi}_{it}^n\right]}{\mathrm{Var}\left[\boldsymbol{\Pi}_{it}^n\right]} = \frac{\mathrm{Cov}\left[\boldsymbol{e}_n^\intercal\boldsymbol{\psi}_{it}, \boldsymbol{e}_n^\intercal\boldsymbol{\Pi}_{it}\right]}{\mathrm{Var}\left[\boldsymbol{e}_n^\intercal\boldsymbol{\Pi}_{it}\right]},$$

where $\boldsymbol{e}_n$ is an $N \times 1$ elementary vector with its n^{th} element being 1 and others being 0. Substituting (19) and (A.8) into the above expression immediately yields (37)–(38). □

Proof of Proposition 8. The certainty equivalent loss for rational and extrapolative investors (41)–(42) are immediately given by using the value functions (A.5) for stock investors and (A.26) for index investors along with the definition of CEL in (39)–(40). □

Table B1. Parameter values. This table reports the parameter values used in our numerical analysis.

Parameter	Variable	Value
Dividend level for stock n	D_{nt}	10
Dividend mean for stock n	μ_{D_n}	0.05
Dividend volatility for stock n	σ_{D_n}	0.25
Supply of stock n	Q_n	5
Number of stocks in the market	N	5
Number of stocks in the index	M	3
Risk-free interest rate	r	0.025
Time discount factor	ρ	0.015
Absolute risk aversion coefficient	γ	0.1
Degree of extrapolation	κ	varying
Population shares of stock investors	(π_e, π_r)	varying
Population shares of index investors	$(\pi_{\mathcal{E}}, \pi_{\mathcal{R}})$	varying

Appendix B: Parameter Values

In this Appendix, we discuss the parameter values employed in our analysis, which are summarized in Table B1. We note that the behaviors of the equilibrium quantities depicted in our Tables and Figures are typical and do not vary much with alternative plausible parameter values.

Given that we adopt the framework in Barberis et al. (2015), we simply follow their calibration for parameters that are common to both models. This means, we also choose the dividend level of $D_{nt} = 10$, the mean dividend change of $\mu_{D_n} = 0.05$, the dividend change volatility of $\sigma_{D_n} = 0.25$, the stock supply of $Q_n = 5$ for each stock n, $n = 1, \ldots, N$, the interest rate as $r = 2.5\%$, the time discount factor as $\rho = 1.5\%$, and the absolute risk aversion coefficient as $\gamma = 0.1$.

For the degree of extrapolation, consistent with its empirical estimates in Barberis et al. (2015) and Cassella and Gulen (2018), we set its baseline value to $\kappa = 0.5$. To study the effects of higher and lower degree of extrapolation, we employ the empirical estimate of its standard deviation of 0.2 in Cassella and Gulen (2018), and also consider its values that are one standard deviation higher (0.70) and lower (0.30) from its baseline value. Finally, we simply take the number of stocks in the market as $N = 5$ and the number of stocks in the index as $M = 3$.

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