Voting and Trading on Public Information*

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Abstract

This paper studies how public information, such as proxy advice, affects shareholder voting and, thus, corporate decision-making. Although public information improves the voting decisions of uninformed shareholders, it also induces privately informed shareholders to sell their shares rather than to vote. As a result, public information impairs information aggregation by voting but improves information aggregation by trading. We show that, overall, public information can undermine corporate decision-making. Furthermore, the effect of more precise public information on corporate decision-making is non-monotonic. Our results give rise to new empirical predictions and have implications for regulation.

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1 Introduction

By voting, shareholders can affect major corporate decisions such as director appointments, M&A deals, and ESG proposals. It is often highly uncertain whether a given proposal will improve firm value. This uncertainty can be alleviated by public information released prior to the vote, such as proxy advice,[1] disclosure by the firm,[2] or news reports. However, public information not only influences how shareholders vote, but also induces them to trade. The empirical literature shows that less informed shareholders vote according to public information such as proxy advice,[3] whereas better informed shareholders leverage their private information to *vote or trade*.[4]

This paper studies how public information affects shareholder voting and, thus, corporate decision-making, taking into account that shareholders can trade their shares. We show that corporate decision-making can be impaired by public information even if it is unbiased and does not affect private information production.[5] Furthermore, increasing the quality of public information can have adverse effects on corporate decision-making. We identify two effects of public information as major drivers of our results. First, public information can influence who votes, by inducing privately informed shareholders to trade prior to the vote. Second, public information can influence the information content of stock market prices before the vote, thereby providing additional information shareholders can use in their voting decisions.

We present a model with a finite number of shareholders, one of which is a privately informed minority blockholder. The other shareholders are small and uninformed. Shareholders need to vote on whether to adopt a proposal. Whether the proposal is value increasing depends on an uncertain state of the world.[6] Before the vote, all shareholders receive an informative (but

[1] Almost all shareholder votes are accompanied by proxy advisor recommendations. For instance, the largest proxy advisor, Institutional Shareholder Services (ISS), issues more than 45,000 proxy analyses in more than 100 global markets. See the ISS website: https://www.issgovernance.com/solutions/proxy-voting-services/.

[2] The firm itself (or management) regularly discloses information and recommendations for shareholders on how to vote. For instance, in the US, before mergers the board is legally required to give a voting recommendation.

[3] Alexander et al. (2010) and Malenko & Shen (2016) show that public proxy recommendations substantially affect vote outcomes. Alexander et al. (2010) find that "the change in the probability of a dissident victory associated with a pro-dissident recommendation is between 14 and 30 percentage points in our sample"(p.4422).

[4] Iliev & Lowry (2015) identify trading in response to proxy advice, and Li et al. (2022) document abnormal trading volume before shareholder meetings.

[5] Malenko & Malenko (2019) and Buechel et al. (2023) study how public information (proxy advice) affects private information production. We keep private information fixed.

[6] Corporate decision making, firm value and welfare are identical in our model and, thus, used interchangeably.

imperfect) *public* signal. In addition, the blockholder receives a private signal and can trade before the vote. For simplicity, the private signal perfectly reveals the state.[7] Noise traders à la Kyle (1985) allow the blockholder to partially camouflage her trade. The small shareholders observe the share price before voting.

In a benchmark without public information, the blockholder does not trade and votes according to her private signal. The small, uninformed shareholders mix between voting for and against the reform in accordance with the prior probability that the reform is value increasing. In equilibrium, the blockholder is likely to swing the vote and improve firm value, making it optimal for her not to sell.

When there is public information but this information is sufficiently imprecise, the blockholder does not sell her shares and votes according to her private signal, as in the benchmark. Small shareholders mix between voting in line with and against the public signal. Though the public signal is informative and unbiased, mixing is a best response because small shareholders condition their voting decisions on the event that their individual vote is decisive (pivotal).

To understand the intuition, consider one small shareholder, and suppose that all other small shareholders vote according to the public signal with more than 50 percent probability. A small shareholder is then pivotal if the votes of the other small shareholders and the blockholder lead to a tie. If the public signal is correct, the blockholder and, likely, the majority of small shareholders, vote according to it. Because small and large shareholders tend to vote in the same direction, a tie in the vote is unlikely in this case and, thus, a small shareholder is unlikely to be pivotal. Conversely, if the public signal is incorrect, the blockholder opposes it, whereas small shareholders are again likely to follow it. Because small shareholders and the blockholder tend to vote in opposite directions, a tie is relatively more likely. Thus, a small shareholders is relatively more likely to be pivotal if the public signal is incorrect.

In equilibrium, small shareholders indeed vote as suggested by the public signal with more

[7]It would be sufficient for our results if the blockholder sometimes had a more informative signal than the public signal. In practice, due to their large stakes, blockholders have substantial incentives to acquire information – especially, relative to proxy advisors or news agencies who do not have a stake. In line with this, Iliev et al. (2021) report: "Consistent with larger investors being better able to recoup the costs of monitoring via their larger portfolio holdings, they conduct significantly more governance research" (p.5584). Michaely et al. (2023) show that a significant portion of institutional investors spends effort on research to develop their independent "voting rationales." Wang (2022) provides evidence on the imperfect quality of proxy advice.

than 50 percent probability. Given shareholders' voting strategy, the public signal, and updating from being pivotal, a small shareholder is indifferent between voting for and against the public signal, making mixing a best response. Thus, in equilibrium, updating from being pivotal offsets updating from the public signal. As the precision of the public signal rises, shareholders update more strongly from it. To keep small shareholders indifferent, they need to follow the public signal increasingly often to update more strongly from being pivotal and, thereby, offset the more precise public signal. Because small shareholders' mixing probability increases in the precision of the public signal, their votes become more correlated (with each other).

For sufficiently low signal precisions, the blockholder does not find it optimal to sell. Because small shareholders' votes are not too correlated,[8] the blockholder is likely to be pivotal and swing the vote toward the correct decision – even if the public signal is incorrect. Further, the blockholder could only sell at a discount because the share price would partially reflect that she does not positively influence the vote when she sells her shares. Hence, not selling is privately optimal for the blockholder. This changes, however, when the public signal becomes too precise. Though a more precise public signal induces the small shareholders to vote correctly more often, it also correlates their votes more strongly, also when the public signal is incorrect. As a result, conditional on the public signal being incorrect, the blockholder anticipates that the vote will likely decrease firm value, even if she votes. Therefore, she prefers to sell her shares when the public signal is more informative.

Whenever the blockholder does not trade (i.e., for sufficiently small signal precisions), public information always dominates the benchmark without public information in terms of firm value and welfare. Intuitively, the blockholder still incorporates her information into the decision by voting, and small shareholders' votes are better informed.

For higher signal precisions, the blockholder sells her shares in equilibrium if the public signal is incorrect. Small shareholders vote in line with the public signal unless they obtain conflicting information from the stock market. The blockholder's selling decision has two consequences. First, by choosing not to vote against the public signal even though she has conflicting information, the blockholder drains the vote of her private information precisely when it would be most

[8] Put differently, shareholders' mixing probability is sufficiently close to 50 percent.

valuable. This impairs information aggregation by voting and decreases the likelihood that the correct decision is implemented. Second, however, by selling her shares, the blockholder incorporates part of her information into the share price, which improves information aggregation by trading. As a result, the remaining shareholders learn additional information from the stock market and improve their voting decision. In particular, if the blockholder's sale of shares is uncovered, small shareholders know in equilibrium that the public signal must be incorrect, and, thus, they vote against the public signal.

Overall, we find that public information can reduce firm value relative to the benchmark without it. This occurs for intermediate signal precisions – those for which public information impairs information aggregation by voting by inducing the informed blockholder to sell her shares rather than to vote. At the same time, these intermediate signal precisions are *not* sufficiently high to make small shareholders' votes informed enough so that they improve corporate decision-making – even in conjunction with the additional information from the stock market. Of course, as the public signal becomes very precise, it eventually improves firm value.

Moreover, firm value can be non-monotonic in the precision of the public signal. For low signal precisions, the blockholder always votes according to her private signal and public information improves the voting decisions of small shareholders. In this case, more precise public information enhances firm value. However, when the signal precision becomes too high, the blockholder prefers to sell her shares as the "correlation-of-votes" effect is too strong. At the point where the public information becomes too precise, welfare can fall discontinuously. More precise public information can thus be harmful for firm value and welfare. After this point, welfare increases again with the precision of the public signal.

Our model is consistent with the existing empirical evidence on shareholder voting and trading, and it gives rise to new empirical predictions. In our model, a public signal, which is informative about the merit of a proposal, endogenously becomes predictive of the vote outcome. This is in line with Alexander et al. (2010), who document empirically that proxy advice is not just informative about the merit of a proposal but also about the vote outcome. Moreover, our model predicts that public information can induce shareholders with conflicting information to sell their shares before the vote. Thus, our model predicts high abnormal trading volume in the days between the release of proxy advice and the shareholder meeting – as has been empirically documented by Li et al. (2022).[9] According to our model, high abnormal trading volume before a vote but after the release of proxy advice should decrease the likelihood that shareholders follow proxy advice.

Our model generates two main, novel predictions: firm value can decrease due to the presence of public information, and more precise public information may harm firm value. To test these hypotheses, exogenous variation in the presence, or the precision, of public information is required. Changes in mandatory disclosure requirements or in the regulation of proxy advice may serve as potential sources for exogenous variation to test our hypotheses empirically.

In practice, institutional investors have an incentive to vote according to proxy advice to limit their exposure to lawsuits alleging a violation of their fiduciary duty.[10] Our model cautions against allowing funds to satisfy the fiduciary duty to their clients purely by following proxy advice. If institutional investors always vote as suggested by the proxy advisor, they correlate their votes more. This, in turn, makes proxy advice a better predictor of the vote outcome and, thus, maximizes the incentives of well-informed shareholders with conflicting information to sell, which can reduce firm value.

Our model highlights that an increase in the correlation of votes of uninformed shareholders may have adverse effects. In practice, this correlation cannot only be induced by public information but also by the rise of delegated investors who vote on behalf of their investors. Dasgupta et al. (2021) document that institutional investor ownership of the US stock market has increase from around 20 percent in 1970 to around 60 percent in 2020. Through the lens of our model, the rise of delegated investment and the concomitant correlation of votes may impair corporate decision-making. In particular, if delegated investors do not increase the informativeness of votes, there is no upside to the correlation of votes. A fortiori, the negative welfare effects will be even more pronounced.

While we derive all results assuming that shareholders are sophisticated and vote strate-

[9]While Li et al. (2022) document substantial pre-vote trading, their empirical analysis focuses on the drivers of post-vote trading due to potential inference problems in the period before a shareholder vote.

[10]In the words Daniel Gallagher, former commissioner of the US Securities and Exchange Commission (SEC), "relying on the advice from the proxy advisory firm became a cheap litigation insurance policy: for the price of purchasing the proxy advisory firm's recommendations, an investment adviser could ward off potential litigation over its conflicts of interest." Malenko & Malenko (2019) discuss this in more detail.

gically (i.e., condition their vote on being pivotal), the adverse affect of public information on firm value does not rely on shareholders' strategic sophistication. In particular, because non-strategic, sincere voting would imply that small shareholders always follow the informative public signal, small shareholders' votes would be perfectly correlated – independent of the informativeness of the public signal. Hence, the blockholder can never swing the vote, and she always sells in equilibrium whenever she knows the public signal to be incorrect. Thus, public information will reduce welfare for low and intermediate signal precisions (a larger set of parameters than with strategic voting). Hence, by allowing shareholders to respond strategically to public information, public information becomes more valuable than in a setting with non-strategic shareholders. With non-strategic voting, however, more precise public information monotonically improves welfare because already the lowest signal precision induces the exit of well-informed shareholders. We discuss and show in extensions that our results are robust to allowing for share purchases, abstentions, communication, and trading after voting.

Related Literature. Our paper contributes to the large literature on information aggregation in voting (e.g., Condorcet 1785, Austen-Smith & Banks 1996, Feddersen & Pesendorfer 1996, 1997, 1998, 1999). More closely related are the recent papers on shareholder voting and public information provision through proxy advice. Malenko & Malenko (2019) analyze how a proxy advisor optimally sells information to voters. They show that proxy advice may undermine private information production. Buechel et al. (2023) show that under some conditions, proxy advice may spur private information production. In contrast to these papers, our mechanism does not rely on the incentives to acquire additional private information. Malenko et al. (2021) study an information-design problem of a proxy advisor who can give a public recommendation and sell a private research report. They show that the proxy advisor may want to create controversy (i.e., send a public recommendation that goes against shareholders' priors) to maximize the proceeds from selling the private research report. None of the above papers considers trading after proxy advice.

The interaction of voting and trading in a setting with preference heterogeneity is analysed by Musto & Yılmaz (2003) and Levit et al. (2021, 2024). We study voting and trading in a setting with information aggregation. Musto & Yılmaz (2003) show how financial markets can affect political elections about redistributive policies. Levit et al. (2021) and Levit et al. (2024) consider settings with heterogeneous shareholder preferences. Share trading renders the shareholder base and the identity of the median voter endogenous, changing the outcome of the vote.

Focusing on information aggregation when shareholders can vote and trade, Meirowitz & Pi (2022) show that shareholders may have an incentive to vote against their information to reap trading profits after the vote has taken place. Meirowitz & Pi (2023) study information acquisition in this setting. Because of shareholders' incentive to manipulate their vote to generate trading profits, the vote outcome may be worse if more shareholders become informed. Bouton et al. (2022) show that disagreement and, thereby, trading volume, after a shareholder meeting may be larger than before. None of these papers considers public information and its effect on information aggregation when shareholders vote and trade. Thus, our mechanism that public information may induce informed shareholders to sell their shares before the vote is novel.

More broadly, we provide the first theory of shareholders trading on information prior to voting. The timing of trading is not a technicality. It creates new economic effects. Our model highlights that trading also has a bright side: if trading occurs before the vote, it incorporates additional information into the share price that can feed back into the voting decisions of shareholders. This effect is absent in the literature that focuses on post-vote trading.

Our paper is also related to Bar-Isaac & Shapiro (2020) and Malenko & Malenko (2023). Bar-Isaac & Shapiro (2020) show that a blockholder may optimally not vote all her shares in order to avoid crowding out the information of other shareholders. In their framework, the blockholder may sell shares because selling is a substitute for abstaining and, hence, improves welfare. By contrast, in our setting, a blockholder sells her shares because a public signal correlates the votes of uninformed shareholders. The blockholder's exit reduces information aggregation by voting and, thus, can harm welfare. As a result, the motive and the welfare consequences of the blockholder's trades are different. Malenko & Malenko (2023) study a model of delegating shareholders' voting decision to large asset managers.[11]

[11] Biased advice is studied by Ma & Xiong (2021) and Levit & Tsoy (2022). Ma & Xiong (2021) show that biased proxy advice might arise because shareholders demand biased advice. Levit & Tsoy (2022) show that a biased

More broadly, our paper relates to the literature on blockholder exit and voice (Hirschman 1970, Admati & Pfleiderer 2009, Edmans 2009, Edmans & Manso 2010, Edmans et al. 2018, Levit 2019, Voss 2022). In the prior literature, blockholder exit works via managerial incentives; by contrast, we study how blockholder exit induced by public information affects corporate decision-making via voting. Voice takes the form of voting in our model. We show that exit undermines voice by the blockholder but may improve voice by the other shareholders through stock market feedback. The idea that exit may undermine voice was first raised by Coffee (1991) and Bhide (1993).

Because our model allows small shareholders to learn from the share price, and to change their vote and, ultimately, firm value, our paper is also related to the literature on feedback effects of financial markets (e.g., Dow & Gorton 1997, Goldstein & Guembel 2008, Bond et al. 2010, Edmans et al. 2015, Dow et al. 2017, Davis & Gondhi 2024, Ahnert et al. 2020, Banerjee et al. 2021, Machado & Elisa Pereira 2023, Terovitis & Vladimirov 2022). None of these papers considers how stock market feedback affects shareholder voting. For an overview of the literature, see Bond et al. (2012) and Goldstein (2023). Because our paper identifies an adverse effect of public information, it is also related to the literature studying when such adverse effects arise in settings with strategic complementarities (e.g., Morris & Shin 2002, Angeletos & Pavan 2007). Our model of voting and trading does not feature strategic complementarities.

2 Model

Consider a single firm with $N+n$ shares outstanding, where $N \in \mathbb{N}$ is odd and $n \in \mathbb{N}$ is even. N shares are held by small shareholders holding one share each. Of these, $N-n$ are "active" small shareholders who vote, and n are "passive" noise traders who do not.[12] Furthermore, there is a large shareholder L who owns $n \in \{2, \ldots, \frac{N-1}{2}\}$ shares, a minority of the actively voted shares. We assume that $N \geq 5$ and henceforth for brevity refer to the N actively voted shares as the

expert, such as a proxy advisor, might use one-size-fits-all recommendations to hide his bias and, thus, increase his influence over decisions. Hu et al. (2024) show that funds receiving proxy advice may purchase additional, customized advice when there is preference heterogeneity.

[12] This assumption is for technical ease only because the noise traders share a common posterior with the small shareholders and, therefore, could not contribute additional information.

total number of shares.[13]

The firm has an upcoming shareholder meeting at which shareholders need to vote on adopting a reform or sticking with the status quo $d \in \{r, q\}$. Whether the reform is value increasing depends on an uncertain state of the world $\omega \in \{R, Q\}$. The common prior that $\omega = R$ is $1/2$. If the decision d matches the state ω, the per-share firm value $u(d, \omega)$ is one, and zero otherwise; i.e., $u(r, R) = u(q, Q) = 1$ and $u(r, Q) = u(q, R) = 0$.

All shareholders receive a public signal $s_{pub} \in \mathcal{S}_{pub} = \{R, Q\}$ with precision $\pi = \mathbb{P}[s_{pub} = R|\omega = R] = \mathbb{P}[s_{pub} = Q|\omega = Q] \in (1/2, 1)$. The large shareholder receives, in addition to the public signal, a private signal $s_{priv} \in \mathcal{S}_{priv} = \{R, Q\}$. As a result, she is better informed about the state than the other shareholders. For simplicity, we assume that s_{priv} reveals the state perfectly, i.e., $\mathbb{P}[s_{priv} = R|\omega = R] = \mathbb{P}[s_{priv} = Q|\omega = Q] = 1$.[14] In Online Appendix D, we consider the case in which L only sometimes becomes informed.

Trading. After observing the public and private signals, L decides whether to exit (sell her shares).[15] Formally, L chooses $h : \mathcal{S}_{pub} \times \mathcal{S}_{priv} \to \{-n, 0\}$. Simultaneously, liquidity traders sell an aggregate amount of n or 0 shares with equal probability. The presence of liquidity traders enables the blockholder to partially camouflage her exit because, in the tradition of Kyle (1985), the market maker only observes the total order flow $O \in \mathcal{O} = \{-2n, -n, 0\}$. The scope for camouflaging is limited because, prior to trading, the liquidity traders' selling decision is not observed by L. The competitive market maker observes the total order flow $O \in \mathcal{O}$ and the public signal, and buys any shares offered at the fair price $P = \mathbb{E}[u(d, \omega)|O, s_{pub}]$.

Voting. After trading closes, voting takes place. Like the market maker, all shareholders observe the total order flow $O \in \mathcal{O}$.[16] Then, all shareholders simultaneously submit their vote. To capture the predominant anonymity among (small) shareholders, we consider symmetric strategies: $p(O) : \mathcal{S}_{pub} \times \mathcal{O} \to [0, 1]$.[17] That is, given some order flow O and public signal s_{pub}, a

[13]For $N = 3$, L cannot be a minority blockholder since $n \geq 2$.

[14]Our results only require that the blockholder is sometimes better informed than the public signal. Clearly, blockholders have stronger incentives to become informed than, for example, proxy advisors (see, e.g., Iliev et al. 2021, for empirical evidence).

[15]We discuss in Section 5.3 and show formally in Online Appendix F that our results are robust to allowing for share purchases.

[16]Because of the symmetry of our model and the stock market feedback, share prices may not be invertible. Thus, we assume that shareholders can directly observe the order flow as, for instance, in Edmans et al. (2015).

[17]This assumption is standard in the literature on shareholder voting (see, e.g., Malenko & Malenko 2019) and

shareholder votes with probability $p(O)$ according to the public signal and votes with probability $1 - p(O)$ against it. The decision with more votes is implemented.

We consider perfect Bayesian equilibria. As standard in the (shareholder) voting literature (e.g., Feddersen & Pesendorfer 1998, Malenko & Malenko 2019), we assume that shareholders do not play weakly dominated strategies at the voting stage. Hence, they vote as-if-pivotal. Moreover, we assume that if L is indifferent between trading and staying, she stays. Last, we assume that the market maker, who buys the shares, does not vote – similar to the noise traders. This assumption is for technical ease only, as the market maker shares a common posterior with the small shareholders and, therefore, could not contribute additional information. The timing is summarized in the Figure 1.

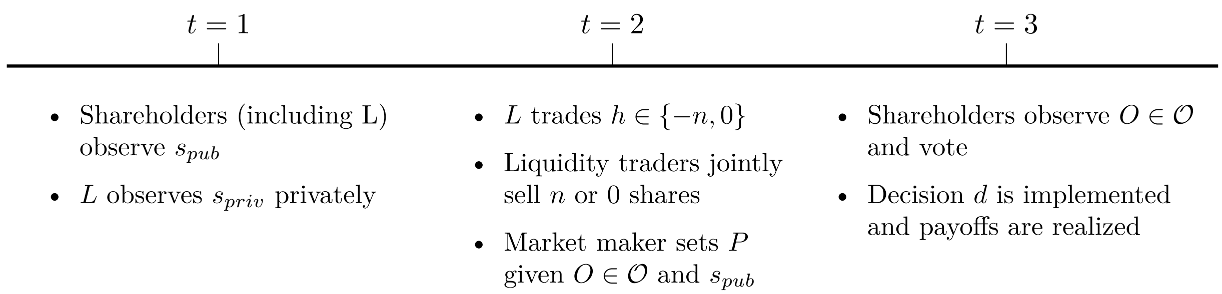

Figure 1: Sequence of Events

Let $\mathcal{W} = \mathbb{E}[u(d, \omega)]$ denote the per-share, ex-ante firm value. Ex-ante firm value and (utilitarian) welfare are equivalent in our model. Last, let $\mathbb{P}(p, N - n, k)$ be the probability that k out of $N - n$ small shareholders vote in line with the public signal when each small shareholder votes in line with the public signal with probability p. Formally,

$$\mathbb{P}(p, N - n, k) = \binom{N - n}{k} p^k \, (1 - p)^{N-n-k}$$

3 Benchmark without Public Information

We start with a benchmark without public information. In this benchmark, small shareholders can base their vote only on their prior and on order flow O. Absent of s_{pub}, a shareholder's strategy is given by $p(O) \in [0, 1]$, which, for ease of exposition and with slight abuse of notation,

in the literature on political voting(see, e.g., Feddersen & Pesendorfer 1998).

denotes the probability of voting for $d = r$. The large shareholder still obtains her private signal s_{priv}.

Proposition 1. *(Benchmark) When there is no public information, there exists an equilibrium in which small shareholders randomize with probability $p(O) = \frac{1}{2}$ for all $O \in \mathcal{O}$. L does not exit and votes according to s_{priv}. Ex-ante firm value is*

$$\mathcal{W}^B = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k). \tag{1}$$

Proof: See Appendix A.1.

Proposition 1 shows that there always exists an equilibrium in which the small shareholders mix with probability $1/2$ between voting for either decision.[18] Because small shareholders lack additional information absent of a public signal, they vote according to their prior.[19]

The large shareholder always remains with the firm and votes according to her private signal. As a result, the correct decision is implemented if enough small shareholders support the large shareholder to reach a majority. In particular, at least $\frac{N+1}{2} - n$ small shareholders need to support L in order for the correct decision to be implemented, yielding (1). The likelihood that the correct decision is implemented increases in n because a larger blockholder is able to swing the vote toward the correct decision more often. As a result, $\mathcal{W}^B$ increases in n.

It is optimal for the large shareholder not to exit. The fact that L has private information about the state ω is not in itself sufficient to create trading incentives. Because shareholders votes are uncorrelated ($p^* = 1/2$), L cannot predict whether the decision will match the state,

[18] Note that this mixed equilibrium can be micro-founded as the unique pure strategy equilibrium in (weakly) undominated strategies in a setup in which every small shareholder obtains a signal with a precision of $1/2 + \epsilon$. Taking $lim_{\epsilon \to 0}$, yields our equilibrium as the limit case. Suppose L stays and all shareholders vote according to their private signals. Then, for a small shareholder, the probability of voting correctly when voting with her signal (conditional on being pivotal) is

$$\frac{(1/2+\epsilon)\binom{N-n-1}{\frac{N-1}{2}-n}(1/2+\epsilon)^{\frac{N-1}{2}-n}(1/2-\epsilon)^{\frac{N-1}{2}}}{(1/2+\epsilon)\binom{N-n-1}{\frac{N-1}{2}-n}(1/2+\epsilon)^{\frac{N-1}{2}-n}(1/2-\epsilon)^{\frac{N-1}{2}} + (1/2-\epsilon)\binom{N-n-1}{\frac{N-1}{2}}(1/2-\epsilon)^{\frac{N-1}{2}-n}(1/2+\epsilon)^{\frac{N-1}{2}}} \geq \frac{1}{2} \tag{2}$$

for all $\epsilon \geq 0$. Hence, it is a best response for all small shareholder to vote according to their private signal.

[19] Note that even if we allowed for abstentions, this would still constitute an equilibrium because shareholders are exactly indifferent between voting for $d = r$ and voting for $d = q$ in equilibrium. If a small shareholder unilaterally deviates to abstaining, then, in the cases where she is pivotal, there is a tie ($N - 1$ is even) which, for any tie-breaking rule, cannot enhance welfare.

which determines firm value. Hence, L has no private information about the expected firm value and, thus, cannot reap trading profits. Moreover, if L were to exit and not able to camouflage, she would incur a strict loss. The reason is that the market maker would anticipate that the expected firm value is below the ex-ante firm value because L has exited and thus does not vote. Thus, absent of public information, L never sells her shares. Trading incentives can arise, however, when there is public information, as we show in Section 4.

The equilibrium in Proposition 1 is the only one in the benchmark which may dominate all equilibria with public information in terms of firm value and welfare. The other potential equilibria in the benchmark are always welfare-dominated by an equilibrium with public information and, thus, are irrelevant for our results on the welfare effects of public information.[20] For the remainder of the paper, we therefore refer to the equilibrium in Proposition 1 as the *benchmark equilibrium*. Note that our welfare results would be even easier to obtain if welfare superior equilibria were to exist in the benchmark.

4 The Effect of Public Information

When an informative public signal s_{pub} is disclosed, there are three equilibria that may maximize firm value. We distinguish them by whether small shareholders vote according to a pure or mixed strategy and by the trading behavior of the blockholder when she knows that the public signal is incorrect $s_{priv} \neq s_{pub}$.

The first equilibrium (EQ^{NE}) exists if and only if the public signal's precision π is sufficiently low. In this equilibrium, L always stays with the firm and votes according to her private information. The non-existence of EQ^{NE} for larger signal precisions will be the source of inefficiencies arising from public information.

Proposition 2. *(EQ^{NE} without exit) There exists a $\underline{\pi}(N,n) \in (1/2, 1)$ such that if and only if $\pi \leq \underline{\pi}(N,n)$ there is an equilibrium in which L never exits and always votes according to s_{priv}. The small shareholders mix between voting as suggested by the the public signal with probability*

[20] In Online Appendix G, we show that two other types of equilibria may exist but that they are dominated either by the equilibrium characterized in Proposition 3 or by the one in Proposition 4.

p^* *and against with probability* $1 - p^*$ *where*

$$p^* = \frac{1}{1 + (\frac{1-\pi}{\pi})^{\frac{1}{n}}} \in (1/2, \pi), \tag{3}$$

for $O \in \{-n, 0\}$. *If* $O = -2n$, *small shareholders vote as suggested by the public signal. Ex-ante firm value is given by*

$$\mathcal{W}^{NE} = \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k).$$

Proof. See Appendix A.2. □

In EQ^{NE}, small shareholders vote with the public signal with probability p^* for $O \in \{-n, 0\}$.[21] Although the public signal is informative and unbiased, it is optimal for a small shareholder to mix because she conditions on the event that her vote is decisive (pivotal). A shareholder's vote is pivotal if there is a tie among the votes of the other shareholders. In EQ^{NE}, a small shareholder can be pivotal in two cases: In the first, the public signal is correct; the blockholder and $\frac{N-1}{2} - n$ small shareholders vote *with* it, but $\frac{N-1}{2}$ small shareholders vote against it. In the second, the public signal is incorrect; the blockholder and $\frac{N-1}{2} - n$ small shareholders vote *against* it, but $\frac{N-1}{2}$ small shareholders vote with it. Whether a small shareholder wants to vote with or against the public signal then depends on the relative likelihood of these two cases. Mixing is a best response if and only if the two case are equally likely.

To understand the equilibrium, suppose that shareholders mix with $p^* \in (\frac{1}{2}, 1)$. Then, if the public signal is correct, the blockholder and, likely, the majority of small shareholders ($p^* > 1/2$), vote according to it. Because small and large shareholders tend to vote in the same direction, a tie in the vote is unlikely, and, thus, a small shareholder is unlikely to be pivotal. Conversely, if the public signal is incorrect, the blockholder opposes it, whereas small shareholders are again likely to follow it. Because small shareholders and the blockholder tend to vote in opposite directions, a tie is relatively more likely. Thus, a small shareholder is more likely to be pivotal if the public signal is incorrect.

[21] For brevity, we omit $O \in \{-n, 0\}$ in the mixing probability p^*.

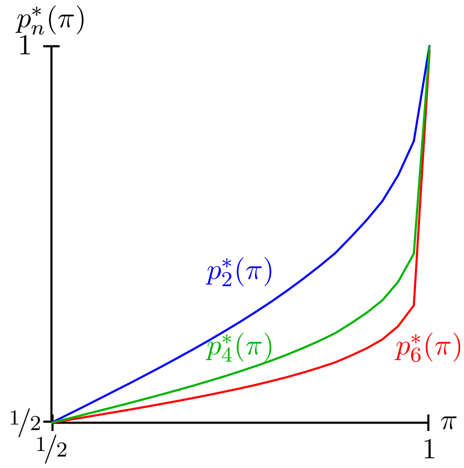

Figure 2: Small Shareholders' Strategy. $p_n^*(\pi)$ denotes small shareholders' strategy in EQ^{NE} for $O \in \{-n, 0\}$ as a function of π and for varying $n \in \{2, 4, 6\}$.

In equilibrium, p^* ensures that, conditional on being pivotal, both states are equally likely so that a small shareholder is indifferent, and mixing is a best response. Thus, in equilibrium, updating from being pivotal offsets updating from the public signal. As the precision of the public signal rises, shareholders update more strongly from it. To keep small shareholders indifferent, they need to follow the public signal increasingly often to update more strongly from being pivotal and thereby offset the more precise public signal. Because small shareholders' mixing probability p^* increases in the precision of the public signal π, their votes becomes more correlated (with each other).

Figure 2 displays the small shareholders' strategy for $n \in \{2, 4, 6\}$. For a given π and a fixed p^*, a larger blockholder stake n increases the probability that the public signal was incorrect conditional on a small shareholder being pivotal. Thus,for a given p^*, a larger stake n induces shareholders to update more strongly from being pivotal; hence, a lower p^* is required to keep shareholders indifferent.

If the public signal is relatively imprecise, L has a low incentive to exit. For low π, p^* is close to one half. Hence, even if s_{pub} is incorrect, the vote is likely to implement the correct decision when L stays and votes because L can swing the vote toward the correct decision. However, as the public signal becomes more precise, L has a high incentive to exit. For larger signal precisions π, p^* rises, and shareholders' votes become very correlated. If s_{pub} is incorrect,

L anticipates that she is unlikely to prevent a value-decreasing vote outcome with her vote, and may thus be better off selling her shares. Of course, whether L wants to sell depends on the price she expects to obtain. If $O = -n$, L's exit remains undetected and the share price equals the ex-ante firm value. Because L does not exit in the conjectured equilibrium, $O = -2n$ induces off-path beliefs. We assume that off-path beliefs after $O = -2n$ do not convey any information about the state; so it is optimal for shareholders to simply follow the public signal and $P(-2n) = \pi$. These off-path beliefs can, for instance, be micro-founded in an extension in which the blockholder only sometimes becomes privately informed. The reason is that L will then exit whenever she is uninformed (see Online Appendix D).[22] Given L's expected profits from exiting, she finds it optimal to deviate from the conjectured equilibrium to selling her shares for all $\pi > \underline{\pi}(N, n) \in (1/2, 1)$. Thus, EQ^{NE} does not constitute an equilibrium anymore, and only the other types of equilibria described below exist.

The following lemma establishes that whenever EQ^{NE} exists, it welfare dominates the benchmark without public information.

Lemma 1. *If EQ^{NE} exists, $\mathcal{W}^{NE} \geq \mathcal{W}^{B}$.*

Proof. See Appendix A.3. □

Formally, Lemma 1 requires that

$$\underbrace{\pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)}_{=\mathcal{W}^{NE}} \geq \underbrace{\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)}_{=\mathcal{W}^{B}}.$$

Intuitively, $\mathcal{W}^{NE} \geq \mathcal{W}^{B}$ holds because in both cases the informed blockholder stays and votes for the correct alternative. The only difference is the probability with which small shareholders vote for the correct alternative. In EQ^{NE} each shareholder puts some weight on the public signal and votes for the correct alternative with $p^* > \frac{1}{2}$, as opposed to $p^* = \frac{1}{2}$ in the benchmark; thus, the probability that an individual shareholder votes for the correct decision is higher in EQ^{NE}. Whenever EQ^{NE} exists, it thus welfare dominates the benchmark. Public information

[22]Note that higher off-path beliefs will shrink the parameter space in which the first equilibrium exists and, thus, make our following welfare results easier to attain.

may therefore only be harmful when EQ^{NE} does not constitute an equilibrium. As Proposition 2 shows, sufficiently precise public information can induce L to trade and, thus, lead to the non-existence of EQ^{NE}. This will give rise to a non-monotonicity in firm value as public information becomes more precise.

The second equilibrium always exists. In it, L sells her stake whenever the public signal is incorrect, and small shareholders never randomize.

Proposition 3. *(EQ^{E}_{NR} with exit and no randomizing) There exists an equilibrium in which L exits if $s_{pub} \neq s_{priv}$ and otherwise stays and votes with the public signal. Small shareholders vote according to s_{pub} if $O \in \{-n, 0\}$, and they vote against s_{pub} if $O = -2n$. Ex-ante firm value is $\mathcal{W}^{E}_{NR} = \frac{(1+\pi)}{2}$.*

Proof. See Appendix A.4. □

In EQ^{E}_{NR}, the large shareholder exits whenever she observes that the public signal is incorrect. By contrast, if it is correct, she stays and votes but is never pivotal. Hence, in contrast to EQ^{NE}, L does not directly influence the vote because small shareholders' votes are perfectly correlated. However, L indirectly affects the vote outcome because stock market feedback may change small shareholders' votes. In EQ^{E}_{NR}, small shareholders are never individually pivotal and, hence, they vote according to their posterior based on the public signal and the total order flow. When $O \in \{-n, 0\}$ they are either certain that L has kept her shares ($O = 0$), or they do not learn new information from the order flow ($O = -n$). Hence, it is optimal for them to follow the informative public signal. Observing $O = -2n$, however, small shareholders infer that the public signal is incorrect and vote against it. Stock market feedback, therefore, allows shareholders to improve their voting decision relative to only following the public signal. Information aggregation in the stock market can thus aid corporate decision-making via voting.

Because small shareholders' votes are perfectly correlated, when L observes that the public signal is incorrect, she anticipates that firm value will be $u(d, \omega) = 0$ if she remains with the firm. Hence, selling her stake is the unique best response – independent of π. When the public signal is correct, she anticipates that staying results in $u(d, \omega) = 1$, which is therefore her unique best response.

The following lemma shows that ex-ante firm value and, thereby, welfare, is larger in the benchmark without public information than in EQ^E_{NR} if and only if π is sufficiently small.

Lemma 2. *$\mathcal{W}^B > \mathcal{W}^E_{NR}$ if and only if $\pi < \overline{\pi}^E_{NR}(N,n)$ where*

$$\overline{\pi}^E_{NR} = 2\Big[\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)\Big] - 1. \quad (4)$$

Further, $\overline{\pi}^E_{NR}(N,n) < \overline{\pi}^E_{NR}(N,n+1)$. There is a $\hat{n} \leq \frac{N-1}{2}$ so that for all $n \geq \hat{n}$, $\overline{\pi}^E_{NR}(N,n) > 1/2$.

Proof. See Appendix A.5. □

In EQ^E_{NR}, small shareholders always vote with the public signal if it is correct. If the public signal is incorrect, small shareholders vote against the public signal if and only if the stock market reveals L's exit ($O = -2n$), which occurs with probability $1/2$. Hence, ex-ante firm value is $\mathcal{W}^E_{NR} = \pi + \frac{1-\pi}{2} = (1+\pi)/2$, which is clearly increasing in π because a higher signal precision means that shareholders vote correctly more often. Moreover, $\mathcal{W}^E_{NR}$ is independent of n. By contrast, in the benchmark there is no public signal so that $\mathcal{W}^B \in (1/2, 1)$ is independent of π, but $\mathcal{W}^B$ increases in n because for a larger stake, L swings the vote more often toward the correct decision. Thus, for sufficiently large n, there exists a unique cutoff $\overline{\pi}^E_{NR}(N,n) \in (1/2, 1)$ such that $\mathcal{W}^B \geq \mathcal{W}^E_{NR}$ if and only if $\pi \leq \overline{\pi}^E_{NR}(N,n)$. This cutoff $\overline{\pi}^E_{NR}(N,n)$ also increases in n.

A third equilibrium may arise with public information. This third equilibrium features stock market feedback and mixing by the small shareholders.

Proposition 4. *(EQ^E_R with exit and randomizing) There is a $\hat{\pi}(N,n) \in (1/2, 1)$ such that if and only if $\pi \geq \hat{\pi}(N,n)$ there exists an equilibrium in which small shareholders vote according to s_{pub} with certainty if $O = 0$. Small shareholders vote according to s_{pub} with probability*

$$p^* = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)^{\frac{2}{n}} \left(\frac{\binom{N-n-1}{\frac{N-n-1}{2}}}{\binom{N-n-1}{\frac{N-1}{2}}}\right)^{\frac{2}{n}}} \quad (5)$$

if $O = -n$, and against s_{pub} with $1 - p^*$. If $O = -2n$, small shareholders vote against the public

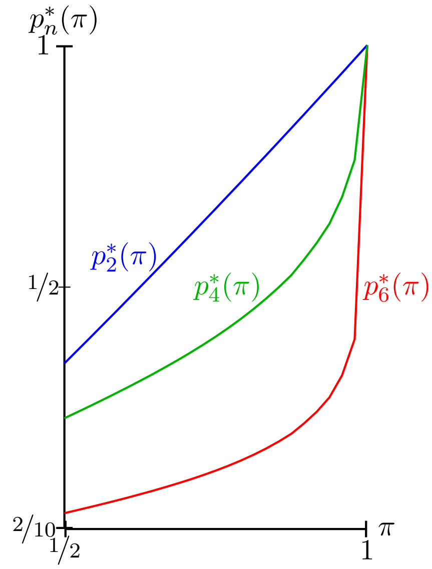

Figure 3: Small Shareholders' Strategy. $p^*(\pi)$ denotes small shareholders' strategy in EQ_R^E for $O = -n$ as a function of π and for varying $n \in \{2, 4, 6\}$.

signal. L stays and votes if $s_{priv} = s_{pub}$ and exits otherwise. Ex-ante firm value is

$$\mathcal{W}_R^E = \frac{1}{2} + \frac{1}{2}\Big(\pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)\Big) \tag{6}$$

Further, $\mathcal{W}_R^E$ increases in π.

Proof. See Appendix A.6. □

As in EQ_{NR}^E, the blockholder exits whenever the public signal is incorrect. By contrast, small shareholders vote in line with the public signal only if the stock market reveals with certainty that L did not exit ($O = 0$). If the stock market reveals L's exit ($O = -2n$), small shareholders vote against the public signal. The difference to EQ_{NR}^E arises for $O = -n$, which leaves small shareholders uncertain about whether L has exited. In this case, they mix between voting in line with or against the public signal according to (5) as illustrated in Figure 3 for $n \in \{2, 4, 6\}$.

The mixing probability admits a different form than in EQ^{NE} because small shareholders update differently from being pivotal. The reason is that rather than staying and voting against the public signal, L now exits whenever s_{pub} is incorrect. Figure 3 shows that, as a result, p^* is below $1/2$ for sufficiently small signal precisions π, even though the public signal is informative.

To understand the intuition, suppose that the public signal is (almost) uninformative ($\pi \approx 1/2$) and that small shareholders mix with $p^* = 1/2$. Then, the vote is more likely to be split if the blockholder has sold her shares. This is because the blockholder does not push the vote toward a certain decision and $p^* = 1/2$, which makes a split vote among the small shareholders likely. Therefore, conditional on a small shareholder being pivotal and because $\pi \approx 1/2$, it is likely that the blockholder has exited. Hence, the shareholder assigns a high posterior probability that the public signal is incorrect, and she wants to vote against the public signal.

By contrast, $p^* < 1/2$ makes a tie in the vote *more* likely when the public signal is correct because the blockholder votes in line with it, whereas small shareholders tend to vote against it. Conversely, it makes a tie in the vote *less* likely if the public signal is incorrect because the blockholder has sold her shares and the small shareholders tend to vote in the same direction. Overall, p^* has to be smaller than $1/2$ to balance updating from being pivotal and the inference from the public signal so that a small shareholder is indifferent between voting for and against s_{pub}, and mixing is a best response.

As the public signal becomes more informative, shareholders update more strongly from it, and they vote in line with it more often (i.e., p^* increases in π). By contrast, a larger blockholder (larger n) induces shareholders to update more strongly from being pivotal and less strongly from the public signal; this results in a lower p^* (see Figure 3).

Equilibrium EQ_R^E only exists if the precision of the public signal is sufficiently large ($\pi \geq \hat{\pi}(N, n)$). For small π, small shareholders are unlikely to vote in line with the public signal if $O = -n$. Thus, it is optimal for the blockholder to deviate from the conjectured equilibrium and exit if the public signal is correct; this is because the likelihood of the decision implied by the public signal being implemented is too small – even if the blockholder stays and votes. For shareholders to vote in accordance with the public signal sufficiently often, a relatively high signal precision is required. Thus, this equilibrium only exists if the public signal is sufficiently precise.

$\mathcal{W}_R^E$ increases in π. This is not immediate because for $p^*(O = -n) < 1/2$, small shareholders tend to vote against the public signal. Increasing π therefore makes it more likely that small shareholders vote for the incorrect decision if $O = -n$. However, a higher π also induces the blockholder to stay and vote more often, which increases firm value. We show that the effect of the blockholder's votes dominates as otherwise it would be optimal for the blockholder to exit when $s_{pub} = s_{priv}$.[23] Thus, for the values of π where EQ_R^E exists, $\mathcal{W}_R^E$ increases in π.

We now compare welfare in this third equilibrium EQ_R^E to that in the benchmark without public information.

Lemma 3. *$\mathcal{W}^B > \mathcal{W}_R^E$ if and only if $\pi < \overline{\pi}_R^E(N,n)$ where $\overline{\pi}_R^E(N,n)$ is the solution to*

$$\overline{\pi}_R^E(N,n) = \frac{2 \times \left[\sum_{k=\frac{N+1}{2}-n}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k)\right] - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)}{\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)} \tag{7}$$

Proof. See Appendix A.7. □

Lemma 3 shows that, similar to the second equilibrium, the third equilibrium with public information is also welfare dominated by the benchmark without public information, provided the signal precision is not too high.

Welfare. Given lemmas 1 - 3, we can rank welfare in the equilibria with public information relative to the benchmark. There may exist multiple equilibria. To stack the deck against our result that public information reduces welfare, we focus on the equilibrium that maximizes firm value.[24]

Proposition 5. *(Welfare) For every $N \geq 9$, there exist a $\underline{n}$ such that for all $n \geq \underline{n}$ there is an intermediate range of signal precision $\pi \in [\underline{\pi}(N,n), \min\{\overline{\pi}_{NR}^E(N,n), \overline{\pi}_R^E(N,n)\}] \subset (1/2, 1)$ for which public information reduces welfare (i.e., the benchmark exhibits higher welfare than any equilibrium with public information).*

Proof. See Appendix A.8. □

Proposition 5 establishes that public information can be detrimental for firm value and aggregate welfare. In particular, if $\underline{\pi}(N,n) < \min\{\overline{\pi}_{NR}^E(N,n), \overline{\pi}_R^E(N,n)\}$, there exists an interval

[23] Of course, p^* increases in π, but since $d\mathcal{W}_R^E/dp^* = 0$ by an envelope condition, this has no effect on welfare.

[24] The equilibrium that maximizes firm value must be one of EQ^{NE}, EQ_{NR}^E, or EQ_R^E, as we show in the proof of Proposition 5 in Appendix A.8.

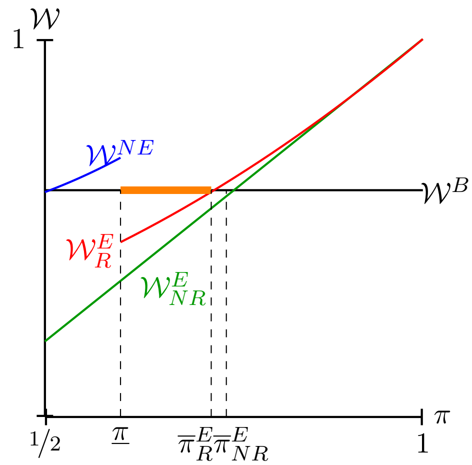

Figure 4: Global Welfare with Stock Market Feedback

of intermediate signal precisions π for which EQ^{NE} does not exist and the benchmark still welfare dominates $\mathcal{W}_{NR}^E$ and $\mathcal{W}_R^E$. If shareholders can coordinate on the firm-value-maximizing equilibrium, this is the only case when public information can reduce welfare since EQ^{NE} always welfare dominates the benchmark. If shareholders coordinate on a different equilibrium, public information can reduce welfare more often. Figure 4 illustrates firm value in all three equilibria with public information and the benchmark equilibrium without public information. For low π, EQ^{NE} exists and results in higher welfare than the benchmark. Increasing π increases welfare with public information for all $\pi < \underline{\pi}(N, n)$. However, at $\underline{\pi}(N, n)$, increasing π further induces the blockholder to sell her shares if the public signal is incorrect because small shareholders' votes are too correlated. Hence, EQ^{NE} ceases to exist, leading to a discontinuous drop in firm value. The non-existence of EQ^{NE} induces firm value to be *non-monotonic* in the informativeness of public information, yielding the following corollary to Proposition 5:

Corollary 1. *(Non-monotonicity) Increasing the precision of public information can (discontinuously) decrease welfare.*

The non-monotonicity arises whenever at $\underline{\pi}(N, n)$, $\mathcal{W}^{NE} > \max\{\mathcal{W}_R^E, \mathcal{W}_{NR}^E\}$, which is a weaker condition than the one required for Proposition 5 because it does not rely on the comparison to the benchmark.

Within EQ_{NR}^E and EQ_R^E, firm value increases in π and eventually dominates the benchmark again. Intuitively, as π approaches one, the public signal becomes perfect and, therefore, always yields the correct decision. Public information cannot be harmful in this case.

Whenever $n \geq \underline{n}$, there is an intermediate range of π where public information reduces welfare.[25] Intuitively, in the benchmark equilibrium, welfare is increasing in n because a larger blockholder is more likely to be pivotal and swing the vote toward the correct decision. By contrast, block size has a smaller effect on welfare in EQ^E_{NR} and EQ^E_R as L exits when the public signal is incorrect. In EQ^E_{NR}, the blockholder is never pivotal if she stays. In EQ^E_R, the blockholder is pivotal only if she stays and $O = -n$. This limits the impact of a larger stake n relative to the benchmark where L always stays and votes. Thus, a larger n increases welfare in the benchmark more strongly than in EQ^E_{NR} or EQ^E_R. Therefore, a large n implies that EQ^E_{NR} and EQ^E_R can only dominate the benchmark if small shareholders are very likely to vote correctly – that is, for very high values of π. We show in the appendix that, as a result, $\underline{\pi}(N,n) < \min\{\overline{\pi}^E_{NR}(N,n), \overline{\pi}^E_R(N,n)\}$ for sufficiently large n. Then, EQ^{NE} ceases to exist before either EQ^E_{NR} or EQ^E_R dominate the benchmark. Overall, our results demonstrate how public information can harm welfare by creating incentives for better informed shareholders to trade out of the firm rather than using their private information to vote.

5 Discussion of Assumptions and Extensions

5.1 Abstaining from Voting

In practice, a large part of votes is taken by delegated investors who must vote to fulfill their fiduciary duty. Otherwise they could be subject to lawsuits by dissatisfied investors.[26] As a result, the literature has often abstracted from the possibility of abstentions (e.g., Malenko & Malenko (2019)).[27] This subsection highlights that our results continue to hold and that the key mechanism identified in our paper continues to operate even if shareholders can abstain.

[25] Appendix C demonstrates in a numerical example that this welfare result can already arise for realistic and small block sizes of 5%.

[26] For example, Iliev & Lowry (2015) write that "[s]hareholder voting is mandatory across all mutual funds" (p.449).

[27] Similarly, Malenko & Malenko (2023) report: "The likely reason why institutions rarely abstain is to avoid being accused of violating fiduciary duties to their clients. For example, based on our calculations using the ISS Voting Analytics database, mutual funds abstain in less than 1% of proposals"(p.25).

Note that our results continue to hold in the presence of retail investors who can abstain as long as institutional investors (who cannot abstain) jointly hold more shares than the privately informed blockholder. Empirically, Dasgupta et al. (2021) show that "today only 38.3% of US corporate equity is directly owned by households. The remainder is indirectly held via different asset managers – commonly referred to as institutional investors" (p. 4).

Our results continue to hold even when *all* shareholders can abstain. To establish this result, Online Appendix E introduces a model in which shareholders can abstain and the blockholder is informed only with some probability. As before, all shareholders receive an informative public signal. In this setting, public information adds additional information that is not attainable in the benchmark, stacking the deck further against value-decreasing public information.

First, we show that even if we allow for abstentions, there is no equilibrium with public information in which the first-best firm value is attained. The first-best firm value would require that the blockholder determines the outcome of the vote if she is informed. If she is not informed, the vote ought to be based on the public signal. The reason first-best firm-value is not attainable is that, if the blockholder is uninformed, she has an incentive to exit rather than vote with the public signal. Small shareholders, who do not know whether the blockholder is privately informed, then will either crowd out the blockholder's votes inefficiently often, or they will not vote with the public signal often enough.

Second, we show that public information can reduce welfare relative to a benchmark without public information. Again, there exist multiple equilibria with public information. However, in any equilibrium, small shareholders do not abstain, or they only abstain if the order flow reveals that L stayed invested ($O = 0$). Small shareholders never abstain if $O = -n$. To see the intuition, suppose that, to the contrary, small shareholders were to abstain if $O = -n$. Then, after observing $O = -n$, a small shareholder is never pivotal if L stays and votes, but is pivotal if L exits (e.g., because L is uninformed). In this latter case, a small shareholder strictly prefers to vote as suggested by the public signal. Thus, after observing $O = -n$ and conditioning on being pivotal, it is *not* optimal for a small shareholder to abstain. Hence, in equilibrium, small shareholders still crowd out the blockholder's private information with positive probability.

5.2 Communication

As is standard in the literature on information aggregation via voting, we abstract from pre-vote communication. In our setting of shareholder voting, financial market regulation creates barriers to communication. For example, Malenko & Malenko (2019) argue that regulation on "forming a group" prevents shareholders from communicating due to the risk of triggering a mandatory tender offer or takeover defenses such as poison pills.[28] These concerns are particularly prevalent for large shareholders, the only entity that could improve firm value by communicating in our model.

Moreover, there are several reasons why communication would not be a panacea in our model, even if it were possible. If communication reaches only some shareholders but not others, then communication acts like an increase in the block size n and can reduce welfare. Intuitively, as long as the number of shares held by informed shareholders is a minority (either held by the blockholder or by small shareholders who received the blockholder's communication), there exists an incentive to exit when the public signal is incorrect.[29]

Lastly, even if communication were to reach all shareholders with certainty, this need not resolve the information-aggregation problem. The reason is that communication by blockholders would create incentives for manipulative short-selling in the spirit of Goldstein & Guembel (2008). If an entity can simply communicate the true state, short sellers have an incentive to build a short position and then communicate the false state to profit on their short position.

5.3 Share Purchases

In principle, one can imagine that the large shareholder may have an incentive to purchase additional shares to swing the vote. In Online Appendix F, we examine the effect of share purchases on our results. We assume that the blockholder does not buy enough shares to take control (i.e., have a majority stake). This can be motivated by the in-practice prevalent wealth constraints (Winton 1993) or by risk-aversion (Admati et al. 1994) and captures the fact that

[28] A similar argument is made by Puchniak & Varottil (2023).

[29] Alternatively, if communication reaches all small shareholders with some probability and no small shareholders with the complementary probability, as in Malenko et al. (2023), then shareholders can in equilibrium achieve firm value of one if communication succeeds; if it does not succeed, then they play the game analyzed in our model, in which exit incentives arise if the public signal is incorrect. Thus, our results continue to hold.

even the most activist blockholders "seldom seek control" (Brav et al. 2008, p.1729).[30]

We show that if the public signal is incorrect the blockholder prefers selling to buying. The reason is that conditional on the public signal being incorrect, shares are overvalued such that selling is more attractive than buying. Hence, our equilibria remain equilibria in a setting in which the blockholder can also buy shares.

One may wonder about the welfare effects of share purchases in general. We show that while share purchases tend to improve welfare because the blockholder can obtain larger voting power, our welfare result that public information may harm welfare remains true.

If stock purchases are possible, the blockholder has an incentive to acquire additional shares when $s_{pub} = s_{priv}$ because shares tend to be undervalued if the public signal is correct. These share purchases directly improve welfare because there are more correct votes cast by the blockholder. However, there are two countervailing, indirect effects. First, public information discourages stock purchases (and encourages exit, as in the main model) by the blockholder when $s_{pub} \neq s_{priv}$ relative to the benchmark without public information. Second, there is a pivotality effect. If the blockholder purchases shares when the public signal is correct she obtains more voting power. Hence, small shareholders are even less likely to be pivotal if the public signal is correct, inducing them to vote in line with the public signal less often. We show that, due to these indirect effects, public information may still reduce welfare.

5.4 Trading After Voting

While we restrict the analysis in the main model to trading before the vote, in practice, shareholders may also trade after the vote. If the true state of the world is not uncovered directly after the vote, the blockholder may still use her private information to reap trading gains after the vote. The blockholder could then vote based on her private information and trade afterward if she observes that the incorrect decision was implemented. One may thus conjecture that the blockholder would prefer to trade only after the vote. However, this need not be true because the blockholder's trading profits are larger before the vote.

Intuitively, pre-vote trading may generate value for the remaining shareholders through stock

[30] Regarding less activist funds, Edmans & Holderness (2017) note that "[d]iversification requirements hinder mutual funds from acquiring the large positions needed to exercise control" (p. 598).

market feedback. The blockholder reaps some of this benefit through the share price she obtains when exiting. In particular, even if her exit is uncovered, the share price is strictly positive due to the feedback it triggers. Post-vote trading has no such beneficial effect as the decision has already been taken. Thus, if exit is uncovered, the share price must be zero to reflect that the incorrect action was taken.

5.5 Non-strategic Voting

Throughout the paper, we assume that shareholders vote strategically (i.e., condition their vote on being pivotal). While this is a standard assumption in the shareholder-voting literature, the adverse affect of public information on firm value does not rely on it. If instead shareholders simply voted sincerely, EQ^E_{NR} would be the only equilibrium when there is public information. In the absence of public information, the benchmark equilibrium would still constitute an equilibrium. Thus, public information will reduce welfare for all signal precisions $\pi < \overline{\pi}^E_{NR}$ (see Lemma 2) – a strict superset of signal precisions that reduce welfare in our model with strategic voting. Hence, by allowing shareholders to respond strategically to public information, public information becomes more valuable than in a setting with non-strategic shareholders. With non-strategic voting, there is no non-monotonicity because the lowest signal precision already induces the exit of well-informed shareholders.

6 Empirical Implications

The key mechanism of our paper is consistent with the existing empirical evidence. Our welfare results provide avenues for future empirical research. While these empirical implications apply to any form of public information, we phrase them in terms of proxy advice - the most prominent form in the empirical literature.

One component of the mechanism in our model is that proxy advice is informative about the vote outcome over and above the information it provides on the quality of the proposal that is voted on. Alexander et al. (2010) provide direct evidence of this. More generally, the fact that proxy advice is correlated with shareholders' votes has been demonstrated in different settings and for different investors, e.g., by Malenko & Shen (2016) for say-on-pay votes, by Iliev & Lowry (2015) for mutual funds and by Brav et al. (2022) for retail investors.[31]

The mechanism of our model implies that public information such as proxy advice can lead to trading *before* a shareholder meeting. In the model, pre-vote trading is driven by privately informed shareholders who disagree with proxy advice and anticipate that the vote outcome will be what the proxy advisor recommended. This prediction is consistent with Li et al. (2022) who document high abnormal trading volume in the days prior to shareholder meetings. While Li et al. (2022) hypothesize that these trades could be driven by information leakage (i.e., by shareholders learning how others have voted), our model highlights that pre-vote trading can be spurred by the predictive effect of proxy advice even when there is no information leakage.[32]

More precisely, our model highlights a *motive* for pre-vote trading: disagreement with the proxy advisor or, more generally, with public information. Iliev & Lowry (2015) document that disagreement with ISS leads funds to reduce their portfolio holding, but their quarterly data prevents them from investigating whether sales occur before or after the vote. Li et al. (2022) find that post-vote trading is not driven by disagreement with ISS. This suggests that trading due to disagreement with ISS takes place before the vote, consistent with our model. Still, our paper indicates that zooming in on the motives for pre-vote trading may be a fruitful avenue for future empirical research.

A further testable, empirical prediction of our model is that the correlation of votes with proxy advice should be particularly large when there are few sale orders before the vote. Testing this would be a first step toward disentangling the direct effect of proxy advice on shareholder voting and the indirect effect from proxy advice-induced trading.

In terms of welfare results, two novel predictions of our model are that firm value can decrease due to the presence of public information and that more precise public information may harm firm value. Clearly, causal inference requires exogenous variation in the presence or the precision of public information. Potential sources of such exogenous variation are changes in mandatory disclosure requirements or in the regulation of proxy advice.

[31] Iliev & Lowry (2015) report that "Over 25% of funds rely almost entirely on Institutional Shareholder Services (ISS) recommendations" (p.446).

[32] More generally, there is ample empirical evidence that governance issues are material enough to trigger trading (e.g., Iliev & Lowry (2015), Gantchev & Giannetti (2021), Li et al. (2022)).

Our model also offers guidance on the interpretation of associations observed in the data. Interestingly, the fact that shareholders do not follow proxy advice does not necessarily mean that proxy advice did not change their voting decision for the better. Without proxy advice, the stock market may have provided less information to shareholders because better informed shareholders would have had lower trading incentives. This cautions against measuring the influence of a proxy advisor by the number of shareholders who follow its recommendations.

7 Conclusion

We study the effect of public information on information aggregation in the corporate setting where shareholders can trade before the vote. Public information can impair information aggregation by voting because it induces informed shareholders with differential information to trade out of the firm rather than to vote. Conversely, by inducing informed shareholders to trade more, public information tends to improve the information content of the firm's stock price. The additional information from the stock market feeds back into the voting decision of other shareholders. Despite this positive feedback effect, we find that unbiased public information can impair overall information aggregation and, thus, reduce firm value.

In our model, trading based on public information makes the shareholder base more homogeneous with respect to their information. This bears some high-level resemblance to Levit et al. (2021, 2024) where trading renders the shareholder base more homogeneous regarding preferences. There is a crucial difference, however. While preference homogeneity tends to increase shareholder welfare, information homogeneity tends to reduce shareholder welfare.

Our model highlights potential pitfalls of public information provision before shareholder meetings (e.g., via firm disclosure or proxy advice). It cautions against allowing institutional investors to rely purely on public proxy recommendations to satisfy their fiduciary duty to their clients. The model delivers several new, testable empirical implications on the relation of public information, trading patterns before shareholder meetings, and firm value.

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A Proofs

A.1 Proof of Proposition 1

Proof. Conjecture EQ^B in Proposition 1 is an equilibrium. Then, L always votes for the correct decision and therefore $d = \omega$ if and only if at least $\frac{N+1}{2} - n$ small shareholders out of $N - n$ vote for the correct decision. Thus, ex-ante firm value is $\mathcal{W}^B = \mathbb{E}[u(d,\omega)] = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$. Consider L's exit decision. L's expected payoff from staying is the ex-ante firm value $\mathbb{E}[u(d,\omega)]$. Her per share exit payoff is $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n)$. $O = -n$ arises in equilibrium only if noise traders sell. Thus, the market maker does not update and $P(O = -n) = \mathbb{E}[u(d,\omega)]$. By contrast, $O = -2n$ reveals that L has traded which never arises on the equilibrium path. The equilibrium can be supported, for instance, by off-path beliefs according to which small shareholders do not update their beliefs about ω from $O = -2n$ so that it is still optimal to mix with $p(-2n) = 1/2$. This off-path belief arises as an on-path belief in a model where the blockholder is informed only with some probability. Then, if L is uninformed, she always sells with positive probability (see Online Appendix D for the analysis) and, thus, small shareholders do not update from $O = -2n$. Hence, $P(O = -2n) = 1/2$. L's per share exit payoff is $\frac{1}{2}\mathbb{E}[u(d,\omega)] + \frac{1}{2}\frac{1}{2} < \mathbb{E}[u(d,\omega)]$. Thus, a deviation to exit is not profitable. After observing $O \in \{0, -n\}$ and conditioning on being pivotal, a small shareholder is indifferent and thus mixing is a best response. Thus, the equilibrium exists.

□

A.2 Proof of Proposition 2

Proof. Suppose shareholders vote according to s_{pub} with probability p^*. A shareholder is only willing to randomize if, conditional on being pivotal, she is indifferent whether to vote for r or q. This implies that both states must be equally, i.e., $\mathbb{P}[\omega = R | s_{pub},\ piv] = \mathbb{P}[\omega = Q | s_{pub},\ piv] = \frac{1}{2}$.

Without loss of generality, let $s_{pub} = R$. Then, by Bayes Theorem, it follows that

$$\begin{aligned}
&\mathbb{P}[\omega = R | s_{pub},\ piv] \\
=&\frac{\mathbb{P}[\omega = R|s_{pub}]\ \mathbb{P}[piv|s_{pub}, \omega = R]}{\mathbb{P}[\omega = R|s_{pub}]\ \mathbb{P}[piv|s_{pub}, \omega = R] + (1 - \mathbb{P}[\omega = R|s_{pub}])\ \mathbb{P}[piv|s_{pub}, \omega = Q]} \\
=&\frac{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}}}{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}} + (1-\pi)\ \binom{N-n-1}{\frac{N-1}{2}}\ {p^*}^{\frac{N-1}{2}}\ (1-p^*)^{\frac{N-1}{2}-n}}. \qquad (8)
\end{aligned}$$

Since indifference requires that $\mathbb{P}[\omega = R|s_{pub},\ piv] = 1/2$, we have

$$\begin{aligned}
&\frac{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}}}{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}} + (1-\pi)\ \binom{N-n-1}{\frac{N-1}{2}}\ {p^*}^{\frac{N-1}{2}}\ (1-p^*)^{\frac{N-1}{2}-n}} = \frac{1}{2} \\
\Longleftrightarrow &\binom{N-n-1}{\frac{N-1}{2}-n} = \frac{1-\pi}{\pi} \binom{N-n-1}{\frac{N-1}{2}} \left(\frac{p^*}{1-p^*}\right)^n \\
\Longleftrightarrow &\frac{\pi}{1-\pi} = \left(\frac{p^*}{1-p^*}\right)^n \\
\Longleftrightarrow &p^* = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)^{\frac{1}{n}}}. \qquad (9)
\end{aligned}$$

Share prices are $P(0) = P(-n) = \mathbb{E}[u(d, \omega)]$ because on the conjectured equilibrium path the blockholder stays and $O = -n$ is attributed to noise traders selling. $O = -2n$ reveals that L sold her stake and we assign off-path beliefs $\mathbb{P}[s_{pub} = \omega|O = -2n] = \pi$. The off-path belief can be rationalized by assuming that L's exit occurs due to a tremble orthogonal to her information. Moreover, it is the on-path belief in a model where the blockholder is informed only with some probability (see Online Appendix D for the analysis). Given these off-path beliefs, following s_{pub} is a best response for small shareholders and leads to $P(-2n) = \pi$. L does then indeed not want to exit after observing $s_{priv} \neq s_{pub}$ if

$$\begin{aligned}
&\mathbb{E}[u(d,\omega)|s_{priv} \neq s_{pub}, L\, stays] \geq \frac{1}{2}\big(P(-n)+P(-2n)\big) \\
\iff &\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \frac{1}{2}\Big(\pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi)\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)\Big) + \frac{1}{2}\pi \\
\iff &\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \frac{\pi}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1}{2}\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \\
&- \frac{\pi}{2}\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) + \frac{\pi}{2}\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) + \frac{\pi}{2}\sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) \\
\iff &\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq 2\pi \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) + \pi \sum_{k=\frac{N+1}{2}-n}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)
\end{aligned} \tag{10}$$

Moreover, L does indeed not want to exit after observing $s_{priv} = s_{pub}$ if

$$\mathbb{E}[u(d,\omega)|s_{priv} = s_{pub}, L\, stays] \geq \frac{1}{2}\big(P(-n)+P(-2n)\big) \tag{11}$$

where $\mathbb{E}[u(d,\omega)|s_{priv} = s_{pub}, L\, stays] = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k)$. Since $p^* \geq \frac{1}{2}$, it follows that $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) \geq \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)$ and thus the IC constraint after observing $s_{priv} = s_{pub}$ holds provided that the IC constraint after observing $s_{priv} \neq s_{pub}$ holds. Evaluating (10) at $\pi = 1/2$, which implies $p(\pi = 1/2) = 1/2$, yields

$$\begin{aligned}
&\sum_{k=0}^{\frac{N-1}{2}} \binom{N-n}{k} (\frac{1}{2})^{N-n} \geq \sum_{k=\frac{N+1}{2}}^{N-n} \binom{N-n}{k} (\frac{1}{2})^{N-n} + \frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{\frac{N-1}{2}} \binom{N-n}{k} (\frac{1}{2})^{N-n} \\
\iff &\sum_{k=0}^{\frac{N-1}{2}} \binom{N-n}{k} \geq \sum_{k=\frac{N+1}{2}}^{N-n} \binom{N-n}{k} + \frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{\frac{N-1}{2}} \binom{N-n}{k} \\
\iff &\binom{N-n}{0} + \binom{N-n}{1} + \cdots + \binom{N-n}{\frac{N-1}{2}} \\
&\geq \binom{N-n}{\frac{N+1}{2}} + \binom{N-n}{\frac{N+3}{2}} + \cdots + \binom{N-n}{N-n} + \frac{1}{2}\Big(\binom{N-n}{\frac{N+1}{2}-n} + \cdots + \binom{N-n}{\frac{N-1}{2}}\Big)
\end{aligned} \tag{12}$$

Since $\binom{N-n}{k} = \binom{N-n}{N-n-k}$, $\binom{N-n}{\frac{N+1}{2}} = \binom{N-n}{N-n-(\frac{N+1}{2})} = \binom{N-n}{\frac{N-1}{2}-n}$, $\binom{N-n}{\frac{N-1}{2}-(n-1)} = \binom{N-n}{\frac{N+1}{2}-n}$, and $\binom{N-n}{\frac{N-1}{2}} = \binom{N-n}{\frac{N+1}{2}-n}$, IC (12) becomes

$$\begin{aligned} &\binom{N-n}{\frac{N+1}{2}-n} + \binom{N-n}{\frac{N+1}{2}-n+1} + \cdots + \binom{N-n}{\frac{N-1}{2}} \geq \frac{1}{2}\Big(\binom{N-n}{\frac{N+1}{2}-n} + \cdots + \binom{N-n}{\frac{N-1}{2}}\Big) \\ \iff &\binom{N-n}{\frac{N+1}{2}-n} + \binom{N-n}{\frac{N+1}{2}-n+1} + \cdots + \binom{N-n}{\frac{N-1}{2}} \geq 0. \end{aligned} \tag{13}$$

Hence, constraint (10) is slack at $\pi = \frac{1}{2}$ and the equilibrium exists. By continuity, the equilibrium also exists for π sufficiently close to $\frac{1}{2}$. By contrast, the equilibrium does not exist for π sufficiently close to 1 because $p(\pi = 1) = 1$ and therefore (10) cannot hold at $\pi = 1$ since the LHS becomes zero and the RHS becomes 2. By continuity, it then follows that there exists at least one cutoff $\underline{\pi} \in (0.5, 1)$ such that (10) holds with equality. We now establish that there exists exactly one such cutoff $\underline{\pi} \in (0.5, 1)$. First, note that for $\pi \in [0.5, 1]$, $p^* \in [0.5, 1]$, $\frac{dp^*}{d\pi} > 0$ and $\sum_{k=0}^{N-n} \binom{N-n}{k} p^k (1-p)^{N-n-k} = 1 \forall p \in [0.5, 1]$. Then, we can rewrite equation (10) as

$$\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + \pi \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) \tag{14}$$

By the FOSD property (which we establish in Proposition 6), the LHS must be decreasing in p^* and both sums on the RHS must be increasing in p^*. Moreover, p^* is increasing in π. Thus, an increase in π decreases the LHS and increases all terms on the RHS. Thus, there is a single crossing and there exists a unique cutoff $\underline{\pi} \in (1/2, 1)$ such that the equilibrium exists if and only if $\pi \leq \underline{\pi}$. □

A.3 Proof of Lemma 1

Proof. We need to establish that

$$\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) \leq \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \tag{15}$$

First note that at $\pi = 1/2$, $p^*(\pi = 1/2) = 1/2$ and that $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k)$. Thus, at $\pi = 1/2$,

$$\pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$$

and (15) holds with equality. Note further that the LHS of (15) is independent of π whereas the RHS is strictly increasing in π. To see this, consider any $\sum_{k=0}^{x} \mathbb{P}(p, N-n, k)$ for some $x < N-n$.

$$\begin{aligned}\sum_{k=0}^{x} \mathbb{P}(p, N-n, k) = &\binom{N-n}{0}(1-p)^{N-n} + \binom{N-n}{1} p(1-p)^{N-n-1} + \\ &\binom{N-n}{2} p^2(1-p)^{N-n-2} + \dots + \binom{N-n}{x} p^x(1-p)^{N-n-x}\end{aligned}$$

Then,

$$\begin{aligned}\frac{d\sum_{k=0}^{x} \mathbb{P}(p, N-n, k)}{dp} = &\binom{N-n}{0}(N-n)(1-p)^{N-n-1}(-1) \\ &+ \binom{N-n}{1}(1-p)^{N-n-1} + \binom{N-n}{1} p(N-n-1)(1-p)^{N-n-2}(-1) \\ &+ \binom{N-n}{2} 2p(1-p)^{N-n-2} + \binom{N-n}{2} p^2(N-n-2)(1-p)^{N-n-3}(-1) \\ &+ \dots \\ &+ \binom{N-n}{x} xp^{x-1}(1-p)^{N-n-x} + \binom{N-n}{x} p^x(N-n-x)(1-p)^{N-n-x-1}(-1)\end{aligned}$$

Since $\binom{N-n}{0}(N-n) = \binom{N-n}{1}$, $\binom{N-n}{1}(N-n-1) = 2\binom{N-n}{2}$, the first term of each line exactly cancels out the last term of the line above. Thus, the derivative simplifies to

$$\frac{d\sum_{k=0}^{x} \mathbb{P}(p, N-n, k)}{dp} = (-1)\binom{N-n}{x}(N-n-x)p^x(1-p)^{N-n-x-1}$$

Using that $\binom{N-n}{x}(N-n-x) = \frac{(N-n)!}{(N-n-x)!x!}(N-n-x) = (N-n)\frac{(N-n-1)!}{(N-n-x-1)!x!} = (N-n)\binom{N-n-1}{x}$, we can rewrite this as

$$\frac{d\sum_{k=0}^{x}\mathbb{P}(p,N-n,k)}{dp} = (-1)(N-n)\binom{N-n-1}{x}p^x(1-p)^{N-n-1-x}$$

To establish that $\frac{dRHS}{d\pi} > 0$, we use that $\frac{dRHS}{d\pi} = \frac{\partial RHS}{\partial\pi} + \frac{dRHS}{dp^*}\frac{dp^*}{d\pi}$ and first consider $\frac{dRHS}{dp^*}$. Notice that $\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k) = 1 - \sum_{k=0}^{\frac{N-1}{2}-n}\mathbb{P}(p^*,N-n,k)$ and thus that $\frac{d\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)}{dp^*} = -\frac{d\sum_{k=0}^{\frac{N-1}{2}-n}\mathbb{P}(p^*,N-n,k)}{dp^*}$. Thus,

$$\begin{aligned}
\frac{dRHS}{dp^*} &= \frac{\pi}{2}\frac{d\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)}{dp^*} + \frac{(1-\pi)}{2}\frac{d\sum_{k=0}^{\frac{N-1}{2}}\mathbb{P}(p^*,N-n,k)}{dp^*}\\
&= \frac{\pi}{2}(N-n)\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} - \frac{(1-\pi)}{2}(N-n)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n}\\
&= \frac{N-n}{2}\Big[\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} - (1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n}\Big]
\end{aligned}$$

Thus, $\frac{dRHS}{dp^*} = 0$ if and only if

$$\begin{aligned}
&\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} - (1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} = 0\\
&\iff \frac{\pi}{1-\pi} = \left(\frac{p^*}{1-p^*}\right)^n\\
&\iff p^* = \frac{1}{1+\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{n}}} \qquad (16)
\end{aligned}$$

and thus holds true by Proposition 2. Thus, we have established that $\frac{dRHS}{dp^*} = 0$. Further, $\frac{\partial RHS}{\partial\pi} > 0$ because $\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p,N-n,k) > \sum_{k=0}^{\frac{N-1}{2}}\mathbb{P}(p,N-n,k)$ for $p > \frac{1}{2}$. Therefore the RHS of (15) is increasing in π since $p^* > \frac{1}{2}$ for all $\pi > \frac{1}{2}$. Thus, (15) holds with equality at $\pi = 1/2$ and as inequality for any $\pi \in (1/2, 1]$. □

A.4 Proof of Proposition 3

Proof. Conjecture that EQ^E_{NR} in Proposition 3 constitutes an equilibrium. Then, $P(0) = 1$, $P(-n) = \pi$, $P(-2n) = 1$. $P(0) = 1$ as $O = 0$ reveals to the market maker that L has not sold her shares which means that the public signal is correct. Moreover, $P(-n) = \pi$ as the market maker assigns probability π to L not having sold her shares which means that the public signal is correct and that the correct decision will be taken. Last $P(-2n) = 1$ as the market maker knows that L has sold, which means that the public information is incorrect and that shareholders will take the correct decision. Moreover,

$$\mathbb{E}[u(d,\omega)] = \mathbb{P}(s_{pub} = \omega) + \mathbb{P}(s_{pub} \neq \omega) \times \mathbb{P}(O = -2n \mid exit) = \frac{1+\pi}{2}, \tag{17}$$

which used that $\mathbb{P}(O = -2n \mid exit) = 1/2$. To establish existence, first consider L's trading decisions. If $s_{priv} = s_{pub}$, L's per-share payoff from staying in the firm is 1 which exceeds her per-share payoff from selling of $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) = \frac{1+\pi}{2}$. If $s_{priv} \neq s_{pub}$, L's payoff from staying is zero while the expected payoff from exit is, as before, $\frac{1+\pi}{2}$. Hence, exit is optimal.

Consider L's voting decision. If $s_{priv} = s_{pub}$, L is never pivotal. If she were pivotal, she would prefer to vote according to her private signal. Small shareholders are never pivotal either. If they were pivotal it would be a best response to vote in line with the public signal when $O \in \{-n, 0\}$ since the public signal is informative ($\pi \geq \frac{1}{2}$). If $O = -2n$, small shareholders are never pivotal. If they were pivotal, it would be a best response to vote against s_{pub} since at $O = -2n$ they know that s_{pub} is incorrect. Thus, EQ^E_{NR} exists. □

A.5 Proof of Lemma 2

Proof. $\overline{\pi}^E_{NR}(N,n)$ is the π that induces the same welfare in EQ^{NE} and the benchmark. Thus,

$$\frac{1+\overline{\pi}^E_{NR}(N,n)}{2} = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$$

$$\iff \overline{\pi}^E_{NR}(N,n) = 2\Big[\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)\Big] - 1. \tag{18}$$

The property that $\overline{\pi}^E_{NR}(N,n) < \overline{\pi}^E_{NR}(N,n+1)$ is equivalent to

$$\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) < \sum_{k=\frac{N+1}{2}-(n+1)}^{N-n-1} \mathbb{P}(\frac{1}{2}, N-n-1, k)$$
$$\iff \binom{N-n}{\frac{N+1}{2}-n} + \binom{N-n}{\frac{N+3}{2}-n} + \cdots + \binom{N-n}{N-n} < 2\left[\binom{N-n-1}{\frac{N-1}{2}-n} + \binom{N-n-1}{\frac{N+1}{2}-n} + \cdots + \binom{N-n-1}{N-n-1}\right]$$

using that by Pascal's triangle $\binom{N-n-1}{\frac{N-1}{2}-n} + \binom{N-n-1}{\frac{N+1}{2}-n} = \binom{N-n}{\frac{N+1}{2}-n}$ etc. we have equally

$$\binom{N-n}{\frac{N+1}{2}-n} + \cdots + \binom{N-n}{N-n} < \binom{N-n-1}{\frac{N-1}{2}-n} + \binom{N-n-1}{N-n-1} + \binom{N-n}{\frac{N+1}{2}-n} + \cdots + \binom{N-n}{N-n}$$
$$\iff 0 < \binom{N-n-1}{\frac{N-1}{2}-n} + \binom{N-n-1}{N-n-1}$$

which holds. This concludes the proof that $\overline{\pi}^E_{NR}(N,n) < \overline{\pi}^E_{NR}(N,n+1)$.

Because $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) < 1$ by $n < \frac{N+1}{2}$, $\overline{\pi}^E_{NR}(N,n) < 1$. We now establish that there is a $\hat{n} \leq \frac{N-1}{2}$ so that for all $n \geq \hat{n}$, $\overline{\pi}^E_{NR}(N,n) > 1/2$. $\overline{\pi}^E_{NR}(N,n) > 1/2$ is equivalent to $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) > \frac{3}{4}$, or

$$\sum_{k=0}^{\frac{N-1}{2}-n} \binom{N-n}{k} (\frac{1}{2})^{N-n} < \frac{1}{4}. \tag{19}$$

For $\hat{n} = \frac{N-2}{3}$, the inequality is $\frac{1}{4} > \sum_{k=0}^{\frac{N+1}{6}} \mathbb{P}(\frac{1}{2}, \frac{2(N+1)}{3}, k)$. Define $x = \frac{2(N+1)}{3}$ and write the inequality as

$$\frac{1}{4} > \sum_{k=0}^{\frac{1}{4}x} \mathbb{P}(\frac{1}{2}, x, k). \tag{20}$$

This inequality holds since the summands in $\sum_{k=0}^{\frac{1}{4}x} \mathbb{P}(\frac{1}{2}, x, k)$ are increasing up to $x/2$ if x is even, or $x-1/2$ if x is odd. Put differently, if the probabilities in the sum were all equally likely, the inequality would hold as equality. But as the binomial distribution with success probability $1/2$ places more weight on k close to $x/2$ rather than on the extremes close to $k=0$, the inequality holds strictly. Combined with the result that $\overline{\pi}^E_{NR}(N,n)$ is increasing in n, we have that $\overline{\pi}^E_{NR}(N,n) > \frac{1}{2}$ for all $n \geq \hat{n} = \frac{N-2}{3}$.

□

A.6 Proof of Proposition 4

Proof. Conjecture EQ^E_R in Proposition 4 is an equilibrium. Then, $\mathbb{P}[s_{pub} = \omega | O = 0] = 1$, $\mathbb{P}[s_{pub} = \omega | O = -n] = \pi$, and $\mathbb{P}[s_{pub} = \omega | O = -2n] = 0$. To establish existence, consider a small shareholder's voting strategy. If $O = 0$ (respectively $O = -2n$), a small shareholder knows that $s_{pub} = \omega$ (respectively $s_{pub} \neq \omega$) and thus voting for s_{pub} (respectively against) is a best response, also if a shareholder were pivotal. If $O = -n$ and conditional on being pivotal a small shareholder is indifferent which way to vote if

$$\mathbb{P}[\omega = R | s_{pub} = R,\ piv, O = -n] = \mathbb{P}[\omega = Q | s_{pub} = R,\ piv, O = -n] = \frac{1}{2}, \tag{21}$$

Without loss of generality, let $s_{pub} = R$. Then, by Bayes Theorem

$$\begin{aligned}
&\mathbb{P}[\omega = R | s_{pub},\ piv, O = -n] \\
=&\frac{\pi\ \mathbb{P}[piv | s_{pub}, O = -n, \omega = R]}{\pi\ \mathbb{P}[piv | s_{pub}, O = -n, \omega = R] + (1-\pi)\ \mathbb{P}[piv | s_{pub}, \omega = Q, O = -n]} \\
=&\frac{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}}}{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}} + (1-\pi)\ \binom{N-n-1}{\frac{N-1-n}{2}}\ {p^*}^{\frac{N-1-n}{2}}\ (1-p^*)^{\frac{N-1-n}{2}}}.
\end{aligned} \tag{22}$$

Setting this equal to 1/2 yields

$$\frac{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}}}{\pi\ \binom{N-n-1}{\frac{N-1}{2}-n}\ {p^*}^{\frac{N-1}{2}-n}\ (1-p^*)^{\frac{N-1}{2}} + (1-\pi)\ \binom{N-n-1}{\frac{N-1-n}{2}}\ {p^*}^{\frac{N-1-n}{2}}\ (1-p^*)^{\frac{N-1-n}{2}}} = \frac{1}{2}$$

$$\iff \frac{\pi}{1-\pi}\ \frac{\binom{N-n-1}{\frac{N-1}{2}-n}}{\binom{N-n-1}{\frac{N-n-1}{2}}} = \left(\frac{p^*}{1-p^*}\right)^{\frac{n}{2}} \tag{23}$$

$$\iff p^*(O = -n) = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\ \frac{\binom{N-n-1}{\frac{N-1-n}{2}}}{\binom{N-n-1}{\frac{N-1}{2}-n}}\right)^{\frac{2}{n}}} \tag{24}$$

Henceforth, we write $p^* := p^*(O = -n)$ for brevity. Note that $p^*(\pi = 0.5) < 0.5$ as $\frac{\binom{N-n-1}{\frac{N-1-n}{2}}}{\binom{N-n-1}{\frac{N-1}{2}-n}} > 0$. Moreover, for a given duple (N, n), p^* strictly increases in π with $p(\pi = 1) = 1$. Next, consider L's trading strategy. In the conjectured equilibrium, share prices are $P(0) = 1$, $P(-2n) = 1$, and

$$P(-n) = \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k). \tag{25}$$

Given a small shareholder's strategy, L does not want to exit when $s_{priv} = s_{pub}$ if

$$\frac{1}{2}1 + \frac{1}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) \geq \frac{1}{2}P(-n) + \frac{1}{2}P(-2n)$$
$$\iff \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) \geq \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k). \tag{26}$$

The LHS of (26) is monotonically increasing in p^* and the RHS is monotonically decreasing in p^* (by the FOSD arguments in Proposition 6). For any $p \geq 1/2$, (26) holds, but for sufficiently low p, (26) is violated. Thus, there exists a unique level of p, called $\hat{p}(N, n)$, for which (26) holds with equality. Then, (26) holds if and only if $p^* \geq \hat{p}(N, n)$.

Since $dp^*/d\pi > 0$, the LHS is increasing in π and the RHS decreasing in π. Thus, there exists at most one level of π where $p^*(\pi) = \hat{p}(N, n)$ and (26) holds with equality. We label this level of π as $\hat{\pi}(N, n)$. Then, (26) holds if and only if $\pi \geq \hat{\pi}(N, n)$. Since $\lim_{\pi \to 1} p^* = 1$ for all (N, n), it must be that $\hat{\pi}(N, n) < 1$. Since $p^*(\pi = 1/2) > 0$ it may be that for some (N, n), $\hat{\pi}(N, n) < 1/2$.

If $s_{priv} \neq s_{pub}$, expected firm value if L stays is

$$\frac{1}{2}0 + \frac{1}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) < \frac{1}{2}P(-n) + \frac{1}{2}P(-2n). \tag{27}$$

Hence, exit is the unique best response. Thus, EQ_R^E exists for all $\pi \geq \hat{\pi}(N, n)$. Ex-ante firm value is

$$\begin{aligned}\mathcal{W}_R^E =&\pi\Big(\frac{1}{2}1+\frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)\Big)+(1-\pi)\Big(\frac{1}{2}\sum_{k=0}^{\frac{N-n-1}{2}}\mathbb{P}(p^*,N-n,k)+\frac{1}{2}1\Big)\\ =&\frac{1}{2}+\frac{1}{2}\Big(\pi\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)+(1-\pi)\sum_{k=0}^{\frac{N-n-1}{2}}\mathbb{P}(p^*,N-n,k)\Big) \qquad (28)\end{aligned}$$

To establish that $\frac{d\mathcal{W}_R^E}{d\pi}>0$, recall from the proof of Proposition 1 that

$$\frac{d\sum_{k=0}^{x}\mathbb{P}(p,N-n,k)}{dp}=(-1)(N-n)\binom{N-n-1}{x}p^x(1-p)^{N-n-1-x}.$$

To establish that $\frac{d\mathcal{W}_R^E}{d\pi}>0$, we use that $\frac{d\mathcal{W}_R^E}{d\pi}=\frac{\partial\mathcal{W}_R^E}{\partial\pi}+\frac{d\mathcal{W}_R^E}{dp^*}\frac{dp^*}{d\pi}$ and first consider $\frac{d\mathcal{W}_R^E}{dp^*}$. Notice that $\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)=1-\sum_{k=0}^{\frac{N-1}{2}-n}\mathbb{P}(p^*,N-n,k)$ and thus that $\frac{d\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)}{dp^*}=-\frac{d\sum_{k=0}^{\frac{N-1}{2}-n}\mathbb{P}(p^*,N-n,k)}{dp^*}$. Thus,

$$\begin{aligned}\frac{d\mathcal{W}_R^E}{dp^*}&=\frac{\pi}{2}\frac{d\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)}{dp^*}+\frac{(1-\pi)}{2}\frac{d\sum_{k=0}^{\frac{N-n-1}{2}}\mathbb{P}(p^*,N-n,k)}{dp^*}\\&=\frac{N-n}{2}\Big[\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}}-(1-\pi)\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}}\Big]\end{aligned}$$

Thus, $\frac{d\mathcal{W}_R^E}{dp^*}=0$ if and only if

$$\begin{aligned}&\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}}=(1-\pi)\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}}\\ \iff&\frac{\pi}{1-\pi}\frac{\binom{N-n-1}{\frac{N-1}{2}-n}}{\binom{N-n-1}{\frac{N-n-1}{2}}}=\Big(\frac{p^*}{1-p^*}\Big)^{\frac{n}{2}} \qquad (29)\end{aligned}$$

which holds since (29) is identical to (23) which defines the mixing probability p^*. Thus, we have established that $\frac{d\mathcal{W}_R^E}{dp^*}=0$. Therefore, $\frac{d\mathcal{W}_R^E}{d\pi}=\frac{\partial\mathcal{W}_R^E}{\partial\pi}+\frac{d\mathcal{W}_R^E}{dp^*}\frac{dp^*}{d\pi}$ simplifies to $\frac{d\mathcal{W}_R^E}{d\pi}=\frac{\partial\mathcal{W}_R^E}{\partial\pi}$.

$$\frac{\partial \mathcal{W}_R^E}{\partial \pi} = \frac{1}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) - \frac{1}{2} \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$$

Thus, $\frac{\partial \mathcal{W}_R^E}{\partial \pi} > 0$ if and only if $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$. This is identical to (26), i.e., the condition that the blockholder wants to stay after observing $s_{priv} = s_{pub}$. Thus, if EQ_R^E exists, this inequality holds. Therefore $\frac{\partial \mathcal{W}_R^E}{\partial \pi} > 0$ and $\frac{d\mathcal{W}_R^E}{d\pi} = \frac{\partial \mathcal{W}_R^E}{\partial \pi} + \frac{d\mathcal{W}_R^E}{dp^*}\frac{dp^*}{d\pi} = \frac{\partial \mathcal{W}_R^E}{\partial \pi} > 0$. □

A.7 Proof of Lemma 3

Proof. Since $\mathcal{W}_R^E$ increases in π (which we established in Proposition 4), and $\mathcal{W}^B$ is independent of π, there exists at most one level of π where $\mathcal{W}^B = \mathcal{W}_R^E$. We refer to this level of π as $\overline{\pi}_R^E(N,n)$. Then, it must be that $\mathcal{W}^B > \mathcal{W}_R^E$ if and only if $\pi < \overline{\pi}_R^E(N,n)$. To characterize $\overline{\pi}_R^E(N,n)$, notice that $\mathcal{W}^B > \mathcal{W}_R^E$ is equivalent to

$$\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) > \frac{1}{2} + \frac{\pi}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1-\pi}{2} \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$$

$$\iff 2 \times (\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)) - 1 > \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k) \tag{30}$$

We can use that

$$\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \sum_{k=\frac{N+1}{2}-n}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k)$$
$$= \frac{1}{2} + \sum_{k=\frac{N+1}{2}-n}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k)$$

and rewrite (30) as

$$2 \times \left[\sum_{k=\frac{N+1}{2}-n}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) \right] - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k) > \pi \left[\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k) \right]$$

If EQ_R^E exists, then $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k) > 0$ and thus have that $\mathcal{W}^B > \mathcal{W}_R^E$ if and only if

$$\frac{2 \times \left[\sum_{k=\frac{N+1}{2}-n}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) \right] - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)}{\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) - \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)} > \pi \tag{31}$$

□

For any (N, n), $\overline{\pi}_R^E(N, n) < 1$. This holds since $\mathcal{W}_R^E$ increases in π and $\lim_{\pi \to 1} \mathcal{W}_R^E = 1$ while $\mathcal{W}^B$ is independent of π and $\mathcal{W}^B < 1$. Thus, for any (N, n), there exists a π sufficiently close to 1 such that $\mathcal{W}_R^E > \mathcal{W}^B$. We show in Appendix A.8 that for n sufficiently large, $\overline{\pi}_R^E(N, n) > \frac{1}{2}$.

A.8 Proof of Proposition 5

This proof has four parts. Part zero shows that for any (N, n, π), the equilibrium which maximizes ex-ante firm value is either EQ^{NE} or EQ_{NR}^E or EQ_R^E. Given this, we can restrict our attention to these three equilibria. In part one, we show that for every N, there exists an n' such that for all $n \geq n'$ an intermediate range of signal precision π exists where $\mathcal{W}^B > \mathcal{W}_{NR}^E$ and EQ^{NE} does not exist, formally, that there exist $\pi \in [\underline{\pi}(N, n), \overline{\pi}_{NR}^E(N, n)] \subset (1/2, 1)$. In part two, we show that for every N, there exists an n'' such that for all $n \geq n''$ an intermediate range of signal precision π exists where $\mathcal{W}^B > \mathcal{W}_R^E$ and EQ^{NE} does not exist, formally, that there exist $\pi \in [\underline{\pi}(N, n), \overline{\pi}_R^E(N, n)] \subset (1/2, 1)$. In part three, we combine these insights to establish that for every N, there exists a $\underline{n}$ such that for all $n \geq \underline{n}$ there exists an intermediate range of signal precision $\pi \in [\underline{\pi}(N, n), \min\{\overline{\pi}_{NR}^E(N, n), \overline{\pi}_R^E(N, n)\}] \subset (1/2, 1)$, i.e., where $\mathcal{W}^B > \mathcal{W}_{NR}^E$ and simultaneously $\mathcal{W}^B > \mathcal{W}_R^E$ and $\mathcal{W}^{NE}$ does not exist.

Part 0: It is sufficient to consider EQ^{NE}, EQ_{NR}^E, and EQ_R^E.

Proof. This part shows that in the model with public information, there exist no welfare optimal equilibria other than EQ^{NE}, EQ^{E}_{NR}, and EQ^{E}_{R}.

In the model in section 4, an equilbrium describes the trading decision the blockholder makes when the public signal is correct and when it is incorrect, as well as the voting decisions small shareholders make at each order flow. Since we assume that shareholders vote as-if pivotal, the blockholder who stays invested always votes based on her private information.

To economize on notation, let the blockholder's trading strategy by a duple $\{T_0, T_1\}$ where T is a trading decision and T_0 denotes the trading decision given that $s_{priv} = s_{pub}$ while T_1 denotes the trading decision given that $s_{priv} \neq s_{pub}$. $T \in \{E, NE\}$ where E denotes exit and NE denotes no exit. Thus, there are four different strategies the blockholder can take ($\{NE, NE\}$, $\{NE, E\}$, $\{E, NE\}$, $\{E, E\}$).

A small shareholder's strategy describes her actions at every order flow. For ease of notation, let a strategy be a vector $\{v \mid O = 0, v \mid O = -n, v \mid O = -2n\}$ where $v \in \{W, M, A\}$ and W denotes voting with the public signal, M denotes mixing, and A denotes voting against the public signal. When small shareholders mix, their mixing probability is uniquely pinned down. This is because to mix, a small shareholder needs to be indifferent and she can only be indifferent when she is pivotal. Suppose, to the contrary, that she is not pivotal. Then, she updates from the public signal $\pi > 1/2$ and from order flow which either perfectly reveals ω or adds no information to the public signal. Thus, if a small shareholder is not pivotal, she has a strict preference and mixing is not a best response.

When a small shareholder updates from being pivotal, she is indifferent and willing to mix if and only if, given that other shareholders mix with probability p, there exist some $x, y \leq N-n-1$ such that

$$\pi \binom{N-n-1}{x} p^x (1-p)^{N-n-1-x} = (1-\pi) \binom{N-n-1}{y} p^y (1-p)^{N-n-1-y}$$

$$\iff \frac{\pi}{1-\pi} \frac{\binom{N-n-1}{x}}{\binom{N-n-1}{y}} = \left(\frac{p}{1-p}\right)^{y-x}, \tag{32}$$

where the LHS is just a number and the RHS is strictly increasing for $y > x$ and strictly decreasing for $y < x$ so that there is a unique solution p^* if it exists. For $y = x$, the indifference condition requires $\pi = 1 - \pi$, a contradiction as $\pi > 1/2$. Note that this implies that there can never be mixing after $O = -2n$ because $O = -2n$ implies that L exited and, thus, $x = y$. Similarly, when $\{E, E\}$, both $O = -n$ and $O = -2n$ imply that L exited. Thus, $x = y$ and mixing cannot be a best response for small shareholders.

$\{E, E\}$. For any equilibrium where the blockholder follows $\{E, E\}$, it must be the case that small shareholders vote as suggested by the public signal when $O = -n$ and $O = -2n$ because $\pi > 1/2$ and mixing cannot be part of an equilibrium by the argument above. Thus, any candidate equilibrium where the blockholder plays $\{E, E\}$ can only result in ex-ante per share firm value of π and is, thus, payoff dominated by EQ^{E}_{NR} in the paper.

$\{NE, NE\}$. Next, we rule out any equilibrium where the blockholder never exits other than EQ^{NE}. No equilibrium with $\{NE, NE\}$ and $v(O = 0) = A$ can exist. The reason is that if $n < {}^{N-1}/2$, a small shareholder is never pivotal at $O = 0$ and voting as-if pivotal then requires her to deviate to voting as suggested by the public signal. If $n = {}^{N-1}/2$, a small shareholder is pivotal if and only if the public signal is correct and thus she strictly prefers to deviate to voting as suggested by the public signal. By the same reasoning, no equilibrium with $v(O = -n) = A$ can exist.

No equilibrium with $\{NE, NE\}$ and $v(O = -2n) = A$ can exist. In any potential equilibrium with $\{NE, NE\}$, $O = -2n$ induces off-path beliefs. In accordance with our refinement, shareholders do not update based on this deviation (we micro-found these off-path beliefs in a model where an uninformed agent always exits, see the extension in Online Appendix D). Hence, small shareholders must vote as suggested by the public signal at $O = -2n$ because small shareholders are not pivotal and $\pi > 1/2$.

No equilibrium with $\{NE, NE\}$ and $\{W, W, W\}$ can exist as conditional on $s_{priv} \neq s_{pub}$ the blockholder strictly prefers to deviate. Staying results in a zero payoff while $P(-n) = \pi > 0$. Similarly, no equilibrium with $\{NE, NE\}$ and $\{W, M, W\}$ or $\{M, W, W\}$ can exist as then conditional on $s_{priv} \neq s_{pub}$ the blockholder strictly prefers to exit as her profit from staying is less than $1/2$ (0 for O that induces W and less than 1 for O that induces M) while her profit from exiting is larger than $1/2$ (as $P(-2n) = \pi$ and $P(-n) = \mathbb{E}[u(d, \omega)] \geq 1/2$). This leaves only one equilibrium with $\{NE, NE\}$, namely EQ^{NE} as characterized in Proposition 2.

$\{NE, E\}$. For $\{NE, E\}$, small shareholder's perfectly learn ω for $O \in \{-2n, 0\}$. Since small shareholders vote as-if pivotal, they must vote according to s_{pub} if $O = 0$ and opposed to s_{pub} if $O = -2n$.

$\{NE, E\}$ and $\{W, A, A\}$ cannot be an equilibrium as the blockholder strictly prefers to exit when $s_{priv} = s_{pub}$. Given $s_{priv} = s_{pub}$, the blockholder's expected payoff from staying is $1/2$ as she knows that the correct decision is taken if $O = 0$ but not if $O = -n$. The exit-payoff exceeds $1/2$ since $P(O = -2n) = 1$ and $P(-n) > 0$. Thus, a deviation to exit is strictly profitable.

There only remain two candidate equilibria where the blockholder plays $\{NE, E\}$. The first is ($\{NE, E\}$, $\{W, W, A\}$), which is EQ^E_{NR} as characterized in Proposition 3. The second is ($\{NE, E\}$, $\{W, M, A\}$) which is EQ^E_R as characterized in Proposition 4.

$\{E, NE\}$. For $\{E, NE\}$, the blockholder exits whenever $s_{pub} = s_{priv}$. Small shareholder's perfectly learn ω when $O \in \{-2n, 0\}$ and, since small shareholders vote as-if pivotal, they vote as opposed to the public signal if $O = 0$ and as suggested by the public signal at $O = -2n$.

There only remain three candidate equilibria where the blockholder plays $\{E, NE\}$. The first is ($\{E, NE\}$, $\{A, W, W\}$) which cannot be an equilibrium as then L strictly prefers to deviate and exit when $s_{priv} \neq s_{pub}$. Given $s_{priv} \neq s_{pub}$, L's expected payoff from staying is $1/2$. The expected payoff from exit exceeds $1/2$ since $P(-2n) = 1$ and $P(-n) > 0$. Thus, the deviation to exit is strictly profitable.

The second is ($\{E, NE\}$, $\{A, A, W\}$) and cannot be an equilibrium because small shareholders want to deviate at $O = -n$. The reason is that after observing $O = -n$, the posterior that the public signal is correct is $\pi > 1/2$ and small shareholders are never pivotal. Thus, they vote as-if pivotal which is to vote as suggested by s_{pub}.

The third candidate equilibrium is ($\{E, NE\}$, $\{A, M, W\}$) which we refer to as the equilibrium with inverted exit and randomization EQ^{IE}_R. In this candidate equilibrium, after observing $O = -n$ small shareholders mix and vote as suggested by the public signal with probability p^*. Given p^*, the public signal and updating from being pivotal, a small shareholder needs to be indifferent. Thus, p^* is the solution to:

$$\pi\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} = (1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} \tag{33}$$

$$\iff \frac{\pi}{1-\pi}\frac{\binom{N-n-1}{\frac{N-n-1}{2}}}{\binom{N-n-1}{\frac{N-1}{2}}} = \left(\frac{p^*}{1-p^*}\right)^{\frac{n}{2}} \tag{34}$$

Notice that $p^*(\pi = \frac{1}{2}) > \frac{1}{2}$ since $\binom{N-n-1}{\frac{N-n-1}{2}}$ is the central binomial coefficient and is thus larger than any other binomial coefficient, including than $\binom{N-n-1}{\frac{N-1}{2}}$. Moreover, $\frac{dp^*}{d\pi} > 0$.

Share prices are $P(0) = 1$, $P(-2n) = 1$,

$$P(-n) = \pi \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi)\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k),$$

and ex-ante per-share firm value is

$$\mathcal{W}_R^{IE} = \frac{1}{2} + \frac{1}{2}\pi \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1}{2}(1-\pi)\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k).$$

EQ_R^{IE} exists only if it is indeed optimal for L to stay after observing $s_{priv} \neq s_{pub}$. The expected payoff from staying is $\frac{1}{2} + \frac{1}{2}\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)$ and the expected payoff from exiting is $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n)$. Thus, staying is optimal if and only if

$$\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k). \tag{35}$$

Equilibrium existence also requires that it is indeed optimal for L to exit after observing $s_{pub} = s_{priv}$. Since staying yields $\frac{1}{2}\sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) < \frac{1}{2}$ and exiting yields $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) > \frac{1}{2}$ exiting is always optimal and this constraint is slack.

Thus, the equilibrium exists if and only if (35) holds. Since the LHS decreases in p^* and the RHS increases in p^* by the usual FOSD arguments, there exists a unique level of p^* such that (35) holds if and only if p^* is below this level and not if it is above. Since p^* increases in π, this means that there exists a cutoff value of π such that the equilibrium exists if and only if π is below this cutoff value.

We now show that $\frac{d\mathcal{W}_R^{IE}}{d\pi} < 0$. To establish this, note that $\frac{d\mathcal{W}_R^{IE}}{d\pi} = \frac{\partial \mathcal{W}_R^{IE}}{\partial \pi} + \frac{d\mathcal{W}_R^{IE}}{dp^*}\frac{dp^*}{d\pi}$ and that $\frac{\partial \mathcal{W}_R^{IE}}{\partial \pi} = \frac{1}{2}\sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) - \frac{1}{2}\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)$ which is negative if and only if $\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) > \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k)$ which is the case whenever EQ_R^{IE} exists by (35). Thus, $\frac{\partial \mathcal{W}_R^{IE}}{\partial \pi} < 0$. Moreover, $\frac{d\mathcal{W}_R^{IE}}{dp^*} = 0$ by the familiar envelope arguments (compare proof of Lemma 1). More precisely,

$$\frac{d\mathcal{W}_R^{IE}}{dp^*} = \frac{\pi}{2}(N-n)\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} - \frac{1-\pi}{2}(N-n)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} \tag{36}$$

and thus $\frac{d\mathcal{W}_R^{IE}}{dp^*} = 0$ if and only if

$$\pi\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} = (1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} \tag{37}$$

which is identical to (34) and thus holds for the equilibrium mixing probability. Thus, $\frac{d\mathcal{W}_R^{IE}}{dp^*} = 0$ and $\frac{d\mathcal{W}_R^{IE}}{d\pi} = \frac{\partial \mathcal{W}_R^{IE}}{\partial \pi} + \frac{d\mathcal{W}_R^{IE}}{dp^*}\frac{dp^*}{d\pi} = \frac{\partial \mathcal{W}_R^{IE}}{\partial \pi} < 0$.

Next, we show that EQ_R^{IE} either does not exist, or, if it exists, is welfare dominated by EQ_R^E which then also exists. For ease of notation, denote the equilibrium mixing probability in EQ_R^{IE} as $p^*(EQ_R^{IE}) = p_R^{IE}$ and in EQ_R^E as $p^*(EQ_R^E) = p_R^E$.

At $\pi = \frac{1}{2}$, we have $p_R^{IE} = 1 - p_R^E$. Thus,

$$\sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p_R^{IE}, N-n, k) = \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p_R^E, N-n, k), \text{ and}$$
$$\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p_R^{IE}, N-n, k) = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p_R^E, N-n, k).$$

Thus, at $\pi = \frac{1}{2}$, if EQ_R^{IE} exists, it must also be the case that EQ_R^E exists as the IC constraints are identical.

First, consider the case where EQ_R^{IE} does not exist at $\pi = \frac{1}{2}$. Since we know that EQ_R^{IE} exists if and only if π is sufficiently low, this means that EQ_R^{IE} does not exist for any $\pi \geq 1/2$

and we therefore do not need to consider it for the global welfare result.

Second, consider the case where EQ_R^{IE} exists at $\pi = \frac{1}{2}$. Then, $\mathcal{W}_R^{IE}(\pi = 1/2) = \mathcal{W}_R^{E}(\pi = 1/2)$. Thus, EQ_R^{IE} and EQ_R^{E} are identical in terms of welfare at $\pi = 1/2$. Since $\mathcal{W}_R^{E}$ increases in π (which we established in Proposition 4) and $\mathcal{W}_R^{IE}$ decreases in π (which is the property we established above), it follows that for all $\pi > 1/2$, $\mathcal{W}_R^{IE} < \mathcal{W}_R^{E}$. Moreover, we know that when EQ_R^{E} exists for some π, it also exists for all larger π. Therefore, in this case where EQ_R^{E} exists at $\pi = 1/2$, it exists at all $\pi \geq 1/2$. Thus, whenever EQ_R^{IE} exists, it is welfare dominated by EQ_R^{E} which then also exists. Hence, EQ_R^{IE} is never the welfare optimal equilibrium with public information.

Thus, the welfare-maximizing equilibrium must be one of the three studied in the paper, i.e. EQ^{NE}, or EQ_{NR}^{E} or EQ_R^{E}. For the remainder of the proof, we therefore restrict our attention to these three equilibria. □

Part 1: $\pi \in [\underline{\pi}(N,n), \overline{\pi}_{NR}^{E}(N,n)]$

Proof. We know that EQ^{NE} exists if and only if

$$\begin{aligned}
&\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + \pi \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) \\
\iff &\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \geq \pi\Big(1 - \sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(p^*, N-n, k)\Big) + \pi \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) \\
\iff &\frac{\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)}{\pi} \geq 1 - \sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(p^*, N-n, k) + \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k)
\end{aligned}$$

Because $N - n - (N+1)/2 = (N-1)/2 - n$, $p > \frac{1}{2}$, and by the symmetry of the binomial probabilities, we know that $\sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(p^*, N-n, k) < \sum_{k=\frac{N+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k)$. Hence, the RHS is larger than 1 so that a sufficient condition for non-existence of EQ^{NE} is

$$\frac{\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)}{\pi} \leq 1. \tag{38}$$

By Lemma 1, $\mathcal{W}^{NE} \geq \mathcal{W}^{B}$ if EQ^{NE} exists and, by Lemma 2, $\mathcal{W}_{NR}^{E} < \mathcal{W}^{B}$ if and only if $\pi < \overline{\pi}_{NR}^E(N,n)$. Thus, to show that, for given (N,n), there exist an interval of π's $\pi \in [\underline{\pi}(N,n), \overline{\pi}_{NR}^E(N,n)]$ where $\mathcal{W}_{NR}^E < \mathcal{W}^B$ and EQ^{NE} does not exist, it is sufficient to show that (38) holds at $\overline{\pi}_{NR}^E(N,n)$, i.e.,

$$\frac{\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*(\overline{\pi}_{NR}^E(N,n)), N-n, k)}{\overline{\pi}_{NR}^E(N,n)} \leq 1$$
$$\iff \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}\Big(p^*(\overline{\pi}_{NR}^E(N,n)), N-n, k\Big) \leq \overline{\pi}(N,n)_{NR}^E \tag{39}$$

Defining the mixing probability p^* for a given $\overline{\pi}_{NR}^E(N,n)$ as $\overline{p}(N,n)$ gives

$$\overline{p}(N,n) = \frac{1}{1 + \left(\frac{1-\overline{\pi}_{NR}^E(N,n)}{\overline{\pi}_{NR}^E(N,n)}\right)^{\frac{1}{n}}}.$$

To establish that the sufficient condition is satisfied, consider equation (39) at $n = \frac{N-1}{2}$. Then, (39) becomes

$$\overline{\pi}_{NR}^E \geq \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\overline{p}, \frac{N+1}{2}, k) = 1 - \overline{p}^{\frac{N+1}{2}} \tag{40}$$

Notice that $\overline{\pi}_{NR}^E(N, \frac{N-1}{2}) = 1 - \frac{1}{2^{\frac{N-1}{2}}}$. Thus, (39) becomes $1 - \overline{p}^{\frac{N+1}{2}} \leq 1 - \frac{1}{2^{\frac{N-1}{2}}}$, which rearranges to

$$\left(\frac{1}{2}\right)^{\frac{N-1}{N+1}} \leq \overline{p} \tag{41}$$

Notice that $\left(\frac{1}{2}\right)^{\frac{N-1}{N+1}}$ is decreasing in N and that

$$\overline{p}\Big(N, \frac{N-1}{2}\Big) = \frac{1}{1 + \frac{1}{\left(2^{\frac{N-1}{2}}-1\right)^{\frac{2}{N-1}}}} = \frac{\left(2^{\frac{N-1}{2}}-1\right)^{\frac{2}{N-1}}}{\left(2^{\frac{N-1}{2}}-1\right)^{\frac{2}{N-1}}+1} \tag{42}$$

is increasing in N. Thus, it is sufficient to show that Inequality (41) holds for the smallest admissible $N = 5$. Substituting $N = 5$ into (41) yields

$$\left(\frac{1}{2}\right)^{\frac{4}{6}} \leq \frac{1}{1 + \frac{1}{(2^2-1)^{\frac{2}{4}}}}$$
$$\iff \left(\frac{1}{2}\right)^{\frac{2}{3}} \leq \frac{\sqrt{3}}{\sqrt{3}+1}$$

which is approx $0.62996 \leq 0.63397$ and thus holds. Thus, we have established that for every $N \geq 5$, there exist some (n, π) combinations where $\mathcal{W}^E_{NR} < \mathcal{W}^B$ and EQ^{NE} does not exist.

□

Part 2: $\pi \in [\underline{\pi}(N, n), \overline{\pi}^E_R(N, n)]$

Proof. As this proof uses the randomization probability which small shareholders use in EQ^{NE} as well as the randomization probability which small shareholders use in EQ^E_R after $O = -n$, we adjust our notation to make the distinction clear. We continue to denote the randomization probability which small shareholders use in EQ^{NE} as p^*, and relabel the randomization probability which small shareholders use in EQ^E_R after $O = -n$ as q^*. Thus, at $n = \frac{N-1}{2}$, we have

$$q^*(N, \frac{N-1}{2}) = \frac{1}{1 + (\frac{1-\pi}{\pi})^{\frac{4}{N-1}} (\frac{\frac{N-1}{2}}{\frac{N-1}{4}})^{\frac{4}{N-1}}} \tag{43}$$

which means that $\frac{dq^*(N, \frac{N-1}{2})}{d\pi} > 0$.

We define $\tilde{\pi}(N, n)$ as the level of π such that $q^*(\tilde{\pi}(N, n), N, n) = \frac{1}{2}$, i.e., $\tilde{\pi}(N, n)$ solves $q^*(\tilde{\pi}(N, n), N, n) = \frac{1}{2}$. To find $\tilde{\pi}(N, n)$, substitute into (43) which yields

$$1 = (\frac{1-\pi}{\pi})^{\frac{4}{N-1}} \left(\frac{\frac{N-1}{2}}{\frac{N-1}{4}}\right)^{\frac{4}{N-1}} \tag{44}$$

which solves to

$$\tilde{\pi}(N, \frac{N-1}{2}) = \frac{(\frac{\frac{N-1}{2}}{\frac{N-1}{4}})}{1 + (\frac{\frac{N-1}{2}}{\frac{N-1}{4}})}. \tag{45}$$

As short-hand, let $B = (\frac{\frac{N-1}{2}}{\frac{N-1}{4}})$. Then, $\tilde{\pi}(N, \frac{N-1}{2}) = \frac{B}{1+B}$.

In addition, define $\tilde{p}(N,n) = p(\tilde{\pi}(N,n), N, n)$. Then, at $n = \frac{N-1}{2}$, $\tilde{p}(N, \frac{N-1}{2}) = \frac{1}{1+\frac{1}{B^{\frac{2}{N-1}}}} = \frac{B^{\frac{2}{N-1}}}{1+B^{\frac{2}{N-1}}}$.

To establish that there exist $\pi \in [\underline{\pi}(N,n), \overline{\pi}^E_R(N,n)]$, the proof focuses on $n = \frac{N-1}{2}$ and takes two steps. First, we establish that EQ^{NE} does not exist at $\tilde{\pi}(N,n)$. Then, we establish that, at $\tilde{\pi}(N,n)$, $\mathcal{W}^B > \mathcal{W}^E_R$ and therefore it must be that $\overline{\pi}^E_R(N,n) > \tilde{\pi}(N,n)$ and that $\underline{\pi}(N,n) < \tilde{\pi}(N,n)$.

For the first step, we know from equation (10) that at $n = \frac{N-1}{2}$, EQ^{NE} does not exist if and only if

$$\frac{1 - p^{\frac{N+1}{2}}}{1 + p^{\frac{N+1}{2}} - (1-p)^{\frac{N+1}{2}}} < \pi \tag{46}$$

Evaluated at $\tilde{\pi}(N,n)$, this becomes

$$\begin{aligned} 1 - \tilde{p}^{\frac{N+1}{2}} &< \frac{B}{1+B}(1 + \tilde{p}^{\frac{N+1}{2}} - (1-\tilde{p})^{\frac{N+1}{2}}) \\ \iff \frac{1 - \tilde{p}^{\frac{N+1}{2}}}{2\tilde{p}^{\frac{N+1}{2}} - (1-\tilde{p})^{\frac{N+1}{2}}} &< B \end{aligned}$$

Substituting in $\tilde{p}$ yields:

$$\begin{aligned} \frac{1 - \left(\frac{B^{\frac{2}{N-1}}}{1+B^{\frac{2}{N-1}}}\right)^{\frac{N+1}{2}}}{2\left(\frac{B^{\frac{2}{N-1}}}{1+B^{\frac{2}{N-1}}}\right)^{\frac{N+1}{2}} - \left(\frac{1}{1+B^{\frac{2}{N-1}}}\right)^{\frac{N+1}{2}}} &< B \\ \iff \frac{(1 + B^{\frac{2}{N-1}})^{\frac{N+1}{2}} - B^{\frac{N+1}{N-1}}}{2B^{\frac{N+1}{N-1}} - 1} &< B \\ \iff (1 + B^{\frac{2}{N-1}})^{\frac{N+1}{2}} + B &< 2 \times B \times B^{\frac{N+1}{N-1}} + B^{\frac{N+1}{N-1}} \end{aligned}$$

Since $B > 1$ and $\frac{N+1}{N-1} > 1$, it must be that $B^{\frac{N+1}{N-1}} > B$. Thus, a sufficient (but not necessary) condition for the inequality above to be true is that

$$(1 + B^{\frac{2}{N-1}})^{\frac{N+1}{2}} < 2 \times B \times B^{\frac{N+1}{N-1}} \tag{47}$$

$$\iff 1 + B^{\frac{2}{N-1}} < 2^{\frac{2}{N+1}} \times B^{\frac{2}{N+1}} \times B^{\frac{2}{N-1}} \tag{48}$$

For small N, this inequality can be computed and holds, e.g. $N = 9$, $N = 13$, $N = 17$, $N = 21$, $N = 25$, etc. To establish that this inequality is also satisfied for large N, take each side in turn. The LHS must be less than 3 for all N. The reason is that B is one binomial coefficient of many and the sum of binomial coefficients satisfies $\sum_{i=0}^{x}\binom{x}{i} = 2^x$. Thus, $B = \binom{\frac{N-1}{2}}{\frac{N-1}{4}} < 2^{\frac{N-1}{2}}$ and thus the LHS is less than $3\forall N$. The RHS approaches 4 for large N. The reason is that B is the central binomial coefficient, which means that, for large N, $B^{\frac{2}{N-1}}$ and $B^{\frac{2}{N+1}}$ approach 2. Thus, the RHS is approximately $2^{\frac{2}{N-1}} \times 2 \times 2 = 2^{\frac{2}{N-1}} \times 4$ where $2^{\frac{2}{N-1}} > 1\forall N$ and thus $RHS > LHS$. Thus, equation (48) holds which establishes that at $\tilde{\pi}(N, n)$, EQ^{NE} does not exist.

The second step is to establish that at $\tilde{\pi}(N, n)$, $\mathcal{W}^B > \mathcal{W}_R^E$. At $n = \frac{N-1}{2}$, we have

$$\mathcal{W}_R^E = \frac{1}{2} + \frac{\pi}{2}\sum_{k=1}^{\frac{N+1}{2}} \mathbb{P}(q^*, \frac{N+1}{2}, k) + \frac{1-\pi}{2}\sum_{k=0}^{\frac{N-1}{4}} \mathbb{P}(q^*, \frac{N+1}{2}, k) \tag{49}$$

Evaluate this at $\tilde{\pi}$, where by definition of $\tilde{\pi}$ we have that $q(\tilde{\pi}) = \frac{1}{2}$.

$$\mathcal{W}_R^E = \frac{1}{2} + \frac{\tilde{\pi}}{2}\sum_{k=1}^{\frac{N+1}{2}} \mathbb{P}(\frac{1}{2}, \frac{N+1}{2}, k) + \frac{1-\tilde{\pi}}{2}\sum_{k=0}^{\frac{N-1}{4}} \mathbb{P}(\frac{1}{2}, \frac{N+1}{2}, k)$$

Since $\sum_{k=0}^{\frac{N-1}{4}} \mathbb{P}(\frac{1}{2}, \frac{N+1}{2}, k) = \frac{1}{2}$ this simplifies to:

$$\mathcal{W}_R^E = \frac{3}{4} - \frac{\tilde{\pi}}{4} + \frac{\tilde{\pi}}{2}\sum_{k=1}^{\frac{N+1}{2}} \mathbb{P}(\frac{1}{2}, \frac{N+1}{2}, k)$$

The goal is to show that $\mathcal{W}^B > \mathcal{W}_R^E$, which is equivalent to showing that

$$1 - \Big(\frac{1}{2}\Big)^{\frac{N+1}{2}} > \frac{3}{4} - \frac{\tilde{\pi}}{4} + \frac{\tilde{\pi}}{2}\Big(1 - \Big(\frac{1}{2}\Big)^{\frac{N+1}{2}}\Big)$$

$$\iff \frac{\frac{1}{4} - \Big(\frac{1}{2}\Big)^{\frac{N+1}{2}}}{\frac{1}{4} - \frac{1}{2}\Big(\frac{1}{2}\Big)^{\frac{N+1}{2}}} > \tilde{\pi}$$

Use that $\tilde{\pi} = \frac{B}{1+B}$ where $B = \binom{\frac{N-1}{2}}{\frac{N-1}{4}}$.

$$\frac{\frac{1}{4} - \left(\frac{1}{2}\right)^{\frac{N+1}{2}}}{\frac{1}{4} - \frac{1}{2}\left(\frac{1}{2}\right)^{\frac{N+1}{2}}} > \frac{B}{1+B} \tag{50}$$

$$\iff 2^{\frac{N-1}{2}} - 2 > B \tag{51}$$

A property of binomial coefficients is that $\sum_{i=0}^{x}\binom{x}{i} = 2^x$ and that this is spread across coefficient with an even or odd index, i.e. $\sum_{i\geq 0}^{x}\binom{x}{2i} = 2^{x-1}$ and $\sum_{i\geq 0}^{x}\binom{x}{2i+1} = 2^{x-1}$. Thus, for any binomial coefficient such as B, it must be that $B < 2^{x-1}$. Here, $B = \binom{\frac{N-1}{2}}{\frac{N-1}{4}}$ must be $B < 2^{\frac{N-1}{2}-1} = 2^{\frac{N-3}{2}}$. Hence, to establish that (51) holds, it is sufficient (but not necessary) to show that

$$2^{\frac{N-1}{2}} - 2 > 2^{\frac{N-3}{2}}$$

$$\iff 2 \times 2^{\frac{N-3}{2}} > 2^{\frac{N-3}{2}} + 2$$

$$\iff 2^{\frac{N-3}{2}} > 2$$

which holds for $N \geq 9$. Thus, we have established that at $\tilde{\pi}(N,n)$, $\mathcal{W}^B > \mathcal{W}_R^E$.

Since $\mathcal{W}_R^E$ is monotonically increasing in π (which we established in Proposition 4), it follows that $\overline{\pi}_R^E(N,n) > \tilde{\pi}(N,n)$ and that $\underline{\pi}(N,n) < \tilde{\pi}(N,n)$. Thus, there exist $\pi \in [\underline{\pi}(N,n), \overline{\pi}_R^E(N,n)] \subset (1/2, 1)$ where $\mathcal{W}^B > \mathcal{W}_R^E$ and EQ^{NE} does not exist. □

Part 3: Combining parts

Proof. From part 1, we know that for $n = \frac{N-1}{2}$, there exist $\pi \in [\underline{\pi}(N,n), \overline{\pi}_{NR}^E(N,n)] \subset (1/2, 1)$ where $\mathcal{W}^B > \mathcal{W}_{NR}^E$. From part 2, we know that for $n = \frac{N-1}{2}$, there exist $\pi \in [\underline{\pi}(N,n), \overline{\pi}_R^E(N,n)] \subset (1/2, 1)$ where $\mathcal{W}^B > \mathcal{W}_R^E$. Thus, for $n = \frac{N-1}{2}$, there exist $\pi \in [\underline{\pi}(N,n), \min\{\overline{\pi}_{NR}^E(N,n), \overline{\pi}_R^E(N,n)\}] \subset (1/2, 1)$ where $\mathcal{W}^B > \mathcal{W}_{NR}^E$, $\mathcal{W}^B > \mathcal{W}_R^E$ and EQ^{NE} does not exist. This concludes the proof. □

B Technical Results

Proposition 6. *For the binomial distribution, an increase in the success probability p leads to an FOSD increase in the CDF.*

Proof. For the binomial CDF, an FOSD increase means

$$\sum_{k=0}^{x} \mathbb{P}(p,N,k) \geq \sum_{k=0}^{x} \mathbb{P}(q,N,k) \ \forall\ p \leq q \tag{52}$$

which is equivalent to

$$\sum_{k=0}^{x} \left[\binom{N}{k} \left(p^k (1-p)^{N-k} - q^k (1-q)^{N-k} \right) \right] > 0 \ \forall\ p \leq q \tag{53}$$

Hence, we need to show that (53) holds $\forall\ p \leq q$. To show this, we first establish a result on the PDF and then derive results on the CDF.

Lemma 4. *Let $y = p^k(1-p)^{N-k}$. Then $\frac{dy}{dp} < 0$ iff $k < Np$.*

Proof.

$$\frac{dy}{dp} = kp^{k-1}(1-p)^{N-k} - p^k(N-k)(1-p)^{N-k-1} \tag{54}$$

Thus $\frac{dy}{dp} < 0$ is equivalent to

$$kp^{k-1}(1-p)^{N-k} - p^k(N-k)(1-p)^{N-k-1} < 0 \tag{55}$$

$$k(1-p) - (N-k)p < 0 \tag{56}$$

$$k < Np \tag{57}$$

□

From Lemma 4, it follows immediately that for all $x < Np$ we have

$$\sum_{k=0}^{x} \mathbb{P}(p,N,k) \geq \sum_{k=0}^{x} \mathbb{P}(q,N,k) \ \forall\ p \leq q \tag{58}$$

i.e. that FOSD holds.

Moreover, from the if and only if relation of Lemma 4, we also know that for all $x > Np$, we have that

$$\sum_{k=x}^{N} \mathbb{P}(p, N, k) \leq \sum_{k=x}^{N} \mathbb{P}(q, N, k) \; \forall \; p \leq q \tag{59}$$

which we can rearrange

$$1 + \sum_{k=x}^{N} \mathbb{P}(p, N, k) \leq 1 + \sum_{k=x}^{N} \mathbb{P}(q, N, k) \; \forall \; p \leq q \tag{60}$$

$$1 - \sum_{k=x}^{N} \mathbb{P}(q, N, k) \leq 1 - \sum_{k=x}^{N} \mathbb{P}(p, N, k) \; \forall \; p \leq q \tag{61}$$

$$\sum_{k=0}^{x-1} \mathbb{P}(q, N, k) \leq \sum_{k=0}^{x-1} \mathbb{P}(p, N, k) \; \forall \; p \leq q \tag{62}$$

i.e. that FOSD holds. Hence, an increase in the success probability p leads to an FOSD improvement in the CDF. □

C Numerical Example

We show in the main text that our global welfare results hold for any $N \geq 9$. This online appendix illustrates our results in an numerical example of $N = 601$ for a varying of the size of the block n. The results are depicted in Figure 5.

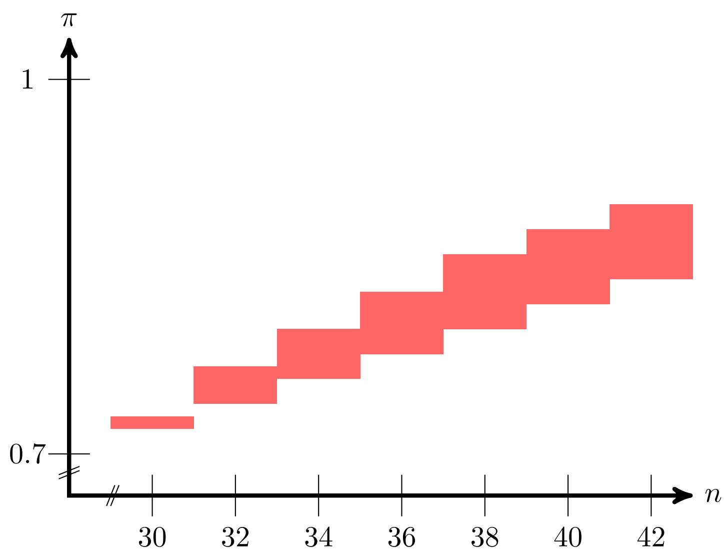

Figure 5: Global Welfare Comparison ($N = 601$). The shaded area denotes the signal precisions $\pi \in [\underline{\pi}(N,n), \min\{\overline{\pi}^E_{NR}(N,n), \overline{\pi}^E_R(N,n)\}] \subset (1/2, 1)$, i.e., where welfare is larger without public information than in any equilibrium with public information.

Areas shaded in red indicate that welfare is larger without public information than in any equilibrium with public information. For lower signal precisions, EQ^{NE} exists and welfare dominates the benchmark. For higher signal precisions, one of EQ^E_R and EQ^E_{NR} exists and welfare dominates the benchmark.

The numerical results in Figure 5 show that public information can reduce welfare even for a moderately sized blockholder. In particular, when there are $N = 601$ actively voting shares, and the blockhkolder has $n = 30$ shares, there exist signal precisions π where public information reduces welfare by our mechanism. This corresponds to the blockholder holding slightly less than five percent of the actively voted shares (and even less of the total number of shares). Empirical research often uses five percent as the minimal number of shares to identify a shareholder as a blockholder and, in practice, almost all companies have such a blockholder (Edmans & Holderness 2017).

Online Appendix

D Partially Informed Blockholder

While our assumption that the blockholder is perfectly informed about the state may seem restrictive, this section shows that public information can induce exit by the blockholder also when she is only partially informed. Moreover, stock market feedback into voting continues to operate. Intuitively, the result that public information induces exit only requires that the blockholder sometimes receives a private signal which is so precise that the blockholder believes it even if the public signal is conflicting. Then, after observing the private signal, the public signal is only informative about the vote outcome and can thus induce exit.

For concreteness, consider a version of the model where the blockholder is informed with probability $\rho \in (0,1)$, rather than with certainty. Whether L is informed is her private information. Apart from L's imperfect information, the model is as in Section 4. The following proposition shows that a modified version of EQ^E_{NR} still exist for all parameter values.

Proposition 7. *(EQ^E_{NR} with partially informed blockholder) There exists an equilibrium in which, if informed, L exits when $s_{pub} \neq s_{priv}$ and stays and votes with the public signal when $s_{pub} = s_{priv}$; if uninformed, L exits with probability $\ell^* \in (0,1]$ and with probability $(1-\ell^*) \in [0,1)$, L stays and votes with the public signal, where*

$$\ell^* = \begin{cases} 1 & \text{if } \rho \geq \frac{2\pi-1}{\pi^2}, \\ \frac{\rho(1-\pi)^2}{(1-\rho)(2\pi-1)} \in (0,1) & \text{otherwise.} \end{cases}$$

Small shareholders vote according to s_{pub} if $O \in \{-n, 0\}$ and vote against s_{pub} if $O = -2n$. Ex-ante firm value is

$$\mathcal{W}^E_{NR} = \begin{cases} \frac{1+\rho\pi}{2} & \text{if } \rho \geq \frac{2\pi-1}{\pi^2}, \\ \pi + \frac{\rho\pi(1-\pi)}{2} & \text{otherwise.} \end{cases}$$

Proof. See Online Appendix H.1 □

The blockholder exits whenever she is informed and observes that the public signal is incorrect. In addition, if L is uninformed, she exits with positive probability $\ell^* > 0$. Her exit probability ℓ^* is increasing in ρ and decreasing in π. Intuitively, if L is more likely to be informed, then, for a given ℓ^*, $O = -2n$ is more indicative of an incorrect public signal which makes voting against it more likely to improve firm value. Hence, a larger ρ increases $P(-2n)$ and makes exit more attractive for the (uninformed) large shareholder. Thus, public information continues to induce L to exit if her private signal conflicts with the public signal. The equilibrium always features some exit by the uninformed blockholder. If $\ell^* = 0$ was conjectured in equilibrium, an uninformed L's profit from exiting is $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n)$ where $P(-n) > \pi$ and $P(-2n) > \pi$, whereas her payoff from staying is π, making an exit a profitable deviation. Hence, $\ell^* = 0$ cannot be part of an equilibrium.

Interestingly, stock market feedback continues to operate even if L is only sometimes informed. In particular, even for ρ close to 0, small shareholders vote against the public signal after observing $O = -2n$. This is surprising since exit by the uninformed blockholder introduces the possibility that the public signal is correct even at $O = -2n$. However, the uninformed blockholder does not exit so often that it becomes optimal for small shareholders to vote as suggested by the public signal after observing $O = -2n$. This derives from L's indifference condition between exit and staying when she is uninformed. If the uninformed blockholder would exit too often, this would reduce her exit payoff so severely that she would prefer to stay. However, as we have discussed above, always staying can not be an equilibrium. Thus, she has to mix according to a probability which ensures that stock market feedback operates.

Overall, welfare increases in ρ and is smaller than in the baseline model. Intuitively, an uninformed L reduces welfare for two reasons. First, when L is uninformed she cannot trade on private information and therefore cannot correct public information via stock market feedback. Second, since only L knows whether she is informed, small shareholders' ability to draw inference from stock markets is reduced. For example, when L is uninformed, exits, and exit is uncovered ($O = -2n$), small shareholders vote against the public signal as they attribute exit to an informed L who knows that the public signal is incorrect. Had small shareholders been able to attribute $O = -2n$ to L being uninformed, they would have preferred to vote as suggested by the public information.

Thus, public information can continue to induce exit and stockmarket feedback can continue to operate when L is only partially informed. Online Appendix E shows that our result that firm value can be lower with public information than without it continues to hold when L is only partially informed.

E Abstaining from Voting

While, in practice, a large part of votes is taken by delegated investors who have to vote due to their fiduciary duty,[33] this online appendix investigates the effect of abstentions on the equilibrium outcome with and without public information. This online appendix highlights that our results apply even in an institutional setting where abstentions are possible. First, we show that equilibrium welfare with public information is bounded away from the first-best. Second, we find that our welfare results are robust in the sense that welfare can still be lower with public information than without it.

To highlight the effect of abstentions, consider the model as in the paper but with two modifications. First, at the voting stage, shareholders cannot just vote for the reform or against it, but additionally can abstain. Second, suppose that L is perfectly informed with probability $1/2$ and otherwise has no private information.[34]

E.1 Benchmark: No Public Information

In the absence of public information, the first-best outcome would be that L's private information determines the decision if L is informed, and that if L is uninformed, shareholders pick any decision, possibly at random. When shareholders can abstain, there exists an equilibrium which achieves the first-best.

Proposition 8. *(EQ_A^B) When there is no public information, there exists an equilibrium in which small shareholders abstain for any order flow they observe. If L is informed, she does not*

[33] For example, Iliev & Lowry (2015) state that "Shareholder voting is mandatory across all mutual funds" (p.449).

[34] As before, the decision with more votes is implemented. When shareholders can abstain, it is possible that the vote is tied. We assume that a tied vote results in the decision being made based on a fair coin toss.

exit and votes according to s_{priv}. If L is uninformed, she exits. Ex-ante per share firm value is $\mathcal{W}_A^B = {}^3/_4$.

Proof. See Online Appendix H.2. ☐

In addition, there exists a second equilibrium in which small shareholders randomize. This second equilibrium is analogous to the benchmark equilibrium of Proposition 1.

Proposition 9. *(EQ_R^B) When there is no public information, there exists an equilibrium in which small shareholders randomize over voting for and against the reform with probability $p = {}^1/_2$ for any order flow they observe. If L is informed, she does not exit and votes according to s_{priv}. If L is uninformed, she exits. Ex-ante per share firm value is*

$$\mathcal{W}_R^B = \frac{1}{4} + \frac{1}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k). \tag{63}$$

Proof. See Online Appendix H.3. ☐

These equilibria can be ranked in terms of ex-ante firm value.

Lemma 5. $\mathcal{W}_A^B > \mathcal{W}_R^B$

Proof. See Online Appendix H.4. ☐

Lemma 5 means that when we consider EQ_R^B, which we do in the paper for institutional reasons, e.g., because of rules which require funds to vote in their clients beset interest, then we are stacking the deck against our result that welfare can be lower with public information than without it.

While EQ_A^B achieves the first-best firm value, there is room for public information to increase firm value further. Intuitively, if L is not informed, firm value could be increased by deciding based on the public signal. Thus, in this version of the model, public information may add otherwise unknown information. The next section studies the case with public information.

E.2 The Effect of Public Information

We show that while public information increases first-best firm value, this cannot be achieved in equilibrium. Moreover, equilibrium firm value with public information can be below equilibrium firm value in the benchmark without public information.

With public information, the first-best outcome would be to base the decision on L's private information, if she is informed, and otherwise base the decision on the public signal. As a result, first-best firm value is $\mathcal{W}^{FB} = \frac{1+\pi}{2}$.

Proposition 10. *There does not exist an equilibrium which achieves first-best firm value.*

Proof. See Online Appendix H.5. □

This impossibility result is driven by L's trading incentives. First, notice that the only equilibria which could implement the first-best require L to use her private information if she is informed, and otherwise to use the public signal while small shareholders abstain. Crucially, the public signal must be used by L and not another shareholder as only L knows whether she is informed or not.[35] However, it is not optimal for L to use the public signal when she is uninformed since, conditional on being uninformed, L strictly prefers to exit. The uninformed L wants to exit even if the public signal is very precise because the share price which is received when exist is uncovered ($O = -2n$) is also increasing in the precision of the public signal.

Moreover, EQ_A^B and EQ_R^B are not an equilibrium when there is public information. The reason why EQ_A^B is not an equilibrium when there is public information is that at $O = -n$ and $O = -2n$ small shareholders are pivotal with strictly positive probability when L has exited. Thus, at $O = -n$ and $O = -2n$ small shareholders strictly prefer to deviate to voting in line with the public signal. Thus, there does not exist an equilibrium where L's private information always affects the vote outcome through L's vote. Similarly, EQ_R^B is not an equilibrium when there is public information as a small shareholder strictly prefers to vote with the public signal if all other small shareholders mix between voting for and against the reform with probability $p = 1/2$ at $O = -n$.

[35] L does not perfectly reveal whether she is informed through trading due to the existence of noise traders.

When there is public information, there can exist three equilibria. Two where the informed blockholder who observes $s_{priv} \neq s_{pub}$ exits and one where she stays.

Proposition 11. *(EQ^E_{NA}) For any triplet (N, n, π), there exists an equilibrium where small shareholders vote as suggested by the public signal if $O = 0$ and $O = -n$, but vote against the public signal if $O = -2n$. If L is informed and the public signal is correct ($s_{priv} = s_{pub}$), L stays and votes according to s_{priv}. If L is informed and the public signal is incorrect ($s_{priv} \neq s_{pub}$), L exits. If L is uninformed, L mixes between exiting with probability ℓ^* and staying and voting with the public signal with probability $1 - \ell^*$, where*

$$\ell^* = \begin{cases} 1 & \text{if } \pi \leq 2 - \sqrt{2}, \\ \frac{(1-\pi)^2}{2\pi - 1} \in (0,1) & \text{otherwise.} \end{cases}$$

and ex-ante firm value is

$$\mathcal{W}^E_{NA} = \begin{cases} \frac{1}{2} + \frac{1}{4}\pi & \text{if } \pi \leq 2 - \sqrt{2}, \\ \frac{5}{4}\pi - \frac{1}{4}\pi^2 & \text{otherwise.} \end{cases}$$

Proof. See Online Appendix H.6. □

This equilibrium features no abstentions and is like the equilibrium described in Proposition 7. Like in all equilibria with exit by the blockholder who observes $s_{priv} \neq s_{pub}$, the reason for L's exit is that public information correlates the votes of small shareholders which creates an incentive for L to exit rather than to vote based on her information.

There exists a second equilibrium where the informed L exits after observing $s_{priv} \neq s_{pub}$. The equilibrium is almost identical to EQ^E_{NA} with the only difference being that small shareholders abstain after observing $O = 0$.

Proposition 12. *(EQ^E_A) For any triplet (N, n, π), there exists an equilibrium where small shareholders abstain if $O = 0$, vote as suggested by the public signal if $O = -n$, and vote against the suggestion of the public signal if $O = -2n$. L's strategy is the same as in EQ^E_{NA}. Ex-ante firm value is as in EQ^E_{NA}, i.e., $\mathcal{W}^E_A = \mathcal{W}^E_{NA}$.*

Proof. See Online Appendix H.7. □

While EQ_A^E features abstentions by small shareholders after they observe $O = 0$, the reason for L's exit and ex-ante welfare are as in EQ_{NA}^E. In EQ_A^E the informed blockholder who observes $s_{priv} \neq s_{pub}$ knows that she may swing the vote if she stayed and noise traders do not exit such that $O = 0$. However, she knows that she will fail to swing the vote if noise traders exit and $O = -n$. As a result, $L's$ expected benefit from staying is larger than in EQ_{NA}^E, but still not large enough to make it optimal for her to stay. The result is that she exits with certainty, which yields the same ex-ante firm value as in EQ_{NA}^E.

There exists a third equilibrium. In this equilibrium, the informed blockholder does not exit after observing $s_{priv} \neq s_{pub}$.

Proposition 13. *(EQ^{NE}) For any N, there exist combinations of n and π such that the following strategies constitute an equilibrium. Small shareholders abstain if $O = 0$, mix between voting with the public signal with probability p^* and against with probability $1 - p^*$ if $O = -n$, vote as suggested by the public signal if $O = -2n$. If L is informed, L stays and votes according to s_{priv}, regardless of whether the public signal is correct or not. If L is uninformed, she exits. Small shareholder's mixing probability p^* is the solution to*

$$\begin{aligned}
&\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} + (1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} \\
=&\pi\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} + (1-\pi)\binom{N-n-1}{\frac{N-n-1}{2}}p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}}
\end{aligned}$$

and satisfies $p^* \geq \frac{1}{2}$ as well as $p^* \geq \frac{1}{1+(\frac{1-\pi}{\pi})^{\frac{1}{n}}}$. Ex-ante firm value is:

$$\begin{aligned}
\mathcal{W}^{NE} =&\frac{1}{4} + \frac{1}{4}\pi \\
&+\frac{1}{4}\pi\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*, N-n, k) + \frac{1}{4}(1-\pi)\sum_{k=0}^{\frac{N-1}{2}}\mathbb{P}(p^*, N-n, k) \\
&+\frac{1}{4}\pi\sum_{k=\frac{N-n+1}{2}}^{N-n}\mathbb{P}(p^*, N-n, k) + \frac{1}{4}(1-\pi)\sum_{k=0}^{\frac{N-n-1}{2}}\mathbb{P}(p^*, N-n, k)
\end{aligned}$$

Proof. See Online Appendix H.8. □

In this equilibrium, the informed blockholder who knows that $s_{priv} \neq s_{pub}$ does not exit because she knows that she will swing the vote for sure if $O = 0$ and has a chance of swinging the vote if $O = -n$. This benefit from staying can be sufficiently large to make it optimal for L to stay.

The mixing probability small shareholders use at $O = -n$ exceeds the mixing probability familiar from EQ^{NE} as described in Proposition 2. Here and in Proposition 2, small shareholders know that they are pivotal if L has private information and $\frac{N-1}{2} - n$ out of $N - n - 1$ small shareholders vote with L, which could be when the public signal is correct or incorrect. Different from Proposition 2, small shareholders can also be pivotal if L is uninformed and has exited. In this case, small shareholders strictly prefer to vote as suggested by the public signal. Thus, the mixing probability needs to be larger than in Proposition 2 as otherwise a small shareholder would strictly prefer to vote in line with the public signal. A probability larger than in Proposition 2 and therefore further from $1/2$ makes it more likely that a small shareholder is pivotal when L is informed, has stayed, and votes against the public signal relative to when L is informed, has stayed, and votes as suggested by the public signal as well as relative to when L is uninformed and has exited. Therefore, for a mixing probability larger than in Proposition 2 shareholders are indifferent which way to vote and thus willing to mix.

Combining the three equilibria which exist when there is public information and the impossibility result that no equilibrium achieves first-best firm value yields a global welfare result. As in the paper, we again assume that shareholders can coordinate on the payoff maximizing equilibrium for a given information structure. Then, the following result shows that even when shareholders can abstain, public information can reduce welfare. Formally,

Proposition 14. *(Global welfare with abstentions) For every N, there exist combinations of n and π such that public information reduces welfare, i.e. where welfare is higher without public information than in any equilibrium with public information.*

Proof. See Online Appendix H.9. □

For the intuition behind Proposition 14, consider the benefits and costs of public information. The benefit is that small shareholders make better decisions when they know that L is uninformed, e.g., once exit is uncovered at $O = -2n$. The cost is that L's private information is not always used when there is public information. This occurs when small shareholders are uncertain whether L has exited or not ($O = -n$) and thus mix in their voting behaviour which, in turn, introduces errors. It leads to cases where the informed L looses the vote and to cases where L was uninformed, exited and small shareholders by chance do not follow the public signal. Since the benefit of better decisions making at $O = -2n$ is small if the public signal is imprecise, but the cost of not using L's private information is large already for an imprecise public signal, there exist cases where public information reduces welfare.

Notice that allowing abstentions increases firm value in the case with and without public information. However, crucially, it does not change the result that public information can reduce firm value.

F Share purchases

Exit is preferred to purchasing. While in the paper, it may appear as if in EQ^E_{NR} the blockholder would benefit from deviating to purchasing additional shares rather than exiting, this online appendix shows that this is not the case. Consider a variant of the model where noise traders trade $\{-n, 0, +n\}$ with equal probability.

Due to the increased noise, there have to be $N + 3n$ total shares outstanding. n continues to be the initial block size and $N - n$ are held by active small shareholders who vote. n shares are owned by noise traders and $2n$ shares are in the market maker's inventory, neither of whom votes. The large shareholder may now also purchase n shares so that her strategy becomes $h : \mathcal{S}_{pub} \times \mathcal{S}_{priv} \to \{-n, 0, +n\}$. We assume that shares are purchased from the market maker's inventory. If the blockholder purchases n additional shares, her total stake is $2n$ out of $N + n$ actively voting shares. Thus, to ensure that the blockholder remains a minority shareholder, we assume that $2n \leq \frac{N+n-1}{2}$. If the blockholder exits, there are $N - n$ actively voting shares, as in the baseline model. If the blockholder stays, there are N actively voting shares. Apart from allowing for share acquisition, the model is as in Section 4.

Consider EQ^E_{NR} in which L exits when $s_{priv} \neq s_{pub}$. We show that L would never like to purchase additional shares instead of exiting. To this end, suppose $s_{priv} \neq s_{pub}$. Given the conjectured equilibrium EQ^E_{NR}, shareholders vote with the public signal unless $O = -2n$. Share prices are given by $P(-n) = P(0) = \pi$ and shareholders vote with the public signal as they do not learn from the order flow O.[36] $P(+n) = 1$ and shareholders vote with the public signal as, in the conjectured equilibrium, $O = +n$ reveals that L did not exit. Therefore, the public signal must be correct. $P(-2n) = 1$ and shareholders vote against the public signal as the market maker and small shareholders learn that L has sold her stake. Because $O = +2n$ does not occur on the equilibrium path, we have to assign off-path beliefs. The off-path beliefs which make purchases most attractive are those in which $O = +2n$ signals to shareholders that public signal is incorrect such that they change their vote, and, $P(+2n) = 1$. Hence, conditional on $s_{priv} \neq s_{pub}$, exit profits are

$$n\Big(\frac{1}{3}P(-2n) + \frac{1}{3}P(-n) + \frac{1}{3}P(0)\Big) = n\frac{2\pi + 1}{3} > 0,$$

where 0 would be the payoff from not trading as the incorrect decision would be implemented. Now suppose L deviates to acquiring n shares if $s_{priv} \neq s_{pub}$. L's deviation profits are

$$\frac{1}{3}(1n + n(1 - P(+2n)) + \frac{1}{3}(0n + n(0 - P(n)) + \frac{1}{3}(0n + n(0 - P(0)) = \frac{1}{3}n - \frac{1}{3}n - \frac{1}{3}\pi n < 0.$$

Conditional on $s_{priv} \neq s_{pub}$, firm value is zero if $O \in \{0, +n\}$ because small shareholders vote according to the public signal. If $O = +2n$ shareholders change their vote, leading to firm value of one. Because L still needs to acquire overvalued shares in two of three cases, she incurs a loss. Hence, this deviation can never be profitable. Intuitively, the stock market feedback after $O \in \{-2n, +2n\}$ induces shareholders to revise their vote - both if L sells and buys shares. Hence, selling and buying shares has the same effect on firm value. However, conditional on $s_{priv} \neq s_{pub}$, shares are overvalued such that selling is more attractive than buying.

[36]This contrasts with EQ^E_{NR} in section 4 where $P(0) = 1$ as $O = 0$ revealed that L did not exit. Here, noise traders sometimes purchase shares which changes the inference drawn from $O = 0$ and thus the share price.

Welfare effects of share purchases. While in the main model it may appear as if allowing the blockholder to purchase additional shares would overturn the result that public information can harm firm value, this online appendix shows that there are two opposing forces. One is that stock purchases by a blockholder who observes that $s_{priv} = s_{pub}$ make shareholders less likely to vote with the public signal (pivotality effect). The other is that public information discourages stock purchases by an informed shareholder relative to the benchmark without public information. We show these forces in a simplified model that abstracts from stock market feedback.

Again suppose that L has n shares and can stay (not trade), exit (sell all shares), or can purchase n additional shares from the market maker. For ease of exposition, suppose that the market is perfectly liquid such that trading has no effect on the share price. As a result, small shareholders can never infer information from the share price and therefore the blockholder's trading does not affect their voting decision.

Proposition 15. *(Benchmark) When there is no public information, the following strategies constitute an equilibrium. L purchases n additional shares and votes according to s_{priv}. Small shareholders mix between voting for R and Q with probability 1/2. Ex-ante per-share firm value is* $\mathcal{W}^B = \sum_{k=\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$.

Proof. See Online Appendix H.10. □

This benchmark is analogous to the benchmark in our main model with a minor scaling difference. The blockholder's stake after purchasing ($2n$ in this online appendix) is analogous to the blockholder's stake in the main model when she does not exit (n in the main model).

When there is public information, there are two equilibria, analogous to EQ_R^E and EQ_{NR}^E. In both, the blockholder purchases if the public signal is correct and sells otherwise.[37]

Proposition 16. *(EQ_{NR}^T) When there is public information, the following strategies constitute an equilibrium. If $s_{pub} = s_{priv}$, L purchases n additional shares and votes according to s_{priv}. If $s_{pub} \neq s_{priv}$, L sells n shares. Small shareholders follow the public signal. Ex-ante per-share firm value is* $\mathcal{W}_{NR}^T = \pi$.

[37] We label them EQ_{NR}^T and EQ_R^T to capture that there is trading (buying and selling) not just exit (selling).

Proof. See Online Appendix H.11. □

Proposition 17. *(EQ_R^T) When there is public information, the following strategies constitute an equilibrium. If $s_{pub} = s_{priv}$, L purchases n additional shares and votes according to s_{priv}. If $s_{pub} \neq s_{priv}$, L sells n shares. Small shareholders mix between voting as suggested by the public signal with probability p^* and otherwise voting against. p^* is defined as the solution to*

$$\pi \binom{N-n-1}{\frac{N+n-1}{2} - 2n} p^{*\frac{N+n-1}{2} - 2n} (1-p^*)^{\frac{N+n-1}{2}} = (1-\pi) \binom{N-n-1}{\frac{N-n-1}{2}} p^{*\frac{N-n-1}{2}} (1-p^*)^{\frac{N-n-1}{2}} \quad (64)$$

where $p^*(\pi = 1/2) < 1/2$ and $dp^*/d\pi > 0$. Ex-ante per-share firm value is

$$\mathcal{W}_R^T = \pi \sum_{\frac{N+n+1}{2} - 2n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k) \quad (65)$$

This equilibrium exists if and only if $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$.

Proof. See Online Appendix H.12. □

In EQ_R^T small shareholders mix. The mixing probability can be less than $1/2$ because of the pivotality effect. This echoes how shareholders mix at $O = -n$ in EQ_R^E in the body of the paper. In both cases, the blockholder votes if and only if $s_{pub} = s_{priv}$ (the difference is only the scaling of block size n vs. $2n$).

To see why $p^* < \frac{1}{2}$ for low π, suppose that instead $p = 1/2$ and π is low. Then, a small shareholders is likely to be pivotal if the public signal is incorrect as then the blockholder has exited and only small shareholders vote. Given $p = 1/2$, a tie is relatively likely. In contrast, a small shareholder is unlikely to be pivotal if the public signal is correct as then the blockholder votes as suggested by the public signal. Thus, if small shareholders split their vote equally, the blockholder decides and a small shareholder is not pivotal. However, she is pivotal if few small shareholders vote as suggested by s_{pub}. Thus, a small shareholder who conditions on being pivotal strictly prefers to vote against s_{pub}.

There exists a $p^* < 1/2$ where small shareholders are indifferent and willing to randomize. This is because $p^* < 1/2$ makes a tie more likely when $s_{priv} = s_{pub}$ and the blockholder stayed, and less likely when $s_{priv} \neq s_{pub}$ and the blockholder has exited. Thus, for low signal precisions the pivotality effect works against the direct effect that share purchases increase the blockholder's stake and thereby result in more votes for the correct decision.

In both equilibria, the blockholder trades based on whether the public signal is correct. When the blockholder knows that the public signal is incorrect, she prefers to sell rather than to purchase additional shares and therefore does not vote. Thus, public information discourages stock purchases by an informed shareholder relative to the benchmark without public information. Interestingly, this holds even when small shareholders mix and therefore the blockholder's votes can change the voting outcome and would be more likely to do so when she purchases additional shares.

Next, we show that no other equilibria exist and that, in particular, no equilibria exist where the blockholder always purchases additional shares. Intuitively, such an equilibrium cannot exist because then small shareholders would vote as suggested by the public signal or mix with a larger weight on the public signal. In both cases, it is not optimal for a blockholder to purchase additional shares when $s_{pub} \neq s_{priv}$ as they are overvalued.

Proposition 18. *When there is public information, there exist no other equilibria than EQ_R^T and EQ_{NR}^T.*

Proof. See Online Appendix H.13. □

Proposition 19. *For every (n, N), there exists $\pi'(N,n) < \pi''(N,n)$ such that if $\pi \in [\pi'(N,n), \pi''(N,n)]$, $\max\{\mathcal{W}_R^T, \mathcal{W}_{NR}^T\} < \mathcal{W}^B$.*

Proof. See Online Appendix H.14. □

Intuitively, in the benchmark the blockholder always influences the decision with her votes and thus her information contributes to firm value. In contrast, when there is public information, the blockholder influences the decision only if $s_{priv} = s_{pub}$. Thus, if the blockholder is sufficiently large or the public signal sufficiently imprecise, then the benchmark results in higher welfare than any equilibrium with public information.

G Other Equilibria in the Benchmark

In this online appendix we consider the other equilibria which may arise in the benchmark without public information in addition to the one discussed in Proposition 1. We show that these are immaterial for our welfare results as they are always welfare-dominated by an equilibrium with public information.

First, there may be pure strategy equilibria which rely on stock market feedback. In such an equilibrium, small shareholders coordinate to vote for the same decision, say $d = r$, if there is no conflicting information from the stock market, i.e., if $O \in \{-n, 0\}$. If $O = -2n$, small shareholders instead vote $d = q$. If L receives $s_{priv} = Q$, she infers that staying will lead all small shareholders to vote for the value-decreasing decision. Therefore, it is optimal for L to exit. In such an equilibrium, ex-ante firm value is $3/4$ and is thus always dominated by the equilibrium from Proposition 3 which yields $\mathcal{W}_{NR}^{E} = \frac{1+\pi}{2} \geq \frac{3}{4}$.

Second, there may be equilibria with stock market feedback and randomization. In these equilibria, L stays for one state, say $\omega = R$, and exits otherwise. Small shareholders vote for $d = r$ if $O = 0$ and for $d = q$ if $O = -2n$. Small shareholders randomize for $O = -n$. The mixing probability admits the following form (derived in Proposition 4 and evaluated at $\pi = 0.5$ because we consider the benchmark without public information)

$$p^*(O = -n) = \frac{1}{1 + \left(\frac{\binom{N-n-1}{\frac{N-1-n}{2}}}{\binom{N-n-1}{\frac{N-1}{2}-n}} \right)^{\frac{2}{n}}}. \tag{66}$$

By the proof of Proposition 4, welfare $\mathcal{W}_{R}^{E}$ increases in π and attains its minimum at $\pi = 0.5$. Hence, if this type of equilibrium exists in the benchmark without public information, it is dominated by EQ_{R}^{E}.

H Proofs of Results in the Online Appendix

H.1 Proof of Proposition 7

Proof. Conjecture EQ_{NR}^{E} in Proposition 7 is an equilibrium. Then, share prices are

$$P(0) = \frac{\pi(\rho + (1-\rho)(1-\ell))}{\pi\rho + (1-\rho)(1-\ell)}, \tag{67}$$

$$P(-n) = \pi, \tag{68}$$

$$P(-2n) = \frac{(1-\pi)(\rho + \ell(1-\rho))}{(1-\pi)\rho + (1-\rho)\ell}. \tag{69}$$

Then, if L is informed and observes $s_{pub} \neq s_{priv}$, exit is a unique best response because $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) > 0$, where the left hand side is the expected exit profit in the conjectured equilibrium whereas the right hand side is the deviation payoff from staying. If L is informed and observes $s_{priv} = s_{pub}$, staying is a unique best response as it results in a payoff of 1 while exit would yield $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) < 1$. An uninformed blockholder, by contrast, is indifferent between exiting and staying if

$$\begin{aligned} &\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) = \pi \\ \iff &\pi\big(\rho(1-\pi) + (1-\rho)\ell\big) = (1-\pi)\big(\rho + (1-\rho)\ell\big) \\ \iff &\ell^* = \frac{\rho(1-\pi)^2}{(1-\rho)(2\pi-1)}. \end{aligned} \tag{70}$$

$\ell^* < 1$ iff $\rho < \frac{2\pi-1}{\pi^2}$ and $\ell^* > 0$ always holds since $\rho > 0$. Hence, if $\rho \geq \frac{2\pi-1}{\pi^2}$, $\ell^* = 1$. Thus, L's trading strategy is indeed optimal.

To show that a small shareholder's voting strategy is optimal, let the common posterior that the public signal is incorrect after observing order flow be denoted by $\xi(O) \in [0,1]$. Then $\xi(0) = \frac{(1-\pi)(1-\rho)(1-\ell)}{\pi\rho+(1-\rho)(1-\ell)} < 1 - \pi < \frac{1}{2}$, $\xi(-n) = 1 - \pi < \frac{1}{2}$, and $\xi(-2n) = \frac{(1-\pi)(\rho+\ell(1-\rho))}{(1-\pi)\rho+(1-\rho)\ell}$. Notice that $\xi(-2n) > 1/2$ holds always true at ℓ^*. To establish this, and therefore also that voting at $O = -2n$ is optimal, note that $\xi(-2n) > 1/2$ simplifies to

$$\frac{(1-\pi)\rho}{(2\pi-1)(1-\rho)} > \ell \tag{71}$$

Recall that $\ell^* = \min(\frac{\rho(1-\pi)^2}{(1-\rho)(2\pi-1)}, 1)$. Substituting $\frac{\rho(1-\pi)^2}{(1-\rho)(2\pi-1)}$ into (71) yields $(1-\pi) > (1-\pi)^2$ which holds for all $\pi > \frac{1}{2}$. Thus (71) holds when $\ell^* = \frac{\rho(1-\pi)^2}{(1-\rho)(2\pi-1)}$ which means that voting against public information at $O = -2n$ is indeed optimal. At $\ell^* = 1$, $\xi(-2n) > 1/2$ whenever $\rho > {}^{2\pi-1}/_{\pi}$ which holds true because $\ell^* = 1$ if and only if $\rho \geq \frac{2\pi-1}{\pi^2}$.

The voting behavior of small shareholders is pinned down by $\xi(O)$ as no agent plays a weakly dominated strategy and thus votes as if pivotal. Hence, the voting strategy in the conjectured equilibrium is a best response. Moreover, the blockholder's voting strategy is a best response as she either votes based on s_{priv} (if that signal is available) or votes in line with the public signal, which is informative and thus following it is optimal as there is no additional information available. □

H.2 Proof of Proposition 8

Proof. Conjecture EQ_A^B in Proposition 8 is an equilibrium. Then, $P(O = 0) = 1$ since the market maker learns that L has stayed which means that L is informed and thus that the correct decision will be taken with certainty. $P(O = -2n) = 1/2$ since the market maker learns that L has exited. Thus, the decision is not based on any information and therefore correct with probability $1/2$. $P(O = -n) = \frac{3}{4}$ as it is equally likely that L is informed and stayed, in which case per share firm value is 1, or that L is uninformed and exited, in which case per share firm value is $1/2$. Ex-ante firm value is $\mathcal{W}_A^B = \frac{3}{4}$ as it is 1 whenever L is informed and $1/2$ otherwise.

Trading. If L exits, her expected pay-off is $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) = \frac{5}{8}$. If L is informed, staying yields payoff 1 and thus is strictly preferred to exiting. If L is uninformed, staying yields $\frac{1}{2}$ and thus exiting is optimal.

Voting. If $O = 0$, then small shareholders are never pivotal. As both states are equally likely, they are indifferent and abstaining is an best response. If $O = -n$, a small shareholder is pivotal if L has exited. In this equilibrium, L's exit only reveals that L was uninformed and thus reveals no information about the state. Thus, a small shareholder's posterior is that both states are equally likely, she is indifferent and, hence, abstaining is a best response. If $O = -2n$, a small shareholder is pivotal and knows that L has exited. Again, this just reveals that L was uninformed and contains no information on the state. Hence, abstaining is a best response.

Thus, for any order flow, abstaining is a best response. If L is informed and stays, she is pivotal and thus voting based on her information is a best response. Thus, the equilibrium exists. □

H.3 Proof of Proposition 9

Proof. Conjecture EQ_R^B in Proposition 9 is an equilibrium. Then, $P(O=0) = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$ since the market maker learns that L has stayed which means that L is informed. $P(O = -2n) = \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \frac{1}{2}$ since the market maker learns that L has exited which must be because L is uninformed. $P(O = -n) = \frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{1}{4}$ because it is equally likely that L stayed, in which case she is informed, and that L exited because she is uninformed. Similarly, $\mathcal{W}_R^B = \frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{1}{4}$.

Trading. If L exits, her expected pay-off is $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) = \frac{1}{4}\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{3}{8}$. If L is informed, staying yields pay-off $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) > \frac{1}{2}$ and thus is strictly preferred to exiting. If L is uninformed, staying yields $\sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \frac{1}{2}$ and thus exiting is optimal.

Voting. If $O = 0$, then a small shareholder is pivotal if the blockholder and $\frac{N-1}{2} - n$ out of $N-n-1$ small shareholders vote for one decision and $\frac{N-1}{2}$ small shareholders for the other. However, as it is equally likely that the blockholder is voting for the reform as it is that the blockholder is voting against the reform and as $p^* = 1/2$, a small shareholder is indifferent which way to vote conditional on being pivotal. Thus, randomizing is a best response.

If $O = -n$, then a small shareholder can be pivotal in several scenarios. It could be that L has exited and $\frac{N-n-1}{2}$ out of $N-n-1$ small shareholders vote one way and $N-n-1$ the other. Or, it could be that L has stayed. In this case, there are two pivotality scenarios as for $O = 0$. As exit by L only reveals that L was uninformed and reveals nothing about the state, and as being pivotal also reveals nothing about the state, a small shareholder is indifferent which way to vote. Overall, at $O = -n$ a small shareholder is indifferent which way to vote conditional on being pivotal. Thus, randomizing is best response.

If $O = -2n$, a small shareholder knows that L has exited and is pivotal if $\frac{N-n-1}{2}$ out of $N-n-1$ small shareholders vote one way and $N-n-1$ the other. As neither L's exit nor conditioning on pivotality are informative about the state, a small shareholder is indifferent between voting for and against the reform. Thus, randomizing is a best response. If L is informed and stays, she may be pivotal and, conditional on being pivotal, it is optimal to vote based on her information. Thus, the equilibrium exists. □

H.4 Proof of Lemma 5

Proof.

$$
\begin{aligned}
& \mathcal{W}_A^B > \mathcal{W}_R^B \\
\iff & \frac{3}{4} > \frac{1}{4} + \frac{1}{2} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) \\
\iff & 1 > \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k),
\end{aligned}
$$

which holds true. □

H.5 Proof of Proposition 10

Proof. As argued in the paper, first-best firm value can only be achieved by equilibria where L uses her private information if she is informed and otherwise votes based on the public signal while small shareholders abstain. Thus, there are two candidate equilibria which achieve first-best firm value.

Candidate equilibrium 1. Small shareholders abstain if $O = 0$ and if $O = -n$, but vote as suggested by the public signal if $O = -2n$. When L is privately informed, she does not exit and votes for s_{priv}. When L is not privately informed, she does not exit and votes in line with the public signal.

Candidate equilibrium 2. Small shareholders abstain for any order flow. When L is privately informed, she does not exit and votes for s_{priv}. When L is not privately informed, she does not exit and votes in line with the public signal.

We now establish that neither of these candidate equilibria exists. Candidate equilibrium 1 would mean that $\mathbb{E}[u(d, \omega)] = \frac{1+\pi}{2}$ and $P(O = 0) = P(O = -n) = \frac{1+\pi}{2}$. $O = -2n$ induces off-path beliefs. As the informed blockholder obtains an equilibrium payoff of 1 by staying invested in the conjectured equilibrium, we assume that off-path beliefs assign probability 1 to the case that L is uninformed.[38] Thus, $P(O = -2n) = \pi$ since the market maker would infer that L has exited because she is uninformed but can draw no inference from that about the state, i.e. the share price would reflect the signal precision. Therefore, L's expected pay-off from exit is $\frac{1}{2}P(O = -n) + \frac{1}{2}P(O = -2n) = \frac{1}{4} + \frac{3}{4}\pi$. If L is uninformed, then staying and voting based on the public signal yields π. Since $\frac{1}{4} + \frac{3}{4}\pi > \pi \,\forall\, \pi < 1$, the uninformed L strictly prefers to deviate and exit. Hence, candidate equilibrium 1 does not constitute an equilibrium.

Candidate equilibrium 2 does not constitute an equilibrium either. The reason is that upon observing $O = -2n$, small shareholders know that L has exited. While this is not informative about the state, it means that a small shareholder is pivotal as all other small shareholders abstain. Thus, a small shareholder strictly prefers to deviate and vote as suggested by the public signal. □

H.6 Proof of Proposition 11

Proof. This equilibrium is the special case of Proposition 7 with $\rho = 1/2$. Thus, we omit a detailed proof. □

H.7 Proof of Proposition 12

Proof. Conjecture EQ_A^E in Proposition 12 exists. Then, $P(0) = \frac{\pi(2-\ell)}{\pi+1-\ell}$, $P(-n) = \pi$, $P(-2n) = \frac{(1-\pi)(1+\ell)}{\ell+(1-\pi)}$ as in Proposition 11.

Trading. To verify that this equilibrium exists, first consider L's exit decision. If L is informed and $s_{priv} = s_{pub}$, staying yields 1. Exit yields $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) < 1$ and thus staying is optimal. If L is informed and $s_{priv} \neq s_{pub}$, then staying yields $\frac{1}{2}$ as L knows that the correct decision is taken if $O = 0$ but that the incorrect decision is taken if $O = -n$. While the payoff from staying is larger than in EQ_{NA}^E, we still have that exit is strictly preferred since exit results in $\frac{1}{2}P(-n) + \frac{1}{2}P(-2n) > \frac{1}{2}$ since $P(-n) = \pi > \frac{1}{2}$ and $P(-2n) = \frac{(1-\pi)(1+\ell)}{\ell+1-\pi} > \frac{1}{2} \iff \frac{1-\pi}{2\pi-1} > \ell$ which holds since $\ell^* = \min(\frac{(1-\pi)^2}{2\pi-1}, 1)$

[38] More formally, this off-path belief would be selected by the intuitive criterion as L obtains an on-path payoff of 1 if she is informed and strictly less than 1 when deviating. This holds for any $P(-2n)$ since it must be that $P(-n) < 1$. In contrast, if L is uninformed, her on-path payoff is less than 1.

Voting. As in Proposition 11, at $O = -n$ and $O = -2n$ small shareholders are never pivotal and thus, by assumption, vote according to their posterior as-if pivotal. At $O = 0$ small shareholders are never pivotal. Abstaining and voting in line with the public signal both result in the same decision, i.e. in the decision recommended by the public signal, as L will vote for it. Thus, abstaining is best response if $O = 0$. If L is informed, she only stays if the public signal is correct. Hence, if $O = 0$, L is pivotal and votes according to her information. If $O = -n$, she is not pivotal and so voting as-if pivotal implies that she votes according to her signal as well. An uninformed L who stays votes according to the public signal which is optimal if she is pivotal ($O = 0$) and optimal as-if she were pivotal ($O = -n$). Therefore, the equilibrium exists. □

H.8 Proof of Proposition 13

Proof. Conjecture EQ^{NE} in Proposition 13 is an equilibrium. Then, $P(O = 0) = 1$, since $O = 0$ reveals that L did not exit which means that L is informed and that the correct action is taken with certainty. $P(O = -2n) = \pi$, since $O = -2n$ reveals that L must have exited and was uninformed. In this case, the action recommended by the public signal is taken for sure and is correct with probability π. The share price after $O = -n$ reflects that there is uncertainty about whether L has exited or not. Formally, since $\mathbb{P}(L \textit{ stayed} \mid O = -n) = \mathbb{P}(L \textit{ exited} \mid O = -n) = \frac{1}{2}$, we have

$$
\begin{aligned}
P(O = -n) =& \frac{1}{2}\pi \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1}{2}(1-\pi) \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \\
&+ \frac{1}{2}\pi \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1}{2}(1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)
\end{aligned}
$$

Randomization probability. To be willing to randomize at $O = -n$, small shareholders must be indifferent between voting with and against the suggestion of the public signal given that $O = -n$ and given that they are pivotal. There are four cases in which small shareholders are pivotal after observing $O = -n$.

The first case is that L has stayed and thus is informed, the public signal is correct, and exactly $\frac{N-1}{2} - n$ out of $N - n - 1$ small shareholders vote with the public signal. In this case, a small shareholder wants to vote with the public signal. The probability of this event, $\mathbb{P}(i)$, is

$$\mathbb{P}(i) = \frac{1}{2}\pi \binom{N-n-1}{\frac{N-1}{2}-n} p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} \tag{72}$$

The second case is that L has stayed and thus is informed, the public signal is incorrect, and exactly $\frac{N-1}{2}$ out of $N-n-1$ small shareholders vote with the public signal. In this case, a small shareholder wants to vote against the public signal. The probability of this event, $\mathbb{P}(ii)$, is

$$\mathbb{P}(ii) = \frac{1}{2}(1-\pi) \binom{N-n-1}{\frac{N-1}{2}} p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n} \tag{73}$$

The third case is that L has exited and thus is uninformed, the public signal is correct and exactly $\frac{N-n-1}{2}$ out of $N-n-1$ small shareholders vote with the public signal. In this case, a small shareholder wants to vote with the public signal. The probability of this event, $\mathbb{P}(iii)$, is

$$\mathbb{P}(iii) = \frac{1}{2}\pi \binom{N-n-1}{\frac{N-n-1}{2}} p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} \tag{74}$$

The fourth case is that L has exited and thus is uninformed, the public signal is incorrect and exactly $\frac{N-n-1}{2}$ out of $N-n-1$ small shareholders vote with the public signal. In this case, a small shareholder wants to vote against the public signal. The probability of this event, $\mathbb{P}(iv)$, is

$$\mathbb{P}(iv) = \frac{1}{2}(1-\pi) \binom{N-n-1}{\frac{N-n-1}{2}} p^{*\frac{N-n-1}{2}}(1-p^*)^{\frac{N-n-1}{2}} \tag{75}$$

Thus, a small shareholder is willing to randomize at $O = -n$ if and only if

$$\mathbb{P}(i \mid piv) + \mathbb{P}(iii \mid piv) = \mathbb{P}(ii \mid piv) + \mathbb{P}(iv \mid piv) \tag{76}$$

$$\iff \mathbb{P}(i) + \mathbb{P}(iii) = \mathbb{P}(ii) + \mathbb{P}(iv) \tag{77}$$

To establish the properties of p^* described in Proposition 13, notice that $\mathbb{P}(iii) > \mathbb{P}(iv) \,\forall\, \pi >$

$\frac{1}{2}$. Thus, a necessary condition for (77) to hold is that $\mathbb{P}(i) < \mathbb{P}(ii)$, which equals

$$\frac{1}{2}\pi\binom{N-n-1}{\frac{N-1}{2}-n}p^{*\frac{N-1}{2}-n}(1-p^*)^{\frac{N-1}{2}} < \frac{1}{2}(1-\pi)\binom{N-n-1}{\frac{N-1}{2}}p^{*\frac{N-1}{2}}(1-p^*)^{\frac{N-1}{2}-n}$$

By the same steps as in the proof of Proposition 2, this reduces to:

$$\frac{1}{1+(\frac{1-\pi}{\pi})^{\frac{1}{n}}} < p^*$$

Thus, the mixing probability must (weakly) exceed the mixing probability from Proposition 2. This also establishes that the mixing probability must be weakly larger than 1/2.

Verify IC constraints. The equilibrium exists if and only if, given p^*, the following three IC constraints are satisfied.

First, it needs to be optimal for L to stay if she is informed and the public signal is correct. The expected pay-off from exiting is $\frac{1}{2}P(O=-n)+\frac{1}{2}P(O=-2n)$. The expected pay-off from staying is $\frac{1}{2}+\frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)$. Thus, the IC constraint is:

$$\frac{1}{2}P(O=-n)+\frac{1}{2}P(O=-2n) \leq \frac{1}{2}+\frac{1}{2}\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k) \tag{78}$$

Second, it needs to be optimal for L to stay if she is informed and the public signal is incorrect. The expected pay-off from exiting is as before but the expected pay-off from staying has changed such that this IC constraint is

$$\frac{1}{2}P(O=-n)+\frac{1}{2}P(O=-2n) \leq \frac{1}{2}+\frac{1}{2}\sum_{k=0}^{\frac{N-1}{2}}\mathbb{P}(p^*,N-n,k) \tag{79}$$

Third, it needs to be optimal for L to exit if she is uninformed. The alternative would be to stay and vote in line with the public signal. Thus, this IC constraint is

$$\frac{1}{2}P(O=-n)+\frac{1}{2}P(O=-2n) > \frac{1}{2}\pi+\frac{1}{2}\Big(\pi\sum_{k=\frac{N+1}{2}-n}^{N-n}\mathbb{P}(p^*,N-n,k)+(1-\pi)\sum_{k=0}^{\frac{N-1}{2}-n}\mathbb{P}(p^*,N-n,k)\Big) \tag{80}$$

Since $p^* \geq \frac{1}{1+(\frac{1-\pi}{\pi})^{\frac{1}{n}}} \geq \frac{1}{2}$, the IC for L after observing $s_{priv} = s_{pub}$ is slack. This holds because $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) \forall p^* > \frac{1}{2}$ and with equality if $p^* = \frac{1}{2}$. Thus, we only need to consider the ICs (79) and (80).

Consider a large blockholder. To establish that for any N, there exist combinations of n and π such that the equilibrium exists, we focus on $n = \frac{N-1}{2}$. Then, the equation which defines p^*, i.e. equation (77), simplifies to

$$\pi(1-p^*)^{\frac{N-1}{2}} + \pi \binom{\frac{N-1}{2}}{\frac{N-1}{4}} p^{*\frac{N-1}{4}} (1-p^*)^{\frac{N-1}{4}} = (1-\pi) p^{*\frac{N-1}{2}} + (1-\pi) \binom{\frac{N-1}{2}}{\frac{N-1}{4}} p^{*\frac{N-1}{4}} (1-p^*)^{\frac{N-1}{4}} \quad (81)$$

For $\pi = \frac{1}{2}$, this simplifies to

$$(1-p^*)^{\frac{N-1}{2}} + \binom{\frac{N-1}{2}}{\frac{N-1}{4}} p^{*\frac{N-1}{4}} (1-p^*)^{\frac{N-1}{4}} = p^{*\frac{N-1}{2}} + \binom{\frac{N-1}{2}}{\frac{N-1}{4}} p^{*\frac{N-1}{4}} (1-p^*)^{\frac{N-1}{4}} \quad (82)$$

and further to $p^* = \frac{1}{2}$.

Thus, for $n = \frac{N-1}{2}$ and $\pi = \frac{1}{2}$ we have that $p^* = \frac{1}{2}$. This simplifies the IC of the blockholder when $s_{pub} \neq s_{priv}$ (79) to

$$\frac{1}{2} + \frac{1}{2} \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) \geq \frac{1}{4} + \frac{1}{4} \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{1}{4} \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) \quad (83)$$

where $\sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) = \frac{1}{2}$ and thus the IC simplifies further to

$$\frac{1}{8} + \frac{1}{4} \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) > 0 \quad (84)$$

which holds. Hence, it is indeed optimal for the blockholder to stay when $s_{pub} \neq s_{priv}$. Moreover, the incentive compatibility constraint of the blockholder who observed $s_{pub} \neq s_{priv}$ is slack which means that it will also be satisfied for π close to but above $\frac{1}{2}$.

Similarly, the IC constraint of the uninformed L, equation (80), simplifies to

$$\frac{1}{4} + \frac{1}{4} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{1}{4} \sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(\frac{1}{2}, N-n, k) < \frac{3}{8} + \frac{1}{4} \sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) \quad (85)$$

where $\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k) = \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$ and thus the inequality becomes $\sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(\frac{1}{2}, N-n, k) < \frac{1}{2}$ which holds. Thus, it is indeed optimal for the uninformed L to exit. Moreover, the constraint is slack which means that it will also be satisfied at π close to but above $\frac{1}{2}$.

As all IC constraints are satisfied, the equilibrium exists. Moreover, as all IC constraints are slack at $\pi = \frac{1}{2}$, the equilibrium also exists for π close to but above $\frac{1}{2}$. □

H.9 Proof of Proposition 14

Proof. Consider $n = \frac{N-1}{2}$ and $\pi = \frac{1}{2}$. Then, we know from the proof of Proposition 13 that $p^* = \frac{1}{2}$. This means that ex-ante firm value of EQ^{NE} is

$$\mathcal{W}^{NE} = \frac{3}{8} + \frac{1}{4} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) + \frac{1}{4} \sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$$

where $\sum_{k=\frac{N-n+1}{2}}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \frac{1}{2}$ and thus ex-ante firm value simplifies to

$$\mathcal{W}^{NE} = \frac{1}{2} + \frac{1}{4} \sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k).$$

Since $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) < 1$, we have that $\mathcal{W}^{NE} < \frac{3}{4}$. Without public information, ex-ante firm value in the equilibrium with abstentions is $\mathcal{W}_A^B = \frac{3}{4}$. Thus, we have $\mathcal{W}^{NE} < \mathcal{W}_A^B$. Moreover, for low π, i.e. $\pi < 2 - \sqrt{2}$, we have that $\mathcal{W}_{NA}^E = \mathcal{W}_A^E = \frac{1}{2} + \frac{1}{4}\pi < \frac{3}{4} = \mathcal{W}_A^B$. Thus, for $n = \frac{N-1}{2}$ and $\pi = \frac{1}{2}$ welfare is lower without public information than with public information. The fact that the inequalities hold strictly indicates that they also hold for π close to but above $\frac{1}{2}$. □

H.10 Proof of Proposition 15:

Proof. Conjecture EQ^B in Proposition 15 is an equilibrium. Then the share price is $\mathcal{W}^B = \sum_{k=\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k)$. If the blockholder follows her conjectured strategy and purchases n additional shares, per-share firm value is $\mathcal{W}^B$. Thus, the blockholder purchases at the fair price. The blockholder strictly prefers purchasing to holding the existing shares as holding would result in per-share firm value of $\sum_{k=\frac{N+1}{2}-n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) < \mathcal{W}^B$. Thus, deviating to holding would result in a lower value of the blockholder's existing shares. The blockholder is indifferent between following the conjectured strategy (purchasing) and selling her shares. Selling would result in $\mathcal{W}^B$ on the existing shares, which is like in the conjectured equilibrium. Moreover, as there is no trading profit in the conjectured equilibrium, the blockholder is indifferent between selling n shares and purchasing n additional shares. Thus, purchasing n additional shares is a best response. As there is no public information and no information contained in being pivotal, voters are indifferent whether to vote in favor of the proposal or against and thus are willing to randomize with probability $p^* = 1/2$ as conjectured. Thus, the equilibrium exists. □

H.11 Proof of Proposition 16:

Proof. Conjecture EQ_{NR}^T in Proposition 16 is an equilibrium. Then, the share price is $\mathcal{W}_{NR}^T = \pi$. When $s_{pub} = s_{priv}$, $\mathbb{E}[W \mid s_{pub} = s_{priv}] = 1$ as the correct decision will be taken for sure. Thus, shares are undervalued. The blockholder therefore strictly prefers purchasing additional shares to holding as both would result in per-share value of the existing shares of 1, but as purchasing generates an additional trading profit on the newly acquired shares. The blockholder also strictly prefers purchasing to selling as she would sell undervalued shares. Thus, purchasing is a best response if $s_{pub} = s_{priv}$.

When $s_{pub} \neq s_{priv}$,, $\mathbb{E}[W \mid s_{pub} \neq s_{priv}] = 0$ as the incorrect decision will be taken with certainty. Thus, shares are overvalued. The blockholder therefore strictly prefers selling shares to holding as holding would yield a per-share payoff of zero. Selling is also strictly preferred to purchasing as the blockholder would make a zero payoff on existing shares and pay a positive price for shares which result in zero payoff. Hence, purchasing shares would result in a loss. Thus, selling is strictly optimal.

Small shareholders are never pivotal and thus vote sincerely, which is to vote as suggested by the public signal. Hence, the conjectured strategy is optimal and the equilibrium exists. □

H.12 Proof of Proposition 17:

Proof. Conjecture EQ_R^T in Proposition 17 is an equilibrium. Then, the share price is $\mathcal{W}_R^T$.

Trading. After a correct public signal and purchasing additional shares as conjectured, $\mathbb{E}[W \mid s_{pub} = s_{priv}] = \sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k)$. An alternative for L is to sell her shares at price $\mathcal{W}_R^T$. Thus, purchasing additional shares is optimal if and only if $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \mathcal{W}_R^T$ or equally $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$. We show below that this holds if π is not too small.

After an incorrect public signal, L can sell her shares at price $\mathcal{W}_R^T$. Holding would result in per-share firm value of $\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k)$. Purchasing would result in per-share firm value of $\sum_{k=0}^{\frac{N+n-1}{2}} \mathbb{P}(p^*, N-n, k)$ and since $\sum_{k=0}^{\frac{N-1}{2}} \mathbb{P}(p^*, N-n, k) < \sum_{k=0}^{\frac{N+n-1}{2}} \mathbb{P}(p^*, N-n, k)$ it is wlog to restrict attention to the deviation of purchasing. Thus, selling is optimal if only if $\mathcal{W}_R^T > \sum_{k=0}^{\frac{N+n-1}{2}} \mathbb{P}(p^*, N-n, k)$. This constraint is slack by the previously established condition that $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$.

Voting. A small shareholder is pivotal in two cases. First, when the public signal is correct, the blockholder has purchased, and $\frac{N+n-1}{2} - 2n$ out of $N-n-1$ small shareholders vote as suggested by the public signal. Second, when the public signal is incorrect, the blockholder has exited, but $\frac{N-n-1}{2}$ out of $N-n-1$ small shareholders vote as suggested by the public signal. This pins down the mixing probability given in the proposition.

To see that for low π, we get $p^* < \frac{1}{2}$, rewrite (64) as:

$$\frac{\pi}{1-\pi} \frac{\binom{N-n-1}{\frac{N+n-1}{2}-2n}}{\binom{N-n-1}{\frac{N-n-1}{2}}} = (\frac{p^*}{1-p^*})^n \tag{86}$$

Since $\binom{N-n-1}{\frac{N-n-1}{2}}$ is the central binomial coefficient, $\binom{N-n-1}{\frac{N-n-1}{2}} > \binom{N-n-1}{\frac{N+n-1}{2}-2n}$ and thus for $\pi = 1/2$ the LHS is less than 1. The RHS can only be less than 1 if $p^* < 1/2$.

Overall. Thus, this equilibrium exists if and only if $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$ which means that p^* cannot be too small and therefore that π cannot be too small. For π close to 1, p^* close to 1, the condition holds and thus the equilibrium exists. □

H.13 Proof of Proposition 18:

In any equilibrium, the share price P^* must satisfy

$$P^* = [W] = \pi\mathbb{E}[W \mid s_{pub} = \omega] + (1-\pi)\mathbb{E}[W \mid s_{pub} \neq \omega] \tag{87}$$

where the expectation is calculated based on equilibrium strategies.

Proof. There cannot exist an equilibrium where the blockholder makes the same trading decision independent of whether the public signal is correct or not.

Suppose to the contrary that the blockholder exits for $s_{priv} = s_{pub}$ and $s_{priv} \neq s_{pub}$. Then a shareholder strictly prefers to vote as suggested by the public signal and the share price is π. Thus, when the blockholder observes that $s_{priv} = s_{pub}$, she knows that the correct decision will be taken with certainty and thus that shares will pay off 1. Thus, shares are underpriced and it is strictly optimal to deviate to purchasing shares.

Suppose that the blockholder purchases n additional shares for $s_{priv} = s_{pub}$ and $s_{priv} \neq s_{pub}$. Then small shareholders either all vote as suggested by the public signal, or mix with the mixing probability as defined in EQ^{NE} (rescaling the post purchase block size to be n) and therefore $p^* > 1/2 \forall \pi > 1/2$. In both cases, shares are overvalued if $s_{priv} \neq s_{pub}$. Thus, when the blockholder observes that $s_{priv} \neq s_{pub}$, she strictly prefers to deviate and exit.

Suppose that the blockholder does not trade, i.e. neither exits nor purchases n additional shares. Then, small shareholders mix as in EQ^{NE} and therefore $p^* > 1/2 \forall \pi > 1/2$. Therefore shares are overvalued if $s_{priv} \neq s_{pub}$. Thus, when the blockholder observes $s_{priv} \neq s_{pub}$, she strictly prefers to deviate and exit.

There cannot exist an equilibrium where the blockholder holds, i.e. does not trade, in some cases. It is optimal to hold if and only if shares are correctly valued after observing the public signal. Suppose in a conjectured equilibrium the blockholder holds if the public signal is incorrect. This is optimal if and only if $\mathbb{E}[W \mid s_{pub} \neq \omega] = \mathbb{E}[W]$ which, by (87), implies that we must have $\mathbb{E}[W \mid s_{pub} = \omega] = \mathbb{E}[W]$ and therefore $\mathbb{E}[W \mid s_{pub} = \omega] = \mathbb{E}[W \mid s_{pub} \neq \omega]$. This can only be satisfied if small shareholders mix, i.e. $p \in (0, 1)$. But if small shareholders mix, it is strictly optimal for the blockholder to deviate at the info set where she was conjectured to hold and purchase additional shares instead of holding. Formally, $\mathbb{E}[W \mid s_{pub} \neq \omega, buy] > \mathbb{E}[W \mid s_{pub} \neq \omega, hold] = P^*$. Thus, there is a contradiction and there cannot exist an equilibrium where the blockholder holds.

There cannot exist an equilibrium where the blockholder sells shares when the public signal is correct and buys shares if it is incorrect. Conjecture that the blockholder sells if the public signal is correct and purchases if it is incorrect. Small shareholders either play a pure or a mixed strategy. The only pure strategy which could be an equilibrium is to vote as suggested by the public signal. However, if small shareholders vote as suggested by the public signal, it is strictly optimal for the blockholder to deviate from the conjectured trading strategy as, when $s_{priv} = s_{pub}$, the blockholder knows that shares will pay off 1 which makes them underpriced and makes it optimal to buy, rather than to sell.

When small shareholders play a mixed strategy, they vote as suggested by the public signal with probability p^* which solves

$$\pi \binom{N-n-1}{\frac{N-n-1}{2}} p^{*\frac{N-n-1}{2}} (1-p^*)^{\frac{N-n-1}{2}} = (1-\pi) \binom{N-n-1}{\frac{N+n-1}{2}} p^{*\frac{N+n-1}{2}} (1-p^*)^{\frac{N-3n-1}{2}} \tag{88}$$

$$\frac{\pi}{1-\pi} \frac{\binom{N-n-1}{\frac{N-n-1}{2}}}{\binom{N-n-1}{\frac{N+n-1}{2}}} = (\frac{p^*}{1-p^*})^{\frac{n}{2}} \tag{89}$$

where $\frac{\pi}{1-\pi} > 1$ since $\pi > \frac{1}{2}$ and $\frac{\binom{N-n-1}{\frac{N-n-1}{2}}}{\binom{N-n-1}{\frac{N+n-1}{2}}} > 1$ since $\binom{N-n-1}{\frac{N-n-1}{2}}$ is the central binomial coefficient and thus is larger than any other binomial coefficient such as $\binom{N-n-1}{\frac{N+n-1}{2}}$. Thus, if small shareholders play a mixed strategy, the mixing probability must satisfy $p^* > 1/2$. We now establish a contradiction between this mixing strategy and the blockholder's trading strategy.

Purchasing shares if $s_{pub} \neq \omega$ is only optimal if $\mathbb{E}[W \mid s_{pub} \neq \omega, buy] \geq P^*$. For the conjectured equilibrium to exist, by (87), we must have $\mathbb{E}[W \mid s_{pub} = \omega, sell] \leq P^*$. However, since $p^* > 1/2$, it must be that $\mathbb{E}[W \mid s_{pub} = \omega, buy] > \mathbb{E}[W \mid s_{pub} \neq \omega, buy]$. Thus, it must be strictly optimal for the blockholder to deviate to purchasing shares when she observes $s_{priv} = s_{pub}$ since $p^* > 1/2$ and therefore $\mathbb{E}[W \mid s_{pub} = \omega, buy] > \mathbb{E}[W \mid s_{pub} \neq \omega, buy] > P^* > \mathbb{E}[W \mid s_{pub} = \omega, sell]$. Thus, there is a contradiction and the conjectured equilibrium does not exist. Therefore, no equilibrium other than EQ^T_{NR} and EQ^T_R can exist. $\square$

H.14 Proof Proposition 19:

Proof. First, compare EQ^T_R and EQ^B. From Proposition 17 we know that there exists a $\pi > \frac{1}{2}$ at which $p^* = \frac{1}{2}$. Consider this π. We now establish that at this π, $\mathcal{W}^B > \mathcal{W}^T_R$. If $s_{pub} = s_{priv}$, $\mathbb{E}[W \mid s_{pub} = \omega, EQ^B] = \mathbb{E}[W \mid s_{pub} = \omega, EQ^T_R]$. In both cases, the blockholder has size $2n$ and votes correctly while small shareholders mix with probability $1/2$. But if, at this level of π, $s_{priv} \neq s_{pub}$, $\mathbb{E}[W \mid s_{pub} \neq \omega, EQ^B] > \mathbb{E}[W \mid s_{pub} \neq \omega, EQ^T_R]$. While in both cases shareholders mix with probability $1/2$, in the benchmark the blockholder has size $2n$ while in the case with public information she has exited and thus has size zero. Thus, there are fewer informed votes. Formally, $\mathbb{E}[W \mid s_{pub} \neq \omega, EQ^B] = \sum_{k=0}^{\frac{N-1}{2}-n} \mathbb{P}(\frac{1}{2}, N-n, k)$ while $\mathbb{E}[W \mid s_{pub} \neq \omega, EQ^T_R] = \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(\frac{1}{2}, N-n, k)$ and thus $\mathbb{E}[W \mid s_{pub} \neq \omega, EQ^T_R] < \mathbb{E}[W \mid s_{pub} \neq \omega, EQ^B]$. Since $\mathcal{W} = \pi \mathbb{E}[W \mid s_{pub} = \omega] + (1-\pi)\mathbb{E}[W \mid s_{pub} \neq \omega]$, it follows that $\mathcal{W}^B > \mathcal{W}^T_R$ for the $\pi > \frac{1}{2}$ at which $p^* = \frac{1}{2}$.

Next, compare EQ^T_{NR} and EQ^B. Since $\mathcal{W}^B > 1/2$ and $\mathcal{W}^T_{NR} = \pi$, it follows that for π sufficiently small $\mathcal{W}^T_{NR} < \mathcal{W}^B$.

To see that there exist π where EQ^B results in higher welfare than any equilibrium with public information, consider $\pi = \frac{1}{2}$. Recall that $\mathcal{W}^T_R = \pi \sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) + (1-\pi) \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$. At $\pi = \frac{1}{2}$, we have $p^* < \frac{1}{2}$ and

$$\mathcal{W}^T_R = \frac{1}{2} \sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) + \frac{1}{2} \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k). \tag{90}$$

Since $p^* < \frac{1}{2}$, it must be that $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) < \sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(\frac{1}{2}, N-n, k) = \mathcal{W}^B$ by the usual FOSD argument. Moreover, EQ^T_R exists if and only if $\sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$. Thus, there are two possible cases:

Either EQ^T_R does not exist at $\pi = \frac{1}{2}$ and then $\mathcal{W}^B > \max\{\mathcal{W}^T_R, \mathcal{W}^T_{NR}\}$ follows from $\mathcal{W}^B > \mathcal{W}^T_{NR}$. Or EQ^T_R exists and then we must have $\mathcal{W}^B > \sum_{\frac{N+n+1}{2}-2n}^{N-n} \mathbb{P}(p^*, N-n, k) > \sum_{k=0}^{\frac{N-n-1}{2}} \mathbb{P}(p^*, N-n, k)$ and thus $\mathcal{W}^B > \mathcal{W}^T_R$ which establishes that $\mathcal{W}^B > \max\{\mathcal{W}^T_R, \mathcal{W}^T_{NR}\}$.

Thus, for every (N, n) there exist π sufficiently small such that $\mathcal{W}^B > \max\{\mathcal{W}_R^T, \mathcal{W}_{NR}^T\}$. $\square$