Debt and Taxes: Revisited in Dynamics

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Abstract

This paper analyzes optimal leverage dynamics with personal and corporate taxes and financial distress costs. Key to the analysis, the marginal tax benefit of debt depends on whether the debt is used for financing real investment or financial restructuring. The theory features continuous leverage adjustments and no security issuance costs. There are two local leverage targets for firms with leverage above or below a threshold. The model generates a leverage distribution that matches the data, including many zero-leverage firms. Policymakers can reduce distress costs without losing tax revenue by raising the personal tax rate and lowering the corporate tax rate.

JEL classification: G32, G34, G38, H21, H25
Keywords: Capital structure, Tax benefits of debt, Trade-off theory, Restructuring, Low-leverage puzzle, Tax efficiency

1 Introduction

A fundamental question in corporate finance is how firms manage their capital structures. The tax shield of debt is traditionally viewed as a key determinant of firms' leverage. However, firms seem to have left money on the table by being underleveraged, as noted by Miller (1977): "For big businesses, at least, the supposed trade-off between tax gains and bankruptcy costs looks suspiciously like the recipe for the fabled horse-and-rabbit stew-one horse and one rabbit." In particular, over 1/5 of firms have close to zero leverage (Strebulaev and Yang, 2013). These low-leverage firms typically have high profits and good liquidity, which suggests a low risk of financial distress. It is, therefore, puzzling that these firms do not lever up to take advantage of the tax shield.

This paper demonstrates that corporate and personal taxes themselves—not other frictions—can explain this puzzle. Most literature and textbooks traditionally interpret the tax shield of debt using Miller's (1977) definition, $(1-\tau_b)-(1-\tau_c)(1-\tau_e)$, which evaluates the tax benefits of firms' outstanding debt by comparing the tax cost difference between delivering each dollar to debt holders and delivering each dollar to equity holders. When firms pay debtholders, the debtholders are charged a personal tax on income from bonds at rate τ_b. When firms pay equity holders, there is a corporate income tax at rate τ_c charged on firms' profits and a personal tax on income from equity at rate τ_e charged on equity holders' capital gains.[1] However, this definition may not properly assess the tax benefits of issuing new debt because it only counts taxes on firms' future cash flow but fails to capture the tax consequence of the use of the proceeds from the debt issue. As the proceeds from debt are available to shareholders and added to the firm's value, they are subject to a personal tax on income from equity, either in the form of a dividend tax if they are directly distributed as dividends or in the form of a capital gains tax otherwise.

I develop a dynamic capital structure model with continuous leverage adjustments and no security issuance costs following DeMarzo and He (2021). The frictions are just corporate and personal taxes and bankruptcy costs. I show that the tax benefits of debt issuance can be defined as the sum of two parts: (1) corporate tax savings minus personal tax costs on

[1]Personal taxes on equity income may take various forms in practice. For example, in the US by 2019, only 24% of stocks are held by directly taxable accounts, 40% of stocks are held by foreign investors paying income taxes to their home countries, 30% of stocks are held by retirement accounts that are usually taxed at distribution, and the rest held by nonprofits and the government that are tax-exempt (Rosenthal and Burke, 2020). Although a substantial part of the stock holdings are not directly taxable, most are still subject to income taxes. Moreover, Lin and Flannery (2013) find that personal tax is an important determinant of firms' leverage by studying evidence from the 2003 tax cut.

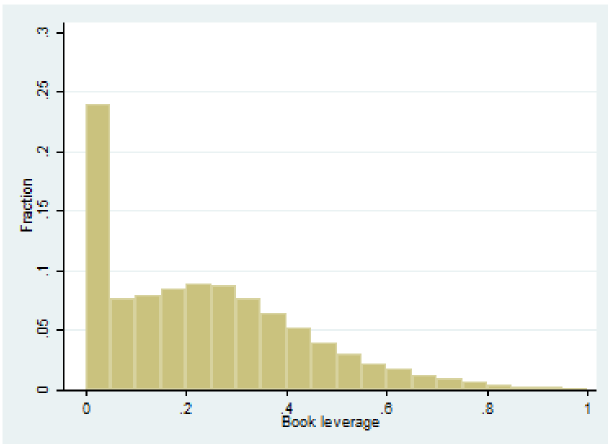
interest expenses and (2) personal tax savings/costs on the net reduction of equity payouts. The model generates leverage dynamics in which capital structure policies depend on firms' financing needs for real investments and debt repayments, and the marginal source of financing may be debt or equity. As a result, a firm's leverage slowly adjusts to one target when leverage is above a threshold or another when leverage is below the threshold, and may switch targets if leverage crosses the threshold due to shocks. With reasonable parameter choices, the model's simulated leverage distribution matches the leverage distribution of Compustat firms, with zero-leverage firms representing about 1/4 of all firms.

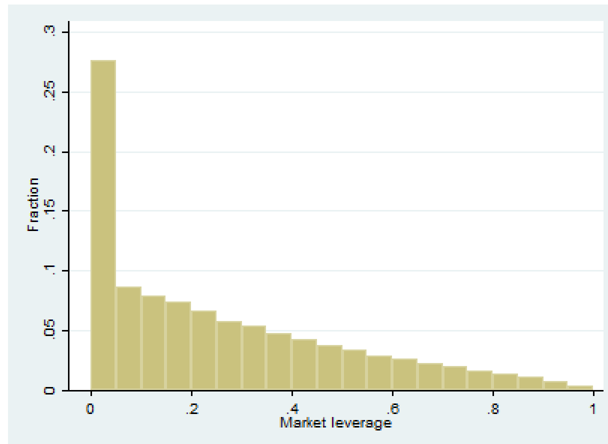
It has been a long-standing puzzle in the literature that firms seem to exploit the tax shield of debt inadequately. Graham (2000) notes low-leverage firms' puzzling choices not to lever up in his conclusion: "Paradoxically, large, liquid, profitable firms with low expected distress costs use debt conservatively." Moreover, Strebulaev and Yang (2013) demonstrate that trade-off models of capital structure in the previous literature (e.g., Goldstein, Ju, and Leland, 2001; Strebulaev, 2007) hardly account for the cross-sectional distribution of corporate leverage ratios, especially the presence of numerous firms with less than 5% leverage. Figure 1 shows the cross-sectional distribution of book and market leverage among Compustat nonfinancial firms in the U.S. from 1962 to 2021. The figure demonstrates that about 1/4 of the firm-year observations have leverage below 5%. Furthermore, the fractions of observations within each 5% bin of market leverage ratios decrease in leverage levels. It is not surprising that standard trade-off models with a single positive leverage target cannot generate this type of distribution. Given the leverage target, we would expect a bell-shaped cross-sectional distribution that tops at the leverage target.

DeMarzo and He (2021) explain the puzzle by a commitment problem: investors expect firms to maintain high leverage and charge a high credit spread so that firms gain no benefit from debt. In their model, firms either never issue debt or continuously issue debt toward a target. In the data, however, many currently low-leverage firms had greater leverage in their earlier years. For example, Nike's and Costco's market leverages, as seen in Figure 2, were as high as 30% in the 1980s, but were lower than 5% in recent years. [2] In addition, firms that slowly adjust to a positive leverage target, as in their model, are unlikely to generate the shape of the fractions of observations in Figure 1 that remain after excluding the zero-leverage firms. A fully satisfactory explanation has yet to be found.

[2] Among firms with less than 5% leverage in the sample of Figure 1, about 2/3 had over 10% leverage, and half had over 20% leverage. DeAngelo et al. (2018) documents that most firms deleverage from their historical peaks to almost zero market leverage. Such deleveraging is suboptimal for firms in the DeMarzo and He (2021) model, as it leads to a value transfer from shareholders to debtholders.

Figure 1: Leverage of Compustat nonfinancial firms in the US, 1962-2021





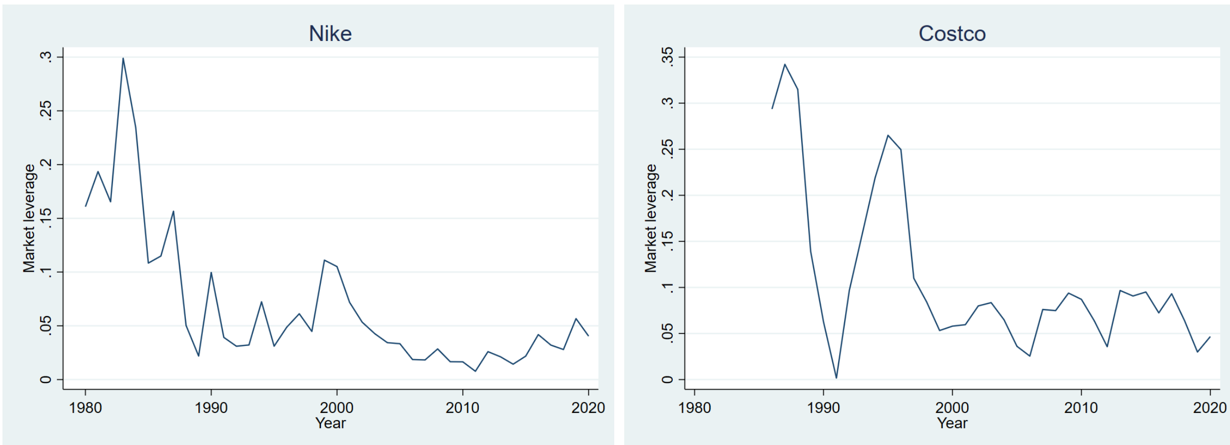
Notes. Panel (a) plots the book leverage, defined by $\frac{\text{Book value of debt } (DLTT+DLC)}{\text{Book value of asset } (AT)}$, of firms headquartered in the U.S. in the Compustat-CRSP merged data set from 1962 to 2020 annually. Firms in the financial industry [Standard Industrial Classification (SIC) codes 6000-6999], utilities (SIC codes 4900-4999), American depository receipts ("ADR") (SIC codes 8800-8999), non-publicly traded firms [stock ownership variable (STKO) 1 or 2], and firm-years with total book value of assets (AT) less than 10 million inflation-adjusted year 2000 dollars are excluded. There are 283,702 firm-year observations. Panel (b) plots the market leverage, defined by $\frac{\text{Book value of debt } (DLTT+DLC)}{\text{Book value of debt } (DLTT+DLC)+\text{Market value of equity } (CSHO\times PRCC\_F)}$ of these firms.

This paper argues that firms may keep their leverage as low as zero—not due to the high costs of debt or precautionary motives, but because they face no tax benefits from issuing debt. Suppose, for example, a low-leverage firm attempts to shield its earnings with more interest expenses (net of interest income). In this case, the firm recapitalizes by issuing debt and distributing the proceeds from debt issuance to shareholders as payouts. Such payouts lead to tax consequences not captured by Miller's definition of the tax benefits of outstanding debt. Shareholders must pay dividend taxes immediately if the proceeds are distributed as dividends. Otherwise, the share repurchase increases the stock value, thereby increasing shareholders' capital gains taxes. [3] Assuming that the firm's payout policy is constant over time, this incremental personal tax cost for shareholders fully offsets their personal tax savings from interest expenses. Indeed, debt issuance does not directly result in shareholders' personal tax savings on income from equity; rather, it only transfers it intertemporally. The tax savings (or costs) for such recapitalization, then, depends on

[3]Here, I consider the increase in outstanding debt and distribution of the proceeds from debt issuance separately. While additional outstanding debt reduces stock value, the share repurchase offsets such reduction.

Figure 2: Time series of Market leverage for Nike and Costco

(a) Nike (b) Costco



Notes. Panel (a) and (b) plot the time series of Nike and Costco's market leverage at the annual frequency, defined by book value of debt / (book value of debt + market value of equity), from their first observation in Compustat data to 2020, respectively.

comparing the firm's corporate tax rate and the bondholders' personal income tax rate. Since the top federal personal tax rates are higher than the top corporate tax rates in most years, it is reasonable that firms have little or no tax incentive for such recapitalization.[4]

In the model, the firm can continuously adjust its leverage without issuance costs and default strategically, as in DeMarzo and He (2021). I solve a smooth equilibrium without discrete leverage adjustments. Equity value and debt price functions are determined by a pair of piecewise non-linear non-homogeneous differential equations with no known closed-form solutions. I develop a novel numerical method using a fourth-order Runge-Kutta-Nystrm algorithm to solve the differential equations. The results of the model are expressed directly as functions of the equity value and debt price functions. These analytical expressions deliver interesting results even before the model is fully solved.

The tax benefits (or costs) of issuing a marginal dollar of debt can be decomposed into two parts. First, each dollar of the firm's EBIT is charged either a corporate tax or bondholders' personal tax, depending on whether or not it is used for interest expenses. The firm, thus, saves the difference between the two tax rates with each dollar of interest payment. Second, debt issuance leads to changes in current and future payouts. Shareholders save the personal tax rate on income from equity with each dollar reduction in net payouts. This part is

[4]The empirical literature (e.g., Ang, Bhansali, and Xing, 2010; Longstaff, 2011) finds that the implicit tax rates priced in bonds are close to the top federal tax rates.

negative when the proceeds from debt are distributed as payouts and positive otherwise. With reasonable corporate and personal tax rates, a marginal debt issuance brings the firm positive or negative tax savings depending on its current and expected future financing margins. When the firm's external financing need is monotone in its leverage, I characterize its optimal financing policies, depending on parameters, by at most four financing rules for different regions of state variables. The marginal sources of financing in each region are equity issuance, debt issuance/repurchase, and dividend distribution.

The model generates leverage dynamics with two local leverage targets for firms with higher and lower leverage than a threshold level. When the corporate tax rate is not higher than the personal tax rate for bond income, high-leverage firms slowly adjust to a high leverage target, and low-leverage firms slowly adjust to zero leverage. Since firms with higher leverage need more external financing due to payments to debtholders, leverage differences between firms persist even if they have identical earnings flow afterward. Such persistence in leverage differences is consistent with empirical evidence in the literature (e.g., Baker and Wurgler, 2002; Lemmon, Roberts, and Zender, 2008). A simulation of the model generates a stationary cross-sectional leverage distribution that matches the leverage distribution in the data, in which about 1/4 of firms have close to zero leverage. In addition, the model generates a novel testable empirical implication that firms with and without external financing need would adjust their leverage differently and have different responses to tax rate changes.

The model has several implications for economic efficiency. First, it allows us to determine the levels at which a policymaker should set the tax rates. This conclusion is possible because the corporate and personal taxes on equity income in the model affect the firm's financing policies differently—in contrast to Miller's formula in which they are always charged together. I decompose the firm's pre-tax value into the values of equity, debt, tax revenue, and expected bankruptcy loss. I find that when a government aims to collect a target level of tax revenue and reduce expected deadweight bankruptcy loss due to leverage distortions, it can improve efficiency by taxing equity holders more at the personal level and less at the corporate level. The firm prefers to issue debt with a longer maturity, but a shorter maturity improves welfare.[5] In an extension with endogenous investment and debt overhang, a corporate tax cut increases investments more than a payout tax cut in the short run.[6]

[5] Long maturity reduces rollover risk as in He and Xiong (2012) and Diamond and He (2014). As a result, the firm takes higher leverage and assumes more bankruptcy risk. In DeMarzo and He (2021), firms are indifferent to longer and shorter maturity since they gain no benefit from debt. Forces that favor shorter maturity, for example, could be investors' liquidity preference (He and Milbradt, 2014) and commitment concerns on long-term debt (Hu et al., 2021).

[6] The short-run effects are characterized by investment changes without change in leverage since leverage

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 discusses the differences between the tax consequences of debt issuance and changes in the tax shield value of outstanding debt. Section 4 sets up the model and characterizes optimal financing policies. Section 5 demonstrates the leverage dynamics and leverage distribution generated by the model. Section 6 discusses the model's welfare implications. Section 7 concludes.

2 Literature Review

This paper contributes primarily to the extensive literature on the trade-off theory of capital structure. Ai, Frank, and Sanati (2020) provide a thorough review of this literature. The literature (for example, Leland, 1994a, 1994b, Goldstein et al., 2001, and DeMarzo and He, 2021) typically assume a constant rate of tax benefits on interest payments, which can be interpreted as the corporate tax or the tax benefits defined by Miller (1977). Exceptions include models with real investment opportunities that incorporate a wide range of frictions (e.g., Hennessy and Whited, 2005, 2007; Gamba and Triantis, 2008) and models that capture the deferral of capital gains tax (Lewellen and Lewellen, 2006; Bolton, Chen, and Wang, 2014). In contrast, I argue that the net tax consequence of debt issuance can be a cost instead of a benefit for firms. That is because, in a leveraged recapitalization, tax costs on distributing the proceeds from debt as payouts can exceed the tax savings from future interest expenses.

My consideration of different tax benefits from debt issuance for financing investments and leveraged restructuring is closely related to Hennessy and Whited (2005), which points out the importance of analyzing the trade-off theory dynamically. In their discrete-time model with one-period debt, Hennessy and Whited (2005) show that marginal debt issuance generates more tax savings when replacing equity issuance than financing distributions. Compared to their quantitative model, my model highlights how taxes affect leverage dynamics in a framework with fewer frictions, showing that taxes can generate the observed leverage distribution without security issuance costs.[7] The continuous-time model with long-

adjustments are slow. In the long run, however, policies also affect investments through firms' leverage changes. Decreasing the tax benefits of debt reduces debt usage, alleviates debt overhang, and improves investment. DeMarzo and He (2021) and Crouzet and Tourre (2021) show that policies cutting the cost of debt may reduce investment in the long run.

[7] Hennessy and Whited (2005) assume the top corporate tax rate to be higher than the personal tax rate on bond income. In their model, firms have low leverage mainly because of precautionary incentives. As a result, their model is unlikely to match the fraction of zero-leverage firms with a reasonable equity flotation

term debt allows me to construct a general definition of tax benefits and analyze more characteristics of capital structure policies. Such a neat setting is also more suitable for studying the implications for tax policies. Ivanov, Pettit, and Whited (2020) also model a nonstandard relationship between tax rates and leverage to rationalize their empirical findings that small private firms' leverage rises after tax cuts. Their model features a different mechanism, in which higher tax rates raise the default threshold since defaults are triggered when firms cannot repay their debt by after-tax earnings, thus increasing the cost of debt.

My theory contributes to a large group of continuous-time dynamic capital structure models pioneered by, for example, Fischer et al. (1989) and Leland (1994). My model is closest to DeMarzo and He (2021), in which firms can adjust capital structure dynamically at no cost.[8] DeMarzo and He (2021) show that the increase in credit spread caused by the leverage ratchet effect can fully offset the tax benefits of debt when firms cannot commit to a leverage policy.[9] I introduce personal taxes—and, most importantly, the personal taxation difference between positive and negative payouts—to the DeMarzo and He (2021) framework. Leverage dynamics in my model differ from those in the previous literature in several ways. First, there are two local leverage targets for firms with higher or lower leverage compared to a threshold, while the standard trade-off theory has a single leverage target. Second, the marginal source of financing can be debt or equity, and firms may repurchase debt at the cost of the leverage ratchet effect. And third, firms gain from the tax shield even without committing to a leverage policy.

Other recent papers that feature non-standard leverage dynamics with multiple regions of financing rules include Malenko and Tsoy (2020) and Bolton et al. (2021). Malenko and Tsoy (2020) study optimal time-consistent debt policies and find that the firm actively adjusts to a leverage target in the stable region but stops adjustments when getting a large enough negative shock and falling into the distress region. Bolton et al. (2021) study leverage dynamics with equity issuance costs, which generates a pecking-order preference due to avoidance of costly equity issuance. Equity issuance costs can also generate asymmetric benefits of debt with positive and negative payouts (e.g., Cooley and Quadrini, 2001; Hennessy and Whited, 2005; Bolton, Wang, and Yang, 2021). The difference is that equity issuance costs increase the benefits of debt when it replaces equity issuance, while a personal tax on

cost.

[8] Earlier models assume firms never restructure (e.g., Leland, 1994a, 1994b) or retire all debt when restructuring (e.g., Fischer, Heinkel, and Zechner, 1989; Goldstein, Ju, and Leland, 2001; Strebulaev, 2007; Dangl and Zechner, 2021).

[9] See, for instance, He and Milbradt (2016), Admati et al. (2018), and Demarzo (2019) for discussions of the commitment problem.

payouts decreases the benefits of debt when it finances payouts. Compared to these papers, my model creates non-standard leverage dynamics by a different mechanism — corporate and personal taxes, and generates a realistic cross-sectional distribution of leverage.

Explaining the cross-sectional distribution of firms' leverage—and especially the levels of low- and zero-leverage firms—has historically posed a critical challenge to the trade-off theory. The benchmark Leland (1994) model, in which firms cannot adjust their debt levels as earnings grow, predicts a leverage ratio of over 70% with reasonable values of parameters, which is way too high compared to the average market leverage of 26% for Compustat firms in the period between 1987 and 2003 (Strebulaev and Yang, 2013). Some papers (e.g. Tserlukevich, 2008) suggest that firms should fully shield their corporate income from taxes by setting interest expenses equal to revenue when leverage adjustments are costless. Models with infrequent leverage adjustments, such as Goldstein et al. (2001), Ju et al. (2005), and Strebulaev (2007), generate more reasonable average leverage ratios, but very few or no firms with leverage below 5%. Finally, models with endogenous investment and fixed costs (e.g., Hennessy and Whited, 2005; Hackbarth and Mauer, 2012; Kurshev and Strebulaev, 2015) can generate zero-leverage firms, but they are unlikely to generate a large proportion of zero-leverage firms with a low cost of debt, as documented in Strebulaev and Yang (2013). In contrast, the simulated leverage distribution of my model can match the cross-sectional leverage distribution in the data, in which about 1/4 of firms have close to zero leverage.

Moreover, this paper relates to the public economic literature on payout taxation. Early works in the "trapped equity" view (also often referred to as the "new" view) of dividend taxation (e.g., King 1974, 1977; Auerbach, 1979, 1981) point out that dividend taxation does not affect firms' decisions when the firm issues no equity.[10] Internal equity is "trapped" in firms and has to be taxed when distributed to shareholders. Auerbach (2002) reviews literature on this topic. This paper applies a similar argument regarding the tax benefits of debt: debt issuance cannot save tax on shareholders' equity income unless it reduces equity issuance. Relatedly, the paper also contributes to the literature on optimal taxation design. For example, Dávila and Hébert (2019) show that governments should tax financially unconstrained firms to maximize the efficiency of investments and production while collecting a given amount of tax revenue. The optimal policy is to tax firms on their payouts to shareholders instead of their income. This paper achieves the same result in a different setting. Besides the investment channel as in Dávila and Hébert (2019), I reveal a new

[10]See, e.g., Poterba and Summers (1984) for discussions of the "traditional" and "new" views of dividend taxation.

channel working through distortions on firms' leverage that makes payout taxation better than corporate income taxation. While raising the same amount of tax revenue, taxes on corporate income lead to more expected deadweight bankruptcy loss than taxes on payouts to shareholders.

3 Tax benefits from debt issuance

Miller (1977) defines the value of tax shield from each dollar (market value) of outstanding perpetual debt by $\left[1 - \frac{(1-\tau_c)(1-\tau_e)}{1-\tau_b}\right]$, which is widely used in the literature and textbooks for evaluating firms' existing debts and the benefits from new debt issuances.[11][12] Changes in the value of tax shields on future earnings, however, may not be the correct measure of tax benefits generated by issuing new debt since they do not capture the potential tax consequences at the debt issuance time. For example, if proceeds from debt issuance are distributed as payouts, taxes on these payouts are not captured by the definition above. Below, I consider tax incentives for real-world scenarios in which firms may issue new debt, and I discuss whether Miller's formula applies in these scenarios.

When considering new debt issuance, a firm may face two scenarios depending on whether an investment opportunity needs to be financed externally and has a positive net present value (NPV) if financed most cheaply. Here I define the tax benefits as the tax savings by issuing a marginal dollar of debt relative to the best alternative without such debt issuance. In the first case, when there is a positive NPV project to finance externally, additional debt issuance leads to a reduction in equity issuance, no change in current payouts or cash on hand, and a reduction in future payouts due to additional interest expenses. Therefore, debt issuance, in this case, does save personal tax for shareholders, and Miller's formula applies in general. [13]

Since positive NPV investment opportunities are limited, there is a second case in which the firm has no positive NPV project to invest in but considers a capital restructure to issue

[11] A scaled definition $(1-\tau_b)-(1-\tau_c)(1-\tau_e)$ describes the difference between after-tax income earned by debtholders and shareholders from each dollar of the firm's earnings. It can be viewed as the tax shield from each dollar of interest expense.

[12] Miller (1977) describes a market equilibrium in which $(1-\tau_b)=(1-\tau_c)(1-\tau_e)$ and there is no optimal capital structure for individual firms. In Online Appendix I, I discuss how the different tax consequences of financing investments by debt and leveraged recapitalization allow firms to gain a surplus from tax shields in a similar market equilibrium.

[13] Tax benefits can be lower than Miller's formula when the project's NPV is negative if financed by equity and when the firm expects to distribute no payouts in the near future.

debt and take advantage of the tax shield.[14] In this case, the firm must distribute the debt proceeds as payouts to reduce taxable income. Debt issuance transfers future payouts to current payouts and does not save personal taxes for shareholders as long as the expected future tax rates on payouts do not exceed the current tax rates. Indeed, the payouts from debt proceeds are double taxed by the personal tax on payouts and the personal tax priced in bonds. We should multiply the first item in Miller's formula by $(1 - \tau_e)$, and the tax benefits become $(1-\tau_b)(1-\tau_e)-(1-\tau_c)(1-\tau_e) = (1-\tau_e)(\tau_c-\tau_b)$. The tax benefits on each dollar of interest expense are the difference between the corporate tax rate and the personal tax rate on bond income, scaled by one minus the rate of unavoidable tax on equity income. Miller's formula, $(1 - \tau_b) - (1 - \tau_c)(1 - \tau_e)$, overstates the tax benefits from debt issuance in this scenario.

The differences are potentially significant and may flip the sign of tax benefits (or costs) from debt issuance. For example, if we apply the top federal rates in 2022 (21% corporate tax, 37% personal tax on bond income, and 23.8% personal tax on equity income) to the definitions above, tax benefits per dollar of interest expense are (1 - 37%) - (1 - 21%)(1 - 23.8%) = 2.8% for financing investments and (21% - 37%)(1 - 23.8%) = -12.19% for capital restructuring. The use of 2017 rates before the tax reform (39% corporate tax, 40.79% personal tax on bond income, and 24.99% personal tax on equity income) results in tax benefits of 13.45% vs. -1.34% for the two cases. A firm facing such tax rates would gain tax benefits from financing investments but not from capital structuring.

4 The model

The following model elucidates firms' optimal capital structure decisions when considering corporate and personal taxes and bankruptcy costs. The firm adjusts leverage freely without commitment as in DeMarzo and He (2021), at no cost of security issuance/repurchase. The key difference between this model and DeMarzo and He (2021) is that this model features personal taxes on bond and equity income. The leverage dynamics differ from DeMarzo and He (2021) in several ways. First, there are two local leverage targets for firms with leverage above and below a threshold; the lower target is 0 when the corporate tax rate τ_c is lower than the personal tax rate on bond income τ_b. Second, if $\tau_c < \tau_b$, the firm repurchases debt

[14] Securities such as treasury bonds have the same risk-adjusted returns as the firm's bond and a zero NPV when fairly priced. When savings are expected to increase the firm's value through, for example, the value of financial flexibility, they are considered positive NPV opportunities in the first scenario, with the alternative being equity issuance.

when: (1) it has existing debt and generates more earnings than expenses or (2) it expects to keep using external financing in a relatively long future. And third, firms gain from tax benefits even without a commitment device.

(a) Model setup

Agents are risk neutral with a discount rate r, and they face a corporate tax at rate τ_c, a personal tax on bond income at rate τ_b, and a personal tax on equity income at rate τ_e. A firm's EBIT follows:

$$dY_t = [\mu(Y_t) + I(Y_t)]\, dt + \sigma(Y_t) dZ_t \tag{1}$$

Investment $I(Y_t)$ can be financed by internal cash flow and issuance of debt and equity. For simplicity, in the baseline model I assume investment is exogenous with a linear cost $\kappa I(Y_t)$. I study endogenous investment choice and debt overhang in Section 6.

The firm issues debt with a coupon rate $c > 0$ that matures exponentially at a rate $m > 0$. Let F_t be the face value of existing debt.[15] Then, the payment flow to debtholders is $(c + m)F_t dt$. Let Φ_t be the endogenous cumulative debt issuance, and $d\Phi_t < 0$ represents a debt repurchase. Existing debt evolves by $dF_t = d\Phi_t - mF_t dt$. Denote the price of debt issuance as $p(Y_t, F_t)$. Debtholders receive an after-tax cash flow of $[(1-\tau_b)c+m]F_t dt$ until the firm goes bankrupt.[16] For simplicity, assume the firm's recovery value is zero at bankruptcy.

Let Γ_t be the endogenous cumulative proceeds from equity issuance, with $\Gamma_t \geq 0$ by definition.[17] In this simple setting, without security issuance costs, the firm cannot get better off with cash holdings than using the cash for debt repurchase, so the firm never holds cash when it has outstanding debt.[18] The payout flow can be written as:

$$d\Delta_t = [Y_t - \underbrace{\tau_c(Y_t - cF_t)}_{\text{corporate tax}} - \underbrace{(c + m)F_t}_{\text{payments to debt}} - \underbrace{\kappa I(Y_t)}_{\text{investment}}]dt + \underbrace{p(Y_t, F_t)d\Phi_t}_{\text{proceeds from debt}} + \underbrace{d\Gamma_t}_{\text{proceeds from equity}} \tag{2}$$

where $d\Delta_t \geq 0$ since payouts are nonnegative. The firm maximizes the expected net payouts

[15] No Ponzi assumption: $F_t < \bar{F}(Y_t)$, where $\bar{F}(Y_t)$ exceeds the unleveraged value of the firm.

[16] Here, I assume only coupons are taxed on a bond and exempt from the corporate tax for tractability, as in Leland (1994b). In practice, interest may differ from the coupon when bonds are not issued at par.

[17] I assume there are no frictions in the equity market, so there is no need to characterize equity issuance prices. Maximizing value for all shareholders is equivalent to maximizing value for existing shareholders. Since payouts are taxed but equity issuance is not, they are modeled separately here.

[18] Consider a firm with both cash holdings and debt outstanding. When investing the cash holdings into fairly priced securities that are not correlated with the firm's default risk, the firm earns the risk-free rate under the risk-neutral measure while paying a higher rate on its outstanding debt.

to shareholders

$$V_t = \max_{T,\Phi,\Gamma} E\left[\int_t^T e^{-r(s-t)}\left[(1-\tau_e)d\Delta_s - d\Gamma_s\right]\right] \tag{3}$$

by choosing optimal capital structure over time and bankruptcy time T.

(b) Security valuations

Substitute (2) into (3), the value function can be written as

$$V(Y_t, F_t) = \max_{T,\Phi,\Gamma}$$
$$E_t\left\{\int_t^T e^{-r(s-t)}\left[(1-\tau_e)\left\{\left[Y_s - \tau_c(Y_s - cF_s) - (c+m)F_s - \kappa I(Y_s)\right]ds + p(Y_s,F_s)d\Phi_s\right\}ds - \tau_e d\Gamma_s\right]\right\} \tag{4}$$

Observe that if a firm issues a dollar of equity and distributes a dollar of payouts simultaneously, there is a net loss of τ_e due to the personal income tax paid on the payouts. We then have the following rule for equity issuance:

Proposition 1. ***(Optimal equity issuance)*** *The firm does not issue equity and distribute payouts at the same time. Optimal equity issuance is uniquely pinned down by optimal debt issuance.*

$$d\Gamma_t = max\left\{-\left[Y_t - \tau_c(Y_t - cF_t) - (c+m)F_t - \kappa I(Y_t)\right]dt - p(Y_t,F_t)d\Phi_t, 0\right\} \tag{5}$$

When the firm issues equity, the proceeds are "trapped" in the firm and cannot be taken back by the shareholders without paying a personal income tax. Therefore, a firm that maximizes the total payoff of all shareholders should never issue equity and distribute payouts simultaneously. Equity financing policies can be characterized by net payouts, with negative payouts representing an equity issuance.

Following DeMarzo and He (2021), I look for a smooth equilibrium with continuous issuance policy, where $d\Phi_t = \phi_t dt$ and $d\Gamma_t = \gamma_t dt$. For simplicity of notations, I compress the time subscripts in the rest of the analysis.

Proposition 2. ***(Smooth equilibrium)*** *$V(Y,F)$ is strictly decreasing in F when $p(Y,F) > 0$. If for any F and F', $V(Y,F) > V(Y,F') + \left(1 - \mathbf{1}_{\{F'-F>0\}}\tau_e\right)p(Y,F)(F'-F)$, then Φ_t and Γ_t are continuous. If $V(Y,F)$ is differentiable in F, then $(1-\tau_e)p(Y,F) \leq -V_F(Y,F) \leq$*

$p(Y, F)$. *A sufficient condition for a smooth equilibrium is that* $V(Y, F)$ *is strictly convex in* F.

Proof. The firm always has the option to adjust debt from F to F'. If $F' < F$, adjusting debt to F' discretely is a debt repurchase financed by equity issuance. Otherwise it is a discrete debt issuance that leads to a payout distribution subject to a personal tax τ_e, so

$$\begin{aligned} V(Y, F) &\geq V(Y, F') + (F' - F)p(Y, F) - \mathbf{1}_{\{F'-F>0\}}\tau_e(F' - F)p(Y, F) \\ &= V(Y, F') + \left(1 - \mathbf{1}_{\{F'-F>0\}}\tau_e\right) p(Y, F)(F' - F) \end{aligned} \tag{6}$$

When the inequality is strict, the firm never discretely adjusts leverage. Then following Proposition 1, there is no discrete equity issuance either. The issuance policies are continuous. If $V(Y, F)$ is differentiable in F, after taking the limit of F' to F, inequality (6) can be reorganized as $(1 - \tau_e)p(Y, F) \leq -V_F(Y, F) \leq p(Y, F)$.

If $V(Y, F)$ is strictly convex in F, that is, $V(Y, F) > V(Y, F') - V_F(Y, F)(F' - F)$, when $F' > F$,

$$V(Y, F) > V(Y, F') - V_F(Y, F)(F' - F) \geq V(Y, F') + (1 - \tau_e)p(Y, F)(F' - F) \tag{7}$$

so there is no discrete debt adjustment. When $F' < F$,

$$V(Y, F) > V(Y, F') - V_F(Y, F)(F' - F) \geq V(Y, F') + p(Y, F)(F' - F) \tag{8}$$

there is no discrete debt adjustment either. Therefore, $V(Y, F)$ strictly convex in F is sufficient for a smooth equilibrium. □

Now I derive optimality conditions for an equilibrium. I look for an equilibrium where V is twice continuously differentiable, p is continuously differentiable, and $p_F(Y, F) < 0$, which means debt price decreases with the amount of outstanding debt given EBIT. Given Proposition 1, it is enough to find conditions for optimal debt policies since payouts and equity issuance are pinned down by the state variables and debt policies.

Let $\bar{\phi}(Y, F) = -\frac{1}{p(Y,F)} [Y - \tau_c(Y - cF) - (c + m) F - \kappa I(Y)]$, representing the debt issuance/repurchase such that there is no payout distribution or equity issuance. Consider when $F > 0$ so that debt repurchase is not bounded by zero. Then the HJB equation for the value function (4) is

$$
\begin{aligned}
rV(Y,F) = \max \Bigg(& \max_{\phi \geq \bar{\phi}} \Bigg\{ (1-\tau_e) \underbrace{\left[Y - \tau_c(Y - cF) - (c+m)F - \kappa I(Y) + p(Y,F)\phi\right]}_{\text{positive net payouts}} \\
& \quad + \underbrace{(\phi - mF)V_F(Y,F)}_{\text{debt evolution}} + \underbrace{\left[\mu(Y) + I(Y)\right]V_Y(Y,F) + \frac{1}{2}\sigma(Y)^2 V_{YY}(Y,F)}_{\text{earnings evolution}} \Bigg\}, \\
& \max_{\phi < \bar{\phi}} \Bigg\{ \underbrace{\left[Y - \tau_c(Y - cF) - (c+m)F - \kappa I(Y) + p(Y,F)\phi\right]}_{\text{negative net payouts (equity issuance)}} \\
& \quad + \underbrace{(\phi - mF)V_F(Y,F)}_{\text{debt evolution}} + \underbrace{\left[\mu(Y) + I(Y)\right]V_Y(Y,F) + \frac{1}{2}\sigma(Y)^2 V_{YY}(Y,F)}_{\text{earnings evolution}} \Bigg\} \Bigg)
\end{aligned} \tag{9}
$$

When $\phi(Y,F) > \bar{\phi}(Y,F)$, the firm is distributing a positive amount of payouts, so a personal tax on equity income at rate τ_e is charged on the payouts. In contrast, when $\phi(Y,F) < \bar{\phi}(Y,F)$, net payouts are negative, representing an equity issuance, and there is no tax charged. If $\phi(Y,F) = \bar{\phi}(Y,F)$, net payouts are zero, and the two parts of the maximization are identical.

A necessary condition for ϕ to be optimal is that either the first order condition holds or a constraint is binding, so an optimal debt policy must satisfy either

$$\phi(Y,F) > \bar{\phi}(Y,F) \quad \text{and} \quad p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e} \tag{10}$$

or

$$\phi(Y,F) = \bar{\phi}(Y,F) \quad \text{and} \quad -V_F(Y,F) \leq p(Y,F) \leq -\frac{V_F(Y,F)}{1-\tau_e} \tag{11}$$

or

$$\phi(Y,F) < \bar{\phi}(Y,F) \quad \text{and} \quad p(Y,F) = -V_F(Y,F) \tag{12}$$

Here the conditions (10) and (12) represent the cases that $\phi(Y,F) > \bar{\phi}(Y,F)$ or $\phi(Y,F) < \bar{\phi}(Y,F)$ and corresponding first order condition holds. The condition (11) represents the case that $\phi(Y,F) = \bar{\phi}(Y,F)$ is binding, and we cannot further characterize the relation between $p(Y,F)$ and $V_F(Y,F)$ than Proposition 2. Since both parts of the value function's HJB equation are linear in $\phi(Y,F)$, these conditions are also sufficient. When the condition (10) holds, the firm distributes payouts and is indifferent to issuing extra debt for distributing payouts. When the condition (11) holds, the firm issues no equity and distributes no payouts, issuing exactly enough debt to finance expenses or repurchasing debt with all free cash flow. When the condition (12) holds, the firm issues equity and is indifferent between debt and equity financing.

(c) Optimal debt policies

Next, I derive debt issuance/repurchase rules when one of the first order conditions holds, using HJB equations for the value function and the debt price. I then determine if these financing policies are feasible, that is, consistent with the inequalities comparing ϕ to $\bar{\phi}$ corresponding to the first order conditions in each case.

The equilibrium debt price satisfies

$$p(Y,F) = E_t\left\{\int_t^T e^{-(r+m)(s-t)}\left[(1-\tau_b)c + m\right] ds\right\} \tag{13}$$

The HJB equation for debt price (13) is

$$\begin{aligned} rp(Y,F) =& (1-\tau_b)c + m[1-p(Y,F)] + [\phi(Y,F) - mF]p_F(Y,F) \\ &+ [\mu(Y) + I(Y)]\, p_Y(Y,F) + \frac{1}{2}\sigma(Y)^2 p_{YY}(Y,F) \end{aligned} \tag{14}$$

When the firm distributes payouts, take derivative of the first part of the maximization in (9) to F and substitute the value function with $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$, we get

$$\begin{aligned} -rp(Y,F) =& \tau_c c - (c+m) + p_F(Y,F)\phi(Y,F) + mp(Y,F) - [\phi(Y,F) - mF]p_F(Y,F) \\ &- [\mu(Y) + I(Y)]\, p_Y(Y,F) - \frac{1}{2}\sigma(Y)^2 p_{YY}(Y,F) \end{aligned} \tag{15}$$

Add (15) to (14), then

$$0 = (\tau_c - \tau_b)c + p_F(Y,F)\phi(Y,F) \tag{16}$$

Hence when $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$, the debt policy is

$$\phi(Y,F) = -\frac{(\tau_c - \tau_b)c}{p_F(Y,F)} = \frac{(1-\tau_e)(\tau_c - \tau_b)c}{V_{FF}(Y,F)} \tag{17}$$

Since $p_F(Y,F) < 0$, $\phi(Y,F)$ has the same sign as $\tau_c - \tau_b$. If $\tau_c < \tau_b$, then $\phi(Y,F) < 0$ and this represents the case that the firm repurchases debt while distributing payouts. The firm earns more than the costs and uses part of the cash left for debt repurchase to save the personal tax priced in debt at the cost of the corporate tax. But due to the leverage ratchet effect, it does not spend all the cash left on debt repurchase but also distributes payouts. If $\tau_c > \tau_b$, then $\phi(Y,F) > 0$ and this represents the case that the firm issues debt while distributing payouts. The firm issues debt to save the corporate tax at the cost of the personal tax priced in debt and distributes the extra cash raised as payouts. Here τ_e is not a key determinant for debt policies because the influence of debt repurchase/issuance on future personal tax costs for shareholders is offset by the changes in current personal tax costs on payouts.

Similarly, when the firm issues equity, take derivative of the second part of the maximization in (9) to F and substitute the value function with $p(Y,F) = -V_F(Y,F)$, we get

$$\phi(Y,F) = -\frac{(\tau_c - \tau_b)c}{p_F(Y,F)} = \frac{(\tau_c - \tau_b)c}{V_{FF}(Y,F)} \tag{18}$$

In this case, ϕ also has the same sign as $\tau_c - \tau_b$. If $\tau_c < \tau_b$, then $\phi < 0$ and this represents the case that the firm repurchases debt while issuing equity. The firm repurchases debt, financing it by equity issuance, to save the personal tax priced in debt at the cost of the corporate tax. If $\tau_c > \tau_b$, then $\phi > 0$ and this represents the case that the firm issues debt and equity at the same time. The firm issues debt to save the corporate tax at the cost of the personal tax priced in debt, but also issues equity due to bankruptcy concerns. Here τ_e is not a key determinant for debt policies because the firm does not expect to distribute payouts in the near future, and debt repurchase/issuance has little effect on shareholders' personal tax costs. Then, I summarize the firm's equilibrium debt policies as the following result.

Proposition 3. ***(Debt strategies conditional on security values)*** *Depending on the comparison between the debt price $p(Y,F)$ and the value function's marginal change to outstanding debt $V_F(Y,F)$, the firm's equilibrium debt policy $\phi(Y,F)$ is given by one of followings:*

1. *If $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$,*

$$\phi(Y,F) = \frac{(1-\tau_e)(\tau_c - \tau_b)c}{V_{FF}(Y,F)}$$

2. If $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$,

$$\phi(Y,F) = \bar{\phi}(Y,F) = -\frac{1}{p(Y,F)}\left[Y - \tau_c(Y - cF) - (c+m)\,F - \kappa I(Y)\right]$$

3. If $p(Y,F) = -V_F(Y,F)$,

$$\phi(Y,F) = \frac{(\tau_c - \tau_b)c}{V_{FF}(Y,F)}$$

In the first and third cases, when the firm distributes payouts or issues equity, debt policy $\phi(Y,F)$ can be represented as the marginal tax benefit of debt divided by the value function's second-order derivative to outstanding debt, as in DeMarzo and He (2021). However, the marginal tax benefit here, either $(1-\tau_e)(\tau_c-\tau_b)$ or $(\tau_c-\tau_b)$ per a dollar of coupon payment, is different from both DeMarzo and He (2021) (τ_c) and the traditional definition with personal taxes $(1-\tau_b)-(1-\tau_c)(1-\tau_e)$. In the first case, proceeds from additional debt issuance are distributed as payouts, so the personal tax on equity income at rate τ_e cannot be saved. In the third case, the firm expects to issue equity continuously and distributes no payouts in the near future, when most coupons on additional debt are paid, so personal tax on equity income at rate τ_e is not paid or saved. The marginal tax benefits in both cases have the same sign as $(\tau_c - \tau_b)$, which is negative for top statutory rates in most years in the U.S. In contrast, the traditional definition of marginal tax benefits—with or without personal taxes—always leads to a positive benefit based on top statutory tax rates.

If $V(Y,F)$ is strictly convex in F, $V_{FF}(Y,F) > 0$ and $\phi(Y,F)$ has the same sign as $\tau_c - \tau_b$ in these two cases, the firm repurchases debt when $\tau_c < \tau_b$ and issues debt otherwise. Although the tax benefits align with reducing bankruptcy risk when $\tau_c < \tau_b$, the firm does not repurchase as much debt as possible. This conclusion follows because the debt ratchet effect means that debt repurchase reduces the risk of existing debt and benefits the debtholders at the cost of shareholders. Such cost increases with debt repurchase, and the firm repurchases debt until it is indifferent with additional debt. With the equilibrium levels of debt repurchase, the firm is indifferent between using a marginal dollar of internal cash for payouts and debt repurchase in the first case when $(1-\tau_e)p(Y,F) = -V_F(Y,F)$, and indifferent between issuing a marginal dollar of debt and equity in the third case when $p(Y,F) = -V_F(Y,F)$.

In the second case when $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$, debt financing is cheaper than external equity but more expensive than internal cash. The firm prefers debt financing to equity financing, and it prefers debt repurchase to payout distribution. It breaks even by issuing or repurchasing debt without any cash flow to or from the shareholders.

Proposition 3 shows the firms' debt policies conditional on the relations between the debt price $p(Y,F)$ and the value function's marginal change on debt $V_F(Y,F)$. We can further characterize debt strategies based on the state variables (Y,F) by checking the conditions at extreme values of parameters and the boundaries between regions of state variables where each of the above debt policies applies. Then I find general relations between the state variables (Y,F) and the comparison of $p(Y,F)$ and $V_F(Y,F)$ by continuity and monotonicity without further specifications of functional forms and without solving $p(Y,F)$ and $V(Y,F)$. Proposition 4 summarizes the results, with proof in the Appendix.

Proposition 4. ***(Optimal financing policies)*** *If $(1-\tau_c)Y - \kappa I(Y)$ monotonically increases in Y, the firm's optimal debt policies on the space of state variables (Y,F) can be described by four regions, with leverage from high to low:*

Region 1 (Equity issuing region): $p(Y,F) = -V_F(Y,F)$, the firm issues equity and issues/repurchases debt if $\tau_c - \tau_b$ is positive/negative.

Region 2 and 4 (Break even by debt regions): $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$, the firm issues no equity and distributes no payouts, and it breaks even by issuing/repurchasing debt if earnings are less/more than expenses.

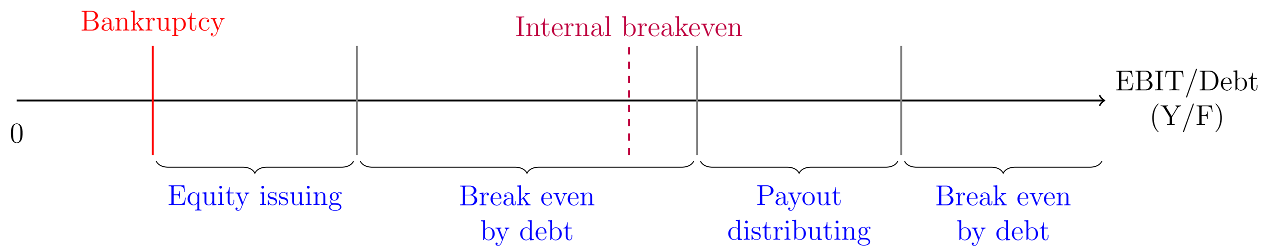
Region 3 (Payout distributing region): This region may contain two types of sub-regions. (1) $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$, the firm distributes payouts and issues/repurchases debt if $\tau_c - \tau_b$ is positive/negative, (2) the firm breaks even by debt as in region 2 and 4. When $\bar{\phi}(Y,F)$ and $-\frac{(\tau_c-\tau_b)c}{p_F(Y,F)}$ satisfies single-crossing condition, (1) is always true in this region.

Regions 2 always exists. Regions 3 must exist if $\tau_c \geq \tau_b$.

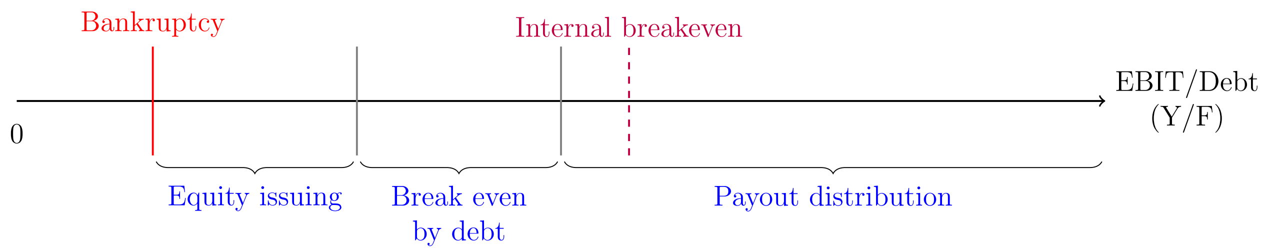
Figure (3) illustrate the results of Proposition 4 indicating the variation in the debt policies with the scaled interest coverage ratio Y/F for any given value of Y, in the cases when $\tau_c < \tau_b$ and $\tau_c \geq \tau_b$. The red line represents the bankruptcy-triggering level of interest coverage ratio. On its right, when leverage is high and close to bankruptcy (*Region 1*), the firm issues equity, and the first order condition $p(Y,F) = -V_F(Y,F)$ holds. The firm's debt policy is given by $\phi(Y,F) = \frac{(\tau_c-\tau_b)c}{V_{FF}(Y,F)}$, representing a debt repurchase if $\tau_c < \tau_b$ or a debt issuance if $\tau_c > \tau_b$. When leverage is lower (*Region 2*), the firm prefers debt financing to equity financing and prefers debt repurchase to payout distribution. Debt price satisfies $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$ and the debt policy is given by $\phi(Y,F) = \bar{\phi}(Y,F)$, so that there is no equity issuance nor payout distribution. The dotted purple line represents when the firm breaks even internally, with earnings exactly meeting expenses. When leverage is further lower (*Region 3*), the firm distributes payouts and the first order condition $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$ holds. The firm's debt policy is given by $\phi(Y,F) = \frac{(1-\tau_e)(\tau_c-\tau_b)c}{V_{FF}(Y,F)}$, representing a debt repurchase if $\tau_c < \tau_b$ or a debt issuance if $\tau_c > \tau_b$. In the case that $\tau_c < \tau_b$, when leverage is low and close to zero (*Region 4*), the firm spends all free cash flow on debt repurchase without distributing payouts until there is no outstanding debt. Debt price satisfies $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$ and the debt policy is given by $\phi(Y,F) = \bar{\phi}(Y,F)$. The firm always issues debt when $\tau_c > \tau_b$ but also repurchases debt when $\tau_c < \tau_b$.

Figure 3: Optimal financing policies

(a) Optimal financing policies with $\tau_c < \tau_b$



(b) Optimal financing policies with $\tau_c \geq \tau_b$



In contrast to standard trade-off models in which the marginal source of financing is always equity (for example, Leland, 1994, DeMarzo and He, 2021), here the marginal source of financing is debt in the break-even by debt regions or equity in the other regions. Such a difference results from a wedge between the costs of internal and external equity due to taxes on payouts. When the cost of debt financing falls between the costs of internal and external equity, the firm has a pecking order preference, prioritizing the use of internal equity to debt to external equity.

Although the model features no commitment to future leverage policies as in DeMarzo and He (2021), the lack of commitment does not fully offset tax benefits. When $-V_F(Y,F) < p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$, the HJB equation of the value function with the optimal debt policy $\phi(Y,F) = \bar{\phi}(Y,F)$ is different from its counterpart with $\phi(Y,F) = 0$. Then, since $\tau_e > 0$ and the high leverage break-even by debt region (*region 2*) always exists, the no-trade valuation in DeMarzo and He (2021) no longer applies in this model. Noticing that no trade of debt is always a feasible option for the firm, we have the following result.

Corollary 1. *As long as $\tau_e > 0$, the firm benefits from the tax shield of debt even if there is no commitment to future leverage policies, $V(Y, F) > V^0(Y, F)$ where $V^0(Y, F)$ represents the value of the firm with no trade of debt.*

(d) Tax benefits

When $\tau_c < \tau_b$, the firm repurchases debt not only when earnings exceed expenses but also when leverage is high and close to the bankruptcy threshold. This behavior occurs because coupons do not save personal equity income tax for shareholders unless they reduce payout distribution. To further understand tax benefits in this model, I discuss the expected value of total and marginal tax benefits below.

The value function can be decomposed as

$$V(Y, F) = V(Y, 0) + TB(Y, F) - BC(Y, F) \tag{19}$$

where the value of net corporate and personal tax savings by debt follows

$$\begin{aligned} rTB(Y, F) =& (\tau_c - \tau_b)cF + \mathbf{1}_{\{\phi > \bar{\phi}\}}\tau_e \{[(1 - \tau_c)c + m] F - p(Y, F)\phi\} \\ &+ (\phi - mF)TB_F(Y, F) + [\mu(Y) + I(Y)]\, TB_Y(Y, F) + \frac{1}{2}\sigma(Y)^2 TB_{YY}(Y, F) \end{aligned} \tag{20}$$

and the bankruptcy cost follows

$$rBC(Y, F) = (\phi - mF)BC_F(Y, F) + [\mu(Y) + I(Y)]\, BC_Y(Y, F) + \frac{1}{2}\sigma(Y)^2 BC_{YY}(Y, F) \tag{21}$$

with $\phi(Y, F), \bar{\phi}(Y, F)$ written as $\phi, \bar{\phi}$ for short.

When the firm does not distribute payouts, the flow payoff is the difference between the corporate tax saved and the personal tax paid on coupons. If $\tau_b > \tau_c$, such flow payoff is negative, representing a tax cost to the firm. When the firm distributes payouts, besides the (negative) saving of $\tau_c - \tau_b$ on coupons discounted by the personal tax on equity income τ_e, the payments to debtholders net of proceeds from debtholders reduce payouts and generate a saving of personal tax on equity income. The firm gains a flow tax benefit from existing debt but not from issuing new debt. When issuing new debt with a face value of a dollar, the firm's expected marginal tax benefit is

$$MTB(Y,F) = \\ -\mathbf{1}_{\{\phi_0 > \bar{\phi}_0\}} \tau_e p(Y,F) + E\left[\int_0^T e^{-(r+m)t} \left\{ (\tau_c - \tau_b)c + \mathbf{1}_{\{\phi_t > \bar{\phi}_t\}} \tau_e \left[(1-\tau_c)c + m \right] \right\} dt \middle| Y, F \right] \tag{22}$$

The marginal tax benefit is higher when the firm is not distributing payouts and when it is more likely to distribute payouts in the near future when coupons and principals are paid to debtholders. In an extreme case, if the firm is not distributing now but is expected to distribute payouts all the time in the future, the marginal tax benefit on each dollar of coupon coincides with Miller's formula $(1-\tau_b) - (1-\tau_e)(1-\tau_c)$. Miller's formula, then, represents an upper bound for the marginal tax benefits of debt after adjusting for personal taxes.

(e) A roadmap for solving the general model

An equilibrium of the general model can be found by:

1. Find the bankruptcy threshold $\{Y,F\}_b$ and the boundaries between the equity issuing region (*Region 1*) and the break-even by debt region (*Region 2*) $\{Y,F\}_e$. Start with arbitrary positive initial values with higher leverage than the internal break-even level, and generate value and price functions by (9) and (14), setting

- $V^b = V^b_Y = V^b_F = 0$, $p^b = 0$,
- $\phi(Y,F) = \frac{(\tau_c - \tau_b)c}{V_{FF}(Y,F)}$ in the equity issuing region (*Region 1*),
- $\phi(Y,F) = \bar{\phi}(Y,F) = -\frac{1}{p(Y,F)}\left[Y - \tau_c(Y - cF) - (c+m)F - \kappa I(Y)\right]$ if $p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$, and $\phi = \frac{(1-\tau_e)(\tau_c-\tau_b)c}{V_{FF}(Y,F)}$ if $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$ in the regions with lower leverage than the equity issuing region (*Region 2, 3 and 4*).

and check if the firm's value and debt price converge to their limits when Y goes to infinity for each F. Adjust the boundaries until reaching convergence.

2. Generate the equilibrium debt issuance ϕ, value function $V(Y,F)$ and debt price $p(Y,F)$ as above using the boundaries found.

3. Verify that $V(Y,F)$ and $p(Y,F)$ are strictly decreasing in F, $V(Y,F)$ is strictly convex in F, and $-V_F(Y,F) \le p(Y,F) \le -\frac{V_F(Y,F)}{1-\tau_e}$.

4. Generate equity issuance γ by (5).

5 Leverage dynamics

Next, I discuss the leverage dynamics when the model is homogeneous, such that all variables can be expressed as a function of a single state variable $y_t = \frac{Y_t}{F_t}$. I focus on the baseline scenario in which $1 - \tau_c \geq 1 - \tau_b > (1 - \tau_c)(1 - \tau_e)$. This relation holds for the U.S. top federal statutory rates in most years.

(a) A homogeneous model

Consider the case that $\mu(Y_t) = \mu Y_t, I(Y_t) = iY_t$, and $\sigma(Y_t) = \sigma Y_t$, with parameters satisfying $\mu + i < r$ and $\tau_c + \kappa i < 1$. Define $y_t \equiv \frac{Y_t}{F_t}, v(y_t) \equiv V\left(\frac{Y_t}{F_t}, 1\right)$, and $p(y_t) \equiv p\left(\frac{Y_t}{F_t}, 1\right)$. Then the model is homogeneous and

$$V(Y_t, F_t) = V\left(\frac{Y_t}{F_t}, 1\right) F_t = v(y_t) F_t \tag{23}$$

$$p(Y_t, F_t) = p\left(\frac{Y_t}{F_t}, 1\right) = p(y_t) \tag{24}$$

Since

$$dY_t = (\mu + i) Y_t dt + \sigma Y_t dZ_t \tag{25}$$

$$dF_t = (\phi_t - mF_t) dt \tag{26}$$

y_t follows

$$\frac{dy_t}{y_t} = \left(\mu + i + m - \frac{\phi_t}{F_t}\right) dt + \sigma dZ_t \tag{27}$$

We can rewrite the HJB equations (9) and (14) in y_t using

$$V_F(Y, F) = v(y) - yv'(y), \quad V_Y(Y, F) = v'(y), \quad V_{YY} = \frac{1}{F} v''(y) \tag{28}$$

$$p_F(Y, F) = -\frac{y}{F} p'(y), \quad p_Y(Y, F) = \frac{1}{F} p'(y), \quad p_{YY} = \frac{1}{F^2} p''(y) \tag{29}$$

Then the HJB equation for the value function (9) becomes

$$
\begin{aligned}
rv(y) = \max \Bigg(& \max_{\phi \geq \bar{\phi}} \Bigg\{ (1-\tau_e) \left[y - \tau_c(y-c) - (c+m) - \kappa i y + p(y)\frac{\phi}{F} \right] \\
& \qquad + \left(\frac{\phi}{F} - m \right) [v(y) - yv'(y)] + (\mu + i) yv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \Bigg\}, \\
& \max_{\phi < \bar{\phi}} \Bigg\{ \left[y - \tau_c(y-c) - (c+m) - \kappa i y + p(y)\frac{\phi}{F} \right] \\
& \qquad + \left(\frac{\phi}{F} - m \right) [v(y) - yv'(y)] + (\mu + i) yv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \Bigg\} \Bigg)
\end{aligned}
\tag{30}
$$

and the HJB equation for the price function (14) becomes

$$
rp(y) = (1-\tau_b)c + m(1-p(y)) - (\phi - mF)\frac{y}{F} p'(y) + (\mu+i) yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \tag{31}
$$

I solve the model by finding the boundaries between regions of financing strategies and the bankruptcy threshold, such that the value and price functions converge to their limits as y_t goes to infinity, following the roadmap in the previous section. Denote y_b as the bankruptcy threshold and $y_e \geq y_b$ as the boundary between the equity issuing region (Region 1) and the high-leverage break-even by debt region (Region 2). In the equity issuing region, when $y < y_e$, $p(y) = -v(y) + yv'(y)$, $\phi = \frac{(\tau_c - \tau_b)cF}{yp'(y)}$ and the firm issues equity, the HJB equations are

$$
(r+m)\, v(y) = (1-\tau_c-\kappa i)y - (1-\tau_c)c - m + (m+\mu+i)\, yv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \tag{32}
$$

$$
(r+m)\, p(y) = (1-\tau_c)c + m + (m+\mu+i)\, yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \tag{33}
$$

In the break-even by debt regions, when $y \geq y_e$, $\frac{1}{1-\tau_e}[-v(y) + yv'(y)] \geq p(y) \geq -v(y) + yv'(y)$, $\phi = -\frac{F}{p(y)}\left[(1-\tau_c-\kappa i)y - (1-\tau_c)c - m\right]$, the HJB equations are

$$
(r+m)\, v(y) = -\frac{1}{p(y)}\left[(1-\tau_c-\kappa i)y - (1-\tau_c)c - m\right][v(y) - yv'(y)] + (m+\mu+i)\, yv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \tag{34}
$$

$$
(r+m)\, p(y) = (1-\tau_b)c + m + \left[(1-\tau_c-\kappa i)y - (1-\tau_c)c - m\right] y\frac{p'(y)}{p(y)} + (m+\mu+i)\, yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \tag{35}
$$

In the payout distributing region, when $y \geq y_e$ and $p(y) = \frac{1}{1-\tau_e}[-v(y) + yv'(y)]$, $\phi = \frac{(\tau_c - \tau_b)cF}{yp'(y)}$ and the firm distributes payouts, the HJB equations are

$$(r+m)\,v(y) = (1-\tau_e)[(1-\tau_c-\kappa i)y - (1-\tau_c)c - m] + (m+\mu+i)\,yv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \tag{36}$$

$$(r+m)\,p(y) = (1-\tau_c)c + m + (m+\mu+i)\,yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y) \tag{37}$$

A solution of the model can be pinned down by the following boundary conditions

$$\lim_{y\to\infty} p(y) = \frac{(1-\tau_b)c + m}{r+m} \qquad \lim_{y\to\infty} v(y) = \frac{(1-\tau_e)(1-\tau_c-\kappa i)y}{r-\mu-i} \tag{38}$$

$$p(y_b) = 0 \qquad v(y_b) = 0 \tag{39}$$

and smooth pasting conditions that $v'(y_b) = 0$, $v'(y)$ and $p'(y)$ are continuous at the boundaries between regions. The differential equations (34) and (35) have no known closed-form solutions. Therefore, I solve $v(y)$ and $p(y)$ numerically by picking y_b, y_e and generating function values from y_b to infinity using the HJB equations and the bankruptcy values, then check if the values converge to their closed-form limits above. The functions are generated by a fourth-order Runge-Kutta-Nystrm algorithm, which allows me to find $v(y+h), v'(y+h), p(y+h), p'(y+h)$ given the HJB equations and $v(y), v'(y), p(y), p'(y)$, where h is the step size. Appendix B shows the details of the algorithm.

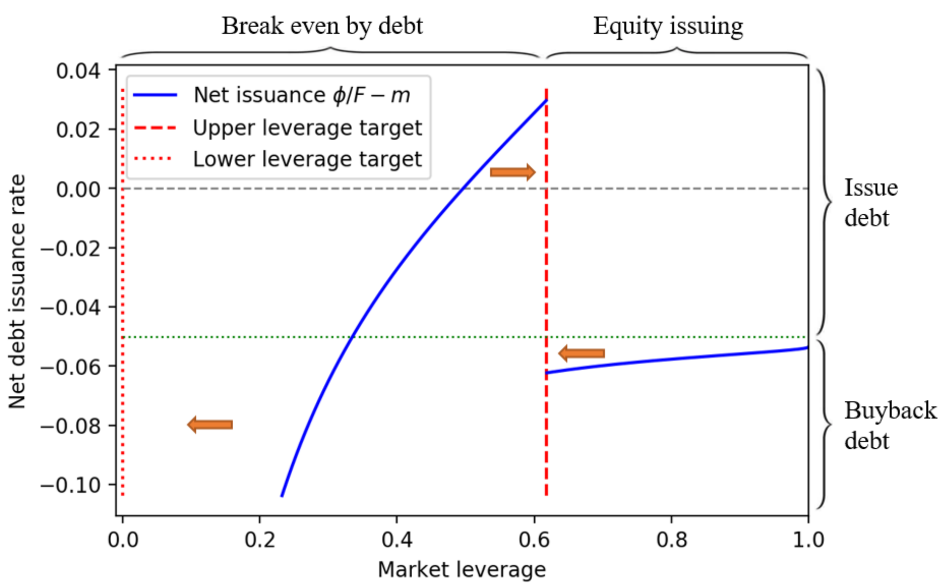
(b) Optimal debt policies and leverage targets

Figure 4 shows the firm's optimal net debt issuance rate $\frac{\phi}{F} - m$ as a function of its market leverage, defined by $\frac{\text{book value of debt}}{\text{market value of equity + book value of debt}} = \frac{F}{V(Y,F)+F} = \frac{1}{v(y)+1}$, in a baseline case with $r = 5\%, \tau_c = 30\%, \tau_b = 35\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 20$. [19] A negative debt issuance rate $\frac{\phi}{F}$, or equivalently, a net debt issuance rate lower than the maturity rate $-m$, represents debt repurchasing. Unlike a firm in the DeMarzo and He (2021) model, which never repurchases debt, the firm repurchases debt both when leverage is high and when leverage is low. Since I am assuming that investments are linear in earnings in this homogeneous case, the firm's net financing need $-[(1-\tau_c-\kappa i)y - (1-\tau_c)c - m]$ is

[19] Parameters of the geometric Brownian motion $\mu + i = 2\%$ and $\sigma = 40\%$ follows DeMarzo and He (2021). $\kappa i = 40\%$ is chosen based on net investments (capital expenditures net of depreciation) to EBIT of Compustat firms with positive net investments.

monotonically increasing in leverage. When financing need is non-positive, the firm cannot save tax on payouts for shareholders by debt since the proceeds are also taxed when distributed to shareholders as payouts. When financing need is high, the firm does not expect to distribute payouts and pay taxes on payouts in the near future. Therefore, in both scenarios, debt issuance depends on the comparison between the corporate tax rate and the personal tax rate on bond income $\tau_c - \tau_b$, and the firm repurchases debt if $\tau_c < \tau_b$. The firm only issues debt in the break-even by debt region when it has a moderate level of leverage. There is a jump in debt issuance at the boundary between the equity issuing region and the break-even by debt region when the firm switches between equity financing and debt financing.

Figure 4: Net debt issuance rate at different levels of leverage



Notes. The figure plots the firm's net debt issuance rate $\phi/F - m$, given its current market leverage $1/(v(y)+1)$. The gray dashed line represents zero net debt issuance and no leverage adjustment. The green dotted line represents no debt issuance or repurchase, and the net debt issuance rate equals the maturity rate of $-m$. The figure shows two local leverage targets at 0.62 (red dashed line) and 0 (red dotted line). The firm's leverage converges to 0.62 when above 0.5 and converges to 0 otherwise. The parameters are $r = 5\%, \tau_c = 30\%, \tau_b = 35\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 20$.

The firm adjusts towards higher leverage when the net debt issuance rate is positive and adjusts towards lower leverage when it is negative. Adjustments are faster when the absolute value of the net debt issuance rate is higher. If the firm adjusts leverage upwards when it is below a level and downwards when it is above that level, I refer to that leverage level as a local leverage target. If the firm always adjusts to a level of leverage regardless of its current leverage, I refer to that leverage level as a global leverage target. In the illustration of Figure 4, there are two local leverage targets, 0.62 and 0. The adjustments are faster when the firm is closer to the leverage targets.

Unlike the traditional wisdom of the trade-off theory that firms should have a single global leverage target, this model allows for two local leverage targets. When leverage is high, the firm's leverage converges to a "upper leverage target"—the boundary between the equity issuing region and the high-leverage break-even by debt region—by issuing debt when leverage is below target and repurchasing debt when leverage is above target. When leverage is low, the firm converges to zero leverage as a "lower leverage target" by repurchasing debt until it reaches zero. Proposition 5 shows the conditions for the model to have two leverage targets.

Proposition 5. ***(Leverage targets)*** *If (1) $\tau_c < \tau_b$ or (2) $m > 0$, $\tau_c = \tau_b$, then 0 is a local leverage target. If $y_e < \frac{(1-\tau_c)c+m[1-p(y_e)]}{1-\tau_c-\kappa i}$, the model has two local leverage targets, and a market leverage of $\frac{1}{v(y_e)+1}$ is also a local leverage target.*

(c) Leverage dynamics

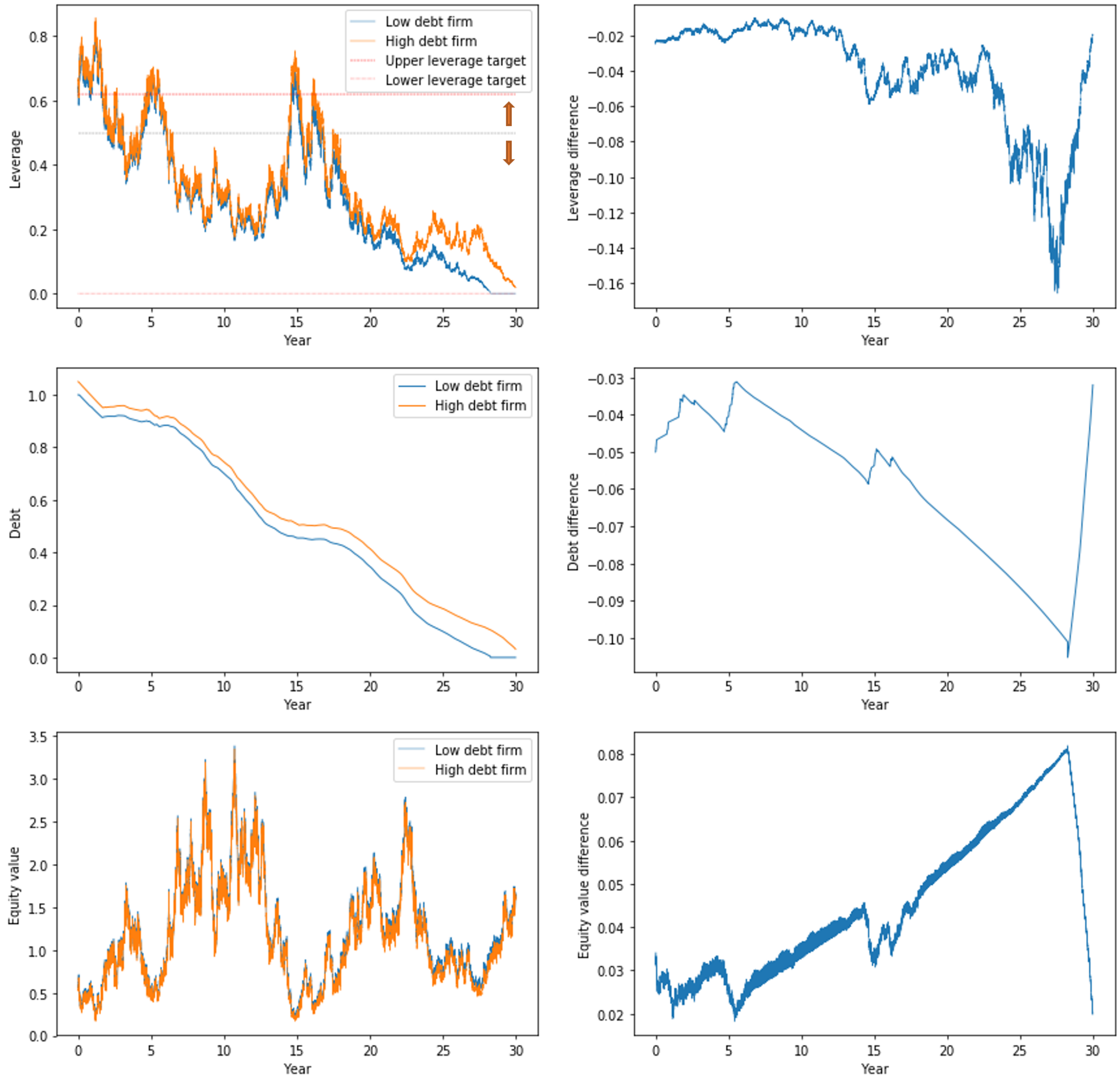
Since marginal tax benefits of debt depend on the firm's financing needs, the firm's financing strategy depends on how well earnings can cover its expenses, including payments to debt holders. The asymmetry in savings on personal equity income tax from debt can be another force—in addition to the debt ratchet effect—that makes debt policies path-dependent. For example, when a firm is in the break-even by debt region, it borrows as much as its financing needs, including payments to debt outstanding.

Figure 5 illustrates an example of the model's simulated leverage dynamics with the baseline parameter values. To highlight the potential persistence of the leverage differences between firms, I show the evolutions of two firms' leverage where both firms have the same earnings process Y_t all-time but a slightly different initial debt level. One has 5% more initial debt than the other, which may arise due to an earnings shock. For example, suppose both firms operate at the upper leverage target before time 0, and one has slightly higher earnings than the other. In that case, a permanent negative earnings shock to the firm with higher earnings can make their earnings equal while leaving them with different levels of outstanding debt. Alternatively, the difference in initial debt may also arise from a temporary shock to one of the ex-ante identical firms that makes the firm raise some additional debt. Subfigures on the left show the evolutions of the two firms' market leverage $\left(\frac{1}{v(y)+1}\right)$, face value of outstanding debt (F) and equity values ($v(y)F$), and subfigures on the right show the differences between the two firms.

Both firms start with leverage close to the upper leverage target. Their leverages deviate from the target after receiving earnings shocks and adjust to targets slowly. The gray dashed line in the upper-left subfigure represents the threshold leverage level above which the firm adjusts to the upper leverage target and otherwise to the lower leverage target. While both firms start at the upper leverage target, they converge to the lower target at the end of the 30-year period, and the low-debt firm reaches zero leverage.[20] Importantly, their leverages can cross the threshold level due to earnings shocks and slow adjustments, so their leverage target can switch between the upper and lower targets.

[20]Such deleveraging is consistent with the empirical findings of DeAngelo et al. (2018) that most firms deleverage from peak market leverage over 0.5 at the median to near zero, which is difficult to reconcile with traditional trade-off theory.

Figure 5: Leverage dynamics and persistence of differences



Notes. The figure plots the simulated leverage dynamics in 30 years of two firms with identical EBIT flows but starting with different levels of debt. The gray dashed line in the upper left subfigure represents the leverage level such that net debt issuance is 0, as in 4. Leverage adjusts to targets slowly. A firm's leverage target can switch from one to the other due to earnings shocks until it reaches zero leverage. Differences in leverage, debt, and equity value can persist for long. The parameters are $r = 5\%, \tau_c = 30\%, \tau_b = 35\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 20$.

Although the two firms have the same earnings flow, their leverage and valuation differences persist throughout the 30-year period and even grow larger. Before the low-debt firm reached zero leverage, the leverage difference reached 16%, and the equity value difference reached 8%—more than double the initial differences that were below 4%. A temporary shock, then, can have long-persisting effects on a firm's capital structure. Such persistence implies that cross-sectional differences in leverage between firms can continue for a long time period, even without differences in earnings.

(d) A Simulation of leverage distribution

To further explore the model's implications for the cross-sectional differences in firms' leverage, I simulate the cross-sectional distribution of firms' leverage in the model and compare it to the data. To generate a stationary leverage distribution and allow zero-leverage firms to fail, I assume that firms face an exogenous random Poisson shock such that EBIT drops to zero. Such a shock can be interpreted as, for example, the firm's product becoming outdated due to competitors technological advance.[21] Each bankruptcy firm, either due to debt payments or the technology shock, is replaced by a new firm that makes a lump-sum investment to enter and chooses the optimal fractions of debt and equity to finance the investment.

Let λ be the density of the Poisson technology shock dN_t. Then we can write the earnings process as

$$dY_t = (\mu + i)Y_t dt + \sigma Y_t dZ_t - Y_t^- dN_t \tag{40}$$

where Y_t^- denotes earnings before the shock. The HJB equations become

[21] In reality, earnings are not always positive as in a geometric Brownian motion. I match the rate of such shocks to the proportion of Compustat firms with earnings that drop from positive to negative and remain negative for at least three consecutive years (about 2%).

$$
\begin{aligned}
rv(y) = \max \Bigg(&\max_{\phi \geq \bar{\phi}} \Bigg\{ (1-\tau_e) \left[y - \tau_c(y-c) - (c+m) - \kappa i y + p(y) \frac{\phi}{F} \right] \\
&\quad + \left(\frac{\phi}{F} - m \right) [v(y) - y v'(y)] + (\mu + i) y v'(y) + \frac{1}{2} \sigma^2 y^2 v''(y) - \lambda v(y) \Bigg\}, \\
&\max_{\phi < \bar{\phi}} \Bigg\{ \left[y - \tau_c(y-c) - (c+m) - \kappa i y + p(y) \frac{\phi}{F} \right] \\
&\quad + \left(\frac{\phi}{F} - m \right) [v(y) - y v'(y)] + (\mu + i) y v'(y) + \frac{1}{2} \sigma^2 y^2 v''(y) - \lambda v(y) \Bigg\} \Bigg)
\end{aligned}
\tag{41}
$$

For a new firm that needs to invest I_0 to enter and receive an earnings flow starting with Y_0, it chooses debt and equity financing to maximize the continuation value of the firm plus the payoff to shareholders at entry, that is,

$$
v(y_0) F_0 + \left(1 - \mathbf{1}_{p(y_0)F_0 - I_0 > 0} \tau_e\right) [p(y_0) F_0 - I_0] \tag{42}
$$

Since $y = \frac{Y}{F}$ is the only state variable in this homogeneous case, I normalize initial earnings as $Y_0 = 1$. When I_0 is large enough, the firm gains a full tax shield without being constrained from saving personal tax on equity income. The following result describes the firm's optimal debt issuance.

Proposition 6. ***(Unconstrained optimal leverage)*** *Let* $F_0^* = \arg\max_{F_0} \left[v\left(\frac{1}{F_0}\right) + p\left(\frac{1}{F_0}\right) \right] F_0$ *be the debt issuance that maximizes the firm's total enterprise value. If* $p\left(\frac{1}{F_0^*}\right) F_0^* \leq I_0 \leq \left[p\left(\frac{1}{F_0^*}\right) + v\left(\frac{1}{F_0^*}\right) \right] F_0^*$*, the firm's optimal debt issuance is* F_0^**.*

Proof.

$$
\begin{aligned}
\left[v\left(\frac{1}{F_0^*}\right) + p\left(\frac{1}{F_0^*}\right) \right] F_0^* - I_0 &\geq \left[v\left(\frac{1}{F_0}\right) + p\left(\frac{1}{F_0}\right) \right] F_0 - I_0 \\
&\geq \left[v\left(\frac{1}{F_0}\right) + p\left(\frac{1}{F_0}\right) \right] F_0 - I_0 - \mathbf{1}_{p(y_0)F_0 - I_0 > 0} \tau_e \left[p\left(\frac{1}{F_0}\right) F_0 - I_0 \right]
\end{aligned}
\tag{43}
$$

□

I assume that the initial investment for new firms satisfies the conditions above. Then, I simulate the leverage dynamics of 5,000 firms that start with the unconstrained optimal leverage. Suppose any firm fails due to the interest coverage ratio y falling below the bankruptcy threshold y_b or the technology shock. In that case, it is replaced by a new firm starting with the unconstrained optimal leverage. I simulate the evolutions of firms' leverage until it reaches a stationary distribution, with parameters $r = 5\%, \tau_c = 30\%, \tau_b = 30\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 1/15, \kappa = 20, \lambda = 2\%$. Here the Poisson shock density λ matches the proportion of Compustat firms with earnings that drop from positive to negative and remain negative for at least three consecutive years.

As exhibited in Figure 6, the simulated leverage distribution matches the leverage distribution of Compustat firms in the data in Figure 1, showing that the mechanism of the model can generate the observed cross-sectional patterns of leverage in the data even if firms are ex-ante identical. Leverage in both the simulation and the data are measured by Book value of debt/(Book value of debt + Market value of equity). As in the data, about 1/4 of firms have lower than 5% market leverage, and the fractions of firms in each 5% bin are decreasing in leverage levels.[22] Such a distribution contrasts with the bell-shaped distribution implied by traditional trade-off models that have a single global leverage target, suggesting the importance of the different tax incentives for financing investments and capital restructuring in understanding the leverage cross-sections.

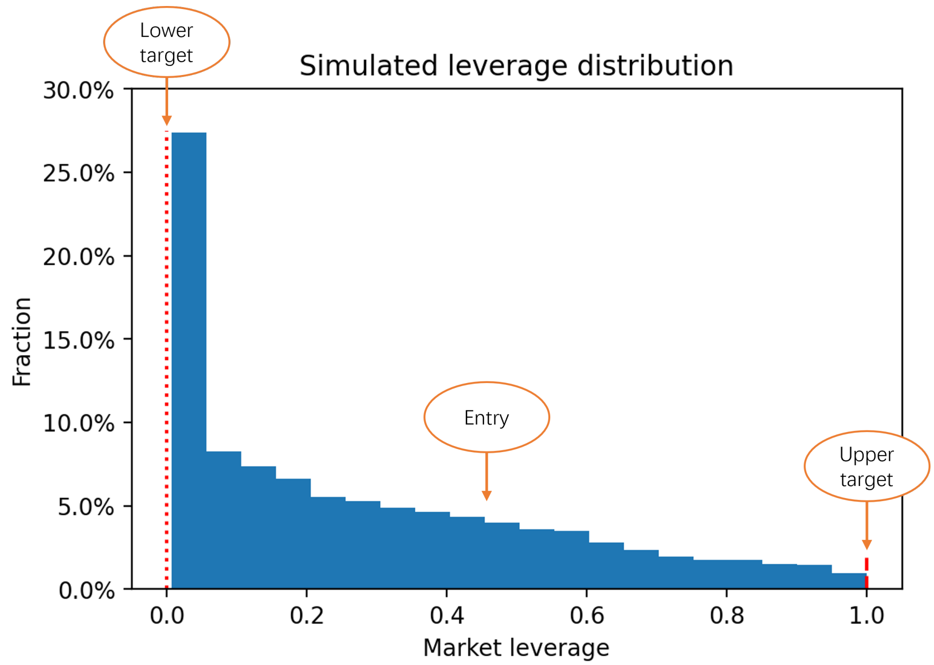
(e) Empirical implications

The model has several empirical implications for firms' financing policies that align with existing empirical evidence or can be tested in the data. First, the model generates a reasonable fraction of zero-leverage firms and helps explain the zero-leverage puzzle (Strebulaev and Yang, 2013). The potentially zero or negative tax benefits of leveraged recapitalization rationalize zero-leverage firms' reluctance to increase leverage. Such an explanation can be tested by measuring firms' marginal tax benefits for issuing additional debt in the data following equation (22). The marginal benefits depend on firms' financing needs and can be negative with reasonable tax rates when there is no need for external financing. The traditional measure $(1 - \tau_b) - (1 - \tau_c)(1 - \tau_e)$ overestimates marginal tax benefits for issuing additional debt, especially for firms without external financing need.

Second, leverage dynamics in the model are path-dependent with persistent differences,

[22] A difference is that the simulated distribution has a thicker tail. The thinner tail in the data may be generated by more realistic assumptions about financially distressed firms, such as allowing for restructuring. I leave that for future work.

Figure 6: Simulated leverage distribution



Notes. The figure plots a simulated stationary distribution of 5,000 firms' market leverage. Failed firms are replaced by new firms entering at the unconstrained optimal leverage. The distribution matches the market leverage distribution in the data shown in Figure 1. The parameters are $r = 5\%, \tau_c = 30\%, \tau_b = 30\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 1/15, \kappa = 20, \lambda = 2\%$.

which is consistent with empirical evidence of persistent effect from past capital structure decisions (e.g., Baker and Wurgler, 2002) and persistent cross-sectional differences (e.g., Lemmon et al., 2008). Both the financing needs to pay debtholders and the debt ratchet effect make firms with higher leverage continue to issue more debt.

Third, in contrast to a traditional trade-off theory, the model features two local leverage targets instead of one. This difference allows the model to generate new empirical implications for firms' leverage adjustments to targets. A firm's leverage target can switch between two targets with significant differences due to changes in its leverage and financing needs. For example, in the model where investment is assumed to be a fixed proportion of EBIT, there is a threshold level of leverage such that the firm adjusts leverage upwards when above that threshold and downwards otherwise, and slow adjustments allow the firm's leverage to cross the threshold due to earnings shocks. Firms actively adjust leverage to targets when leverage is high (equity issuing region) regardless of financing needs, but they only passively adjust leverage with a financing pecking order of

$internal\ cash \prec debt\ financing \prec equity\ financing$ when leverage is low. Such behavior essentially differs from the traditional understanding of the trade-off theory that a firm always actively—although perhaps slowly or infrequently—adjusts to a single leverage target.

Lastly, the dependence of tax benefits on firms' financing needs implies that firms less able to finance their investments with internal cash can better take advantage of the tax benefits and should have higher leverage. This implication is consistent with Denis and McKeon (2012)'s finding that firms' leverage evolution mainly depends on financial surplus, and firms predominantly cover deficits with debt. It is also consistent with the empirical characterization of the long-standing low-leverage puzzle that low-leverage firms typically have good profit and liquidity (e.g., Graham, 2000; Strebulaev and Yang, 2013). Such an inverse correlation between profitability and leverage has been viewed as evidence against the static trade-off theory (e.g., Myers, 1993). Future research can test whether firms with more external financing needs, that is, more investments and less profits, generally have higher leverage if their taxable income is still positive.

6 Welfare analysis and optimal taxation

In the traditional definition, interest expenses generate tax savings at a constant rate $(1 - \tau_b) - (1 - \tau_c)(1 - \tau_e)$. In that case, taxing shareholders at the firm and personal levels are equivalent, given a fixed value of $(1 - \tau_c)(1 - \tau_e)$. However, this paper shows that corporate and personal taxes on equity income affect firms' financing policies differently. As a result, analyzing the effect of tax rates τ_c and τ_e on economic efficiencies in this model can lead to important policy implications. In addition, I discuss optimal maturity from the firm and social welfare perspectives.

(a) Tax efficiency

In order to analyze the distortion of taxes on firms' capital structure and the resulting inefficient bankruptcy loss, I first decompose the firm's pre-tax unleveraged value into the values of equity, debt, expected tax revenue, and expected bankruptcy loss. The pre-tax value of a leveraged firm, including equity value, debt value, and expected tax revenue, equals the firm's pre-tax unleveraged value net of the expected bankruptcy loss. Normalizing $F = 1$, the unleveraged pre-tax value of the firm equals $v_0^{pre-tax}(y) = y(1 - \kappa I)/(r - I)$. Equity and debt values are $v(y)$ and $p(y)$. Expected bankruptcy loss is

$$BL(y) = BC(y)\frac{1 - \kappa I}{(1 - \tau_e)(1 - \tau_c - \kappa I)} \tag{44}$$

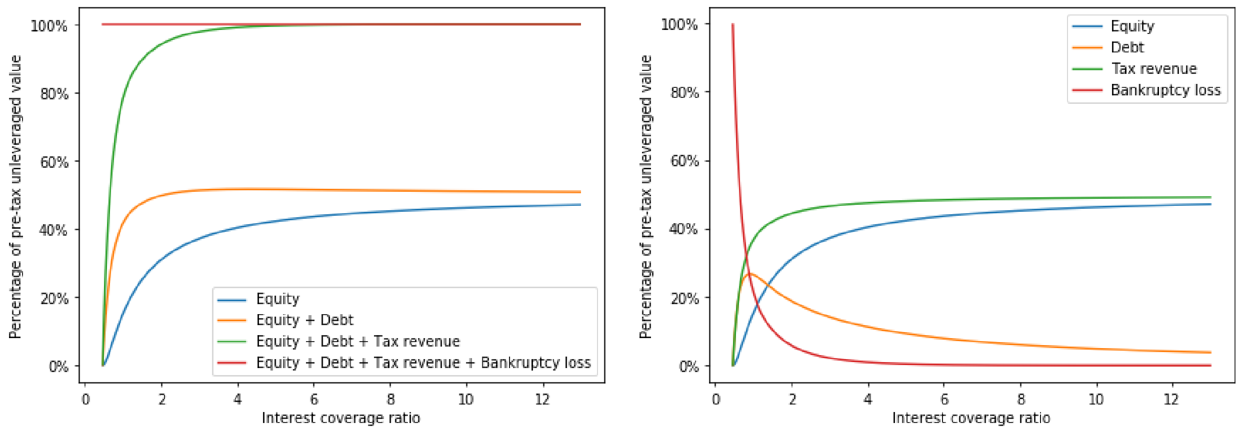
where $BC(y)$ solves HJB equation (21). Then the expected tax revenue equals

$$TR(y) = v_0^{pre-tax}(y) - v(y) - p(y) - BL(y) \tag{45}$$

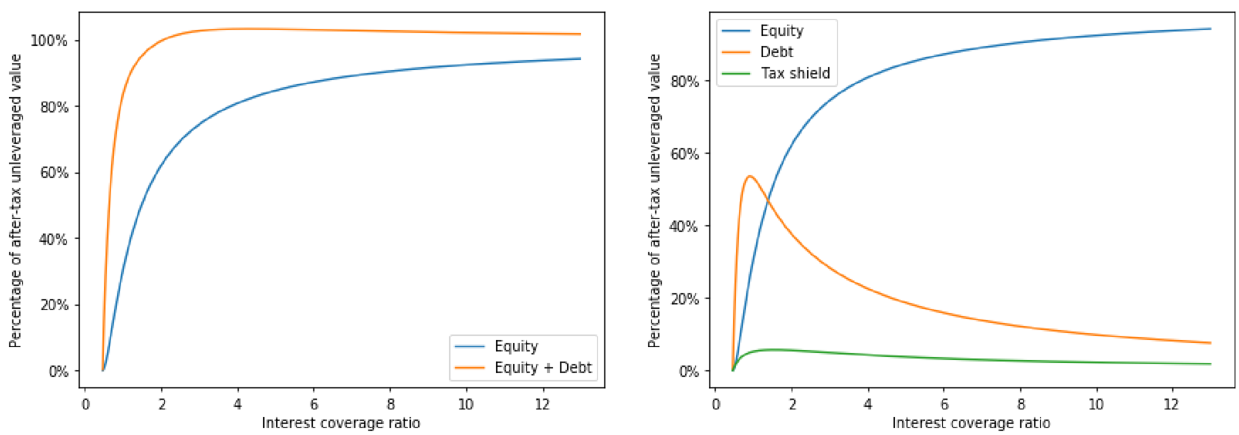
Figure 7 shows the decompositions of the firm's pre- and after-tax values at different interest coverage ratios (y/c) with parameters $r = 5\%, \tau_c = 30\%, \tau_b = 35\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 10$. Panel (a) decomposes the firm's pre-tax unleveraged value into equity value, debt value, expected tax revenue, and expected bankruptcy loss following equation (45). Panel (b) presents the firm's equity value, debt value, and tax shield value as percentages of its after-tax unleveraged value. As the interest coverage ratio increases, there is lower bankruptcy risk, higher expected tax revenue, and a shift of the firm's capital structure from debt to equity. Taxes take up to 50% of the firm's earnings net of investments. The value of debt peaks at 26.73% when the interest coverage ratio is 0.92. Total enterprise value (equity + debt) peaks at the unconstrained optimal leverage when the interest coverage ratio is 4.31. In that case, the firm earns 3.26% of its after-tax unleveraged value from the tax shield of debt net of expected bankruptcy cost. The tax shield of debt is worth 5.58% of the firm's after-tax unleveraged value at the maximum, lower than $(1 - \tau_b) - (1 - \tau_c)(1 - \tau_e) = 9\%$ in the traditional definition of tax shield.

Figure 7: Decomposition of the firm's value

(a) Pre-tax value decomposition



(b) After-tax value decomposition



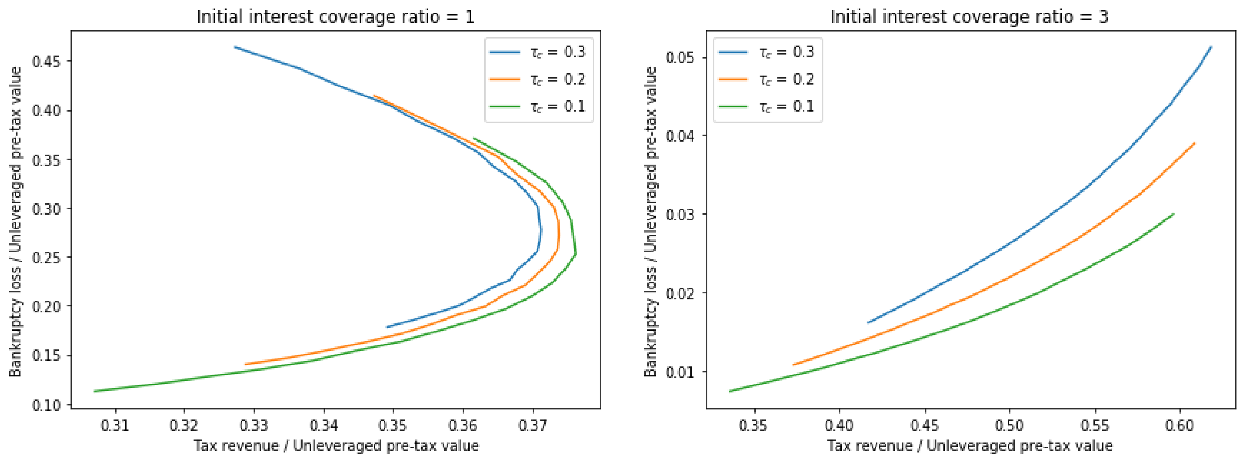
Notes. This figure shows decompositions of the firm's value at different interest coverage ratios (y/c) before and after taxes. Panel (a) decomposes the firm's pre-tax unleveraged value into the values of equity, debt, expected tax revenue, and expected bankruptcy loss. Panel (b) plots the firm's equity and debt value normalized by its after-tax unleveraged value. Pictures on the left plot the cumulative sum of the components, and pictures on the right plot values of each component. The parameters are $r = 5\%, \tau_c = 30\%, \tau_b = 35\%, \tau_e = 20\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 10$.

A government may want to collect more tax revenue or a target level of tax revenue while minimizing the deadweight bankruptcy loss caused by leverage distortions. The following analysis focuses on the case in which the policymaker chooses a combination of corporate tax rate τ_c and personal tax rate on equity income τ_e, given a fixed personal tax rate on bond income τ_b. If the government can choose all three tax rates τ_c, τ_b, τ_e freely, it can set τ_b high enough that firms never use debt. The government can then tax an arbitrary proportion of the firm's earnings without causing inefficiency since the only source of inefficiency in this model is the deadweight bankruptcy loss. In the real world, however, personal tax rates on bond income are usually the same as the rates on wages. It is reasonable to take such personal tax rates as given in the problem discussed here since these rates involve other redistributive concerns that are not covered in this paper.

Figure 8 plots feasible pairs of expected bankruptcy loss and tax revenue, normalized by the firm's unleveraged pre-tax value for different combinations of corporate tax rate τ_c and personal tax rate on equity income τ_e. Assume that the personal tax rate on bond income is fixed at $\tau_b = 35\%$. Each line plots the feasible sets with different values of τ_e when the corporate tax rate τ_c is 10%, 20%, and 30%. Pictures on the left and right plot the cases in which the firm's interest coverage ratio is 1 and 3, as examples for high and low leverage firms. The other parameters are $r = 5\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 10$. The government aims to achieve outcomes in the lower right with higher tax revenue and lower expected bankruptcy loss. The upward-sloping parts of the lines represent the desirable choices where the government faces a trade-off between tax revenue and expected bankruptcy loss. The figure shows that for both high- and low-leverage firms, a lower corporate tax rate can push the line of feasible outcomes to the right, which is preferred.

In Figure 9, I study the optimal choice of corporate tax rate when the policymaker has a fixed tax revenue target. The figure plots the expected bankruptcy loss normalized by the firm's unleveraged pre-tax value when the policymaker collects a target tax revenue with different corporate tax rates τ_c. In this case, the personal tax rate on equity income τ_e is automatically pinned down by the target tax revenue and the other tax rates. The picture on the left plots the case in which the firm's interest coverage ratio is one, and the target tax revenue is 35% of the firm's unleveraged pre-tax value. This case serves as an example of a high-leverage firm. The picture on the right plots the case in which the firm's interest coverage ratio is three, and the target tax revenue is 45% of the firm's unleveraged pre-tax value. This case is an example of a low-leverage firm. Other parameters are the same as above. Given the tax revenue targets, expected bankruptcy loss increases with the corporate tax rate in both cases. Therefore, in these cases, the optimal approach to collecting tax revenue is to set the corporate tax rate at 0 and only tax the shareholders with the personal tax on equity income. In general, the government can reduce expected bankruptcy loss due to leverage distortions without losing tax revenue by taxing shareholders more at the personal level and less at the corporate level. That is because when the firm cannot reduce payouts

Figure 8: Feasible pairs of expected bankruptcy loss and tax revenue



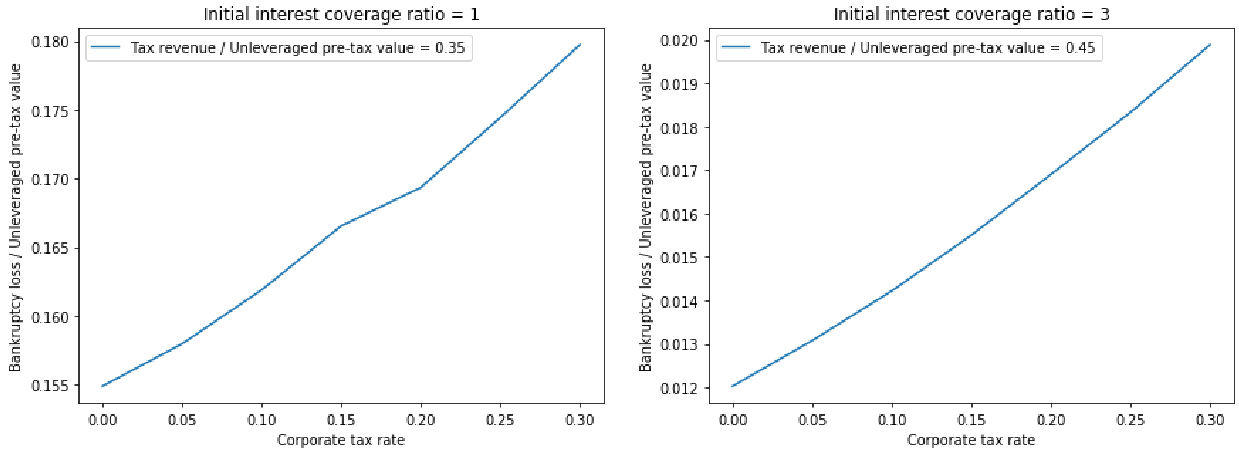
Notes. This figure shows feasible pairs of expected bankruptcy loss and tax revenue, both normalized by the firm's unleveraged pre-tax value, for different combinations of corporate tax rate τ_c and personal tax rate on equity income τ_e. Personal tax rate on bond income is fixed at $\tau_b = 35\%$. Each line plots the feasible sets with different values of τ_e given $\tau_c = 10\%/20\%/30\%$. Pictures on the left and right plot the cases when the firm's interest coverage ratio is 1 and 3, as examples for high- and low-leverage firms. The other parameters are $r = 5\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 10$.

by additional debt issuance, the tax on payouts allows the government to collect tax revenue without incentivizing higher leverage. I illustrate this intuition in a simple two-period model in Appendix C.

(b) Optimal maturity

Similarly, we can assess the firm's maturity preference by comparing the values of securities with different maturity rates. Figure 10 plots the firm's security values when it issues debt with different fixed maturity rates. The picture on the left plots equity values normalized by the firm's after-tax unleveraged value at different leverage levels when the expected maturity is 1, 5, 20 years, or infinity. The picture on the right plots the total enterprise value (debt + equity) normalized by the firm's after-tax unleveraged value at different leverage levels when the expected maturity is 1 year, 5 years, 20 years, or infinity. The equity value increases with expected maturity, but total enterprise value decreases with expected maturity because longer maturity lowers rollover risk for firms and transfers risk from shareholders to debtholders. Without other frictions, firms issue long-term debt, which is suboptimal from a social welfare perspective and leads to higher leverage and bankruptcy risk.

Figure 9: Optimal corporate tax rate given target tax revenue



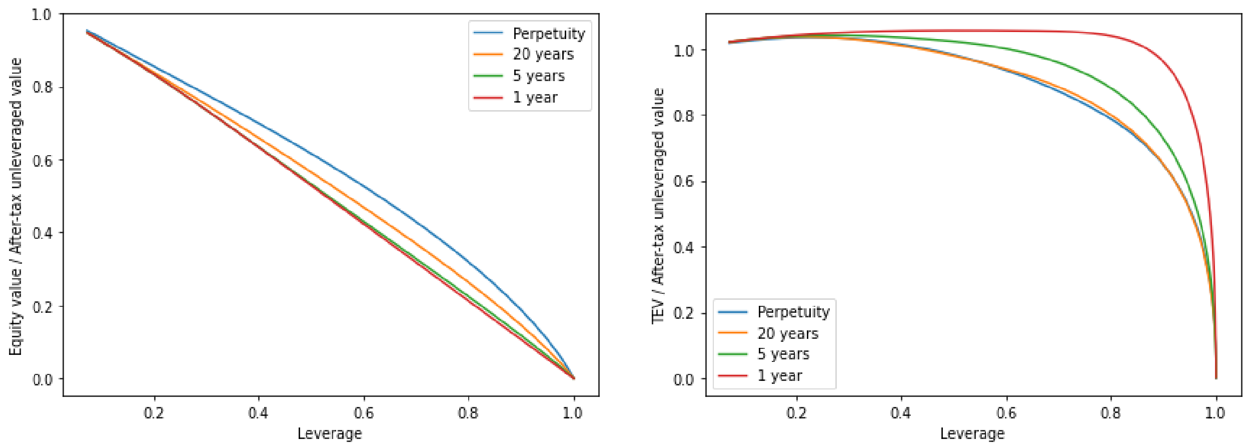
Notes. This figure shows the expected bankruptcy loss normalized by the firm's unleveraged pre-tax value when the policymaker collects a target tax revenue with different corporate tax rates τ_c. Personal tax rate on bond income is fixed at $\tau_b = 35\%$. Personal tax rate on equity income τ_e is automatically pinned down by the target tax revenue and the other tax rates. The picture on the left plots the case in which the firm's interest coverage ratio is 1 and the target tax revenue is 35% of the firm's unleveraged pre-tax value as an example of a high-leverage firm. The picture on the right plots the case in which the firm's interest coverage ratio is 3 and the target tax revenue is 45% of the firm's unleveraged pre-tax value as an example of a low-leverage firm. The other parameters are $r = 5\%, \mu = 0, i = 2\%, \sigma = 40\%, c = \frac{r}{1-\tau_b}, m = 5\%, \kappa = 10$.

(c) Endogenous investment and debt overhang

Here, I assume that investments are endogenous with quadratic costs $\frac{1}{2}\kappa i^2 Y$, and I study optimal investment rates and debt overhang. In this case, we can rewrite the HJB equation as

$$\begin{aligned} rv(y) = \max \Bigg(&\max_{\phi \geq \bar{\phi}} \Bigg\{ (1-\tau_e) \left[y - \tau_c(y-c) - (c+m) - \frac{1}{2}\kappa i^2 y + p(y)\frac{\phi}{F} \right] \\ &\qquad + \left(\frac{\phi}{F} - m \right) [v(y) - yv'(y)] + iyv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \Bigg\}, \\ &\max_{\phi < \bar{\phi}} \Bigg\{ \left[y - \tau_c(y-c) - (c+m) - \frac{1}{2}\kappa i^2 y + p(y)\frac{\phi}{F} \right] \\ &\qquad + \left(\frac{\phi}{F} - m \right) [v(y) - yv'(y)] + iyv'(y) + \frac{1}{2}\sigma^2 y^2 v''(y) \Bigg\} \Bigg) \end{aligned} \tag{46}$$

Figure 10: Security values with different debt maturity rates



Notes. This figure shows the firm's security values when it issues debt with different fixed rates of maturity. The picture on the left plots equity values normalized by the firm's after-tax unleveraged value at different levels of leverage, when the expected maturity is 1 year, 5 years, 20 years, or infinity. The firm's equity value is higher when debt maturity is longer, given any level of leverage. The picture on the right plots total enterprise value (debt + equity) normalized by the firm's after-tax unleveraged value at different levels of leverage, when the expected maturity is 1 year, 5 years, 20 years, or infinity. The firm's total enterprise value is higher when debt maturity is shorter, given any level of leverage.

Taking the first order condition to i, the optimal investment is

$$i = \begin{cases} \frac{v'(y)}{\kappa} & \text{if } \phi < \bar{\phi} \\ \frac{v'(y)}{\kappa} \frac{p(y)}{-[v(y) - yv'(y)]} & \text{if } \phi = \bar{\phi} \\ \frac{v'(y)}{(1-\tau_e)\kappa} & \text{if } \phi > \bar{\phi} \end{cases} \tag{47}$$

In addition to the standard result, the optimal investment rate is multiplied by $\frac{p(y)}{-V_F(Y,F)} = \frac{p(y)}{-[v(y)-yv'(y)]}$ when $\phi = \bar{\phi}$ and by $\frac{1}{1-\tau_e}$ when $\phi > \bar{\phi}$. Since $-V_F(Y,F) \leq p(y) \leq \frac{-V_F(Y,F)}{1-\tau_e}$ according to Proposition 2, these multipliers imply that the firm invests more given the marginal gain $v'(y)$ when the marginal source of financing is internal equity than when the marginal source of financing is debt than when the marginal source of financing is external equity. The costs of financing have a pecking order relation. Internal equity, which is "trapped" in the firm and taxed when distributed, is cheaper than external equity. When debt serves as the marginal source of financing, it is cheaper than external equity and more expensive than internal equity.

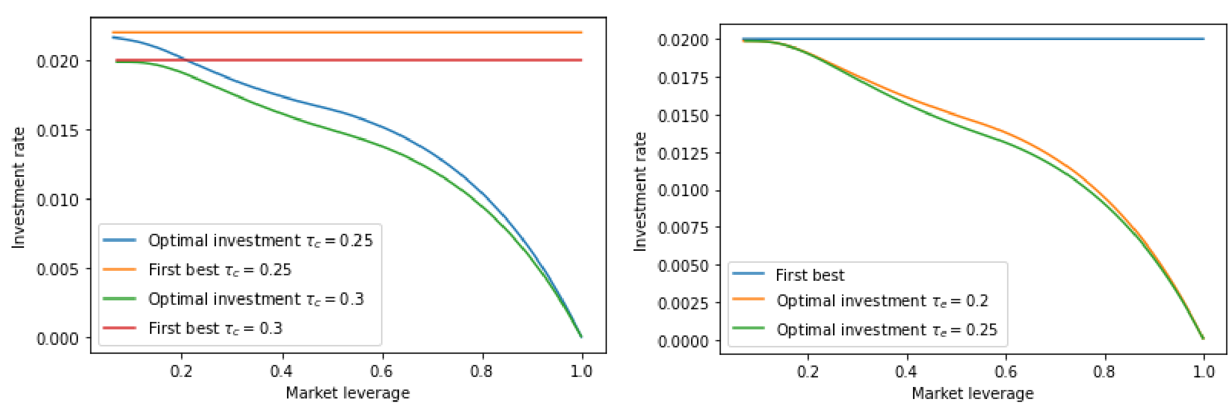
When the firm is unleveraged, the optimal investment rate is

$$i_{unlev} = r - \mu - \sqrt{(r-\mu)^2 - \frac{2(1-\tau_c)}{\kappa}} \tag{48}$$

Figure 11 plots firms' optimal investment rates at different levels of leverage, compared to the first best investment without leverage, and compared across different tax rates. Panel (a) plots the investment rates when the corporate tax rate is 25% and 30%. Panel (b) plots the investment rates when the personal tax rate on equity income is 20% and 25%. Investment rates decrease in leverage due to debt overhang. Given the level of leverage, a 5% corporate tax cut improves investment much more than a 5% cut on payout tax. This can be interpreted as the short-term effect of tax cuts since leverages adjust slowly, and the indirect effect of tax cuts through their impact on leverages happens in the long term. Notice that only τ_c affects the investment rate of an unleveraged firm since when investments are financed internally, the firm always pays τ_e whether it distributes the money or invests it.

Figure 11: Equilibrium investment rates with differnt tax rates

(a) Investment with different corporate tax rates (b) Investment with different payout tax rates



Notes. This figure shows the equilibrium investment rates compared to the first best investment rates at different levels of leverage. Panel (a) plots the investment rates when the corporate tax rate is 25% and 30%. Panel (b) plots the investment rates when the personal tax rate on equity income is 20% and 25%. Investment rates decrease in leverage due to debt overhang. A higher corporate tax rate leads to lower equilibrium and first best investment rates. A higher personal tax rate on equity income leads to slightly higher investment rates.

7 Conclusion

This paper shows that the tax benefits of debt for financing investment and leveraged recapitalization are different. A dynamic trade-off model with corporate and personal taxes and bankruptcy costs features two local leverage targets. Depending on a firm's leverage compared to a threshold, it adjusts to one target or the other and may switch targets due to earnings shocks. When the corporate tax rate is lower than the personal tax rate on bond income, the lower leverage target is 0, which helps explain the zero-leverage puzzle (Strebulaev and Yang, 2013) that over 1/5 of firms have close to zero leverage. A simulation of the model generates a cross-sectional leverage distribution that matches the data.

The paper also studies policymakers' choice of tax rates with a trade-off between tax revenue and expected deadweight bankruptcy loss due to leverage distortions. I show that policymakers can reduce expected bankruptcy loss without losing tax revenue by taxing shareholders more at the personal level and less at the corporate level. A corporate tax cut also improves investments more than a personal tax cut on equity income of the same size.

References

Admati, Anat R, Peter M DeMarzo, Martin F Hellwig, and Paul Pfleiderer (2018), "The leverage ratchet effect," *The Journal of Finance*, 73, 145–198.

Ai, Hengjie, Murray Z Frank, and Ali Sanati (2020), "The trade-off theory of corporate capital structure,".

Ang, Andrew, Vineer Bhansali, and Yuhang Xing (2010), "Taxes on tax-exempt bonds," *The Journal of Finance*, 65, 565–601.

Auerbach, Alan J (1979), "Share valuation and corporate equity policy," *Journal of Public Economics*, 11, 291–305.

Auerbach, Alan J (1981), "TAX INTEGRATION AND THE" NEW VIEW" OF THE CORPORATE TAX: A 1980'S PERSPECTIVE," In Proceedings of the Annual Conference on Taxation Held under the Auspices of the National Tax Association-Tax Institute of America, 21–27, JSTOR.

Auerbach, Alan J (2002), "Taxation and corporate financial policy," *Handbook of public economics*, 3, 1251–1292.

Baker, Malcolm and Jeffrey Wurgler (2002), "Market timing and capital structure," *The journal of finance*, 57, 1–32.

Bolton, Patrick, Hui Chen, and Neng Wang (2014), "Debt, taxes, and liquidity," Technical report, National Bureau of Economic Research.

Bolton, Patrick, Neng Wang, and Jinqiang Yang (2021), "Leverage dynamics under costly equity issuance," *NBER Working Paper*, 26802.

Cooley, Thomas F and Vincenzo Quadrini (2001), "Financial markets and firm dynamics," *American economic review*, 91, 1286–1310.

Crouzet, Nicolas and Fabrice Tourre (2021), "Can the cure kill the patient? Corporate credit interventions and debt overhang," *Corporate credit interventions and debt overhang (June 1, 2021)*.

Dangl, Thomas and Josef Zechner (2021), "Debt maturity and the dynamics of leverage," *The Review of Financial Studies*, 34, 5796–5840.

Dávila, Eduardo and Benjamin M Hébert (2019), "Optimal Corporate Taxation Under Financial Frictions," Technical report, National Bureau of Economic Research.

DeAngelo, Harry, Andrei S Gonçalves, and René M Stulz (2018), "Corporate deleveraging and financial flexibility," *The Review of Financial Studies*, 31, 3122–3174.

Demarzo, Peter M (2019), "Presidential address: Collateral and commitment," *The Journal of Finance*, 74, 1587–1619.

DeMarzo, Peter M and Zhiguo He (2021), "Leverage dynamics without commitment," *The Journal of Finance*, 76, 1195–1250.

Denis, David J and Stephen B McKeon (2012), "Debt financing and financial flexibility evidence from proactive leverage increases," *The Review of Financial Studies*, 25, 1897–1929.

Diamond, Douglas W and Zhiguo He (2014), "A theory of debt maturity: the long and short of debt overhang," *The Journal of Finance*, 69, 719–762.

Fischer, Edwin O, Robert Heinkel, and Josef Zechner (1989), "Dynamic capital structure choice: Theory and tests," *The journal of finance*, 44, 19–40.

Gamba, Andrea and Alexander Triantis (2008), "The value of financial flexibility," *The journal of finance*, 63, 2263–2296.

Goldstein, Robert, Nengjiu Ju, and Hayne Leland (2001), "An EBIT-based model of dynamic capital structure," *The Journal of Business*, 74, 483–512.

Graham, John R (2000), "How big are the tax benefits of debt?," *The journal of finance*, 55, 1901–1941.

Hackbarth, Dirk and David C Mauer (2012), "Optimal priority structure, capital structure, and investment," *The Review of Financial Studies*, 25, 747–796.

He, Zhiguo and Konstantin Milbradt (2014), "Endogenous liquidity and defaultable bonds," *Econometrica*, 82, 1443–1508.

He, Zhiguo and Konstantin Milbradt (2016), "Dynamic debt maturity," *The Review of Financial Studies*, 29, 2677–2736.

He, Zhiguo and Wei Xiong (2012), "Rollover risk and credit risk," *The Journal of Finance*, 67, 391–430.

Hennessy, Christopher A and Toni M Whited (2005), "Debt dynamics," *The journal of finance*, 60, 1129–1165.

Hennessy, Christopher A and Toni M Whited (2007), "How costly is external financing? Evidence from a structural estimation," *The Journal of Finance*, 62, 1705–1745.

Hu, Yunzhi, Felipe Varas, and Chao Ying (2021), "Debt maturity management," Technical report, Working Paper.

Ivanov, Ivan, Luke Pettit, and Toni M Whited (2020), "Taxes depress corporate borrowing: Evidence from private firms," *Available at SSRN 3694869*.

Ju, Nengjiu, Robert Parrino, Allen M Poteshman, and Michael S Weisbach (2005), "Horses and rabbits? Trade-off theory and optimal capital structure," *Journal of Financial and Quantitative analysis*, 40, 259–281.

King, Mervyn A (1974), "Taxation and the cost of capital," *The Review of Economic Studies*, 41, 21–35.

King, Mervyn A (1977), Public policy and the corporation 3, London: Chapman and Hall; New York: Wiley.

Kurshev, Alexander and Ilya A Strebulaev (2015), "Firm size and capital structure," *Quarterly Journal of Finance*, 5, 1550008.

Leland, Hayne et al. (1994), "Bond prices, yield spreads, and optimal capital structure with default risk," Technical report, University of California at Berkeley.

Leland, Hayne E (1994), "Corporate debt value, bond covenants, and optimal capital structure," *The journal of finance*, 49, 1213–1252.

Lemmon, Michael L, Michael R Roberts, and Jaime F Zender (2008), "Back to the beginning: persistence and the cross-section of corporate capital structure," *The journal of finance*, 63, 1575–1608.

Lewellen, Jonathan and Katharina Lewellen (2006), "Internal equity, taxes, and capital structure," *Unpublished Working Paper, Dartmouth College.*

Lin, Leming and Mark J Flannery (2013), "Do personal taxes affect capital structure? Evidence from the 2003 tax cut," *Journal of Financial Economics*, 109, 549–565.

Longstaff, Francis A (2011), "Municipal debt and marginal tax rates: Is there a tax premium in asset prices?," *The Journal of Finance*, 66, 721–751.

Malenko, Andrey and Anton Tsoy (2020), "Optimal time-consistent debt policies," *Available at SSRN 3588163.*

Miller, Merton H (1977), "Debt and taxes," *the Journal of Finance*, 32, 261–275.

Myers, Stewart C (1993), "Still searching for optimal capital structure," *Journal of applied corporate finance*, 6, 4–14.

Poterba, James M and Lawrence H Summers (1984), "The economic effects of dividend taxation," Technical report, National Bureau of Economic Research.

Rosenthal, Steve and Theo Burke (2020), "Whos Left to Tax? US Taxation of Corporations and Their Shareholders," *Draft. Presented at New York University School of Law. October*, 6.

Strebulaev, Ilya A (2007), "Do tests of capital structure theory mean what they say?," *The journal of finance*, 62, 1747–1787.

Strebulaev, Ilya A and Baozhong Yang (2013), "The mystery of zero-leverage firms," *Journal of Financial Economics*, 109, 1–23.

Tserlukevich, Yuri (2008), "Can real options explain financing behavior?," *Journal of Financial Economics*, 89, 232–252.

Appendix A: Proofs

Proof of proposition 4

I characterize the relations between regions of financing strategies where different equilibrium conditions hold by checking the limiting conditions for extreme values of state variables and the boundary conditions between regions. By continuity, the regions in which conditions 1 and 3 in Proposition 3 hold (equity issuing region and payout distributing region) do not connect. Therefore, in the followings, I check equilibrium conditions at bankruptcy, when $\frac{Y}{F}$ converges to infinity and at the boundaries where a break-even by debt region connects to an equity issuing region or a payout distributing region.

When bankruptcy is triggered, both the debt price and the first order partial derivative of the value function are zero, that is, $p^b(Y,F) = V_F^b(Y,F) = 0$, where $p^b(Y,F), V^b(Y,F)$ denote the debt price and the value function at a pair of (Y,F) such that bankruptcy is triggered.

When $\frac{Y}{F} \to \infty$, the firm's securities converge to the risk-free values,

$$p^{rf}(Y,F) = \frac{(1-\tau_b)c+m}{r+m} \tag{49}$$

$$V^{rf}(Y,F) = V^{rf}(Y,0) - (1-\tau_e)\frac{(1-\tau_c)c+m}{r+m}F \tag{50}$$

where $p^{rf}(Y,F), V^{rf}(Y,F)$ denote the risk-free limit of the debt price and the value function. If $\tau_b > \tau_c$, then $-\frac{V_F^{rf}(Y,F)}{1-\tau_e} > p^{rf}(Y,F) > -V_F^{rf}(Y,F)$, and $\phi^{rf}(Y,F) = \bar{\phi}^{rf}(Y,F)$. The firm spends all free cash on debt repurchase when $\frac{Y}{F}$ is large enough, to save the difference between personal income tax rates priced in bonds and the corporate tax rates. The tax benefits dominate the debt ratchet effect of debt repurchase when the remaining debt is low enough. If $\tau_b < \tau_c$, then $-\frac{V^{rf}(Y,F)}{1-\tau_e} < p^{rf}(Y,F)$ and the equilibrium security prices cannot converge to the risk-free valuations since otherwise the firm will issue debt discretely to take advantage of the high debt price until $-\frac{V^{rf}(Y,F)}{1-\tau_e} = p^{rf}(Y,F)$. That is because investors always expect the firm to lever up when leverage is low.

Then I analyze the boundary conditions between regions. Suppose there exists a region of (Y,F) such that the first order condition $p(Y,F) = -V_F(Y,F)$ holds (equity issuing region). Then at the boundary between this region and the region in which $-\frac{V^{rf}(Y,F)}{1-\tau_e} > p(Y,F) >$

$-V_F(Y,F)$ (break-even by debt region),

$$-rV_F(Y,F) = (1-\tau_c)c - p_F(Y,F)\phi + m[1+V_F(Y,F)] - (\phi - mF)V_{FF}(Y,F) \\ - [\mu(Y)+I(Y)]\,V_{FY}(Y,F) - \frac{1}{2}\sigma(Y)^2 V_{FYY}(Y,F) \tag{51}$$

where $p(Y,F) = -V_F(Y,F)$, $p_F(Y,F) = -V_{FF}(Y,F)$, $p_Y(Y,F) = -V_{FY}(Y,F)$ by smooth pasting conditions. $\phi(Y,F) = \bar{\phi}(Y,F)$ in the break-even by debt region, and $\phi(Y,F) = \frac{(\tau_b-\tau_c)c}{p_F(Y,F)}$ in the equity issuing region. Compare (51) with (14), if $\bar{\phi}(Y+\epsilon_Y, F+\epsilon_F) > \frac{(\tau_b-\tau_c)c}{p_F(Y+\epsilon_Y,F+\epsilon_F)}$ within a neighborhood of the boundary in the break-even by debt region, then $-V_{FYY}(Y+\epsilon_Y,F+\epsilon_F) < p_{YY}(Y+\epsilon_Y,F+\epsilon_F)$. Then, since $p(Y,F) > -V_F(Y,F)$ in the break-even by debt region, this region must be on the "lower-right" side (with higher $\frac{Y}{F}$) of the equity issuing region. Otherwise if $\bar{\phi}(Y+\epsilon_Y,F+\epsilon_F) < \frac{(\tau_b-\tau_c)c}{p_F(Y+\epsilon_Y,F+\epsilon_F)}$ within a neighborhood of the boundary in the break-even by debt region, the break-even by debt region must be on the "upper-left" side (with lower $\frac{Y}{F}$) of the equity issuing region. However, since equity issuance is positive at the "upper-left" boundary of the equity issuing region, by continuity, the firm should issue more debt when not issuing equity, that is, $\bar{\phi}(Y+\epsilon_Y,F+\epsilon_F) > \frac{(\tau_b-\tau_c)c}{p_F(Y+\epsilon_Y,F+\epsilon_F)}$. That leads to a contradiction. Therefore, the break-even by debt region must be on the "lower-right" side of the equity issuing region. There is at most one continuous equity issuing region where $p(Y,F) = -V_F(Y,F)$.

Similarly, if there exists a region of (Y,F) such that the first order condition $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$ holds (payout distributing region), at the boundary between this region and the break-even by debt region,

$$-rV_F(Y,F) = (1-\tau_e)\left[(1-\tau_c)c - p_F(Y,F)\phi + m\right] + mV_F(Y,F)) - (\phi - mF)V_{FF}(Y,F) \\ - (\mu(Y)+I(Y))\,V_{FY} - \frac{1}{2}\sigma(Y)^2 V_{FYY}(Y,F) \tag{52}$$

where $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$, $p_F(Y,F) = -\frac{V_{FF}(Y,F)}{1-\tau_e}$, $p_Y(Y,F) = -\frac{V_{FY}(Y,F)}{1-\tau_e}$ by smooth pasting conditions. $\phi(Y,F) = \bar{\phi}(Y,F)$ in the break-even by debt region, and $\phi(Y,F) = \frac{(\tau_b-\tau_c)c}{p_F(Y,F)}$ in the payout distributing region. Compare (52) with (14), if $\bar{\phi}(Y+\epsilon_Y,F+\epsilon_F) > \frac{(\tau_b-\tau_c)c}{p_F(Y+\epsilon_Y,F+\epsilon_F)}$ within a neighborhood of the boundary in the break-even by debt region, then $-V_{FYY}(Y+\epsilon_Y,F+\epsilon_F) < p_{YY}(Y+\epsilon_Y,F+\epsilon_F)$. Then since $p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$ in the break-even by debt region, this region must be on the "upper-left" side of the payout distributing region. If $\bar{\phi}(Y+\epsilon_Y,F+\epsilon_F) < \frac{(\tau_b-\tau_c)c}{p_F(Y+\epsilon_Y,F+\epsilon_F)}$ within a neighborhood of the boundary in the break-even by debt region, then $-V_{FYY}(Y+\epsilon_Y,F+\epsilon_F) > p_{YY}(Y+\epsilon_Y,F+\epsilon_F)$. Then since

$p(Y,F) < -\frac{V_F(Y,F)}{1-\tau_e}$ in the break-even by debt region, this region must be on the "lower-right" side of the payout distributing region. When $\bar{\phi}(Y,F)$ and $\frac{(\tau_b-\tau_c)c}{p_F(Y,F)}$ satisfies single-crossing condition, a break-even by debt region cannot be on the "lower-right" of one payout distributing region while being on the "upper-left" of another payout distributing region. Then there is at most one continuous payout distributing region where $p(Y,F) = -\frac{V_F(Y,F)}{1-\tau_e}$.

Then we can summarize the regions of financing policies as in Proposition 4.

Proof of proposition 5

By Proposition 4, when $y = \frac{Y}{F}$ is large enough, $\phi(y) = \bar{\phi}(y) < 0$ if $\tau_c < \tau_b$ and $\phi(y) = 0$ if $\tau_c = \tau_b$. Therefore, the firm's leverage converges to zero when it is low enough.

If $y_e < \frac{(1-\tau_c)c+m[1-p(y_e)]}{1-\tau_c-\kappa i}$, let $y^0 > y_e$ be a solution of $y = \frac{(1-\tau_c)c+m[1-p(y)]}{1-\tau_c-\kappa i}$, which exists by the continuity of $p(y)$. Then for $y \in (y_e, y^0)$, $\phi(y) = \bar{\phi}(y) = -\frac{F}{p(y)}\left[(1-\tau_c-\kappa i)y_e - (1-\tau_c)c - m\right] > mF$. The firm's leverage converges to the leverage at y_e. When $y < y_e$, $\phi(y) = \frac{(\tau_c-\tau_b)cF}{yp'(y)} < mF$. The firm's leverage also converges to the leverage at y_e. Therefore, the leverage ratio at y_e is also a local leverage target.

By Proposition 4 and the monotonicity of $\bar{\phi}(y)$, there cannot be other leverage targets.

Appendix B: The algorithm for solving the model numerically

Here I describe the algorithm for solving the HJB differential equations for the security values $v(y)$ and $p(y)$.

Step 1. Start with a guess of the bankruptcy threshold $\hat{y}_b$.

Step 2. Make a guess of the boundary $\hat{y}_e$ between the equity issuing region and the break-even by debt region, which is larger than y_b.

Step 3. Starting with $\hat{v}(\hat{y}_b) = \hat{v}'(\hat{y}_b) = \hat{p}(\hat{y}_b) = 0$ and $\hat{p}'(\hat{y}_b)$ found by closed-form solutions in the equity issuing region, generate $\hat{v}(y), \hat{p}(y)$ by the following algorithm.

A fourth-order Runge-Kutta-Nystrm algorithm for $v''(y) = \mathscr{G}(y, p(y), p'(y), v(y), v'(y))$ and $p''(y) = \mathscr{H}(y, p(y), p'(y), v(y), v'(y))$:

(1) Let

$$l_1^v = \mathscr{G}(y, p(y), p'(y), v(y), v'(y)) \tag{53}$$
$$l_1^p = \mathscr{H}(y, p(y), p'(y), v(y), v'(y)) \tag{54}$$
$$v_1' = v'(y) + l_1^v h/2 \tag{55}$$
$$p_1' = p'(y) + l_1^p h/2 \tag{56}$$
$$v_1 = v(y) + (v'(y) + v_1')/2 \times h/2 \tag{57}$$
$$p_1 = p(y) + (p'(y) + p_1')/2 \times h/2 \tag{58}$$

where h is a small step size.

(2) Let

$$l_2^v = \mathscr{G}(y + h/2, p_1, p_1', v_1, v_1') \tag{59}$$
$$l_2^p = \mathscr{H}(y + h/2, p_1, p_1', v_1, v_1') \tag{60}$$
$$v_2' = v'(y) + l_2^v h/2 \tag{61}$$
$$p_2' = p'(y) + l_2^p h/2 \tag{62}$$
$$v_2 = v(y) + (v'(y) + v_2')/2 \times h/2 \tag{63}$$
$$p_2 = p(y) + (p'(y) + p_2')/2 \times h/2 \tag{64}$$

(3) Let

$$l_3^v = \mathscr{G}(y + h/2, p_2, p_2', v_2, v_2') \tag{65}$$
$$l_3^p = \mathscr{H}(y + h/2, p_2, p_2', v_2, v_2') \tag{66}$$
$$v_3' = v'(y) + l_3^v h \tag{67}$$
$$p_3' = p'(y) + l_3^p h \tag{68}$$
$$v_3 = v(y) + (v'(y) + v_3')/2 \times h \tag{69}$$
$$p_3 = p(y) + (p'(y) + p_3')/2 \times h \tag{70}$$

(4) Let

$$l_4^v = \mathscr{G}(y+h, p_3, p_3', v_3, v_3') \tag{71}$$

$$l_4^p = \mathscr{H}(y+h, p_3, p_3', v_3, v_3') \tag{72}$$

$$v'(y+h) = v'(y) + h/6 \times (l_1^v + 2l_2^v + 2l_3^v + l_4^v) \tag{73}$$

$$p'(y+h) = p'(y) + h/6 \times (l_1^p + 2l_2^p + 2l_3^p + l_4^p) \tag{74}$$

$$v(y+h) = v(y) + h/6 \times (v_1' + 2v_2' + 2v_3' + v'(y+h)) \tag{75}$$

$$p(y+h) = p(y) + h/6 \times (p_1' + 2p_2' + 2p_3' + p'(y+h)) \tag{76}$$

Here $\mathscr{G}(y, p(y), p'(y), v(y), v'(y))$ and $\mathscr{H}(y, p(y), p'(y), v(y), v'(y))$ are determined by reorganizing the HJB equations in each region.

Then iterate for $y+h$, until y reachs a large enough threshold such that the security values are close enough to their limits for y converging to infinity.

Step 4. Check if $\hat{p}(y)$ converges to $\frac{(1-\tau_b)c+m}{r+m}$. If not, adjust $\hat{y}_e$ and repeat steps 3-4 until convergence.

Step 5. Check if $\hat{v}(y)$ converges to $\frac{(1-\tau_e)(1-\tau_c-\kappa i)y}{r-\mu-i}$. If not, adjust $\hat{y}_b$ and repeat steps 2-4 until convergence.

Step 6. Check if the results satisfy the equilibrium conditions.

Appendix C: Tax efficiency problem in a two-period model

Here I illustrate why it is more efficient to tax shareholders at the personal level (τ_e) than the corporate level (τ_c) in a simple two-period model.

Suppose a firm is making an investment that needs external financing I in period 0, with $I = 0$ representing the investment can be self-financed. The investment generates an uncertain profit that is revealed in period 1. For simplicity, assume the profit is earned in perpetuity with EBIT $Y \in (0, \infty)$ per period. Let $h(Y)$ and $H(Y)$ be the probability density function and cumulative distribution function of Y. In period 0, the firm issues perpetual debt with coupon c per period to maxmize the after-tax profits for the shareholders. In period 1, the firm defaults if Y is lower than the interest expense c and there is no recovery value at bankruptcy. Assume all agents are risk neutral and the risk-free rate is r. Then we can write the firm's problem as

$$\max_c (1 - \mathbf{1}_{\{p(c)-I>0\}}\tau_e)(p(c) - I) + \frac{1}{r}(1-\tau_e)(1-\tau_c)\int_c^\infty h(Y)\,(Y-c)\,dY \tag{77}$$

and the debt price is

$$p(c) = \frac{1}{r}(1-\tau_b)c\,[1 - H(c)] \tag{78}$$

Then the first order condition is

$$\left[1 - \frac{(1-\tau_e)(1-\tau_c)}{(1-\mathbf{1}_{\{p(c)-I>0\}}\tau_e)(1-\tau_b)}\right][1 - H(c)] = ch(c) \tag{79}$$

representing the trade-off between the marginal tax benefits on the left and the marginal bankruptcy cost on the right. When $\frac{ch(c)}{1-H(c)}$ is increasing in c, higher marginal tax benefits leads to higher leverage and expected bankruptcy cost. For example, when Y follows an exponential distribution, the optimal leverage c^* is linear in $(1-\mathbf{1}_{\{p(c^*)-I>0\}}\tau_e)(1-\tau_b)-(1-\tau_e)(1-\tau_c)$.

The expected tax revenue is

$$\mathbf{1}_{\{p(c)-I>0\}}\tau_e(p(c) - I) + \frac{1}{r}\int_c^\infty h(Y)\,\{[1-(1-\tau_e)(1-\tau_c)]\,(Y-c) + \tau_b c\}\,dY \tag{80}$$

Notice that optimal leverage c is determined by either $\frac{(1-\tau_e)(1-\tau_c)}{1-\tau_b}$ or $\frac{1-\tau_c}{1-\tau_b}$ depending on I and the tax rates. When keeping $(1-\tau_e)(1-\tau_c)$ and τ_b fixed, increasing τ_e and decreasing τ_c correspondingly increases tax revenue and decreases the expected bankruptcy cost when $p(c^*)-I>0$, and has no effect otherwise. Therefore, taxing shareholders at the personal level (τ_e) is generally more efficient than the corporate level (τ_c). While the corporate tax τ_c can always be shielded by interest expenses and encourages leverage, sometimes the government can gain tax revenue from taxing payouts at τ_e without affecting leverage.

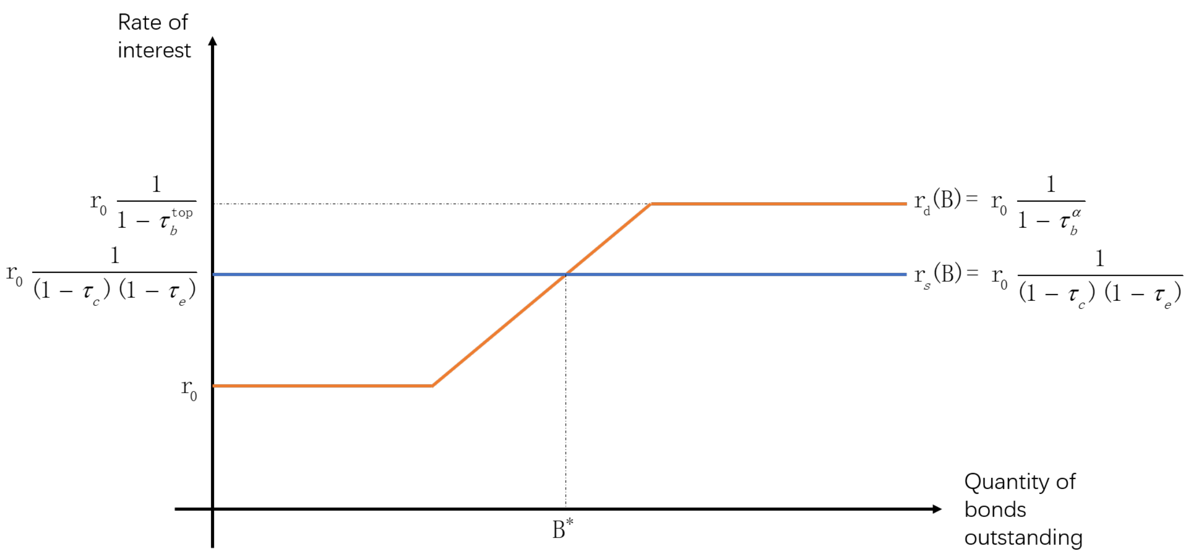
Online Appendix of "Debt and Taxes: Revisited in Dynamics", Hu (2024)

(Not Intended for Publication)

Online Appendix I: Extended discussion on Miller (1977)

Miller (1977) describes a market equilibrium where $(1-\tau_c)(1-\tau_e) = 1-\tau_b^{\text{marginal bondholder}}$. In this equilibrium, firms gain no tax benefits on their values. There is no optimal leverage ratio for individual firms but only an equilibrium leverage ratio for the whole corporate sector. Cross-sectional leverage differences are determined by the clientele of firms' bonds with different personal tax rates. The figure below plots all firms' and investors' supply and demand of bonds in this equilibrium following Figure 1 in Miller (1977). There are no frictions except taxes. r_0 is the interest rate of tax-exempt bonds. The upward-sloping part of the demand curve represents that interest rates have to increase to attract investors in higher tax brackets as the amount of debt outstanding grows. Investors with low personal tax rates gain all the surplus.

Figure: Market equilibrium in Miller's (1977) framework



Such equilibrium requires shareholders to make tax rates on capital gains small enough by tax concessions. "In the limiting case ... that $(1-\tau_c)(1-\tau_{ps})$ implied a value for τ_{pb}^{α} greater than the top bracket of the income tax, then no interior market equilibrium would be

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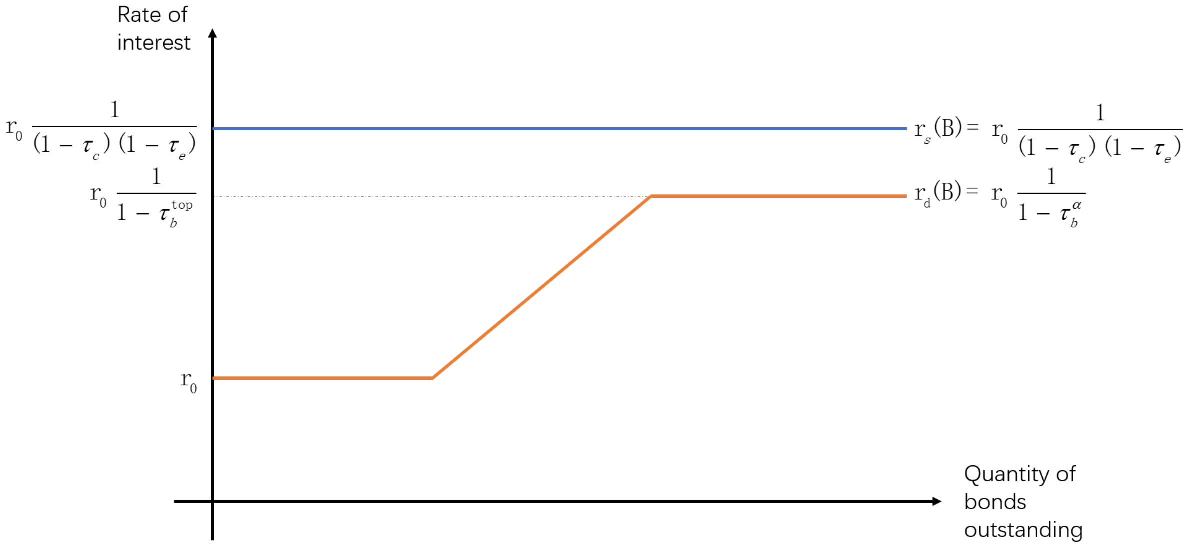
possible." However, empirical measures of effective tax rates on equity income are typically not small enough for the equation to hold without τ_b exceeding the top rates. Then the supply and demand curves become the following .[1]

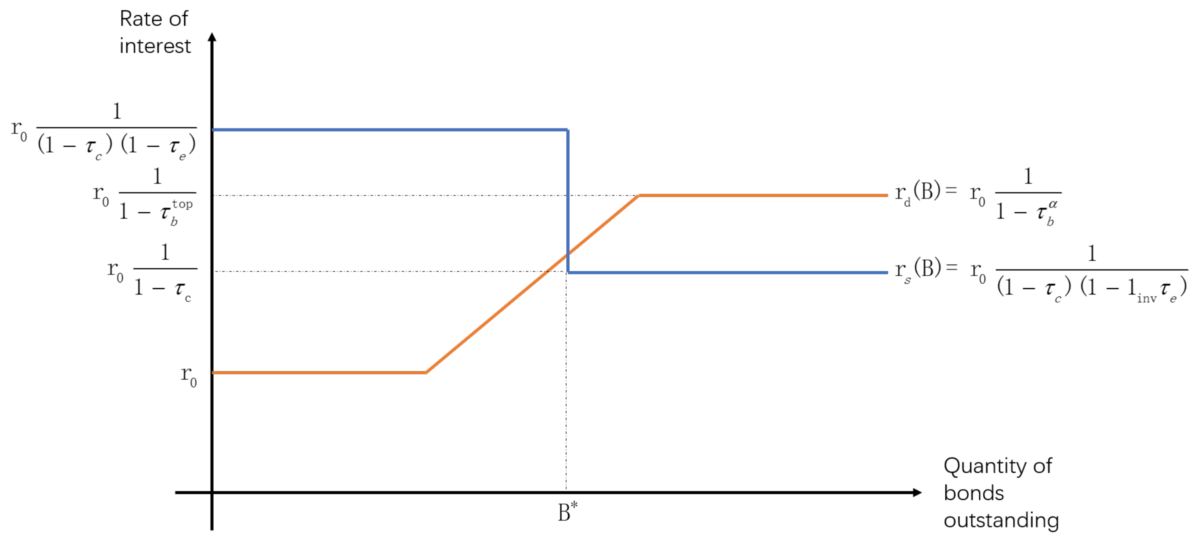
Figure: No interior equilibrium in Miller's (1977) framework



The different tax consequences of financing investments by debt and leveraged recapitalizations imply a different shape of the supply curve. Therefore, firms supply bonds at rate $r_0 \frac{1}{(1-\tau_c)(1-\tau_e)}$ only for financing investments. When recapitalizing for tax shields, firms offer rate $r_0 \frac{1}{(1-\tau_c)}$. The figure below plots a revised equilibrium in this manner. In such an equilibrium, the marginal bondholders' personal tax rate can be anywhere between τ_c and $1-(1-\tau_c)(1-\tau_e)$. Firms gain a surplus from debt. Cross-sectional distributions of leverage depend on firms' external financing need.

[1] One way to recover an interior equilibrium here is to consider gradient corporate tax rates, which make the supply curve downward sloping.

Figure: Market equilibrium in revised framework



Online Appendix II: A model without leverage adjustments (following Leland 1994)

Here I model the key mechanism of this paper into a stylized model without dynamic leverage adjustments following Leland (1994b). A firm earns an exogenous cash flow following a lognormal process, issues debt at time 0, and rolls over the debt. In addition, I assume the firm needs external financing at the beginning and considers personal taxes. I solve the model in closed form and show that the firm's capital structure choice largely depends on the amount of external financing needed at time 0 due to the tax benefit differences between external financing and recapitalizing. When the firm needs no external financing, as in Leland (1994b), opposite to the traditional result without personal taxes, the firm issues no debt if the personal income tax rate on interest payments is no less than the corporate tax rate.

(a) Model setup

Investors and the firm are risk neutral. There exists a risk-free asset paying a constant rate of return r after tax. A firm's before-tax cash flow follows

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t \tag{1}$$

where $\mu < r$. At time 0, the firm needs to finance an investment $I \geq 0$ by issuing debt or equity to earn the cash flow.[2] Assume that $I < \frac{(1-\tau_c)(1-\tau_e)Y}{r-\mu}$, the investment does not exceed the firm's unleveraged value. The firm issues homogeneous debt with a coupon rate c and total principal F that matures exponentially at rate $m \geq 0$. It rolls over matured debt until bankruptcy.

There are three taxes at constant rates: a corporate tax at rate τ_c, a personal income tax on bonds at rate τ_b, and a personal income tax on equity at rate τ_e. By a constant rate of personal tax on equity, I am assuming that the firm's distribution strategy and shareholders' tax deferral streategy are fixed over time, so that each dollar available to shareholders are taxed equally.[3] For simplicity, assume the firm holds no cash and distributes all free cash flow as dividends.The firm maximizes the total value of after-tax dividends for shareholders and claims bankruptcy when it is optimal. At bankruptcy, a fraction $\alpha \in [0, 1)$ of the firm's unleveraged after-tax value $v_{unlev}(Y_b) = \frac{(1-\tau_c)(1-\tau_e)Y_b}{r-\mu}$ can be recovered and paid to debtholders, where Y_b denotes pre-tax earnings at bankruptcy.

(b) Optimal debt issuance

Denote $v(Y)$ as the firm's equity value after dividends or equity issuance for $t \geq 0$ and $v_0(Y)$ as the equity value before dividends or equity issuance at time 0. Let $p(Y)$ be the price of debt with a unit face value that rolls over until bankruptcy, equaling the after-tax value of payments earned by debtholders, and $\tilde{p}(Y)$ be the value of the firm's payments to this debt before personal income tax on bonds. Then $v_0(Y)$ can be written as the sum of time 0 after-tax dividends (with negative value representing equity issuance) and the equity value after dividends or equity issuance $v(Y)$, where $v(Y)$ equals the unleveraged cash flow value $v_{unlev}(Y)$ plus tax benefits for saving corporate tax $\mathcal{TB}_c(Y)$ and personal income tax on equity $\mathcal{TB}_e(Y)$ on the cash flow minus bankruptcy costs $\mathcal{BC}(Y)$ and the value of payments to debtholders $\tilde{p}(Y)F$. At time 0, the firm chooses a face value of debt F to maximize

$$\begin{aligned} v_0(Y) &= \left(1 - \mathbf{1}_{\{p(Y)F - I \geq 0\}}\tau_e\right)[p(Y)F - I] + v(Y) \\ &= \left(1 - \mathbf{1}_{\{p(Y)F - I \geq 0\}}\tau_e\right)[p(Y)F - I] + v_{unlev}(Y) + \mathcal{TB}_c(Y) + \mathcal{TB}_e(Y) - \mathcal{BC}(Y) - \tilde{p}(Y)F \end{aligned} \tag{2}$$

[2]If $I = 0$, the firm starts with no need for external financing as in Leland (1994b).

[3]Deferring the realizations of personal taxes on equity by stock repurchases or cash holdings can be represented by a lower value of the parameter τ_e, as long as the firm's distribution strategy is fixed over time.

Here $\mathbf{1}_{\{p(Y)F-I\geq 0\}}$ equals 1 if the firm distributes dividends at time 0 and equals 0 if the firm issues equity. Let $\mathcal{TC}_b(Y)=[\tilde{p}(Y)-p(Y)]F$ be the personal income tax costs on bonds, then we can rewrite (2) as

$$v_0(Y)=-I-\mathbf{1}_{\{p(Y)F-I\geq 0\}}\tau_e[p(Y)F-I]+v_{unlev}(Y)+\mathcal{TB}_c(Y)+\mathcal{TB}_e(Y)-\mathcal{TC}_b(Y)-\mathcal{BC}(Y) \tag{3}$$

Besides tax benefits and costs on the cash flow, personal income taxes also reduce shareholders' payoff at time 0 by $\tau_e[p(Y)F-I]$ if there is a dividend payment. Therefore, the net tax benefits of debt are reduced if the firm issues more debt than needed for financing the investment.

Solving the value function

Next, I solve each component of $v_0(Y)$ by their HJB equations. The price of debt follows

$$\underbrace{rp(Y)}_{\text{required return}}=\underbrace{(1-\tau_b)c}_{\text{after-tax coupon}}+\underbrace{m[1-p(Y)]}_{\text{rollover gain}}+\underbrace{\mu Yp'(Y)+\frac{1}{2}\sigma^2Y^2p''(Y)}_{\text{cash flow evolution}} \tag{4}$$

with boundary conditions at infinity $p(\infty)=\frac{(1-\tau_b)c+m}{r+m}$, and at bankruptcy $p(Y_b)=\frac{1}{F}\alpha v_{unlev}(Y_b)$. Then the after-tax value of a par bond is [4]

$$p(Y)=\frac{c(1-\tau_b)+m}{r+m}\left[1-\left(\frac{Y}{Y_b}\right)^{\gamma_1}\right]+\frac{1}{F}\alpha v_{unlev}(Y_b)\left(\frac{Y}{Y_b}\right)^{\gamma_1} \tag{5}$$

where

$$\gamma=\frac{-\left(\mu-\frac{1}{2}\sigma^2\right)\pm\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2(m+r)}}{\sigma^2} \tag{6}$$

The value of payments to a unit face value of debt before personal income taxes $\tilde{p}(Y)$ follows

$$\underbrace{r\tilde{p}(Y)}_{\text{required return}}=\underbrace{c}_{\text{pre-tax coupon}}+\underbrace{m[1-p(Y)]}_{\text{rollover gain}}+\underbrace{\mu Y\tilde{p}'(Y)+\frac{1}{2}\sigma^2Y^2\tilde{p}''(Y)}_{\text{cash flow evolution}} \tag{7}$$

[4] Here, I assume only coupons are taxed on a bond for tractability, as in Leland (1994b). The value of a unit principal bond that is taxed only on its coupon is $\frac{m+c(1-\tau_b)}{m+yield(1-\tau_b)}$, while that of a bond taxed on its yield is $\frac{m+c-\tau_b yield}{m+yield(1-\tau_b)}$. There is a difference $\frac{(yield-c)\tau_b}{m+yield(1-\tau_b)}$ that makes the simplifying assumption increase the debt price and decrease the tax shield for lower coupon rates. It slightly increases the rollover gain when the firm is close to bankruptcy and the yield is high if $\tau_b-\tau_c>0$.

Substract (4) from (7) and multiply by F, we get

$$r\mathcal{TC}_b(Y) = \tau_b cF + \mu Y \mathcal{TC}_b'(Y) + \frac{1}{2}\sigma^2 Y^2 \mathcal{TC}_b''(Y) \tag{8}$$

with boundary conditions at infinity $\mathcal{TC}_b(\infty) = \frac{\tau_b cF}{r}$, and at bankruptcy $\mathcal{TC}_b(Y_b) = 0$. Then the personal income tax cost on bonds is

$$\mathcal{TC}_b(Y) = \frac{\tau_b cF}{r}\left[1 - \left(\frac{Y}{Y_b}\right)^{\eta_1}\right] \tag{9}$$

where

$$\eta = \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) \pm \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} \tag{10}$$

Since $r > \mu$, $m \geq 0$, $\gamma_1 \leq \eta_1 < 0 < 1 < \eta_2 \leq \gamma_2$.

Similarly, the tax benefit from saving corporate taxes is

$$\mathcal{TB}_c(Y) = \frac{\tau_c cF}{r}\left[1 - \left(\frac{Y}{Y_b}\right)^{\eta_1}\right] \tag{11}$$

The value of corporate tax savings (11) differs from the value of personal income tax cost on bonds (9) only by the tax rates, because they are both based on coupon payments. Each dollar of the firm's cash flow is taxed either by the personal income rate τ_b or the corporate rate τ_c, depending on whether it is used for coupon payments.

The bankruptcy cost follows

$$r\mathcal{BC}(Y) = \mu Y \mathcal{BC}'(Y) + \frac{1}{2}\sigma^2 Y^2 \mathcal{BC}''(Y) \tag{12}$$

with boundary conditions at infinity $\mathcal{BC}(\infty) = 0$, and at bankruptcy $\mathcal{BC}(Y_b) = (1-\alpha)v_{unlev}(Y_b)$. Then

$$\mathcal{BC}(Y) = (1-\alpha)v_{unlev}(Y_b)\left(\frac{Y}{Y_b}\right)^{\eta_1} \tag{13}$$

The tax benefit from saving personal tax on equity follows [5]

$$r\mathcal{TB}_e(Y) = \tau_e(1-\tau_c)cF + \tau_e m[1-p(Y)]F + \mu Y \mathcal{TB}_e'(Y) + \frac{1}{2}\sigma^2 Y^2 \mathcal{TB}_e''(Y) \tag{14}$$

[5] Here I abstract from the tax differences between payouts and equity issuance for tractability, assuming that all cash flow between the firm and equity holders faces a flat rate of τ_e. I model this difference in the next section.

with boundary conditions at infinity $\mathcal{TB}_e(\infty) = \frac{\tau_e F}{r}\{(1-\tau_c)c + m[1-p(\infty)]\}$, and at bankruptcy $\mathcal{TB}_e(Y_b) = 0$. Then

$$\mathcal{TB}_e(Y) = \tau_e \left[\tilde{p}(Y)F - \mathcal{TB}_c(Y) - \frac{\alpha}{1-\alpha}\mathcal{BC}(Y)\right] \tag{15}$$

The firm saves personal income tax for equity holders by reducing cash available to them, that is, payments to debtholders net of corporate tax and payment at bankruptcy.

Substitute (4)(11)(15)(9)(13) into (3) and reorganize. If $p(Y)F - I \geq 0$, then

$$\begin{aligned} v_0(Y) = & \underbrace{-(1-\tau_e)I}_{\text{investment cost}} + \underbrace{\frac{(1-\tau_c)(1-\tau_e)Y}{r-\mu}}_{\text{unleveraged value}} + \underbrace{(\tau_c-\tau_b)(1-\tau_e)\frac{cF}{r}\left[1-\left(\frac{Y}{Y_b}\right)^{\eta_1}\right]}_{\text{tax benefits on cash flow net of costs at issuance}} \\ & - \underbrace{[(1-\alpha)+\tau_e\alpha]\frac{(1-\tau_c)(1-\tau_e)Y_b}{r-\mu}\left(\frac{Y}{Y_b}\right)^{\eta_1}}_{\text{bankruptcy cost including prepaid tax}} \end{aligned} \tag{16}$$

If $p(Y)F - I \leq 0$, then

$$\begin{aligned} v_0(Y) = & -\underbrace{I}_{\text{investment cost}} + \underbrace{\frac{(1-\tau_c)(1-\tau_e)Y}{r-\mu}}_{\text{unleveraged value}} + \underbrace{[(1-\tau_b)-(1-\tau_c)(1-\tau_e)]\frac{cF}{r}\left[1-\left(\frac{Y}{Y_b}\right)^{\eta_1}\right]}_{\text{tax benefits on cash flow net of costs at issuance}} \\ & + \underbrace{\tau_e F\left\{\frac{(1-\tau_b)c+m}{r+m}\left[1-\left(\frac{Y}{Y_b}\right)^{\gamma_1}\right] - \frac{(1-\tau_b)c}{r}\left[1-\left(\frac{Y}{Y_b}\right)^{\eta_1}\right]\right\}}_{\text{tax savings from rollover}} \\ & - \underbrace{\left\{(1-\alpha)\left(\frac{Y}{Y_b}\right)^{\eta_1} + \tau_e\alpha\left[\left(\frac{Y}{Y_b}\right)^{\eta_1} - \left(\frac{Y}{Y_b}\right)^{\gamma_1}\right]\right\}\frac{(1-\tau_c)(1-\tau_e)Y_b}{r-\mu}}_{\text{bankruptcy cost including prepaid tax}} \end{aligned} \tag{17}$$

The tax benefits for generating each dollar of interest expense is $(\tau_c - \tau_b)(1-\tau_e)$ when it is generated by recapitalizing and is $(1-\tau_b) - (1-\tau_c)(1-\tau_e)$ when it is generated by financing investment by debt. The difference $\tau_e(1-\tau_b)$ is due to personal income tax on equity charged on time 0 dividends. In the first case, when $p(Y)F - I \geq 0$, all terms in (16) are scaled by $(1-\tau_e)$ – as in the literature about the trapped equity view of dividend taxation,[6] when equity issuance is bounded at 0 and cannot be further reduced, all the firm's cash flow is subject to personal income tax on equity. Besides the deadweight loss at bankruptcy, there is an additional tax cost on the recovery value because the recovery value is priced in debt

[6]See, for example, Auerbach (1981).

and added to the dividends at time 0. In the second case, when $p(Y)F - I \leq 0$, debt reduces the cash flow available to shareholders by both interest expenses and rollover losses, leading to an additional term of personal income tax savings on equity.

When $\tau_c \leq \tau_b$, (16) is no larger than the firm's unleveraged value since the tax benefits are negative, so the firm always wants to issue less debt if $F > \frac{I}{p(Y)}$. Then we have the following result

Proposition 7. ***(no recapitalization)*** *If $\tau_b \geq \tau_c$, optimal debt issuance $F^* \leq \frac{I}{p(Y)}$, the firm never issues more debt than needed for financing investments.*

This is because the marginal tax benefit of debt issuance becomes negative when equity issuance drops to 0 and cannot be further reduced. Proceeds from additional debt have to be distributed to equity holders and taxed by τ_e, so the tax benefit only depends on comparing the corporate tax rate to the personal income tax rate on coupons.

Optimal default

The bankruptcy threshold Y_b in (16)(17) is chosen endogenously such that the firm claims bankruptcy when equity value and it's dirivative to earnings reaches 0, ie.e, $v(Y_b) = 0$ and $v'(Y_b) = 0$. The equity value after time 0 is

$$v(Y) = v_{unlev}(Y) + \mathcal{TB}_c(Y) + \mathcal{TB}_e(Y) - \mathcal{BC}(Y) - \tilde{p}(Y)F \tag{18}$$

$$= \underbrace{\frac{(1-\tau_c)(1-\tau_e)Y}{r-\mu}}_{\text{unleveraged value}} + \underbrace{\frac{(1-\tau_e)(\tau_c-\tau_b)cF}{r}\left[1-\left(\frac{Y}{Y_b}\right)^{\eta_1}\right]}_{\text{tax benefits on cash flow}} - \underbrace{(1-\tau_e)\frac{c(1-\tau_b)+m}{r+m}F\left[1-\left(\frac{Y}{Y_b}\right)^{\gamma_1}\right]}_{\text{flow payments to debtholders}}$$

$$+ \underbrace{(1-\tau_e)\alpha v_{unlev}(Y_b)\left[\left(\frac{Y}{Y_b}\right)^{\eta_1} - \left(\frac{Y}{Y_b}\right)^{\gamma_1}\right] - v_{unlev}(Y_b)\left(\frac{Y}{Y_b}\right)^{\eta_1}}_{\text{bankruptcy cost to equity holders'}} \tag{19}$$

By the smooth pasting condition $v'(Y_b) = 0$, we get

$$Y_b = \frac{\frac{(\tau_c-\tau_b)c}{r}\eta_1 - \frac{(1-\tau_b)c+m}{r+m}\gamma_1}{\frac{1-\tau_c}{r-\mu}\left[1-(1-\tau_e)\alpha(\gamma_1-\eta_1)-\eta_1\right]}F \tag{20}$$

When debt is perpetual, i.e., $m = 0$, there is no rollover of debt and $\gamma_1 = \eta_1$, then $Y_b = \frac{-(r-\mu)\eta_1 c}{r(1-\eta_1)}F$, the same as in Leland (1994a). Tax rates does not affect default decisions. When debt has finite maturity, i.e, $m > 0$, then the bankruptcy threshold Y_b increases with

τ_b and decreases with τ_e, since debtholders' personal income tax decreases the rollover gain while equity holders' personal income tax increases the rollover gain.

Optimal debt issuance

We can solve the optimal debt issuance F by substituting (20) into ((16)) and (17), then maximize the time-0 value function over F. For simplicity, I focus on the case when debt is perpetual so that $m = 0$. [7] I denote $\widetilde{F}$ as the optimal debt issuance when the firm issues equity at time 0 and refer to it as the opitmal financing leverage. When $p(Y)F - I \leq 0$, debt issuance that maximizes (17) is

$$\widetilde{F}^* = \min\left\{\widetilde{F}, \frac{I}{p(Y)}\right\} \tag{21}$$

where

$$\widetilde{F} = \frac{r(1-\eta)}{-\eta(r-\mu)c}\left[1-\eta-\frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_b)-(1-\tau_c)(1-\tau_e)}(1-\alpha)\eta\right]^{\frac{1}{\eta}} Y \tag{22}$$

The optimal financing leverage is increasing in the corporate tax rate τ_c and the personal income tax rate on equity τ_e, and decreasing in the personal income tax rate on bond τ_b. It coincides with a Leland model setting the constant tax benefit as Miller's formula $(1-\tau_b)-(1-\tau_c)(1-\tau_e)$ and the unleveraged value of the firm as $\frac{(1-\tau_c)(1-\tau_e)}{r-\mu}Y$. Next, I denote $\widehat{F}$ as the optimal debt issuance when the firm distributes dividends at time 0 and refer to it as the opitmal recapitalizing leverage. When $p(Y)F - I \geq 0$, debt issuance that maximizes (16) is

$$\widehat{F}^* = \max\left\{\widehat{F}, \frac{I}{p(Y)}\right\} \tag{23}$$

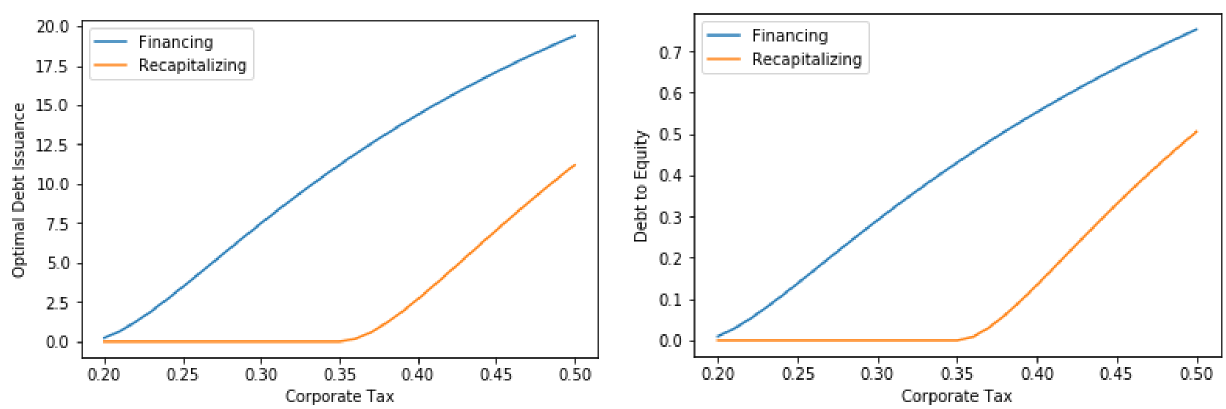
where

$$\widehat{F} = \frac{r(1-\eta)}{-\eta(r-\mu)c}\left[1-\eta-\frac{1-\tau_c}{\tau_c-\tau_b}[(1-\alpha)+\tau_e\alpha]\eta\right]^{\frac{1}{\eta}} Y \tag{24}$$

[7] When $m > 0$, optimal debt issuance cannot be solved in closed form due to the rollover gains term of equity holders' personal tax. I discuss results with rollover and optimal maturity in the next section with endogenous leverage adjustments and leave the analysis without leverage adjustments in the appendix.

The optimal recapitalizing leverage is increasing in the corporate tax rate τ_c and decreasing in the personal income tax rates τ_b and τ_e. Personal income tax on equity becomes a cost rather than benefit for recapitalizing since the recovery value at bankruptcy is priced in debt and included in the proceeds from debt at time 0, which is paid to equity holders and taxed. The optimal recapitalizing leverage is strictly lower than the optimal financing leverage when $\tau_e > 0$, $\tau_c < 1$, and $(1-\tau_b) > (1-\tau_c)(1-\tau_e)$. The Figure below plots the optimal financing leverage and optimal recapitalizing leverage with different corporate tax rates τ_c and fixed personal tax rates $\tau_b = 35\%$, $\tau_e = 20\%$. When $\tau_c < \tau_b$, $\widehat{F} < 0$ and the firm does not recapitalize. [8] The difference between two leverage targets is largest when the corporate tax rate τ_b is close to the personal tax rate on bond τ_b. Since the corporate tax rates and personal income tax rates are usually close in practice, the model implies that leverage targets for a firm facing an investment problem and a recapitalization problem are very different.

Figure: Leverage targets without leverage adjustments



The firm chooses between the optimal financing leverage $\widetilde{F}^*$ and the optimal recapitalizing leverage $\widehat{F}^*$ to maximize $v_0(Y)$.

Proposition 8. ***(optimal debt issuance)*** *The optimal financing leverage $\widetilde{F}$ is higher than the optimal recapitalizing leverage $\widehat{F}$. Let $\underline{F}$ be the smallest F such that $I = p(Y,F)F$ if such*

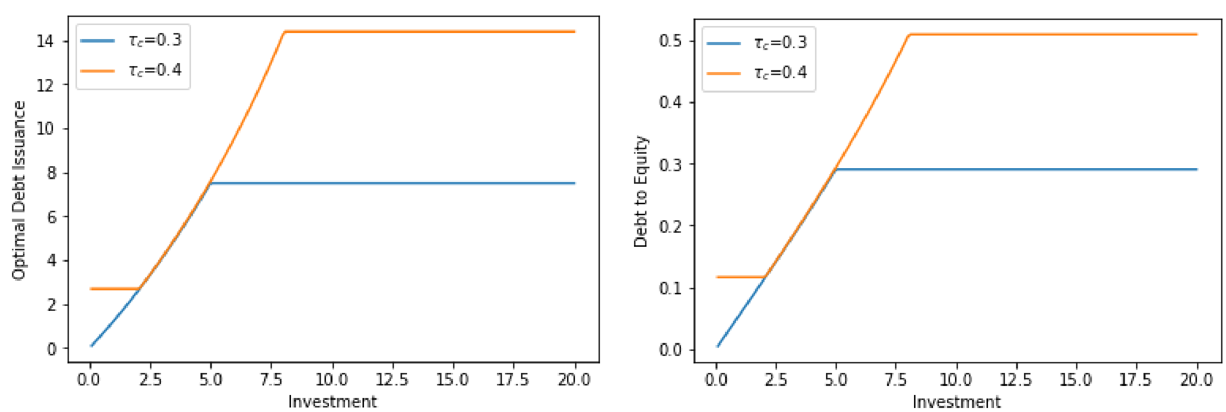
[8] Since the firm starts with no debt, negative debt issuance at time zero is infeasible.

$\underline{F}$ exists. The firm's optimal debt issuance is

$$F^* = \begin{cases} \widehat{F} & if\ \frac{I}{p(Y,\widehat{F})} \leq \widehat{F} \\ \underline{F} & if\ \frac{I}{p(Y,\widehat{F})} > \widehat{F}\ and\ \frac{I}{p(Y,\widetilde{F})} \leq \widetilde{F} \\ \widetilde{F} & if\ \frac{I}{p(Y,\widetilde{F})} > \widetilde{F} \end{cases} \tag{25}$$

The firm chooses the optimal recapitalizing leverage when it is enough to finance the investment. Otherwise, the firm chooses the lowest level of debt that can exactly finance the investment if proceeds from the optimal financing leverage exceeds the financing need for investment, and chooses the optimal financing leverage if it is not enough the finance the investment. The Figure below plots optimal debt issuance with different investment I and fixed personal tax rates $\tau_b = 35\%$, $\tau_e = 20\%$ when the corporate rate is $\tau_c = 30\%$ and $\tau_c = 40\%$. The firm issues debt at the recapitalizing target when required investment I is low, and at the financing target when required investment I is high. Besides the two leverage targets, the firm issues (the minimum level of) debt that exactly meets the investment need without paying dividends or issuing equity when facing a moderate level of investment. Debt issuance is lower than the financing target so issuing debt is cheaper than issuing equity and reducing debt issuance is suboptimal. On the other hand, debt issuance is higher than the recapitalizing target, so issuing more debt and distributing the proceeds as dividends is also suboptimal.

Figure: Optimal Debt issuance without leverage adjustments



(c) Debt policies with existing debt

Debt issuance is bounded at 0 when there is no financing need for investment and the personal tax rate is higher than the corporate tax rate. Now we relax this bound by assuming that the firm has existing debt at time 0 with principal F_0 and the same c and m as the new debt. The initial debt has no covenants and does not restrict the firm's issuance of new debt. Also, we allow I to be negative here, representing the financing need net of internal cash at time 0. When I is negative, there are some internal cash available for payouts or debt repurchase. Let F be the total principal of debt after time 0. Then equity holders' payoff at time 0 becomes $\left(1 - \mathbf{1}_{\{p(Y)(F-F_0)-I\geq 0\}}\tau_e\right)[p(Y)(F - F_0) - I]$ and the time-0 value function is

$$\begin{aligned} v_0(Y) = &- p(Y)F_0 - I - \mathbf{1}_{\{p(Y)(F-F_0)-I\geq 0\}}\tau_e[p(Y)(F - F_0) - I] + v_{unlev}(Y) \\ &+ \mathcal{TB}_c(Y) + \mathcal{TB}_e(Y) - \mathcal{TC}_b(Y) - \mathcal{BC}(Y) \end{aligned} \tag{26}$$

The initial debt decreases the firm's value by its value at the current price. A lower debt price impairs the existing debt holders and benefits the equity holders. Such friction between equity holders and debt holders leads to the debt ratchet effect, making the firm take higher leverage.

Proposition 9. ***(Optimal debt issuance with existing debt)*** *If the firm has existing debt with face value $F_0 > 0$ at the beginning of time 0, the optimal debt issuance $F^{**}(Y, F_0) > F^*(Y) - F_0$. When $\tau_b > \tau_c$ and $I < 0$, the firm repurchases debt if*

$$F_0 < Y\frac{(1-\eta)r}{-\eta(r-\mu)c}\left(\frac{\tau_b - \tau_c}{\tau_b - \tau_c - \frac{\eta}{1-\eta}(1-\tau_c)(1-\tau_e)\alpha}\right)^{-\frac{1}{\eta}} \tag{27}$$

When the net financing need $I < 0$, equity is trapped with equity issuance bounded at 0, so all payouts to shareholders are taxed at τ_e. Debt repurchase earns a marginal tax benefit proportional to $\tau_b - \tau_c$. Such benefit dominates the debt ratchet effect if the existing debt F_0 is not too large, leading to a debt repurchase. The figure below plots the optimal debt issuance when $F_0 = 1$, compared to the issuance that adjusts debt from F_0 to the leverage targets $\widetilde{F}$ and $\widehat{F}$ without existing debt. The firm repurchases debt when $I < 0$ and issues debt otherwise. Debt issuance/repurchase are fixed at target levels when I is large/small enough, with both targets higher than targets without exisitng debt. As before, the firm issues debt that exactly meets the financing need for moderate levels of I.

Figure: Optimal Debt issuance with existng debt