Tech Dollars and Exchange Rates*

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Abstract

We document a strong positive correlation between U.S. innovation and the growth of the real dollar index. Examining wealth fluctuations across countries, we observe a (re)connection between exchange rate movements and relative changes in aggregate quantities, such as consumption and output growth, once wealth changes are controlled for. Moreover, relative wealth changes are positively correlated with aggregate quantities. In addition, we find that U.S. innovation is associated with an increase in foreign capital inflows at both the aggregate and firm levels. These observations motivate our theoretical analysis of how technological innovation affects exchange rate movements. We introduce a minimal deviation from the standard endowment economy model of exchange rate: in an economic boom, new firms are created, but they are randomly distributed to a small part of the population. Our calibrated model successfully replicates key features of the data, specifically, the joint dynamics of exchange rates, stock returns, real output and consumption growth, and trade flows.

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How do technological innovation and productivity shocks affect exchange rates? In a complete market, technological innovations typically drive economic booms, leading to increases in macroeconomic variables such as consumption and output. This should cause a depreciation of the real exchange rate due to decreased marginal utility. However, empirical evidence indicates a weak or even positive correlation between these macroeconomic variables and exchange rates, a phenomenon known as the cyclicality puzzle (Backus and Smith, 1993; Kollmann, 1995). More generally, exchange rates seem disconnected from the macro variables that should affect them Obstfeld and Rogoff (2001).

In this paper, we document a strong positive correlation between U.S. innovation and the growth of the real dollar index. Our measure of U.S. innovation is positively correlated with U.S. TFP growth. This suggests limited risk-sharing of innovation and productivity shocks, implying that wealth is not allocated in a way that facilitates risk-sharing across countries. Examining wealth fluctuations, we document a (re)connection between exchange rate movements and relative changes in aggregate quantities, such as consumption and output growth, once wealth changes are controlled for. Additionally, we find that relative wealth changes are positively correlated with real quantities. We interpret this as an indication that the economic forces driving wealth changes are imperfectly shared productivity shocks. Furthermore, we show that U.S. innovation is associated with an increase in foreign capital inflows at both the aggregate and firm levels. These findings motivate our theoretical analysis of how technological innovation affects exchange rate movements. We argue, both empirically and theoretically, that our findings are informative about the impact of technological innovation on exchange rates, real quantities and capital flows.

We begin our analysis by examining the relationship between U.S. innovation and the growth of the real dollar index. The annual U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan, Papanikolaou, Seru, and Stoffman, 2017) to the total stock market value. Focusing on the post-Bretton Woods era, we find a positive correlation between U.S. innovation and the growth of the real dollar index. A one-standard-deviation increase in U.S. innovation is associated with approximately 3 to 4 log points of exchange rate appreciation at the annual level. Moreover, our innovation measure is positively correlated with U.S. productivity growth, measured by the utilization adjusted TFP series (Fernald (2014)).

In a complete market, one would expect a negative correlation between innovation shocks and the growth of the real exchange rate. Specifically, if high innovation in the U.S. represents favorable states for U.S. households, the U.S. would transfer wealth to the rest of the world, leading to a depreciation of the real exchange rate and a reduction in U.S. relative wealth. However, the observed positive correlation between innovation and the real exchange rate indicates imperfect risk-sharing of innovation shocks. This suggests that wealth fluctuations are not allocated to equalize marginal utilities across countries. These findings motivate our analysis of the role of wealth fluctuations in driving exchange rate movements.

We find a (re)connection between exchange rates and macroeconomic fundamentals: when controlling for relative wealth changes, the correlation between macroeconomic variables and exchange rate changes becomes negative, consistent with predictions from risk-sharing models[1].

For wealth changes to reconnect exchange rate movements with relative consumption growth, wealth changes must correlate with both exchange rate movements and aggregate consumption growth. Examining their correlation, we find that wealth changes are positively correlated with both consumption growth and output growth. This indicates that wealth changes are associated with positive productivity shocks. Importantly, these shocks are not perfectly shared, highlighting the role of incomplete markets.

Why does innovation in the U.S. lead to an appreciation of the dollar? We provide evidence that U.S. innovation is associated with increased foreign capital inflows. At the aggregate level, we find a significant positive correlation between U.S. innovation, foreign direct investment inflows, and portfolio equity inflows. At the firm level, we find that U.S. firms experiencing an innovation shock — measured by the grant of significant novel patents — subsequently observe a significant increase in foreign institutional ownership. These findings support the notion that technological innovations are associated with adjustments in international financial flows. They indicate that heightened demand for U.S. assets, which embody frontier technology, drives up demand for dollars.

The main goal of our subsequent economic analysis is to rationalize these empirical facts in a theoretical framework. To do so, we introduce a minimal deviation to the standard endowment economy model: in addition to the standard endowment shock in each country, countries can now each experience displacive innovation shocks that reallocate output among agents. This mechanism is a reduced-form version of a model of endogenous production and creative destruction; in such models periods of economic growth can be associated with significant reallocation (see, e.g., Kogan, Papanikolaou, and Stoffman, 2020).

To build intuition, we start with a set of minimal ingredients: a two-country endowment model in which each household has logarithmic preferences and home bias in consumption. In addition to a shock to the aggregate endowment, each country experiences a shock constructed to mimic the properties of creative destruction as in Gârleanu, Panageas, Papanikolaou, and Yu (2016). In particular, growth is partially driven by the arrival of new projects (firms) that potentially displace the incumbents. The key feature of the model is incomplete markets: ownership of the new projects does not accrue to the shareholders but is instead randomly allocated to a (measure zero) subset of the population. The key friction is that households cannot sell claims on their future potential endowment of these new projects. As a result, shocks to the relative profitability of new projects lead to the redistribution of wealth from the existing firm owners to the new entrepreneurs. This wealth redistribution increases the cross-sectional dispersion of consumption growth—the majority of households incur small losses while a fortunate few experience substantial increases in their wealth.

[1]In contrast, the unconditional correlation between macroeconomic variables and exchange rates is weak, as is well-documented in the literature (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2001).

Since households' marginal utility is convex, the displacement shock raises the stochastic discount factor and therefore leads to an appreciation of the real exchange rate (this mechanism is similar in spirit to Constantinides and Duffie, 1996). The real exchange rate appreciation benefits domestic assets, and therefore elevating the country's total wealth, as in Dahlquist, Heyerdahl Larsen, Pavlova, and Penasse (2023).

Our model generates the negative correlation between exchange rates and macroeconomic quantities when conditioned on wealth changes, with a minimal deviation from the standard setting. The key to replicating this pattern is the model's implication that changes in wealth summarize the impact of imperfect risk-sharing associated with displacement shocks. Moreover, the model can also generate the unconditional exchange rate disconnect: depending on the relative magnitude of the displacement shocks and the neutral shocks, the correlation between exchange rate and aggregate consumption and output can be positive or negative.

The displacement shock in our model is related to the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. This gap arises because investors holding the market portfolio must continually liquidate some of their holdings to purchase shares of new firms entering the market, maintaining the self-financing nature of the strategy. As new firms enter each period, the growth of the market portfolio falls short of the growth of the aggregate market cap. Guided by this insight, we construct a displacement shock series for the U.S. market and show that it is positively correlated with the growth of the real U.S. dollar index. Notably, during periods of significant innovation, when many new firms emerge, the dollar appreciates in real terms.

In addition, this simple model yields testable implications. Specifically, a positive displacement shock in the model leads to an increase in income inequality. As a result, the model implies a positive correlation between exchange rates and changes in (relative) income inequality. This prediction is consistent with the data. In a panel regression of 11 countries covering the post-Bretton Woods era, we find a positive and statistically significant correlation between changes in bilateral exchange rates and changes in relative income inequality.[2] For instance, focusing on the coefficients from the pooled regression, a one-standard deviation increase in income inequality in a foreign country relative to the United States is associated with a 1.7 log point appreciation of its currency relative to the US dollar. Similarly, a one-standard deviation increase in income inequality is associated with a 2 log point appreciation of its relative wealth growth.

We then explore the ability of our mechanism to quantitatively account for the key correlations in the data. To do so, we extend the model along several dimensions, specifically we allow for recursive preferences over relative consumption and relax the assumption of extreme inequality—a positive

[2]The sample covers the 1974 to 2022 period and a combination of G-10 currency countries and G-7 countries: Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, France, Italy and the United States. Kocherlakota and Pistaferri (2008) also document a positive correlation between real exchange rate growth and between-country differences in the growth rates of right-tail inequality.

measure set of households receive new projects. In addition, we allow for the distribution of the displacement shock to vary over time. Though these modifications are not needed to qualitatively explain the key patterns in the data, they help the model deliver realistic quantitative predictions. We calibrate the model to the data by choosing parameters that minimize the distance between the data and model-implied statistics, essentially a form of the simulated method of moments (SMM).

Our model successfully replicates the first two moments of aggregate consumption and output growth, exchange rates, and stock returns, while generating low and relatively smooth risk-free rates. Our model reproduces the three key 'anomalies' in the exchange rate literature: the volatility puzzle of Brandt, Cochrane, and Santa-Clara (2006), the Backus-Smith correlation puzzle, and the violation of the uncovered interest rate parity (UIP). Key to replicating the failure of the UIP is the time-varying distribution of the displacement shock.

The quantitative success of the model does not come at the cost of unrealistic parameters. In terms of preference parameters, the model calibration requires a degree of relative risk aversion of 6.3 and an elasticity of inter-temporal substitution (EIS) equal to 1.6, which are largely in line with the literature. The preference weight on relative consumption is rather high (0.82) though it comes with a high standard error, implying that the model solution is not very sensitive to this particular value. Further, the model requires a highly persistent and risk-skewed displacement shock. To ensure that the magnitudes of displacement shocks are realistic, we also target the mean level of observed income inequality as part of the model calibration, which helps discipline the volatility of the displacement shock. Last, just like most existing models (Colacito and Croce, 2013), our calibration requires a high degree of home bias in household preferences (0.988). A high degree of home bias is needed in order to generate sufficiently volatile exchange rates given the high level of correlation in consumption growth across countries.

The empirical evidence supports our model's predictions regarding the correlation between exchange rate and trade flows. First, in both the model and in the data, net exports are counter-cyclical. In addition, there is a significant negative correlation across countries between changes in top income shares and changes in current account balances.

Moreover, in the model, shifts in the degree of technological innovation across countries also generate movements in financial flows due to diversification motives, together with a positive correlation between capital inflows and currency appreciation. In particular, a positive displacement shock in the home country is associated with the creation of new firms (projects) which are initially owned by a small subset of households (entrepreneurs). Entrepreneurs sell their shares to diversify their holdings and foreign investors buy some of these shares to rebalance their portfolio as the share of the home country in the world market portfolio increases. The net effect is that the home country experiences net capital inflows and its currency appreciates, which is consistent with the findings of Hau and Rey (2006); Camanho, Hau, and Rey (2020); Rey, Rousset Planat, Stavrakeva, and Tang (2024).

In sum, we develop a quantitative general equilibrium model that successfully replicates the joint dynamics of exchange rates, consumption growth, trade flows, and stock returns. Our model thus contributes to a voluminous literature studying the determination of exchange rates in two-country equilibrium models (see, e.g., Chari, Kehoe, and McGrattan, 2002; Alvarez, Atkeson, and Kehoe, 2002, 2009; Corsetti, Dedola, and Leduc, 2008; Pavlova and Rigobon, 2007, 2008, 2010, 2011; Verdelhan, 2010; Colacito and Croce, 2011, 2013; Farhi and Gabaix, 2016; Stathopoulos, 2016; Ready, Roussanov, and Ward, 2017; Colacito, Croce, Gavazzoni, and Ready, 2018).

At a broader level, our mechanism can also be re-interpreted through the lens of the Balassa-Samuelson hypothesis (Balassa, 1964; Samuelson, 1964). The Balassa-Samuelson effect is that, if productivity increases in the tradable sector tend to be higher than those in the nontradable sector, then the conventionally constructed real exchange rates—that is, using a price index of a combination of both tradable and non-tradable prices as the price deflator—will comove with the cross-country differences in the relative speed of productivity increases between tradable and non-tradable sectors. Our mechanism is distinct from the Balassa-Samuelson effect in that it operates through limited risk-sharing of innovation and the resulting wealth effects, which shift demand across sectors. In contrast, the Balassa-Samuelson effect works through the equalization of wages and production costs. As a result, our mechanism does not require productivity shocks to occur in the tradable sector. Productivity improvements in non-tradable sectors, such as services, can also lead to real exchange rate appreciation, albeit with an offsetting effect from increased non-tradable goods supply. Furthermore, empirically testing the Balassa-Samuelson hypothesis has proven challenging. Its reliance on the law of one price for tradable goods, which is a key assumption of the Balassa-Samuelson hypothesis, has not been supported by empirical evidence (Chari et al., 2002). In addition, the observed productivity differentials between tradable and non-tradable sectors have not been successfully linked to exchange rate movements in the data.

Displacive innovation shocks in our model contribute to economic growth and, due to imperfect risk-sharing, increase income inequality. This mechanism integrates into the broader literature on the relationship between exchange rates, economic growth, and inequality. Specifically, Gavazzoni and Santacreu (2020) explores the international spread of long-run productivity shocks. Kocherlakota and Pistaferri (2007) examines the implications of limited risk-sharing on exchange rates. This paper connects these two strands of literature by demonstrating that displacive innovation shocks could be a common underlying cause.

Our work also relates to the literature that analyzes the weak correlation between exchange rates and fundamentals (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2001; Yu, 2013; Colacito, Croce, Liu, and Shaliastovich, 2021; Jiang, Krishnamurthy, Lustig, and Sun, 2021; Lewis and Liu, 2022; Zhang, 2021; Chernov, Haddad, and Itskhoki, 2023; Colacito, Croce, Liu, and Shaliastovich, 2023). We contribute to this literature by documenting that when adjustments for wealth changes are accounted for, the correlation between exchange rates and economic fundamentals is more pronounced. Our work therefore highlights the importance of recognizing imperfect risk-sharing of productivity shocks in understanding exchange rate dynamics.

Recent work by Jiang, Krishnamurthy, and Lustig (2023b) shows that resolving classical exchange rate puzzles is challenging when investors can frictionlessly trade home and foreign currency risk-free bonds[3]. However, their definition of cyclicality is relatively restrictive—it is defined as the correlation between exchange rate movement and SDF differentials, whereas our model is about the linkages between SDF and fundamentals.

The exchange rate disconnect puzzle has also led to the development of models with segmented asset markets (e.g. Farhi and Werning (2014); Itskhoki and Mukhin (2021); Fang (2021); Fang and Liu (2021); Fukui, Nakamura, and Steinsson (2023); Kekre and Lene (2024)). Hau and Rey (2006); Camanho et al. (2020) study a model of segmentation in equity markets that links portfolio flows in equities to exchange rates. Greenwood, Hanson, Stein, and Sunderam (2020); Gourinchas, Ray, and Vayanos (2020) focus on the bond market. Gabaix and Maggiori (2015) develop a tractable general-equilibrium model in which portfolio-rebalancing motives drive exchange rate movements. Bacchetta and Van Wincoop (2006) examines the implications of exchange rates of agents who infrequently rebalance their bond portfolios. Fang (2021) and Fang and Liu (2021) analyze how financial intermediaries affect exchange rate movements. Lilley, Maggiori, Neiman, and Schreger (2019) provide supporting evidence that links the purchase of US bonds by foreigners to the dollar exchange rate. Lustig and Verdelhan (2019) discuss the limitation of market incompleteness in resolving the volatility, cyclicality, and risk premium puzzles. Jiang et al. (2023b) highlight the role of bond euler equation in understanding exchange rate dynamics.

Although our model also features incomplete markets, the mechanisms are quite distinct. In these models, capital flows (rebalancing needs) are in general exogenously assumed; these models focus instead on how these capital flows can impact exchange rates in the absence of complete risk sharing. By contrast, our model does not feature any exogenous movements in capital flows; rather, capital flows are determined in equilibrium. That said, in our model, shifts in the degree of technological innovation across countries generate movements in financial flows due to diversification motives, together with a positive correlation between capital inflows and currency appreciation, consistent with the findings of Hau and Rey (2006); Camanho et al. (2020); Rey et al. (2024).

Our work contributes to the growing body of literature that studies the special role of dollar assets in international markets. Previous studies have focused on several aspects: the U.S.'s role as a global insurance provider and its exorbitant privilege, the convenience yield of holding U.S. assets, and more recently the impact of U.S. fiscal policy (e.g., Gourinchas and Rey (2007a,b); Gourinchas, Rey, and Govillot (2010); Atkeson, Heathcote, and Perri (2022), Dahlquist et al. (2023);

[3]Several other papers (Jiang, 2023; Sandulescu, Trojani, and Vedolin, 2021) have also examined the extent to which various forms of incomplete markets can help resolve well-known puzzles in international finance, including the Backus and Smith (1993) puzzle, the disconnect puzzle Meese and Rogoff (1983), and the volatility puzzle Brandt et al. (2006).

Sauzet (2023),Jiang, Krishnamurthy, and Lustig (2021); Jiang et al. (2021); Koijen and Yogo (2020); Chen, Jiang, Lustig, Nieuwerburgh, and Xiaolan (2023); Jiang, Krishnamurthy, and Lustig (2023a); Van Nieuwerburgh, Jiang, Lustig, and Xiaolan (2021), Jiang (2021); Kim (2023))[4]. Our model can potentially speak to the strength of the dollar as well. Specifically, it suggests that a major factor in dollar exchange rate dynamics is the difference in rates of creative destruction between the U.S. and the rest of the world (ROW). Over recent decades, many significant innovations have originated in the U.S. [5]. The U.S. higher rate of creative destruction could be a key contributor to the strong demand for dollar assets.

The existence of common risk factors in exchange rates has been the subject of considerable debate (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018; Jiang, 2023). Richmond (2019); Lustig and Richmond (2019); Jiang and Richmond (2019) emphasize the importance of international trade linkages in generating comovement across currencies. To the extent that technology spillovers are correlated with trade flows, our framework provides a new perspective on the importance of trade network linkages.

The main mechanism in our paper is closely related to Gârleanu et al. (2016); Kogan et al. (2020) and Huang, Kogan, and Papanikolaou (2023). Kogan et al. (2020) build a general equilibrium model with capital embodied technology shocks in which benefits of innovation are distributed asymmetrically across the economy. The key friction is that potential innovators cannot contract ex ante to share the economic rents that their ideas generate. As a result, financial market participants capture only part of the benefits, despite bearing all of the costs of creative destruction. The reallocative impact on household wealth implies that improvements in technology can reduce households indirect utility. This displacive effect on indirect utility is amplified when households care about their consumption relative to the economy-wide average, since household dislike being 'left behind'. Kogan et al. (2020) show that the resulting displacement risk can lead to increased demand for insurance (an increase in the stochastic discount factor) and can help rationalize certain cross-sectional features of asset returns. Huang et al. (2023) examine this mechanism in a multi-region model of a monetary union and study its implications for regional inflation dynamics. Gârleanu et al. (2016) embed a reduced-form of this mechanism in a standard endowment model and study its implications for the equity risk premium. Kogan, Papanikolaou, Schmidt, and Song (2020) present complementary evidence that surges in innovation correlate with higher labor income risks for incumbent workers, leading to a stronger demand for insurance.

Last, our work is highly complementary to Chahrour, Cormun, De Leo, Guerrón-Quintana, and Valchev (2024), who identify a shock to future TFP growth in the US using VARs, and show that this shock leads to an appreciation of the dollar over the short run. These patterns are somewhat

[4] Atkeson et al. (2022) suggest that the U.S. may have already depleted its exorbitant privilege, attributing this to the decline in the U.S. net foreign asset position from 2007 to 2022, driven by the rising valuation of U.S. corporations.

[5] Periods of significant technological breakthroughs in the U.S.—such as the rise of personal computers in the 1980s, the emergence of internet companies around 2000, and recent advancements in artificial intelligence and large language models—have coincided with times of a strong dollar.

hard to reconcile with models with complete markets (for example Colacito and Croce, 2013), in which positive news on future productivity would lead to lower marginal utility of US investors and therefore to a depreciation of the dollar. By contrast, the patterns in Chahrour et al. (2024) are entirely consistent with our model, in which a positive innovation shock in the US leads to both higher output growth and an appreciation of the dollar.

1 Dollar Exchange Rates and U.S. Innovation

We begin by analyzing the correlation between U.S. innovation and the growth of the dollar exchange rate in real terms. Below, we briefly discuss the sources of main variables in our analysis and refer the reader to Appendix A for additional details.

1.1 Data Sources

We obtain data on consumption, GDP, and net exports from the World Bank, specifically, the World Development Indicators. We use household final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real. Inflation rates are calculated using Consumer Price Index (CPI) from the World Bank. We obtain end-of-year nominal exchange rates from the IMF. The real exchange rate is calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country. Data on aggregate foreign direct investment and portfolio equity flows are obtained from the World Bank. Data on interest rates comes from Global Financial Data. Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized inflation using annual changes in CPI. Data on equity index returns (MSCI series) are obtained from Datastream. We measure income inequality using the top 1% income share and obtain data on both the top income share and the country's total net wealth from the World Inequality Database [6].

Our sample is dictated by data availability and consists of a combination of G-10 currency countries and G-7 countries. Specifically, it includes Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, France, Italy and the United States. We take the domestic country to be the United States and define the exchange rate as the units of foreign currency per dollar. The sample period covers the post-Bretton Woods era. The sample covers the period from 1974 to 2022.

[6] The World Inequality Database provides each country's total net wealth (code = mpweal) in local currency. To express this wealth in U.S. dollars, we multiply the total net wealth by the corresponding exchange rate.

1.2 Real Dollar Index Growth and U.S. Innovation

We calculate the growth of the real dollar index as the equal-weighted average of the log growth rates of the real dollar exchange rates against the currencies in our sample[7]. We measure the annual U.S. innovation level as the log of the ratio of the total economic value of patents (Kogan et al., 2017) each year to the total stock market capitalization at the end of the year.

We examine the relationship between dollar index growth and U.S. innovation by estimating the following specification

$$\Delta \log e_{t-s,t}^{USD} = \alpha + \beta_1 Inno_{US,t-s,t} + \beta_2 X_{t-s} + \varepsilon_t \tag{1}$$

The dependent variable is the growth in the (log) dollar index level from $t - s$ to t, where $s = 1, 3$. The independent variable is the sum of U.S. innovation between year $t - s$ and t. Depending on the specification, we include the lagged dollar index level, lagged U.S. innovation, or both as control variables. Standard errors are calculated using the Newey-West procedure, with a bandwidth of one or three years, respectively.

Table 1 shows an economically and statistically significant positive correlation between the level of U.S. innovation and the growth of the U.S. dollar index. A one-standard-deviation increase in U.S. innovation is associated with approximately 3 to 4 log points of exchange rate appreciation at the annual level. This positive relationship remains consistent when focusing on a 3-year horizon and is robust to different specifications.

To visualize the time-series variation in real dollar growth and U.S. innovation, Figure 1 plots the residualized series of real dollar index growth and U.S. innovation[8], corresponding to column (3) in Table 1. We observe that (1) the two series are highly correlated, and (2) during periods of significant innovation, such as the internet boom around the 2000s and the AI boom during the 2020s, U.S. innovation is high, and the dollar appreciates in real terms. Figure 2 presents the residualized series over a 3-year horizon, corresponding to column (6) in Table 1. Smoothing over this longer horizon reduces volatility. The strong correlation persists, with particularly pronounced alignment during the 2000s and 2020s.

We next examine the robustness of this correlation using an alternative measure of U.S. innovation. Specifically, we define annual U.S. innovation as the log of the ratio of the total real economic value of patents (Kogan et al. (2017)) to the total number of patents. Table B.1 presents the results, showing that this alternative measure is positively correlated with the growth of the real dollar index.

Lastly, we examine this correlation at each bilateral country pair. To do so, we estimate the

[7] The level of the dollar index is obtained by accumulating these growth rates over the past years.

[8] Specifically, we regress both real dollar index growth $\Delta \log e_{t-s,t}$ and U.S. innovation $\text{Inno}_{US,t-s,t}$ on the lagged U.S. innovation $\text{Inno}_{US,t-s}$ and lagged U.S. index $\log e_{t-s}^{USD}$ and obtain the residuals.

following specification:

$$\Delta \log e_{F,t-1,t} = \alpha + \beta_1 Inno_{US,t-1,t} + \beta_2 X_{t-s} + \varepsilon_{F,t} \tag{2}$$

Compared with (1), the dependent variable now is the growth in the bilateral exchange rate e between the foreign country F and the U.S. The control variables X_{t-s} include the lagged innovation level and the lagged exchange rate level.

Table B.3 presents the results for each country pair as well as the panel regression. The panel regressions include country fixed effects, with standard errors computed using the Driscoll and Kraay (1998) methodology to account for heteroskedasticity, serial correlation, and cross-sectional dependence. In the panel regression, the estimated coefficient for innovation is 0.034, with a standard error of 0.017. When estimating equation (2) for individual countries, the point estimates are again positive in 10 out of the 11 cases.

1.3 Innovation, Productivity and Exchange Rate

Technological innovation is arguably the main driver of productivity growth. The positive correlation between U.S. innovation and the real dollar index growth relates to the literature emphasizing the importance of news about TFP shocks as a key driver of exchange rates (Nam and Wang, 2015; Chahrour et al., 2024). In particular, Chahrour et al. (2024) argue, using a vector auto-regression, that news shocks about future TFP are the dominant driver of exchange rates.

We examine the relationship between our innovation measure and U.S. TFP. Similar to Chahrour et al. (2024), we use the Fernald (2014) series on utilization-adjusted U.S. TFP. Specifically, we analyze the relationship by estimating the following univariate regression:

$$\Delta A_{t,t+s} = \alpha + \beta_s Inno_{t,t+1} + \varepsilon_{t+s} \tag{3}$$

where the dependent variable $\Delta A_{t,t+s}$ is the growth in U.S. TFP between t to $t+s$. Figure B.1 plots the coefficients $\frac{\beta_s}{s}$ for horizons s ranging from one to five years. We see that the innovation measure exhibits a statistically and economically significant positive correlation with U.S. TFP growth. Specifically, a one-standard-deviation increase in U.S. innovation is associated with an annual growth of 1 to 1.5 log points in U.S. TFP.

Beyond the intuitive connection in the TFP shocks, we note that, unlike Chahrour et al. (2024), who find that news about TFP drives exchange rate movements while TFP innovation responds with a delay, our direct measure of innovation exhibits a more immediate and contemporaneous comovement. We next discuss the theoretical framework in which innovation and exchange rate can be positive correlated.

1.4 The Role of Incomplete Market

In a complete market where innovation risks are perfectly shared, we would expect a negative correlation between innovation shocks and the real exchange rate. Specifically, if high innovation in the U.S. represents good states for U.S. households, the U.S. would transfer wealth to the rest of the world, leading to a depreciation of the real exchange rate.

However, the observed positive correlation between innovation and the real exchange rate suggests that innovation risk-sharing is imperfect. This observation relates to the well-known Backus-Smith correlation and the exchange rate disconnect puzzle (Backus and Smith (1993); Meese and Rogoff (1983)), which rely on the assumption of perfect risk-sharing— wealth fluctuations are shared to facilitate the equalization of marginal utilities across countries. In reality, certain risks are not perfectly shared, and wealth fluctuations between countries can affect the relationship across exchange rates and macroeconomic fundamentals.

Next, we explore how these wealth fluctuations influence exchange rate movements. To this end, we examine the following specification:

$$\log e_{F,t} - \log e_{F,t-1} = \alpha + \beta_1 \left(\log \frac{x_t^{US}}{x_t^F} - \log \frac{x_{t-1}^{US}}{x_{t-1}^F} \right) + \beta_2 \left(\log \frac{W_t^{US}}{W_t^F} - \log \frac{W_{t-1}^{US}}{W_{t-1}^F} \right) + \beta_3 X_{t-1} + \varepsilon_{F,t}. \quad (4)$$

Here, $x^c \in \{C, Y\}$ refers to consumption and output in country c. W^{US}, W^F are the nominal wealth of the U.S. and foreign country in dollars. Controls X_{t-1} include lagged dependent and independent variables: the lagged exchange rate level $\log e_{F,t-1}$, the lagged level of $\log \frac{x_{t-1}^{US}}{x_{t-1}^F}$, and lagged wealth ratios $\log \frac{W_{t-1}^{US}}{W_{t-1}^F}$.

Panel A of Table 2 reports the results. First, we observe that the estimated slope coefficient for relative wealth is positive in both individual country regressions and the panel regression. This indicates that an increase in wealth is often associated with currency appreciation, which aligns with findings in Dahlquist et al. (2023) that currency appreciation typically benefits domestic assets, leading to increased domestic wealth. In other words, the total wealth denominated in local currency does not adjust enough to offset the movement from the nominal exchange rate [9]. Consequently, fluctuations in the nominal exchange rate lead to significant changes in a country's wealth measured in dollars, reflected in the high R-squared values in our estimated regressions.

More importantly, the coefficient on consumption growth turns negative and is statistically significant. Focusing on the panel regression result, a one-standard deviation increase in consumption

[9] For instance, if the nominal dollar index appreciates by 10%, the value of U.S. assets denominated in dollars does not decline sufficiently to offset the wealth effects caused by the nominal exchange rate movement. There are several reasons why U.S. investors do not sell enough domestic assets to purchase foreign assets when the dollar index rises, one of which is trading costs. Significant evidence suggests that investors act as though they encounter substantial transaction costs when purchasing foreign securities Lewis (1995). Additionally, recent findings provide strong evidence of home-currency bias in bond investments Maggiori, Neiman, and Schreger (2020).

growth differentials is associated with a 2 log point depreciation of its currency against the dollar. Examining results for individual countries, we note that the correlation is negative in 10 out of the 11 countries.

Panel B of Table 2 shows that the estimates for output are similar. The estimated coefficient of wealth is always positive. In the panel regression, the estimated coefficient of output is equal to -0.016 with a Driscoll and Kraay (1998) standard error of 0.004. When estimating (4) separately for each country, we again see that the point estimates of output are all negative.

These results show that, conditional on imperfectly shared wealth fluctuations, there is a (re)connection between exchange rate and macroeconomic fundamentals: agents in different countries engage in risk-sharing — marginal utilities are high when macroeconomic fundamentals are low.

These patterns align with recent work by Aguiar, Itskhoki, and Mukhin (2024), which shows that risk-sharing across countries is better than implied by the Backus-Smith correlation when analyzed through consumption allocation and trade shares, as these are less influenced by financial market imperfections. Unlike Aguiar et al. (2024), who use trade shares to circumvent imperfectly shared shocks and test risk-sharing through quantities, our method accounts for these imperfectly shared shocks by conditioning on relative wealth across countries[10].

Notably, while nominal exchange rates and wealth ratios are highly correlated, controlling for nominal exchange rates does not produce a negative conditional correlation between consumption growth and the real exchange rate growth. Table B.2 reexamines the panel regression of specification (4) by replacing changes in wealth ratios with nominal exchange rate growth (column 2) and by considering both nominal exchange rates and wealth changes in local currency units (column 3). As shown in Table B.2, the negative conditional correlation between consumption growth and real exchange rate growth does not emerge with these alternative specifications. This provides evidence that nominal exchange rate fluctuations driven by imperfectly shared financial shocks do not fully explain the reconnect pattern we recover.

The fact that the coefficients on consumption and output change from positive to negative suggests that wealth changes must correlate with both exchange rates and aggregate consumption growth. In a complete market where economic booms represent good states, risk-sharing implies that countries experiencing positive productivity shocks should transfer wealth to the rest of the world, resulting in a decline in their relative wealth. In other words, wealth changes and aggregate quantities are expected to be negatively correlated. However, we don't observe this in the data. Specifically, we examine this prediction of risk-sharing by regressing relative wealth changes on macroeconomic fundamentals:

[10]We note that, imperfectly shared shocks within a country can affect international risk-sharing and lead to wealth fluctuations across countries. In the next section, we present a model with this feature.

$$\left(\log \frac{W_t^{US}}{W_t^F} - \log \frac{W_{t-1}^{US}}{W_{t-1}^F}\right) = \alpha + \beta \left(\log \frac{x_t^{US}}{x_t^F} - \log \frac{x_{t-1}^{US}}{x_{t-1}^F}\right) + \gamma X_{t-1} + \varepsilon_{F,t}. \tag{5}$$

Where $x^c \in \{C, Y\}$ refers to consumption and output in country c. Controls X_{t-1} included lagged dependent and independent variables. The results are reported in Table 3.

Panel A of Table 3 shows that wealth and consumption are positively correlated. In the panel regression, a one standard deviation increase in consumption growth is associated with a 4.5 log point increase in relative wealth. For individual countries, all coefficients are positive. Moreover, 10 out of 11 coefficients are statistically significant.

Similarly, Panel B of Table 3 presents analogous estimates for output. The estimated coefficient for output growth is consistently positive. In the panel regression, the coefficient for output growth is 0.035, with a standard error of 0.01. In individual country regressions, we see that all coefficients are positive, with 7 out of 11 being statistically significant.

In summary, these results suggest that the shocks underlying these patterns are imperfectly shared productivity shocks. One plausible source for these positive productivity shocks is technological innovation. Improvement in technology is arguably the main driver of productivity, but its benefits are not shared equally. In fact, as we observe in Table 1, U.S. technological progress is associated with an appreciation of the dollar in real terms.

1.5 Innovation and Capital Flows

Why does innovation in the U.S. lead to an appreciation of the dollar? In a risk-sharing setup, upon receiving innovation shocks, the U.S. should transfer wealth to the rest of the world. As a result, we would observe an increase in capital outflow from the U.S. In what follows, we examine the relationship between U.S. innovation and capital flows in the data.

We first examine the relationship between U.S. innovation, U.S. foreign direct investment (FDI) inflows, and U.S. portfolio equity inflows at the aggregate level. Panel A of Figure 3 shows the trends in U.S. innovation intensity and FDI inflows over time. The correlation between U.S. innovation and FDI inflows is 0.309. U.S. innovation experienced a significant boom around 2000, driven by advancements in internet technology, and this period was associated with a large increase in FDI inflows.

Panel B plots the relationship between U.S. innovation and aggregate portfolio equity flows. We observe a significant positive correlation between the two time series: correlation = 0.281. Moreover, periods of higher innovation intensity are associated with increased portfolio equity inflows.

To delve deeper, we shift our focus to portfolio equity flows at the firm level. Specifically, we examine how foreign institutional capital responds to firm-level innovation shocks. This analysis allows us to understand whether innovation at the micro level attracts foreign investment. To quantify this relationship, we estimate the following specification:

$$\Delta \text{IO\_FOR}_{i,t,t+1} = \beta \log(inno)_{i,t-1,t} + \gamma X_{i,t} + \varepsilon_{i,t+1} \tag{6}$$

Where the dependent variable $\Delta \text{IO\_FOR}_{i,t,t+1}$ is the change in foreign institutional ownership for firm i between t and $t+1$. The foreign institutional ownership data are from the FactSet Lionshare database. The independent variable is the (log) number of important patents granted to firm i in the previous year $t-1$, according to three innovation measures to capture patent quality. First, we adjust the number of patents based on their number of forward citations. Second, we adjust patents for their economic value, according to Kogan et al. (2017). Breakthrough patents are defined as being in the top 20% based on citations or economic value in each year. Lastly, we use the patent breakthrough characterization of Kelly, Papanikolaou, Seru, and Taddy (2021), who use textual analysis to identify significant novel patents. The breakthrough patents are defined as being in the top 20% of the distribution in terms of their backward and forward similarities, using a five-year window. When *inno* is equal to zero, we replace $\log(inno)$ by zero and add a dummy equal to one if *inno* is equal to 0, thereby preventing the removal of the observation from the data. The vector of controls $X_{i,t}$ includes foreign institutional ownership at time t, $\text{IO\_FOR}_{i,t}$, firm and year fixed effects. We also estimate a specification where we add the firm's sales (log) and size at time t as additional controls. The sample covers 2000-2017, allowing patents a five-year window for citation accumulation. Table 4 presents the results.

We see that foreign institutional ownership increases after firms are granted novel patents. In terms of magnitude, a one-standard deviation in important patent grants is associated with approximately a 0.2 percentage point increase in the ownership of foreign institutional investors. Moreover, these estimates remain robust even after controlling for firm size and revenue (Columns 4-6 of Table 4), as well as for foreign institutions' time-varying preferences for specific sectors (Columns 7-9 of Table 4). These findings are consistent with the notion that technological innovations attract international equity capital flows. The heightened demand for U.S. stocks, which embody frontier technology, drives up the demand for dollars.

So far, we document a robust positive correlation between U.S. innovation and real dollar index growth, indicating limited risk-sharing of innovation shocks. Moreover, wealth changes across countries are positively correlated with macro fundamentals, suggesting that the poorly shared shocks are positive supply shocks. In addition, U.S. innovation is associated with increased demand for U.S. capital at both the aggregate and firm levels. Motivated by these observations, we next introduce a model that replicates these patterns. The model's mechanism relies on incomplete markets and the unequal distribution of benefits from innovation.

2 A Simple Model

To fix ideas, we begin our analysis with the minimal set of modeling ingredients that are necessary. As a result, our goal in this section is to provide some analytic intuition for the main mechanism in the paper. Section 3 presents a more general model that can be calibrated to fit the data.

2.1 Setup

We begin with a discussion of the modeling setup. The economy consists of two countries, home (H) and foreign (F), and two goods, X and Y. Time is discrete and is indexed by t.

2.1.1 Firms

There is a continuum of productive units in each country that produce output. We term these production units firms, but that definition is somewhat arbitrary since firm boundaries are ill-defined. We can also think of these as individual projects.

Firms in each respective country only produce the local good. That is, the firms in the home country only produce the X good, while foreign firms only produce the Y good. There is an expanding measure of firms in each country, indexed by (i, s, c) where s denotes the date at which the firm is created, $i \in [0, 1]$ denotes the index of the firm within its cohort in each country, and $c \in \{H, F\}$ denotes the country.

A firm characterized by (i, s, H) produces a flow of output $x^{i,H}_{t,s}$ at time t, according to

$$x^{i,H}_{t,s} = a^{i,H}_{t,s} X_t \tag{7}$$

The setup is symmetric in both the home and foreign countries; hence, a firm in the foreign country (i, s, F) produces output $y^{i,F}_{t,s}$

$$y^{i,F}_{t,s} = a^{i,F}_{t,s} Y_t \tag{8}$$

Here, $a^{i,H}_{t,s}, a^{i,F}_{t,s} \in [0, 1]$ denote the fraction of aggregate output accruing to a firm i located in the home and foreign country, respectively. By construction, these shares add to one

$$\sum_{s \leq t} \int_{i \in [0,1]} a^{i,c}_{t,s} = 1, \quad c \in \{H, F\} \tag{9}$$

The model has an element of creative destruction, in which new productive units displace existing ones. We model this in reduced form, following Gârleanu et al. (2016). Each period a new set of firms arrive exogenously in each country. These new firms, indexed by $i \in [0, 1]$, are heterogeneous in their productivity. The productivity of a newly arriving firm i in country $c \in \{H, F\}$ satisfies

$$a_{t,t}^{i,c} = (1 - e^{-u_t^c}) dL_t^{i,c} \tag{10}$$

where u_t^H, u_t^F are random, non-negative, shocks in home and foreign countries, affecting all firms in each country at time t. The components $L_t^{i,H}, L_t^{i,F}$ denotes cross-sectional measures and its increment $dL_t^{i,H}, dL_t^{i,F}$ are random, non-negative, idiosyncratic productivity components, which are determined at time t and satisfies $\int_{i\in[0,1]} dL_t^{i,H} = 1$ and $\int_{i\in[0,1]} dL_t^{i,H} = 1$. It follows that the total fraction of output produced by the cohort of firms born at time t is equal to

$$\frac{\int_{i\in[0,1]} x_{t,t}^{i,H}}{X_t} = 1 - e^{-u_t^H} \tag{11}$$

$$\frac{\int_{i\in[0,1]} y_{t,t}^{i,F}}{Y_t} = 1 - e^{-u_t^F} \tag{12}$$

The random shocks u_t^c reallocate revenue from incumbents to new entrants. Collectively, the fraction of output produced by existing firms is $e^{-u_t^H}$ for the home country and $e^{-u_t^F}$ for the foreign country. Specifically, the output share of an incumbent firm created at a time $s < t$ in country $c \in \{H, F\}$ is given by

$$a_{t,s}^{i,c} = a_{s,s}^{i,c} e^{-\sum_{n=s+1}^{t} u_n^c} \tag{13}$$

2.1.2 Aggregate Output

The aggregate output in each country evolves exogenously according to

$$\Delta \log X_{t+1} = \mu + \varepsilon_{t+1}^H + \delta u_{t+1}^H \tag{14}$$

$$\Delta \log Y_{t+1} = \mu + \varepsilon_{t+1}^F + \delta u_{t+1}^F \tag{15}$$

Notice that each output process is driven by two country-specific shocks, ε and u. The first shock, ε, affects the output (and dividends) of all firms symmetrically. The second shock, u, is the 'displacive' shock discussed above, which reallocates market share from existing to new firms. We allow this shock to affect aggregate output—motivated by standard models of endogenous growth—and parameterize its impact by $\delta \in (0, 1)$.

2.1.3 Households

Each country is populated by a unit measure of infinitely-lived agents, indexed by (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ denotes their country. At time zero, households are equally endowed with all firms in existence at that time. Households have access to the financial market and maximize their expected utility of consumption

$$U_{i,t}^c = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^s \log(C_{i,s}^c). \tag{16}$$

Household consumption C_t^c is an aggregate of the two goods produced by the home country (X) and the foreign country. Importantly, households exhibit 'home bias', that is, they tilt their consumption basket to the domestically produced good. That is, the consumption basket of each household living in country $c \in \{H, F\}$ at date t is given by

$$C_t^H = (x_t^H)^\alpha (y_t^H)^{1-\alpha} \tag{17}$$

$$C_t^F = (x_t^F)^{1-\alpha} (y_t^F)^\alpha. \tag{18}$$

Here, x_t^c and y_t^c denote the consumption of good X and good Y in country $c \in \{H, F\}$ at date t. The parameter $\alpha \in (\frac{1}{2}, 1)$ captures the degree of home bias in household preferences.

Last, we normalize the price of the home consumption good (the numeraire) to one; hence,

$$\alpha p_{x,t} + (1-\alpha) p_{y,t} = 1 \tag{19}$$

where $p_{x,t}, p_{y,t}$ are the price of the two goods X and Y, respectively. We denote the price of X and Y as $p_{x,t}, p_{y,t}$. The numeraire is α units of X good and $(1-\alpha)$ units of Y good, i.e.,

2.1.4 Creative Destruction and New Firms

Each period, households innovate with some probability. Successful innovation leads to the creation of new firms. The key feature of the model is that households cannot share this risk ex-ante, that is, they cannot sell claims against their future endowment of these new firms, as in Kogan et al. (2020). As a result, a shock to the relative profitability of new firms u leads to the redistribution of wealth from the owners of existing firms to the new entrepreneurs.

In particular, at time zero, agents are equally endowed with all the firms in existence at that time. From that point onward, agent (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ receives firm (i, t, c) at time t, i.e., a new firm with productivity proportional to $a_{t,t}^{i,c}$. For tractability, we closely follow Gârleanu et al. (2016) and focus on the limiting case in which firm creation generates extreme inequality. Specifically, we assume that only a set of measure zero of firms manages to produce non-zero profits; by contrast, the vast majority of new firms are worthless.[11] Consequently, when making consumption and saving decisions, households attach zero probability to the event that they

[11] More formally, we assume that, for every t, the distribution of idiosyncratic productivity $dL_t^{i,c}$ consists exclusively of point masses. That is, we assume that L_t^c is a discrete measure on $[0, 1]$, so that it is an increasing, right-continuous, left-limits process that is constant on $[0, 1]$ except on a countable set, where it is discontinuous. Both the magnitudes of the jumps of L_t, and the locations of the points of discontinuity are random. This assumption ensures that only a set of measure zero of consumers obtains the profitable new firms.

receive a profitable firm.[12]

2.1.5 Financial Markets

Households can trade a complete set of securities contingent on the realization of aggregate shocks. That is, they can trade equity claims on existing firms and risk-less, zero-net-supply bonds in either country. Consumers can also trade claims to the realizations of the displacement shocks (u^H_{t+1}, u^F_{t+1}) and output growth (X_{t+1}, Y_{t+1}). Importantly, however, a key market is missing: consumers cannot enter contracts that are contingent on the realized value of their future endowments of new firms.

This market incompleteness is a key part of the mechanism, as it introduces a wedge between aggregate consumption growth and the marginal utility of the average investor.

2.2 Equilibrium

Our definition of equilibrium is standard. An equilibrium is a set of price processes, consumption choices, and asset allocations such that (a) consumers maximize expected utility over consumption and asset choices subject to their dynamic budget constraint, (b) all asset and goods markets clear.

Markets are incomplete, hence households' marginal utilities are not equalized across states. To solve for the competitive equilibrium, we construct a representative agent whose preferences are a weighted average of household utilities in each country

$$\max_{\{x^H_t, y^H_t, x^F_t, y^F_t\}} \sum_t \beta^t \left(\log C^H_t + \lambda_t \log C^F_t\right) \tag{20}$$

Importantly, the Pareto-Negishi weight λ_t is stochastic in our model. This representative agent maximizes her utility subject to the following resource constraints,

$$x^H_t + x^F_t = X_t \tag{21}$$

$$y^H_t + y^F_t = Y_t \tag{22}$$

along with the consumption aggregator in (17).

Here, we note that even though households in both countries are heterogeneous in their wealth, consumption-wealth ratios are equalized within each country which facilitates aggregation. Hence, the representative consumer in each country solves the same optimization problem. That said, it is important to emphasize that even though we construct the preferences of each representative household as a function of the country-level consumption variables C^H_t and C^F_t, no household actually consumes that amount as markets are incomplete. Given our assumption, the effect of

[12] More precisely, what matters for household portfolio decisions is the physical probability of obtaining a new firm times the marginal utility of consumption in that state. Not only is the physical probability of receiving a new firm equal to zero, but also is the marginal utility of wealth (and consumption) since each firm is extremely valuable.

market incompleteness collapses into a scaling factor λ_t^c—and without loss of generality we have normalized $\lambda_t^H = 1$. See Appendix A.2 for more details.

In brief, λ_t is the time-varying ratio of marginal utilities of either good in the two countries at time t. It is equal to the wealth ratio between the two countries, and it varies over time as the result of market incompleteness

$$\lambda_t = \frac{W_{F,t}}{W_{H,t}} \tag{23}$$

where $W_{c,t} = \int_{i\in[0,1],c} w_t^{i,c}$ is the total wealth of households in country $c \in \{H, F\}$. In equilibrium, the ratio of wealth λ_t between the foreign and the home country affects both real allocations as well as the terms of trade. For example, the relative price of the foreign good Y in units of the domestic good X is equal to

$$p_t \equiv \frac{p_{y,t}}{p_{x,t}} = \frac{X_t}{Y_t}\frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t}, \tag{24}$$

and depends not on only on aggregate quantities Y_t and X_t, but also on the countries' relative wealth λ_t.

2.3 Displacement Risk and the SDF

The presence of displacement risk introduces a wedge between aggregate consumption growth and the stochastic discount factor. To understand why this is the case, note that, because of incomplete markets, the marginal utility of the 'representative' household is not only determined by aggregate consumption, but also by the realization of the displacement shock.

To see this, consider the following simplified version of the model, in which a) households have extreme home bias preferences $\alpha = 1$ (or equivalently single-country version of the model) and b) the value of all new firms is equally and randomly allocated to a measure π of the population. In this case, we can divide all households at each point in time into two groups, those that receive profitable new firms and those that do not. Agents have a constant consumption to wealth ratio, hence their consumption process is directly linked to the dividends of the firms they own. Hence, the equilibrium stochastic discount factor can be written as

$$\frac{M_{t+1}^H}{M_t^H} = \beta\left(\frac{X_{t+1}}{X_t}\right)^{-1}\left((1-\pi)\,e^{u_{t+1}^H} + \pi\left(\frac{1-e^{-u_{t+1}^H}}{\pi}\right)^{-1}\right). \tag{25}$$

Recall that we have assumed that income inequality is extreme, that is, L_t^i is comprised of point masses or equivalently $\pi \to 0$. In this case, the expression for the SDF simplifies to

$$\frac{M_{t+1}^H}{M_t^H} = \beta\left(\frac{X_{t+1}}{X_t}\right)^{-1} e^{u_{t+1}}. \tag{26}$$

In brief, we see that incomplete markets introduce a wedge between our stochastic discount factor and the one arising in a standard, Lucas-tree endowment economy. This additional term, given by $e^{u_{t+1}}$ adjusts for the fact that not all households experience the same growth rate in consumption; a set of measure zero experiences a dramatic increase as they receive new firms. Since marginal utility is a convex function of consumption, an increase in the dispersion of consumption growth raises the stochastic discount factor, similar in spirit to Constantinides and Duffie (1996).

In our model, the dynamics of the stochastic discount factor in each country is given by

$$\frac{M^H_{t+1}}{M^H_t} = \beta \frac{C^H_t}{C^H_{t+1}} \frac{1}{b_{H,t+1}} \quad \text{and} \quad \frac{M^F_{t+1}}{M^F_t} = \beta \frac{C^F_t}{C^F_{t+1}} \frac{1}{b_{F,t+1}}, \tag{27}$$

where $b_{H,t+1}$ and $b_{F,t+1}$ are the wealth shares of the people in home and foreign country who did not receive profitable firms at $t+1$,

$$b_{H,t+1} = \frac{\int_{i\in[0,1],a^{i,H}_{t+1,t+1}=0} w^{i,H}_{t+1}}{\int_{i\in[0,1]} w^{i,H}_t} \quad \text{and} \quad b_{F,t+1} = \frac{\int_{i\in[0,1],a^{i,F}_{t+1,t+1}=0} w^{i,F}_{t+1}}{\int_{i\in[0,1]} w^{i,F}_t} \tag{28}$$

The difference between (27) and equation (26) above is due to the fact that households own both domestic as well as foreign stocks, which implies that $b_{H,t+1}$ depends on both the domestic as well as the foreign displacement shocks u_H and u_F. That said, the relationship between b and u depends on the state of the economy, specifically, the relative wealth of the two countries, as captured by λ. For instance, when λ is high then country F is richer than country H. In this case, a small u_H shock will likely lead to a larger change in b_H than would be the case if country H were richer than F—since the new trees created in country H constitute a large share of wealth relative to the wealth of H households.

Overall, these movements in the stochastic discount factors of the home and foreign country in response to the displacement shocks u^c_t have direct implications for exchange rates, which we explore next.

2.4 Exchange Rates

We next characterize the behavior of exchange rates in the model. Because financial markets are integrated between the two countries, absence of arbitrage implies that the value of the exchange rate-is equal to the ratio of the two countries stochastic discount factors,

$$e_t = \frac{M^H_{t+1}}{M^F_{t+1}}. \tag{29}$$

The change in the exchange rate (in logs) can be written as

$$\Delta \log e_{t+1} = \Delta \log C^F_t - \Delta \log C^H_t + \Delta \log W_{H,t+1} - \Delta \log W_{F,t+1} \tag{30}$$

$$= \Delta \log C^F_{t+1} - \Delta \log C^H_{t+1} + \log b_{F,t+1} - \log b_{H,t+1}. \tag{31}$$

Equation (31) summarizes the main result in this paper. In the case of log utility, if markets were complete, λ would be a constant. In that case, bilateral exchange rate movements are purely determined by movements in the relative consumption growth between the home and foreign country. More generally, the ratio λ_t could vary over time, but its movements would still be determined by movements in relative consumption growth (either in the short run or in the long run). As a result, these models imply that exchange rates are *counter-cyclical*: an economic boom in the home country (an increase in X_t and thus, due to home-bias, C^H_t) leads to a decline in e, that is , a *depreciation* of the home currency relative to the foreign currency.

By contrast, in our model, there is an additional factor in play that arises due to market incompleteness: displacement risk, which is captured by $b_{H,t+1}$ and $b_{F,t+1}$. To obtain some intuition, we can approximate the evolution of λ_t around its long-run mean using a first-order Taylor expansion,

$$\log \frac{\lambda_{t+1}}{\lambda_t} = \Delta \log \frac{W_{F,t+1}}{W_{H,t+1}} = \log \frac{b_{H,t+1}}{b_{F,t+1}} \approx u^F_{t+1} - u^H_{t+1}. \tag{32}$$

Consistent with the discussion above, the wealth share λ_t varies over time as a result of incomplete markets and the displacement shock. A positive realization of u^F_{t+1} implies that a measure-zero of households in the foreign country received claims to new firms. Due to the limited risk-sharing, these households were not able to share these claims with the other households—either in the foreign or the domestic country. As a result, the relative wealth of the foreign country rises. See Section A.5 of the Appendix for more details on the derivation of (32).

Importantly, equation (31) suggests that, changes in countries' wealth ratio should be positively correlated with exchange rate. In addition, once we control for wealth fluctuations between countries , we should be able to observe a negative correlation between consumption growth and the exchange rate growth, as predicted by risk-sharing. This is exactly what we documented in Table 2.

As a result, the log growth rate of the exchange rate can be approximated as

$$\begin{aligned}\Delta e_{t+1} &\approx \Delta c^F_{t+1} - \Delta c^H_{t+1} + u^H_{t+1} - u^F_{t+1} \\ &\approx \underbrace{(2\alpha - 1)(1 - \delta)}_{>0}(u^H_{t+1} - u^F_{t+1}) + (1 - 2\alpha)(\varepsilon^H_{t+1} - \varepsilon^F_{t+1}).\end{aligned} \tag{33}$$

Consistent with the discussion so far, a positive displacement shock u^H_{t+1} will lead to an appreciation of the exchange rate, while a positive 'neutral' shock ε^H_{t+1} will cause the exchange rate to depreciate. Since country output and consumption depend on both shocks, exchange rates in the model can be either positively or negatively correlated with consumption or output growth.

To see how the model can generate the exchange rate disconnect, consider the log growth in the relative country output,

$$\Delta x_{t+1} - \Delta y_{t+1} = \delta(u^H_{t+1} - u^F_{t+1}) + \varepsilon^H - \varepsilon^F \tag{34}$$

which is increasing in both u^H_{t+1} and ε^H_{t+1}. Similarly, the growth in relative consumption can be written as

$$\Delta c^H_{t+1} - \Delta c^F_{t+1} \approx (1 - 2\alpha)(1 + \delta - 2\alpha)(u^H_{t+1} - u^F_{t+1}) + (2\alpha - 1)(\varepsilon^H - \varepsilon^F). \tag{35}$$

Importantly, assuming that

$$\delta < 2\alpha - 1 \tag{36}$$

implies that aggregate consumption growth in the home country is positively correlated with the displacive shock in that country, u^H_{t+1}. This is consistent with the pattern documented in Table 3.

Examining equations (33) and (34), we can see that the presence of the neutral shock ε tends to make exchange rates counter-cyclical, just like the standard model. By contrast, as long as (36) holds, the displacement shock u leads to positive co-movement between exchange rates, aggregate output and consumption. Thus, the unconditional correlation (disconnect) between exchange rates, country output and consumption depends on model parameters, for instance, the relative variance of the two aggregate shocks.

2.5 The Stock Market

The previous section illustrates that the model can generate a pro-cyclical exchange rate. But if that is the case, can the model also simultaneously generate a negative correlation between a country's exchange rate and its local stock market? The answer is that it can, and the reason again is due to the disconnect between the value of the stock market, that is, claims to existing firms, and aggregate consumption growth.

In particular, consider the value of existing trees in each country (the stock market) in country $c \in \{H, F\}$,

$$S^H_t = p_{x,t}X_t + \mathrm{E}_t[M^H_{t,t+1}(S^H_{t+1}e^{-u^H_{t+1}})] = p_{x,t}X_t(1 + pd^H_t) \tag{37}$$

$$S^F_t = p_{y,t}Y_t + \mathrm{E}_t[M^F_{t,t+1}(S^F_{t+1}e^{-u^F_{t+1}})] = p_{y,t}Y_t(1 + pd^F_t) \tag{38}$$

Take home country for example, the log return of holding the market portfolio is

$$\begin{aligned} r^H_{t+1} &= \log\left(\frac{X_{t+1}e^{-u^H_{t+1}}}{X_t}\frac{1 + pd^H_{t+1}}{pd^H_t}\right) \\ &= \mu + (\delta - 1)u^H_{t+1} + \varepsilon^H_{t+1} + \log\left(\frac{1 + pd^H_{t+1}}{pd^H_t}\right) \end{aligned} \tag{39}$$

Equation (39) highlights an important feature of our model: the distinction between aggregate dividend growth X_t and the growth of dividends that accruing to the stock market portfolio. The reason for this distinction is that aggregate dividends do not constitute the gains from holding the stock market: investing in the stock market at time t only generates $X_{t+1}e^{-u^H_{t+1}}$ dividends at $t+1$. A positive displacement shock increases aggregate dividends by introducing new firms, but also dilutes the shares of the existing firms. On the other hand, following a positive displacement shock the price-dividend ratio also decreases. As a result, a positive displacement shock leads to a decline in stock market returns.

2.5.1 An alternative proxy for displacement shocks

In this section, we construct a proxy for the displacement shock that closely aligns with our theoretical model. Similar to Gârleanu et al. (2016), the displacement shock in our model is related to the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. Guided by this insight,we construct a series of displacement shocks for the U.S. market.

To this end, we characterize the distinction between aggregate market capitalization S_t and the return of the market portfolio. Let $P_{t,t+1}$ represent the value at time $t+1$ of a market portfolio consisting of all firms existing at time t. Given the arrival of new firms, we can express the aggregate value of the stock market at $t+1$ as

$$\log S_{t+1} = \log P_{t,t+1} + u_{t+1} \tag{40}$$

This representation captures the displacement shock u, which highlights the discrepancy between the returns on a portfolio invested in incumbent firms and the growth in aggregate market capitalization.

Each month, we calculate the value of a portfolio that holds the entire market, excluding all dividend payments but adjusted for stock splits (CRSP item RETX). We then compare this with the aggregate market cap at the end of each month. The log difference between the two values represents the magnitude of the displacement shock u for that month. We aggregate all monthly displacement shocks over a year to obtain the annual displacement series. We aggregate these monthly displacement shocks over a year to construct the annual displacement series. To account for the lock-up period post-IPO, we lag the annual displacement series by one year, reflecting the delayed wealth effects associated with the emergence of new firms.

The difference between the growth of aggregate market cap and the growth of portfolio of all existing firms arises because an investor holding the market portfolio must pay to acquire new firms entering the market. To maintain the self-financing nature of the strategy, the investor must continually liquidate some of the shares she holds to purchase shares of new firms. Since new firms enter the market each period, the growth of the market portfolio falls short of the growth of the aggregate market cap.

Figure 4 plots the displacement shock against the growth of the real U.S. dollar index. We observe that the displacement shocks are positively correlated with the growth of real U.S. dollar index (correlation = 0.349). Notably, during periods of significant innovation, many new firms emerge, and the dollar appreciates in real terms.

Table 5 presents the results of a univariate regression of exchange rate growth on U.S. displacement shocks. The results confirm that bilateral exchange rates are significantly correlated with U.S. displacement shocks. Notably, the coefficients for all countries are positive. In terms of magnitude, a one standard deviation increase in the displacement shock is associated with a 4 log point appreciation in dollar exchange rates.

2.6 Exchange Rates and the Growth of Top Incomes

The presence of the displacement shock u captures the idea that the benefits of economic growth are not shared equally. In the model, u captures the reallocation (creative destruction) that occurs between owners of existing firms and those who create new firms (entrepreneurs). Specifically, each period, a measure zero of the population receives profitable new firms. If we were to treat this transfer as capital income, fluctuations in u would translate into fluctuations into income inequality in the model.

Here, we develop this idea further and connect the displacive shock u in the model to an observable quantity, the top 1% share of income. In particular, the top 1% income consists of two groups of households.

The first group consists of the households that receive new firms in the current period. The total capital gains from new firms in the home country, as a fraction of total income is

$$I_{H,t}^{capital} = \frac{S_t^H(1-e^{-u_t^H})}{W_{H,t}\xi + (p_{x,t}X_t e^{-u_t^H} + p_{y,t}Y_t e^{-u_t^F})\frac{W'_{H,t}}{W'_{H,t}+W'_{F,t}} + S_t^H(1-e^{-u_t^H})}. \tag{41}$$

Where $W'_{H,t}$ and $W'_{F,t}$ are the total wealth of the two countries excluding new projects, that is

$$W'_{H,t} = W_{H,t} - (1-e^{-u_t^H})S_t^H$$
$$W'_{F,t} = W_{F,t} - (1-e^{-u_t^F})S_t^F$$

We can see that the size of the u shock at time t determines the amount of wealth that is transferred from existing firms to the new firms. The value of these new firms constitutes a capital gain for the successful entrepreneurs, and they are randomly distributed to a small part of the population. Hence, some of it is part of the income share of the top 1%.

The second group is the households who have had received projects in the past and consequently earn a large capital income on those wealth. These households derive capital income equal to

$$W_{H,t}\xi + (p_{x,t}X_t e^{-u_t^H} + p_{y,t}Y_t e^{-u_t^F})\frac{W'_{H,t}}{W'_{H,t} + W'_{F,t}}. \tag{42}$$

These capital gains and annuity income are proportionally distributed to all the population. Therefore, the top income inequality is a function of both the current displacement shock and the current wealth inequality. The wealth inequality, in turn, is a function of past displacement shocks.

The above discussion illustrates how the joint dynamics of income inequality and exchange rates can inform us about the quantitative impact of the displacement shock. In particular, recall equation (31), which states that exchange rate growth is determined by relative consumption growth and changes in the wealth share of households that are displaced in each country b_H and b_F, which are primarily driven by the displacive shock u_H and u_F, respectively. To the extent that income inequality is a useful proxy for the u shock in the model, the correlation between exchange rates and income inequality would reveal the importance of the displacive shock u as a driver of exchange rates.

To explore this idea further, we estimate the following specification,

$$\Delta Y_t = \alpha + \beta_{ineq}\left(\log\frac{Q_t^{US}}{Q_{t-1}^{US}} - \log\frac{Q_t^F}{Q_{t-1}^F}\right) + \log e_{F,t-1} + \varepsilon_{F,t}. \tag{43}$$

Where dependent variable is equal to the exchange rate growth and wealth changes: $\Delta Y_t \in \{\log e_{F,t} - \log e_{F,t-1}, \Delta\log\frac{W_{US}}{W_F}\}$. Independent variable Q_{US}, Q_F are inequality measures in the United States and country F, respectively. As our baseline case, we take Q to refer to the income share of the top 1%. We estimate equation (43) and Panel A of Table 6 presents the results for the panel regression (43) together with country-by-country estimates.

Focusing on the panel regression, we see that the estimated coefficient β is positive and statistically significant. That is, increases in income inequality in the foreign country are associated with an appreciation of its currency relative to the U.S. In terms of magnitude, a one-standard deviation increase in income inequality in the foreign country is associated with a 1.7 log point appreciation of its currency relative to the US dollar. Examining the country-level regressions, we observe a consistent pattern. The individually estimated β_{ineq} coefficients are generally positive (9 out of 11), though not always statistically significant (3 out of 11).

Panel B of Table 6 presents the results with wealth changes as the dependent variable. Here, the estimated coefficient β is also positive and statistically significant, showing a comparable magnitude to the exchange rate results. Additionally, the individual country regressions generally exhibit positive coefficients.

We examine the robustness of these findings using alternative measures of inequality and estimating equation (43) without the consumption term. We obtain similar results. Appendix

Table B.4 shows that we obtain similar findings if we measure inequality as the top 0.1% share of income.

In brief, the correlation between income inequality and exchange rates is comparable in magnitude to the correlation between exchange rates and consumption growth. Here, we note that even after controlling for income inequality, the estimated coefficient for consumption is positive and statistically significant in the panel regression. This is not particularly surprising: even under the null of the model, income inequality is likely to be a noisy proxy for the displacive shock u as it is affected by other quantities as equation (41) shows. Nevertheless, when calibrating the model, we will take these positive correlations into account: we will include the estimated slope coefficients in (43), specifically β_{ineq} and the slope coefficient on consumption growth β_c in our calibration targets.

3 The Full Model

So far, we have presented a stylized model that allows us to highlight the key mechanism in the paper. Though transparent, however, the model is not rich enough to quantitatively capture all the interesting aspects of the data. Here, we introduce several additional features and aim for a full quantitative exploration of the mechanism.

3.1 Setup

To conserve space, we only highlight the differences with the simpler model in the previous section.

3.1.1 Agents' Preferences and Demographics

We make three changes relative to the previous setup.

First, we introduce finite lives. This helps ensure that the level of inequality in each country remains stationary. For simplicity, the size of population in each country is normalized to one. At each date, a mass ξ of agents, chosen randomly, die, and a mass of ξ of agents are born, so that the population remains constant. There is an annuity market so that households who do not die receive ξ of their wealth from the annuity. For the wealth of people who die at period t, a $1-\xi$ fraction will be used to finance the annuity within the country, and the remaining ξ fraction will be distributed uniformly to the new borne agents. This way, we do not need to keep track of the time at which the agents are born.

Second, we modify household preferences. Agents have non-time separable preferences; in addition, they care about both their own absolute level of consumption but also their consumption relative to an index. In particular, households' continuation utility at time t is given by

$$U_{i,t}^c = \left[(1-\beta)(\hat{C}_{i,t}^c)^{1-\frac{1}{\psi}} + \beta\,\mathrm{E}_t\left[(U_{i,t+1}^c)^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}. \tag{44}$$

That is, households have recursive preferences of the Epstein-Zin form. The parameters γ and ψ measure the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS), respectively. The coefficient β is the effective time-preference parameter, which also incorporates the probability of death, that is, $\beta = \tilde{\beta}(1-\xi)$ where ξ is the probability of death and $\tilde{\beta}$ is the households' subjective time discount factor.

In addition, $\hat{C}^c_{i,t}$ refers to a composite good that depends both on the households' own consumption $C^c_{i,t}$ but also its level relative to the aggregate $\bar{C}^c_t$ in their country,

$$\hat{C}^c_{i,t} = \left(C^c_{i,t}\right)^h \left(\frac{C^c_{i,t}}{\bar{C}^c_t}\right)^{1-h}. \tag{45}$$

Here, $C^c_{i,t}$ is the agent i's own consumption bundle in country $c \in \{H, F\}$—defined in (17)—which is comprised of both home and foreign goods. The parameter h denotes the strength of the relative preference effect. When $h = 1$, these preferences specialize to the standard Epstein-Zin preferences. In general, for $h \in [0, 1]$ agents place a weight h on their own consumption and a weight $1 - h$ on their consumption relative to average consumption in country $c \in \{H, F\}$.

Households can hedge their mortality risk using a competitive annuity market. Households are risk averse, hence they all purchase annuities. The annuity issuer collects the wealth of deceased households ξW and distributes the proceeds to the surviving population and the newly born agents.

Finally, we relax the assumption of extreme inequality, by assuming that the measure of population that receives the value of new firms is non-negligible, that is, $\pi > 0$. Though this modification makes the model significantly less tractable, it helps the model match the observed patterns of inequality in the data.

3.1.2 Aggregate Output

The evolution of aggregate output in each country is still given by equations (14) and (15). We next make distributional assumptions about these shocks.

First, we allow for the displacement shocks in each country to be correlated, possibly due to technology spillovers. That is, the effective displacement shock in each country u is a weighted average of each country's idiosyncratic displacement shock $\bar{u}$,

$$\begin{aligned} u^H_{t+1} &= (1-\rho_u)\,\bar{u}^H_{t+1} + \rho_u\,\bar{u}^F_{t+1} \\ u^F_{t+1} &= (1-\rho_u)\,\bar{u}^F_{t+1} + \rho_u\,\bar{u}^H_{t+1}. \end{aligned}$$

The idiosyncratic displacement shocks in each country $\bar{u}^c_t, c \in \{H, F\}$ follow a Markov chain with three states $[u_1, u_2, u_3]$ and transition matrix given by

$$T = \begin{bmatrix} \nu_{1,1} & \nu_{1,2} & \nu_{1,3} \\ \nu_{2,1} & \nu_{2,2} & \nu_{2,3} \\ \nu_{3,1} & \nu_{3,2} & \nu_{3,3} \end{bmatrix}, \qquad \sum_{j=1}^{3} \nu_{i,j} = 1. \tag{46}$$

Second, we assume that the 'neutral' shocks are i.i.d. and jointly normally distributed $[\varepsilon^h, \varepsilon^f] \in N(0, \Sigma)$, where

$$\Sigma = \begin{bmatrix} \sigma_e^2 & \rho_e \sigma_e^2 \\ \rho_e \sigma_e^2 & \sigma_e^2 \end{bmatrix} \tag{47}$$

$\rho_e > 0$ is the correlation between the neutral shocks between two countries.

3.2 Equilibrium

The equilibrium in the full model is largely similar to the one in the simplified model, even as the algebra is somewhat more involved.

Given our assumptions, the stochastic discount factor in country c is given by (see Appendix B.3 for derivation)

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi}+h-1} \tilde{b}_{c,t+1} \left(\frac{U_{c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{48}$$

where

$$\tilde{b}_{c,t+1} = \pi \left(\frac{b_{c,t+1}\pi + 1 - b_{c,t+1}}{\pi} \right)^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}} + (1-\pi) b_{c,t+1}^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}} \tag{49}$$

As before, exchange rates are equal to the ratio of stochastic discount factors. We have

$$\begin{aligned} \Delta \log e_{t+1} &= \Delta \log M_{t+1}^H - \Delta \log M_{t+1}^F \\ &= \left(\frac{h}{\psi} + 1 - h \right) (\Delta \log C_{t+1}^F - \Delta \log C_{t+1}^H) + \Big(\log(\tilde{b}_{F,t+1}) - \log(\tilde{b}_{H,t+1}) \Big) \\ &\quad + \frac{1/\psi - \gamma}{1-\gamma} \left(\log \frac{U_{H,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{H,t+1}^{1-\gamma}]} - \log \frac{U_{F,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{F,t+1}^{1-\gamma}]} \right) \end{aligned} \tag{50}$$

Examining (50), we note the similarities with the log utility case—equation (31). That is, exchange rate dynamics are still driven by relative consumption growth in the two countries, as well as the relative degree of displacement in the current period ($b_{H,t+1}$ and $b_{F,t+1}$). The key difference with the time-separable case is that now the shocks to the future distribution of these variables matters, as encoded into households' continuation utility $U_{H,t+1}$ and $U_{F,t+1}$.

3.3 Estimation

In this section, we describe how we calibrate the model to the data. Given the degree of non-linearity in our model, solution methods that are based on log-linearizations around the steady state are not necessarily reliable. As such, we solve for the global solution of the model by discretizing the state-space and using a combination of value and policy function iteration. See Appendix C for a brief description of our numerical procedure.

To reduce the number of parameters, we make simplifying restrictions on the dynamics of u shocks. First, we assume that $u_2 = u_1 + \varepsilon$, with $\varepsilon > 0$, and consider the limiting case as $\varepsilon \to 0$, that is $u_1 = u_2$. Hence, a transition from u_1 to u_2 mostly affects the future distribution of u (as the transition probabilities change), rather than the current level of displacement. We interpret the transition from u_1 to u_2 as a manifestation of the learning process in the early stages of a potential technological revolution. Second, we assume that the matrix T corresponds to the transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters—the corresponding autocorrelation parameter p and q. Specifically, we assume that the transition matrix has the following form

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix} \tag{51}$$

Where p^2 is the probability of staying in the lowest state once already there and q^2 is the probability of staying in the highest state once there [13].

After restricting the evolution of u, the full model has a total of 16 parameters. We estimate the parameters of the model using an indirect inference method (Lee and Ingram, 1991). Specifically, given a vector of X of target statistics in the data, we obtain parameter estimates by

$$\hat{p} = \arg\min_{p \in \mathcal{P}} \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right)' W \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right) \tag{52}$$

Where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model.

The matrix W determines the importance of each statistic in the distance criterion to be minimized. In general, we choose to penalize proportional deviations of the model statistics from their empirical counterparts, so $W = diag(XX')^{-1} I_W$. The diagonal matrix I_W allows us to introduce some exceptions to this criterion based on the importance that the existing literature places on matching certain features of the data—but also moments that are revealing of our model mechanism. As such, we apply a factor of 10 on the UIP coefficient, the stock market excess return,

[13] Conversely, $(1-p)^2$ is the probability of transitioning from the lowest to the highest state and $(1-q)^2$ is the probability of transitioning from the highest to the lowest. When $p \neq q$, there is conditional heteroscedasticity in the shocks. For the case when $p = q$, the discrete process has the first-order persistence as q.

the level of top 1% income share, and the volatility of exchange rate. The remaining elements on the diagonal of I_W are normalized to one.

Our estimation targets are reported in the first column of Table 7. They include a combination of first and second moments of aggregate quantities, asset prices and exchange rates. In addition to these standard international moments in the literature, we also target a set of correlations between wealth changes and real variables. In the model, the neutral shock and displacement shock have different implications for the cyclicality of exchange rates and relative wealth ratios. Thus, the set of correlation between wealth changes and consumption, output, together with the set of bilateral correlations, is informative about the relative magnitude of these two shocks. We also include the correlation between dollar index growth and U.S. innovation, where innovation is proxied by displacement shocks as described in Section 2.5.1. In addition, we target the average top 1% income inequality of the United States and the exchange rate reconnect pattern – i.e., the set of estimated coefficients of regressions 4. In the model, we consider the stock market as a levered claim of domestic consumption goods by a factor of two. See Appendix D for more details on the construction of the target moments.

3.4 Model Fit

Table 7 shows that the baseline model fits data reasonably well. Most of the empirical moments are close to their model counterparts and fall within the 5th to 95th intervals from simulations. Our model reproduces the realist patterns of both aggregate consumption and output growth. On the asset pricing side, the model generates the realistic levels of equity risk premium and volatility of the stock market. The volatility of the realized interest rate in the data is more volatile than the simulated data, but this may be largely driven by the high inflation around 1980s.

On the international side, our model successfully replicates the three key anomalies in the literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of the UIP. Moreover, the model generates positive correlation between wealth changes and consumption. This is because the u shock is not only positively correlated to the aggregate consumption and output, and is also associated with significant wealth transfer due to imperfect risk sharing. That is, our model can replicate the pattern in the data where shocks that drive up wealth ratios are positive supply shocks. The key to the replication of the UIP anomaly is the time-varying volatility—more precisely, the time-varying distribution of the effective size of the u-shock—that endogenously arises in the equilibrium. Despite the fact that consumption, output and stock market are highly correlated, the exchanges rate in our model is as volatile as in the data due to a high level of home-bias. Finally, net exports in our model are counter-cyclical, as in the data.

In addition, the replication of international puzzles does not require a unrealistic magnitude of technology shocks. In fact, Our model generates a realistic level of income inequality. The correlation between wealth and inequality in the data falls within the 5th and 95th percentiles of the simulation intervals. Given that most dynamics in our model are driven by the displacement shock, whose magnitude is indirectly linked to observed income inequality, these results are reassuring. Furthermore, our model reproduces the estimated coefficients of the bivariate regression of exchange rate growth on wealth changes, consumption, and output. That is, once the wealth changes associated with displacement shocks are controlled for, the risk-sharing associated with neutral shocks becomes apparent—a feature in the data that our model can also replicate.

3.5 Parameter Estimates

Table 8 reports the parameter estimates of the model. Examining the set of parameter estimates, there are several points worth making. In terms of preference parameters, the model calibration requires a reasonable set of preference parameters: degree of relative risk aversion (6.3) and the elasticity of inter-temporal substitution (1.6). That said, the standard errors in both parameters are relatively high, which implies that the model solution is not particularly sensitive to these parameters. In addition, the model requires a very high level of home bias (0.988), similar to Colacito and Croce (2013), in order to generate volatile exchange rates. In addition, the preference weight on relative consumption is rather high (0.81) though again, the relatively high standard error implies the model solution is not very sensitive to this particular value. Second, in terms of the distribution of shocks, we see that the calibration requires a highly persistent ($p = 0.930, q = 0.831$) and right-skewed displacement shock to fit the data.

To get a deeper understanding of how these parameters are identified from the data, we also compute the Gentzkow and Shapiro (2014) measure of elasticity of parameters to moments. To conserve space, we only briefly discuss these results here, and relegate the full set of results to Appendix Figures D.1 to D.6.

In terms of technology shocks, the mean μ and volatility σ_e of the neutral shock is identified by the first two moments of consumption and output growth. The distribution of the displacement shock u is primarily identified by the volatility of exchange rate and the stock market (since the spread between u_1 and u_3 affects the volatility of the SDF in the model), as well as the level of top 1% income share (since it directly affects the average top 1% income share in the model), as well as. The parameter governing the importance of the displacement shock to output δ is also primarily identified by the correlation between wealth and output, and the volatility of exchange rates, since it determines the joint dynamics of the SDF and output growth. The two parameters governing the correlation between the home/foreign shocks (ρ_e and ρ_u) are primarily identified by the correlation of home and foreign consumption growth, output, and the stock market. These two correlation parameters also affect the effective size of these two shocks. Last, the persistence of the displacement shock u is primarily identified by the equity premium and volatility of stock returns, since the u shock is a key driver of stock returns.

In terms of preference parameters, the degree of home bias α is identified primarily by the volatility of the exchange rates. The coefficient of relative risk aversion γ is identified from the mean and volatility of stock returns, as well as the volatility of the risk-free rate. The subjective discount factor β and the probability of death ξ are jointly mainly identified by the mean of the risk-free rate and the level of inequality. In general, these two parameters play a similar role in most model quantities, with the exception of inequality: higher β implies a higher price-dividend ratio and therefore a lower share of top income from "accumulated wealthy" people; by contrast, a higher death rate ξ implies less concentration of wealth and dividend income which lowers income inequality. As they generate somewhat different implications for the relation between top income share and u shock, the moments on inequality help determine these two parameters. The weight on own h consumption is primarily identified by the volatility of exchange rate and the correlation between wealth and consumption growth. As h falls, households place a higher emphasis on relative consumption, thereby increasing the significance of the displacement shock.This displacement shock is the main factor driving both the volatility of the Stochastic Discount Factor (SDF) and the positive correlation between wealth and consumption. The elasticity of intertemporal substitution (EIS) affects the volatility of interest rates and hence is primarily identified by the volatility of excess returns.

4 Model Implications

Here, we examine the model's implications. First, we focus on the key mechanisms in the model, that is, how the key quantities in the model respond to the two exogenous shocks u and ε. Second, we examine the forces that allow the model to replicate some of the stylized facts in the literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of UIP.

4.1 Model Mechanism

Figure 5 present the response of key model quantities to the two shocks in the model: the displacement shock u (Panel A) and the neutral shock ε (Panel B). For brevity, we examine responses to shocks in the home country only; shocks to the foreign country are exactly symmetric.

4.1.1 Quantities

First, consider Figure 5. The first two columns show the response of the exchange rate and consumption growth to the two shocks. As we can see, a positive ε shock in the home country leads to a depreciation of the currency and an increase in consumption growth. This is the standard shock in most models and the reason why exchange rates are counter-cyclical. By contrast, a positive u shock leads to an appreciation of the exchange rate as well as an increase in consumption growth.

The next two columns of Figure 5 illustrate why the exchange rate appreciates in response to a positive u shock. Recall equation (50) in the full model. Columns three and four of the Figure 5 illustrate how the last two terms of the equation respond to the shocks in the model. Specifically, an increase in u_H leads to a decline in the wealth share of the owners of incumbent firms in the home country b_H and therefore to an appreciation of the exchange rate. Similarly, the fourth column shows that an increase in u_H leads to a decline in the continuation utility of households in the home country U_H, which also contributes to the appreciation of the home currency. Put differently, both of these latter forces lead to an increase in the stochastic discount factor in the home country, as can be seen from equation (48), which causes the home currency to appreciate.

Last, this figure illustrates why exchange rates are in general counter-cyclical even in models with recursive preferences (e.g., Colacito and Croce, 2013). As we can see in the top right panel, a positive shock to ε leads to an increase in households' continuation utility, which contributes to the home currency depreciation. Though the neutral shock ε is i.i.d. in the model, this result is much more general: any shock which increases households' continuation utility will lead to a depreciation of the currency. Persistent shocks to consumption growth (long-run risk) fall into this category.

4.1.2 Assets Prices

Next, we examine the impact of two shocks on financial assets. Figure 6 plots impulse responses for log-SDF, stock market return r_{ex}, risk-free rate r_f and volatility of log-SDF for both countries. The first column shows that the neutral shock and the displacement shock have an opposite effect on the growth of log-SDF: a positive displacement (neutral) shock leads to an increase (decrease) of the log-SDF growth. This means that the displacement shock u has a negative risk premium while the neutral shock carries a positive risk premium.

Consistent with the analyses above, the difference in how the SDF responds to two shocks stems primarily from how the benefits of technological progress are shared among households. Both shocks u and ε lead to an increase in the aggregate output, which causes SDF to fall. However, in case of displacement shock, the fall in consumption and continuation utility due to unequal sharing of technological progress is sufficiently large to offset the benefits of higher aggregate consumption.

The third column depicts the response of stock market and highlights an important feature of our model: the difference between aggregate dividend growth X_t and the growth of dividends accruing to the investment in the stock market. The reason for this difference is that aggregate dividends do not constitute the gains from holding the stock market: investing in the stock market at time t only generate $X_{t+1}e^{-u^H_{t+1}}$ dividends at time $t+1$. A positive displacement shock increases the aggregate dividends by introducing new firms, but also dilutes the shares of the existing firms. As a result, a positive displacement shock leads to a decline in the returns to incumbent firms on impact.

4.1.3 Output, Trade and Financial Flows

Finally, we examine the impact of two shocks on aggregate output and international financial flows. Figure 7 plots impulse responses for consumption share λ_t, wealth share w_t, output growth $\Delta\log(X)$ and $\Delta\log(Y)$, net export scaled by output and net international investment position scaled by country's wealth.

First, note that the second column of Figure 7 shows that both the neutral shock and displacement shock contribute to an increase in the aggregate output – they are positive productivity shocks. However, they have different implications for exchange rate movements: displacement shocks lead to exchange rate appreciation, whereas neutral shocks are associated with exchange rate depreciation.

In the model, the home and foreign country's net exports as a fraction of total output are

$$\frac{NX_t^H}{X_t} = \frac{p_{x,t}X_t - p_{x,t}x_t^H - p_{y,t}y_t^H}{p_{x,t}X_t} = 1 - \frac{1}{\alpha + (1-\alpha)\lambda_t} \tag{53}$$

$$\frac{NX_t^F}{Y_t} = \frac{p_{y,t}Y_t - p_{y,t}Y_t^F - p_{y,t}x_t^H}{p_{y,t}Y_t} = 1 - \frac{\lambda_t}{1-\alpha+\alpha\lambda_t} \tag{54}$$

And the net international investment position (NIIP) scaled by the country's wealth is

$$\frac{A_t^H}{W_t^H} = \frac{W_t^H - S_t^H}{W_t^H} \tag{55}$$

$$\frac{A_t^F}{W_t^F} = \frac{W_t^F - S_t^F}{W_t^F} \tag{56}$$

The third and the fourth column of Figure 7 show that the dynamics of the international flows are mostly driven by the displacement shocks.

Specifically, the third column shows that following a positive displacement shock, the net export declines and the country becomes an importer. We can see from (53) that the balance of trade is purely determined by λ. In the model, the large country is the net importer and the small country is the net exporter. As λ_t decreases, home country becomes wealthier and its households want to consume more. Therefore, home country exports less of domestic goods and imports more of the foreign goods. Home country's balance of trade deteriorates and home currency appreciates. Thus, the model is able to reproduce the counter-cyclical net export.

Moreover, displacement shocks lead to increased inequality. As a result, we should expect that countries experiencing larger increases in current account deficits also see greater inequality growth. Figure 9 illustrates this pattern. From 1980 to 2022[14], there is a strong negative cross-country correlation between changes in top income shares and changes in the current account balance. In terms of magnitude, an increase of one percentage point in the top 1% income share over the period corresponds to a deterioration of the current-account-to-GDP ratio by 1.7 percentage points.

[14]The current account balance data is from the World Economic Outlook database, which is available from 1980 onward.

Next we turn to capital flows. The fourth column of Figure 7 shows that a positive displacement shock leads to capital inflows. Recall that each period, investors who hold the market portfolio needs to pay to acquire the new firms that enter the market. When the home country receives a larger displacement shock than the foreign country, there are more new firms in home country than the foreign. Households receiving these new firms (entrepreneurs) are motivated to sell their stakes to rebalance their portfolio. Part of these firms are acquired by foreigners who wish to rebalance their portfolio. The net result is that foreign demand for home assets increases relative to home demand for foreign assets, and therefore the home country experiences net capital inflows as its wealth increases. These inflows are associated with currency appreciation in the model, consistent with the evidence in Camanho et al. (2020); Hau and Rey (2006); Rey et al. (2024).

These predictions receive support in the data. At the aggregate level, Figure 3 shows that U.S. foreign direct investment inflows correlates positively with U.S. innovation intensity. We next turn to portfolio flows. According to the model, the arrival of an innovative project should attract foreign capital inflows to these firms, leading to an increase in foreign ownership. This prediction aligns with the pattern shown in Table 4.

4.2 Implications for Exchange Rate 'Puzzles'

The exchange rate literature has traditionally focused on various "puzzles" in exchange rate behavior. One important puzzle is the seemingly disconnect between exchange rate movements and macroeconomic variables such as consumption and output Backus and Smith (1993); Obstfeld and Rogoff (2001). In addition, interest rate differences do not predict changes in exchange rates with the correct sign to enforce the uncovered interest rate parity (Fama (1984)). In what follows, we examine the extent to which our model can address these longstanding puzzles.

4.2.1 Aggregate consumption and the Backus-Smith Puzzle

Given the analyses in the first part of Section 4.1, it follows that our model is able to generate a positive correlation between countries' differences in consumption growth, and exchange rate growth, resolving the Backus-Smith anomaly. Recall that the displacement shocks produce a positive comovement between consumption and exchange rate, while neutral shocks generates a negative correlation between two variables. The replication of the Backus-Smith correlation thus requires that the impact of displacement shock dominates that of neutral shock. The final quantitative impact of the displacement shock depends on the calibration of its displacement effect δ and households relative preference h, as well as the relative magnitude between two shocks.

4.2.2 Productivity Shocks and Exchange Rates

Recent study by Chahrour et al. (2024) revisited the exchange rate disconnect puzzle and used a vector auto-regression to argue that news about future U.S. productivity drives exchange rate movements[15]. Specifically, Chahrour et al. (2024) show that shocks to expectations about future U.S. productivity explain a large fraction of the variation in both exchange rates and real macroeconomic quantities, though at different horizons.

Despite both focusing on medium-to-long-term productivity growth, Chahrour et al. (2024) argue that the traditional long-run risk model (e.g., Colacito and Croce (2013)) is inconsistent with their empirical findings, particularly the positive correlation between consumption and medium-to-long-term productivity growth. This is because, in long-run risk models, perfect risk-sharing implies that domestic consumption declines in response to improvements in long-term productivity growth.

In contrast, displacement shocks in our model are imperfectly shared. As a result, they generate a positive correlation between output growth and the exchange rate, and home consumption rises. To illustrate, consider a transition between u_1 and u_2 (Panel B of Figure 8), which alters the conditional distribution of future displacement shocks rather than the current level of productivity. This transition represents to a positive shock to the expectation of future productivity growth (as discussed in Chahrour et al. (2024)), resulting in an appreciation in the real exchange rate.

Additionally, Chahrour et al. (2024) find that the U.S. current account deteriorates in anticipation of future U.S. productivity improvements. As discussed in the last section, displacement shocks at home increase home households' wealth (Column 1 of Figure 7), leading them to export less and import more, which leads to a deterioration in the trade balance (Column 3 of Figure 7). At the same time, displacement shocks at home are also associated with strong foreign demand for home equity, so the home country experiences an increase in capital inflows (Column 4 of Figure 8).

Overall, the patterns documented in Chahrour et al. (2024) are entirely consistent with the mechanism of our model. We interpret this as supportive evidence that recognizing the limited risk-sharing of displacive innovation shocks is essential for understanding the relationship between exchange rates, productivity growth, and real quantities.

4.2.3 The Forward Premium Anomaly

Uncovered interest rate parity (UIP) states that the expected change in exchange rates should be equal to the interest rate differential between two countries, and that the currency with lower interest rate tends to appreciate. Therefore, the regression coefficient of future exchange rates growth on interest rate differential should be equal to one. Empirically, the coefficient is much smaller than one and even negative. The violation of the UIP is often referred to as the forward premium puzzle. Fama (1984) and Backus, Foresi, and Telmer (2001) note that time-varying volatility of the SDFs is a necessary condition for the replication of this anomaly. We next show that in our model the failure of the UIP is an endogenous equilibrium outcome.

The upper panels (Panel A) of Figures 5 through 7 shows the responses of key model quantities to a displacive shock (the economy moves from u_2 to u_3). Figure 8 shows the responses of exchange

[15] Stavrakeva and Tang (2024) argue that macroeconomic news accounts for most of the variation in exchange rates.

rate, log-SDF, risk-free rate and the volatility of log-SDF following a shock from u_1 to u_3 and a shock from u_1 to u_2. Upon the realization of a positive displacement shock to the home country, the home currency appreciates and the total wealth of the home country increases relative to the foreign country. Recalling the discussion of the mean-reversion of λ, in the future the effective size of the u-shock in the foreign country is expected to be greater than that of the home country. As a result, the foreign currency is subsequently expected to appreciate (Figure 8, column 1). Turning our attention to the risk-free rate, we see two forces in opposite directions. On the one hand, due to the difference in effective size of displacement shocks, foreign households expect a lower consumption growth than home households.

Without the endogenous time-varying higher moments that arise in equilibrium, foreign interest rate will be lower than domestic interest rate and that the UIP coefficient would be exactly one. However, home households face a higher level of uncertainty than foreign households. Column 3 of Figure 8 shows that following a positive shock, the volatility of domestic log-SDF will be higher than that of foreign in the following periods. Since X-good denominated assets are more valuable than that of the foreign country, home displacement shock has a larger impact on foreign households than the impact of foreign displacement shock on home households. Taking u shocks from both countries into consideration, foreign households' uncertainty about future displacement impact is smaller than that of home country. This leads to a lower interest rate at home country[16]. In sum, the time-varying λ_t gives rise to the time-varying conditional distribution of the effective size of future u shocks. This, in turn, implies a time-varying volatility of SDF, which weakens the UIP. Depending on which effect dominates, the home interest rate could be either lower or higher than the foreign interest rate.

To quantify the failure of UIP in the model, we next estimate the standard UIP regression in simulated data. We initialize the model at the symmetric steady state and is simulated for 150 periods, repeated 10000 times. We use the last 50 periods for each simulated sample to perform the UIP regression. Figure 10 displays the distribution of UIP coefficients. Examining the figure, we observe that UIP is largely violated in the model: the sample average of the UIP coefficient closely aligns with its counterpart in the data.

Hassan, Mertens, and Wang (2024) argue that the difference in currency returns should arise mostly from interest rate differentials, as exchange rates are notoriously difficult to predict empirically Meese and Rogoff (1983). They point out that the inverse relationship between the mean and higher moments of log-SDFs in the international finance literature is inconsistent with the observed unpredictability of exchange rates.

In our model, the distribution of log-SDFs is highly skewed. Specifically, the states u_1 and u_2 differ mainly in their expectations of future innovation shocks, which could reflect early-stage

[16] For example, the foreign households expect a larger but less volatile displacement shock, as u shock on home country also leads to a big displacement effect. In contrast, domestic households expect a small but skewed distribution of u–only a big displacement shock at home has sizable impact.

learning during a technological revolution. As a result, although the two states exhibit different means of log-SDFs, exchange rate predictability remains challenging due to the infrequent nature of these shocks. This is also consistent with the pattern documented in Chahrour et al. (2024), who argue that shocks to the expectation of future TFP changes are important for resolving UIP puzzles.

5 Conclusion

In this paper, we first document a positive correlation between U.S. innovation and the growth of real dollar index. We show that exchange rate movements reconnect with relative changes in aggregate quantities, such as consumption and output growth, once wealth changes are accounted for. Moreover, we find that countries' wealth fluctuations are positively correlated with macroeconomic fundamentals, indicating that the underlying shocks driving these fluctuations are positive productivity shocks. Finally, we show that foreign institutional ownership of firms increases following the granting of significant patents.

We provide a quantitative general equilibrium model that successfully replicates these patterns, as well as the joint dynamics of exchange rates, consumption growth, trade flows, and stock returns. We introduce a minimal deviation to the standard endowment economy model: in addition to the standard endowment shock in each country, countries can now each experience displacive shocks that reallocate output among agents. This minimal deviation from the standard model is sufficient to generate the documented patterns.

Our calibrated model successfully replicates the first two moments of aggregate consumption and output growth, exchange rates, and stock returns while generating low and relatively smooth risk-free rates. Our model replicate the three key 'anomalies' in the exchange rate literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of the uncovered interest rate parity (UIP).

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Figures and Tables

Table 1: Dollar Index growth and U.S. Innovation

Depende Variable = Dollar Index Growth						
	1-Year	1-Year	1-Year	3-Year	3-Year	3-Year
KPSS/MKT	0.026**	0.035**	0.039**	0.046**	0.044**	0.060***
	(0.011)	(0.017)	(0.015)	(0.019)	(0.017)	(0.020)
Lagged Dollar Index	YES	NO	YES	YES	NO	YES
Lagged Innovation	NO	YES	YES	NO	YES	YES
Observations	49	49	49	47	47	47
R-squared	0.191	0.100	0.220	0.476	0.178	0.526

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e^{USD}_{t-s,t} = \alpha + \beta_1 Inno_{US,t-s,t} + \beta_2 X_{t-s} + \varepsilon_t$$

The sample period is 1974-2022. U.S. innovation is measured by the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Control variable X_{t-s} includes lagged innovation and lagged Dollar Index level at $t - s$. Both series are in logs. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors (in parentheses) are obtained using Newey-West with one/three period lag. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 2: Exchange Rate Growth, Wealth Growth and Consumption and Output Growth

Panel A. Real Exchange Rate Growth, Consumption Growth and Wealth Changes

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Wealth change	0.107***	0.103***	0.099***	0.108***	0.131***	0.106***	0.116***	0.094***	0.101***	0.101***	0.082***	0.114***
	(0.004)	(0.007)	(0.009)	(0.010)	(0.009)	(0.006)	(0.008)	(0.010)	(0.006)	(0.007)	(0.015)	(0.007)
Consumption growth	-0.020***	-0.010	-0.015*	-0.027**	-0.024**	-0.005	-0.027***	-0.001	-0.032***	-0.011	0.003	-0.022**
	(0.003)	(0.007)	(0.008)	(0.010)	(0.009)	(0.008)	(0.008)	(0.009)	(0.008)	(0.008)	(0.023)	(0.008)
Observations	420	49	49	31	25	25	49	25	49	42	27	49
R2	0.840	0.860	0.779	0.870	0.950	0.961	0.870	0.893	0.881	0.867	0.843	0.882

Panel B. Real Exchange Rate Growth, Output Growth and Wealth Changes

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Wealth change	0.103***	0.103***	0.096***	0.105***	0.129***	0.105***	0.109***	0.095***	0.100***	0.099***	0.102***	0.106***
	(0.004)	(0.007)	(0.008)	(0.010)	(0.008)	(0.006)	(0.008)	(0.011)	(0.007)	(0.007)	(0.014)	(0.007)
GDP growth	-0.016***	-0.011*	-0.015*	-0.039***	-0.019**	-0.007	-0.014*	-0.001	-0.026***	-0.010	-0.036	-0.007
	(0.004)	(0.006)	(0.008)	(0.013)	(0.007)	(0.005)	(0.008)	(0.010)	(0.007)	(0.010)	(0.021)	(0.008)
Observations	420	49	49	31	25	25	49	25	49	42	27	49
R2	0.832	0.865	0.811	0.862	0.950	0.965	0.849	0.878	0.874	0.863	0.860	0.856

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log wealth ratio and log consumption growth ratio:

$$\log e_t - \log e_{t-1} = \alpha + \beta_1 \Delta \log W_{t-1,t} + \beta_2 \Delta \log C_{t-1,t} + \gamma X_{t-1} + \varepsilon_t$$

where the vector of controls X_{t-1} includes lagged relative levels $\log e_{t-1}, \log W_{t-1}, \log C_{t-1}$. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with five periods lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Panel B repeats the analysis, replacing consumption with the country's GDP. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 3: Wealth changes, consumption growth and output growth

Panel A. Independent variable = consumption growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Consumption Changes	0.045***	0.035*	0.035**	0.047**	0.055**	0.084**	0.044**	0.049**	0.015	0.044**	0.139***	0.068***
	(0.010)	(0.017)	(0.015)	(0.019)	(0.021)	(0.034)	(0.017)	(0.022)	(0.021)	(0.020)	(0.025)	(0.021)
Observations	439	49	49	49	25	25	49	25	49	43	27	49
R-squared	0.190	0.165	0.171	0.269	0.470	0.270	0.232	0.262	0.146	0.227	0.610	0.262

Panel B. Independent variable = gdp growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
GDP Changes	0.035***	0.027*	0.013	0.056**	0.034*	0.042	0.034*	0.052**	0.011	0.030	0.141***	0.034*
	(0.010)	(0.016)	(0.018)	(0.022)	(0.019)	(0.027)	(0.019)	(0.023)	(0.018)	(0.023)	(0.028)	(0.020)
Observations	432	49	49	42	25	25	49	25	49	43	27	49
R-squared	0.144	0.147	0.079	0.210	0.399	0.157	0.157	0.257	0.157	0.165	0.561	0.135

Notes: Panel A of the table reports regression results of the growth of the log wealth ratio on the log consumption growth ratio.

$$\log W_t - \log W_{t-1} = \alpha + \beta \Delta \log x_{t-1,t} + \gamma X_{t-1} + \varepsilon_t$$

where $x = consumption$ and the vector of controls X_t includes lagged relative levels of both dependent and independent variables: $\log x_{t-1}, \log W_{t-1}$. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France, and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Panel B repeats the analysis with $x = GDP$. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 4: Innovation and Foreign Institutional Ownership in the US

	Dependent variable = Change in Foreign Institutional ownership								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
KPSS (Top 20%)	0.182***			0.138**			0.159**		
	(0.061)			(0.062)			(0.065)		
Cites (Top 20%)		0.221***			0.190***			0.210***	
		(0.045)			(0.049)			(0.056)	
KPST(Top 20%)			0.192**			0.170**			0.186**
			(0.067)			(0.071)			(0.066)
Firm Controls	No	No	No	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	No	No	No
Industry × Year	No	No	No	No	No	No	YES	YES	YES
Observations	67986	67986	67986	65761	65761	65761	64723	64723	64723
Adj R2	0.788	0.787	0.787	0.795	0.794	0.794	0.796	0.796	0.796
Within R2	0.386	0.384	0.384	0.399	0.398	0.398	0.368	0.367	0.367

Notes: This table reports regression coefficients (times 100) of :

$$\Delta \text{IO\_FOR}_{i,t,t+1} = \beta \log(inno)_{i,t-1,t} + \gamma X_{i,t} + \varepsilon_{i,t+1}. \tag{57}$$

The dependent variable $\Delta\text{IO\_FOR}_{i,t,t+1}$ is the change in foreign institutional ownership for firm i between t and $t+1$. The foreign institutional ownership data are from FactSet Lionshare database. The independent variable is the (log) number of important patents granted to firm i in the last year $t-1$, according to various innovation measures. When *inno* is equal to zero, we replace log(*inno*) with zero and add a dummy equal to one if *inno* is equal to 0, thereby preventing the removal of the observation from the data. The vector of control $X_{i,t}$ includes foreign institutional ownership at time t, $\text{IO\_FOR}_{i,t}$, firm and year fixed effects. In columns (4)-(6), we add firm size and sales (log) at $t-1$ as additional controls. In columns (7)-(9), we replace year fixed effects with industry × year fixed effects. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors in parentheses are clustered at the SIC industry and year level. $^{***}p < 0.01$,$^{**}p < 0.05$,$^{*}p < 0.1$.

Table 5: Exchange rate growth and U.S. Innovation

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Displacement	0.039***	0.047***	0.030**	0.029*	0.038	0.051*	0.034*	0.042	0.031	0.050***	0.057***	0.063***
	(0.011)	(0.017)	(0.011)	(0.017)	(0.028)	(0.026)	(0.018)	(0.026)	(0.021)	(0.016)	(0.019)	(0.019)
Observations	467	49	49	49	25	25	49	25	49	49	49	49
R-squared	0.194	0.297	0.248	0.286	0.250	0.297	0.222	0.262	0.105	0.237	0.278	0.230

Notes: The table reports regression results of the growth of log exchange rate on displacement shocks:

$$\log e_t - \log e_{t-1} = \alpha + \beta_1 Inno_{US,t} + \beta_2 X_{t-1} + \varepsilon_t$$

The sample period is 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Control variable X_{t-1} includes lagged innovation and lagged exchange rate at $t-1$. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 6: Inequality growth, wealth growth and exchange rate

Panel A. Exchange rate and inequality growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.017***	0.065***	-0.007	0.015	0.058	0.027	0.037**	0.093**	0.023	-0.004	0.008	0.014
	(0.005)	(0.021)	(0.017)	(0.023)	(0.064)	(0.029)	(0.017)	(0.038)	(0.022)	(0.013)	(0.012)	(0.019)
Observations	418	49	49	42	18	25	42	18	42	42	49	42
R2	0.124	0.412	0.138	0.189	0.202	0.176	0.302	0.385	0.080	0.094	0.161	0.114

Panel B. Wealth change and inequality growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.020***	0.072***	-0.008	0.007	0.041	0.049	0.035*	0.066	0.045*	0.001	0.012	0.009
	(0.005)	(0.023)	(0.019)	(0.025)	(0.060)	(0.032)	(0.019)	(0.045)	(0.024)	(0.014)	(0.020)	(0.021)
Observations	396	49	49	42	18	25	42	18	42	42	27	42
R2	0.106	0.428	0.101	0.092	0.166	0.242	0.139	0.315	0.207	0.162	0.149	0.117

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log income inequality growth ratio.

$$\log e_t - \log e_{t-1} = \alpha + \beta \Delta \log I_{t-1,t} + \gamma X_{t-1} + \varepsilon_t$$

where $\Delta \log I_{t-1,t}$ is the growth of the ratio of top 1% income share. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Panel B repeats the analysis with the dependent variable equal to the growth of wealth ratios. Exchange rate, consumption, and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 7: Moments used in Model Estimation

	Data	Model		
		Median	5%	95%
Aggregate Quantities				
Consumption growth, mean	0.014	0.015	0.009	0.020
Consumption growth, volatility	0.022	0.015	0.013	0.034
Output growth, mean	0.014	0.015	0.011	0.020
Output growth, volatility	0.021	0.014	0.013	0.017
Mean top 1% income share	0.158	0.220	0.156	0.297
Asset prices				
Risk-free rate, mean	0.016	0.023	-0.010	0.032
Risk-free rate, volatility	0.031	0.014	0.005	0.043
Excess stock returns, mean	0.037	0.036	0.014	0.098
Excess stock returns, volatility	0.252	0.115	0.055	0.280
Exchange rate, volatility	0.115	0.076	0.034	0.181
Correlations (regression slopes)				
Consumption growth and wealth growth	0.008	0.008	0.002	0.054
Output growth and wealth growth	0.005	0.002	-0.002	0.005
Wealth growth and inequality growth	0.020	0.053	-0.003	0.156
Bi-variate correlations (regression slopes)				
Exchange rate and				
—relative c-growth	-0.020	-0.013	-0.045	-0.005
—relative wealth growth	0.107	0.085	0.036	0.215
Exchange rate and				
—relative output-growth	-0.016	-0.009	-0.014	-0.003
—relative wealth growth	0.103	0.077	0.034	0.180
Correlations				
Consumption growth (H and F)	0.454	0.749	0.199	0.904
Output growth (H and F)	0.596	0.845	0.036	0.215
Stock Returns (H and F)	0.598	0.148	-0.228	0.628
Trade surplus (as % of output) growth and c-growth	-0.299	-0.165	-0.905	0.255
Uncovered Interest Parity				
UIP slope	-0.215	-0.224	-4.366	1.915
Dollar index growth and US innovation, correlation	0.349	0.431	-0.621	0.751

Notes: This table reports both empirical moments computed using the G-7 & G-10 data set and simulated moments from the model. All the parameters are estimated as in Table 8.

Table 8: Parameter Estimates

Description	Symbol	Value	SE
Preferences:			
Home bias	α	0.988	0.012
Preference for own consumption	h	0.187	0.646
Subjective discount rate	β	1.058	0.156
Risk aversion	γ	6.352	2.546
Elasticity of intertemporal substitution	ψ	1.643	1.650
Death	ξ	0.079	0.140
Endowments:			
Displacement shock productivity	δ	0.202	0.064
Measure of projects-receiver	π	0.107	0.675
Mean of output growth	μ	0.012	0.008
Displacement shock low state	u_1	0.001	0.029
Displacement shock high state	u_3	0.143	0.099
Persistence of displacement shock			
— low state persistence	p	0.930	0.054
— high state persistence	q	0.831	0.132
Volatility of neutral shock	σ_e	0.013	0.002
Technology spillover	ρ_u	0.711	0.132
Correlation of neutral shock	ρ_e	0.862	0.160

Notes: This table reports the estimated parameters of the model. See the main text and the Appendix D for details on the estimation of the model.

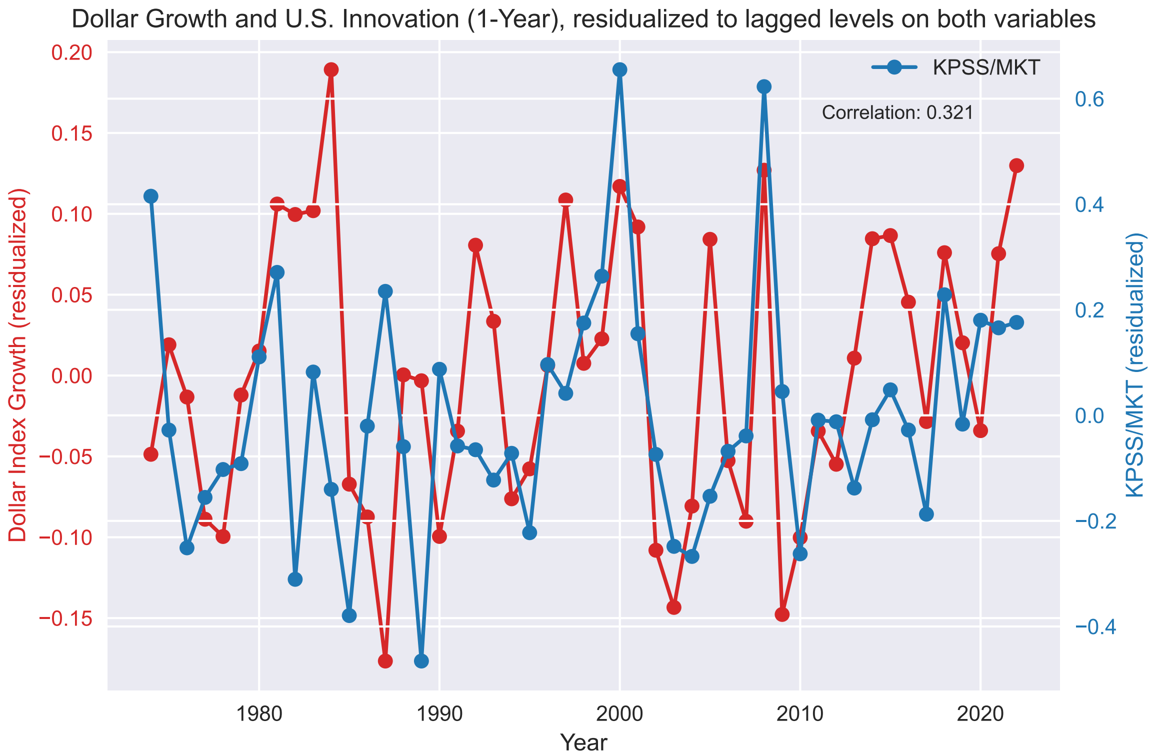


Figure 1: This figure plots the dollar index growth against U.S. innovation from 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Both series are in logs. The log growth is computed over 1-year. Both series are residualized to the lagged levels, corresponding to the regression specification in (1).

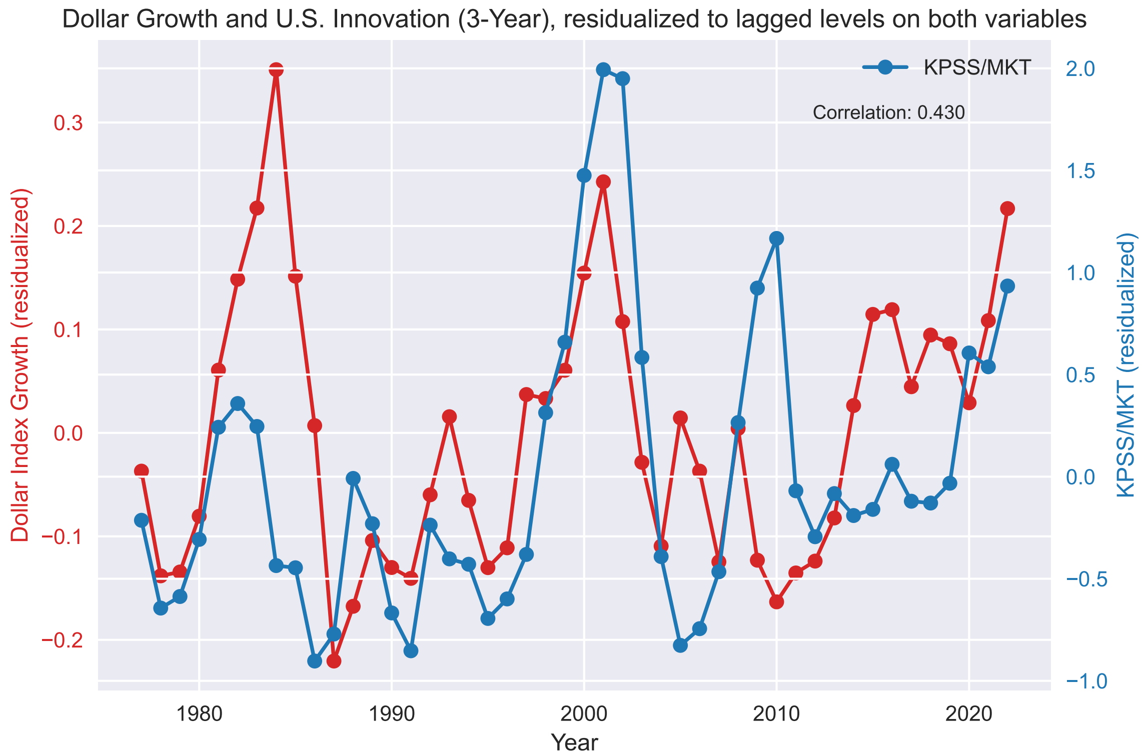
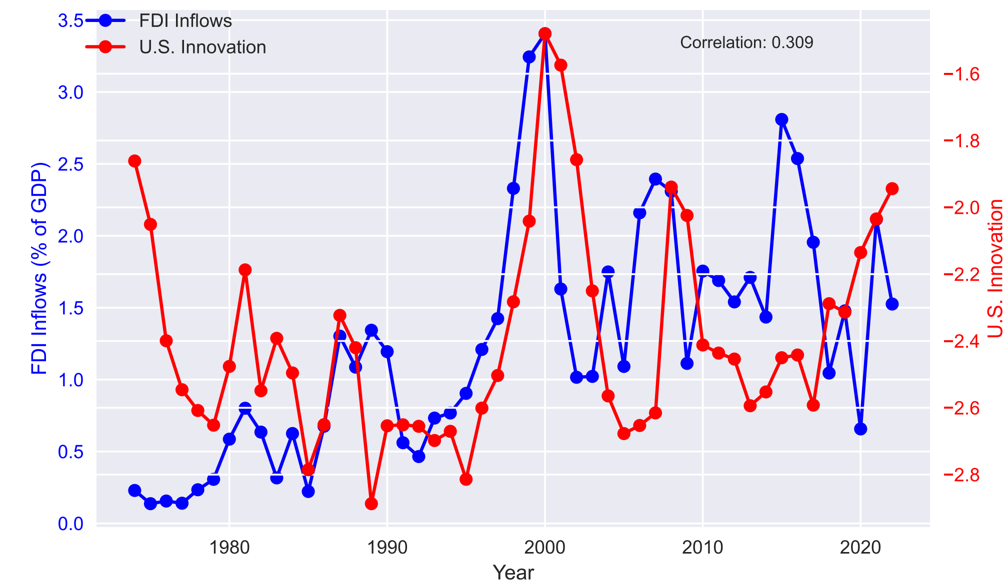


Figure 2: This figure plots the dollar index growth against U.S. innovation from 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Both series are in logs. The log growth is computed over 3-year window. Both series are residualized to the lagged levels, corresponding to the regression specification in (1).

Panel A. U.S. Innovation and U.S. FDI Inflows



Panel B. U.S. Innovation and U.S. Portfolio Equity Inflows

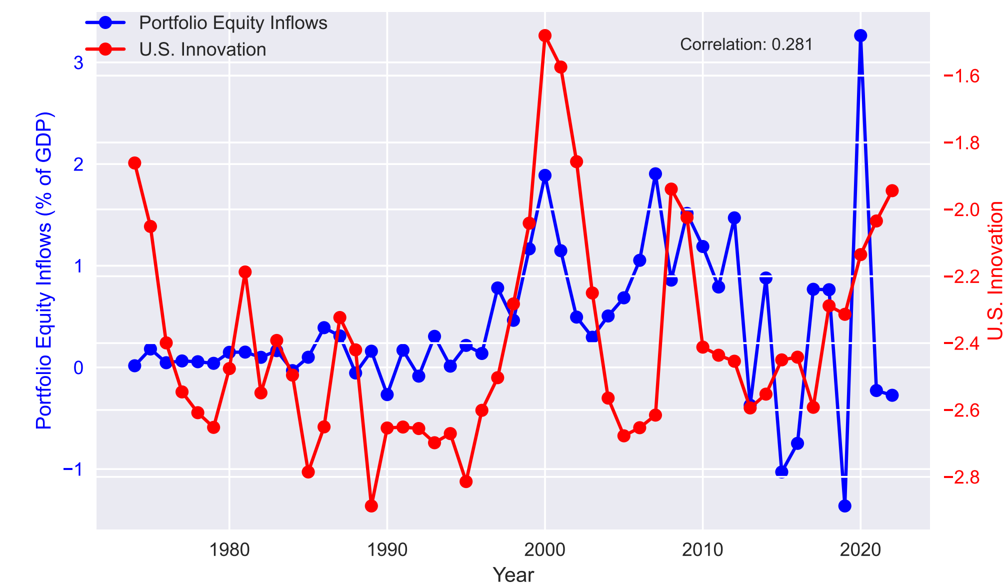


Figure 3: This figure plots the U.S. innovation index and foreign direct investment inflows (in Panel A) and portfolio equity inflows (in Panel B) in the U.S. The U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The aggregate FDI inflows and portfolio equity inflows are obtained from the World Bank. See A for details.

The real dollar index and the US innovation (Alt. Displacement shocks)

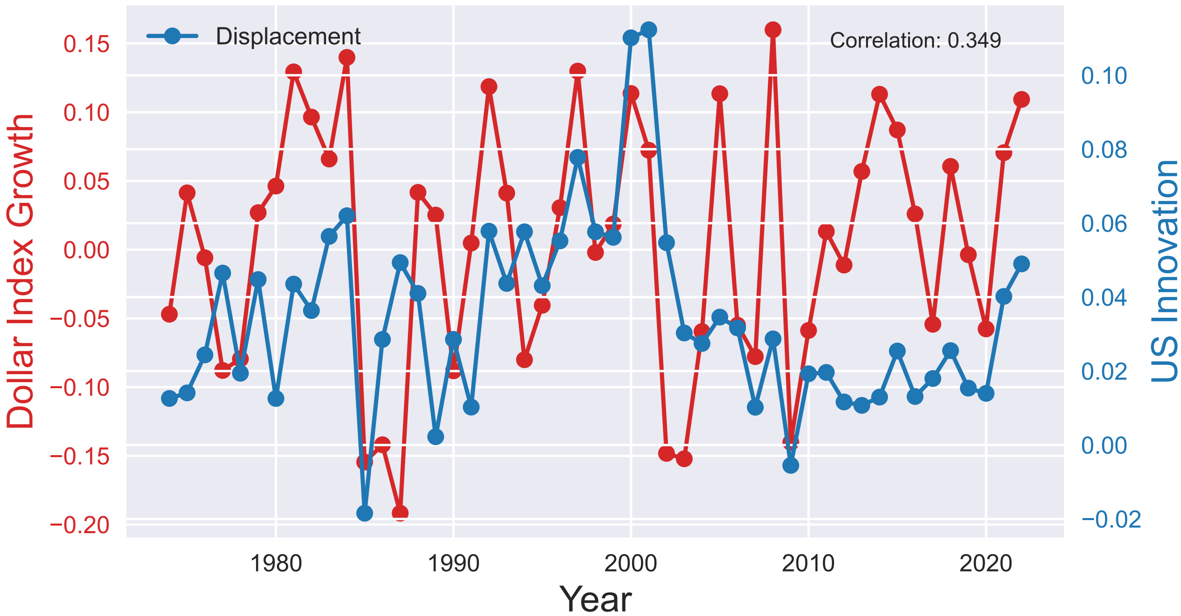


Figure 4: This figure plots the growth of the real dollar index (red) and the alternative US displacement series (blue) constructed in Section 2.5.1 that is based on the difference between the aggregate market capitalization growth and the returns from holding the market portfolio. The dollar index in red is the equal-weighted real dollar indexes.

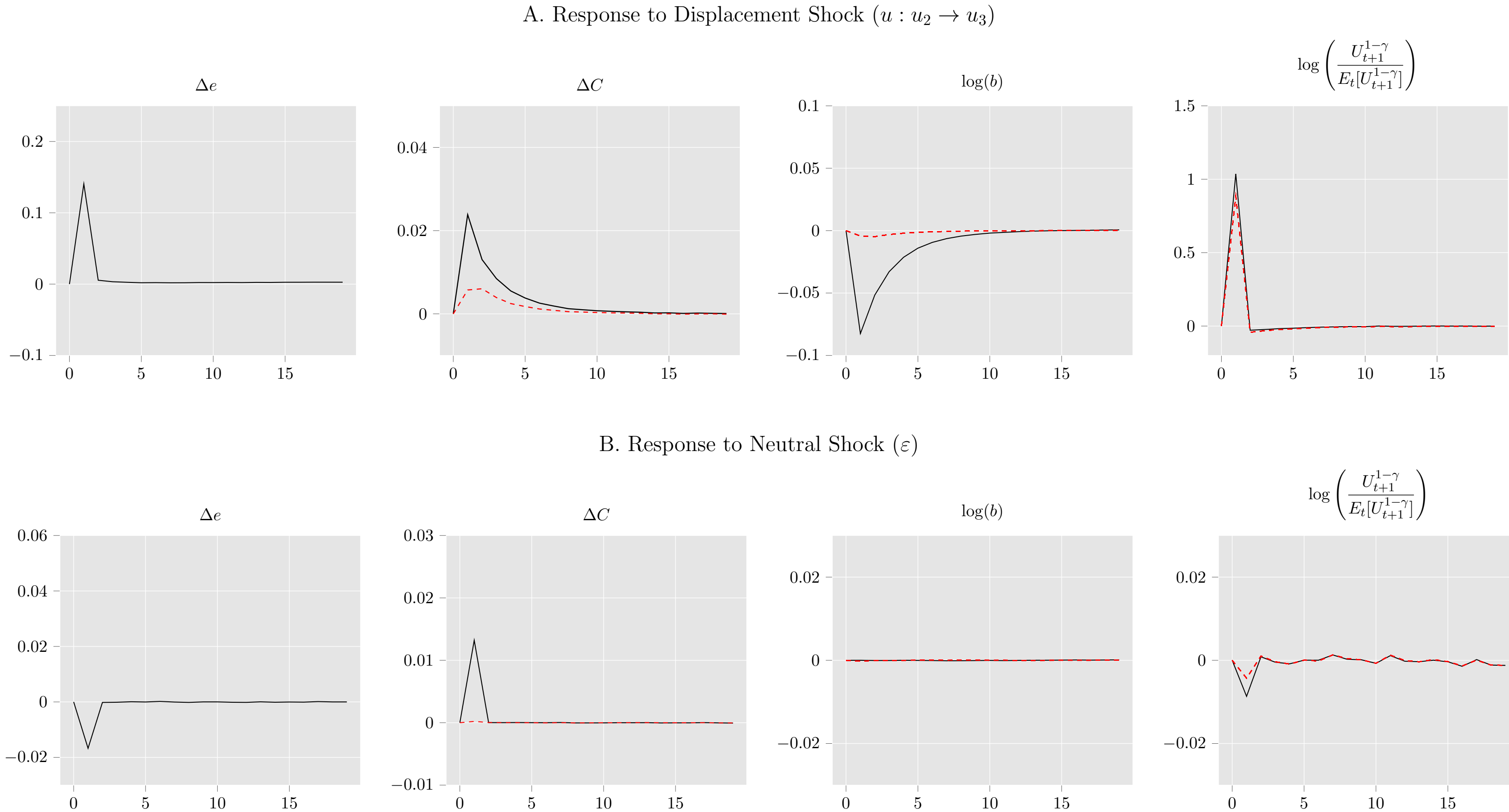


Figure 5: This figure plots the impulse response of variables to a shock to the home country (u in Panel A and ε in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 8. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

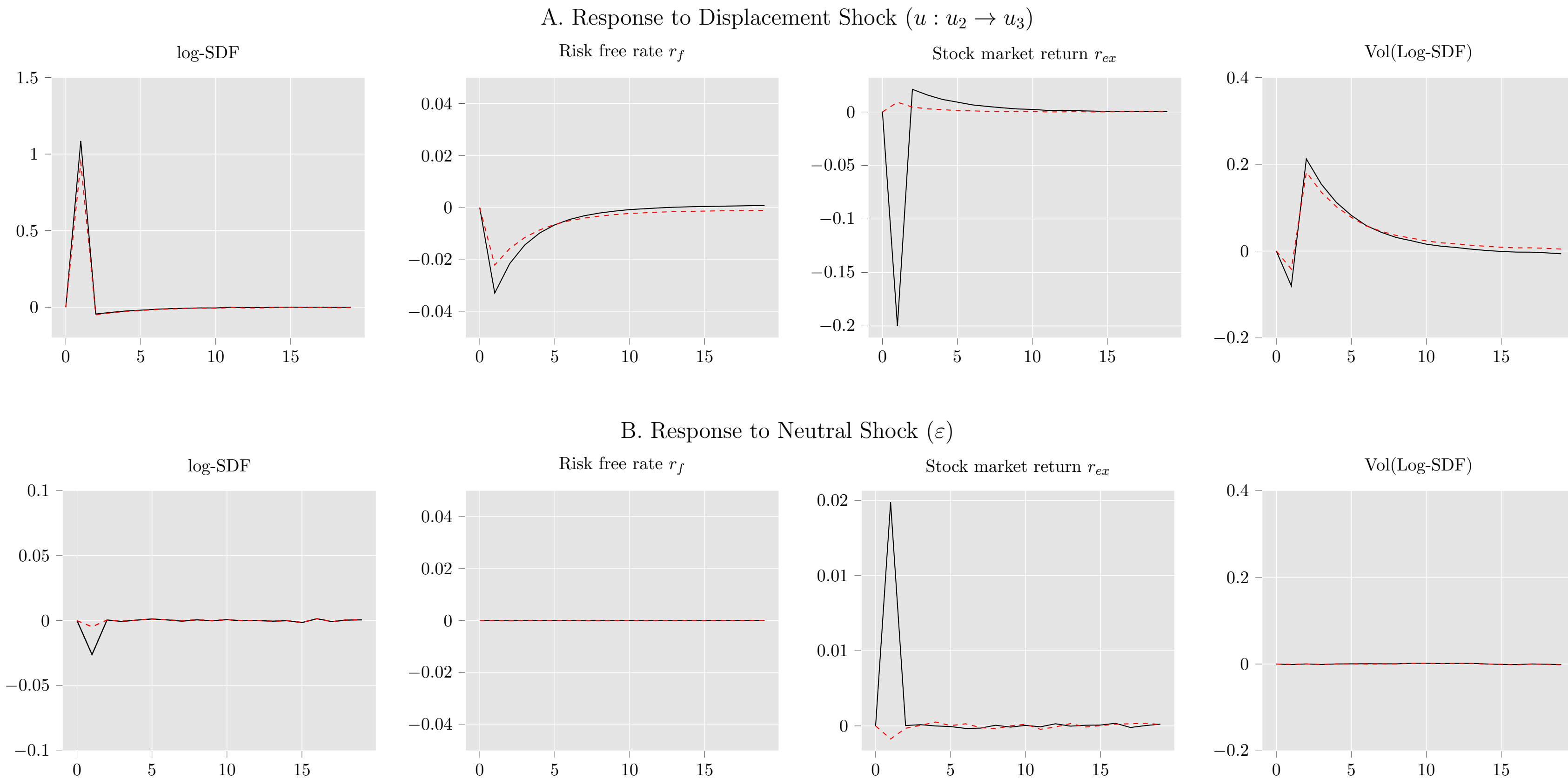


Figure 6: This figure plots the impulse response of variables to a shock to the home country (u in Panel A and ε in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 8. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

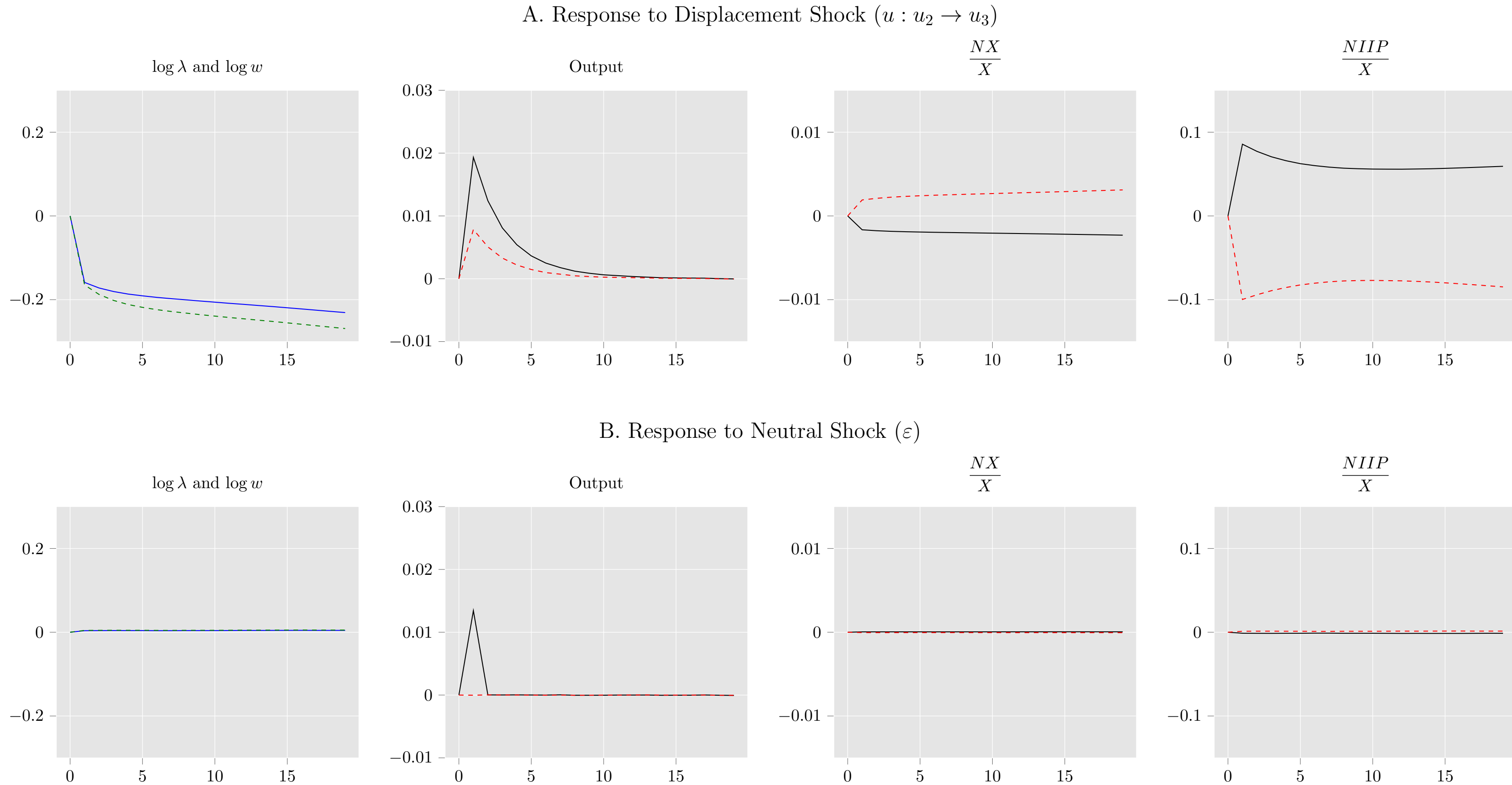


Figure 7: This figure plots the impulse response of variables to a shock to the home country (u in Panel A and ε in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 8. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

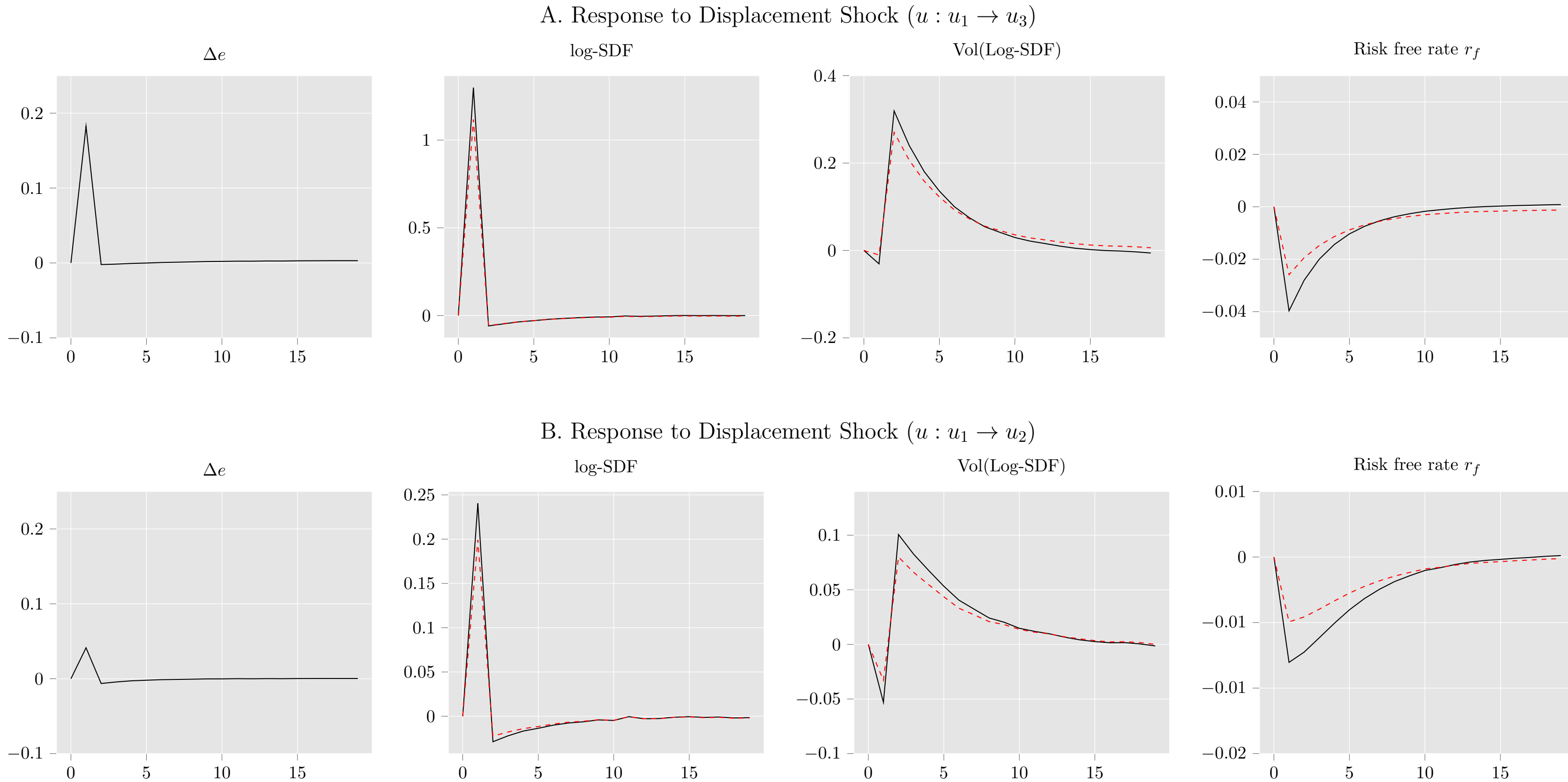


Figure 8: This figure plots the impulse response of variables to a shock to the home country ($u_1 \to u_3$ in Panel A and $u_1 \to u_2$ in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 8. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse responses are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

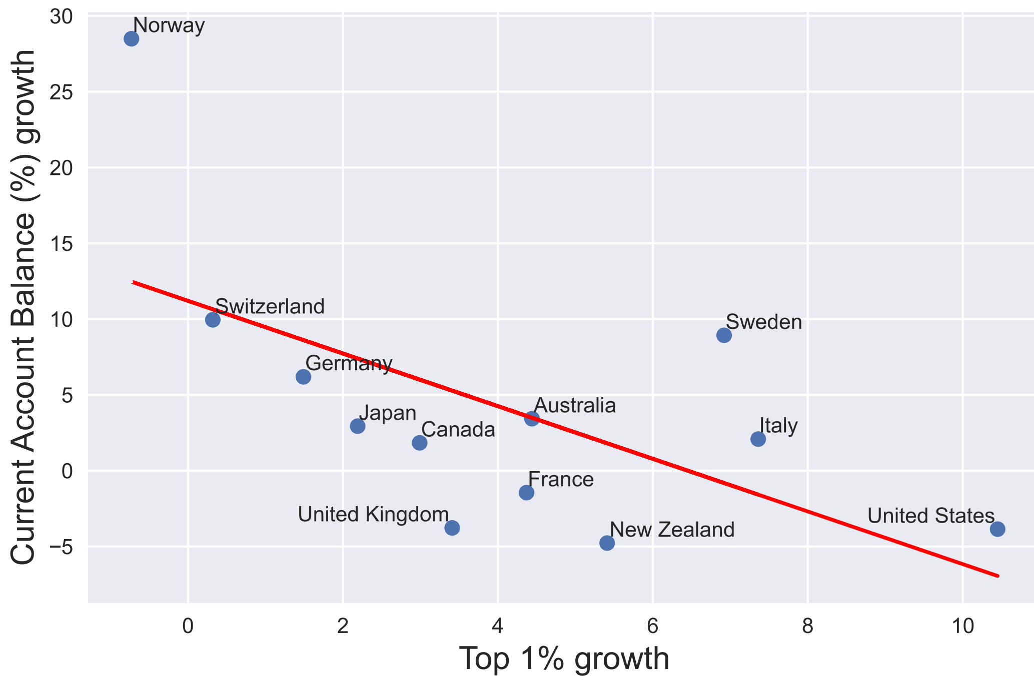


Figure 9: This figure plots the changes between top 1% income share and current account balance (%) from 1980-2022. Income inequality data is from the world inequality database, and the current account balance data is from the world economic outlook database.

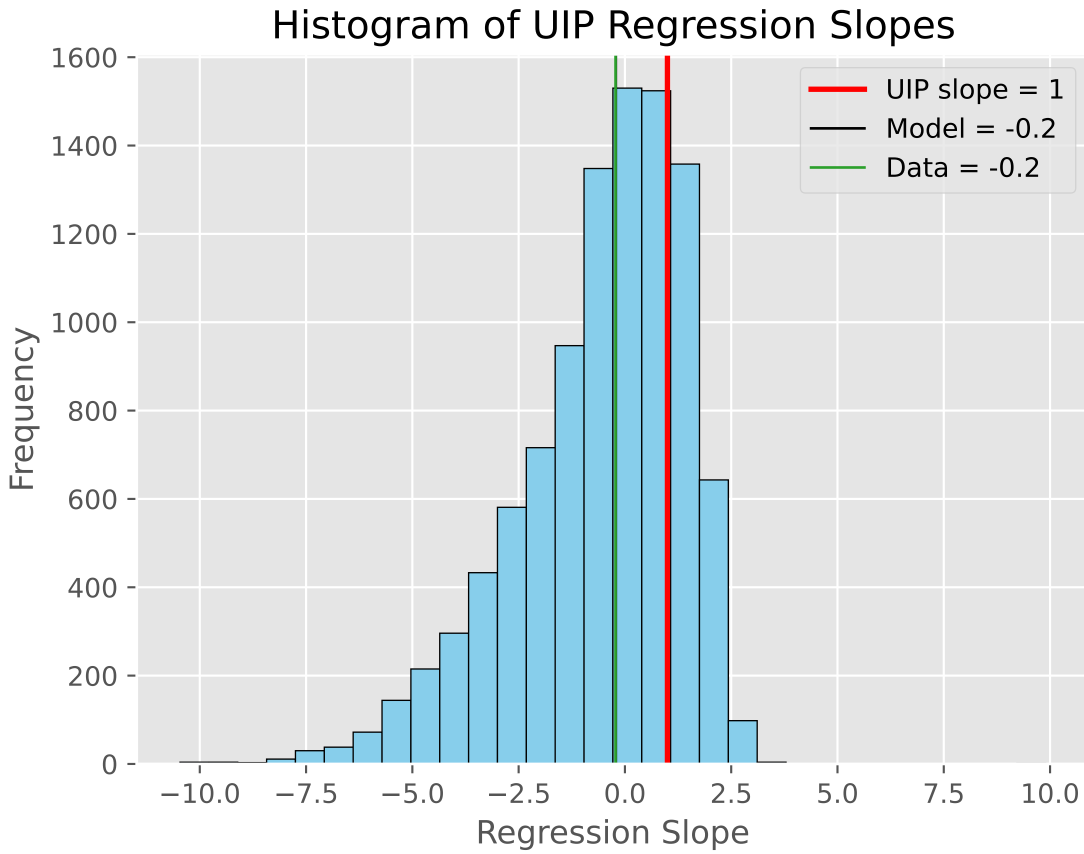


Figure 10: This figure shows the histogram of UIP regression slopes from individual simulations. The calibrated model is simulated 10,000 times, each spanning 150 periods, with UIP regressions performed using the last 50 years of each simulation. The black line represents the mean of the estimated UIP coefficients across all simulations, while the blue line represents the estimate from the data.

Appendix

A Solution to the Simplified Model

A.1 Representative agents

A.1.1 The representative agent in each country

First, we show that within a country, finding optimal solutions for heterogeneous agents is equivalent to finding optimal solution for a representative agent.

In each country, even though agents are heterogeneous in their wealth, because of homethetic preference consumption-wealth ratios are equalized.

Consider H country for example, we define the representative agent as

$$U_t^H = \int_{i \in [0,1]} U_{i,t} w_t^{i,H}$$

where $U_{i,t}$ and $w^{i,t}$ are the utility and wealth share of household i. That is, the representative agent takes the country-level endowment and the wealth distribution as given and maximizes the wealth-weighted utility.

Because all agents within a country are solving the same optimization problem up to their wealth, so is the wealth-weighted representative agent. Put differently, the representative agent behaves the same way as the individual agent, but scaled up to a wealth that is equal to the country's aggregate wealth. Thus, solving for the equilibrium solutions for heterogeneous agents within a country is equivalent to finding the optimal solution for the representative agent.

Denote C_t^c as the country-level aggregate consumption, the utility of representative agent can be written as

$$U_t^H = \lambda_t^H U(C_t^H)$$

Where $U(x)$ is the utility function for individual household – $U(x) = \log(x)$ in this case. That is, the utility of representative agent is proportional to an fictitious agent who consumes country-level aggregate consumption. The time-vaying scaling factor λ_t^H reflects the change of wealth distribution $w_t^{i,c}$ within the country. If market is complete, wealth distribution is invariant and λ_t^H would be a constant.

Now the equilibrium allocation problem reduces to a problem with two (representative) agents and incomplete markets.

A.1.2 Aggregation with log preference

The H's representative agent's utility can be written as

$$U_t^H = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s^H$$

With incomplete markets, the usual construction of a planner's utility as a weighted sum, with constant weights, of individual representative utility function is not possible. Instead, we are going to employ a fictitious planner with stochastic weights.

This fictitious representative agent maximizes his utility subject to the resource constraints:

$$\begin{aligned}
\max_{\{x_t^H, y_t^F, x_t^F, y_t^F\}, t=0,1,2,..} \quad & \sum_t \beta^t \left(\log C_t^H + \lambda_t \log C_t^F\right) \\
\text{s.t.} \quad & x_t^F + x_t^F = X_t \\
& y_t^H + y_t^F = Y_t \\
& C_t^H = (x_t^H)^\alpha (y_t^H)^{1-\alpha} \\
& C_t^F = (x_t^F)^{1-\alpha} (y_t^F)^\alpha
\end{aligned}$$

where we have normalized the weight on the Home representative agent to be equal to one and assigned the weight λ to the foreign representative agent. λ_t is the marginal utilities of either good of the two countries.

A.2 Allocations

For concreteness, we focus on the exposition on the Home consumer. First, at each t, we derive the consumer's demands for X and Y goods, keeping overall consumption expenditure $\mathcal{C}_H$ fixed.

$$\max_{\{x_t^H, x_t^H\}} \alpha \log x_t^H + (1-\alpha) \log y_t^H \tag{58}$$

$$\text{s.t.} \quad p_{x,t} x_t^H + p_{y,t} y_t^H = \mathcal{C}_H \tag{59}$$

We obtain the following demands

$$x_t^H = \frac{\alpha \mathcal{C}_H}{p_{x,t}}, y_t^H = \frac{(1-\alpha)\mathcal{C}_H}{p_{y,t}} \tag{60}$$

The indirect utility function defined as $U_H(\mathcal{C}_H, p_{x,t}, p_{y,t})$ is then given by

$$U_H(\mathcal{C}_H, p_{x,t}, p_{y,t}) = \log(\mathcal{C}_H) + F(p_{x,t}, p_{y,t}) \tag{61}$$

Function F depends only on variables that are exogenous from the viewpoint of the consumer and therefore, because of the separability, it drops out the portfolio choice.

Hence, the optimization problem of consumer is equivalent to the single-good consumption-investment problem, with consumption expenditure $\mathcal{C}_H$ replacing the consumption. Importantly, it implies that the prices of individual goods $p_{x,t}, p_{y,t}$ do not pose a risk that the consumer desires to hedge.

With log preference, consumers have constant consumption-to-wealth ratio. Thus, the pareto weights λ_t is equal to the consumption expenditure ratio, which in turn is equal to the wealth ratio between two countries $\lambda_t = \frac{W_{F,t}}{W_{H,t}}$. Substituting the demand functions in the budge constraints, we get the allocations (62)-(65).

$$x_t^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_t} X_t \tag{62}$$

$$x_t^F = \frac{(1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_t} X_t \tag{63}$$

$$y_t^H = \frac{1-\alpha}{1-\alpha+\alpha\lambda_t} Y_t \tag{64}$$

$$y_t^F = \frac{\alpha\lambda_t}{1-\alpha+\alpha\lambda_t} Y_t. \tag{65}$$

A.3 SDFs and Asset Prices

SDF. Let $\mathcal{N}_{t+1}^c$ denote the set of all indices of agents in country c who receives worthless ideas at time $t+1$. In what follows, we will focus on the exposition on the Home consumer. By definition,

$$\frac{M_{t+1}^H}{M_t^H} = \beta \mathrm{E}\left(\frac{c_{t+1}^{i,H}}{c_t^{i,H}}\right)^{-1} = \beta\left(\frac{\int_{i\in\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i\in\mathcal{N}_{t+1}^H} dC_t^{i,H}}\right)^{-1} \tag{66}$$

where the first equation follows from the consumer's Euler equation and the second equation follows from the probability of receiving a profitable firm being zero. As a result, households' anticipated consumption growth coincides with the consumption growth of the cohort $\mathcal{N}_{t+1}^H$. Market clearing implies:

$$C_{t+1}^H = \int_{i\in\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} + \int_{i\notin\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} \tag{67}$$

Note that $\mathbf{1}_{i\notin\mathcal{N}_{t+1}^H} \times \mathbf{1}_{i\notin\mathcal{N}_t^H} = 0$ almost surely, so

$$\int_{i\in\mathcal{N}_{t+1}^H} dC_t^{i,H} = C_t^H \tag{68}$$

Combining (66)-(68) along with the allocation rules (62)-(65) we have that

$$\frac{M_{t+1}^H}{M_t^H} = \beta(\frac{X_{t+1}}{X_t})^{-\alpha}(\frac{Y_{t+1}}{Y_t})^{\alpha-1}\left(\frac{\alpha+(1-\alpha)\lambda_t}{\alpha+(1-\alpha)\lambda_{t+1}}\right)^{-\alpha}\left(\frac{\alpha\lambda_t+1-\alpha}{\alpha\lambda_{t+1}+1-\alpha}\right)^{\alpha-1}\left(1-\frac{\int_{i\notin\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i\in[0,1]} dC_{t+1}^{i,H}}\right)^{-1} \tag{69}$$

Note that with log preference, consumption bundles is proportional to consumption expenditure, which in turn is proportional to wealth. Therefore the last term can be written as

$$b_{H,t+1} = \frac{\int_{i\in\mathcal{N}_{t+1}^H} w_{t+1}^{i,H}}{\int_{i\in[0,1]} w_{t+1}^{i,H}} = \frac{\int_{i\in\mathcal{N}_{t+1}^H} d\mathcal{C}_{t+1}^{i,H}}{\int_{i\in[0,1]} d\mathcal{C}_{t+1}^{i,H}} = 1 - \frac{\int_{i\notin\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i\in[0,1]} dC_{t+1}^{i,H}} \tag{70}$$

Substituting back we obtain (27). Similarly, we can derive the SDF for foreign consumers.

$$\frac{M^F_{t+1}}{M^F_t} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{\alpha-1} \left(\frac{Y_{t+1}}{Y_t}\right)^{-\alpha} \frac{1}{b_{F,t+1}} \frac{\lambda_t}{\lambda_{t+1}} \left(\frac{\alpha+(1-\alpha)\lambda_t}{\alpha+(1-\alpha)\lambda_{t+1}}\right)^{\alpha-1} \left(\frac{\alpha\lambda_t+1-\alpha}{\alpha\lambda_{t+1}+1-\alpha}\right)^{-\alpha} \tag{71}$$

Asset Prices. Let us first focus on the stock market in Home country. The SDF can be used to price the risky stocks by no arbitrage:

$$S^H_t = p_{x,t}X_t + \mathrm{E}_t[\sum_{t+1}^{T} M^H_s p_{x,s} X_s e^{-\sum_{t+1}^{s} u^H_j}] \tag{72}$$

Note that M^H_t is the SDF using consumption bundles of the home country, if we define ζ^H_t as the SDF using local goods of home country, then we have

$$M^H_t p_{x,t} = \zeta^H_t$$

Note that the first-order condition of X-good for consumers gives:

$$\zeta^H_s = \beta^{s-t}\frac{\alpha}{c^H_{x,s}} \tag{73}$$

where $c^H_{x,s}$ is the total consumption of X goods by the households who have not received any profitable firms between $t+1$ and s, which has a probability of one. Therefore,

$$c^H_{x,s} = \frac{\alpha}{\alpha+(1-\alpha)\lambda_s} X_s \Pi^s_{t+1} b_{H,s} \tag{74}$$

Substituting (73) and (74) into (72), we have

$$S^H_t = p_{x,t}X_t \mathrm{E}_t[\sum_{t+1}^{T} \beta^{s-t} \frac{\Pi^s_{t+1}\frac{1}{b_{H,s}}\left(\alpha+(1-\alpha)\lambda_s\right)}{\frac{1}{b_{H,t}}\left(\alpha+(1-\alpha)\lambda_t\right)} e^{-\sum_{t+1}^{s} u^H_j}] + p_{x,t}X_t \tag{75}$$

The derivation for foreign country's stock market is similar.

A.4 The Change of Wealth Distribution

In A.2 we show that the optimization of consumer is equivalent to the single-good consumption -investment problem, with consumption expenditure $\mathcal{C}$ replacing the consumption. Moreover, the consumers do not hedge the prices of individual goods $p_{x,t}, p_{y,t}$.

This implies that the consumers in home and foreign are solving the same portfolio-choice problem. As a result, their optimal portfolios and wealth growth are the same across different states. Hence, the wealth ratio at $t+1$ is given by

$$\lambda_{t+1} = \frac{\int_{i\in[0,1]} w^{i,F}_{t+1}}{\int_{i\in[0,1]} w^{i,H}_{t+1}} = \frac{\int_{i\in\mathcal{N}^F_{t+1}} w^{i,F}_{t+1} + \int_{i\notin\mathcal{N}^F_{t+1}} w^{i,F}_{t+1}}{\int_{i\in\mathcal{N}^H_{t+1}} w^{i,H}_{t+1} + \int_{i\notin\mathcal{N}^H_{t+1}} w^{i,H}_{t+1}} \tag{76}$$

Note that the total value of profitable firms at $t+1$ is related to the displacement shocks u^H_{t+1}, u^F_{t+1}. From 11 and 12 it follows that the total value of new firms are:

$$\int_{i\notin\mathcal{N}^H_{t+1}} w^{i,H}_{t+1} = S^H_{t+1}(1-e^{-u^H_{t+1}}) \tag{77}$$

$$\int_{i\notin\mathcal{N}^F_{t+1}} w^{i,F}_{t+1} = S^F_{t+1}(1-e^{-u^F_{t+1}}) \tag{78}$$

And the total value of old firms is

$$\int_{i\in\mathcal{N}^F_{t+1}} w^{i,F}_{t+1} + \int_{i\in\mathcal{N}^H_{t+1}} w^{i,H}_{t+1} = S^H_{t+1}e^{-u^H_{t+1}} + S^F_{t+1}e^{-u^F_{t+1}} \tag{79}$$

Because the consumers in home and foreign hold the same portfolio, the wealth ratio for the households that do not receive new firms are the same at t and $t+1$. Hence,

$$\lambda_t = \frac{\int_{i\in[0,1]} w^{i,F}_t}{\int_{i\in[0,1]} w^{i,H}_t} = \frac{\int_{i\in\mathcal{N}^f_{t+1}} w^{i,F}_t}{\int_{i\in\mathcal{N}^h_{t+1}} w^{i,H}_t} \tag{80}$$

Combining (76)-(80) we obtain

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(S^H_{t+1}e^{-u^F_{t+1}} + S^F_{t+1}e^{-u^F_{t+1}}\right) + \frac{1}{\lambda_t}S^F_{t+1}(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}\left(S^H_{t+1}e^{-u^F_{t+1}} + S^F_{t+1}e^{-u^F_{t+1}}\right) + S^H_{t+1}(1-e^{-u^H_{t+1}})} \tag{81}$$

A.5 Approximation

We now derive the approximate analytical solutions near the long-term steady state. That is, when $\lambda_t = 1$ and when u^H_{t+1}, u^F_{t+1} are small.

By symmetry, when $\lambda_t = 1$ the price-dividend ratio of the stock markets are the same. Let us denote this ratio as C_{pd}, i.e.,

$$\left(\frac{S^H_t}{p_{x,t}X_t}\right)_{\lambda_t=1} = \left(\frac{S^F_t}{p_{y,t}Y_t}\right)_{\lambda_t=1} = C_{pd} \tag{82}$$

Using the price ratio relation given by (24), we can rewrite (81) as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}pd^H_{t+1} + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}e^{-u^F_{t+1}}pd^F_{t+1}\right) + \frac{1}{\lambda_t}\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}pd^H_{t+1} + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}e^{-u^F_{t+1}}pd^F_{t+1}\right) + (1-e^{-u^H_{t+1}})pd^H_{t+1}} \tag{83}$$

where pd^c_{t+1} is the price-dividend ratio of the stock market in country $c \in \{H, F\}$ at $t+1$. To further simplify, we use the fact that u^H_{t+1}, u^F_{t+1} are small so that $pd^c_{t+1} \approx C_{pd}$ for $c \in \{H, F\}$. Denote the total wealth of stock market as $\bar{W} = W_H + W_F$, we make the following observation:

$$\bar{W} = S^H_{t+1} + S^F_{t+1} \tag{84}$$

$$\frac{S^F_{t+1}}{S^H_{t+1}} \approx \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} \tag{85}$$

The second equation is because $pd^c_{t+1} \approx C_{pd}$. It follows that

$$S^H_{t+1} = \frac{\alpha+(1-\alpha)\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W}, \qquad S^F_{t+1} = \frac{1-\alpha+\alpha\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W} \tag{86}$$

The dynamics of wealth distribution can thus be written as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) + \frac{1}{\lambda_t}(1-\alpha+\alpha\lambda_{t+1})(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) + (\alpha+(1-\alpha)\lambda_{t+1})(1-e^{-u^H_{t+1}})} \tag{87}$$

Denote the common terms in both the numerator and denominator as

$$B = \left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) \tag{88}$$

Some algebra gives

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}B + \frac{1-\alpha}{\lambda_t}(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}B + \alpha(e^{-u^F_{t+1}} - e^{-u^H_{t+1}}) + (1-\alpha)\lambda_{t+1}(1-e^{-u^H_{t+1}})} \tag{89}$$

To progress further, we use the result from A.2 that consumers in both countries have the same portfolios and therefore the same wealth growth. At $t+1$, the wealth of households in both countries who do not receive profitable firms is

$$\int_{i\in N^h_{t+1}} w^{i,H}_{t+1} + \int_{i\in N^f_{t+1}} w^{i,F}_{t+1} = \left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) \tag{90}$$

$$\frac{1}{1+\lambda_{t+1}}\bar{W} \tag{91}$$

Because consumers hold the same portfolio, we have

$$\frac{\int_{i\in N^H_{t+1}} w^{i,H}_{t+1}}{\int_{i\in N^F_{t+1}} w^{i,F}_{t+1}} = \frac{\int_{i\in[0,1]} w^{i,H}_t}{\int_{i\in[0,1] w^{i,F}_{t+1}}} = \frac{1}{\lambda_t} \tag{92}$$

Therefore

$$\int_{i\in N^H_{t+1}} w^{i,H}_{t+1} = \frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right)\frac{1}{1+\lambda_{t+1}}\bar{W} \tag{93}$$

On the other hand, by definition

$$\int_{i\in[0,1]} w_{t+1}^{i,H} = \int_{i\in N_{t+1}^H} w_{t+1}^{i,H} + \int_{i\notin N_{t+1}^H} w_{t+1}^{i,H} \tag{94}$$

so that

$$\begin{aligned}\int_{i\in N_{t+1}^H} w_{t+1}^{i,H} &= \int_{i\in[0,1]} w_{t+1}^{i,H} - \int_{i\notin N_{t+1}^H} w_{t+1}^{i,H} \\ &= \frac{1}{1+\lambda_{t+1}}\bar{W} - (1-e^{-u_{t+1}^H})\frac{\alpha+(1-\alpha)\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W} \end{aligned}\tag{95}$$

Substituting (93) and (95) into (89), after some algebra we get

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1-\alpha+\alpha e^{-u_{t+1}^H}+(1-\alpha)(1-e^{-u_{t+1}^F})\frac{1}{\lambda_t}}{1-\alpha+\alpha e^{-u_{t+1}^F}+(1-\alpha)\lambda_t(1-e^{-u_{t+1}^H})} \tag{96}$$

Using the fact that $e^x \approx 1+x$ and $\lambda_t = 1$, we have

$$\begin{aligned}\Delta\log\lambda_{t+1} &= \log\frac{1-\alpha+\alpha e^{-u_{t+1}^H}+(1-\alpha)(1-e^{-u_{t+1}^F})\frac{1}{\lambda_t}}{1-\alpha+\alpha e^{-u_{t+1}^F}+(1-\alpha)\lambda_t(1-e^{-u_{t+1}^H})} \\ &\approx u_{t+1}^F - u_{t+1}^F \end{aligned}\tag{97}$$

To get the approximate expression for log growth of consumption ratio, substituting (62)-(65) into (17), we have

$$\Delta c_{t+1}^H - \Delta c_{t+1}^F = (2\alpha-1)[\Delta\log X_{t+1} - \Delta\log Y_{t+1} + \Delta\log\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}] - \Delta\log\lambda_{t+1}$$

note that $\lambda_{t+1} \approx 1+\Delta\log\lambda_{t+1}$, so we have

$$\Delta\log\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} \approx (2\alpha-1)\Delta\log\lambda_{t+1} \tag{98}$$

substituting back we get (35). The derivation for log growth of output ratio is straightforward from definitions.

B Appendix Figures and Tables

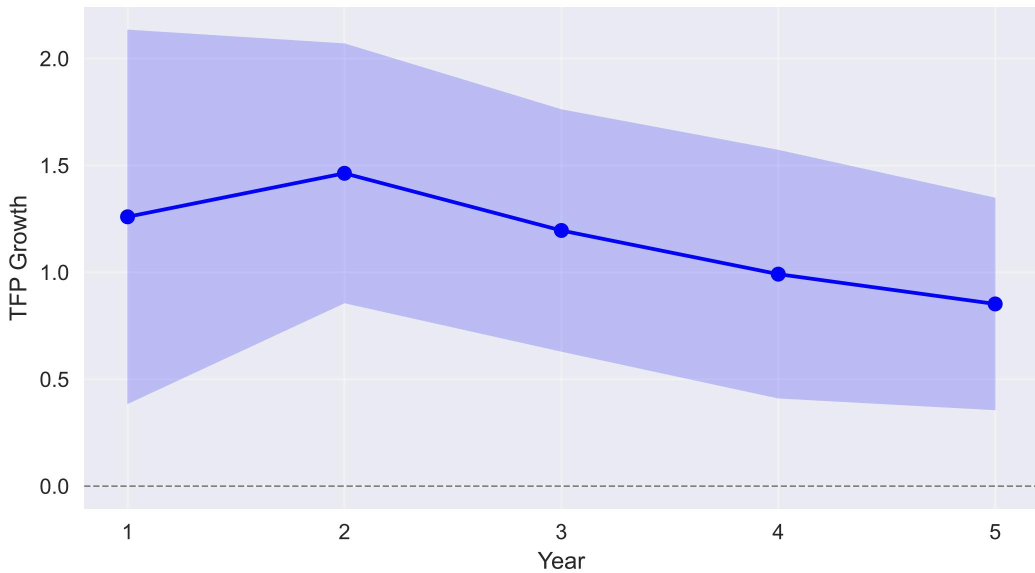


Figure B.1: This figure plots the coefficients of TFP growth $\frac{\beta_s}{s}$ from t to $t+s$ (for $s = 1, 2, 3, 4, 5$) regressed on U.S. innovation from t to $t+1$. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. TFP growth is based on annual utilization-adjusted U.S. TFP Fernald (2014).

Table B.1: Dollar Index growth and U.S. Innovation

Depende Variable = Dollar Index Growth						
	1-Year	1-Year	1-Year	3-Year	3-Year	3-Year
KPSS(avg)	0.028**	0.038	0.031	0.051**	0.114***	0.106***
	(0.012)	(0.025)	(0.023)	(0.022)	(0.021)	(0.017)
Lagged Dollar Index	YES	NO	YES	YES	NO	YES
Lagged Innovation	NO	YES	YES	NO	YES	YES
Observations	49	49	49	47	47	47
R-squared	0.188	0.053	0.188	0.493	0.239	0.581

Notes: The table reports regression results of the growth of log dollar index on U.S. innovation:

$$\Delta \log e^{USD}_{t-s,t} = \alpha + \beta_1 Inno_{US,t-s,t} + \beta_2 X_{t-s} + \varepsilon_t$$

The sample period is 1974-2022. U.S. innovation is measured as the log of the ratio of the average real value of patent each year (Kogan et al. (2017)) . The dollar Index is computed as an equal weighted average real value of the US dollar against the group of currencies in our sample. Control variable X_{t-s} includes lagged innovation and lagged Dollar Index level at $t-s$. Both series are in logs. The sample consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. Standard errors (in parentheses) are obtained using Newey-West with one/three period lag. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table B.2: Exchange Rate Growth, Wealth Growth and Consumption Growth

	(1)	(2)	(3)
C growth	-0.018***	-0.002	-0.001
	(0.003)	(0.002)	(0.002)
Nominal FX growth		0.115***	0.115***
		(0.001)	(0.002)
Wealth($) growth	0.109***		
	(0.004)		
Wealth(LCU) growth			-0.032
			(0.040)
Observations	439	439	439
R-squared (within)	0.846	0.976	0.977

Notes: The table reports regression results of the growth of log exchange rate on log wealth ratio and log consumption growth ratio:

$$\log e_{t+1} - \log e_t = \alpha + \beta_1 \Delta \log W_{t,t+1} + \beta_2 \Delta \log C_{t,t+1} + \gamma X_t + \varepsilon_{t+1}$$

where the vector of controls X_t includes lagged relative levels $\log e_t, \log W_t, \log C_t$. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with five periods lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table B.3: Exchange rate growth and U.S. Innovation

Dependent Variable = FX Growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
KPSS/MKT	0.032**	0.045**	0.039***	0.012	0.025	0.017	0.030	0.010	-0.001	0.045**	0.057**	0.053**
	(0.016)	(0.021)	(0.014)	(0.022)	(0.039)	(0.038)	(0.022)	(0.036)	(0.025)	(0.021)	(0.024)	(0.025)
Observations	467	49	49	49	25	25	49	25	49	49	49	49
R-squared	0.145	0.215	0.251	0.207	0.192	0.197	0.219	0.266	0.102	0.160	0.221	0.131

Notes: The table reports regression results of the growth of log dollar exchange rate on U.S. innovation:

$$\log e_{t+1} - \log e_t = \alpha + \beta_1 Inno_{US,t+1} + \beta_2 X_t + \varepsilon_{t+1}$$

The sample period is 1974-2022. U.S. innovation is measured as the log of the ratio of the total value of patents each year (Kogan et al. (2017)) to the total market value. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Control variable X_t includes lagged innovation and lagged exchange rate at t. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Data on income inequality and wealth is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table B.4: Inequality growth and exchange rate

Panel A. Exchange rate and inequality growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.019***	0.056***	-0.010	0.023	0.049	0.043	0.039**	0.097**	0.025	-0.002	0.013	0.010
	(0.005)	(0.017)	(0.017)	(0.025)	(0.062)	(0.028)	(0.017)	(0.040)	(0.023)	(0.014)	(0.013)	(0.017)
Observations	418	49	49	42	18	25	42	18	42	42	49	42
R2	0.131	0.399	0.132	0.203	0.188	0.233	0.303	0.405	0.084	0.093	0.152	0.122

Panel B. Wealth change and inequality growth

	Panel	AUS	CAN	CHE	DEU	FRA	GBR	ITA	JPN	NOR	NZL	SWE
Inequality growth	0.022***	0.064***	-0.013	0.016	0.035	0.070**	0.037*	0.069	0.045*	0.002	0.019	0.003
	(0.006)	(0.018)	(0.019)	(0.027)	(0.057)	(0.030)	(0.019)	(0.046)	(0.025)	(0.015)	(0.020)	(0.020)
Observations	396	49	49	42	18	25	42	18	42	42	27	42
R2	0.107	0.416	0.124	0.108	0.164	0.355	0.142	0.331	0.198	0.154	0.122	0.112

Notes: Panel A of the table reports regression results of the growth of log exchange rate on log income inequality growth ratio.

$$\log e_t - \log e_{t-1} = \alpha + \beta \Delta \log I_{t-1,t} + \gamma X_{t-1} + \varepsilon_t$$

where $\Delta \log I_{t-1,t}$ is the growth of the ratio of top 0.1% income share. The sample period is 1974-2022. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and we report Driscoll and Kraay (1998) standard errors in parentheses. Panel B repeats the analysis with the dependent variable equal to the growth of wealth ratios. Data on income inequality and wealth is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Online Appendix for

"Tech Dollars and Exchange Rates"

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A Data Source

The consumption, GDP and net export data come from the World Bank. We use households final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real. We use end-of-period exchange rate data from the International Financial Statistics (IMF). Sample period is 1974-2022.

Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.

Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized inflation using annual changes in CPI. The interest rates series for New Zealand and Switzerland starts from 1978 and 1980, respectively. For the rest, the sample period is 1974-2022.

Data on equity index returns (MSCI series) is obtained from Datastream. Equity returns data for New Zealand starts from 1980. Data on top 1% (0.1%) percentage income share and country's total wealth is from World Inequality Database. To calculate each country's wealth in dollars, we multiply the total wealth (data code = mpweal) from the World Inequality Database, denominated in local currency, by the corresponding nominal exchange rate.

The current account data is from the world economic outlook database, spanning from 1980-2022. Data on foreign direct investment net inflows (as a percentage of GDP) and portfolio equity investment net inflows are obtained from the World Development Indicators (World Bank)[1]. We divide portfolio equity net inflows by the corresponding year's GDP to measure equity inflows. The foreign institutional ownership is from the Factset Lionshare database. Patents data is from Kogan et al. (2017). U.S. firms fundamentals are from Compustat.

B Model with Epstein-Zin preference

With Epstein-Zin preference, we can construct the representative agent because the aggregation property only depends on the homotheticity of the preference. So in this case, the representative agent constructed above behaves the same as an individual agent in a country but scaled up to the country-level wealth.

B.1 Dynamics of the Consumption Ratio

Denote $W_t^c = W(\hat{C}_t^c, U_{t+1}^c)$ as the utility of the representative agent of country c. Denote the partial derivatives with respect to composite consumption and continuation utility as $W_{1,t}^c, W_{2,t}^c$, we have

$$\frac{\partial W_t^c}{\partial \bar{C}_t^c} = \frac{\partial W_t^c}{\partial \hat{C}_t^c}\frac{\partial \hat{C}_t^c}{\partial \bar{C}_t^c} = W_{1,t}^c(\bar{C}_t^c)^{h-1}$$

[1]Data is from World Development Indicators. Data code for foreign direct investment is BX.KLT.DINV.WD.GD.ZS; Data code for porfolio equity net inflows is BX.PEF.TOTL.CD.WD.

$$\frac{\partial W_t^c}{\partial U_{t+1}^c} = W_{2,t}^c$$

The intertemporal marginal rate of substitution of representative agent of country c is

$$M_{t,t+1}^c = \frac{\frac{\partial W}{\partial U}^c_{t+1} \frac{\partial W}{\partial \bar{C}}^c_{t+1}}{\frac{\partial W}{\partial \bar{C}}^c_t} = \frac{W_{2,t}^c W_{1,t+1}^c}{W_{1,t}^c} (\frac{\bar{C}_{t+1}^c}{\bar{C}_t^c})^{h-1} \tag{B.1}$$

International trade of X good implies that the marginal utilities of good X for $t = 1, 2, ...$ in each possible state is

$$\left(\prod_{j=0}^{t-1} W_{2,j}^H\right) W_{1,t}^H \bar{C}_t^H \frac{\alpha}{x_t^H} (\bar{C}_t^H)^{h-1} = (\bar{C}_t^F)^{h-1} \frac{1-\alpha}{x_t^F} \bar{C}_t^F W_{1,t}^F \left(\prod_{j=0}^{t-1} W_{2,j}^f\right) \tag{B.2}$$

Define the date t Pareto weights as

$$\begin{aligned} \Lambda_t^c &= \Lambda_0^c \left(\prod_{j=0}^{t-1} W_{2,j}^c\right) W_{1,t}^c \bar{C}_t^c (\bar{C}_t)^{h-1} \\ &= \Lambda_{t-1}^c W_{2,t-1}^c \frac{W_{1,t}^c}{W_{1,t-1}^c} (\frac{\bar{C}_t^c}{\bar{C}_{t-1}^c})^{h-1} \frac{\bar{C}_t^c}{\bar{C}_{t-1}^c} = S_{t-1}^c M_{t-1,t}^c \exp(\Delta c_t^c) \end{aligned}$$

Since the economy starts with a symmetric setup $\Lambda_0^H = \Lambda_0^F$. We can rewrite (B.2) as

$$\Lambda_t^H \frac{\alpha}{x_t^H} = \frac{1-\alpha}{x_t^F} \Lambda_t^F$$

Denote $\lambda_t = \frac{\Lambda_t^F}{\Lambda_t^F}$ as the ratio of Pareto weights. The optimality condition can be written as

$$\lambda_t = \frac{\alpha x_t^F}{(1-\alpha) x_t^H} \tag{B.3}$$

Similar to the log case, note that with Cobb-Douglas preference over different goods, households consumption expenditure share for each good is fixed. That is, foreign households spend $1-\alpha$ on X-good and home households spend α on X-good. Therefore, (B.3) shows that λ_t is also the consumption expenditure between foreign and home. That is, $\lambda_t = \frac{p_F C_F}{p_H C_H} = \frac{\mathcal{C}_{F,t}}{\mathcal{C}_{H,t}}$. Also, we have that

$$\lambda_{t+1} = \lambda_t \frac{M_{t,t+1}^F e^{\Delta c_{t+1}^F}}{M_{t,t+1}^H e^{\Delta c_{t+1}^H}} \tag{B.4}$$

B.2 Allocations and Exchange Rate

Similar to the log case, since the ratio of consumption expenditure is $\lambda_t = \frac{\mathcal{C}_{F,t}}{\mathcal{C}_{H,t}}$, we have

$$x_t^H = \frac{\alpha \mathcal{C}_{H,t}}{p_{x,t}}, y_t^H = \frac{(1-\alpha)\mathcal{C}_{H,t}}{p_{y,t}}, x_t^F = \frac{(1-\alpha)\mathcal{C}_{F,t}}{p_{x,t}}, y_t^F = \frac{\alpha \mathcal{C}_{F,t}}{p_{y,t}}$$

substituting these demands into resource constraints, we get the allocations (62), (63), (64), (65). Given these allocations, we can calculate the consumption bundles:

$$\bar{C}_{H,t} = (x_t^H)^{\alpha}(y_t^H)^{1-\alpha} \tag{B.5}$$

$$\bar{C}_{F,t} = (x_t^F)^{1-\alpha}(y_t^F)^{\alpha} \tag{B.6}$$

We can also compute the price of consumption bundles in home and foreign countries:

$$p_t^H = \frac{p_{x,t}x_t^H + p_{y,t}y_t^H}{C_{H,t}} \tag{B.7}$$

$$p_t^F = \frac{p_{x,t}x_t^F + p_{y,t}y_t^F}{C_{F,t}} \tag{B.8}$$

Note that the relative price of good Y in terms of good X is

$$p_t = \frac{X_t}{Y_t}\frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t} \tag{B.9}$$

By definition, the exchange rate is the ratio of price of consumption bundles:

$$E_t = \frac{p_t^H}{p_t^F} = \frac{\bar{C}_{F,t}}{\bar{C}_{H,t}}\frac{1}{\lambda_t} \tag{B.10}$$

The exchange rate growth is

$$\frac{E_{t+1}}{E_t} = \frac{\lambda_{t+1}}{\lambda_t}\frac{\bar{C}_{F,t+1}/\bar{C}_{F,t}}{\bar{C}_{H,t+1}/\bar{C}_{H,t}} \tag{B.11}$$

Note that (B.4) and (B.11) shows that in our model exchange rate growth is equal to the growth of SDF, as the model has an integrated financial markets.

B.3 SDF

Let us focus on the home country. The derivation for foreign country is similar. Since preference is homothetic, consumption is proportional to wealth. To calculate the SDF of the representative agent, we need to consider two groups of population: the population that receive the new firms in the current period (with measure π, denote as N); and the population that does not receive the new firms in the current period (with measure $1-\pi$, denote as O).

To this end, first note that $b_{i,t+1}$ is the fraction of wealth account for by the cohort that does not receive profitable projects from period t to $t+1$ in country i. The wealth shares of these two groups within the home country are

$$b_{H,t}(1-\pi), b_{H,t}\pi + 1 - b_{H,t}$$

The consumption growth and relative consumption growth for group O are $\frac{\bar{C}_{t+1}}{\bar{C}_t}b_{H,t+1}$ and $b_{H,t+1}$.

And the consumption growth and relative consumption growth for group O are $\frac{b_{H,t+1}\pi+1-b_{H,t+1}}{\pi}\frac{\bar{C}_{t+1}}{\bar{C}}$ and $\frac{b_{H,t+1}\pi+1-b_{H,t+1}}{\pi}$. Therefore, the growth in the composite consumption for two groups {O, N} are (we omit the country index H from now on)

$$\frac{\hat{C}_{t+1}}{\hat{C}_t}_O = \left(\frac{\bar{C}_{t+1}b_{t+1}}{\bar{C}_t}\right)^h (b_{t+1})^{1-h} = \frac{\bar{C}_{t+1}^h}{\bar{C}_t^h}b_{t+1}$$
$$\frac{\hat{C}_{t+1}}{\hat{C}_t}_N = \left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\frac{b_{t+1}\pi+1-b_{t+1}}{\pi}\right)^h (\frac{b_{t+1}\pi+1-b_{t+1}}{\pi})^{1-h}$$
$$= (\frac{\bar{C}_{t+1}}{\bar{C}_t})^h \frac{b_{t+1}\pi+1-b_{t+1}}{\pi}$$

Similarly, we can derive the growth in continuation utility for these two groups. Since the consumption to utility ratio are equalized across two groups, we have

$$\frac{\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)}{\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)} = \frac{b_{t+1}}{\frac{b_{t+1}\pi+1-b_{t+1}}{\pi}} \tag{B.12}$$

The SDF of these two groups can be written as

$$M_{O,t,t+1} = \beta(\frac{\hat{C}_{t+1}}{\hat{C}_t})_O^{-\frac{1}{\psi}}(\frac{\bar{C}_{t+1}}{\bar{C}_t})^{h-1}\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$
$$M_{N,t,t+1} = \beta(\frac{\hat{C}_{t+1}}{\hat{C}_t})_N^{-\frac{1}{\psi}}(\frac{\bar{C}_{t+1}}{\bar{C}_t})^{h-1}\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

In this economy, each investor's own inter-temporal marginal rate of substitution is a valid SDF. Hence the cross-sectional average of investors' inter-temporal marginal rates of substitution is a valid stochastic discount factor. That is,

$$M_{t,t+1} = (1-\pi)M_{O,t,t+1} + \pi M_{N,t,t+1}$$
$$= \beta\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\frac{h}{\psi}+h-1}\left(\pi\left(\frac{b_{t+1}\pi+1-b_{t+1}}{\pi}\right)^{-\frac{1}{\psi}}\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} + (1-\pi)b_{t+1}^{-\frac{1}{\psi}}\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}\right)$$

Combining with (B.12), we have (48).

B.4 Wealth ratio

First, note that the marginal utility of consumption of the representative agent in each country is

$$\frac{\partial\tilde{U}}{\partial C} = (1-\tilde{\beta})\tilde{U}^{\frac{1}{\psi}}\hat{C}^{-\frac{1}{\psi}}\bar{C}^{h-1}$$

we can compute the wealth of households who didn't receive projects at t, in units of local consumption bundles:

$$\begin{aligned}\hat{W}_H &= \frac{\tilde{U}}{\frac{\partial \tilde{U}}{\partial C_H}} \\ &= \frac{1}{1-\beta}(\tilde{U})^{1-1/\psi}(\hat{C}_{H,t})^{\frac{1}{\psi}}\bar{C}_{H,t}^{1-h} \\ &= \frac{1}{1-\beta}(\frac{\tilde{U}_{H,t}}{\hat{C}_{H,t}})^{1-1/\psi}\hat{C}_{H,t}\bar{C}_{H,t}^{1-h} \\ &= \frac{1}{1-\beta}(\frac{\tilde{U}_{H,t}}{\hat{C}_{H,t}})^{1-1/\psi}\bar{C}_{H,t}\end{aligned}$$

Similarly we can derive the wealth for foreign country,

$$\hat{W}_F = \frac{1}{1-\beta}\tilde{U}_{F,t}^{1-1/\psi}\hat{C}_{F,t}^{1/\psi}\bar{C}_{F,t}^{1-h} = \frac{1}{1-\beta}(\frac{\tilde{U}_{F,t}}{\hat{C}_{F,t}})^{1-1/\psi}\bar{C}_{F,t} \tag{B.13}$$

Note that the wealth above are calculated in the units of local consumption bundles, so the ratio of two countries' wealth should be adjusted by the price of their respective consumption bundles

$$\frac{W_F}{W_H} = \frac{\hat{W}_F}{\hat{W}_H}\frac{p_F}{p_H} = \left(\frac{\frac{U_{F,t}}{\hat{C}_{F,t}}}{\frac{U_{H,t}}{\hat{C}_{H,t}}}\right)^{1-1/\psi} \lambda_t \tag{B.14}$$

The second equation comes from the fact that $\lambda = \frac{p_F \bar{C}_F}{p_H \bar{C}_H}$ (Recall (B.3)).

B.5 Asset Prices

Similar to the log case, we have

$$\begin{aligned}S_t^H &= p_{x,t}X_t + \mathrm{E}_t[M_{t,t+1}^H S_{t+1}^H] \\ pd_t^H &= \mathrm{E}_t[M_{t+1}^H \frac{p_{x,t+1}X_{t+1}}{p_{x,t}X_t}(1+pd_{t+1}^H)e^{-u_{t+1}^H}] \\ S_t^F &= p_{y,t}Y_t + \mathrm{E}_t[M_{t,t+1}^F S_{t+1}^F] \\ pd_t^F &= \mathrm{E}_t[M_{t+1}^H \frac{p_{y,t+1}Y_{t+1}}{p_{y,t}Y_t}(1+pd_{t+1}^F)e^{-u_{t+1}^F}]\end{aligned}$$

B.6 Trade and Capital Flows

The net export as a fraction of total output is

$$\frac{NX_t^H}{X_t} = \frac{p_{x,t}X_t - p_{x,t}x_t^H - p_{y,t}y_t^H}{p_{x,t}X_t} = 1 - \frac{1}{\alpha + (1-\alpha)\lambda_t} \tag{B.15}$$

$$\frac{NX_t^F}{Y_t} = \frac{p_{y,t}Y_t - p_{y,t}Y_t^F - p_{y,t}x_t^H}{p_{y,t}Y_t} = 1 - \frac{\lambda_t}{1-\alpha+\alpha\lambda_t} \tag{B.16}$$

The net international investment position scaled by country's wealth, is

$$\frac{A_t^H}{p_{x,t}X_t} = \frac{W_t^H - S_t^H}{W_t^H} \tag{B.17}$$

$$\frac{A_t^F}{p_{y,t}Y_t} = \frac{W_t^F - S_t^F}{W_t^F} \tag{B.18}$$

C Numerical Procedure

Here, we briefly describe the numerical procedure for solving the model.

C.1 All equations to solve

The equilibrium is obtained by jointly solving the set of non-linear equations that describe the equilibrium conditions: (14), (15),(17),(18),(24), (28), (37), (38), (62),(63),(64),(65), (B.14), (48). We put all the equations here, as below:

On the aggregate level, we have

$$d\log X = \mu + \delta u_H + \varepsilon_H$$
$$d\log Y = \mu + \delta u_F + \varepsilon_F$$

For each country's allocation we have (62)-(65).

$$x_t^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_t} X_t \tag{C.19}$$

$$x_t^F = \frac{(1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_t} X_t \tag{C.20}$$

$$y_t^H = \frac{1-\alpha}{1-\alpha+\alpha\lambda_t} Y_t \tag{C.21}$$

$$y_t^F = \frac{\alpha\lambda_t}{1-\alpha+\alpha\lambda_t} Y_t. \tag{C.22}$$

The displacement effect

$$b_{H,t+1} = 1 - \frac{(1+pd_{H,t+1})(1-e^{-u_{H,t+1}})}{\left(1+pd_{H,t+1} + (1+pd_{F,t+1})\frac{p_{y,t+1}Y_{t+1}}{p_{x,t+1}X_{t+1}}\right)\frac{1}{1+w_{t+1}}} \tag{C.23}$$

$$b_{F,t+1} = 1 - \frac{(1+pd_{F,t+1})(1-e^{-u_{F,t+1}})}{\left((1+pd_{H,t+1})\frac{p_{x,t+1}X_{t+1}}{p_{y,t+1}Y_{t+1}} + (1+pd_{F,t+1})\right)\frac{w_{t+1}}{1+w_{t+1}}} \tag{C.24}$$

where the dividend ratio is

$$\frac{p_{y,t}Y_t}{p_{x,t}X_t} = \frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t}$$

Post-Dividend price-dividend ratio are given by

$$pd_t^H = \mathrm{E}_t[M_{t,t+1}^H \frac{p_{x,t+1}X_{t+1}}{p_{x,t}X_t}(1+pd_{t+1}^H)e^{-u_{t+1}^H}] \tag{C.25}$$

$$pd_t^F = \mathrm{E}_t[M_{t,t+1}^F \frac{p_{y,t+1}Y_{t+1}}{p_{y,t}Y_t}(1+pd_{t+1}^F)e^{-u_{t+1}^F}] \tag{C.26}$$

Aggregate consumption is given by

$$C_t^H = (x_t^H)^{\alpha}(y_t^H)^{1-\alpha} \tag{C.27}$$

$$C_t^F = (x_t^F)^{1-\alpha}(y_t^F)^{\alpha} \tag{C.28}$$

The two SDFs are given by (48),

$$\frac{M_{t+1}^c}{M_t^c}_o = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1} b_{o,c,t+1}^{-\frac{1}{\psi}}\left(\frac{U_{o,c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{C.29}$$

$$\frac{M_{t+1}^c}{M_t^c}_n = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1} b_{n,c,t+1}^{-\frac{1}{\psi}}\left(\frac{U_{n,c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{C.30}$$

where

$$b_n = \frac{b\pi+1-b}{\pi}$$

$$b_o = b$$

$$\frac{U_{n,t+1}}{C_t} = \frac{U_{n,t+1}}{C_{n,t+1}}\frac{\bar{C}_{t+1}}{\bar{C}_t}(\frac{b\pi+1-b}{\pi})$$

$$\frac{U_{o,t+1}}{C_t} = \frac{U_{o,t+1}}{C_{o,t+1}}\frac{\bar{C}_{t+1}}{\bar{C}_t}b$$

We use cross-sectional average as the aggregate SDF. I.e.,

$$\frac{M_{t+1}}{M_t} = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1}\left(\pi\left(\frac{b_{c,t+1}\pi+1-b_{c,t+1}}{\pi}\right)^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}}+(1-\pi)b_{c,t+1}^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}}\right)\left(\frac{\bar{U}_{c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

and wealth ratio is given by (B.14) and the lambda ratio is given by (B.4). Recursively definition of continuation utility are given by (C.31) - (C.32)..

These are all the equations.

Specifically, we need to numerically solving four functions for any given state: Price-Dividend ratio of H/F and the expected continuation Utility of H/F. The price-dividend ratios are recursively defined above. Next we derive the recursive definition for expected continuation utility:

We focus on the case for the home country and omit the country index. Consider a household with wealth share ω_i, his continuation utility is

$$V_{i,t+1} = [(1-\beta)\hat{C}_{i,t+1}^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}[V_{i,t+2}^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

where

$$\hat{C}_{i,t} = (\omega_{i,t+1}\bar{C}_{i,t+1})^h (\frac{\omega_{i,t+1}\bar{C}_{i,t+1}}{\bar{C}_{i,t+1}})^{1-h} = \omega_{i,t+1}\bar{C}_{t+1}^h$$

So we have the utility as a function of wealth share

$$V_{i,t+1}(\omega_i) = [(1-\beta)(\omega_{i,t+1}\bar{C}_{t+1}^h)^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}[V_{i,t+2}(\omega_{i,t+2}|\omega_{i,t+1})^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

Normalize it we have

$$\begin{aligned}
\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h} &= \left[(1-\beta)(\omega_{i,t+1})^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+1}^h}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left[(1-\beta)(\omega_{i,t+1})^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h}\frac{\bar{C}_{t+2}^h}{\bar{C}_{t+1}^h}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}$$

Due to homotheticity, we know that $\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h}$ is linear in $\omega_{i,t+1}$, also it implies that $\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h}$ is linear in $\omega_{i,t+2}$. Dividing both sides by $\omega_{i,t+1}$

$$\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h\omega_{i,t+1}} = \left[(1-\beta) + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h\omega_{i,t+2}}\frac{\bar{C}_{t+2}^h\omega_{i,t+2}}{\bar{C}_{t+1}^h\omega_{i,t+1}}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

So we can write the utility-consumption ratio as

$$UC_{i,t+1}(\lambda_{t+1}) = \left[(1-\beta) + \beta \mathrm{E}_{t+1}\left[\left(UC_{i,t+2}(\lambda_{t+2})\frac{\bar{C}_{t+2}^h\omega_{i,t+2}}{\bar{C}_{t+1}^h\omega_{i,t+1}}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$= \left[(1-\beta) + \beta \underbrace{\mathrm{E}_{t+1} \left[\left(UC_{i,t+2}(\lambda_{t+2}) \frac{\hat{C}_{t+2}}{\hat{C}_{t+1}} \right)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}_{Q(\lambda_{t+1})^{\frac{1-1/\psi}{1-\gamma}}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{C.31}$$

And the expected continuation utility, normalized by current consumption, is

$$\begin{aligned} Q_t &= \mathrm{E}_t \left[\left(UC_{i,t+1}(\lambda_{t+1}) \frac{\hat{C}_{t+1}}{\hat{C}_t} \right)^{1-\gamma} \right] \\ &= \mathrm{E}_t \left((1-\pi) \left(UC_{i,t+1}(\lambda_{t+1}) (\frac{\bar{C}_{t+1}}{\bar{C}_t})^h b \right)^{1-\gamma} + \pi \left(UC_{i,t+1}(\lambda_{t+1}) (\frac{\bar{C}_{t+1}}{\bar{C}_t})^h \frac{b_H \pi + 1 - b_{H,t}}{\pi} \right)^{1-\gamma} \right) \end{aligned} \tag{C.32}$$

For continuation utility, we have recursively defination given by (C.31) - (C.32).

C.2 Solving on the Grids

Grids. In order to solve the model numerically, we need to set up the grids for state and shocks. The neutral shock has two grid points $[-\sigma_e, \sigma_e]$ where σ_e is its standard deviation. The displacement shock has three grid points $[u_1, u_2, u_3]$. The transition matrix of displacement shock is parametrized by two parameters p, q. The log λ_t is discretized on 45 grid points. We set the bounds for $\log \lambda$ at -5.0 and 5.0.

Algorithm. We solve the equilibrium using policy iteration. This algorithm is based on the fact that value function is the solution of a fixed point problem generated by a contraction mapping.

To initiate the process, we need to start with an initial guess of price-dividend ratio and utility-consumption ratio. We use the static steady state values for the initial guess.

For any point on the grid, we need to solve a set of non-linear equations. Specifically, at time t, given a combination of shocks $u_t^H, u_t^F, \varepsilon_t^H, \varepsilon_t^F$, we need to solve λ_t, w_t. To do so, we first need to guess a value $\hat{\lambda}_t$ and then interpolate the price-dividend ratio and utility-consumption ratio using $\hat{\lambda}_t, u_t^H, u_t^F$. Then, using these imputed values, we solve $\tilde{\lambda}_t$, $\tilde{w}_t$ from the set of non-linear equations.

The difficulty is the fact that for some guessed values, the solution for the set of non-linear equations does not exist. So we try different random guesses starting with the state λ_{t-1}. In particular, we search for guess with the following form:

$$\lambda_{t-1} + \hat{\varepsilon} r |\lambda_{t-1}|$$

where $\hat{\varepsilon} \in N(0,1)$ is a random normal variable. $|\lambda_{t-1}|$ is the abosolute value of λ_{t-1}. r is the variable that starts with 0.05 and it increases by 0.15 for every 3000 attempts. And once it increase by 0.15, the threshold of attempts also raises by 1500. That is, after r becomes 0.2, it will need additional 4500 attempts to be raised again to 0.35, and so on.

If the solution from the above guess is not equal to our initial guess, i.e., $\tilde{\lambda}_t \neq \hat{\lambda}_t$, we update our guess according to $\hat{\lambda}' = (1 - w_u)\lambda_{t-1} + w_u\tilde{\lambda}_t$ and repeat previous step. The weight on the new solution starts with $w_u = \frac{1}{2}$ and get updated every 5 iterations. In essence, we also make w_u random so that it helps the convergence. We iterate the previous step until $\hat{\lambda}_t$ and $\tilde{\lambda}_t$ converges.

Finally, we do not directly solve on the grid of 45 points for $\log \lambda_t$. Instead, we first solve for the log-linearised version of the model, on a subset of the state space. Then, we extrapolate the log-linearized solution on a grid of 5 points with a larger subset of state space and use it as an initial guess and solve for the solution on the grid. Next, we extrapolate the solution of previous step (5 points) on a 7 points grid and use that as an initial guess and solve for the solution on the 7-points grids, and so on.

In summary, we do it iteratively. Gradually, we obtain the solution of the 45 points grid on the full state space. Doing so means that we only update the solution marginally at each step. In theory, for each adjacent steps, the solutions are very close in the function space. As a result, this practice does not only increase the probability of solving the model at each step, but also speeds up the process significantly.

D Estimation

D.1 Identification

In Figure (D.4)–(D.6) we plot the slope of the model's implied moments $X(\theta)$ to small changes in parameters around the optimum $\hat{\theta}$. Specifically, we report

$$E_{i,j} = \frac{dX^j(\theta)}{d\theta^i} \tag{D.33}$$

Which is the numerical gradient computed using a five-point stencil around $\hat{\theta}$.

In addition, in Figure (D.1)–(D.3) we plot the Gentzkow and Shapiro (2014) measure of sensitivity of parameters to moments. We report the measure in elasticity form

$$\hat{\lambda}_{i,j} = \lambda_{i,j}\frac{X^j(\theta)}{\theta^i}$$

where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j. The matrix Λ is computed as

$$\Lambda = -(G'WG)^{-1}G'W$$

Where G is the numerical gradient of the sample moments $g(\theta) = X - \mathcal{X}$ and W is the weighting matrix.

In what follows, we summarize the main patterns in these Figures.

- The parameter δ is identified by

- The mean and volatility of risk-free rate, the volatility of exchange rates, and the volatility of stock markets. The volatility of the exchange rate is mostly determined by the effective displacement effect in the market. When δ is small, the growth impact of u shock is small relative to its displacement effect. As a result, a lower level of δ leads to a higher impact of displacement effect and therefore more volatile pricing kernels. When δ increases, it increase the growth rate and therefore the mean of the risk-free rate.
- The correlation between output and wealth changes. When δ is small, the growth impact of u shock is small relative to its displacement effect. As a result, a lower level of δ leads to a lower level of correlation between wealth changes and relative output growth.

- The parameter μ is identified by
  - The mean of consumption growth and output growth. The higher the μ, the higher the consumption and the output growth.
- The parameter h is identified by
  - The level of exchange rate volatility. As h increases, the effective size of u shock diminishes. Since the exchange rate movement is mostly driven by the displacement shock, its magnitude decreases with h.
  - The correlation between wealth changes and consumption growth. The higher the h, the less of the households perceived impact of u shocks, as they put less weight on the effect of declined wealth share. Since the u (ε) shock contributes to most of the variation in wealth changes, the correlation wealth and consumption helps determine the level of relative consumption.
- The parameter β and ξ are identified by
  - The mean of risk-free rate and the level of income inequality. The risk-free rate in the model is directly linked to the effective discount rate, which itself is a product of discount rate and the survival rate (one minus death rate). These two parameters play a similar role in the model, as they both contribute to the effective discount rate. But they affect inequality in a slightly different way: the higher the discount rate, the higher the price-dividend ratio and therefore a lower share of top income – as the accumulated wealthy people earn less dividend on their wealth. On the other hand, as the death rate increases, it lowers wealth inequality – as people have less expected "time" to accumulate their wealth before they die. Consequently, the level of inequality and the risk-free rate help determine these two parameters.
- The parameter α is identified by
  - The volatility of exchange rate. The higher the home-bias, the larger the effective size of u shock at its own country whose risks can not be diversified away. Hence it weakens the effect of trade and therefore increases the volatility of SDF.

- The mean and volatility of stock returns. The higher level of home-bias increases the volatility of pricing kernels and therefore the volatility of stock markets.

- The parameter γ is identified by
  - The mean and volatility of excess returns. This parameter is not well identified – has relatively large standard errors.

- The parameters ψ is identified by
  - The volatility of risk-free rate and stock markets. A higher willingness for inter-temporal substitute leads to lower variability in risk-free rate.

- The parameter p, q are identified by
  - The correlation between net export and consumption , as well as the correlation between the dollar index and innovation. As p increase, the effective size of u is reduced. Displacement shocks contribute to the negative correlation between net export and consumption, as a result it helps identify the parameter p.
  - The mean and volatility of excess returns. With a recursive preferences, a more persistent shock leads a higher compensation for risks in equilibrium.
  - The coefficients of consumption and output in bi-variate regression with wealth, as it effectively controls the size of (large) displacement shock.

- The parameter u_1, u_2 are identified by (relatively well identified)
  - The volatility of risk-free rate and stock market. The difference between u_1 and u_2 determines the volatility of u shock, which in turn determines the volatility of most of the dynamics in the model. All else equal, a more dispersed u_1 and u_2 increase the volatility of u shock. That is, a small u_1 and a large u_2.
  - The level of top income share. The displacement shock determines the level of inequality. All else equal, a larger magnitude of displacement shocks lead to a higher level of top income share.
  - The coefficient of wealth in the bi-variate regression of exchange rate on wealth and consumption.

- The parameter σ_e is identified by
  - The volatility of consumption growth and output growth. Given that the estimated δ is relatively small, the volatility of aggregate output and aggregate consumption is mostly driven by the neutral shock.

- The correlation between wealth changes and output growth. A larger σ_e weakens the positive correlation generated by the displacement shocks.

- The parameter ρ_e is identified by

  - The correlation between aggregate bilateral consumption and bilateral output. As mentioned above, the volatility of aggregate consumption and output are determined by the neutral shock. Consequently, the correlation between aggregate consumption and aggregate output are determined by the correlation of neutral shocks.
  - The coefficients of wealth in the bi-variate regression with consumption. Recall that the neutral shock contributes to the counter-cyclicality of exchange rate while displacement shock generates opposite forces. A more correlated neutral shock weakens the effective size of neutral shock on the exchange rate and therefore strengthens the correlation between wealth and exchange rate.
  - UIP slope. Displacement shocks contribute to the violation of UIP, whereas neutral shocks help strength the UIP coefficient.

- The parameter ρ_c is identified by

  - The volatility of stock markets and exchange rate. As ρ_c increase, u shocks are becoming less correlated. This leads to a more volatile exchange rate.
  - The correlation between the stock markets. Recall in figure 6 that stock market volatility is mostly driven by u shock and that u shock contributes to the negative correlation in the stock market. Therefore, the positive correlation between the stock market in the data is informative about the amount of technological spillover – that is, the positive correlation between u shocks in the model.
  - The coefficients of wealth and output in the bi-variate regressions. This is because the wealth changes are mostly driven by displacement shocks.

- The parameter π is identified by

  - The volatility of excess returns and exchange rate. A higher π lowers the effectiveness of displacement shocks. As the excess returns are mostly driven by the displacement shock, the size of it helps determine the size of population that is affected.
  - The coefficient of inequality in the wealth-inequality regression. A higher level of π lowers the concentration of capital income and therefore weakens the relation between u shocks and top income share. As a result, a higher π lowers the correlation between top income share and exchange rate.
  - The level of income inequality. The wealth effects of displacement shocks decrease in π.

D.2 Estimation Methodology

The model has a total of 16 parameters. We put two restrictions on the dynamics of u shocks to reduce the number of parameters. First, we assume that $u_1 = u_2$. Hence, a transition from u_1 to u_2 only affects the future distribution of u (as the transition probabilities change) rather than the current level of displacement. Second, we assume that the matrix T corresponds to transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters—the corresponding autocorrelation parameter p and q. Specifically, we assume that the transition matrix has the following form

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix} \tag{D.34}$$

Where p^2 is the probability of staying in the lowest state once already there and q^2 is the probability of staying in the highest state once there. We estimate the remaining parameters of the model using a simulated minimum distance method Lee and Ingram (1991). Specifically, given a vector of X of target statistics in the data, we obtain parameter estimates by

$$\hat{p} = \arg\min_{p\in\mathcal{P}} \left(X - \frac{1}{S}\sum_{i=1}^{S} \hat{X}_i(p) \right)' W \left(X - \frac{1}{S}\sum_{i=1}^{S} \hat{X}_i(p) \right) \tag{D.35}$$

Where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model. Our choice of weighting matrix $W = diag(XX')^{-1} I_W$ penalizes proportional deviations of the model statistics from their empirical counterparts. I_W is a diagonal matrix that adjusts for the relative importance of the statistics in our estimation. We apply a factor of 10 on the equity risk premium, the volatility of exchange rate, the level of top 1% share and the UIP slope. The rest elements on the diagonal of I_W are normalized to one.

We use different weights on the diagonal of I_W to reflect the relative importance of the following moments: equity risk premium, level of top 1% income share, the volatility of exchange rate and the UIP slope. We do this because the magnitude of these moments are relatively well documented in the literature, and also speaks directly to the model's mechanism. For instance, the level of income inequality is directly linked to the size of u shock that drives most of the dynamics in the model.

Our estimation targets are reported in the first column of Table 7. They include a combination of first and second moments of aggregate quantities, asset prices and exchange rates. In additional to these standard international moments in the literature, we also target a set of correlations. The neutral shock and displacement shock have different implications for the cyclicality of the exchange rates. Thus, the set of correlation between exchanges rates and consumption, output and stock market, together with the set of bilateral correlations, are informative about the relative magnitude of these two shocks.

In addition, we target the average top 1% income inequality of the United States and the estimated coefficients of bi-variate regressions (43). In the model, we consider the stock market as a levered claim of domestic consumption goods by a factor of 2.

We simulate the model at annual frequency. For each simulation, we first simulate 100 years data as burn-in, to remove the samples' dependencies on the initial condition. Then, we simulate the data for 50 years – the same length as our empirical sample. The simulation starts with the symmetric steady state where the displacement shocks are at the middle state and $\lambda = 1$. In each iteration we simulate 10000 samples, and simulate pseudo-random variables using the same seed in each iteration.

We compute standard errors for the vector of parameter estimates $\hat{p}$ as

$$V(\hat{p}) = (1 + \frac{1}{S}) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p) \right)^{-1} \frac{\partial}{\partial p} \mathcal{X}(p)' W' V_X(\hat{p}) W \frac{\partial}{\partial p} \mathcal{X}(p) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p) \right)^{-1} \tag{D.36}$$

where

$$V_X(\hat{p}) = \frac{1}{S} \sum_{i=1}^{S} (\hat{X}_i(p) - \mathcal{X}(\hat{p}))(\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p}))'$$

is the estimate of the sampling variation of the statistics in X computed across simulations.

The standard errors calculated in (D.35) are computed using the sampling variation of the target statistics across simulations (D.36).

Solving each iteration of the model is costly, and thus computing the minimum (D.35) using standard methods is infeasible. We therefore use the Radial Basis Function (RBF) algorithm in Björkman and Holmström (2000). The Björkman and Holmström (2000) algorithm first fits a response surface to data by evaluating the objective function at a few points. Then, it searches for a minimum by balancing between local and global search in an iterative fashion. We use a commercial implementation of the RBF algorithm that is available through the TOMLAB optimization package.

D.3 Construction of Estimation Targets

Consumption, output and net export. Output is gross domestic product. Consumption is households and NPISHs final consumption expenditure (private consumption). Net export is the exports of goods and services minus the imports of good and services.

Standard deviation of aggregate quantities. We first calculate the standard deviation for each US-foreign country pair, and then we take the average and use that as our target.

Correlations between aggregate quantities. Similar to the standard deviation, we first calculate the correlation for each US-foreign country pair, and then we take the average and use that as our target.

Real exchange rate. Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.

Risk free rate and Stock market returns. Risk free rate is constructed using three-month

T-bills yield, adjusting for realized inflation using annual changes in CPI. Stock market returns are obtained using MSCI indexes from Datastream.

UIP coefficient. for each US-foreign country pair, we regress the exchange rate growth from t to $t+1$ on the interest rate differentials at t:

$$\Delta e_{US,F,t,t+1} = \alpha_F + \beta_{UIP,F}(r_{F,t} - r_{US,t}) + \varepsilon_{F,t}$$

Then we take an average of the estimated $\beta_{UIP,F}$ across all countries F in our sample.

Inequality, wealth and the coefficients in regression. Income inequality and wealth data is from World Inequality Database, the top 1% income share including capital income. We use the estimated coefficients of the panel regression with country fixed effects, as in Table 3, Table 2 and Table 6. In these regressions, independent variables are standardized using unconditional moments.

Dollar and Innovation. The correlation between equal-weighted dollar index and U.S. innovation as measured by total value of patent Kogan et al. (2017) divided by total market value. Stock market value is from CRSP. We use the year-end value of the stock market.

Figure D.1: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j}\frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

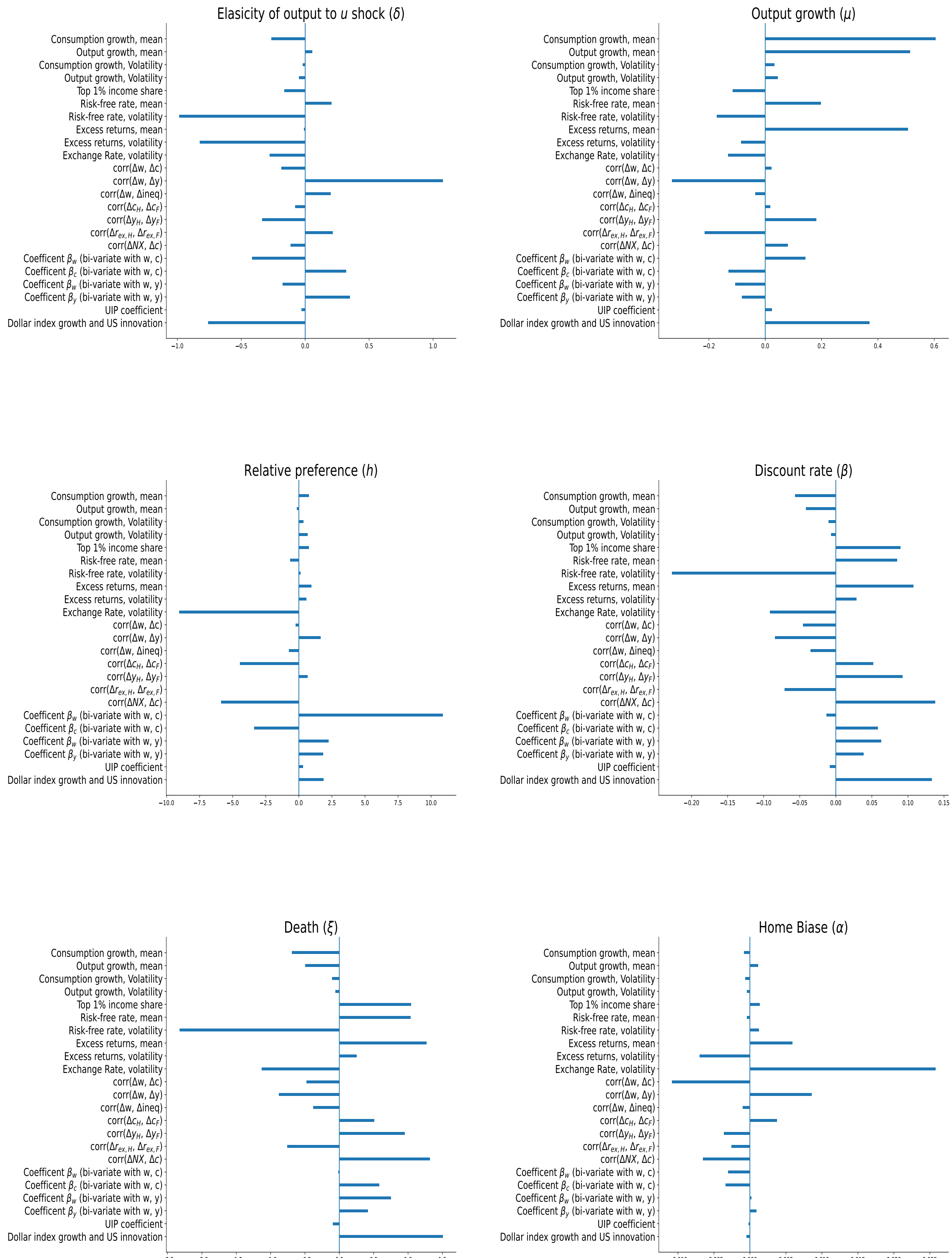

Figure D.2: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j} \frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

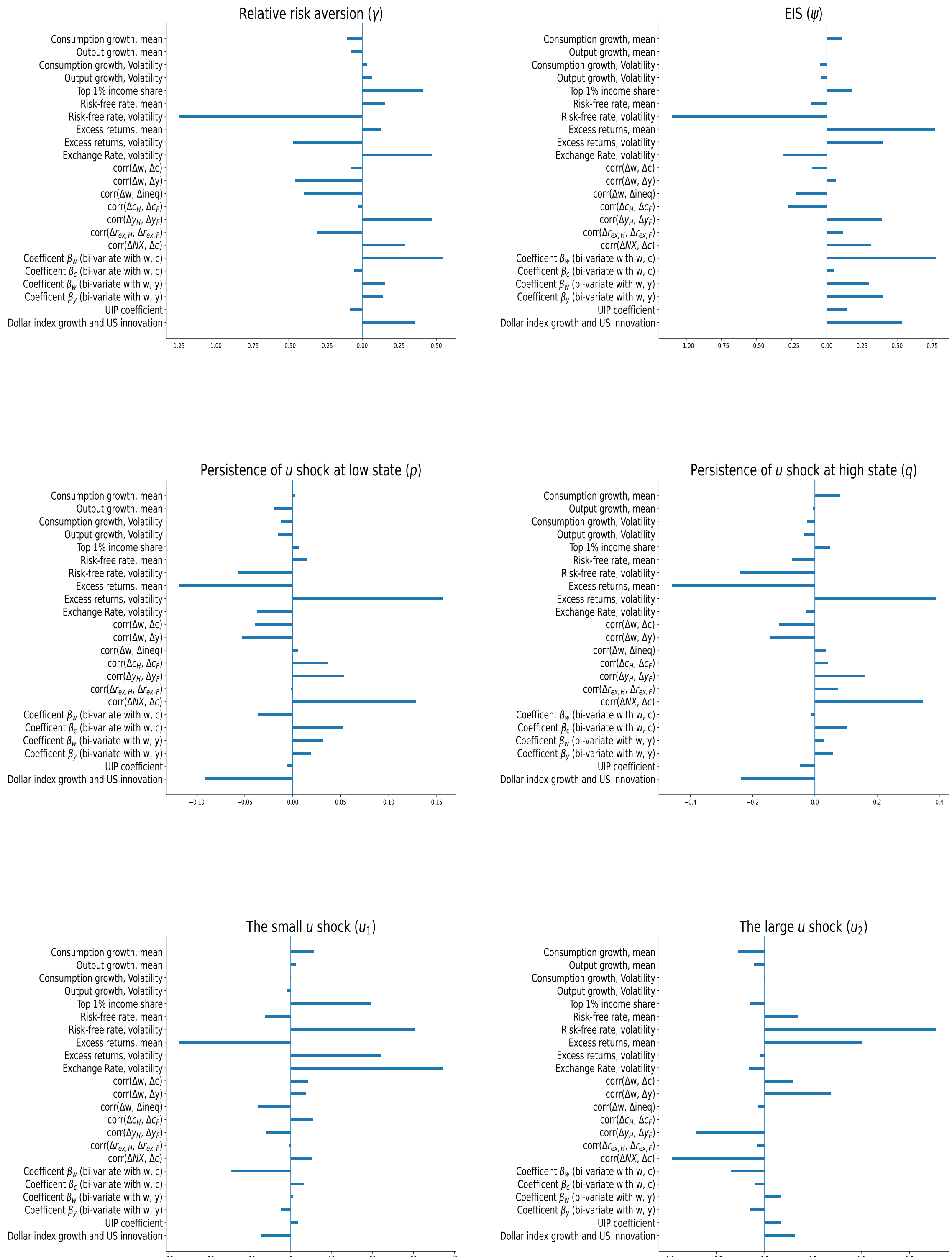

Figure D.3: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j}\frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

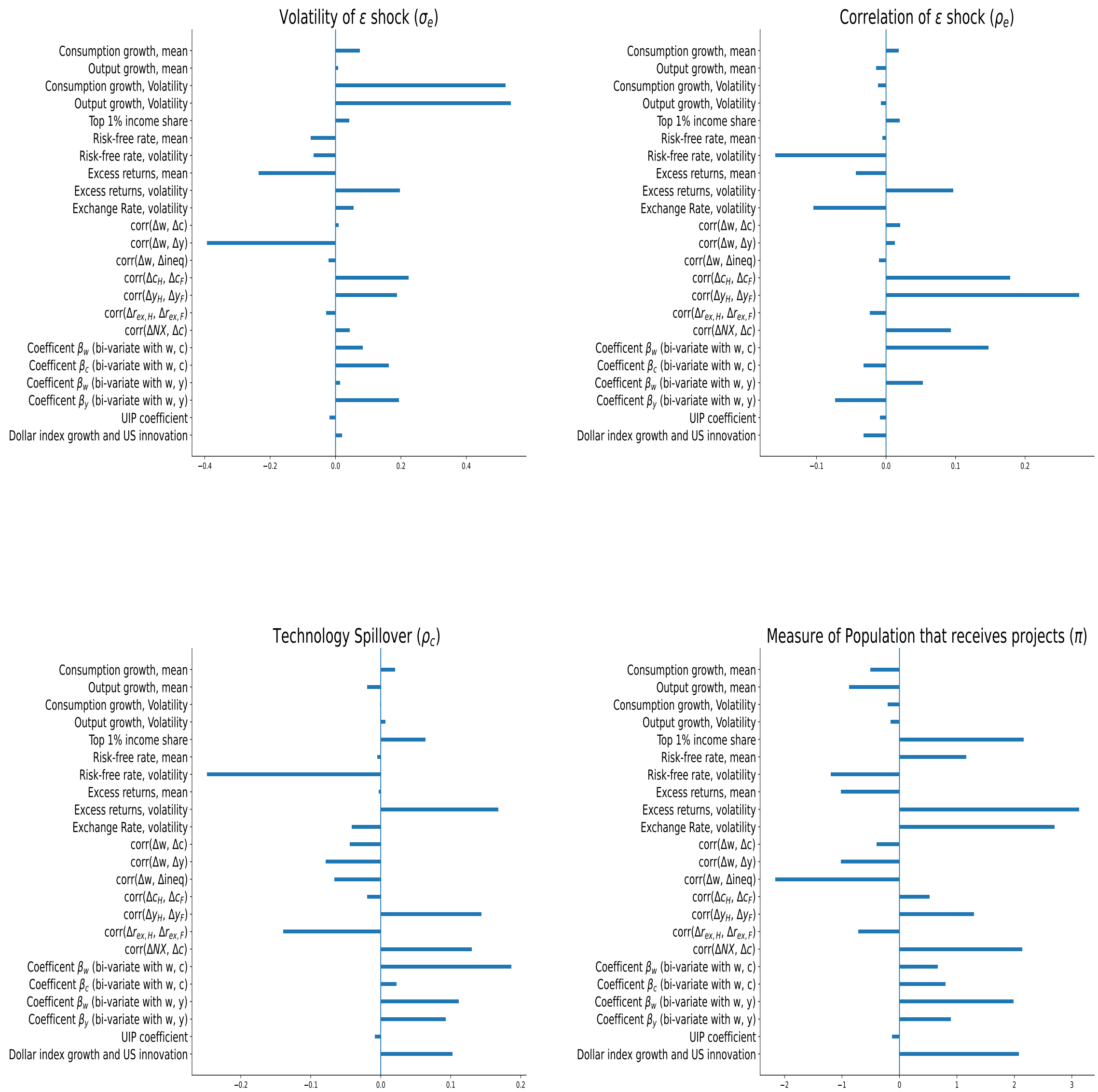

Figure D.4: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$– computed using a 5-point stencile – of moments j to parameter i.

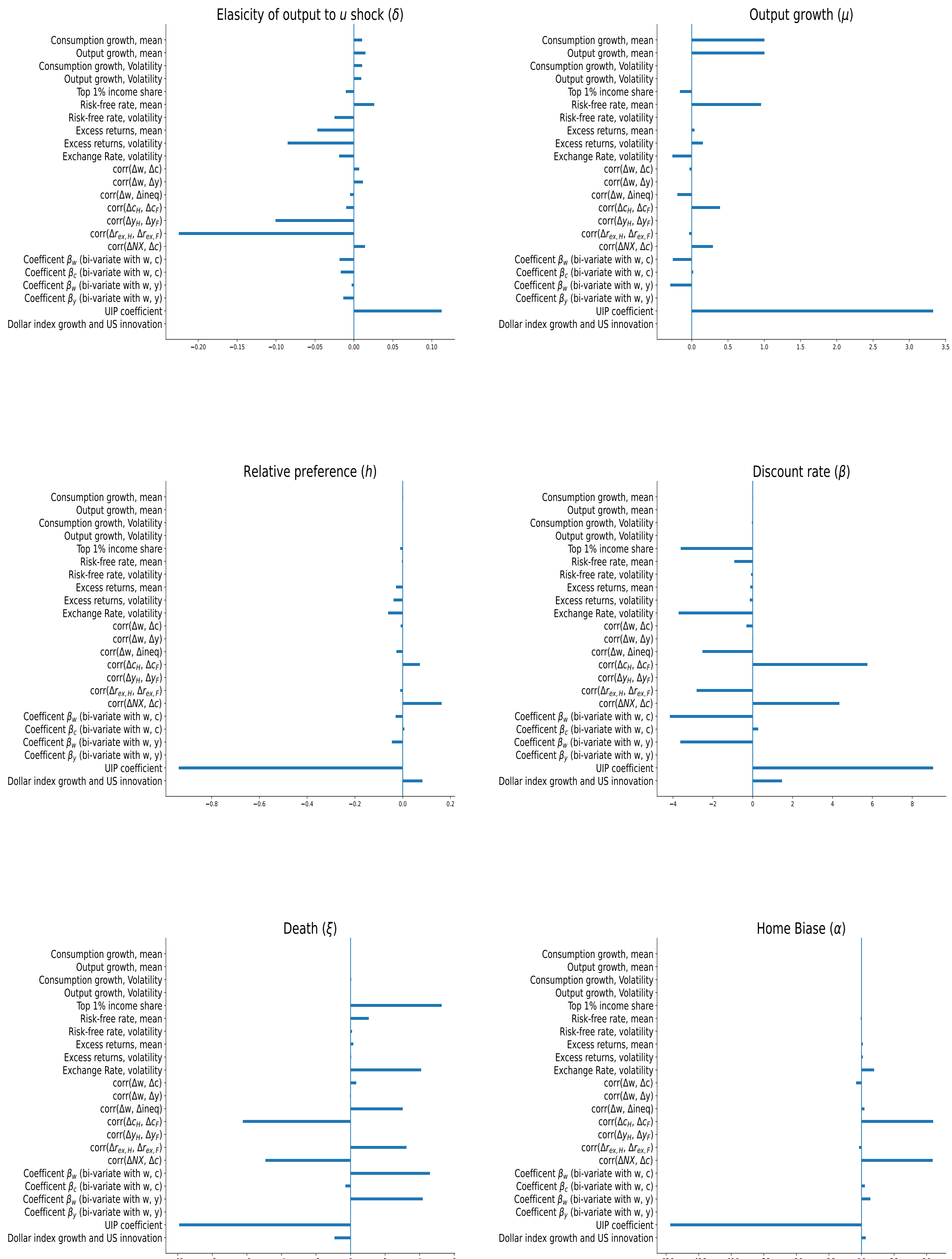

Figure D.5: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$ – computed using a 5-point stencile – of moments j to parameter i.

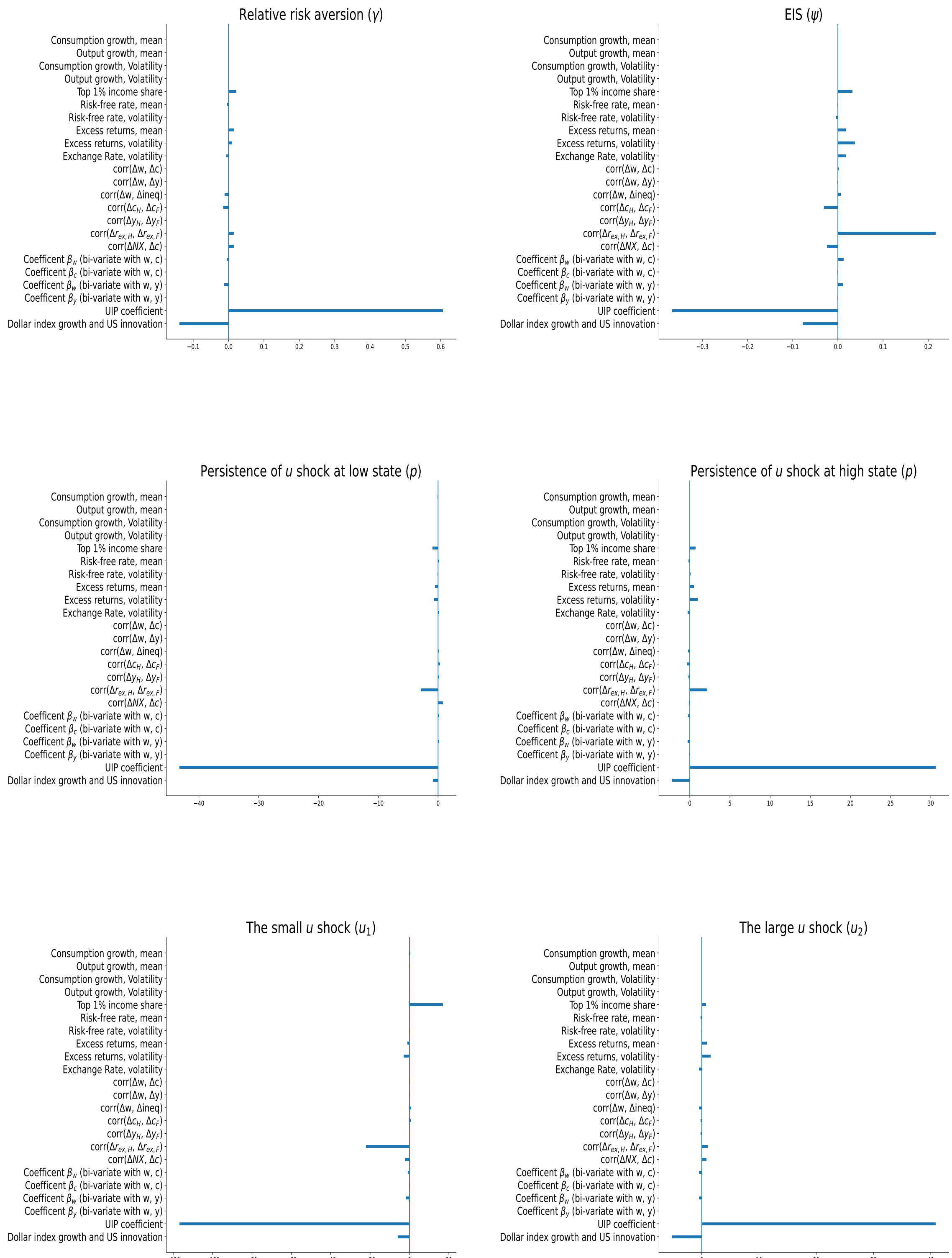

Figure D.6: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$ – computed using a 5-point stencile – of moments j to parameter i.